A Simple Bargaining Mechanism That Elicits Truthful Reservation Prices

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Abstract

We describe a simple 2-stage mechanism that induces two bargainers, a Buyer and a Seller, to be truthful in reporting their reservation prices in a 1st stage. If the Buyer reports a higher reservation price than the Seller, then the referee announces that there a possibility for trade, and the bargainers proceed to make offers in a 2nd stage. The average of the 2nd-stage offers becomes the settlement if they both fall into the interval between the reported reservation prices; if only one offer falls into this interval, it is the settlement, but is implemented with probability $\frac{1}{2}$; if neither offer falls into the interval, there is no settlement.

Keywords: Bargaining, truth-telling mechanisms, probabilistic implementation, incomplete information.

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1 Introduction

How to induce players to go to their "bottom lines" in bargaining is an age-old problem in the design of a bargaining mechanism. A solution for sealed-bid auctions is a second-price, or Vickrey, auction [13], whereby the high bidder pays the second-highest bid, rendering what the winner pays independent of what he or she bid. The extension of this idea to Vickrey [13] -Clarke [3]-Groves [5] (VCG) mechanisms in bargaining likewise induces honesty, because the settlement does not depend directly on what a player offers. Similarly, Brams and Kilgour [2] show that when players bid for rooms in a house in which they share the rent, the "gap procedure" creates a kind of partial independence, motivating the housemates to make truthful bids that sum to the total rent of the house.

When only two bargainers haggle over the price of some good or service, averaging their offers ("splitting the difference") does not induce honesty, because it gives them incentives to exaggerate in opposite directions. Suppose that the bargainers' *reservation prices*—the settlement prices that would make each indifferent between an agreement or none—are private information. When reservation prices are uniformly distributed, Chatterjee and Samuelson [4] show that this mechanism has a simple symmetric equilibrium in which exaggeration is piecewise linear. In their game, the final price is an average of the offers if they *overlap* (i.e., if the buyer's offer does not fall below the seller's); otherwise, there is no agreement, and the players get nothing. The Chatterjee-Samuelson mechanism is linear and symmetric, and maximizes efficiency for the symmetric uniform case (Myerson and Satterthwaite [8]); there is also an infinity of asymmetric equilibria (Leininger, Linhart, and Radner [6]). For background information on mechanism design, see [9].

In this paper, we give a simple two-stage mechanism that induces two bargainers truthfully to reveal their reservation prices to a referee in stage 1. If the 1st-stage offers overlap, there is the *potential* for an agreement, which is realized—at the mean of the 2nd-stage offers—if *both* bargainers' offers fall in the overlap interval; if *only one* bargainer's offer falls in this interval, this offer becomes the agreement with probability $\frac{1}{2}$, but otherwise not; if *neither* bargainer's offer falls in the overlap interval, there is no agreement.

As we will show, this procedure, like another probabilistic mechanism [1] that we discuss in section 5, is not maximally efficient (Myerson and Satterthwaite [8]). In particular, if the players have uniform distributions over each others' reservation prices, it is not as efficient as the the procedure of Chatterjee and Samuelson [4], in part because of the random draw when exactly one offer falls in the overlap interval.

But even if no agreement is *realized*, our procedure does reveal—if it continues to stage 2—that the reservation prices of the bargainers *allow* for a mutually profitable agreement. This information may be useful for other parties that bargain under similar circumstances. While a VCG mechanism will also induce truthful-reporting that is weakly dominant, it cannot be both ex-post budget balanced or ex-post individually rational.[1]

Another benefit of our mechanism is that there is a positive probability of settlement even for extreme reservation values; in contrast, bargainers with extreme values have no incentive to bargain under the Chatterjee-Samuelson procedure, because there is no possibility of any agreement. Finally, the simplicity of both the rules and the equilibrium should appeal to both ex-

[1]The Revelation Principle (see Myerson [7]) implies that any equilibrium can be converted into a truth-telling mechanism by creating for each player a "robot" programmed to play the equilibrium strategy corresponding to the value it receives from the player. But, in such a scheme, truth-telling may not be a weakly dominant strategy.

perimentalists and practitioners.

2 The Mechanism

We consider the possible sale of an object by a Seller to a Buyer. If they can agree on a price p, the object will be transferred from Seller to Buyer, and the Seller will receive p as compensation. Of course, Seller prefers a higher p, whereas Buyer prefers a lower p. If they cannot agree on a p, there is no sale.

For definiteness, our discussion is phrased in terms of a possible sale, but our mechanism has many applications, such as to the settlement of a claim by an insured party against an insurer. In that case, the insured party, preferring a higher settlement, plays the role of Seller, with the insurer in the role of Buyer.

We model the Seller's reservation price for the object, S, as the value of a random variable with cumulative distribution function F_S. The Buyer's reservation price, B, is the value of a random variable with cumulative distribution function F_B. Both F_S and F_B have support $[C, D]$. Both players' reservation prices are private information, and their utilities are quasi-linear, so that if a sale takes place at price p, Buyer will receive $B - p$ and Seller will receive $p - S$. If there is no sale, both players receive 0. The players are risk-neutral.

The mechanism is a two-stage procedure:

Stage 1. The players submit *reserves* to the referee: Seller submits $\widehat{S}$ and Buyer submits $\widehat{B}$. The reserves may or may not equal the corresponding reservation prices (i.e., the 1st-stage submissions are not necessarily truthful). If $\widehat{S} \leq \widehat{B}$, the overlap interval is $[\widehat{S}, \widehat{B}]$, and the procedure moves to stage

2. If $\widehat{S} > \widehat{B}$, the reserves do not overlap, there is no settlement, and the procedure ends.

Stage 2. The players submit *offers* to the referee: Seller submits s, and Buyer submits b. If both s and b fall in the overlap interval defined in stage 1, there is a sale at price $p = \frac{s+b}{2}$. If only one of s and b falls in the overlap interval, the name of one player is selected at random; if the selected player's offer is the one in the overlap interval, then it is sale price; if not, there is no sale. If neither offer is in the overlap interval, there is no sale.

This mechanism determines (i) whether there is a sale and (ii) if there is a sale, at what price. As usual, we model each player as privately learning its own (true) reservation price (S or B) prior to stage 1, and using this information to choose its strategy: $(\widehat{S}, s)$ for Seller; $(\widehat{B}, b)$ for Buyer. Thus, a strategy for Seller is a pair of functions $\widehat{S}(S)$ and $s(S)$ that give the values of its strategic variables as a function of its reservation price. Similarly, Buyer's strategy can be thought of as two functions, $\widehat{B}(B)$ and $b(B)$.

One strategy for a player *weakly dominates* another strategy for that player if the first yields an expected utility that is at least as great as the second, no matter what strategy is chosen by the opponent. A (Bayesian-Nash) *equilibrium* is a profile of strategies with the property that each player's equilibrium strategy maximizes the player's expected utility, given that the opponent plays according to its equilibrium strategy.

To represent our mechanism, we use two functions,

$$t : \mathbb{R}^4 \longrightarrow [0,1] \text{ and } p : \mathbb{R}^4 \longrightarrow \mathbb{R}$$

with the interpretation that $t(\widehat{S}, s, \widehat{B}, b)$ is the probability that an agreement is reached if the 1$^{\text{st}}$-stage reserves are $\widehat{S}$ and $\widehat{B}$, and the 2$^{\text{nd}}$-stage offers are

s and b; similarly, $p(\widehat{S}, s, \widehat{B}, b)$ is the price. Note that both $\widehat{S}$ and s are functions of Seller's true reservation price S; we have written $\widehat{S}$ instead of $\widehat{S}(S)$, and s instead of $s(S)$, for notational simplicity. Observe that the value of p is irrelevant if $t = 0$. Using the functions t and p, we can describe our mechanism formally, as follows:

$$(t,p) = \begin{cases} (1, \frac{s+b}{2}) & \text{if } \widehat{S} \le s, b \le \widehat{B}, \\ (\frac{1}{2}, b) & \text{if } \widehat{S} \le b \le \widehat{B} < s, \\ (\frac{1}{2}, s) & \text{if } b < \widehat{S} \le s \le \widehat{B}, \\ (0,0) & \text{otherwise.} \end{cases} \tag{1}$$

We will assume below that players always choose 2^{nd}-stage strategies that are at least as "aggressive" as their 1^{st}-stage strategies, i.e., that $b \le \widehat{B}$ and $s \ge \widehat{S}$. This assumption is innocuous because, by (1), the players' payoffs are certain to be 0 if these inequalities do not hold, while strategies satisfying them ensure a payoff of at least 0.

We can construct a strategically equivalent mechanism by retaining stage 1 and replacing stage 2 by

Stage $2'$**.** One player, Seller or Buyer, is chosen at random. If Seller is chosen, and if Seller's 2^{nd}-stage offer s satisfies $s \le \widehat{B}$, then the transaction takes place at price $p = s$; if $s > \widehat{B}$, then there is no sale. Similarly, if the player chosen is Buyer, and if Buyer's 2^{nd}-stage offer b satisfies $\widehat{S} \le b$, then the transaction takes place at price $p = b$; if $b < \widehat{S}$, there is no transaction.

We assume that the random selection of a player in stage $2'$ is independent of the players' reservation prices. The equivalence of the two mechanisms arises because, if stage $2'$ is followed, the players' expected utilities are exactly as in (1). For example, if $\widehat{S} \le s, b \le \widehat{B}$, then Seller's expected utility is

$$\frac{1}{2}(s - S) + \frac{1}{2}(b - S) = \frac{s + b}{2} - S = p - S,$$

where p is determined by the first condition of (1). A similar relation holds for Buyer. The verification is immediate if the 2^{nd}-stage offer of only one player, or none, falls in the overlap interval. Below, we will use the stage 2 and stage $2'$ formulations interchangeably.

We assume that the players' reservation-price distributions, F_S and F_B, which have support $[C, D]$, are continuous and independent and have strictly positive densities. Then it follows that, to maximize expected utility, the four strategy functions, $\widehat{S}(S)$ and $s(S)$ for Seller and $\widehat{B}(B)$ and $b(B)$ for Buyer, can be assumed differentiable and strictly increasing in the players' reservation prices.[2]

3 A Simple Truth-Telling Mechanism

We define truth-telling as follows:

Definition 1 *Seller's strategy* $(\widehat{S}, s)$ *is* ***truth-telling*** *if* $\widehat{S}(S) = S$ *for all* $S \in [C, D]$*. Buyer's strategy* $(\widehat{B}, b)$ *is* ***truth-telling*** *if* $\widehat{B}(B) = B$ *for all* $B \in [C, D]$*. A strategy profile* $(\widehat{S}, s; \widehat{B}, b)$ *is a* ***truth-telling equilibrium*** *if it is an equilibrium and both players' strategies are truth-telling.*

Note that truth-telling refers to the players' reserve strategies (1^{st} stage), not their offer strategies (2^{nd} stage).

Buyer's monotone hazard rate condition is satified iff Buyer's cumulative distribution function, $F_B(x)$, satisfies $\frac{d}{dx}\frac{F_B'(x)}{1-F_B(x)} \geq 0$ for all $x \in [C, D]$.

[2]To prove this, use the stage $2'$ formulation and adapt Theorem 1 of Chatterjee and Samuelson [4] to each of the four strategies in turn.

Similarly, *Seller's monotone hazard rate* condition is satisfied iff Seller's cumulative distribution function, $F_S(x)$, satisfies $\frac{d}{dx}\frac{F_S'(x)}{F_S(x)} \leq 0$ for all $x \in [C, D]$. If both Buyer's and Seller's monotone hazard rate conditions are satisfied, our procedure has a unique truth-telling equilibrium, as shown next.

Proposition 1 *Any strategy of Seller, $(\widehat{S}, s)$, is weakly dominated by the truth-telling strategy (S, s). Any strategy of Buyer, $(\widehat{B}, b)$, is weakly dominated by the truth-telling strategy (B, b). There is a truth-telling equilibrium in which the players' 2^{nd}-stage offers, s^* and b^*, are solutions of $1 - F_B(s) = (s - S)F_B'(s)$ and $F_S(b) = (B - b)F_S'(b)$, respectively. Moreover, if $F_S(\cdot)$ and $F_B(\cdot)$ satisfy monotone hazard rate conditions, then the solutions s^* and b^* are unique, and the only truth-telling equilibrium is $(S, s^*; B, b^*)$.*

Proof. First we consider the Seller's expected utility, which we calculate using the procedure of stage $2'$. The Seller knows the value of S and determines strategy $(s, \widehat{S})$ using the functions $s(S)$ and $\widehat{S}(S)$. The expectation must be taken over the Buyer's value B and the random selection of Seller or Buyer. Therefore, Seller's expected utility given S is

$$\frac{1}{2}\int_{b^{-1}(\widehat{S})}^{D} (b(B) - S)\, dF_B(B) + \frac{1}{2}\int_{\widehat{B}^{-1}(s)}^{D} (s - S)\, dF_B(B), \tag{2}$$

where the first integral is associated with the random selection of Buyer (so the price is b) and the second with the selection of Seller (so the price is s). The first integral must be restricted to values of B such that $b(B) \geq \widehat{S}$, which is equivalent to $B \geq b^{-1}(\widehat{S})$, as indicated in the lower limit. Similarly, the second integral is restricted to those values of B for which $s \leq \widehat{B}(B)$, as reflected in the lower limit.

Consider the information provided by (2) about Seller's choice of strategy functions. The first integral of (2) depends on $\widehat{S}(S)$ but not s, and the second integral of (2) depends on $s(S)$ but not $\widehat{S}$. Therefore, Seller maximizes its expected utility by choosing $\widehat{S}$ to maximize the first integral and s to maximize the second.

The first integral of (2) is

$$I_1(\widehat{S}) = \frac{1}{2}\int_{b^{-1}(\widehat{S})}^{D} (b(B) - S)\, dF_B(B).$$

This integral depends on $\widehat{S}$ only through its lower limit, so it can be differentiated to produce

$$\begin{aligned}\frac{d}{d\widehat{S}} I_1(\widehat{S}) &= -\frac{1}{2}\left(b(b^{-1}(\widehat{S})) - S\right) F_B{}'(b^{-1}(\widehat{S}))\frac{db^{-1}(\widehat{S})}{d\widehat{S}} \\ &= -\frac{1}{2}\left(\widehat{S} - S\right) F_B{}'(b^{-1}(\widehat{S}))\frac{db^{-1}(\widehat{S})}{d\widehat{S}}.\end{aligned}$$

Now $F_B{}'(b^{-1}(\widehat{S}))$ can be assumed positive, as $F_B(\cdot)$ is strictly increasing, and $b^{-1}(\cdot)$ is an increasing function because $b(\cdot)$ is. It follows that $I_1(\widehat{S})$ is a strictly increasing function of $\widehat{S}$ if $\widehat{S} < S$, and a strictly decreasing function of $\widehat{S}$ if $\widehat{S} > S$. Thus, for any 2^{nd}-stage strategy (offer) $s = s(S)$ of Seller, it follows that Seller's expected utility using strategy $(\widehat{S}, s)$, where $\widehat{S} \neq S$, is not greater than Seller's expected utility using strategy (S, s). Therefore, we can conclude that any strategy for Seller is weakly dominated by a truth-telling strategy, and that we can assume that $\widehat{S}(S) = S$ at equilibrium. A similar argument for Buyer leads to the conclusion that $\widehat{B}(B) = B$ can be assumed at equilibrium.

Now we address Seller's choice of $s(S)$ to maximize the second integral of (2). With the substitution $\widehat{B}(B) = B$, this integral becomes

$$\begin{aligned}I_2(s) = &\ \tfrac{1}{2}\textstyle\int_s^D (s - S)\, dF_B(B) \\ = &\ \tfrac{1}{2}(s - S)(1 - F_B(s)).\end{aligned}$$

Because $S < D$, the maximum of $I_2(s)$ must be interior, as $I_2(S) = I_2(D) = 0$ but $I_2(\frac{S+D}{2}) > 0$. Hence, the maximum must be a value of s that solves the first-order condition $1 - F_B(s) = (s - S)F'_B(s)$. This equation always has at least one solution. Moreover, under the monotone hazard rate condition for Buyer, this solution is unique; we denote it $s^*(S)$.

Buyer's optimal offer, and its relation to the monotone hazard rate condition for Seller, are analogous. In particular, if both monotone hazard rate conditions are satisfied, there is a unique truth-telling equilibrium, which we denote $(S, s^*; B, b^*)$.[3] ■

Our proof that $(S, s^*; B, b^*)$ is an equilibrium thus relies on the fact that strategies that are not truth-telling are weakly dominated by strategies that are, so there must be a truth-telling equilibrium. Then the offer strategies are obtained by maximizing the players' expected utilities under the assumption of truth-telling. To understand why truth-telling dominates, note that each player benefits from maximizing the width of the overlap interval, up to its reservation price—Seller from below and Buyer from above—in order to ensure, insofar as possible, that the 2$^{\text{nd}}$-stage bids, s and b, fall in the interval, thereby meeting a necessary condition for an agreement.

In fact, truthfully reporting one's reservation price in stage 1 is analogous to bidding one's reservation price in a Vickrey auction: Just as a player cannot win in a Vickrey auction without being the highest bidder, a bargainer cannot reach a settlement unless there is an overlap interval, leading to stage 2. In each case, a player goes to its bottom line for two reasons: (i) failing to do so in stage 1 could preclude a favorable outcome in stage 2 (or cause an unfavorable outcome) and (ii) once in stage 2, the outcome does not depend on what the player reported in stage 1.

[3] Riley and Zeckhauser [12], among others, solve a problem similar to finding s to maximize $I_2(s)$, which can be interpreted as a monopolist's optimal take-it-or-leave-it offer. The problem is also equivalent to finding a buyer's optimal take-it-or-leave-it offer.

A player's utility, if positive, does not depend on the reserves, $\widehat{S}$ and $\widehat{B}$, submitted in stage 1 but, instead, on its bid, s or b, submitted in stage 2. The independence between a player's reserve and its offer implies that it can "afford" to be truthful in stage 1. In fact, a player cannot do worse by reporting its reservation price truthfully in stage 1, and may do better, so we say that under our mechanism each player has an incentive to report its reservation prices truthfully.

The story is different, however, in stage 2: Each player will have an incentive to shade its offer, depending on the distribution of the opponent's reservation price. In the next section, we illustrate, for two specific distribution functions, how much shading is optimal.

4 Examples

For our examples, we assume $C = 0$ and $D = 1$, so that $0 \leq S, B \leq 1$.

Example 1 *Uniform distribution:* $F_S(x) = F_L(x) = x$.

The players' optimal offers, $s^*(S) = \frac{1+S}{2}$ and $b^*(B) = \frac{B}{2}$, are shown in Figure 1 below. Notice that each of these strategies halves the distance between the reservation prices, S and B (shown as the line $S = B$), and the endpoints, 1 and 0, respectively, of the bargaining range. In particular, Seller never offers below $\frac{1}{2}$, and Buyer never offers above $\frac{1}{2}$.

It is clear that truthfully reporting one's reservation price in the 1^{st} stage is weakly but not strictly dominant. For example, if Seller's true value is $S = \frac{3}{4}$, then, as can be seen from Figure 1, Seller's 2^{nd}-stage bid will be $s = s^*(\frac{3}{4}) = \frac{7}{8}$ at equilibrium. If $B > \frac{7}{8}$, there will be a sale with probability $\frac{1}{2}$; otherwise, there is no possibility of a sale. Hence, in the 1^{st}-stage, Seller

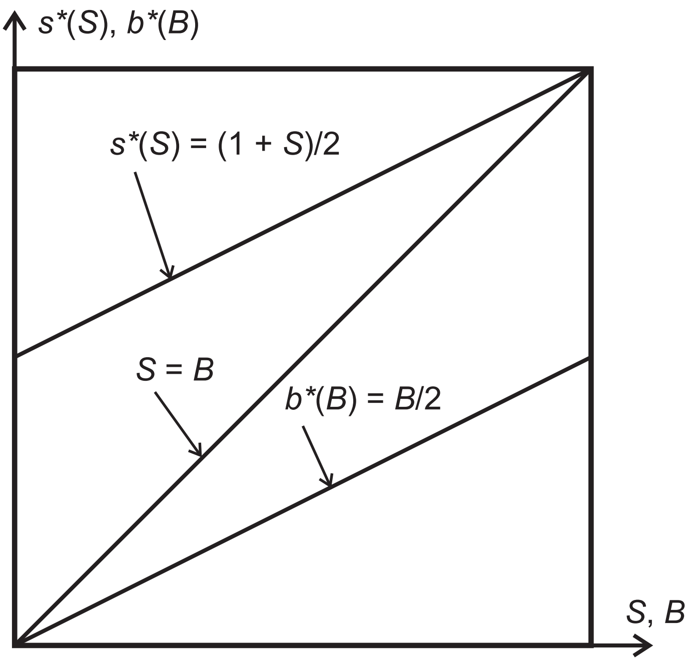

Figure 1: Offer Strategies in Example 1

will be indifferent between reporting $\frac{3}{4}$ and, say, $\frac{5}{8}$ (provided the 2^{nd}-stage bid remains $s = \frac{7}{8}$).

Figure 2 graphs the results of the equilibrium strategies we have identified for all possible values of S and B. A sale occurs with certainty when $B < 2S$ and $B > \frac{1+S}{2}$; these two conditions define the region with darker shading in Figure 2. This region has small values of S and large values of B, with the difference between them so great that the offers, s^* and b^*, fall in the overlap interval.

A transaction occurs with probability $\frac{1}{2}$ when $2S < B < \frac{1+S}{2}$ and when $\frac{1+S}{2} < B < \min\{2S, 1\}$, which are the two regions with lighter shading in Figure 2. In the first of these regions (lower left), $s^* > B$ but $b^* > S$, so there is a sale at $p = b^*$ when Buyer's name is drawn in stage $2'$, and no sale otherwise. Similarly, in the upper right region, $s^* < B$ and $b^* < S$, so there is a sale at $p = s^*$ when Seller's name is drawn in stage $2'$, and no sale otherwise.

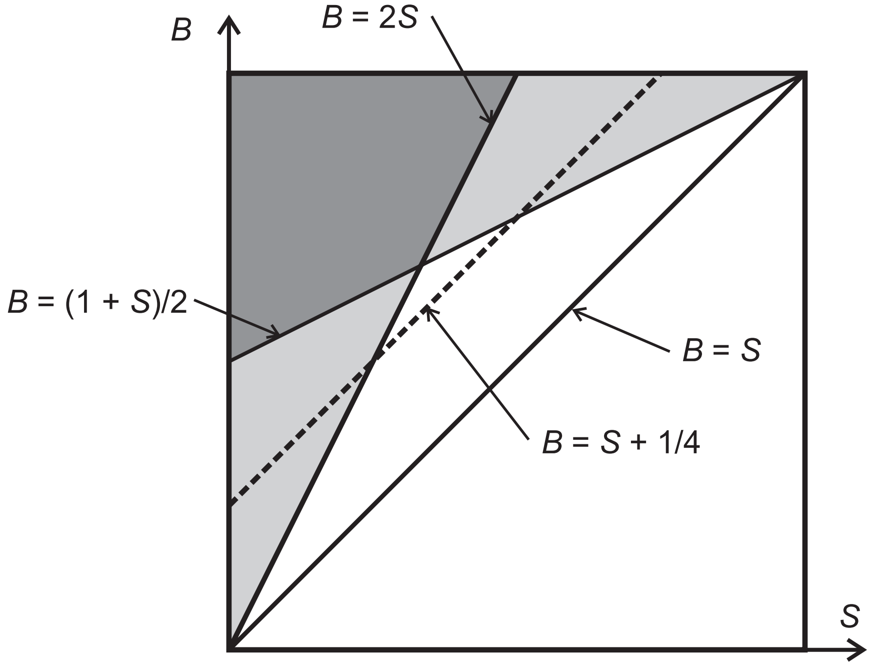

Figure 2: Conditions for a Transaction in Example 1

It is instructive to compare our mechanism with the Chatterjee-Samuelson procedure [4], which produces a transaction, for certain, if and only if $B \geq S + \frac{1}{4}$. This inequality defines the area above the dashed line in Figure 2. We compare mechanisms using the expected surplus they produce, which because of our assumptions equals the total expected utility of Buyer and Seller after the transaction, if any. For an "ideal" procedure, which produces a settlement whenever the players' reservation prices overlap, the total surplus is

$$\int_0^1 \int_S^1 (B - S)\, dS dB = \frac{1}{6}.$$

Myerson and Satterthwaite [8] demonstrated that no mechanism can produce a larger surplus than the Chatterjee-Samuelson procedure, which gives

$$\int_0^{\frac{3}{4}} \int_{S+\frac{1}{4}}^1 (B-S)\, dBdS = \frac{9}{64}.$$

The surplus from our mechanism is

$$\frac{1}{2}\int_0^1 \int_{\frac{1+S}{2}}^1 (B-S)\, dBdS + \frac{1}{2}\int_0^{\frac{1}{2}} \int_{2S}^1 (B-S)\, dBdS = \frac{1}{8},$$

which is $\frac{8}{9} = 88.9\%$ of the maximum possible surplus.

But there are other ways to compare mechanisms. One positive aspect of ours is the potential for trade at all possible values of S and all possible values of B, a feature that the Chatterjee-Samuelson procedure does not share. For instance, if $S = 0.8$ and $B \geq 0.9$, a sale occurs with probability 0.5 under our mechanism, but probability 0 under the Chatterjee-Samuelson mechanism.

Example 2 *Power distribution:* $F_S(x) = x^\alpha$, $F_B(x) = 1 - (1-x)^\beta$, *for* $\alpha, \beta > 0$.

It is easy to verify that these distributions satisfy the monotone hazard rate conditions. Buyer's optimal offer is $b^* = \frac{\alpha B}{1+\alpha}$ and Seller's is $s^* = \frac{1+\beta S}{1+\beta}$, in agreement with Example 1, which corresponds to $\alpha = \beta = 1$. For example, when $\alpha = \beta = 2$, $b^*(B) = \frac{2}{3}B$ and $s^*(S) = \frac{1}{3} + \frac{2}{3}S$, and when $\alpha = \beta = \frac{1}{2}$, $b^*(B) = \frac{1}{3}B$ and $s^*(S) = \frac{2}{3} + \frac{1}{3}S$.

5 Conclusion

We have demonstrated a simple and elegant 2-stage mechanism that induces two bargainers to be truthful in reporting their reservation prices in the 1^{st} stage; if these prices criss-cross, the referee reports that there is an overlap interval, and the bargainers make offers in a 2^{nd} stage. The mean of these offers becomes the settlement if they both fall in the overlap interval. If only one offer does, it is implemented as the settlement price with probability $\frac{1}{2}$, whereas if neither offer does, there is no settlement. The resulting equilibrium offers also are relatively simple functions of the opponent's distribution.

While our mechanism is less efficient than the Chatterjee-Samuelson mechanism in the case of the uniform distribution, it does have several features to recommend it, including the possibility of a transaction even for extreme reservation prices. Also, under monotone hazard rate conditions, there is a unique truth-telling equilibrium (and, even without these conditions, it is almost always unique). Under our mechanism, truth-telling in the first stage is always optimal, but other behavior may produce the same results. But for a player whose value falls within the range of 2^{nd}-stage offers of the opponent, the equilibrium strategy of truth-telling is strictly optimal.

Part of the inefficiency of our mechanism stems from the random implementation of a 2^{nd}-stage offer, s or b, as the exchange price when only one offer falls in the overlap interval. Randomizing the implementation of a single inside offer is the penalty one pays to render a player's reserve independent of its offer in the expected-payoff calculation, thereby making it optimal for the player to report truthfully its reservation price. This independence would be broken, and it would be suboptimal for a player truthfully to report its reservation price, if single inside offers were implemented with certainty.

Brams and Kilgour [1] analyze other mechanisms that induce two bargainers to be truthful, including a "bonus procedure" in which a third party induces the bargainers to be truthful by paying them a bonus when their bids criss-cross. But it is their "penalty procedure" that is closest to the present mechanism in inducing truth-telling behavior.

Under it, the bargainers make simultaneous offers in a single stage, with the proviso that the probability of implementation of a settlement is a function of the *degree* of overlap, if any, in the bids: the greater the overlap, the higher this probability.[4] It yields a surplus of $\frac{1}{12}$, which is 50% of the maximum possible surplus, and thus falls short of the 88.9% achieved by the present mechanism. Moreover, unlike the present mechanism, the players never learn whether their failure to settle was because (i) their reservation prices did not criss-cross (as in stage 1), or (ii) they did criss-cross but probabilistic implementation prevented a settlement in the 2^{nd} stage. In principle, however, they could be told whether (i) or (ii) prevented a settlement; if (ii), they might be motivated to try again, but not using the present mechanism (see discussion below).

An advantage of the present mechanism is that the players *always* learn if stage 2 is reached and, therefore, that there is an overlap interval and the *potential* for a mutually profitable settlement. While our mechanism does not reveal the amount of regret—for example, how close the 2^{nd}-stage offers are to the overlap interval—we see no reason why the values of $\widehat{S} = S$, $\widehat{B} = B$, s, and b could not be revealed by the referee, making public the reason why implementation failed in the 2^{nd} stage.

The knowledge that the optimality of shading one's "bottom line" in stage 2 was all that prevented a settlement might motivate other bargainers in a similar situation to try to find an agreement *by other means* (e.g., face-to-face informal bargaining, mediation, etc.). We stress, however, that under our model, the bargainers must assign probability 0 to the possibility that

[4]The probability of a *certain* settlement in the present mechanism also increases as the overlap of the 1^{st}-stage reserves increases, because a greater overlap increases the likelihood that *both* players' 2^{nd}-stage offers will fall into the overlap interval, ensuring a settlement.

they could benefit from the procedure when it produces no agreement.[5]

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[5]Thus, even if there is an overlap interval from stage 1, the failure of the procedure to give a settlement in stage 2 would mean that its further use is foreclosed. (If this were not the case, truthfulness would not be optimal in stage 1.) But there is nothing in the model, after the procedure has been unsuccessfully tried, that prevents the bargainers from continuing to negotiate with each other—in effect, to transcend the limitations of the procedure. It would be interesting to run an experiment that gives subjects the choice of walking away or continuing to negotiate if there is no settlement in stage 2. How many would elect to continue, and how successful would they be? Besides answering this question, the simple linear solution that our mechanism provides could be tested in an experiment that assumes two-sided incomplete information; see, for example Radner and Schotter [10] and Rapoport, Daniel and Seale [11] for tests of other bargaining mechanisms that make this assumption.

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