INTERNAL GOVERNANCE SECURITY PRICE AND FIRM DYNAMICS*

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Abstract

This paper studies the role of internal governance in dynamic context of the firm using continuous-time dynamic contract model. The internal governance limits the amount of capital diversion, but the executive's decisions on effort and capital diversion are unobservable. The optimal contract induces optimal effort provision and no capital diversion because the continuation value incorporates the benefit of control and increases with firm's performance. An implementation of optimal contract shows the value of internal governance as a part of the security price. With the same governance mechanism, its value changes over time due to the expected longevity of the firm. An increase in internal governance intensifies the dynamic incentive alignment between the executive compensation and firm's performance. It lowers the periodic benefit of control but enhance the return from firm's growth in the executive's incentive. A change in internal governance level is in essence a reallocation of the weight between instantaneous and intertemporal incentive of the executive.

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1 Introduction

This paper studies the effects of internal governance of the firm in dynamic context by using continuous-time dynamic agency model. It considers the dynamic contracting problem when the internal governance limits the scope of capital diversion, in addition to the law, but the investor cannot observe the executive's decision on effort provision and capital diversion. The study of internal governance in dynamic context raises many important issues which cannot be investigated in the static framework. How does the level of governance affect the executive's benefit and investor's profit along their dynamic relationship? How the internal governance level influence the dynamic incentive on effort provision and capital diversion of the executive? Does the value of internal governance change along the firm's dynamics? How does a change in governance level affect the firm's growth and investor's profit at different stage of the firm? Why does not the security price of the firm increase when the governance mechanisms improve? The model provided in this paper answers these questions.

The model also gives explanation on the puzzle about the effect of governance mechanism on security price. When the investors value the firm's governance, the security price is supposed to increase when the governance mechanism improves. However, according to the recent empirical studies[1] of Core, Guay and Rusticus (2006), Johnson, Moorman and Sorescu (2009) and Price, Roman and Rountree (2011), an improvement of governance mechanism does not significantly contribute to an increase in equity price. In our analysis, we show that an enhancement of firm's governance will rise the investor's profit and security price only when the change occurs at the initial stage. When the firm reaches the mature stage, an increase in governance mechanism will be very costly and hence reduce the investor's profit, hence security price. The empirical puzzle is the result of a mixture of samples in different stages of the firm.

The model studies the dynamic contract and agency conflict between the investor and the executive, or the principal and the agent respectively, in line with Jensen and Meckling (1976). It considers the stochastic dynamic investment technology in which the outcome depends on the level of managerial effort and corporate capital. The outcome, firm's cash-flow, is observable and verifiable to principal and agent. However, the agent decision on the level of effort and capital diversion are unobservable. Under stochastic environment, the

[1] The recent studies are more precise and robust because they improve the methodology of the empirical investigation of previous studies, e.g. Gompers, Ishii and Metrick (2003) and Bebchuk, Cohen and Ferrell (2008), which confirm the positive relationship between firm's governance and security price.

principal cannot precisely infer the high cashflow to high effort and low capital diversion, due to uncertainty. The principal maximizes his benefit by designing the optimal dynamic contract to incentivize the agent to provide high effort and divert less capital under the governance structure of the firm.

In this paper, the firm governance, indicating the level of investor protection, is determined by law and internal governance. The law gives the identical fundamental investor protection to all firms in the the economy, as in the *Law and Finance* literature (La Porta, Lopez-de-Silanes, Shleifer and Vishny, hereafter LLSV (1998)). In line with the *Law and Finance*, the internal governance, considered in this paper, is the corporate rules and restrictions imposed in managerial discretion beyond the law level. The internal governance is the agreement on the investor's right and extent of managerial discretion between the investor and the executive[2]. The better internal governance mechanism provides better investor protection of the firm in addition to the law. Because it is not required by the law, different firm could have distinct level of internal governance. It depends on the agreement between the investor and the executive of the firm. In the model, the firm governance constituted by both law and internal governance restraints the capital diversion for private benefit of the executive. The better governance mechanism implies higher investor protection level and lesser capital diversion and private benefit.

The optimal contract composes of agreement on internal governance between the investor and the executive, the investor's recommended investment and compensation processes and the terminal time of the contract. Given the mutually agreed governance level, the investor designs the investment and compensation rules by taking into account two types of incentive constraints. The first is an instantaneous incentive constraint. It motivates the executive away from committing capital diversion. As a result, the periodic compensation includes the benefit of control. The second is the intertemporal incentive. The investor aligns his benefit to the executive's continuation value, proposed by Spear and Srivastava (1987). Using martingale method on dynamic continuation value, pioneered by DeMarzo and Sannikov (2006) and Sannikov (2008), the executive's incentives are aligned with the investor's profit and the growth of the firm though continuation value and optimal compensation processes. The optimal investment follows the marginal-q. The contract terminates when the executive

[2] The internal governance considered in this paper composes of the rules and restrictions on the organization of the corporation and the rights of investors. These rules are delineated in the corporate charter and legally enforceable. The internal governance here is de factor the agreed contract between the investor and the executive which limits the set of managerial decisions within the firm.

expects future benefit to be equal to his outside option, which is normalized to zero, and when the investor cannot gain benefit from the executive under the optimal contract and agreed governance level.

The analysis of governance mechanism in dynamic incentive framework indicates that there is no free cashflow problem, as considered in Jensen (1986), under optimal contract because the executive would not have a motivation to hoard free cashflow in order to gain private benefit. The optimal dynamic contract incorporates the private benefit of control into the executive compensation, according to instantaneous incentive constraint and revelation principle. However, the benefit of control portion in the compensation is limited by the internal governance, which makes this portion negotiable between the investor and the executive. If both parties agree to lessen the benefit of control by raising the internal governance level, the executive will be compensated by higher pay from future growth of the firm. Consequently, the optimal contract motivates the executive to invest all the available capital for future incoming cashflow of the firm in order to gain higher future compensation. There is no agency cost of free cashflow under the optimal dynamic contract.

An implementation of optimal contract illustrates the dynamic value of internal governance as a part of the security price. It illustrates that that the security price composes of four parts; expected investor's profit, expected executive compensation, country's discount and firm's governance premium. The country's discount term captures the effect of investor protection derived from the laws of a country. The firm's governance premium reflects the value of internal governance of a specific firm. Another interesting result is that the value of the country's discount and firm's governance premium can be different overtime, even though the law and internal governance mechanism are constant. The dynamic valuation of the corporate governance elements in security price depends on the longevity of the firm, which is the distance between the current time and the expected termination period[3]. Considering governance mechanism in the dynamic contract provides a method to evaluate the dynamic price of corporate governance from both the country's law and firm's internal governance in a single framework.

An increase in internal governance level intensifies the incentive alignment of the executive's compensation to the firm's profit by lessening instantaneous benefit of control while

[3]This insight is consistent to the market sentiment. The financial market gives a high value on the governance mechanism for a firm with long future. On the contrary, investors in financial market does not concern the corporate governance mechanism when a firm is going to wind up or in reorganization process.

enhancing continuation value due to higher effort cost. As a result, it shifts the weight of executive's compensation from instantaneous benefit of control to future compensation of the firm's profit. This is equivalent to a reallocation of the weight in executive compensation from instantaneous to intertemporal incentive. This is beneficial to the investor when the firm is small and in its initial stage. Under the optimal contract, the higher internal governance motivates the executive to invest more in order to accelerate the firm's growth for hi larger benefit in the future. At the initial stage, investor's profit and executive's continuation value rise together according to the incentive alignment effect. However, in the mature stage of the firm, when the compensation is relatively high, the incentive to induce the executive's effort become very costly. Motivating the effort to generate firm's growth increases the the executive's continuation value, but lowers the investor's profit. The effect of wealth transfer, from firm's profit to executive's compensation, dominates the incentive alignment effect. As a consequence, the investor and the executive could mutually agree to decrease the firm's governance mechanism. This is consistent with the topsy-turvy effect[4], as shown in Tirole (2006). In contrast to previous studies, our present model claims that the decrease in governance level in later stage is mutually beneficial and agreed between investor and the executive.

From the theoretical point of view, an increase in internal governance alleviates the contract incompleteness by reducing the discretionary power of the executive on important issues concerning investors. With higher level of internal governance, investor have more power in managerial decisions and broader issues to vote and approve before their implementation. Under the optimal contract, it makes the contract about managerial decision more complete and change the nature of problem from contractual incompleteness into hidden-action problem.

This paper relates to two main strands of literature. In the recent study on continuous-time dynamic agency literature, this paper follows the line of research of DeMarzo and Sannikov (2006) for corporate finance, Sannikov (2008) for martingale method of dynamic incentive and DeMarzo, Fishman, He and Wang, hereafter DFHW (2010), on dynamic investment model. The theoretical contribution of this paper is to break the direct link between agent's effort decision and the private benefit of control, considered in He (2009) and

[4]The topsy-turvy incentive means that a firm would have high governance standard at the initial stage, but lowers the standard when the firm is mature and stable. This paper is different from previous studies, e.g. Acharya and Volpin (2010), in the sense that a decrease in governance level is mutually agreed between the investor and the executive, not the unilateral decision from the executive only.

DFHW (2010). This disengagement gives another choice for modeling the dynamic agency problem. The agency rent needs not to be derived from agent effort only. Furthermore, the model of this paper is the first to incorporate the instantaneous incentive constraint with the dynamic incentive structure. This methodology can be used for the model involving the static incentive constraint and the revelation principle, such as laws, government regulation and taxation.

In corporate finance and governance literature, this paper is among a few that study the corporate governance in the dynamic context. The studies of corporate governance are broad in both theoretical and empirical investigations, but most of them focus on the static framework. It hence cannot address important issues considered here. The reviews of Becht, Bolton and Roell (2007), Denis and McConnell (2003) and Shleifer and Vishny (1997) give excellent summaries of the previous studies. In this paper, using continuous-time dynamic agency model broadens the analysis of firm's governance on the entire relationship between the investor and the executive. Technically, it gives a clear analytical solution and clean comparative static analysis. In addition, it merges the analysis of dynamic agency with the asset valuation model into a single framework. We can see how governance mechanism affects agency problem in parts of the firm and how firm's value change accordingly. By considering governance mechanism in dynamic context, we can also explore effects of the corporate governance on firm's profit and executive compensation at the different stages of the firm. Further, we can evaluate the dynamic aspect of the governance valuation in security price also. These two issues can not be studied systematically in a static framework. Furthermore, the model provides a method to evaluate the dynamic price of corporate governance from both the country's law and firm's internal governance in a single framework. To the best of my knowledge, this paper is the first to offer theoretical investigation on these issues.

The organization of the paper is following. Section 2 describes the structures of the model. It delineates the investment dynamics, production technology and characteristics of the principal and agent. The formulation of contractual relationship and agency problem is explained here. In Section 3, we derive the dynamic optimal contract using martingale approach on dynamic continuation value and dynamic programming method, the HJB equation. In Section 4, we derive the security price by an implementation of the optimal contract and exhibits the value of governance mechanism as parts of the price. The security price incorporate the effect of agency cost and effect of internal governance as separate elements. Section 5 considers the effects of change in governance on the dynamic relationship between investors' profit and executive compensation. We also discuss the results and implication for governmental intervention on firm's governance. Section 6 concludes. All proofs are provided in the appendix.

2 The Model

In this section, we explain the concept of internal governance and formulate the dynamic agency problem with stochastic investment. In the dynamic contracting framework, we start with describing the capital dynamics, productivity and cashflow processes. We next present the utility and decisions of the investor and the executives, as the principal and the agent respectively. We then explain the contracting circumstance and the role of governance mechanism in the model.

2.1 Internal Governance Mechanism

The internal governance considered here means the firm's rules and requirements that describe the extent of investor's vote to decide or approve important corporate decision and policies. They must be consistent with the law and regulation. These rules and requirements are delineated in the corporate charter and hence legally binding to all contracting parties of the firm, particularly the investor and the executive. In this aspect, the internal governance is the agreement between investors and executives on the scope to operate the firm. This definition gives two important implications.

Firstly, the alteration of the internal governance must have the consent from investors and the executives. The executive cannot unilaterally change the internal governance without consent of the investors. This implication indicates that the mechanism is not the result of optimal decisions of the executive. It is the contract between investors and executives. In the model, the internal governance is a parameter that both parties agree upon before initiating the contractual relationship.

Secondly, the internal governance captures effect of the investor protection mechanism beyond the legal requirement. The law and regulation provides legal infrastructure on many aspects of the mechanisms[5]. However, for a specific firm, investors could find an inadequacy

[5] Becht, Bolton and Roell (2007) summarizes the governance mechanisms into five groups; partial concen-

of protection and would increase the investor protection by extending the requirements in various mechanisms[6]. The internal governance in this paper reflects the firm's additional protection through extension on mechanisms beyond the law. Based on the methodology provided by LLSV (1998), we summarize the effect of internal governance by the parameter g. The higher level of g represents the better investor protection level from internal governance. If the firm does not provide additional protection from the legal requirement, the internal governance is zero, $g = 0$.

The internal governance puts restriction on the set of managerial decisions available to the executive and reduce the potential expropriation and agency cost. The internal governance creates both cost and benefit to both investor and the executive. On the cost side, when internal governance level increases, the executive put more effort to keep up the same level of performance due to more stringent requirement on managerial decisions. The investor has to compensate for higher cost in order to motivate the effort. Moreover, due to the lack of formal enforcement mechanism for the internal governance, the firm incurs additional cost in order to incentivize the executive away from the violation of the agreement. On the benefit side, the firm and investors have lower *potential* cost of expropriation, hence have more resources for corporate investment. We explain the details of cost and benefit of internal governance on the investor and the executive below.

In essence, the internal governance transfers the power of managerial discretion from the executive to investors. In contract theory terminology, the internal governance mitigates the inefficiency from contract incompleteness by assigning the decision power to the principal's decision. The shift of decision rights protects the investors from the risk of expropriation. Even though, the investor have better protection from higher level of internal governance, he

tration of ownership and control, hostile takeovers, board of directors, executive compensation and fiduciary duties of the executives.

[6]The executives have broad range of discretion on the usage of corporate resources that potentially leads to expropriation without legal restraints, for instance the discretion to determine executive compensation, the power to consider self-dealing transactions, ability to commit mixed-motive actions, the authority on making empire-building investments and the decision for managerial entrenchment. These corporate activities are tainted with agency conflict but cannot be limited by law because the executives are protected by the *Business Judgment Rule*. From Clark (1986), pp. 123, the business judgment rule is *the business judgment of the directors (and executives) that will not be challenged or overturned by courts or shareholders, and the directors (and executives) will not be liable for the consequences of their exercise of the business judgment – even for judgments that appear to have been clear mistakes– unless certain exceptions apply.* The rule puts investors on the various aspects of expropriation unrestrained by the law. Hence the internal governance, by providing additional investors right to determine important managerial decisions and policies or to approval crucial corporate decisions, can provide more protection against the risk of expropriation due to the business judgment rule.

still have the problem from unobservable actions of the executive. The investor, as principal of the contract, still need to design the dynamic contract to induce the executive to work optimally. We model the dynamic contract with internal governance mechanism next.

2.2 Investment and Production Technology

We formulate the dynamic investment model similar to DFHW (2010). The firm uses capital and agent's effort to generate the stream of cashflow. The capital is considered as a numeraire. The capital dynamics is determined as follow,

$$dK_t = (I_t - \delta K_t - b_t\zeta(g)K_t)\,dt \tag{1}$$

where K_t is capital, I_t is investment, $\delta > 0$ is capital depreciation rate. The $\zeta(g)$ denotes the rate of capital distortion as a function of internal governance level; denoted by g. We assume $\zeta'(g) < 0$, saying that rate of capital distortion is decreasing in internal governance level. Note that when the firm only follows the legal requirement on governance mechanism, we have $g = 0$. It implies the imperfection in governance mechanism required by the law at the firm level. We assume that $\zeta(0) > 0$ meaning that the rate of capital distortion is positive when a firm does not put additional governance beyond the law. This is consequence of the aforementioned business judgment rule, that opens the room for an expropriation. We currently do not assume the curvature of the $\zeta(\cdot)$ function. With this interpretation of distortion function, we can consider the governance mechanism at both country and firm levels. The variable $b_t \in \{0, 1\}$ is a binary decision choice of the agent's decision to respect the overall level of governance. When $b = 0$, it implies agent abides by the governance mechanism and not committing conflicting behavior and $b = 1$ otherwise.

The capital dynamics then depends on the investment net of depreciation and the agent's decision to expropriate. The expropriation would divert capital away and retard the process of capital accumulation of the firm.

We assume the linear production technology. The cashflow dynamics derives from the increment of the firm's cashflow from production ($K_t dA_t$) net of cost of capital adjustment ($K_t L(i_t)dt$). The firm's cashflow process is following

$$dY_t = K_t\,(dA_t - L(i_t)dt) \tag{2}$$

where Y_t is the cumulative cashflow at time t, dA_t is instantaneous productivity to be defined below. The function $L(i_t)$ is the adjustment cost of investment per capital, where $i_t = \frac{I_t}{K_t}$ is the investment per capital. It is defined as $L(i_t) = \frac{L(I_t,K_t)}{K_t} + \frac{I_t}{K_t}$, in which $L(I_t, K_t) = \frac{\theta}{2}\frac{I_t^2}{K_t}$. Then we can write the adjustment cost in the investment per capital unit as $L(i_t) = \frac{\theta}{2}i_t^2 + i_t$ which is the standard convex adjustment cost of investment.

We model the cumulative productivity as A_t, and hence dA_t denotes the instantaneous productivity at time t. The productivity follows the diffusion process in which the drift is determined by the agent's unobservable effort, $a_t \in \mathbb{R}_+$. The volatility of the productivity is constant and denoted by $\sigma > 0$. The differential form of productivity process reads as follow,

$$dA_t = a_t dt + \sigma dZ_t \tag{3}$$

where Z_t is the standard Brownian motion. Notice that the productivity is subject to uncertainty, due to diffusion term, and the firm possibly incurs loss in spite of positive agent's effort.

The firm is established as contractual relationship between investor and the executive, as the principal and agent respectively. The contract can be ceased when a party unilaterally revoke the relationship and the firm is consequently terminated.

2.3 Modelling Principal and Agent

Due to the separation of ownership and control, the investor, as the principal, employs the executive, or the agent, to run the firm given the technological and agreed governance circumstance. The principal recommends the investment level at any period to the agent for his own profit. The agent controls the drift of productivity process following equation (3), in order to generate the stream of cashflow, which is publicly observable and described in equation (2), as a return to the principal. At any instant, the agent can also choose whether to divert the capital for his private benefit. The distortion in capital dynamics from agent's diversion is limited by the level of governance, as described in equation (1). The principal cannot observe actions of the agent, both the effort determining the drift of productivity process (a_t), and the decision on capital diversion (b_t). In return, the principal gives compensation process to the agent as a return for his effort in order to induce the optimal decisions of the agent for maximization of the expectation of discounted profit.

The agent derives utility from compensation process, denoted by U_t, and provides the stream of effort (a_t) affecting the average productivity level. The cost of effort depends on the effort level and governance mechanism. As previously defined, the internal governance is the agreement between the principal and agent on the rules and requirements of investor rights and votes on important decisions and policy. The high governance requirement, high level of g, would put the stringency on available actions for the agent in order to keep up the performance. This makes it harder for the agent to provide efforts. It hence induces higher cost for higher governance level given an effort level. Moreover, the agent could decide whether to take private benefit from capital diversion. We summarize the assumptions and decisions of agent as follows.

$$(U_t - H(a_t; g))\, dt + b_t \zeta(g) K_t dt \tag{4}$$

The effort cost function, $H(\cdot;\cdot)$, is increasing and convex in effort (a_t), and governance (g) level. We assume complementarity between two arguments; $\frac{\partial^2 H(a,g)}{\partial a \partial g} > 0$. As previously mentioned, at the higher level of internal governance, it takes higher cost for a given level of effort. For $g_1 > g_0, H(a_t; g_1) > H(a_t; g_0)$.

We also assume that the agent would work as long as the net utility from working is not lower than his outside option. This is a standard individual rationality, or participation, constraint. The constraint must be satisfied in any period until the terminal date, τ. We normalize the outside option to zero. We write the individual rational constraint as following,

$$U_t - H(a_t; g) \geqslant 0; \qquad \forall t \in [0, \tau]. \tag{5}$$

Because the decision to divert capital is unobservable, even though the rate of capital diversion can be known, the principal would provide the incentive for the agent to ensure that he would not choose to divert capital at any instance[7]. We then have the instantaneous incentive compatibility constraint,

$$U_t^g \geqslant U_t + \zeta(g) K_t; \qquad \forall t \in [0, \tau], \tag{6}$$

where U_t^g denotes the compensation process that satisfies the instantaneous incentive

[7] This incentive can be incorporated into the contract in the sense that, given the incentive provision the agent would not commit any acts on capital diversion. Otherwise, when the principal can verify the committed diversion, the agent will be executed with prohibitively considerable punishment.

constraint given the governance mechanism and U_t represents the compensation process according to the individual rationality constraint, equation (5). The condition says that the principal would compensate the agent with U_t^g to guarantee no capital diversion. We can write two instantaneous constraints altogether as follow.

$$U_t^g - H(a_t; g) \geqslant \zeta(g)K_t; \qquad \forall t \in [0, \tau] \tag{7}$$

The compensation process (U_t^g) takes into account the benefit from agency rent. It hence satisfies both individual rationality and incentive compatibility constraints to guarantee no expropriation and participation of the agent at the same time. Because the compensation to agent is costly to the principal, he would choose the incentive-compatible compensation process to make the equation (7) hold with equality under the optimal contract.

The principal derives the benefit from the cashflow process and compensates the agent for his effort. We describes the benefit in each period in the following equation.

$$dY_t - U_t^g dt; \qquad \forall t \in [0, \tau]. \tag{8}$$

The principal recovers the termination value when the the firm is ceased at the terminal date. We assume that when the relationship ends, the principal, or the investor, can redeem the value of capital lK_τ, where $l > 0$ is liquidation rate and τ is the terminal date.

2.4 Formulation of Dynamic Contractual Relationship

In the model, both parties maximize their expectation of the discounted benefit over the length of firm, equivalently contractual relationship. The capital and cashflow are observable, while the decisions of agent are not. We assume, without loss of generality, that the discount rate of agent is greater to that of the principal, $\gamma \geqslant r$ respectively[8], and the principal's discount rate is equal to the risk-free rate.

We assume that the principal has the bargaining power[9] and offers the contract to the agent. The dynamic contractual relationship is form by the contract specifying investment

[8]We can also assume $\gamma > r$, the discount rate of the agent is strictly greater than that of the principal. What is necessary for our analysis is that the discount rate of the principal must not be greater than that of the agent. Otherwise, the principal would postpone the compensation to agent indefinitely. However, the difference between strictly and weakly lower discount rate does not change our insight on the analysis of internal governance

[9]This assumption can be relaxed later on. It would affect the boundary value of involved parties, but not the optimal contract characterization

(I_t), incentive-compatible compensation to the agent (U_t^g) and optimal terminal time (τ), after an agreement on the internal governance mechanism (g). The governance mechanism would not change along the dynamics as it represents the limited scope of decisions available to the agent. The governance could change when both parties agree to renegotiate for a new mechanism and consequently the contract restart from the new level of governance. The contract $\{I_t, U_t^g, \tau; g\}$ would depend on the history of cashflow process reflecting agent's effort and performance.

Given the contract and governance level, the agent determines the effort level and decision to divert firm's capital to maximize,

$$\sup_{\{a_t, b_t; t\in[0,\tau]\}} \mathbb{E}^a \left[\int_0^\tau e^{-\gamma t} \left(U_t^g - H(a_t; g) + b_t \zeta(g) K_t \right) dt \right]$$

where the expectation is made under the probability measure generated by the effort process ($a_t; t \in [0, \tau]$). The expectation of discounted total benefit composes of two parts, the net benefit from effort and the private benefit from capital distortion.

Suppose that at the time of contracting, the initial capital is $K_0 > 0$ and the agent's initial payoff is denoted by W_0, the objective function of the principal is formulated as follow.

$$F(W_0, K_0) = \sup_{\{I_t, U_t^g; t\in[0,\tau]\}} \mathbb{E} \left[\int_0^\tau e^{-rt} dY_t - \int_0^\tau e^{-rt} U_t^g dt + e^{-r\tau} l K_\tau \right].$$

which comprises of two elements; the accumulated cashflow net of compensation and the terminal value.

In order to formulate the dynamic optimal contract, the principal must take into account the instantaneous individual and incentive compatibility constraints, shown in equation (5) and (6). Furthermore, the principal must concern the intertemporal incentive of the agent in order to motivate the optimal growth of the firm. We consider the characterization in the next section.

Notice that the cashflow, as the benefit to the principal, depends on both capital level and productivity of the agent. The principal need to give recommendations for optimal investment and effort in order to recommend the optimal trajectories of capital and productivity level respectively. For the cost to principal, the compensation process incorporates both effort cost and private benefit of control. The effort cost derives from cost function and individual rationality constraint. The private benefit of control, or agency cost, is from the distortion function and capital level, which are known to the principal. We assume that the principal knows the effort cost function $H(\cdot; g)$, distortion function $\zeta(\cdot)$ and governance level (g), but does not observe the agent's decisions (a_t, b_t). The optimal contract is formulated by the principal's inference on cashflow as a signal of unobservable decisions of the agent. In the next section, we describe the method of optimal contract characterization based on the formulation and information structure described above.

3 Optimal Contract Characterization

In this section, we derive the dynamic optimal contract. We use the method of stochastic control (HJB equation), the continuous-time dynamic programming, to characterize the optimal contract from principal's perspective. The optimal contract is characterized as a differential equation and boundary conditions. Due to unobservable decisions of the agent, the principal would formulate the contract to motivate optimal decisions over time and to ensure no capital diversion with in any period. To formulate the contract, the principal consider agent's continuation value and capital level as state variables in order to optimize over their stochastic processes. Proposed by Spear and Srivastava (1987), the continuation value at any time encapsulates the expectation of discounted future benefit until the termination of contract. The dynamic continuation value plays an important role of optimal contract characterization.

Under unobservable actions situation, the principal needs to recover the necessary conditions for recommendations to agent's actions and intertemporal motivation. These conditions pinpoint the evolution of the continuation value and induce the optimal incentive to the agent. We will recover the necessary conditions of the incentive compatible actions and the stochastic differential equation of the continuation value. We employ the martingale representation theorem to recover the dynamic continuation value and the necessary conditions. This method is proposed by Sannikov (2008). It provides an intuitive interpretation and clear presentation of the elements in the continuation value process. Through this process, we will clearly see the effects of capital distortion and governance mechanism in the agent's motivation and principal's profit. The optimal contract will lay down the differential equation, its properties and the necessary conditions for the dynamic continuation value to establish the dynamic incentive contract.

This section proceeds as follow. We start with describing the evolution of continuation value. We show that the continuation value is the expectation of discounted net benefit of the agent from current period to the terminal date. The expectation bases on the probability measure generated by effort that determines the drift of productivity process. We then establish the stochastic differential equation of the continuation value based on martingale representation theorem. We next find the necessary conditions for the incentive compatible decisions of the agent and the requirement on continuation value dynamics. With well-behaved continuation value dynamics, we characterize optimal contract by applying HJB equation to the principal's problem. To avoid a complication of many state variable in the system, we use the continuation value per capital as state variable, instead of having two state variables in the contract. As a consequent, the optimal contract is formulated as ordinary differential equation of principal's profit function over the continuation value per capital and necessary conditions for the elements of the contract. The elements of optimal contract, investment dynamics and compensation process, will be the function of continuation value. To pinpoint the solution of the ODE of profit function, we describe the boundary conditions and its important property, its curvature. We conclude this section with a collection of conditions for dynamic optimal contract.

3.1 Continuation Value Dynamics

We define the continuation value of the agent with respect to the instantaneous individual rationality and incentive compatibility constraints. Given the incentive compatibility contract $\{I_t, U_t^g, \tau\}$, the agent's continuation value at time $t \in [0, \tau]$ is following.

$$W_t = \mathbb{E}^a \left[\int_t^\tau e^{-\gamma(s-t)} \left(U_s^g - H(a_s; g) \right) dt \right] \tag{9}$$

The continuation value is the expectation of the discounted net utility of each period under the contract and history of performance from time t to the terminal period τ. At a given period, the principal determines the level of U_t^g with respect to instantaneous participation and incentive compatible constrains. The compensation process must also be aligned with the dynamic incentive under the recommended effort process (a_t) given the current governance mechanism. Notice that the continuation value of the agent does not take into account the private benefit of control explicitly. However, the agency rent is incorporate in directly through the instantaneous incentive compatible constraint.

To recover the evolution of continuation value, we consider the value function of the agent's benefit. The value function of the agent at any time t describes the the net benefit from the initial period up to time t and the expected net benefit in the future through the continuation value. Under the optimal contract and recommended decisions, decided by the principal, the value function is martingale. Intuitively, this is because, after the principal choose the recommended actions to be done by the agent, the principal would compensate just enough to implement the actions. The principal would give a discounted average compensation to the agent, under recommended paths of decisions, just to compensate the costs of actions. Hence, the value function of the agent would be a martingale, meaning that its expectation is zero over time. We write the value function explicitly as follow.

$$V_t = \mathbb{E}^a \left[\int_0^t e^{-\gamma s} (U_s^g - H(a_s; g)) dt + e^{-\gamma t} W_t \right]$$

Based on martingale representation theorem, we claim that there is a stochastic process λ_t that its value at given time depends on the information from previous periods only and it makes the evolution of continuation value a martingale process. The detail of derivation is proven and shown in the appendix. We summarize the result in the following lemma.

Proposition 1 (**Dynamic Continuation Value of the Agent**). *There exists the progressively measurable process λ_t such that the continuation value of the agent is*

$$W_t = W_0 + \int_0^t e^{-\gamma s} \left(\gamma W_s - (U_t^g - H(a_s; g)) \right) ds + \int_0^t e^{-\gamma s} \lambda_s \sigma K_s dZ_s$$

In the differential form,

$$dW_t = \gamma W_t dt - (U_t^g - H(a_t; g)) dt + \lambda_t \sigma K_t dZ_t.$$

or equivalently,

$$dW_t = \gamma W_t dt - (U_t^g - H(a_t; g)) dt + \lambda_t \left(dY_t - K_t (a_t - L(i_t)) dt \right).$$

This proposition says that the evolution of continuation value composes of two parts. The first part is the average growth of the continuation value. It is the summation of the return according to discount rate and the agency rent from each period. We have agency rent here because the principal must incentivize the agent to respect the governance mechanism due to the lack of private enforcement. At the higher level of governance mechanism, the agency rent is reduced, but principal would pay for higher cost of effort through the compensation process instead. We will consider the compensation process under instantaneous constraints under optimal contract when we investigate the optimal profit characterization below. We write the expected evolution of continuation value as follow

$$\mathbb{E}\left[dW_t + (U_t^g - H(a_t; g))dt\right] = \gamma W_t dt.$$

The second part is the volatility element. The volatility term reflects the sensitivity of the continuation value from the cashflow. The progressively measurable process λ_t in the volatility term can be considered as a multiplier of the sensitivity from the cashflow process. Hence, it plays a role to induce optimal effort (a_t) of the agent through maximizing the term $(dY_t - K_t(a_t - L(i_t))dt)$.

We next consider the incentive compatible decisions of the agent given the dynamic continuation value. We also pinpoint the optimal level of progressively measurable process because it is possible that there are many processes that satisfy the requirement of proposition 1.

To characterize the incentive compatible decisions $\{a_t, b_t\}$, the idea bases on the one-shot deviation principle, in the sense that a deviation from the path of optimal decisions at any moment would reduce the total benefit of the agent. Then the condition that guarantee no deviation at any moment can be extent to the whole path of contract. We then use the derived condition to characterize the incentive compatible path of decisions. The detail of derivation is proven and shown in the appendix. We state the necessary conditions for the optimal decisions here.

Under the optimal dynamic contract, the incentive compatible effort must satisfy the following condition at $t \in [0, \tau]$,

$$(\lambda_t a_t - H(a_t; g)) \geqslant (\lambda_t a_t' - H(a_t'; g)), \quad \forall a_t' \neq a_t. \tag{10}$$

For the decision on capital diversion (b_t), we require that

$$b_t(1 + \lambda_t)\zeta(g)K_t \leqslant 0, \quad \forall t \in [0, \tau]. \tag{11}$$

Knowing that $\lambda_t \geqslant 0$, $\zeta(g) > 0$ and $K_t > 0$, this condition is satisfied only if $b_t = 0, \forall t \in [0, \tau]$. Hence, no capital diversion is incentive compatible to agent's decision.

Given the incentive compatible decisions of the agent, we consider the progressive measurable process for the optimal contract. From the requirement of property of the process in proposition 1, it is possible to have many processes satisfying the property. However, the motivation through the continuation value dynamics is costly to the principal because it involves a transfer of cashflow from him to the agent. The principal would choose the lowest possible level of progressively measurable process that satisfies the requirement in proposition 1 and the incentive compatible effort. We state the condition to pinpoint the progressively measurable process for optimal contract as follow.

$$\lambda_t = min\{\tilde{\lambda_t} \in [0, \infty) : a_t \in argmax_{\{\tilde{a} \in [0,\infty)\}}\{\lambda \tilde{a}_t - H(\tilde{a}_t; g)\}\} \tag{12}$$

The necessary conditions for incentive compatible decisions and progressively measurable process in the optimal contract are summarized in the proposition 2.

Proposition 2 (Incentive Compatible Decisions). *From the dynamics of continuation value described in proposition 1, the optimal effort and decision on capital diversion must satisfy equations (10) and (11) respectively, and λ_t meets the condition in equation (12). In short, we require that*

$$a_t \in argmax\{\lambda_t \tilde{a}_t - H(\tilde{a}_t; g)\} \quad \forall t \in [0, \tau], ,$$
$$b_t = 0; \quad \forall t \in [0, \tau],$$
$$\lambda_t = min\{\tilde{\lambda_t} \in [0, \infty) : a_t \in argmax_{\{\tilde{a} \in [0,\infty)\}}\{\lambda \tilde{a}_t - H(\tilde{a}_t; g)\}\}$$

The proposition 2 describes incentive compatible decisions of the agent. Under the optimal contract, no capital diversion ($b_t = 0$) is optimal because the compensation process incorporate the agency rent required by the incentive compatibility constraint. The agent hence does not have motivation to divert more capital from the firm. The optimal effort level (a_t) equilibrates the marginal benefit and marginal effort cost in every period, $\lambda_t = \frac{\partial H(a_t;g)}{\partial a_t}$. Notice that the marginal benefit is chosen at the lowest possible level that satisfies this condition and proposition 1, implying that λ_t makes the agent's continuation value to be a martingale. This proposition states that at any given period t the optimal contract induce efficient decisions of the agent cross-sectionally and dynamically. We next consider the profit function and optimal contract designed by principal, based on the derived dynamic continuation value.

3.2 Principal's Profit Function

We now characterize the optimal dynamic contract. We proceed by using continuous-time dynamic programming method through the Hamilton-Jacobi-Bellman (HJB) equation of the principal's profit function $F(W_t, K_t)$. To simplify the technicality of differential equation, we scale down the continuation value by the capital unit, denoted as $w_t = \frac{W_t}{K_t}$, and consider the optimal contract in the scaled profit function, $f(w_t) = \frac{F(W_t,K_t)}{K_t}$. We also scale the related variables in term of unit per capital. The elements of dynamic contract are characterized by optimality and boundary conditions of the HJB equation on scaled profit function and continuation value per capital unit.

From continuation value dynamics and scaled profit function, the HJB equation for the the scaled profit function is following, denoting $u_t^g = \frac{U_t^g}{K_t}$ and $i_t = \frac{I_t}{K_t}$,

$$\begin{aligned} rf(w_t) = \sup_{\{i_t, u_t^g\}} \{a_t - u_t^g - L(i_t) + \big((\gamma - (i_t - \delta))w_t - (u_t^g - h(a_t;g))\big) f'(w_t) \\ + \frac{1}{2} f''(w_t)\lambda_t^2\sigma^2 + f(w_t)(i_t - \delta)\}. \end{aligned} \tag{13}$$

From the HJB equation (13), the optimal investment per capital is determined by the necessary condition

$$f(w_t) - w_t f'(w_t) = L'(i_t). \tag{14}$$

This is the Euler equation for investment per capital. It is consistent with the marginal-q theory of investment. The condition requires that the optimal investment per capital equilibrates the marginal benefit of investment to the value of the firm, or marginal-q, to the marginal cost of capital adjustment. To illustrate the condition, we define the marginal-q as the derivative of total value of the firm with respect to capital, $q_t = \frac{\partial(F(K_t,W_t)+W_t)}{\partial K_t}$ where $F(W_t, K_t) = K_t f(w_t)$. Then $\frac{\partial F(K_t,W_t)}{\partial K_t} = q_t = -w_t f'(w_t) + f(w_t)$. The marginal cost of capital adjustment described previously is $L'(i_t)$. The equation (14) describes this optimality condition.

To characterize the dynamic contract, we illustrate the optimal investment in term of scaled profit function. From the optimal investment condition, we have $q_t = f(w_t) - w_t f'(w_t)$. Substitute the explicit form of marginal capital adjustment cost, $L'(i_t) = 1 + \theta i_t$, the necessary condition requires that the optimal investment per capital at any period must satisfy

$$i_t^* = \frac{q_t - 1}{\theta} = \left(\frac{f(w_t) - w_t f'(w_t) - 1}{\theta}\right). \tag{15}$$

For the optimal compensation process, we know that the first-order condition from the HJB equation (13) gives us the corner solution, $f'(w_t) = -1$. There is no direct link on optimal dynamic compensation over time. This result is intuitive. We know that the compensation process is costly to the principal and it would be decreased as low as possible at any instant. It is hence determined by the instantaneous constraints, rather than the dynamic optimality condition. However, the compensation process, satisfying instantaneous constraints, would induce the optimal effort over time if it meet the cost of effort induced by the incentive compatible effort (a_t) and the progressively measurable process (λ_t) described in proposition 2 and 1, respectively. This is because the instantaneous constraints force the compensation process to meet the periodic effort cost at any instance. It, in turn, generates the optimal dynamic effort through the λ_t inducing optimal dynamic incentive in the evolution of continuation value. In short, the compensation process then has indirect link to the optimal dynamic effort through the participation and incentive compatibility constraints when proposition 1 and 2 hold.

We summarize the differential equation describing the dynamic movement of the principal's profit function and the conditions that characterize contract $\{i_t, u_t^g\}$ in the proposition 3. We substitute the optimal investment into the HJB equation. The optimal compensation process is determined by instantaneous constraints, and hence denoted by u_t^g in the differential equation to underline that there is no optimal dynamic link over time. The detail of derivation of differential equation and necessary conditions is given in the appendix.

Proposition 3 (ODE of Profit Functions and Elements of Optimal Contract). *The evolution of principal's profit function is described in the form of an ordinary differential equation,*

$$(r+\delta)f(w_t) = a_t - u_t^g + \frac{(q_t-1)^2}{2\theta} + \left((\gamma+\delta)w_t - (u_t^g - h(a_t;g))\right)f'(w_t) + \frac{1}{2}\lambda^2\sigma f''(w_t).$$

The optimal investment per capital is determined by

$$i_t^* = \frac{f(w_t) - w_t f'(w_t) - 1}{\theta}.$$

The optimal compensation process is derived from the equalities of instantaneous individual rationality and incentive compatibility constraints given the optimal level of effort and agency cost under governance level.

From the proposition 3, the profit function has the form of second-order ODE and, consequently, we are guaranteed that the solution to the ODE exists. We need the boundary conditions to pinpoint the exact solution of the ODE. We now investigate the conditions and the important property of the profit function, the curvature.

We first consider the lower boundary condition. We assume that the outside option of the agent is zero. When the continuation value reaches the zero ($W = 0$), implying agent's expectation for future benefit is zero, and equal to outside option, the agent would not have incentive to work for future benefit. The principal will liquidate the firm and redeem the existing capital[10]. From the setting, the terminal value of the firm is lK_τ. The lower boundary condition is $F(0, K_\tau) = lK_\tau$, or, in the scaled profit function,

$$f(0) = l. \tag{16}$$

where l is the liquidation rate of the capital and K_τ is the capital at terminal date when $W_\tau = 0$.

When the continuation value is high enough, the principal would find it unprofitable to remain in the contract with the agent because it is too costly for the principal to motivate the agent to work. It is optimal to terminate the existing contract[11], which is consistent with Spear and Wang (2005) and Sannikov (2008). To characterize such level of continuation value, we consider the benefit of principal and the transfer to agent. Because the principal compensate the agent in the form of transfer of incoming cashflow, the highest compensation the principal is willing to make is the total incoming cashflow. This compensation is de facto the transfer from the principal's profit to the agent's continuation value. It costs the principal at most one unit of the profit $f(w_t)$ to increase a unit of agent's wealth, w_t, similar

[10]The liquidation of the firm is equivalent to terminate the contract and restructuring the firm, e.g. change the agent. Because the principal employs the agent to work for him, he will terminate the contract when the agent does not have incentive to work for him.

[11]Equivalently, the principal might cease the existing contract and change the condition of payment to the agent, while keep the contractual relationship between them going.

to DeMarzo and Sannikov (2006). As a consequence, the slope of profit function over the domain of continuation value is a negative unit, implying a unit decrease in principal's profit becomes a unit increase in principal's wealth. This is the condition to pinpoint the upper boundary of the continuation value. We denote it by $\bar{w}_t$. The condition is also called *smooth pasting condition*[12], which requires that, at upper boundary $\bar{w}$,

$$f'(\bar{w}) = -1. \tag{17}$$

Another requirement on upper boundary is *Super Contact Condition*. It describes the behavior of the change in profit function at the upper boundary. The idea is intuitive. If the principal can transfer the incoming cashflow to the agent in a continuous fashion, at the optimal upper boundary the slope of profit function, that is the change in profit by such transfer, should not be affected when the principal can transfer the cashflow in an infinitesimal amount. This condition is that of the determination the boundary of the state variable in instantaneous stochastic control problem, as proven in Dumas (1991) and Dixit (1993). The condition requires that

$$f''(\bar{w}) = 0. \tag{18}$$

From the lower and upper boundary conditions, we claim that scaled continuation value under the optimal contract has a range from zero to the upper boundary, $w_t \in [0, \bar{w}]$. The profit function is defined accordingly. Another important property of the profit function over this domain is the curvature. We claim that the profit function is strictly concave over the range, $w_t \in (0, \bar{w})$. The proof is given in the appendix. We summarize the properties of profit function, it curvature and boundary conditions, in the following proposition.

Proposition 4 (**Boundary Conditions and Concavity of Profit Function**). *The scaled profit function is strictly concave over the interval $w_t \in (0, \bar{w})$ with the following boundary conditions*

1. *Lower Boundary Condition :* $f(0) = l$
2. *Smooth Pasting Condition :* $f'(\bar{w}) = -1$

[12]This idea is similar to instantaneous control problem in which the the principal starts transferring excess cashflow above the upper boundary of the continuation value. The upper boundary is the reflecting boundary in this circumstance.

3. Super Contact Condition : $f''(\bar{w}) = 0$

where l is the liquidation rate of capital.

We end this section with the proposition 5 describing the characteristics of the dynamic optimal contract. The optimal contract composes of the principal's profit function and recommended actions, agent's dynamic continuation value and the elements of contract. After stating the results, we give detailed explanation and the role of the internal governance in the optimal contract.

Proposition 5 (**Characteristics of Optimal Contract**). *There exists an investor' s profit function $F(W_t, K_t)$ which is homogeneous degree one and proportional to capital K_t; $F(W_t, K_t) = f(w_t)K_t$. The function $f(w_t)$ is the scaled profit function.*

The scaled profit function is strictly concave and evolves according to the ODE

$$(r+\delta)f(w_t) = a_t - u_t^g + \frac{(q_t-1)^2}{2\theta} + \left((\gamma+\delta)w_t - (u_t^g - h(a_t;g))\right) f'(w_t) + \frac{1}{2}\lambda_t^2\sigma f''(w_t)$$

for $w_t \in (0, \bar{w})$. It satisfies boundary conditions $f(0) = l$, $f'(\bar{w}) = -1$ and $f''(\bar{w}) = 0$.

The continuation value of the agent evolves according to

$$dW_t = \gamma W_t dt - (U_t^g - H(a_t;g))dt + \lambda_t \sigma K_t dZ_t.$$

where $\lambda_t = min\{\tilde{\lambda_t} \in [0,\infty) : a_t \in argmax_{\{\tilde{a}\in[0,\infty)\}}\{\lambda\tilde{a}_t - H(\tilde{a}_t;g)\}\}$, K_t evolves according to $dK_t = (I_t - \delta K_t)\, dt$ and $w_t = \frac{W_t}{K_t}$.

Under the optimal contract, the investor recommends the decisions $a_t = a(W_t), b_t = b(W_t)$ to the agent according to

$$a_t \in argmax_{\{\tilde{a}\in[0,\infty)\}}\{\lambda_t\tilde{a}_t - H(\tilde{a}_t;g)\},$$
$$b_t = 0; \quad \forall t \in [0,\tau],$$

The contract comprises of $\{I_t = I(W_t), U_t^g = U^g(W_t), \tau\}$ satisfying

$$\frac{I_t^*}{K_t} = i_t^* = \frac{q_t - 1}{\theta} = \left(\frac{f(w_t) - w_t f'(w_t) - 1}{\theta}\right); \quad \forall t \in [0,\tau]$$

$$U_t^g = H(a_t; g) + \zeta(g)K_t; \quad \forall t \in [0, \tau]$$

due to binding instantaneous constraints, equation (7), and τ is the terminal time determined by the condition $W_\tau = w_\tau = 0$.

Under the optimal contract, principal's profit derives from three parts. The first part is the contribution from expected cashflow, or level effect. It is the net effect of productivity and investment in excess of the compensation. From the biding instantaneous constraint equation (7), the compensation is equal to the cost of effort and agency rent, $u_t^g = h(a_t; g) + \zeta(g)$ respectively. The agency rent lowers the principal's profit at the level effect, hence an improvement the governance mechanism would directly increase the level of the profit.

The second part derives from the first-order effect of the profit, the slope effect. It composes of a benefit from incentive alignment to agent's continuation value and the cost from agency rent. The magnitude of the change in the profit is a consequence of how much the optimal contract align benefit of the agent to the principal's profit, $(\gamma + \delta)w_t$, over the rate of capital distortion, $\zeta(g) = u_t^g - h(a_t; g)$. Because the agency rent is incorporated into the first-order effect and the incentive alignment term is positive, the higher governance level would increase the rate of change in principal's profit at any instance. Intuitively, the role of governance here is incorporated through the capital accumulation process.

The third part is the second-order effect of the profit, the curvature effect, which is the contribution from the dynamic incentive effect through λ_t^2. It affects how much the profit function adjusts to the rate of change from the cashflow fluctuation and its realization. Under the optimal contract, the intertemporal incentive factor is equal to the marginal cost of effort and the marginal cost of effort is increasing in the governance level, the governance mechanism hence indirectly determines the second-order effect also. The higher governance mechanism is, the larger adjustment of the speed of change in profit function will be.

In our model, the agent has the private benefit of control according to revelation principal and instantaneous constraints. The agency rent affects the principal's profit directly through the lower profit and retarded capital accumulation process. Respectively, the role of governance mechanism against the agency rent appears directly on the level and the first-order effects. A higher governance level boosts the profit level and enhance the capital accumulation for the incoming profit. However, the cost of governance level appears indirectly in the second-order effect in the form of higher marginal cost of effort. In effect, the governance level between the principal and agent, as a agreement at the outset of contract, is in fact the determination on the weight between instantaneous and intertemporal incentive under optimal contract. We will discuss the weight between two types of incentive with more detail when we investigate the change in governance. Before considering governance change, we will study the effect of agency rent and governance in the firm's security price in the next section.

4 Security Price and Governance Premium

This section studies the effect of agency rent and governance mechanism in a security price. We show that the firm's security price, as a result of an implementation of the optimal contract, takes into account the agency rent and governance premium. In our model, the rent is the amount of capital distortion from its full capacity in the firm. The agency rent is the private benefit of control due to imperfect investor protection. As previously mentioned, legal infrastructure of a country does not provide a perfect investor protection and exposes investors to expropriations. The internal governance is the additional mechanism from the legal requirement, being agreed between investors and the executives of the firm, that limits the scope of conflicts and reduce the agency rent. Considering the consequence of legal infrastructure and internal governance together, we see that imperfect legal investor protection induces the agency rent as a possible capital distortion within a firm, while internal governance limits the scope of potential loss. From this perspective, we simultaneously consider the governance mechanism from country and firm level in a systematic approach. This section exhibits that the security price incorporates the effect of investor protection from legal requirement and internal governance. This framework can also explain international difference of the security price due to legal investor protection given internal governance level.

To illustrate that the agency rent and benefit of internal governance is included into the security price, we proceed in two steps. Firstly, we show that an implementation of the optimal contract[13], as provided in proposition 5, leads to the security price of a firm which composes of principal's profit, agent's continuation value and the effect from the capital distortion of the firm. The first two elements are the standard result, while the

[13]The implementation of optimal contract is not unique, as shown in DeMarzo and Sannikov (2006). The implementation we use in this section follows the one from DFHW (2010).

third one derives from our model. Then, secondly, by giving a specific form of capital distortion function $\zeta(g)$, we can capture of the effect of agency rent from imperfect legal investor protection and the benefit of internal governance of a firm. Notice that the effect of legal investor protection varies across countries and legal framework, while constant across firms within a country. For a country, firms and their investors face the same conflict of interest. What makes the security price among firms different is the level internal governance, ceteris paribus. We illustrate these elements in the security price being derived from an implementation of the optimal contract. It provides a theoretical rationale for the effects of investor protection at country and firm levels in the same framework, and, furthermore, their effects in dynamic context of the security price.

The optimal contract described in proposition 5 can be implemented by a specific capital structure and financial flexibility which traces the path of continuation value dynamics. Because the implementation of the optimal contract through capital structure is not unique, we use the idea of pure-equity firm and financial slack dynamics similar to DFHW (2010) in this section. We show that under such capital structure the security price, defined as the discounted dividend accumulation over the firm's longevity and the terminal value, includes the effect of imperfect investor protection of a country and benefit of internal governance of a firm.

We assign that the firm is financed only by equities. The investors require the the dividend to be paid out as a minimum periodic return D_t as follow,

$$dD_t = K_t\left(a_t - L(i_t)\right)dt - (\gamma - r)M_t dt. \tag{19}$$

The first term on the right-hand side is the expected incoming cashflow and the second term is the adjustment term for difference between discounting term of the agent and the principal, which is equal to interest rate[14]. The later term is in the form of financial flexibility M_t. The dividend process is an obligation of the agent to pay the principal as his return on investment. In our implementation, if the obligation is not met, the contract and the firm are terminated.

We define financial flexibility dynamics as follow.

[14] We can assume the equality of two discounting terms without loss of generality. In essence, we require that the dynamic dividend payment captures the expected incoming cashflow of the firm.

$$dM_t = rM_t dt + dY_t - dD_t - dX_t \tag{20}$$

where, dX_t is the cashflow reserved for compensation to the agent, dY_t is the dynamics of incoming cashflow from our model setting, $dY_t = K_t\left(a_t - L(i_t)\right)dt + \sigma K_t dZ_t$.

The financial flexibility is the extent of the liquidity capacity of the firm in order to operate without financial problem. It can be considered as cash, credit line or other forms of working capital[15]. For our purpose, we define the dynamic financial flexibility equal to the sum of return on risk-free money market ($rM_t dt$) and incoming cashflow (dY_t) in excess of minimum dividend payout (dD_t) and compensation to the agent (dX_t).

For our purpose of optimal contract implementation, we want the financial flexibility to trace the movement of dynamic continuation value in the sense that the financial flexibility would reach zero whenever the continuation value does. We then assign the financial flexibility to be equal to the continuation value per unit of risk, $M_t = \frac{W_t}{\lambda_t}$. It implies the financial buffer for the short-run downward fluctuation of the productivity and cashflow[16]. The contractual relationship and the firm would continue as long as the financial flexibility does not reach zero. We show in the appendix as a proof of proposition 6 that the dynamic financial flexibility defined in equation (20) and minimum dividend payout process defined in equation (19) would implement the optimal contract and decisions as described in the proposition 5.

We next recover the equity prices from the implementation induced by the dividend process and financial flexibility. Using the standard definition of security price as a expectation of discounted future dividends and the terminal value, we write the explicit form of the security price at time t as,

$$S_t = \mathbb{E}\left[\int_t^{\tau} e^{-r(s-t)} dD_s + e^{-r(\tau - t)} l K_\tau\right]. \tag{21}$$

Substitute dD_t and dM_t defined in equation (19) and (20) respectively, we decompose the security price at time t into parts,

[15] There are many forms and interpretations of the financial flexibility, e.g. credit line (DeMarzo and Sannikov (2006)), cash (BMPR (2007))

[16] When $M_t = \frac{W_t}{\lambda_t}$, or equivalently $m_t = \frac{w_t}{\lambda_t}$, from the dynamic continuation value, we have $w_t - 0 = \lambda_t \sigma (Z_t - Z_0)$, then $m_t = \frac{w_t}{\lambda_t} = \sigma Z_t$

$$
\begin{aligned}
S_t = & \mathbb{E}\left[\int_t^{\tau} e^{-r(s-t)}(dY_s - U_s^g ds) + e^{-r(\tau-t)} l K_\tau\right] \\
& + \mathbb{E}\left[\int_t^{\tau} e^{-r(s-t)}(rM_s ds - dM_s)\right] \\
& + \mathbb{E}\left[\int_t^{\tau} e^{-r(s-t)}(-\zeta(g)K_s)ds\right].
\end{aligned} \tag{22}
$$

We show, in the appendix, that the first part is the profit of the principal ($F(W_t, K_t)$), the second part is continuation value per unit of risk ($\frac{W_t}{\lambda_t}$) and the last part is the negative effect from capital distortion. We now extend this part to capture the effect of imperfect legal investor protection and benefit from internal governance of the firm.

We define the capital distortion function as follow.

$$
\zeta(g) = \Omega - \psi(g) \tag{23}
$$

where Ω is a constant term reflecting the level of capital distortion when the internal governance is zero. This constant captures the possible distortion from imperfect investor production due to laws and legal infrastructure of a country where the firm is situated. A country with good legal infrastructure and high level of investor protection from laws and regulation has a low level of Ω and vice versa. Notice that if we indicate a country with high legal investor protection with i and another country with low protection with j, we have $\Omega_i < \Omega_j$. We define $\psi(g)$ as a measure for the investor protection from the internal governance mechanism (g) of the firm. As previously mentioned, the internal governance gives investor protection against agency conflict in addition to the law. In our model, it lowers the possible capital distortion in the capital accumulation process. We assume that $\psi(0) = 0$ and $\psi'(g) > 0$. Hence, our assumption $\zeta'(g) = -\psi'(g) < 0$ holds with the definition of $\zeta(\cdot)$ function. With these two terms, we put corporate governance at country and firm level into the same perspective.

Substitute the definition of capital distortion from equation (23) into the security price, we now express the security price at time t from equation (22) in the term of investor's expected profit, agent's continuation value, country-level discount and firm's governance premium, respectively as follow,

$$S_t = F(W_t, K_t) + \frac{W_t}{\lambda_t} - \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}\Omega K_s ds\right] + \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(\psi(g))K_s ds\right]. \qquad (24)$$

The security price hence composes of four terms. The first two terms are the standard result; investor's profit and executive compensation. The third term on the right-hand side reflects the agency cost at country level due to the inefficient law and legal enforcement for investor protection. This is the *country's discount term.* The fourth term represents the effect of internal governance of the firm. It captures the amount of capital saved from agency rent due to internal governance. We call this term *governance premium* of the security price. Other things equal, the security price in the country with weak legal investor protection, large Ω, is lower than the country with the stronger one. Within a country, having identical legal infrastructure, a firm with higher internal governance level, high g and $\psi(g)$, has higher security price than the firm with lower level, *ceteris paribus.*

We summarize the implementation of the optimal contract and the security price in the following proposition.

Proposition 6 (**Security Price and Governance Premium**). *Suppose that the firm is financed solely by equity. The firm must pay out the required dividend according to equation (19) and maintain the financial flexibility according to equation (20) to be greater than zero in order to continue the operation. The optimal contract shown in preposition (5) is implemented under the required dividend and financial flexibility processes. The security price of the firm under the implemented optimal contract composes of four parts, as shown in equation (24). It incorporates the country's discount term from imperfect investor protection due to the country's legal infrastructure. The higher the discount term is, the lower security price would be. The security price also includes the governance premium derived from internal governance mechanism, as an agreement between investors and the executive, of the firm. The higher internal governance level is, the larger governance premium and, consequently, the higher the security price would be.*

From the proposition 6, there are three important implications. We discuss them here. Firstly, the value of governance premium depends on the *time distance from termination* $(\tau - t)$. This capture the dynamic aspect of governance premium of the firm. It implies that, on the one hand, the governance premium is large when the firm is in the stage of the growth or the mature stage, when $(\tau - t)$ is large. On the other hand, it is small when the firm is close to termination, financial distress or reorganization due to the expected short time to the termination. This is consistent with the fact that when a firm is closed to financial distress, its equity price in the financial market takes into account only the liquidation value and executive compensation, the first and second elements of the security price in equation (24) respectively.

Secondly, from $\zeta(g) = \Omega - \psi(g)$, we can use the same framework to capture the difference of investor protection at the country level by varying the constant term Ω. For example, for a country j with weak legal investor protection (law level governance), we denote the possible distortion of the capital within the firm as Ω_j and for a country i with stronger investor protection from the law and legal enforcement, the constant is $\Omega_i < \Omega_j$. By varying the value of the constant term, we can consider both effect of governance mechanism for both country and firm levels within the same model. This provides the consistent empirical framework to consider the effect of law and firm's governance level on security prices.

Finally, from $\zeta(g)$ and $\psi(g)$, we have not assumed the shape and curvature of the function. Only requirement is $\zeta'(g) < 0$, or $\psi'(g) > 0$. The shape of the $\psi(g)$ could be a subject to empirical investigation and the legal strategy to enhance the internal governance level. Define $\psi(\cdot) : G \rightarrow [0, \Omega]$, with $\psi'(g) > 0$, the function could have the curvature as linear, concave, convex or quasi-concave properties.The characteristics of the curvature raise an important question *What matter in internal governance?* We employ the investor protection approach in corporate governance, which bases heavily on the rules and agreements between principal and agent to enhance the requirement on investors' vote and requirement on investors' approval on important corporate decisions and policies. The rules and requirements would be the mechanism that protects investors from agency conflict and reduce agency rent. The extent of reduction in agency rent from the mechanism would be subject to empirical investigation.

We next consider the effects of a change in governance in the optimal contract, including profit, continuation value, and optimal decisions. Considering the internal governance change gives us an insight of the shifting between instantaneous and intertemporal incentive structure of the contract. This insight has important implication on how much a government or a regulator can rely on internal governance being determined within a firm.

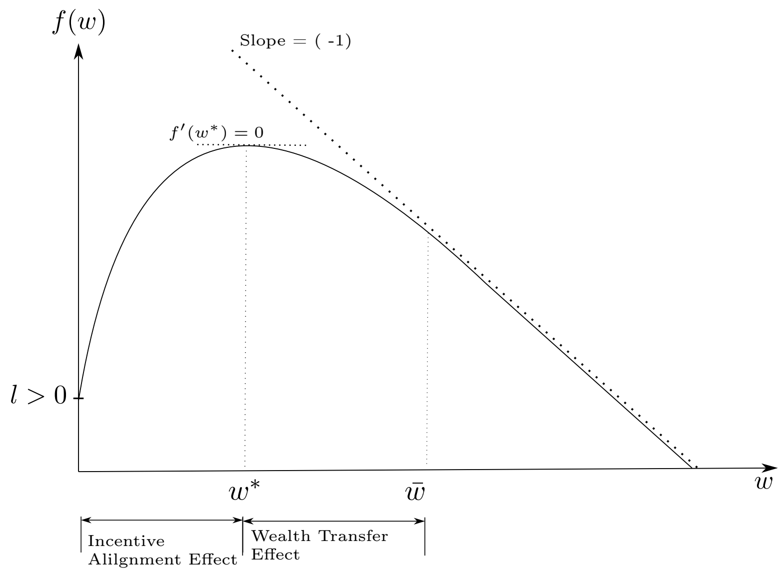

Figure 1: The shape of investor's profit function

5 Change in Governance and Firm Dynamics

This section consider the consequence of a change in internal governance level on firm dynamics. We study how an increase in internal governance mechanism affects the dynamic relationship between principal's profit and agent's continuation value. This relationship can be illustrated as a graph of $(w_t, f(w_t))$ over the interval defined by lower and upper boundaries, $(0, \bar{w})$. As explained in DFHW (2010), the graph of optimal contract is determined by incentive alignment and wealth transfer effects. In the comparative static analysis, we use the method proposed by DeMarzo and Sannikov (2006) to show that an increase in the internal governance enhances the incentive alignment effect and hence accelerates the wealth transfer from the principal's profit to agent's continuation value. This is because the the rise of internal governance shifts the weight of the incentive structure from the instantaneous to the intertemporal one. We then discuss the implications of the change in internal governance and firm dynamics on executive compensation, investors' profit and the role of legal infrastructure on investor protection for different industries.

Before studying the comparative static analysis, we now illustrate the graph of the investor's profit function under optimal contract as described in proposition 5 in figure (1). The principal's profit function is (strictly) concave over $w_t \in (0, \bar{w})$. Similar to DeMarzo and Sannikov (2006) and Sannikov (2008), the curvature of $f(w_t)$ has two parts. The first part has positive slope for $w_t \in (0, w^*)$ and the second part negative slope over $w_t \in (w^*, \bar{w})$. The value w^* pinpoint the change of curvature of the profit function and denote its highest value, $f'(w^*) = 0$. The concavity of the profit function is the result of two effects. The first is the *incentive alignment effect*. Under the optimal contract, an increase in w_t motivate the agent to create total surplus available for distribution between the principal and agent. This effect dominates over $w_t \in (0, w^*)$. An increase in agent's benefit in this range induces higher principal's profit. This effect accounts for the positive slope for the profit function. The second is the *wealth transfer effect* implying that the higher agent's benefit would decrease the investor's profit given the total surplus unchanged. This result bases on the Neoclassical benchmark in investment. This effect results in the negative slope of profit function over $w_t \in (w^*, \bar{w})$.

We now consider an increase in the internal governance level and its effects of optimal contract. The analysis of a decrease in internal governance is similar and hence skipped here. To focus on the effect of internal governance, the ODE of principal's profit function and dynamic continuation value under optimal contract can be rewritten respectively as follows,

$$(r+\delta)f(w_t) = a_t - h(a_t;g) - \zeta(g) + \frac{(q_t-1)^2}{2\theta} + ((\gamma+\delta)w_t - \zeta(g))\, f'(w_t) + \frac{1}{2}\lambda_t^2 \sigma f''(w_t) \quad (25)$$

$$dW_t = \gamma W_t dt - \zeta(g) K_t dt + \lambda_t \sigma K_t dZ_t. \quad (26)$$

where $\lambda_t = \frac{\partial H(a_t;g)}{\partial a_t}$. The change in governance does not affect the boundary conditions. Both lower and upper boundary conditions hold the same under the circumstance of increased internal governance. We now consider its effect on the graph of dynamic relationship within the firm.

From the setting, an increase in internal governance (g) would reduce the rate of capital distortion $(\zeta(g))$ and raise the marginal cost of effect $\left(\frac{\partial H(a_t;g)}{\partial a_t}\right)$. Under the optimal contract, the volatility part of the continuation value dynamic, equation (26), captures the intertemporal motivation that aligns the benefits of the principal and the agent. However it reduces the potential benefit of private benefit of control. Under the revelation principal and instantaneous constraints, the second term of the right-hand side of equation (26) represents the benefit derived from instantaneous incentive for not taking agency rent. As a result, an increase in internal governance level, as an agreement between the principal and the agent, is in fact a shift the weight in agent's benefit from instantaneous to intertemporal incentive.

In the principal's perspective, an increase in internal governance has many consequences on the profit function. To focus on the dynamic aspect of the results, we neutralize the effect of of governance change on executive compensation. We assume that he level effect of principal's profit is unchanged due to an offset of $\frac{\partial H(a_t;g)}{\partial g} > 0$ and $\frac{d\zeta(g)}{dg} < 0$. We then consider the consequence of increasing internal governance on the first-order and second-order effects of the profit function. From equation (25), both first-order and second-order effects are more pronounced due to lower rate of capital distortion and higher marginal effort cost. Given the same level of scaled continuation value, the investor would reach the highest profit at the lower level of the continuation value; for $g_2 > g_1, w^*(g_2) < w^*(g_1)$. Intuitively, an increase in internal governance intensifies the incentive alignment effect. At the low value of continuation value $w_t \in (0, w^*)$, an agreement to strengthen governance mechanism helps investors to achieve the highest profit sooner and less costly in term of the agent's continuation value.

However, after reaching the highest profit, an increase in internal governance also intensifies the wealth transfer effect due to the the higher intertemporal incentive. At the high level of continuation value, $w_t \in (w^*, \bar{w})$, the profit is lower when the internal governance is raised. The effect of increase in internal governance in the dynamic relationship between principal and agent is illustrated in figure 2.

From our analysis, we consider the time for contract renegotiation on internal governance mechanism that will yield benefit to the investors. Because enhanced internal governance is equivalent to reallocation of the weight from instantaneous to intertemporal incentive. The principal can gain benefit from the reallocation only when the incentive align effect persists. We state that the enhancement of internal governance is beneficial to the principal only at the low level of continuation value. Otherwise, the principal's profit is decreased from higher level of internal governance under the optimal contract. The proof of the following claim is provided in the appendix.

$$\frac{\partial f(w)}{\partial g} \begin{cases} > 0; & \text{if } w \in (0, w^*) \\ < 0; & \text{if } w \in (w^*, \bar{w}) \end{cases} \tag{27}$$

In addition, an increase in internal governance also has influence on initial continuation

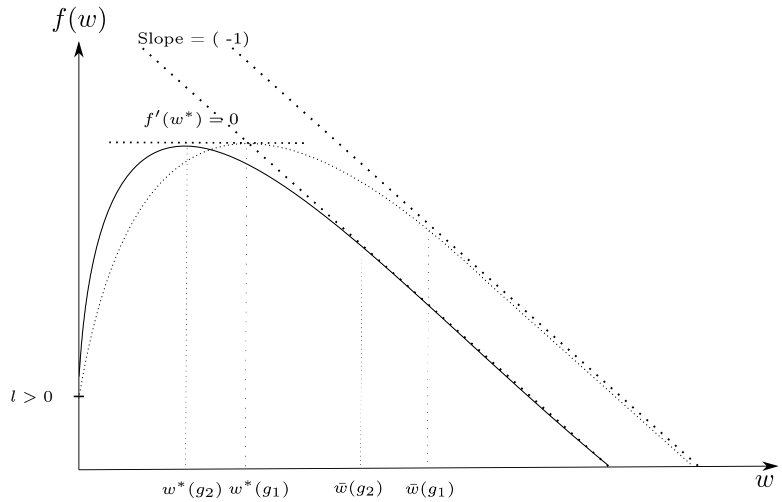

Figure 2: The shape of investor's profit function with increased level of internal governance keeping the effect on level constant; $g_2 > g_1$.

value (w_0). The initial continuation value is the level of continuation value in which the agent expects to have at the beginning of the contract. The higher internal governance level would lower the initial continuation value. This is because, with higher internal governance, the agent will be better compensated for his marginal cost of effort along the dynamic relationship. Given his discounting rate unaltered, higher compensation along the dynamics would yield a higher expected benefit. Other thing equal, the agent would be willing to enter into the contract with lower initial continuation value with the higher incoming benefit along the continuation of the firm.

We conclude the results on the effects of changes in internal governance on the firm dynamic relationship in the proposition 7. We discuss the theoretical implications and the effects on executive compensation, investor's profit after the proposition. The detailed proofs are provided in the appendix.

Proposition 7 (**Change in Internal Governance and Firm Dynamics**). *An increase in internal governance level, as an agreement between the principal and the agent, has an effect of shifting the weight in agent's benefit from instantaneous to intertemporal incentive. The enhance governance level intensifies the incentive of the agent to create value of the firm, as a consequence investor's profit reaches the maximum at lower agent's continuation value,* $\frac{\partial w^*}{\partial g} < 0$. *Moreover, the initial continuation value will be lower with higher governance level,*

$\frac{\partial w_0}{\partial g} < 0$. *The investors will benefit from better governance only when firm is in the initial stage, when incentive alignment effect persists* ($w \in (0, w^*)$)*, and it is better off to lower the governance requirement when firm is in the mature stage, when incentive alignment effect disappears* ($w \in (w^*, \bar{w})$)*, as shown in equation (27).*

Theoretical Implication

From the perspective of contract theory, an increase in internal governance alleviates the contract incompleteness by reducing the authority of the agent to exercise his discretionary power on the important issues concerning investors. With higher level of internal governance, investors have broader scope of issues that require vote and approval before its implementation. Under the optimal contract, it makes the contract about managerial decision more complete and change the nature of problem from contractual incompleteness into hidden-action problem which can be solved with optimal dynamic contract as described in proposition 5. An increase in internal governance beyond legal infrastructure in essence improves the contractual circumstance as required by requirement of the law.

We can consider the enhancement of internal governance as an exchange. When two parties agree on higher level of internal governance beyond the legal requirement, the principal would have lower authority in managerial decisions because he needs to ask for principal's approval before the implementation. He has lower ability to gain potential private benefit of control from sacrificed authority, lower level of $\zeta(g)$. In return, the principal would pay for the additional power over the extent of managerial decision by compensating the agent through higher marginal cost of effort, higher level of $\frac{\partial h(a_t;g)}{\partial g}$. The agreement on an increase in internal governance is in fact the exchange of authority on managerial decisions of the firm with marginal benefit $\zeta'(g)$ and marginal cost $\frac{\partial^2 h(a_t;g)}{\partial g^2}$ over the longevity of the firm.

Implication on Investor's Profit and Executive Compensation

We can see from the ODE of profit function that the agency conflict decreases the current level and future increase of the profit. Intuitively, this is because the principal must compensate at any period for agency rent to cope with instantaneous incentive and this rent derives from existing capital to be used to generate future cashflow. As a result, by lowering the agency rent, the governance mechanism can enhance both current and future profit of the firm, shown in proposition 5. The cost of increased governance level is the higher marginal effort cost, which in fact is the intertemporal incentive for the agent. This cost also grows with the size of the firm according to dynamic continuation value. Consequently, the principal must balance the benefit and cost of governance at any period over the longevity of the firm.

However, we know that the enhancement of governance mechanism is beneficial to investors as long as the incentive alignment effect persists. This effect help increases the growth of the firm and boost the profit along the growth path. This effect occurs when the firm has high potential of the future profit. This normally happens in the initial stage of the firm or the recovery phrase after reorganization. It then suggests that the determination, or increase, in governance mechanism is beneficial to investors at the initial stage of the firm, in which the investors expect the growth of the firm in coming future.

For the agent's compensation, under the optimal contract, the compensation composes of two parts, effort cost and agency rent. The effort cost depends on the past performance through the history-dependent continuation value, while the agency rent does not. This structure shows two aspects of the agent's motivation. Firstly, the effort cost reflects the intertemporal incentive and secondly the agency rent exhibits the instantaneous one. The change in governance mechanism, as an agreement between principal and agent, reallocates the proportion of these incentives.

An enhancement of governance put more weight on the intertemporal incentive and make the compensation subject to performance. It also reduce the agency rent, the instantaneous incentive, at the same time. The shift of incentive is beneficial to the principal only when he expect the future contribution to the growth of the firm as a result of agent's effort. This idea underlines the importance of incentive alignment effect. Otherwise, if the firm is mature and enter into the declining stage with high burden of executive compensation due to previous growth and success, the principal could be better off to decrease the governance level. It then reallocates the proportion of compensation back from intertemporal to instantaneous components. The firm can save high compensation from the executive's motivation and tolerate smaller agency rent. In simple words, for the mature firm to decrease internal governance level, it is better to allow perks than to pay for lucks. An increase in governance level at the initial stage and a decrease in the later stage of the firm is consistent with the observed phenomena in financial markets in the sense that firms enhance governance level at the beginning and relax it in when it is mature and expected low contribution from executives for future benefit.

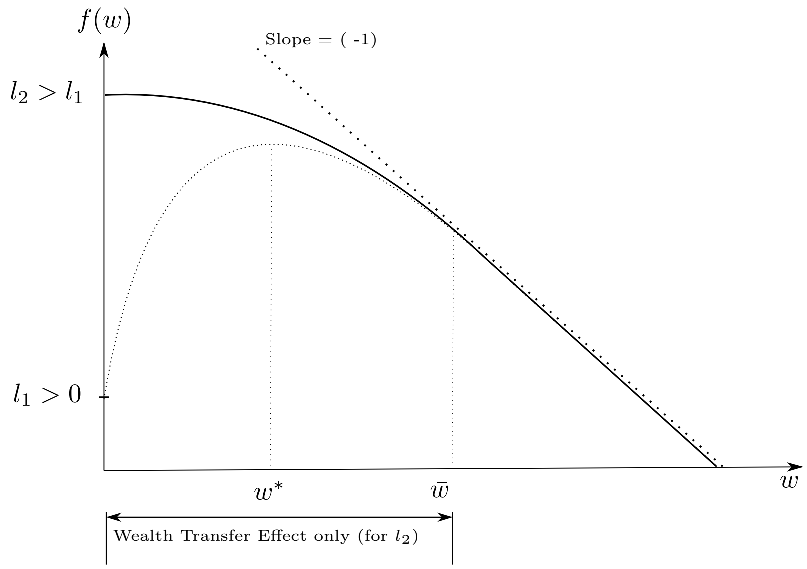

Figure 3: The shape of investor's profit functions with high and low liquidation rate ($l_2 > l_1$)

Implication for Governmental Intervention on Firm's Governance

The investors would benefit from increase in internal governance beyond the law requirement only when the incentive alignment effect persists. Without governmental intervention, we can expect an enhancement of governance level beyond the legal requirement only when there is mutual benefit between the principal and agent. Otherwise, it is possible that firm's governance will be lowered down to the legal requirement.

To consider the governmental intervention on the governance level of a firm, we must consider the private motivation to improve the internal governance whether the firm's relevant parties have an incentive to improve the governance level beyond the law. If the contractual parties have private interest, the government or regulator shall not intervene on firm's governance level because they would agree on optimal level of internal governance of the firm. However, in some kinds of firms or industries, the existing legal governance level is inadequate for investor protection and there is no mutually agreeable possibility for an improvement in firm's governance . We now discuss the nature of business or industries that have this problem.

Under optimal contract, a firm with high liquidation rate will have small incentive alignment effect and this effect disappears when the liquidation rate is high enough. The figure 3 shows the profit functions of two different firms with low (l_1) and high (l_2) liquidation rates, similar to DeMarzo and Sannikov (2006), Sannikov (2008) and DFHW (2010). The firm with high liquidation does not have the incentive alignment effect. The principal, or investors of this firm, will not have benefit from an increase in internal governance. The higher internal governance would deteriorate principal's profit along the firm dynamics, as shown in the figure 4. This is the business that would call for governmental intervention in order to strengthen investor protection of the firm. The high liquidation rate of the firm provides a rationale of industry regulations on firm's governance mechanism by the government.

The high rate of liquidation means that the market value of capital is as high as its value under firm's operation and vice versa[17]. For example, the value of money in the financial market is as same as the value of money capital for the money management business, such as mutual fund. The money management industry would have a high liquidation rate and call for addtional governmental intervention if the existing governance mechanism is inadequate to protect investors. On the contrary, high technological firms have low liquidation rate. The market value of capital used by hi-tech firm is much lower than its value under firm's operation. Consequently, there is considerable possibility for mutual agreement between investors and the executive to improve the governance mechanism of the firm. To conclude, as long as the optimal contract is in use, distinct businesses would require different degree of governmental intervention on firm's governance mechanism.

6 Conclusions

This paper studies the role of internal governance in the context of dynamic relationship between investors and executives, as principal and agent respectively, with continuous-time dynamic agency model. Both parties agree on the level of internal governance before entering into the contract. The optimal dynamic contract is designed based on the agreed level of internal governance in order to cope with the unobservable efforts and decision to expropriate the asset of the firm as the agent's private benefit of control. We show how the internal governance affects the investor's profit, executive compensation and incentives structure of the agent.

For an implementation of the optimal contract, we shows that the security price composes of principal's profit, executive compensation, country's discounting term and firm's governance. The first two elements are standard results, while the last two elements are

[17]The high liquidation rate also implies the low contribution of the agent in the production and profit.

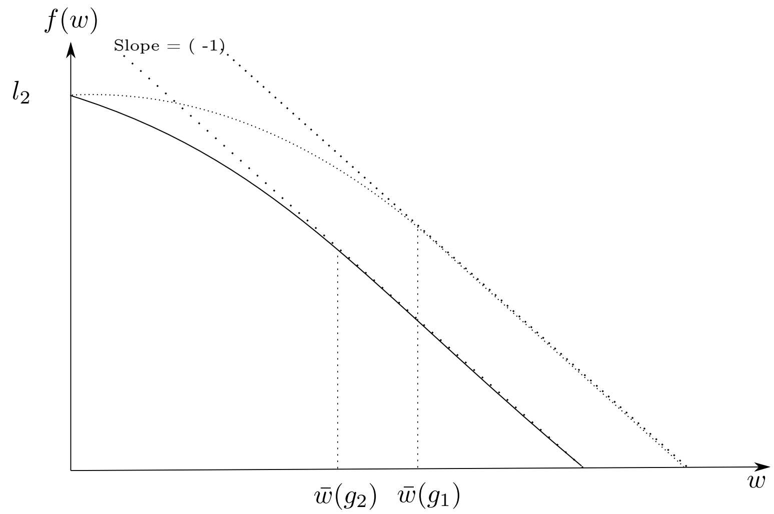

Figure 4: The shape of investor's profit function without incentive alignment effect and higher governance $g_2 > g_1$

new. The model in this paper considers the effect of governance mechanism at both country and firm levels in the security price within a single framework. These two elements also have a dynamic aspect on the security price and valuation of the governance mechanism.

In contrast to most studies in which the firm's governance mechanism is a result of optimal decision of the executive, this paper uses the concept of internal governance mechanism as an agreement on the scope of authority that the agent can have discretion over managerial decisions. The internal governance defined in the model is a set of rules and agreements beyond the laws and legal requirement of a country. With this definition, we can distinguish the effect of corporate governance from country's law and additional mechanism agreed between parties of the firm. We consequently study the effect of law and firm's corporate governance simultaneously and consistently.

We incorporate the the role of internal governance as mechanism against expropriation. The expropriation is in the form of the agent's decision to distort the investment capital of the firm for his private benefit. The internal governance lowers the level of possible distortion. In our model, the agent makes unobservable decisions on effort and expropriation decision. The principal offers a dynamic contract that composes of the compensation, recommended investment and time to terminate the firm, or equivalently the relationship between the principal and the agent. Using martingale method on continuation value, the optimal contract depends on past performance, which is the realized cashflow, and future expected benefit. Even though the effect of internal governance against expropriation is static by its nature because the rules do not change overtime, in our model the optimal contract incorporate the effect of internal governance into the formulation of dynamic incentive. Consequently, the internal governance mechanism affects both instantaneous and intertemporal incentive of the agent and also dynamic behavior of principal's profit.

The key insight comes from the role of capital distorting function and agent's effort cost function. We assume that capital distortion function is a function of internal governance with negative slope. The better governance would lessen the amount of capital distortion. The cost of effort is increasing and convex in internal governance. Intuitively, given a level of effort, a higher level of internal governance puts more restriction on available actions of the agent and hence the cost is higher for the same level of effort. With revelation principle for static incentive and martingale method for dynamic incentive, the optimal contract binds both the capital distortion function and cost of agent's effort to principal's profit and agent's continuation value in dynamic context. Consequently, the static nature of internal governance insinuate into behavior of firm dynamics and security price.

An implementation of the optimal contract by using financial flexibility and assuming pure-equity capital structure shows four important elements of the security price. The first two elements are the standard result; investor's profit and executive compensation. The third element is the agency cost at country level due to the inefficient law and legal enforcement for investor protection. We call it *country's discount term*. The fourth element represents the effect of internal governance of the firm. It captures the amount of capital saved from agency rent due to internal governance. We denote this term the *governance premium* of the security price. We show that the value of governance premium depends on the *time distance from termination*. This capture the dynamic aspect of governance premium of the firm. It implies that, on the one hand, the governance premium is large when the firm is in the stage of the growth or the mature stage. On the other hand, it is small when the firm is close to termination, financial distress or reorganization due to the expected short time to the termination. This is consistent with the fact that when a firm is closed to financial distress, its equity price in the financial market takes into account only the liquidation value and executive compensation.

The analysis of change in internal governance level shows that an increase in governance put more weight on the intertemporal incentive, due to higher marginal cost of agent's effort, and make the compensation subject to performance. It also reduce the agency rent, the instantaneous incentive, at the same time. The shift of incentive is beneficial to the principal only when he expect the future contribution to the growth of the firm as a result of agent's effort. However, if the firm is mature and enter into the declining stage with high burden of executive compensation due to previous growth and success, the principal could be better off to decrease the governance level. It then reallocates the proportion of compensation back from intertemporal to instantaneous components. The firm can save high compensation from the executive's motivation and tolerate smaller agency rent. In simple words, for the mature firm to decrease internal governance level, it is better to allow perks than to pay for lucks. An increase in governance level at the initial stage and a decrease in the later stage of the firm is consistent with the observed phenomena in financial markets in the sense that firms enhance governance level at the beginning and relax it in when it is mature and expected low contribution from executives for future benefit.

The result of our analysis give a systematic view of the effects of corporate governance to various aspects of the firm and also its dynamic context. The insights from our model and the analysis contribute to the literature on corporate finance and dynamic agency problem. For corporate finance and governance literature, this paper is among a few that study the corporate governance in the dynamic context. Using continuous-time dynamic agency model broadens the analysis of firm's governance on the entire relationship between the investor and the executive. It merges the analysis of dynamic agency with the asset valuation model into a single framework. We can see how governance mechanism affects agency problem in parts of the firm and how firm's value change accordingly. By considering governance mechanism in dynamic context, we can also explore effects of the corporate governance on firm's profit and executive compensation at the different stages of the firm. Further, we can evaluate the dynamic aspect of the governance valuation in security price also. These two issues can not be studied systematically in a static framework. Furthermore, the model provides a method to evaluate the dynamic price of corporate governance from both the country's law and firm's internal governance in a single framework.

For the continuous-time dynamic agency literature, this paper breaks the direct link between agent's effort decision and the private benefit of control. This disengagement gives modeling flexibility in dynamic agency problem. The agency rent needs not to be derived from agent effort only. Furthermore, the model of this paper is the first to incorporate the instantaneous incentive constraint with the dynamic incentive structure. This methodology can be used for the model involving the static incentive constraint and the revelation principle, such as laws, government regulation and taxation.

The result of our analysis also gives rational answer to the empirical conundrum about the effect of governance mechanism on security price. The conundrum derives from the inconsistency between investor's valuation of governance and proportion of governance in security price. When the investors positively value the firm's governance, the security price is supposed to increase when the governance mechanism improves. However, recent studies of Core, Guay and Rusticus (2006), Johnson, Moorman and Sorescu (2009) and Price, Roman and Rountree (2011) shows no effect of an improvement of governance mechanism in equity price. From our analysis, we show that an enhancement of firm's governance will rise the investor's profit and security price only when the change occurs at the initial stage. When the firm reaches the mature stage, an increase in governance mechanism will be very costly to the investor in order to motivate the executive's effort at this stage. Hence an improvement of internal governance reduces the investor's profit, hence security price. This insight suggests an inclusion of firm's growth stage as an explanatory variable in the empirical study of the effect of corporate governance on security price.

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Appendix

A Proofs of Propositions

Proof of Proposition 1. To recover the dynamic continuation value, we write the value of total expected benefit given information at time t,

$$V_t = \mathbb{E}^a \left[\int_0^t e^{-\gamma s}(U_s^g - H(a_s; g))dt + e^{-\gamma t} W_t \right]$$

Differentiation with respect to time gives us,

$$dV_t = e^{-\gamma t}(U_t^g - H(a_t; g))dt + d(e^{-\gamma t} W_t)$$

or, equivalently

$$dV_t = e^{-\gamma t} \left[(U_t^g - H(a_t; g))dt + dW_t - \gamma W_t dt \right] \tag{A.1}$$

where $d(e^{-\gamma t} W_t) = e^{-\gamma t} dW_t - \gamma W_t e^{-\gamma t} dt$.

Similarly, we can write the value of total expected benefit in another form. By martingale representation theorem, similar to Sannikov (2008), there exists λ_t that makes the total expected benefit a martingale.

$$V_t = V_0 + \int_0^t e^{-\gamma s} \lambda_t \left(dY_t - K_t(a_t - L(i_t))dt \right)$$

in which $\sigma K_t dZ_t = dY_t - K_t(a_t - L(i_t))dt$ [18]

By taking differentiation, we write another form of the dynamics of agent's total value.

$$dV_t = e^{-\gamma t} \lambda_t \sigma K_t dZ_t \tag{A.2}$$

From the equality of (A.1) and (A.2),

$$e^{-\gamma t} \left[(U_t^g - H(a_t; g))dt + dW_t - \gamma W_t dt \right] = e^{-\gamma t} \lambda_t \sigma K_t dZ_t,$$

we write the dynamics of continuation value as follow,

[18] We have this relationship from the cashflow and the productivity processes. From $dY_t = K_t(dA_t - L(i_t)dt)$ and $dA_t = a_t dt + \sigma dZ_t$, we write $dY_t = K_t(a_t - L(i_t))dt + \sigma K_t dZ_t$.

$$dW_t = \gamma W_t dt - (U_t^g - H(a_t; g))dt + \lambda_t \sigma K_t dZ_t. \tag{A.3}$$

This is the differential form of continuation value dynamics. We can show the integral form this stochastic differential equation by integrating the differential form with respect to time (t) given its initial value (W_0). The result is summarized in the proposition 1. □

Proof of Proposition 2. Given the dynamic continuation value, we need to ensure that the decisions are incentive compatible in the sense that $\{b_t = 0; \forall t \in [0, \tau]\}$ and a_t maximizes the profit. We consider the deviation from the optimal decision during the time 0 and t. Suppose that the agent deviates from optimal decisions and chooses $\{a'_t \neq a_t, b_t = 1\}$ from time $0 \to t$ and chooses the optimal decisions $\{a_t, b_t = 0\}$ from time $t \to \tau$. The total benefit of the agent is following.

$$V_t = \int_0^t e^{-\gamma s}(U_s^g - H(a'_s; g) + \zeta(g)K_s)ds + e^{-\gamma t}W_t$$

We consider the value in the deviation period. We show that the deviation from the optimal decisions would never be positive. The agent does not have incentive to deviate from optimal decisions under the dynamic continuation value described in proposition 1. Taking derivative on V_t gives us,

$$dV_t = e^{-\gamma t}(U_t^g - H(a'_t; g) + \zeta(g)K_t)dt - e^{-\gamma t}(U_t^g - H(a_t; g))dt + e^{-\gamma t}\lambda_t \sigma K_t dZ_t$$

where $\left(-e^{-\gamma t}(U_t^g - H(a_t; g))dt + e^{-\gamma t}\lambda_t \sigma K_t dZ_t\right) = d(e^{-\gamma t}W_t)$, using result from proposition 1.

From $\sigma K_t Z_t(a) = \sigma K_t Z_t(a'_t) + \int_0^t ((a'_s - a_s)dt + b_s \zeta(g)K_s dt)$, we write the diffusion term as $\sigma K_t dZ_t(a) = \sigma K_t dZ_t(a') + (a'_t - a_t)dt + b_t \zeta(g)K_t dt$. We reformulate the dV_t as follow

$$\begin{aligned} dV_t = \quad & e^{-\gamma t}\left[(U_t^g - H(a'_t; g) + \zeta(g)K_t)dt - (U_t^g - H(a_t; g))dt\right] \\ & + e^{-\gamma t}\left[\lambda_t \sigma K_t dZ_t(a') + \lambda_t(a'_t - a_t)dt + \lambda_t b_t \zeta(g)K_t dt\right] \end{aligned}$$

and canceling terms gives

$$\begin{aligned} dV_t = \quad & e^{-\gamma t}\left[H(a_t;g) - H(a'_t;g) + \lambda_t(a'_t - a_t) + (1+\lambda_t)b_t\zeta(g)K_t\right]dt \\ & + e^{-\gamma t}\lambda_t\sigma K_t dZ_t(a'). \end{aligned} \tag{A.4}$$

To consider the incentive compatible decisions $\{a_t, b_t\}$, we consider the case that expectation of the deviation would yield non-positive return, $\mathbb{E}(dV_t) \leqslant 0$. We then consider the drift term of equation (A.4).

For the effort choice (a_t), we require that

$$\big((\lambda_t a'_t - H(a'_t;g)) - (\lambda_t a_t - H(a_t;g))\big) \leqslant 0$$

or, equivalently,

$$(\lambda_t a_t - H(a_t;g)) \geqslant (\lambda_t a'_t - H(a'_t;g)), \quad \forall a'_t \neq a_t. \tag{A.5}$$

For the decision on capital diversion (b_t), we require that

$$b_t(1+\lambda_t)\zeta(g)K_t \leqslant 0. \tag{A.6}$$

The requirement holds only when $b_t = 0, \forall t \in [0,\tau]$.

Note that from the equation (A.5), λ_t is not unique and can take different processes. Because the λ_t determines the agent's incentive through the volatility term of the continuation value dynamics, it is costly to the principal for high value of λ_t. In addition to the incentive compatible decisions, we also require that

$$\lambda_t = min\{\tilde{\lambda_t} \in [0,\infty) : a_t \in argmax_{\{\tilde{a}\in[0,\infty)\}}\{\lambda\tilde{a}_t - H(\tilde{a}_t;g)\}\} \tag{A.7}$$

□

Proof of Proposition 3. Defining the scaled continuation value denoted by w_t, the scaled profit function satisfies the following condition[19]

$$f(w_t) = F(1,w_t) = \frac{1}{K_t}F(K_t,W_t).$$

[19] From homogeneity degree one in profit function, $F(\beta K_t, \beta W_t) = \beta F(K_t,W_t)$ in which $\beta = \frac{1}{K_t}$

Similarly, we scale down the dynamics of continuation value by the application of the Ito's lemma to dW_t, from proposition 1, and capital dynamics (dK_t). The derivation of the dynamics of scaled continuation value is following.

$$
\begin{aligned}
dw_t &= d(\frac{W_t}{K_t}) = d(W_t \cdot K_t^{-1}) \\
&= W_t dK_t^{-1} + K_t^{-1} dW_t + dW_t \cdot dK_t^{-1}
\end{aligned}
$$

Under incentive compatible decisions, the capital dynamics is $dK_t = (I_t - \delta K_t)dt$, then we have $dK_t^{-1} = -K_t^{-1}(i_t - \delta)dt$ where $i_t = \frac{I_t}{K_t}$. Let $\frac{U_t^g}{K_t} = u_t^g$ and $\frac{H(a_t;g)}{K_t} = h(a_t;g)$. We write the dynamic scaled continuation value as follow.

$$
\begin{aligned}
dw_t &= -\frac{W_t}{K_t}(i_t - \delta)dt + \frac{1}{K_t}\left(\gamma W_t - (U_t^g - H(a_t;g))\right)dt + \frac{1}{K_t}\lambda_t \sigma K_t dZ_t \\
&= (\delta - i_t)w_t dt + (\gamma w_t - (u_t^g - h(a_t;g)))dt + \lambda_t \sigma dZ_t \\
&= \left((\gamma + \delta - i_t)w_t - (u_t^g - h(a_t;g))\right)dt + \lambda_t \sigma dZ_t
\end{aligned} \tag{A.8}
$$

We now consider the optimal decisions of the principal using HJB equation of the scaled profit function. We use Ito lemma to transform profit function into the scaled profit function; $F(K_t, W_t) = K_t f(w_t)$.

$$
dF(K_t, W_t) = d(K_t f(w_t)) = K_t d(f(w_t) + f(w_t)dK_t
$$

where $dK_t = (I_t - \delta K_t)dt$ in which $b_t = 0$. From $df(w_t) = f'(w_t)dw_t + \frac{1}{2}f''(w_t)(dw_t)^2$, we have

$$
\begin{aligned}
df(w_t) &= \left((\gamma + \delta - i_t)w_t - (u_t^g - h(a_t;g))\right)f'(w_t)dt + f'(w_t)\lambda_t \sigma dZ_t \\
&\quad + \frac{1}{2}f''(w_t)\lambda^2\sigma^2 dt \\
&= \left((\gamma + \delta - i_t)w_t - (u_t^g - h(a_t;g))f'(w_t) + \frac{1}{2}f''(w_t)\lambda_t^2\sigma^2\right)dt + f'(w_t)\lambda_t \sigma dZ_t
\end{aligned}
$$

We formulate the HJB equation of the profit function, $F(K_t, W_t)$. The instantaneous return of the principal is $K_t(a_t - L(i_t) - u_t^g)dt$ [20], where $u_t^g = \frac{U_t^g}{K_t}$,

$$\begin{aligned} rF(K_t, W_t) &= \sup_{\{i_t, u_t^g\}} K_t(a_t - L(i_t) - u_t^g) + \mathbb{E}[K_t df(w_t) + f(w_t)dK_t] \\ &= \sup_{\{i_t, u_t^g\}} K_t(a_t - L(i_t) - u_t^g) + K_t \mathbb{E}[df(w_t) + f(w_t)(i_t - \delta)dt] \end{aligned}$$

Dividing through by K_t and substitute $f(w_t)$ from above, we have HJB equation in the form of scaled profit function as follow.

$$\begin{aligned} rf(w_t) = \sup_{\{i_t, u_t^g; t\in[0,\tau]\}} &\{a_t - u_t^g - L(i_t) + \left((\gamma - (i_t - \delta))w_t - (u_t^g - h(a_t; g))\right) f'(w_t) \\ &+ \frac{1}{2} f''(w_t)\lambda_t^2\sigma^2 + f(w_t)(i_t - \delta)\} \end{aligned} \tag{A.9}$$

We now investigate the optimal decisions of the principal. From the HJB equation (A.9), the necessary condition for optimal investment reads,

$$f(w_t) - w_t f'(w_t) = L'(i_t). \tag{A.10}$$

This is the Euler equation for investment per capital. The necessary condition coincides with the classical marginal-q theory saying that the optimal investment equalizes the marginal-q and the marginal cost of capital adjustment. We define the marginal-q as the derivative of total value of the firm with respect to capital, $q_t = \frac{\partial(F(K_t,W_t)+W_t)}{\partial K_t}$ where $F(K_t, W_t) = K_t f(w_t)$. Then $\frac{\partial F(K_t,W_t)}{\partial K_t} = q_t = -w_t f'(w_t) + f(w_t)$. Hence $q_t = f(w_t) - w_t f'(w_t)$. Substitute the marginal cost of capital adjustment, $L'(i_t) = 1 + \theta i_t$, the necessary condition requires that

$$f(w_t) - w_t f'(w_t) = 1 + \theta i_t,$$

or, equivalently, in term of optimal investment

$$i_t^* = \frac{q_t - 1}{\theta} = \left(\frac{f(w_t) - w_t f'(w_t) - 1}{\theta}\right). \tag{A.11}$$

[20] This term is from the drift of $dY_t - U_t^g dt = K(a_t - L(i_t) - u_t^g)dt + K_t\sigma dZ_t$

The optimal investment at any period is determined by usual necessary condition, the equality of marginal benefit and cost. The agency conflict deteriorate the level of scaled profit $f(w_t)$ through $u_t^g > 0$ and $u_t^g - h(a_t, g) > 0$ as shown in equation (A.9) and hence the marginal-q, the marginal benefit of investment. The effect of agency conflict slow down the investment dynamics. With corporate governance to limit the extent of agency conflict, it consequently increases the optimal level of investment by increasing profit at any instance.

For the optimal compensation process, we know that the first-order condition from the HJB equation (13) gives us the corner solution, $f'(w_t) = -1$. There is no optimal dynamic link of the compensation process. This result is intuitive. We know that the compensation process is costly to the principal and would be decreased as low as possible at any time. It is hence determined by the instantaneous constraints, rather than the dynamic optimality condition. In other words, the optimal compensation process derives from the instantaneous participation and incentive compatibility constraint at all periods[21].The optimal condition for the compensation process consequently is derived from the optimal effort, individual rationality and incentive compatibility constraints. From equation (8), we see that compensation is costly to the the principal and hence the optimal compensation process is the result from the equalities of the instantaneous constraints given the optimal effort level. The optimal compensation is derived from the equalities of equation (6) and (5).

We substitute the optimal investment into HJB equation (A.9). With a rearrangement, we have

$$
\begin{aligned}
(r+\delta)f(w_t) =& \{a_t - u_t^g + \big((\gamma+\delta)\big)w_t - (u_t^g - h(a_t;g))\big)\, f'(w_t) + \frac{1}{2}\lambda_t^2\sigma^2 f''(w_t) \\
& - L(i_t^*) - i_t^* w_t f'(w_t) + i_t^* f(w_t)\}
\end{aligned}
$$

With explicit functional form of cost of capital adjustment, we rewrite the terms related to optimal investment. Suppressing asterisk, we write $-L(i_t) - i_t w_t f'(w_t) + i_t f(w_t) = -i_t - \frac{\theta}{2} i_t^2 + i_t(f(w_t) - w_t f(w_t))$, and from $f(w_t) - w_t f(w_t) = q_t = (1+\theta i_t)$, we have

[21] This result is consistent with the previous works of DFHW (2010), which does not consider the compensation process explicitly. However their result states that the principal does not pay any compensation to the agent until the state variable reach the upper boundary, which is also the boundary condition as shown in this paper.

$$
\begin{aligned}
-L(i_t) + i_t q_t &= -i_t - \frac{\theta}{2} i_t^2 + i_t(1+\theta i_t) \\
&= \frac{\theta}{2} i_t^2 \quad = \frac{\theta}{2} \frac{(q_t-1)^2}{\theta^2} \\
&= \frac{(q_t-1)^2}{2\theta}
\end{aligned}
$$

□

Proof of Concavity of Profit Function in Proposition 4. With previous boundary conditions, we investigate its property through the second-order derivative over the interval $w_t \in (0, \bar{w})$. We differentiate the ODE of profit function, described in proposition 3, with respect to the scaled continuation value.

$$
\begin{aligned}
(r+\delta) f'(w_t) =& \frac{1}{2\theta} \frac{d\left(f(w_t) - w_t f'(w_t) - 1\right)^2}{dw_t} + (\gamma+\delta) \frac{d(w_t f'(w_t))}{dw_t} \\
& - (u_t^g - h(a_t; g)) \frac{d(f'(w_t))}{dw_t} + \frac{\lambda^2\sigma^2}{2} \frac{(df''(w_t))}{dw_t}
\end{aligned}
$$

Rearranging the terms gives us

$$
\begin{aligned}
(r+\delta) f'(w_t) =& \frac{1}{\theta} (f(w_t) - w_t f'(w_t) - 1)(f'(w_t) - w_t f''(w_t) - f'(w_t)) \\
& + (\gamma+\delta)(w_t f''(w_t) + f'(w_t)) - (u_t^g - h(a_t; g)) f''(w_t) + \frac{\lambda^2\sigma^2}{2} f'''(w_t) \\
=& \frac{1}{\theta} (f(w_t) - w_t f'(w_t) - 1)(-w_t f''(w_t)) \\
& + (\gamma+\delta)(w_t f''(w_t) + f'(w_t)) - (u_t^g - h(a_t; g)) f''(w_t) + \frac{\lambda^2\sigma^2}{2} f'''(w_t).
\end{aligned}
$$

We then evaluate the ODE at the upper boundary $\bar{w}$ and use boundary conditions, equation (17) and (18). We have the following result.

$$
\begin{aligned}
-(r+\delta) &= -(\gamma+\delta) + \frac{\lambda^2\sigma^2}{2} f'''(\bar{w}) \\
(\gamma - r) &= \frac{\lambda^2\sigma^2}{2} f'''(\bar{w})
\end{aligned}
$$

From the assumption $\gamma \geqslant r$, we consider the case of strict inequality here $\gamma > r$, without loss of generality. we conclude that $f'''(\bar{w})$ is positive. We consider the behavior of $f''(\bar{w})$ at the upper boundary. We know that $f''(\bar{w})$ is locally increasing around $\bar{w}$ due to $f'''(\bar{w}) > 0$ and the super contact condition guarantee that $f''(\bar{w}) = 0$, then we conclude that $f''(\bar{w}-\epsilon) < 0$ for $\epsilon > 0$. We can extend the value of ϵ over the interval $(0, \bar{w})$. We then claim that the profit function is concave on the range of interest[22].

□

Proof of Security Price and Governance Premium in Proposition 6. The financial slack is proportional to λ_t which is equal to marginal effort cost under optimal contract. So we can see that $\lambda_t = \frac{\partial h(a;g)}{\partial a}$. The financial flexibility depends on the past performance (w_t) and inversely varies to marginal effort cost. The financial flexibility can take many forms such as cash reserve or credit line. Basically it reflects the firm's asset to absorb the short-run fluctuation without termination or change of managerial control.

To implement the optimal contract through design of financial securities, we want to find the combination of financial assets that induce the optimal decisions in the same way as the optimal contract does. Hereafter, we interpret the financial flexibility as cash reserve of the firm that gains the return at risk-free rate (r). We then assign the dynamic of financial flexibility as follow.

$$dM_t = rM_t dt + dY_t - dD_t - dX_t \tag{A.12}$$

where, dX_t is the cashflow reserved for compensation to the agent, dY_t is the dynamics of incoming cashflow,

$$dY_t = K_t \left(a_t - L(i_t)\right) dt + \sigma K_t dZ_t,$$

dD_t is the dividend process dynamics that investor requires as periodic return to his investment,

$$dD_t = K_t \left(a_t - L(i_t)\right) dt - (\gamma - r) M_t dt. \tag{A.13}$$

[22]We can also check the strict concavity of the profit function over $(0, \bar{w})$ by considering the non-existence of $\tilde{w} < \bar{w}$ in which $f''(\tilde{w}) = 0$ and $f'''(\tilde{w}) > 0$. However, the key argument depends the one we already use, namely $f''(\bar{w} - \epsilon) < 0$ for $\epsilon > 0$. We then skip the formal prove of strict concavity of profit function. The interested reader is referred to the complete proof in DFHW (2010)

The first term on RHS is the expected cashflow and the second term is the adjustment term for difference between discounting term and interest rate.

We write the explicit form of financial flexibility dynamics by substitute the composing dynamics as follow.

$$\begin{aligned} dM_t &= rM_t dt + K_t(a_t - L(i_t))dt + \sigma K_t dZ_t - K_t(a_t - L(i_t))dt + (\gamma - r)M_t dt - dX_t \\ &= rM_t dt + (\gamma - r)M_t dt - dX_t + \sigma K_t dZ_t \end{aligned}$$

Notice that $(\gamma - r)M_t dt$ is an adjustment term for the different between agent's discount term and risk-free rate, which is equal to principal's discount term. At this step, we can assume without loss of generality that $\gamma = r$ in order to skip the term in the implementation[23].

Assuming the equality of the agent's discount and risk-free rate, the dynamic financial flexibility reads

$$dM_t = rM_t dt - dX_t + \sigma K_t dZ_t. \tag{A.14}$$

We write the scaled financial flexibility dynamics $m_t = \frac{M_t}{K_t}$ and $x_t = \frac{X_t}{K_t}$,

$$dm_t = (\gamma - (i_t - \delta))\, m_t dt - dx_t + \sigma dZ_t. \tag{A.15}$$

We verify that dM_t will induce optimal decisions in the sense that dM_t lead to dW_t in the optimal contract as derived above.

We define the continuation value of agent as a value function, $W_t = V(W_t, K_t)$. Then, from $M_t = \frac{W_t}{\lambda_t}$, we have $\lambda_t M_t = V(M_t, K_t)$. We verify dM_t by constructing HJB equation and check the decision induced by the such HJB equation based on dM_t. From $dK_t = (I_t - \delta K_t - b_t \zeta(g) K_t)dt$, equation (1), and $dM_t = rM_t dt - dX_t + \sigma K_t dZ_t$, equation (A.14), at any point in time, we must have $\lambda M = V(M, K)$. We have the results from partial derivatives such that $V_M = \lambda, V_{MM} = 0, V_K = 0, V_{KK} = 0, V_{MK} = 0$.

By Ito lemma, we have

[23] We previously assume that $\gamma > r$ in the derivation of optimal contract. We can maintain the assumption for the analysis of implementation. However, the main result and insight from the implementation do not change when we assume the equality of the two.

$$\begin{aligned} dV(M,K) &= V_M dM_t + V_K dK_t + \frac{1}{2}V_{MM}(dM_t)^2 + \frac{1}{2}V_{KK}(dK_t)^2 + V_{MK}(dM_t \cdot dK_t) \\ &= \lambda\left(\delta M_t dt - dX_t + \sigma K_t dZ_t\right) \end{aligned}$$

We assign that the instantaneous return is equal to λdX_t. We write the HJB induced by M_t with agent's discount term γ as

$$\begin{aligned} \gamma\lambda M_t &= \sup_{\{a_t\in[0,\infty),b_t\in\{0,1\}\}} \{\lambda dX_t + \lambda(\gamma M_t - dX - t)\} \\ &= \sup_{\{a_t\in[0,\infty),b_t\in\{0,1\}\}} \{\lambda\gamma M_t\} \end{aligned}$$

We hence conclude that the financial flexibility, dM_t defined in equation (A.14), implements the decisions induced by the optimal contract.

We now recover the equity prices from the implementation induced by the financial flexibility. From the definition of security price as a expectation of discounted dividend and the terminal value for the investor, we write the explicit form of the security price at time t as,

$$S_t = \mathbb{E}\left[\int_t^\tau e^{-r(s-t)} dD_s + e^{-r(\tau-t)} l K_\tau\right]. \tag{A.16}$$

From dM_t and dD_t defined in equation (A.12) and (A.13) respectively, we have security price at time t in the form of

$$\begin{aligned} S_t =& \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(dY_s - U_s^g ds) + e^{-r(\tau-t)} l K_\tau\right] \\ &+ \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(rM_s ds - dM_s)\right] \\ &+ \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(-\zeta(g)K_s)ds\right]. \end{aligned} \tag{A.17}$$

The security price comprises of three parts. Firstly, we know from the objective of the principal that

$$F(W_t, K_t) = \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(dY_s - U_s^g ds) + e^{-r(\tau-t)} l K_\tau\right].$$

Secondly, we apply the integration by part to the second term of the right hand side of equation (A.17),

$$\begin{aligned}\int_t^\tau e^{-r(s-t)} dM_s =& e^{-r(s-t)} dM_s|_t^\tau + \int_t^\tau r e^{-r(s-t)} M_s ds \\ =& \left(e^{-r(\tau-t)} M_\tau - M_t\right) + \int_t^\tau r e^{-r(s-t)} M_s ds \\ =& - M_t + \int_t^\tau r e^{-r(s-t)} M_s ds,\end{aligned}$$

because $M_\tau = 0$. We then have,

$$\begin{aligned}\mathbb{E}\left[\int_t^\tau e^{-r(s-t)} r M_s ds - dM_s\right] =& \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(r M_s ds) + M_t - \int_t^\tau e^{-r(s-t)} r M_s ds\right] \\ =& E[M_t] \\ =& M_t\end{aligned}$$

where $M_t = \frac{W_t}{\lambda_t}$ by definition.

Finally, we see that the third term on RHS of equation (A.17) is the discounted agency rent from the time t to the terminal time. From our definition of corporate governance as a mechanism to reduce the agency conflict and cost, we can transform this term to highlight the role of corporate governance. We define the distortion function as follow.

$$\zeta(g) = \Omega - \psi(g) \tag{A.18}$$

where Ω is a constant term reflecting the level of capital distortion when the internal governance is zero. This constant captures the possible distortion from inefficient governance mechanism based on the country's legal environment and investor protection. Then, a country with good legal infrastructure for investor protection, the level of Ω is low. We define $\psi(g)$ as a measure for the investor protection from the internal governance mechanism of the firm. We assume that $\psi'(g) > 0$. Hence, our assumption $\zeta'(g) = -\psi'(g) < 0$ holds with the definition of $\zeta(\cdot)$ function.

From our definition of distortion function $\zeta(g)$, we write the third term on RHS of equation (A.17), as follow,

$$\begin{aligned}\mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(-\zeta(g)K_s)ds\right] =&\mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(\psi(g)-\Omega)K_s ds\right] \\ =&-\mathbb{E}\left[\int_t^\tau e^{-r(s-t)}\Omega)K_s ds\right] \\ &+\mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(\psi(g))K_s ds\right]\end{aligned} \tag{A.19}$$

The first term reflects the agency cost at country level due to the inefficient law and legal enforcement for investor protection. This is the country's discount term. The second term capture the effect of internal governance for each firm. It represents the amount of capital saved from agency rent. We call the second term the *governance premium* of the security price. This term captures the effect of internal governance of the firm that reduces the agency rent for a given country's legal infrastructure. The value of internal governance is equal to the discounted capital being recovered from possible distortion under optimal contract. With two terms put corporate governance at country and firm level into the same perspective.

We now write the security price at time t in the term of investor's expected profit, agent's continuation value, country-level discount and firm's governance premium, as follow,

$$S_t = F(W_t, K_t) + \frac{W_t}{\lambda_t} - \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}\Omega K_s ds\right] + \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(\psi(g))K_s ds\right]. \tag{A.20}$$

Other things equal, the security price in the country with weak legal investor protection, Ω is large, is lower than the country with the stronger one. Within a country, having identical legal infrastructure for investor protection, a firm with higher internal governance level, g and $\zeta(g)$ are large, has higher security price than the firm with lower level, *ceteris paribus.*

Equivalently, we can also express the security price in term of the scaled profit function and financial flexibility as

$$S_t = \left(f(\lambda_t M_t) + m_t - \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}\Omega ds\right] + \mathbb{E}\left[\int_t^\tau e^{-r(s-t)}(\psi(g))ds\right]\right)K_t.$$

□

Proof of Comparative Static Analysis in Proposition 7. We conduct the comparative analysis to highlight the consequences of change in internal governance on the principal's profit function. We follow the methodology used in DeMarzo and Sannikov (2006). We proceed in two steps. The first step is to consider the effect of change in governance[24] on the profit for the whole path of continuation value using Keynman-Fac formula. The second step is to consider the total derivative on boundary value or interested value of agent's profit to consider the change in governance on

For the first step, we denote the parameter of interest, including governance level, as ϕ[25]. We differentiate ODE of the profit function with respect to ϕ holding continuation value (w) constant and then evaluate at the upper boundary, $\bar{w}$. Then from ODE of profit function, differentiation with respect to the parameter and using upper boundary conditions, $f'(\bar{w}) = -1$ and $f''(\bar{w}) = 0$, gives

$$
\begin{aligned}
(r+\delta)\frac{\partial f(w)}{\partial \phi} + f(w)\frac{\partial (r+\delta)}{\partial \phi} =& \frac{\partial a}{\partial \phi} - \frac{\partial u^g}{\partial \phi} + \frac{f(w) - wf'(w) - 1}{\theta}\left(\frac{\partial f(w)}{\partial \phi} - w\frac{\partial f'(w)}{\partial \phi}\right) \\
&+ (\gamma+\delta)w\frac{\partial f'(w)}{\partial \phi} + wf'(w)\frac{\partial(\gamma+\delta)}{\partial \phi} \\
&- \zeta(g)\frac{\partial f'(w)}{\partial \phi} - f'(w)\frac{\partial \zeta(g)}{\partial \phi} \\
&+ \frac{\lambda^2\sigma^2}{2}\frac{\partial f''(w)}{\partial \phi} + \frac{f''(w)}{2}\frac{\partial(\lambda^2\sigma^2)}{\partial \phi}.
\end{aligned}
$$

From the upper boundary conditions, we have $\frac{\partial f'(w)}{\partial \phi}|_{w=\bar{w}} = 0$ and $\frac{\partial f''(w)}{\partial \phi}|_{w=\bar{w}} = 0$. We now rearrange the terms in order to apply the Feynman-Kac formula as the solution to PDE,

$$
\begin{aligned}
(r+\delta)\frac{\partial f(w)}{\partial \phi} =& - f(w)\frac{\partial (r+\delta)}{\partial \phi} + \frac{\partial a}{\partial \phi} - \frac{\partial u^g}{\partial \phi} + \left(\frac{f(w) - wf'(w) - 1}{\theta}\right)\frac{\partial f(w)}{\partial \phi} \\
&+ wf'(w)\frac{\partial(\gamma+\delta)}{\partial \phi} - f'(w)\frac{\partial \zeta(g)}{\partial \phi} + \frac{f''(w)}{2}\frac{\partial(\lambda^2\sigma^2)}{\partial \phi}.
\end{aligned}
$$

[24] The method can be applied to the comparative static analysis of other parameters.

[25] We denote ϕ as a representative of general parameters of the model, including discount rate of the agent (γ), variance of the cashflow process (σ^2), liquidation rate (l) and internal governance level (g).

Applying Feynman-Kac formula to express the solution of $\frac{\partial f(w_t)}{\partial \phi}$, with terminal period τ in which investor derives the terminal value at the rate of liquidation (l),

$$\begin{aligned}\frac{\partial f(w_t)}{\partial \phi} = \mathbb{E}^{w_0=w}\Bigg[\int_0^\tau e^{-(r+\delta)t}\Bigg(&-f(w_t)\frac{\partial(r+\delta)}{\partial\phi}+\frac{\partial a_t}{\partial\phi}-\frac{\partial u_t^g}{\partial\phi}\\ &+\left(\frac{f(w_t)-w_tf'(w_t)-1}{\theta}\right)\frac{\partial f(w_t)}{\partial\phi}+wf'(w)\frac{\partial(\gamma+\delta)}{\partial\phi}\\ &-f'(w)\frac{\partial\zeta(g)}{\partial\phi}+\frac{f''(w)}{2}\frac{\partial(\lambda^2\sigma^2)}{\partial\phi}\Bigg)dt\\ &+e^{-(r+\delta)\tau}\frac{\partial l}{\partial\phi}\Bigg]\end{aligned} \tag{A.21}$$

For $\phi = g$ and from the instantaneous constraints, we have $u_t^g = h(a_t; g) + \zeta(g)$ and $\frac{\partial u_t^g}{\partial g} = \frac{\partial h(a_t;g)}{\partial g} + \frac{d\zeta(g)}{dg}$. From the assumptions, $\frac{\partial h(a_t;g)}{\partial g} > 0$ and $\frac{d\zeta(g)}{dg} < 0$. To focus on the effect of governance change on firm dynamics, we neutralize the effect on executive compensation. We assume that the $\frac{\partial h(a_t;g)}{\partial g} = \frac{d\zeta(g)}{dg}$, implying that change in distortion from governance change is equal to the change in effort cost of the executives. Consequently, the first step of comparative statics from governance gives the result

$$\frac{\partial f(w)}{\partial g} = \mathbb{E}^{w_0=w}\left[\int_0^\tau e^{-(r+\delta)t}(-f'(w_t))\zeta'(g)dt\right] \tag{A.22}$$

The sign of derivative depends on the initial value $w_0 = w$, because the value of derivative is evaluated for the change in profit function for the entire path of w_t. We know that $\zeta'(g) < 0$ and the value of $f'(w) > 0$ for $w \in (0, w^*)$ and $f'(w) < 0$ for $w \in (w^*, \bar{w})$. Then we conclude that

$$\frac{\partial f(w)}{\partial g}\begin{cases} > 0; & \text{if } w \in (0, w^*),\ \text{then } f'(w) > 0\\ < 0; & \text{if } w \in (w^*, \bar{w}),\ \text{then } f'(w) < 0\end{cases} \tag{A.23}$$

In the second step of comparative static analysis, we consider the boundary and the relevant conditions. Our interest is the effect of change in governance on the investor's profit and the firm dynamics. We then consider the conditions related to three important point of the agent's continuation value; the initial continuation value[26], the turning point

[26]This is the continuation value when agent enters into the contractual relationship with the principal under agreed governance and other parameters.

and the upper boundary, $\{w_0, w^*, \bar{w}\}$ respectively. We use total derivative on the related conditions and the previous results to derive the conclusion.

For w_0, we use the condition $f(w_0) = \alpha$ saying that the outside option of the principal, or investor, does not depend on the continuation value of the agent and can be represented by a constant term, α. The total derivative is then $\frac{\partial f(w_0)}{\partial \phi} + f'(w_0)\frac{\partial w_0}{\partial \phi} = 0$. We then have the effect of change in governance on the initial continuation value as,

$$\frac{\partial w_0}{\partial \phi} = -\frac{\left(\frac{\partial f(w_0)}{\partial \phi}\right)}{f'(w_0)} \tag{A.24}$$

where $f'(w_0) = \frac{df(w)}{dw}\Big|_{w=w_0}$.

For w^*, we use the condition $f'(w^*) = 0$, characterizing the point that the slope of profit function changes from positive to negative. By total derivative, we have $\frac{\partial f'(w^*)}{\partial \phi} + f''(w^*)\frac{\partial w^*}{\partial \phi} = 0$, and

$$\frac{\partial w^*}{\partial \phi} = -\frac{\left(\frac{\partial f'(w^*)}{\partial \phi}\right)}{f''(w^*)} \tag{A.25}$$

where $\frac{\partial f'(w^*)}{\partial \phi} = \frac{\partial^2 f(w^*)}{\partial \phi \partial w}\Big|_{w=w^*}$.

For $\bar{w}$, we use the ODE evaluated at the upper boundary value as the condition. Applying the total derivative to ODE of the profit function at upper boundary, $(r+\delta)f(\bar{w}) = a_t - u_t^g + \frac{(f(\bar{w})+\bar{w})^2}{2\theta} - (\gamma+\delta)\bar{w}$, we have

$$\begin{aligned}
(r+\delta)\left[\frac{\partial f(\bar{w})}{\partial \phi} + f'(\bar{w})\frac{\partial \bar{w}}{\partial \phi}\right] =& \frac{da(w)}{dw}\frac{\partial w}{\partial \phi}|_{w=\bar{w}} - \frac{du^g(w)}{dw}\frac{\partial w}{\partial \phi}|_{w=\bar{w}} \\
&+ \frac{1}{\theta}(f(\bar{w})+\bar{w})\frac{d(f(\bar{w})+\bar{w})}{d\phi} - (\gamma+\delta)\frac{\partial \bar{w}}{\partial \phi} \\
(r+\delta)\left[\frac{\partial f(\bar{w})}{\partial \phi} - \frac{\partial \bar{w}}{\partial \phi}\right] =& \frac{1}{\theta}(f(\bar{w})+\bar{w})\left[\frac{\partial f(\bar{w})}{\partial \phi} + f'(\bar{w})\frac{\partial \bar{w}}{\partial \phi} + \frac{\partial \bar{w}}{\partial \phi}\right] - (\gamma+\delta)\frac{\partial \bar{w}}{\partial \phi} \\
(r-\gamma)\frac{\partial \bar{w}}{\partial \phi} =& \left[(r+\delta) - \frac{(f(\bar{w})+\bar{w})}{\theta}\right]\frac{\partial f(\bar{w})}{\partial \phi}.
\end{aligned}$$

We then have

$$\frac{\partial \bar{w}}{\partial \phi} = \left[\frac{r+\delta}{r-\gamma} - \frac{(f(\bar{w})+\bar{w})}{\theta(r-\gamma)}\right]\frac{\partial f(\bar{w})}{\partial \phi}.$$

We cannot determine the sign of the coefficient term of $\frac{\partial f(\bar{w})}{\partial \phi}$ because it depends on the quantitative value of the parameters, profit function and continuation value.

We now consider the specific case for $\phi = g$ for $\{w_0, w^*\}$. We then need another auxiliary result.

$$\frac{\partial f'(w^*)}{\partial g} = \frac{\partial \left(\frac{\partial f(w)}{\partial g}\right)}{\partial w}\Bigg|_{w=w^*} = \mathbb{E}^{w_0=w}\left[\int_0^{\tau} e^{-(r+\delta)t}(-f''(w_t))\zeta'(g)dt\right] < 0 \tag{A.26}$$

because $f''(w_t) < 0$ from concavity and $\zeta'(g) < 0$ by the assumption.

The effect of change in governance on the turning point (w^*) is following

$$\frac{\partial w^*}{\partial g} = \frac{-\left(\frac{\partial f'(w^*)}{\partial g}\right)}{f''(w^*)} < 0 \tag{A.27}$$

due to equation (A.26) and concavity of profit function.

The effect of change in governance on the initial continuation value of the agent (w_0) depends on the sign of $f'(w_0) = f'(w)\big|_{w=w_0}$ and $\frac{\partial f(w_0)}{\partial g} = \frac{\partial f(w)}{\partial g}\big|_{w=w_0}$. From equation (A.23), we consider the possible value of relevant elements. For $w_0 \in (0, w^*)$, $f'(w_0) > 0$ and $\frac{\partial f(w_0)}{\partial g} > 0$, we then have $\frac{\partial w_0}{\partial g} < 0$. For $w_0 \in (w^*, \bar{w})$, $f'(w_0) < 0$ and $\frac{\partial f(w_0)}{\partial g} < 0$, the result is $\frac{\partial w_0}{\partial g} < 0$. We conclude that under the optimal contract, the effect of change in governance on initial continuation value is negative,

$$\frac{\partial w_0}{\partial g} = \frac{-\left(\frac{\partial f(w_0)}{\partial g}\right)}{f'(w_0)} < 0 \tag{A.28}$$

□