Risk Aversion in a Model of Endogenous Growth

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Abstract

Despite substantial evidence on incomplete financial markets and substantial risk being borne by innovators, current models of growth through creative destruction model innovators' utilities as risk neutral. In answer to this simplification, the present paper introduces risk averse agents into a model of endogenous growth. In particular, finitely lived agents decide at the beginning of their lives whether to work for a given wage or whether to become innovators and face the risk of whether or not their research will be successful. The present paper shows that under low levels of risk aversion there does not exist a constant growth rate of researchers that would be consistent with positive population growth and expectations on the future path of research. Analytical results only confirm that any growth rate of researchers is bounded strictly between zero and the rate of population growth. It remains to investigate whether this result holds true for other levels of risk aversion as well.

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1 Introduction

The literature on endogenous growth, which has been born in many ways with the seminal papers of Romer (1986) and Romer (1990), has received almost innumerable additions and refinements over the last two decades. Different models yield endogenous growth through AK-style production functions, learning-by-doing or different forms of technological change[1]. One class of models that relate economic growth to technological change are models based on the Schumpeterian idea of *creative destruction* (Schumpeter (1943)), i.e., over time the quality of existing goods increases (alternatively, the cost of producing goods of the same quality decreases) and higher-quality goods (lower cost goods) push older goods of lower quality (higher costs) out of the market. This is the class of models the present paper considers, more specifically, it focuses on models of *quality ladders*, as introduced by Aghion and Howitt (1992) and Grossman and Helpman (1991b) and further developed in e.g. Kortum (1997)[2].

One of the major contributions of these models is that they combine microeconomic incentives for R&D[3] with economic growth: a good that has a higher quality (or lower costs in production) but is otherwise identical to competing goods will have a competitive advantage (in the case of otherwise completely homogeneous goods it will be able to claim the whole market), so there is an incentive to discover such a good. Furthermore, once this good *is* discovered, the incentive to innovate further still exists.

On the other hand, the early models of quality-ladders (as the ones by Aghion and Howitt and Grossman and Helpman) share at least one major shortcoming: They predict that the aggregate growth rate of an economy is an increasing function of the level of population, an effect that Jones (2005) terms "strong" scale effects. This prediction also implies that as soon as population grows at a positive rate, the growth rate of income per capita explodes, which is

[1]Some very good overviews of the main theories of endogenous growth can be found in e.g. Barro and Sala-i-Martin (2003), Grossman and Helpman (1991a), Aghion and Howitt (1998) and recently Acemoglu (2009)

[2]Other prominent papers that consider growth through creative destruction are Segerstrom (1998) and Segerstrom, Anant, and Dinopoulos (1990).

[3]These incentives are the focus of the literature on patent races, which relates market structure to incentives to do R&D. Important papers in this area include Loury (1979), Lee and Wilde (1980) and Dasgupta and Stiglitz (1980), while Reinganum (1985) incorporates the idea that a monopoly position from successful R&D might only be temporary due to further innovations.

clearly not observed in reality. Later models like Kortum (1997), Segerstrom (1998) and Jones (1995) are able to reduce scale effects to a "weak" form, in which the level of population does not influence the growth rate, but only the *level* of output per capita. This prediction, while also not without critics (after all, countries like China and India are amongst the most populous, but also amongst the poorest in the world), at least predicts a constant growth rate, so it fits better with the data[4].

The present paper focuses on an aspect that basically all previous paper on growth through creative destruction share, and that has received little comment so far: the assumption that agents are risk neutral. Although the models built implicitly or even explicitly concede that research is a risky undertaking (in existing models it is generally not known *ex ante* whether research is going to be successful, sometimes not even how good an invention might be), it is by and large assumed that the risk associated with each research project is idiosyncratic and can therefore be hedged against. The typical argument in this case is that R&D is mainly done by firms, whose shareholders should be well able to diversify the risk inherent in a single R&D project.

For those models like Kortum (1997), in which research is performed by individuals choosing their occupation (working in production for a given wage vs. doing R&D), the argument that the risk they face is idiosyncratic is less intriguing. Furthermore, the assumption that the agents in such a setting can hedge against it perfectly is questionable both on theoretical and empirical grounds. Outside the endogenous growth literature, the interplay between entrepreneurial risk, risk attitudes and occupational choice has been discussed by, e.g., Banerjee and Newman (1991), who link occupational choice to the wealth distribution. Incomplete insurance markets for R&D are usually modeled to arise from problems of asymmetric information and/or moral hazard. The potential researcher will in most scenarios possess much more information on the expected profit and the risk inherent in his research than any investor. This implies that there is a problem

[4] Probably the most ingenious paper that finds evidence for weak scale effects is Kremer (1993), but there also exist a number of papers that find more recent cross-country evidence for scale-effects like Alcalá and Ciccone (2004). For a detailed discussion of strong and weak scale effects, including criticism, see Jones (2005).

of asymmetric information, which implies that the market for investing into R&D projects can face the problems of the "lemons" market of Akerlof (1970). In addition, once research funding is granted, it might be the case that the researcher does not put the optimal amount of effort into the project, since perfect supervision by the investor is highly unlikely (again due to asymmetric information).

If, on the other hand, R&D expenditure is decided inside a quoted firm, the typical principal-agent problem arises: Even though the shareholders are able to hedge perfectly against the risk of R&D expenditure, the agent that takes the final decision of how much / in which project to spend it, is probably less insured against failure of each single R&D project. In a worst case scenario, investment into a risky project that did not pay off could cost the decision maker his job, while a well diversified shareholder will hardly suffer, which implies a problem of moral hazard.

Both possible problems of asymmetric information and moral hazard imply that it might not be all that easy for a potential innovator (individual agent or firm) to hedge against the risk inherent in doing R&D. The theoretical reasoning is backed up by empirical evidence of high costs of financing R&D, much of which is very nicely summarized in Hall (2002). The assumption of perfect capital markets that previous papers relied on for diversifying R&D risk is empirically rejected as well (see e.g. David Card and Weber (2007)), and Evans and Jovanovic (1989) find that wealthier individuals are more likely to become entrepreneurs than less wealthy ones, which they link to binding liquidity constraints in opening up a business.

Once one acknowledges the fact that the risk of researching can not be perfectly hedged against, risk aversion on the side of the potential innovator has to be taken into account. As mentioned before, this aspect of growth through R&D has achieved little to no attention. Two notable exceptions are Zeira (2005) and García-Peñalosa and Wen (2006), both of which explicitly incorporate risk averse agents into a model of endogenous growth. García-Peñalosa and Wen though have a very simple setup in which there exists only one sector on which R&D can be targeted on and the focus of their paper is on the effect of redistribution on growth, rather than on risk aversion on growth. Zeira on the other hand focuses on how incentives for risk-sharing will affect the rate of innovation and duplication in an economy, rather than on the impact of varying degrees of risk aversion.

The present paper focuses on the effects of different levels of risk aversion in a model of endogenous growth whose assumptions about technological progress and occupational choice are based on the technological aspects of a trade model by Eaton and Kortum (2001). This particular setup lends itself to analyzing the effect of risk aversion as it is a model of occupational choice, which is a more intuitive framework than to introduce risk averse firms. In the present model, agents face a constant death probability as in Yaari (1965). At the beginning of their life, they decide whether to work for a given wage in production, or whether to occupy themselves with doing research instead. If their research is successful, they will be compensated by the expected net present value of their innovation. If they are unsuccessful, they will receive nothing and consume nothing as well[5]. Working with possibly zero consumption implies an upper bound on the level of risk aversion and the present model therefore only examines the effect of risk aversion up to this upper bound. The path of current R&D is determined by expected future R&D intensity: A higher expected number of researchers in the future will decrease the expected lifetime of an innovation and lower its expected value. With risk-neutral agents, this process leads to a balanced growth path in which a constant fraction of the population chooses to become researchers. With risk-averse agents, however, this result is not an equilibrium anymore. For the levels of risk aversion that are admissible within the structure of the present model, the only equilibrium that exists in constant growth rates is that both the measure of researchers and the population stay constant, i.e., there is no growth. For positive population growth, the model cannot be solved with a constant growth rate of researchers, not even zero. Theoretically, it is possible that the measure of researchers will increase at a decreasing rate, but such a rate makes the model intractable and impossible to solve analytically.

[5]This assumption simplifies the mathematical analysis considerably. Obviously, the marginal utility of an unsuccessful researcher goes to infinity and if an individual could choose *how much* of his labor endowment he should invest in R&D, he would always choose to work at least part-time in production. However, as his choice in the present paper is a discrete one, instead of marginal utilities, it is expected utilities that matter for the occupational choice.

The remainder of the paper is structured as follows: Section 3 introduces the production structure of the model and explains how innovation takes place. Sections 2 and 4 introduce consumers and firms operating in the economy, while section 5 solves for the equilibrium number of researchers and shows that there does not exist a unique growth rate of R&D. Section 6 concludes.

2 Consumers

2.1 Endowments

The economy is populated by a mass L_t of agents. Following Yaari (1965) and Blanchard (1985), each agent faces a Poisson death rate of $\upsilon \in (0, \infty)$. Each agent is endowed with one unit of labor, which they supply inelastically at the instant they are born, τ. The agent chooses between supplying his labor in the production of consumption goods and becoming a researcher. In the former case the agent will receive the fixed wage of 1[6]. The pay-off from being a researcher is uncertain. The probability to obtain an idea within τ is given by the Poisson parameter α, while both the expected length of any profits accruing to a given idea and the probability that the idea will be used in production in the first place are determined by the equilibrium path of researchers in the economy.

2.2 Preferences

Agents are risk averse and aggregate the available goods in a Cobb-Douglas fashion. Independent of the occupation that an individual agent choses, I assume that he is credit-constrained: He can save to smooth consumption over his (expected) lifetime, but cannot borrow. This very strict borrowing constraint arises naturally in the current setup in which expected future income of all the agents is 0 for any $t > \tau$. Agents in debt would never be able to repay their debts. A

[6] The increasing efficiencies from technological progress will translate into lower prices which in turn will increase the real wages of production workers.

less restrictive setup would equip agents with a unit flow of labor at each t. Even under this assumption the natural borrowing constraint for a researcher is 0, as the worst possible outcome for him is never to innovate at all during his lifetime. Allowing for the possibility of multiple innovations during a lifetime would severely complicate the utility maximization problem[7].

While agents are credit-constrained, they are allowed to save any unconsumed income to smooth consumption. I assume that there exists a financial intermediary that acts as an economy-wide mutual fund. This fund is in possession of the ownership claims of all firms that operate in the economy. Upon receiving of their income in τ, each agent i deposits his unconsumed income with the intermediary ($a_{i\tau|\tau}$). This deposit gives agent i a claim on the future earnings of the firms. The fund is also an intermediary between successful innovators and the firms. I.e., a successful innovator sells his idea to the intermediary, who credits him with a deposit equal to the present discounted value of all future profits pertaining to the idea. Then, the intermediary will give the right to the idea to a firm. Deposited income pays an interest of r_t, which is the rate at which future profits of firms are discounted. Accruing profits are used to pay interests on deposits. In this way, the intermediary re-allocates the resources across agents at each t, and the resource constraint of the economy implies that the entire production is consumed each t.

Due to uncertain lifetimes, an agent typically will die while holding positive deposits with the intermediary, and I assume that his deposits become property of the intermediary on this occasion. Death occurs with probability υ at t, independent of an agent's current age. As this is the flow rate with which the agent loses his assets (by dying), the intermediary will compensate him for this risk by paying him a return of $r_t + \upsilon$ on his deposits. Under these assumptions, the maximization problem of agent i born at time τ can be expressed as

$$U_{i\tau} = E_\tau \left[\int_\tau^\infty e^{-(\rho+\upsilon)(t-\tau)} \frac{\left[exp \int_0^1 ln\ x_{it|\tau}(j)dj \right]^{1-\sigma}}{1-\sigma} dt \right] \tag{1}$$

[7]See, e.g., Levhari and Srinivasan (1969), Merton (1969), or Samuelson (1969)

where $x_{it|\tau}(j)$ is the quantity that agent i, born at τ, consumes of good j at time t. ρ is the subjective discounting factor, which is assumed to be constant. Future consumption is discounted both by this factor as well as υ as this gives the rate at which future consumption is lost due to death. σ is the level of risk aversion. As researchers face a possibility of zero consumption, the occupational choice problem is not well defined for $\sigma > 1$. The individual maximizes (1) subject to the following constraints:

$$\dot{a}_{it|\tau} = (r_t + \upsilon)a_{it|\tau} - P_t X_{it|\tau}, \tag{2a}$$

$$a_{it|\tau} \geq 0, \quad X_{it|\tau} \geq 0, \tag{2b}$$

$$\lim_{t\to\infty} e^{-(\bar{r}_{t\tau}+\upsilon)(t-\tau)} a_{it|\tau} = 0, \tag{2c}$$

where

$$\bar{r}_{t\tau} \equiv \frac{1}{t-\tau} \int_{\tau}^{t} r_s \, ds$$

is the average interest rate between τ and t, $a_{it|\tau}$ is the amount of savings the agent has deposited at t, P_t is the aggregate price index in the economy and $X_{it|\tau}$ is the consumption index of agent i. These last two quantities are derived from the static optimization problem of the agent, given in section 2.3.1 below. The initial amount of deposit that an agent has, $a_{i\tau|\tau}$ will depend on his choice of occupation. It will be equal to the wage if he works in production, equal to the value of a patent (equal to V_τ, which will be determined later) if he is a successful researcher, and 0 if he is an unsuccessful researcher. Note that the uncertainty about the initial amount of deposits is the only uncertainty that the agent faces. Once this is resolved, his lifetime utility becomes certain. The values of deposits and consumption at each t depend on when an agent was born, and are therefore indexed by the cohort τ to which the agent belongs to.

2.3 Optimization

2.3.1 Static Optimization Problem

Given the utility function in equation (1) and the constraints given in (2), an agent maximizes his utility in two steps. First, he decides how to allocate his income across his expected lifetime. Second, given his income at any t, he decides how to allocate this income across all types j of goods. It is easier, however, to solve first for the static problem, given the income $y_{it|\tau}$ that results from an optimal intertemporal allocation of consumption. Under this assumption, at each t, the agent's optimization problem then is to maximize

$$\frac{\left[exp \int_0^1 ln\ x_{it|\tau}(j)dj\right]^{1-\sigma}}{1-\sigma} \tag{3}$$

subject to the constraint that

$$\int_0^1 p_t(j)x_{it|\tau}(j)\ dj \leq y_{it|\tau}. \tag{4}$$

The constraint in (4) will hold with equality in equilibrium. This problem is the limit case of the well-known Dixit-Stiglitz model (Dixit and Stiglitz (1977)), and it is easy to show that the first order conditions imply equal expenditure shares for all varieties j:

$$\frac{x_{it|\tau}(j)}{x_{it|\tau}(j')} = \frac{p_t(j')}{p_t(j)}.$$

Given this allocation of expenditure at t, it will be convenient to define general consumption and price indices before turning to the intertemporal maximization problem. I define the consumption index for agent i as

$$X_{it|\tau} = exp \int_0^1 ln\ x_{it|\tau}(j)dj \tag{5}$$

and denote P_t as the ideal price index corresponding to (5), i.e., the price index such that the following first order condition for the consumption index is satisfied:

$$\frac{x_{it|\tau}(j)}{X_{it|\tau}} = \frac{P_t}{p_t(j)} \quad \forall j. \tag{6}$$

The price index satisfying (6) is

$$P_t = exp \int_0^1 ln\, p_t(j) dj \tag{7}$$

and the static constraint at t can be written as

$$P_t X_{it|\tau} = y_{it|\tau},$$

which is the constraint used in the first line of equation (2).

2.3.2 Intertemporal Optimization Problem

The first order conditions of maximizing (1) subject to (2) yield the solution for consumption growth as

$$\frac{\dot{X}_{it|\tau}}{X_{it|\tau}} = \frac{1}{\sigma}\left[r_t - \rho - \frac{\dot{P}_t}{P_t}\right]. \tag{8}$$

Lifetime wealth for any agent is equal to his initial deposits, $a_{i\tau|\tau}$. Again, it is noteworthy that, given the realization of this initial deposit, there is no further uncertainty as to future consumption. As there exists no bequest motive in this economy, integration of the first line of equation (2) yields the condition that optimization requires the present discounted value of lifetime consumption to be equal to $a_{i\tau|\tau}$:

$$a_{i\tau|\tau} = \int_\tau^\infty e^{-(\bar{r}_{t\tau}+\upsilon)(t-\tau)} P_t X_{it|\tau}\, dt. \tag{9}$$

Integrating (8) gives consumption at every t,

$$X_{it|\tau} = X_{i\tau|\tau} \cdot exp\{1/\sigma(\bar{r}_{t\tau} - \rho - \dot{P}_t/P_t)(t-\tau)\}, \tag{10}$$

and using this result in (9) allows optimal consumption to be expressed as

$$X_{i\tau|\tau} = \frac{a_{i\tau|\tau}}{P_\tau} \cdot \mu_\tau, \tag{11}$$

where $\mu_\tau = (\bar{r}_{t\tau} - \dot{P}_t/P_t)(1 - 1/\sigma) + \rho/\sigma + \upsilon$.

All agents of cohort τ will consume according to (10) and (11), regardless of their occupation. The only factor in which their consumption plans will differ is the amount of initial deposits, $a_{i\tau|\tau}$, which will depend on their income. To determine the income of a successful researcher, the behavior of firms and the market for ideas will be introduced in the following section.

3 Output

3.1 Production

The production and innovation side of the economy is a closed economy (one country) version of the model introduced by Eaton and Kortum (2001). Time is continuous. The economy produces at each t a continuum of goods, indexed by $j \in [0, 1]$. Labor is the only input in production. How many units of good j can be produced with a unit input of labor depends on the best available efficiency of production, $z(j)$, and differs across goods j. $z(j)$ represents the state of the art of production in sector j, and $\{z(j)|j \in [0, 1]\}$ is referred to as the technological frontier of the economy. The frontier is common knowledge. Innovative activity in the economy is focused on expanding this frontier.

3.2 Innovation

Let R_t be the measure of individuals that are researchers at t. Researchers in the economy obtain ideas about how to produce goods more efficiently. To each researcher, ideas arrive as a Poisson process with parameter α[8]. Each idea is the realization of two random variables, namely the good j to which it belongs (i.e., research is undirected), and the efficiency $q(j)$ with which j can be produced. While the type of good is drawn from the uniform distribution over $[0, 1]$, it is assumed that the efficiency $q(j)$ is drawn from the Pareto distribution $H(q) = 1 - q^{-\theta}$, which is the same for all sectors[9].

Any new idea of efficiency $q(j)$ will only be used in the production of j if its efficiency is beyond the current state of the art, i.e., if $q(j) > z(j)$. For determining the expected return to R&D, it is imperative to know the probability with which this occurs. It is easiest to treat the individual $z(j)$'s as realizations of a random variable Z drawn from a distribution F. To derive this distribution, denote the total stock of ideas in the economy by time t as $T_t = \alpha \int_0^t R_s \, ds$. As there is a unit interval of goods and T_t is the total stock of ideas, it follows that the number of ideas that have arrived for a specific good j follows a Poisson distribution with parameter T_t. Due to the Poisson arrival of ideas, the probability that k ideas for producing good j have arrived by t is

$$\frac{T_t^k e^{-T_t}}{k!}.$$

If k ideas have arrived, the probability that the best one is less than a given z is $[H(z)]^k$. Putting these two facts together and summing over all possible numbers of ideas results in the technological frontier of the economy:

$$F_t(z) = \sum_{k=0}^{\infty} \frac{T_t^k e^{-T_t}}{k!} H(z)^k = e^{-T_t[1-H(z)]} = e^{-T_t z^{-\theta}}, \quad z \geq 1. \tag{12}$$

[8]This parameter can be seen as the efficiency of research.

[9]The parameter θ governs the variation in efficiencies of production.

3.3 Costs of Production

Given a state of the art $z(j)$, and using the wage as the numeraire, the cost of producing one unit of good j in period t is given by

$$c(j) = \frac{1}{z(j)}. \tag{13}$$

A higher efficiency in production, $z(j)$, implies lower production costs. Knowing that the $z(j)$'s are drawn from $F_t(z)$ as given in equation (12), the costs of production are realizations drawn from

$$G_t(c) = 1 - e^{-T_t c^{\theta}}. \tag{14}$$

With the distribution of production costs given by $G_t(c)$, it is possible to calculate the probability that a new idea $q(j)$ will be used in production, which is the probability that $1/q(j)$ is the lowest production cost in sector j. As this is once again the same for all sectors, the probability of having discovered the lowest cost idea can be given by $1 - G_t(1/q)$. Define $m = q(j)/z(j)$ as the inventive step of the new idea $q(j)$, i.e., an idea will be used in production if its inventive step is at least 1. Then, the probability that an idea q will have an inventive step of at least $m \geq 1$ is accordingly $1 - G_t(m/q)$. Integrating over the Pareto distribution of q gives the probability $b_t(m)$ that an idea will have an inventive step of at least m:

$$b_t(m) = \int_1^{\infty} [1 - G(m/q)] dH(q) \approx \frac{1}{T_t m^{\theta}} \tag{15}$$

and setting $m = 1$ gives the probability $b_t(1) = 1/T_t$ of idea q surpassing the previous state of the art at all[10].

This implies that the probability of any state of the art idea z of still being the state of the art by time $s > t$ is simply $b_s(1)/b_t(1) = T_t/T_s$.

[10] The approximation in (15) follows the argument in Eaton and Kortum (2001), and is based on the handling of efficiencies that are less than 1.

The markup of a new idea, m, is the realization of the random variable M. The distribution of the markup, conditional on an idea having surpassed the previous state of the art, is given by

$$Pr[M \leq m | M \geq 1] = \frac{b_t(1) - b_t(m)}{b_t(1)} = H(m), \tag{16}$$

i.e., it is itself Pareto and independent of time.

4 Profits and Prices

In this economy, firms have the production technology to turn labor input into consumption goods. There exists a continuum of firms, but not all of these are going to be active at any particular time t. Active firms compete à la Bertrand. Under standard Bertrand competition, firms set prices to marginal costs and make zero profit. However, if any single firm can use an idea with efficiency $q(j) > z(j)$, this firm can make a positive profit by charging the marginal costs of any competitor producing with efficiency $z(j)$. Therefore, in equilibrium there will only be one active firm in each sector j that makes strictly positive profits.

Let $Y_t = \int_0^t \int_i y_{it|\tau} \; di \; d\tau$ be total expenditure at time t. As there is a unit continuum of goods, Y_t is also the expenditure per variety of good. The markups that firms can charge over their costs are the m's drawn from $H(m)$, and therefore total profits earned are

$$\Pi_t = Y_t \int_0^1 [1 - m(j)^{-1}] dj = Y_t \int_1^\infty (1 - m^{-1}) H(m) = \frac{Y_t}{1+\theta}. \tag{17}$$

Y_t, in return, is the sum of total profit and total wage income in the economy at t[11]. While total profit income is determined by (17), total wage income depends on the measure of individuals who choose to work in the production sector. Individuals only earn income the moment

[11] This result goes back to the assumption about the financial intermediary just reallocating funds in the economy.

they are born, τ. Population grows at rate n, and people die at rate υ. The measure of agents that are born at time t is then given by $n \cdot L_t$. Out of these, a certain fraction β_t will choose to become researchers. Given the wage of 1, total expenditure / income is

$$\begin{aligned} Y_t &= (1-\beta_t)nL_t + \frac{Y_t}{1+\theta} \\ Y_t &= \frac{1+\theta}{\theta}(1-\beta_t)nL_t \end{aligned} \tag{18}$$

Finally, the assumption of Bertrand competition also allows me to exactly determine the ideal price index from (7). As prices are set to marginal costs, this price index can be determined as a function of production costs,

$$P_t = exp \int_0^1 ln\ p_t(j)dj \ = exp \int_0^1 ln\ c_t(j)\ dj \ = exp \int_0^\infty ln(c)\ dG(c) \ = \gamma\ T_t^{-1/\theta}, \tag{19}$$

where γ is Euler's constant.

Together, (18) and (19) show that average consumption per person (x_t) can change either because of a change in the fraction of researchers in the population, or because of a change in the price index:

$$x_t = \frac{Y_t/L_t}{P_t} = \frac{\frac{1+\theta}{\theta}(1-\beta_t)n}{\gamma T_t^{-1/\theta}}. \tag{20}$$

In particular, it is likely to change over time as the price index depends negatively on the aggregate stock of R&D in the economy, T_t. Therefore, any equilibrium with a growing stock of ideas will see a decreasing price index and a growth of average consumption per capita.

5 Equilibrium

5.1 Equilibrium Condition

An equilibrium in which both workers and researchers are active requires that the utility from working is equal to the expected utility from doing R&D. Using (10) and (11), this condition becomes:

$$\int_{\tau}^{\infty} e^{-(\rho+\upsilon)(t-\tau)} \frac{\left[\frac{1}{P_\tau}\mu_\tau e^{1/\sigma(r_t-\rho-\dot{P}_t/P_t)(t-\tau)}\right]^{1-\sigma}}{1-\sigma}\, dt =$$

$$\frac{\alpha}{T_\tau}\int_{\tau}^{\infty} e^{-(\rho+\upsilon)(t-\tau)} \frac{\left[\frac{V_\tau}{P_\tau}\mu_\tau e^{1/\sigma(r_t-\rho-\dot{P}_t/P_t)(t-\tau)}\right]^{1-\sigma}}{1-\sigma}\, dt + \left(1-\frac{\alpha}{T_\tau}\right)\cdot 0$$

which simplifies to

$$1 = \frac{\alpha}{T_\tau} V_\tau^{1-\sigma}, \tag{21}$$

and where the probability to innovate (i.e., to draw an idea with an efficiency beyond the current state of the art) takes into account the arrival rate of ideas as well as the probability to have a draw beyond the current state of the art.

5.2 Value of R&D

The equilibrium condition of (21) holds for any t. The equilibrium measure of researchers will determine the equilibrium consumption growth. To determine it, V_t must be determined. It has to take into account the profit flows, as well as the interest rate, and the decreasing probability of being the state of the art over the lifetime of an idea. As it simplifies the analysis considerably, in the derivations I assume that the equilibrium interest rate, r_t, will be a constant r. This assumption will be true in equilibrium, and the interest rate will be explicitly solved for in section 5.5.

The value of an idea is equal to the present discounted value of the future streams of profits that this idea grants. Apart from profit flows, Π_t, this value has to take into account the discount rate (r), the probability of the payoffs ending at any $s > t$ if a better idea arrives $(b_s(1)/b_t(1))$, and the fact that the price level will change over time. This implies that the value of an innovation is given by

$$\begin{aligned} V_t &= P_t \int_t^{\infty} e^{-r(s-t)} \frac{\Pi_s}{P_s} \frac{b_s(1)}{b_t(1)} ds \\ &= \frac{P_t}{1+\theta} \int_t^{\infty} e^{-r(s-t)} \frac{Y_s}{P_s} \frac{T_t}{T_s} ds. \end{aligned} \tag{22}$$

Making use of (18), this expression can be further re-written as

$$V_t = \frac{P_t}{\theta} \int_t^{\infty} e^{-r(s-t)} \frac{(1-\beta_s) n L_s}{P_s} \frac{T_t}{T_s} ds. \tag{23}$$

5.3 The Stock of Ideas

The stock of ideas in the economy depends on the path of researchers. Assume that in equilibrium the "gross" amount of researchers in the economy grow at a constant rate g, and face the same death rate υ as the entire population, i.e., the total measure of researchers grows at rate $g - \upsilon$. As with workers, researchers are only active when they are first born, i.e., the measure of active researchers at t is not R_t, but rather gR_t. The level of technology evolves according to

$$\dot{T}_t = \alpha \dot{R}_t = \alpha g R_t, \tag{24}$$

i.e., it increases with the active number of researchers at each t. This implies that

$$T_t = \alpha \int_0^t g e^{(g-\upsilon)s} R_0 \, ds = \frac{\alpha g}{g-\upsilon} R_t - \frac{\alpha g}{g-\upsilon} R_0 \tag{25}$$

and as $t \to \infty$, the growth rate of technology converges to $\dot{T}_t / T_t = g - \upsilon$.

As population grows at rate $n - \upsilon$, the level of technology relative to population converges to

$$\frac{T_t}{L_t} = \frac{\alpha g}{g-\upsilon} \frac{R_t}{L_t} = \frac{\alpha g \Omega_0}{g-\upsilon} e^{at} \tag{26}$$

where $\Omega_0 = R_0 / L_0$ and $a = g - n$.

As the price index in the economy is $P_t = \gamma T_t^{-1/\theta}$, the growth rate of the price index is determined by the growth rate of the stock of ideas

$$\frac{\dot{P}_t}{P_t} = -\frac{g-\upsilon}{\theta}, \tag{27}$$

i.e., the overall price level will decrease if $g - \upsilon > 0$. This will facilitate growth in consumption.

5.4 Solution

The expected present discounted value of a successful idea from equation (22) depends on the expectations on the future path of research, through its influence on the future level of technology. An equilibrium path of research is one such that the expected path of research stimulates the amount of research to make the expectation valid. If the perceived growth rate of the measure of researchers is $g - \upsilon$, making use of the results from the previous section, the value of a successful innovation from equation (23) can be re-written as

$$
\begin{aligned}
V_t &= \frac{P_t}{\theta} \int_t^{\infty} e^{-r(s-t)} \frac{(1-\beta_s) n L_s}{P_s} \frac{T_t}{T_s} \, ds \\
&= \frac{n}{\theta} \left[-L_t \frac{1}{\frac{g-\upsilon}{\theta} - r - a} + R_t \frac{1}{\frac{g-\upsilon}{\theta} - r} \right].
\end{aligned} \tag{28}
$$

To obtain a finite solution for the value of R&D, the above derivations assume that the parameter values satisfy $r > max\{\frac{g-\upsilon}{\theta} - (g-n), \ \frac{g-\upsilon}{\theta}\}$. The exact steps and definitions utilized in (28) can be found in the appendix.

Under these assumptions, equation (21) can be solved for the number of researchers active in period t:

$$
\begin{aligned}
1 &= \left[\frac{\alpha}{T_t}\right]^{1/(1-\sigma)} \frac{n}{\theta} \left[L_t \frac{1}{r + a - \frac{g-\upsilon}{\theta}} - R_t \frac{1}{r - \frac{g-\upsilon}{\theta}} \right] \\
\frac{\theta}{n} \left[\frac{T_t}{\alpha}\right]^{1/(1-\sigma)} &= L_t \frac{1}{r + a - \frac{g-\upsilon}{\theta}} - R_t \frac{1}{r - \frac{g-\upsilon}{\theta}} \\
R_t &= \frac{r - \frac{g-\upsilon}{\theta}}{r + a - \frac{g-\upsilon}{\theta}} L_t - (\frac{\theta r}{n} - \frac{g-\upsilon}{n}) \left[\frac{T_t}{\alpha}\right]^{1/(1-\sigma)}
\end{aligned} \tag{29}
$$

The solution in (29) is for a given perception of the future growth rate of R&D, namely that the measure of researchers grows at rate $g - \upsilon$. An equilibrium requires that the actual growth rate of the measure of researchers in (29) is equal to this perceived growth rate. In the long run, the determining factor in the evolution of (29) is whether the growth rate of L_t is bigger (in absolute value) than the growth rate of $T_t^{1/(1-\sigma)}$ or not. This implies that the actual growth rate of R_t will, in the long run, be either $n - \upsilon$ (from L_t) or $\frac{1}{1-\sigma}(g-\upsilon)$ (from $T_t^{1/(1-\sigma)}$). There are two cases to consider under which the perceived and the actual growth rates of R_t are identical:

1. $n - \upsilon \geq \frac{1}{1-\sigma}(g-\upsilon)$: Under this condition, the actual growth rate of R_t will go to $n - \upsilon$ and

the perceived and the actual growth rate of R_t are identical if $n - \upsilon = g - \upsilon$. This implies that $n = g$. However, as the occupational choice problem in this setup is only well defined if $\sigma < 1$, the condition that $n - \upsilon \geq \frac{1}{1-\sigma}(g - \upsilon)$ can hold with $n = g$ only if $n = g = \upsilon$. I.e., perceived and actual growth rate are only identical if there is neither growth in the population nor in the measure of researchers.

2. $\frac{1}{1-\sigma}(g-\upsilon) > n-\upsilon$: Under this condition, R_t will indeed *decrease*, at a rate of $\frac{1}{1-\sigma}(g-\upsilon)$, and will asymptotically go to zero. Actual and perceived growth of R_t are only equal if $g - \upsilon = -\frac{1}{1-\sigma}(g - \upsilon)$. For any $\sigma < 1$, this can only hold if $g = \upsilon$. However, if $g = \upsilon$, the condition for this case to occur is only satisfied for $n < \upsilon$, i.e., population would actually have to *decrease* over time.

The two possible cases that govern the behavior of R_t in equilibrium are indeed peculiar. It is particularly interesting to note that there exists no single value of positive population growth under which the perceived and actual growth rate of R_t will be identical. The result that a no-growth equilibrium results for the case of constant population is, on the other hand, standard for the "semi-endogenous" growth models and is independent of the introduction of risk aversion.

5.5 Consumption Growth and Parameter Values

The solution found in (29) is in many ways unsatisfactory from an analytical point of view. Also, it is based on the assumption of a constant interest rate, which has yet to be verified. To do so, the path of consumption (per capita) has to be determined. A second question to consider is whether there are any parameter values for which there exists a balanced growth path with constant (possibly zero) consumption growth.

Equation (8) together with (27) determine the consumption growth for an individual born at τ. This, however, is not necessarily equal to the average consumption growth per capita, as individuals of different age will hold different amounts of deposits. To derive the average consumption growth per capita, consumption of the individuals must be integrated over all cohorts τ. I assume that $L_0 = 1$, and that all individuals at this point are young. This implies that the measure of individuals born at τ is equal to $L_{\tau|\tau} = n\ exp\{(n-\upsilon)t\}$ and that the measure of individuals born at τ and still alive at $t > \tau$ is $L_{t|\tau} = n\ exp\{(n-\upsilon)t - \upsilon(t-\tau)\}$. Consumption at t for an agent born at τ is given by (11) as

$$X_{it|\tau} = \frac{a_{it|\tau}}{P_t} \cdot \mu_t. \tag{30}$$

By the definition of $\mu_t = \bar{r}_{t\tau} + (g-\upsilon)/\theta(1-1/\sigma) + \rho/\sigma + \upsilon$, it does not depend on τ (nor t) if the interest rate is constant, and neither does the price level. Using $\mu_t = \mu$ and integrating (30) over τ yields average consumption per capita, x_t:

$$x_t = \int_0^t X_{it|\tau} n e^{-\upsilon(t-\tau)+(n-\upsilon)t}\ d\upsilon = \frac{\mu}{P_t} a_t, \tag{31}$$

where

$$a_t = \int_0^t a_{it|\tau} n e^{-\upsilon(t-\tau)+(n-\upsilon)t}\ d\upsilon$$

is the average level of deposits per capita.

By (31), the growth rate of average consumption per capita is

$$\frac{\dot{x}_t}{x_t} = \frac{1}{x_t}\left[-\mu\frac{1}{P_t^2}\dot{P}_t a_{it} + \frac{\mu}{P_t}\dot{a}_{it}\right] \tag{32}$$

and as average deposits per capita evolve according to $\dot{a}_{it} = r \cdot a_{it} - P_t x_t$, this becomes

$$\frac{\dot{x}_t}{x_t} = r + \frac{g-\upsilon}{\theta} - \mu, \tag{33}$$

which makes use of the definition of $\dot{P}_t/P_t$. (33) implies that a constant growth rate of average consumption per capita requires a constant interest rate and vice versa, i.e., the assumption of a constant interest rate will be verified if average consumption per capita grows at a constant rate.

Using (20), the resource constraint of the economy implies that the growth rate of average consumption per capita is

$$\frac{\dot{x}_t}{x_t} = -\frac{\dot{\beta}_t}{1-\beta_t} + \frac{g-\upsilon}{\theta}. \tag{34}$$

The result in (34) implies that in the short run, consumption per capita only grows at a constant rate if $g = n$, under which condition the fraction of researchers stays constant ($\beta_t = \beta$) and the first term in (34) disappears. In this case, $r = \mu$ will hold and the interest rate will solve as $r = \frac{n-\upsilon}{\theta}(\sigma - 1) + \rho + \upsilon\sigma$, i.e., the interest rate will be constant, and consumption per capita will grow at a rate of $(n-\upsilon)/\theta$. In the long run, however, any value in which $n \neq g$ will also imply a constant growth rate of consumption per capita, as the fraction of researchers, β_t, will either go to 0 or to 1, at which point $\dot{\beta}_t$ will be 0, and the first term of (34) also disappears. The assumptions made to obtain (29) are therefore always verified in the long run, independent of the actual values of n and g.

As the interest rate does turn out to be constant in equilibrium, the question remains how much can be deduced about the growth rate of researchers (and therefore consumption per capita) under risk aversion. Figure 1 summarizes graphically a number of points that can be made. First of all, the increase of researchers under risk aversion is strictly less than under risk neutrality; it is bounded above by the growth rate of researchers under risk neutrality. Second, a constant measure of researchers constitutes a lower bound. Third, no constant growth rate between 0 and $n - \upsilon$ will be an equilibrium either. This leaves as a possible smooth candidate for an equilibrium a decreasing growth rate, as shown by the dashed line in figure 1. If this was the case, e.g., if the growth rate of researchers was of the form $C \cdot t^{\gamma-1}$,$\gamma < 1$, then the measure of researchers would still increase without bounds, although its growth rate would tend to zero as $t \to \infty$ and the overall fraction of researchers would also go to zero. In this case, the growth rate of consumption per capita would, consequently, tend toward zero as well.

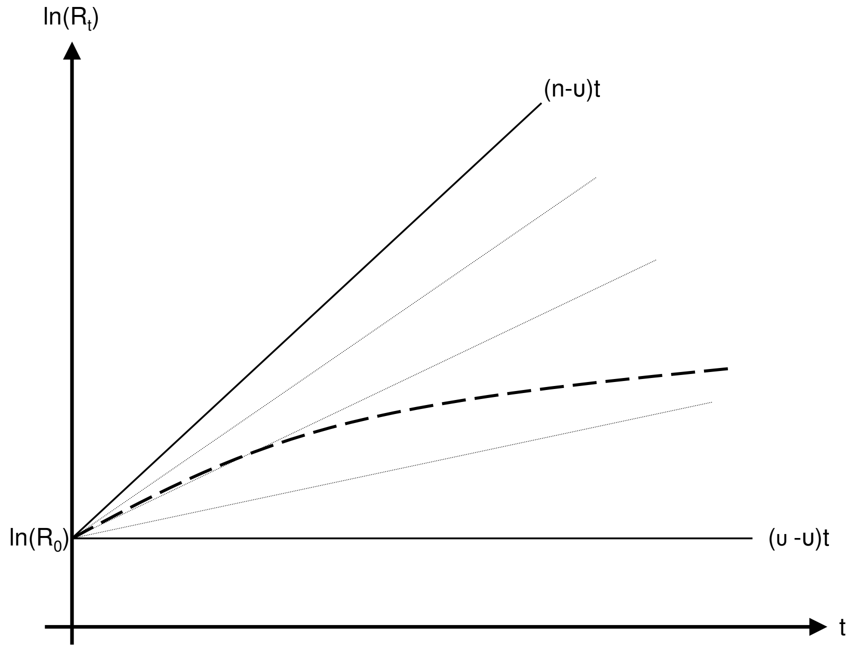

Figure 1: Possible Values of Growth in Researchers

Unfortunately, a non-constant growth rate of researchers leaves the model analytically intractable. At this stage, probably the most important conclusion that can be drawn is that the introduction of risk aversion not only lowers the equilibrium growth rate of researchers (and therefore consumption per capita), but it also prohibits any constant growth rate at all.

6 Conclusions

The vast majority of the work on endogenous growth shares the assumption that agents are risk neutral. This is a valid assumption under perfect capital markets, where agents can borrow against their expected future income from innovation, or if innovation is carried out by large organizations that can diversify the risk inherent in R&D. However, in reality it seems likely that there exists an information asymmetry between private innovators and financial investors.

Innovators are supposedly better informed than investors about both the likelihood of success and the payoffs associated with their innovation. Neither can individuals who engage in R&D diversify the inherent risk. Even if R&D is done mainly by quoted firms, where the shareholders can hold well-diversified portfolios, the decision to engage in any specific R&D project lies with a limited number of agents within the firm, who might bear more of the risk than the actual shareholders. Empirical studies have indeed shown that the costs to finance R&D are high, and that capital markets are not really perfect. Under these conditions, it seems important to study whether the introduction of risk aversion affects the basic conclusions of endogenous growth models.

To study this question, the present paper incorporates risk averse agents in a model of occupational choice which is based on a model by Eaton and Kortum (2001). This model in turn is closely related to Kortum (1997), and its results on economic growth are in line with the "semi-endogenous" growth models: consumption growth per capita ultimately depends on the population growth rate, and technological parameters that are usually exogenously given. The equilibrium balanced growth path encompasses a constant fraction of researchers. The introduction of risk averse agents calls for a further change in the analysis, namely, finitely-lived agents. From a theoretic point of view, finite lives are necessary to ensure that innovators cannot borrow against expected future income. Over an infinite horizon, the future income from R&D could be calculated and the natural borrowing constraint would be the expected income from R&D. Finite lives, however, do not affect the main analysis, i.e., the occupational choice between working for a wage and working as a researcher, in any other way than in setting the borrowing constraint.

Ex ante, it was assumed that risk averse agents should in some way decrease the fraction of the population that chooses to become researchers, as this is the riskier occupation. Indeed, one result that has been found is that there does not exist an equilibrium anymore in which a constant fraction of the population becomes researchers. The actual growth rate of the measure of researchers is always bounded from above by the growth rate of the population. At the same time, no other exponential increase in researchers, nor a constant measure of researchers, turns out to be an equilibrium in which the perceived and the actual growth rate of researchers are identical. Analytically, it can only be stated that if there exists an equilibrium rate at which the measure of researchers increases, then this value must be (i) a non-constant (positive) rate, and (ii) a rate strictly less than the rate at which total population increases. This rate could not be analytically determined. For the future, it seems necessary to investigate to which extent these results are based on the assumption that the level of risk aversion is bounded between 0 and 1. If, however, the results pertain even with no restrictions on the level of risk aversion, it seems that a simulation of large parts of the economy would be needed to find the particular rate at which the measure of researchers grows. This as well is outside the scope of the present paper, but might prove interesting in further studies.

A The Value of R&D

The actual steps that are taken to arrive at the final value of (28) are as follows:

$$
\begin{aligned}
V_t &= \frac{P_t}{\theta}\int_t^\infty e^{-r(s-t)}\frac{(1-\beta_s)nL_s}{P_s}\frac{T_t}{T_s}\,ds \\
&= \frac{T_t}{\theta}\int_t^\infty e^{((g-\upsilon)/\theta-r)(s-t)}(1-\beta_s)n\frac{L_s}{T_s}\,ds \\
&= \frac{T_t}{\theta}\int_t^\infty e^{((g-\upsilon)/\theta-r)(s-t)}(1-\beta_t e^{a(s-t)})n\frac{L_t}{T_t}e^{-a(s-t)}\,ds \\
&= \frac{nL_t}{\theta}\int_t^\infty e^{((g-\upsilon)/\theta-r-a)(s-t)}-\beta_t e^{((g-\upsilon)/\theta-r)(s-t)}\,ds \\
&= \frac{nL_t}{\theta}\left\{\left[\frac{1}{(g-\upsilon)/\theta-r-a}e^{((g-\upsilon)/\theta-r-a)(s-t)}\right]_t^\infty-\beta_t\left[\frac{1}{(g-\upsilon)/\theta-r}e^{((g-\upsilon)/\theta-r)(s-t)}\right]_t^\infty\right\} \\
&= \frac{n}{\theta}\left[-L_t\frac{1}{\frac{g-\upsilon}{\theta}-r-a}+R_t\frac{1}{\frac{g-\upsilon}{\theta}-r}\right].
\end{aligned}
$$

The first step takes advantage of the fact that $P_s = P_t e^{-((g-\upsilon)/\theta)(s-t)}$ from (27). The second to third line in turn uses the result on the ratio T_t/L_t from (26). The final result then follows from the solution to the integral.

References

Acemoglu, D. (2009): *Introduction to Modern Growth Theory*. The MIT Press.

Aghion, P., and P. Howitt (1992): "A Model of Growth Through Creative Destruction," *Econometrica*, 60(2), 323–351.

——— (1998): *Endogenous Growth Theory*. Cambridge, MA: MIT Press.

Akerlof, G. A. (1970): "The Market for "Lemons": Quality Uncertainty and the Market Mechanism," *The Quarterly Journal of Economics*, 84(3), 488–500.

Alcalá, F., and A. Ciccone (2004): "Trade and Productivity," *The Quarterly Journal of Economics*, 119(2), 613–646.

Banerjee, A. V., and A. F. Newman (1991): "Risk-Bearing and the Theory of Income Distribution," *The Review of Economic Studies*, 58(2), 211–235.

Barro, R. J., and X. Sala-i-Martin (2003): *Economic Growth*. MIT Press, 2 edn.

Blanchard, O. J. (1985): "Debt, Deficits, and Finite Horizons," *The Journal of Political Economy*, 93(2), 223–247.

Dasgupta, P., and J. Stiglitz (1980): "Uncertainty, Industrial Structure, and the Speed of R&D," *The Bell Journal of Economics*, 11(1), 1–28.

David Card, R. C., and A. Weber (2007): "Cash-on-Hand and Competing Models of Intertemporal Behaviour: New Evidence from the Labor Market," *The Quarterly Journal of Economics*, 122(4), 1511–1560.

Dixit, A. K., and J. E. Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity," *The American Economic Review*, 67(3), 297–308.

Eaton, J., and S. Kortum (2001): "Technology, trade, and growth: A unified framework," *European Economic Review*, 45, 742–755.

Evans, D. S., and B. Jovanovic (1989): "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints," *The Journal of Political Economy*, 97(4), 808–827.

García-Peñalosa, C., and J.-F. Wen (2006): "Redistribution and Entrepreneurship with Schumpeterian Growth," .

Grossman, G. M., and E. Helpman (1991a): *Innovation and Growth in the Global Economy*. Cambridge: MIT Press.

——— (1991b): "Quality Ladders in the Theory of Growth," *The Review of Economic Studies*, 58(1), 43–61.

Hall, B. H. (2002): "The Financing of Research and Development," *Oxford Review of Economic Policy*, 18(1), 35–51.

Jones, C. I. (1995): "R&D-Based Models of Economic Growth," *The Journal of Political Economy*, 103(4), 759–784.

——— (2005): "Growth and Ideas," in *Handbook of Economic Growth*, ed. by P. Aghion, and S. Durlauf, vol. 1, chap. 16, pp. 1063–1111. Elsevier, 1 edn.

Kortum, S. (1997): "Research, Patenting, and Technological Change," *Econometrica*, 65(6), 1389–1419.

Kremer, M. (1993): "Population Growth and Technological Change: One Million B.C. to 1990," *The Quarterly Journal of Economics*, 108(3), 681–716.

Lee, T., and L. L. Wilde (1980): "Market Structure and Innovation: A Reformulation," *The Quarterly Journal of Economics*, 94(2), 429–436.

Levhari, D., and T. N. Srinivasan (1969): "Optimal Savings Under Uncertainty," *The Review of Economic Studies*, 36(2), 153–163.

Loury, G. C. (1979): "Market Structure and Innovation," *The Quarterly Journal of Economics*, 93(3), 395–410.

Merton, R. C. (1969): "Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case," *The Review of Economics and Statistics*, 51(3), 247–257.

Reinganum, J. F. (1985): "Innovation and Industry Evolution," *The Quarterly Journal of Economics*, 100(1), 81–99.

Romer, P. M. (1986): "Increasing Returns and Long-Run Growth," *The Journal of Political Economy*, 94(5), 1002–1037.

——— (1990): "Endogenous Technological Change," *The Journal of Political Economy*, 98(5, Part 2), S71–S102.

Samuelson, P. S. (1969): "Lifetime Portfolio Selection By Dynamic Stochastic Programming," *The Review of Economics and Statistics*, 51(3), 239–246.

Schumpeter, J. A. (1943): *Capitalism, Socialism and Democracy*. London: G. Allen & Unwin.

Segerstrom, P. S. (1998): "Endogenous Growth witout Scale Effects," *The American Economics Review*, 88(5), 1290–1310.

Segerstrom, P. S., T. Anant, and E. Dinopoulos (1990): "A Schumpeterian Model of the Product Life Cycle," *The American Economic Review*, 5(5), 1077–1091.

Yaari, M. E. (1965): "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer," *The Review of Economic Studies*, 32(2), 137–150.

Zeira, J. (2005): "Innovations, Patent Races and Endogenous Growth," Discussion paper, C.E.P.R. Discussion Papers.