Inter-sectorial Knowledge Diffusion and Scale Effects

Elie Gray*, André Grimaud†

October 2011

Abstract

Knowledge diffusion, both within and across sectors, shapes the pools of knowledge used by R&D activities. Exploiting the formalization of Salop (1979), we propose a Schumpeterian growth model explicitly considering that each innovation influences a more or less significant range of sectors. Besides encompassing most models of the related literature, this framework enables us to reassess the issue of scale effects, showing that most frameworks developed to remove scale effects amount to assuming no inter-sectorial diffusion of knowledge, and that it is possible to eliminate scale effects while maintaining the effects of public policies, but without suppressing knowledge diffusion.

Keywords: Economic Growth, Schumpeter, Scale Effects, Knowledge Accumulation, Inter-sectorial Knowledge Diffusion, R&D, Knowledge Spillovers, Non Rivalry.

JEL Classification: O30, O31, O33, O40, O41.

*Université de Toulouse, Toulouse Business School. *Address:* Toulouse Business School - Groupe ESC Toulouse, 20 Boulevard Lascrosses, BP 7010 31068 Toulouse Cedex 7. *Email:* e.gray@esc-toulouse.fr

†Université de Toulouse, Toulouse School of Economics (IDEI, LERNA) and Toulouse Business School. *Address:* Toulouse School of Economics (LERNA), Manufacture des Tabacs, 21 allée de Brienne, 31000 Toulouse. *Email:* grimaud@cict.fr

We thank Philippe Aghion, Hippolyte d'Albis, Jean-Pierre Amigues, Jean-Luc Gaffard, Maurizio Iacopetta, Pietro Peretto, Gilles Saint-Paul, and Francesco Vona for comments and suggestions.

1 Introduction

Technological progress plays a major part in the striking phenomenon of long-run growth. This process of knowledge accumulation has been formalized by the endogenous growth theory based on innovation through two complementary paradigms. The first one considers that growth is driven by a diversification of the variety of intermediate goods, as in Romer (1990), for instance. The second one, developed notably in Grossman & Helpman (1991) or in Aghion & Howitt (1992), introduces quality improving innovations based on stochastic and sequential R&D races. The keystone of knowledge accumulation lies in the presence of the externality triggered by the non rivalry property of knowledge, commonly referred to as "knowledge spillovers": knowledge produced in any given sector can potentially spill from this sector over other sectors. In other words, once knowledge has been generated by a research and development (R&D) activity, it diffuses across others R&D activities. The significance of the interactions between sectors (*i.e.* of inter-sectorial knowledge spillovers) has universally been underlined. In particular, several empirical studies stress that R&D performed in one sector may produce positive spillovers effects in other sectors (see, for instance, Griliches, 1992, 1995; or Hall, Mairesse & Mohnen, 2010). As stated in Hall, Mairesse & Mohnen (2010), "such spillovers are all the more likely and significant as the sender and the receiver are closely related". Moreover, as argued by Hall (2004), "it is safe to say that without diffusion, innovation would have little social or economic impact. In the study of innovation, the word diffusion is commonly used to describe the process by which individuals and firms in a society/economy adopt a new technology, or replace an older technology with a newer". Furthermore, the term "diffusion" has often been used to refer to the phenomenon involving that, as stated by Chari & Hopenhayn (1991), "there is a lag between the appearance of a technology and its peak usage". This temporal dimension of knowledge diffusion is undoubtedly important; in this paper, however, we abstract away from it by considering instantaneous diffusion. Our analysis focuses on the spatial dimension of knowledge diffusion: in direct line with the above empirical statements, we explicitly introduce inter-sectorial knowledge diffusion in a canonic fully endogenous Schumpeterian growth model. More precisely, we consider that the knowledge produced in each sector can potentially influence the R&D activity of any other sector.

In the standard growth literature, innovations are produced drawing previously created knowledge from a given *pool of knowledge*. Moreover, the intensity of knowledge spillovers varies considerably from one framework to the other. In Romer (1990), or in Jones (1995b), all the knowledge accumulated so far in the economy is used to produce new knowledge. Similarly, in Aghion & Howitt (1992), in Young (1998), in Howitt (1999), in Segerstrom (2000), or in Garner (2010), spillovers depend on the knowledge level of the frontier firms. In those models, knowledge spillovers are implicitly assumed to be global. In other words, the pool of knowledge used by each R&D activity comprises the whole knowledge accumulated so far in the economy. To some extend, this theory implicitly takes into account the generally agreed fact that "one sector gets ideas from the research and experience of others"[1]. In contrast, in Grossman & Helpman (1991), in Segerstrom (1998), in Peretto (1999), or in Aghion & Howitt (2009), for example, spillovers are assumed to be intra-sectorial only (*i.e.* knowledge is sector-specific). In Peretto (1998), Dinopoulos & Thompson (1998), or Li (2000, 2003), spillovers depend on the average knowledge in the economy. In spite of the fact that knowledge spillovers are ubiquitous in modern growth literature, their formalization has been somehow under-investigated. Notable exceptions are the models of Li (2000, 2002, 2003) and Peretto & Smulders (2002). Li emphasizes the role of these spillovers and shows that semi-endogenous growth theory is more general than fully endogenous growth theory in that the latter relies on knife-edge conditions. The model of Peretto & Smulders is based on the concept of spillovers networks. It focuses on knowledge spillovers within firms and industries and on the role of market structure in shaping them. In particular, the authors introduce a concept of "technological distance" to examine the interaction between firms.

In the present framework, we expressly assume that each piece of knowledge produced in any given sector can diffuse with more or less intensity in the economy. As in Peretto & Smulders (2006), we can define a technological distance. As expected, this distance is decreasing with the average intensity of diffusion of knowledge within the whole economy. Depending on its extent, an innovation can potentially influence the creation of knowledge in sectors that are more or

[1] Aghion & Howitt (1998, Ch. 3).

less technologically distant. Firstly, this allows us to propose a more accurate formalization of the aforementioned knowledge spillover externality. Secondly, it enables us to shed a light on the mechanism by which the pools of knowledge in which R&D activities draw from in order to produce new knowledge are shaped.

Besides knowledge spillovers, the non rivalry property of knowledge has another significant implication. Because a piece of knowledge can be used infinitely without any additional cost, once its production cost has been incurred, technologies (e.g. final good or knowledge production functions) using knowledge as an input exhibit increasing returns to scale. As stated by Eicher & Turnovsky (1999) or by Jones (1999, 2005), non rivalry of knowledge is at the source of a non desirable scale effets property in the seminal endogenous growth models: the models of Romer (1990), Grossman & Helpman (1991), Aghion & Howitt (1992), among others, all have in common to predict that the economy's long-run per capita growth rate increases in its size, measured by the level of population. The presence of scale effects raises both an empirical and a theoretical problem. It is commonly agreed that the presence of this property is strongly inconsistant with twentieth century observed stylized facts. Indeed, empirical evidence both for the United States (Backus, Kehoe & Kehoe, 1992) and for OECD countries (Jones, 1995a) have invalidated the fact that, the larger the scale of the economy is, the stronger growth will be. Furthermore, if population grows at a positive and constant rate, then the economy's per capita growth rate increases exponentially over time and eventually becomes infinite in the steady-state.

The issue of scale effects in growth models has been reviewed in a large body of literature[2]. However, to the best of our knowledge, it has not been systematically investigated in the light of inter-sectorial knowledge diffusion[3], except in some papers as for instance Peretto & Smulders (2002). Nevertheless, because non rivalry and diffusion of knowledge, on the one hand, and non rivalry and scale effects, on the other, are intrinsically linked, it appears natural to analyse the issue of scale effects under this perspective. A purpose of the paper consists in showing that inter-sectorial knowledge diffusion is a key point to reconsider the scale effects property.

Two major ranges of models suppressing scale effects can be identified. Jones (1995b) removes the scale effects property from Romer (1990)'s variety-based model. Kortum (1997) or Segerstrom (1998), among others, also provide scale-invariant growth models, in which the long-run growth is proportional to the exogenous rate of population growth. These first approaches are based on the notion of "diminishing technological opportunities" and gave birth to the "semi-endogenous growth" literature which exhibits the theoretical link between the growth rate of the economy and the growth rate of the population. This range of models displays "weak scale effects": scale effects are still present in the determination of the variables levels but no longer of their growth rates[4]. Yet, in this theory, economic policies (especially R&D subsidies) turn to have an impact only on the levels of economic variables, not on the long-run growth rates. Moreover, in the absence of positive population growth, the growth rate of the economy is nil.

Contrasting with this growth theory, an alternative range of literature, that one often refers to as "endogenous growth without scale effects theory", appeared through the impulse of Aghion & Howitt (1998), Dinopoulos & Thompson (1998), Peretto (1998), Young (1998), Howitt (1999) or Peretto & Smulders (2002), among others. These "fully endogenous Schumpeterian" growth models restore the effect of economic policies on long-term growth, without displaying the scale effects property, which is eliminated through a "variety expansion mechanism" which introduces an increasing R&D difficultly. As stated by Dinopoulos & Sener (2007), "horizontal product differentiation takes the form of variety accumulation and removes the scale-effects property from these models [...]. Vertical product differentiation takes the form of quality improvements or process innovations and generates endogenous long-run growth". In this class of double differentiation models, sectors proliferation dilutes R&D effort in a larger number of different sectors,

[2]For overviews and accurate expositions of the growth theory related body of literature, see for instance Jones (1999), Li (2000, 2002, 2003), Laincz & Peretto (2006), Dinopoulos & Sener (2007), or Ha & Howitt (2007).

[3]Schulstad (1993) studies the link between temporal diffusion of knowledge and scale effects in an endogenous growth model *à la* Grossman & Helpman (1991).

[4]More precisely, one can consider two forms of scale effects: "strong scale effects", as in the first generation of endogenous growth models, and "weak scale effects". The latter is displayed in the semi-endogenous growth models, in which the growth rate of the the economy is an increasing function of the growth rate of the population. For more details on this distinction, see, for instance, Jones (2005). In the present paper, like in most of the literature tackling this issue, we focus on "strong scale effects" to which one generally refers to as "scale effects". From now on, we use the same language abuse.

thus dissipating its effect on the overall rate of productivity growth. It appears clearly that this theory relies on two assumptions[5]. Firstly, the scale of the economy, as measured by the level of its population, has an impact on the number of sectors. More precisely, an increase in population results in a proportionate increase in the variety of goods (*i.e.* in the number of sectors, assumed to be a measure of the R&D difficultly) in the economy. Regarding this first key point, there are several ways to justify this assumption, whether it is empirically (Laincz & Peretto, 2006) or theoretically (Dinopoulos & Sener, 2007). The second assumption is more intricate; it relates to the nature of the pools of knowledge used by each sector that is generally considered in these fully endogenous growth models. In Peretto & Smulders (2002), it is accurately described as follows: "R&D productivity depends on some measure of accumulated public knowledge that is independent of the number of firms and hence of the scale of the economy. This independence may stem from the assumption that (a) spillovers across firms are absent (e.g. Peretto, 1999), that (b) spillovers depend on average knowledge (e.g. Smulders & Van de Klundert, 1995; Peretto, 1998; Dinopoulos & Thompson, 1998), or that (c) spillovers depend on the knowledge of the most advanced firm (e.g. Young, 1998; Aghion & Howitt, 1998; Howitt, 1999). All these models have the property that a large economy replicates the structure of a small economy. [...] Moreover, although they allow for spillovers, all these models assume that a larger number of firms undertaking independent R&D projects does not support a larger aggregate stock of public knowledge. The implicit assumption is that all public knowledge is replicated". In this paper, we go further in this direction, arguing that scale effects are eliminated by implicitly wiping out inter-sectorial knowledge diffusion. We show that, in order to eliminate this property while maintaining the effects of public policies on the growth rate of the economy, these models introduce a normalization assumption which implies that the knowledge production function in any given sector intrinsically depends on the level of knowledge in this sector only. This feature seams at odds with the common view on how knowledge springs into existence, since it somehow eludes part of its fundamental non rivalry property.

Is it possible to eliminate scale effects while maintaining inter-sectorial knowledge diffusion? The present paper notably tackles this central issue; in particular, we show that, under some reasonable assumptions, the answer is positive. The model we present provides a foundation for a general class of Schumpeterian growth models, without changing the main insights of the standard literature. The new feature of our framework is twofold. Firstly, we derive a general formalization of knowledge accumulation from commonly agreed assumptions. Secondly, as stated above, we introduce explicitly the role played by knowledge diffusion in the constitution of the pools of knowledge used by R&D activities to create new knowledge.

Foremost, our analysis is in the direct line of the seminal works of Romer, Grossman & Helpman, or Aghion & Howitt, even though we present a formalization of knowledge accumulation which is slightly different from theirs. The process of knowledge production is decomposed in two steps: the occurrence of new ideas and the consecutive knowledge increase. We keep the commonly shared assumption of stochastic arrival of innovations: as in Grossman & Helpman (1991), or Aghion & Howitt (1992), we assume a Poisson arrival rate of new ideas, which depends positively on the research effort. Regarding knowledge increases, we consider that, in each sector, they depend positively on a given pool of knowledge in which R&D activities draw from to produce new knowledge. From those two basic assumptions, we derive the law of motion of knowledge in each sector. The latter generalizes many of the various laws of motion considered in the literature. Indeed, making assumptions on the functional relationships and on the nature of the pools previously mentioned enables us to obtain most standard growth models, whether fully endogenous (with or without scale effects), or semi-endogenous.

Besides the fact that we propose a quite general formalization of the process of knowledge accumulation, we explicit a mechanism through which the pools of knowledge evoked above are shaped, introducing knowledge diffusion. In this respect, we exploit the formalization of circular product differentiation model of Salop (1979) to take into account the fact that knowledge can either be specific to the sector in which it has been created or diffuse variously among R&D sectors, ranging from local to global diffusion.

[5]See, for instance, the overviews provided by Jones (1999, 2005), Peretto & Smulders (2002), Laincz & Peretto (2006), Dinopoulos & Sener (2007), Ha & Howitt (2007), or Aghion & Howitt (2009, Ch.4).

Each point of the clockwise oriented circle corresponds to an intermediate sector, having its own R&D activities producing innovations. Allowing for miscellaneous scopes of diffusion of knowledge has a critical aftermath on the composition of the pool of knowledge used by each R&D activity. We consider three possible types of knowledge diffusion. Once an innovation has occurred, the diffusion of the inherent knowledge can be intra-sectorial (sector specific innovation), narrow or wide. The limit case of wide inter-sectorial diffusion is global diffusion, in which knowledge is used by all the sectors in the economy. It is noteworthy that the seminal frameworks proposed in the beginning of the nineties implicitly consider particular cases of knowledge diffusion. For instance, a specificity of the model of Grossman & Helpman (1991) consists in that each sector makes use of the knowledge produced within this sector only, *i.e.* it has his own specific pool of knowledge. On the contrary, the Romer (1990) model assumes that each sector uses the same pool of knowledge, which consists of the whole disposable knowledge. In other words, the former considers that there is no inter-sectorial knowledge diffusion (solely intra-sectorial knowledge diffusion), and the latter assumes global inter-sectorial knowledge diffusion. Whereas the fully endogenous Schumpeterian literature generally considers one of these two polar cases of knowledge diffusion, our framework allows us to analyse all intermediate cases in between. Moreover, various scopes of diffusion can coexist within a single model.

We characterize the set of decentralized equilibria with creative destruction *à la* Aghion & Howitt (1992) as functions of two types of public tools: subsidies to each intermediate good demand, and subsidies (or, potentially, taxes) to R&D activities. The first basic result exhibits a new determinant of growth: the growth rate of the economy is increasing in the intensity of knowledge diffusion within the economy. The second standard result, which is in line with the fully endogenous growth theory, consists in that public policies have an impact on the equilibrium growth rate. The main results of the paper relate to the presence of the counterfactual property of scale effects. In order to quantify their significance, we introduce a measure, which allows us to clarify why it is possible to eliminate scale effects while maintaining knowledge diffusion in a fully endogenous Schumpeterian growth model.

The paper is organized as follows. In Section 2, we present the model. We pay a particular attention to the description of the formalization of the way knowledge accumulates and diffuses among R&D activities. Accordingly, we explain how the pools of knowledge come into existence. Furthermore, we describe the decentralized economy. Section 3 focuses on showing how the standard theory fits into our model, starting with seminal endogenous growth with scale effects. Then we propose an explanation, based on knowledge diffusion analysis, of how scale effects have been removed in fully endogenous Schumpeterian models. In Section 4, we study the interaction between the size of the economy, inter-sectorial knowledge diffusion and scale effects. Considering several cases, we show in particular that the significance of scale effects depends on the speed at which the scope of knowledge diffusion increases with the size of the economy. We present three characteristic cases. Firstly, we consider the elementary case in which the scope of knowledge diffusion does not increase with the size of the economy. In this case, scale effects are nil. Secondly, the scope of knowledge diffusion is assumed to spread, but less quickly than the size of the economy: scale effects decrease over time and asymptotically vanish. Finally, we introduce the fundamental case of global diffusion, which can somehow be related to the issue of general purpose technologies. Accordingly, the scope of knowledge diffusion increases at the same speed as the size of the economy. In this limit case, even if scale effects remain, it can be argued that they are not significant since the probability of occurrence of general purpose technologies is low. We conclude in Section 5. The Appendix is in Section 6; it provides the detailed analysis of the decentralised economy. In particular, we fully characterize the set of equilibria as functions of public policy tools in the two cases of no population growth and of constant population growth.

2 Model

This section presents the fundamentals of the economy studied in the paper. We consider a continuous-time Schumpeterian growth model, in which innovations can diffuse, with more or less intensity, within the sectors R&D activities. In this respect, we exploit the formalization of a circular product differentiation model of Salop (1979). The key ingredients of the model developed in this paper lie in the formalization of knowledge accumulation and diffusion within the economy.

2.1 Knowledge Accumulation

There is a continuum Ω_t, of measure N_t, of intermediate sectors uniformly distributed on a clockwise oriented circle. Each intermediate sector ω, $\omega \in \Omega_t$, is characterized by an intermediate good ω, produced in quantity x_ω, and by a level of knowledge χ_ω. It has its own R&D activity which is dedicated to the creation of innovations. Each innovation successively increases the amount of knowledge inherent in this sector. Accordingly, we define the whole disposable knowledge in the economy, at each date t, as:

$$\mathcal{K}_t = \int_{\Omega_t} \chi_{\omega t}\, d\omega \tag{1}$$

It is commonly agreed that new knowledge is produced using two types of inputs: rival goods (e.g. labor, physical capital, final good), and a non rival one (a stock of knowledge previously created). In the present model, the mechanism at the source of the creation of knowledge relies on two core assumptions.

Firstly, the innovation process is uncertain. We assume (Assumption 1) that, if $l_{\omega t}$ is the amount of labor devoted to R&D at date t in any intermediate sector ω, $\omega \in \Omega_t$, to move on to the next quality of intermediate good ω, innovations occur randomly with a Poisson arrival rate $\lambda(l_{\omega t})$, where $\lambda(.)$ is an increasing function of class $\mathcal{C}^1$.

In the standard Schumpeterian growth theory, there is a quality ladder for each intermediate good: each innovation takes the intermediate good quality up by one rung on this ladder. Each R&D activity creates new knowledge (innovations) making use of previously created knowledge. The second assumption (Assumption 2) formalizes this idea by considering that, in order to produce new knowledge, each sector ω draws from a specific *pool of knowledge*, denoted $\mathcal{P}_t^\omega$. Formally, for any intermediate good ω, $\omega \in \Omega_t$, if an innovation occurs at date t, the increase in knowledge, $\Delta\chi_{\omega t}$, (*i.e.* the quality improvement) depends on the current size of the pool of knowledge in which this sector's R&D activity draws from: $\Delta\chi_{\omega t} = \sigma(\mathcal{P}_t^\omega)$, $\forall \omega \in \Omega_t$, where $\sigma(.)$ is an increasing function. From those two assumptions, one derives the law of motion of the average knowledge inherent in any sector ω, as expressed in Proposition 1 below.

Proposition 1. *Under Assumptions* 1 *and* 2*, on average, the law of motion of the knowledge characterizing any intermediate sector ω is:*

$$\dot{\chi}_{\omega t} = \lambda(l_{\omega t})\, \sigma(\mathcal{P}_t^\omega)\ ,\ \forall \omega \in \Omega_t$$

Proof. Consider any given sector ω, $\omega \in \Omega_t$, and a time interval $(t, t+\Delta t)$. At date t, the knowledge in this sector is $\chi_{\omega t}$. Let k, $k \in \mathbb{N}$, be the number of innovations that occur during the interval $(t, t+\Delta t)$. Under assumptions 1 and 2, the knowledge at date $t+\Delta t$, $\chi_{\omega\, t+\Delta t}$, is a random variable taking the values $\{\chi_{\omega t} + k\sigma(\mathcal{P}_t^\omega)\}_{k\in\mathbb{N}}$ with associated probabilities $\left\{\frac{\left(\int_t^{t+\Delta t}\lambda(l_{\omega u})du\right)^k}{k!} e^{-\int_t^{t+\Delta t}\lambda(l_{\omega u})du}\right\}_{k\in\mathbb{N}}$.

Accordingly, the expected level of knowledge at date $t+\Delta t$ is:

$$\mathbb{E}[\chi_{\omega\, t+\Delta t}] = \sum_{k=0}^{\infty} \frac{\left(\int_t^{t+\Delta t}\lambda(l_{\omega u})\,du\right)^k}{k!} e^{-\int_t^{t+\Delta t}\lambda(l_{\omega u})du}\left[\chi_{\omega t} + k\sigma(\mathcal{P}_t^\omega)\right]$$

$$= \left[\chi_{\omega t}\sum_{k=0}^{\infty}\frac{\left(\int_t^{t+\Delta t}\lambda(l_{\omega u})\,du\right)^k}{k!} + \sigma(\mathcal{P}_t^\omega)\left(\int_t^{t+\Delta t}\lambda(l_{\omega u})\,du\right)\sum_{k=1}^{\infty}\frac{\left(\int_t^{t+\Delta t}\lambda(l_{\omega u})\,du\right)^{k-1}}{(k-1)!}\right] e^{-\int_t^{t+\Delta t}\lambda(l_{\omega u})du}$$

The MacLaurin series $\sum_{k=0}^{K}\frac{\left(\int_t^{t+\Delta t}\lambda(l_{\omega u})du\right)^k}{k!}$ converges to $e^{\int_t^{t+\Delta t}\lambda(l_{\omega u})du}$ as $K\to\infty$. Thus, one gets:

$$\mathbb{E}[\chi_{\omega\, t+\Delta t}] = \left[\chi_{\omega t} e^{\int_t^{t+\Delta t}\lambda(l_{\omega u})du} + \sigma(\mathcal{P}_t^\omega)\left(\int_t^{t+\Delta t}\lambda(l_{\omega u})\,du\right) e^{\int_t^{t+\Delta t}\lambda(l_{\omega u})du}\right] e^{-\int_t^{t+\Delta t}\lambda(l_{\omega u})du}$$

$$\Leftrightarrow \mathbb{E}\left[\chi_{\omega\, t+\Delta t}\right] = \chi_{\omega t} + \left(\int_t^{t+\Delta t} \lambda\left(l_{\omega u}\right) du\right) \sigma\left(\mathcal{P}_t^{\omega}\right)$$

Let $\Lambda_{\omega u}$ denote a primitive of $\lambda\left(l_{\omega u}\right)$ with respect to the time variable u. Finally, rewriting the previous expression, we can exhibit the Newton's difference quotients of $\mathbb{E}\left[\chi_{\omega t}\right]$ and of $\Lambda_{\omega t}$:

$$\frac{\mathbb{E}\left[\chi_{\omega\, t+\Delta t}\right] - \chi_{\omega t}}{\Delta t} = \frac{\Lambda_{\omega t+\Delta t} - \Lambda_{\omega t}}{\Delta t} \sigma\left(\mathcal{P}_t^{\omega}\right)$$

Finally, letting Δt tend to zero, one gets $\frac{\partial \mathbb{E}[\chi_{\omega t}]}{\partial t} \equiv \mathbb{E}\left[\dot{\chi}_{\omega t}\right] = \lambda\left(l_{\omega t}\right) \sigma\left(\mathcal{P}_t^{\omega}\right)$. This proves that the expected knowledge in any intermediate sector ω, $\omega \in \Omega_t$, is a differentiable function of time. Its derivative gives the law of motion of the expected knowledge as given in Proposition 1 above, in which the expectation operator is dropped to simplify notations. □

The law of motion of knowledge derived in Proposition 1 encompasses most of the ones assumed in the standard growth theory. Depending on the specifications of the functions $\lambda(.)$ and $\sigma(.)$, and on the choice of the pools $\mathcal{P}_t^{\omega}$, most standard growth frameworks can be obtained within the present framework. Several illustrations are presented below, in Particular cases 1 and 2.

The overviews provided notably by Jones (1999), Laincz & Peretto (2006), or Dinopoulos & Sener (2007), propose a classification of the various growth models according to their key result with respect to the presence of scale effects. Three classes of models emerge: endogenous growth models exhibiting this non desirable property (*e.g.* the models of Romer, 1990; Grossman & Helpman, 1991; or Aghion & Howitt, 1992), semi-endogenous growth models introducing decreasing returns to scale to suppress scale effect (*e.g.* the models of Jones, 1995b; Kortum, 1997; or Segerstrom, 1998), and fully endogenous growth models, which eliminate scale effects by allowing for expansion in the number of sectors (*e.g.* the models of Aghion & Howitt, 1998, Ch. 12; Dinopoulos & Thompson, 1998; Peretto, 1998; Young, 1998; Howitt, 1999; Peretto, 1999; or Aghion & Howitt, 2009, Ch. 4). As underlined by Jones, Laincz & Peretto and Dinopoulos & Sener, the main distinction between these models lies in the specification of the knowledge production technology.

The general framework we introduce in this paper allows us to come across this classification again. In this respect, we distinguish semi-endogenous and fully endogenous (whether with or without scale effects) growth theory. Moreover, we propose an alternative classification among fully endogenous growth models, which adds up to the previous one: we classify the models developed in the literature according to the considered pool of knowledge.

Particular case 1: semi-endogenous growth theory. Assume that $\lambda\left(l_{\omega t}\right) = \lambda l_{\omega t}$ $(\lambda > 0)$, $\sigma\left(\mathcal{P}_t^{\omega}\right) = \sigma {\mathcal{P}_t^{\omega}}^{\Phi}$ $(\sigma > 0, \Phi < 1)$, and $\mathcal{P}_t^{\omega} = \mathcal{K}_t, \forall \omega \in \Omega_t$, (the pool of knowledge used by each sector ω is the whole knowledge accumulated so far). One gets $\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} {\mathcal{K}_t}^{\Phi}$, $\forall \omega \in \Omega_t$. Furthermore, assuming $N_t = N$, one has $\Omega_t = \Omega, \forall t$. Therefore, summing on Ω, one obtains:

$$\dot{\mathcal{K}}_t = \lambda \sigma L_t^R {\mathcal{K}_t}^{\Phi},$$

where $L_t^R = \int_{\Omega} l_{\omega t}\, d\omega$ is the overall amount of labor dedicated to research in the economy.

This law of motion, obtained as a particular case of the one derived in Proposition 1, is formally identical to those assumed in the semi-endogenous growth theory. Indeed, this theory is presented using a similar expression in Jones (1999, equation (4)), in Laincz & Peretto (2006, equation (5)), in Dinopoulos & Sener (2007, equations (2) and (6)), in Ha & Howitt (2007, equation (3)), as well as in Acemoglu (2009, Ch. 13, equation (13.34)).

Particular case 2: fully endogenous growth theory. Assume that $\lambda\left(l_{\omega t}\right) = \lambda l_{\omega t}$ $(\lambda > 0)$, and that $\sigma\left(\mathcal{P}_t^{\omega}\right) = \sigma \mathcal{P}_t^{\omega}$ $(\sigma > 0)$ (*i.e.*, when an innovation occurs in sector ω, the increase in knowledge $\Delta\chi_{\omega t}$ is proportional to the current size of the pool of knowledge used by this sector). Consequently, the law of motion of the knowledge characterizing any intermediate sector ω derived from Proposition 1 is:

$$\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \mathcal{P}_t^{\omega}\ ,\ \forall \omega \in \Omega_t \tag{2}$$

The law of motion (2) is quite general. Indeed, choosing particular specifications of the pools $\mathcal{P}_t^{\omega}$, enables us to obtain several laws of knowledge accumulation commonly used in the fully endogenous growth Schumpeterian theory. We propose to classify the various models proposed in this literature in four main ranges according to the considered pools of knowledge (*i.e.* the considered types of knowledge spillovers).

Global knowledge spillovers. A first range of models assumes that each sector uses the whole disposable knowledge in the economy, that is $\mathcal{P}_t^\omega = \int_{\Omega_t} \chi_{ht}\, dh = \mathcal{K}_t$. Accordingly, one gets the following knowledge production function:

$$\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \mathcal{K}_t, \forall \omega \in \Omega_t \tag{3}$$

In this type of model, knowledge spillovers are assumed to be global. In the models of Aghion & Howitt (1992), Young (1998), Howitt (1999), Segerstrom (2000), or Garner (2010), among others, the increase in knowledge consecutive to the occurrence of an innovation in sector ω at date t depends on the level of knowledge reached in the most advanced sector. This type of framework can be directly obtained from our formalization. Indeed, assuming $\mathcal{P}_t^\omega = \chi_t^{max}$, where $\chi_t^{max} \equiv \max\{\chi_{\omega t}, \omega \in \Omega_t\}$, one gets $\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \chi_t^{max}$, $\forall \omega \in \Omega_t$. To some extent, their works, insofar as they consider spillovers depending on the knowledge level of the frontier firms ("leading-edge technology"), are closely related to this first range of models[6].

It is noteworthy that the expression of the knowledge production (3), which is endogenously derived from assumptions made in a stochastic quality ladders model, leads to a law of motion of the whole disposable knowledge which is formally identical to the knowledge production function initially introduced by Romer (1990). Indeed, assuming $N_t = N$, and thus $\Omega_t = \Omega, \forall t$, differentiating (1) with respect to time, and plugging (3) yields:

$$\dot{\mathcal{K}}_t = \int_\Omega \dot{\chi}_{\omega t}\, d\omega = \lambda \sigma \left(\int_\Omega l_{\omega t} d\omega \right) \mathcal{K}_t \Leftrightarrow \dot{\mathcal{K}}_t = \lambda \sigma L_t^R \mathcal{K}_t$$

Similarly, in the case of "leading-edge technology" (*i.e.* if $\mathcal{P}_t^\omega = \chi_t^{max}$), one obtains the following aggregated law of motion: $\dot{\mathcal{K}}_t = \lambda \sigma L_t^R \chi_t^{max}$.

No inter-sectorial knowledge spillovers (only intra-sectorial knowledge spillovers). In the models proposed by Grossman & Helpman (1991), Segerstrom (1998), Peretto (1999), Acemoglu (2009, Ch. 14), or Aghion & Howitt (2009, Ch. 4), among others, it is assumed that the pool of knowledge used in each sector comprises only the knowledge previously accumulated within this sector, that is $\mathcal{P}_t^\omega = \chi_{\omega t}, \forall \omega \in \Omega_t$. One gets the following knowledge production functions $\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \chi_{\omega t}, \forall \omega \in \Omega_t$. In this type of model, there are no spillovers across sectors; they are only intra-sectorial.

Knowledge spillovers depending on average knowledge. The models of Dinopoulos & Thompson (1998), Peretto (1998), Li (2003), among others, consider firm-specific knowledge production functions such that, as stated by Laincz & Peretto (2006), "spillovers depend on average knowledge". Surveying this literature, these authors formalize this assumption in equation (9) of their paper. One can equivalently refer to equations (7) and (9) in Jones (1999), to equations (13) and (14) in Dinopoulos & Sener (2007), to equation (5) in Ha & Howitt (2007), or to the framework used in Aghion & Howitt (2009, Ch. 4). Using our notations, this normalization assumption gives the following knowledge production function:

$$\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t} \mathcal{P}_t^\omega, \text{ where } \mathcal{P}_t^\omega = \int_{\Omega_t} \frac{\chi_{ht}}{N_t}\, dh, \forall \omega \in \Omega_t \tag{4}$$

Here, the new knowledge produced in any given sector depends on a knowledge aggregator, which is the average knowledge within the whole economy.

Furthermore, to solve the models, one generally considers the symmetric case[7] in which $\chi_{\omega t} = \chi_t, \forall \omega \in \Omega_t$. Hence, one has $\mathcal{P}_t^\omega = \frac{\chi_t}{N_t} \int_{\Omega_t} dh = \chi_t, \forall \omega \in \Omega_t$. Eventually, it appears

[6] A similar interpretation of the original framework proposed by Aghion & Howitt (1992) can be found in Jones (1999), in Dinopoulos & Sener (2007), or in Ha & Howitt (2007).

[7] See, for instance, Aghion & Howitt (1992 or 1998, Ch. 3), or Peretto & Smulders (2002). The relevancy of the symmetric equilibrium is discussed in details in Peretto (1998, 1999), or in Cozzi, Giordani & Zamparelli (2007).

that the cases in which knowledge spillovers are only intra-sectorial and those in which they depend on average knowledge are closely related. This issue will be further developed in Section 3.2 (see, in particular, Proposition 5 and its proof).

No knowledge spillovers (neither inter nor intra-sectorial knowledge spillovers). In Barro & Sala-i-Martin (2003, Ch. 6) or in Peretto (2007), for instance, the knowledge production technology uses final good only. In this extreme case, there are neither inter-sectorial nor intra-sectorial knowledge spillovers. A similar framework in which new knowledge is produced only with private inputs can also be considered using our formalization. Assume that $\mathcal{P}_t^\omega = 1$; accordingly, one has $\dot{\chi}_{\omega t} = \lambda \sigma l_{\omega t}, \forall \omega \in \Omega_t$. In this case, the only input used in the production of knowledge is labor[8].

In this paper, we focus only on fully endogenous growth theory. In this respect, the law of knowledge accumulation considered from now on is given by (2). However, as it will be developed in Subsection 2.2 below, we are not going to set arbitrarily the pools of knowledge used in each sector; we precise the mechanism through which these pools are formed.

2.2 Knowledge Diffusion and Pools of Knowledge

In the previous subsection, we have sketched the way knowledge accumulates. Now, taking into account knowledge diffusion, we propose a mechanism formalizing how the pools of knowledge are shaped. Knowledge inherent in a given sector can diffuse variously among R&D sectors, ranging from local to global diffusion. Therefore, the constitution of the pools relies on the influence that R&D activities have on each other.

Each sector is simultaneously a *sender* and a *receiver* of knowledge: in the following, the index h, $h \in \Omega_t$, is used to point out a sector from which knowledge χ_h diffuses (the sender); the index ω, $\omega \in \Omega_t$, is used to point out the sector that may potentially use this knowledge (the potential receiver). For any R&D activity ω, $\omega \in \Omega_t$, the disposable pool of knowledge, $\mathcal{P}_t^\omega$, is composed of the knowledge produced in this sector so far, as well as of knowledge diffused from other sectors.

Regarding the process of knowledge diffusion, we assume that, when an innovation occurs in any sector, it can either be specific to the R&D sector in which it has been created (*"sector specific innovation"*), or, it can diffuse locally to R&D activities (*"narrow innovation"*), or finally, it can diffuse more broadly on a larger set of R&D activities (*"wide innovation"*).

Let us define the *scope of diffusion* of an innovation as the measure of the subset of sectors of Ω_t which are able to use the knowledge inherent in this innovation[9], or, in other words, as the measure of the neighborhood of diffusion of this innovation. Formally, when an innovation occurs in any intermediate sector h, $h \in \Omega_t$, its scope of diffusion is a random variable θ which can take three values[10]: 0, $\underline{\theta}$ or $\overline{\theta}_t$, respectively with probabilities p_0, p_n and p_W, where $1 < \underline{\theta} < \overline{\theta}_t \leqslant N_t$ and $p_0 + p_n + p_W = 1$. Accordingly, the expected scope of diffusion of innovations is:

$$\mathbb{E}[\theta]_t = p_n \underline{\theta} + p_W \overline{\theta}_t \tag{5}$$

It is a measure of the intensity of inter-sectorial knowledge spillovers, which is comprised between zero (no inter-sectorial spillovers / only intra-sectorial spillovers) and N_t (only global inter-sectorial spillovers).

In the case of sector specific innovations ($\theta = 0$), there is no sector using knowledge χ_h besides sector h itself. In the case in which there is inter-sectorial knowledge diffusion ($\theta = \underline{\theta}$ or $\theta = \overline{\theta}_t$), one assumes that knowledge diffuses symmetrically over the circle Ω_t. Thus, the neighborhoods of diffusion of knowledge χ_h inherent in sector h (*i.e.* the subset of sectors of Ω_t which are able

[8] Alternatively, going back to the general expression given in Proposition 1, one can also obtain this framework under more meaningful assumptions: $\lambda(l_{\omega t}) = \lambda l_{\omega t}$ ($\lambda > 0$) and $\sigma(\mathcal{P}_t^\omega) = \sigma$ ($\sigma > 0$). The latter assumption amounts to consider that any jump in quality is independent of previously created knowledge.

[9] The fact that a lag can be involved by technology adoption remains to be explored within our model and is left for further research. One could for instance consider that the more distant two sectors are, the longer the lag in technology adoption.

[10] In order to simplify the model, we assume that, without loss of generality, $\underline{\theta}$ is independent of time, whereas $\overline{\theta}_t$ potentially depends on time. Indeed, we will consider different cases in which $\overline{\theta}_t$ is independent of time or not, depending on the nature of the diffusion of wide innovations. We name *"global innovation"*, the limit case of a wide innovation which diffuses to the whole economy: $\overline{\theta}_t = N_t$.

to use the knowledge inherent in sector h), $h \in \Omega_t$, in the case of narrow innovations and of wide innovations are respectively: $\underline{\Omega^h} \equiv [h - \underline{\theta}/2\,;\, h + \underline{\theta}/2]$ and $\overline{\Omega^h_t} \equiv \left[h - \overline{\theta}_t/2\,;\, h + \overline{\theta}_t/2\right]$, where $\underline{\Omega^h} \subseteq \overline{\Omega^h_t} \subseteq \Omega_t$. Figure 1 below illustrates this formalization.

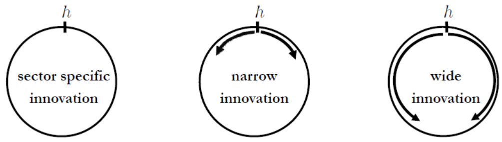

Figure 1: Three types of innovation

Accordingly, one gets the expression of $\mathcal{P}^\omega_t$, given in Proposition 2 below.

Proposition 2. *At each date t, in any intermediate sector ω, $\omega \in \Omega_t$, the expected pool of knowledge used by the R&D activity is:*

$$\mathcal{P}^\omega_t = (1 - p_n - p_W)\chi_{\omega t} + p_n \int_{\underline{\Omega^\omega}} \chi_{ht}\, dh + p_W \int_{\overline{\Omega^\omega_t}} \chi_{ht}\, dh\ ,\ \forall \omega \in \Omega_t \tag{6}$$

Proof. Let us refer to an innovation in any given sector h as to "innovation h". As stated above, any innovation h, $h \in \Omega_t$, consists either in a sector specific innovation with probability $p_0 = 1 - p_n - p_W$, in a narrow innovation with probability p_n, or in a wide innovation with probability p_W.

In the first case, the only sector specific innovation h, $\forall h \in \Omega_t$, reaching the location of R&D activity ω is innovation ω itself. The corresponding amount of knowledge received by R&D activity ω is then $\chi_{\omega t}$.

In the second case, solely narrow innovations h which are located in the nearby neighborhood $\underline{\Omega^\omega}$ can get to R&D activity ω. The amount of knowledge inherent in narrow innovations which can be used by R&D activity ω is then $\int_{\underline{\Omega^\omega}} \chi_{ht}\, dh$.

Finally, all wide innovations h which are located in the neighborhood $\overline{\Omega^\omega_t}$ reach R&D activity ω. Thus, the amount of knowledge inherent in wide innovations and received by R&D activity ω is $\int_{\overline{\Omega^\omega_t}} \chi_{ht}\, dh$. Consequently, the expected total amount of knowledge used by any R&D activity ω, $\omega \in \Omega_t$, is given by the expression (6) above. □

The expression (6) underlines the fact that the R&D activity of a given sector always uses the knowledge accumulated so far in this sector and potentially captures part of the mass of the ideas created in all other ones. This subset of the whole disposable knowledge, $\mathcal{K}_t$, is more or less large, depending on the scope of knowledge diffusion. As mentioned previously, the issue of knowledge spillovers has been tackled in Peretto & Smulders (2002). They consider that the "extent to which a firm can take advantage of the public knowledge created by other firms decreases with the technological distance between the creator and the user of such knowledge". In their paper, the average technological distance between an arbitrary pair of firms increases with the size of the economy. The formalization of knowledge diffusion that we have introduced enables us to propose a similar concept of technological distance, which is also increasing in the size of the economy (as measured by the number of sectors). Furthermore, we argue that the average technological distance should be decreasing in the expected scope of diffusion of innovations. Accordingly, a simple measure of technological distance within the present framework can be assumed to be:

$$\mathcal{D}_t = \frac{N_t}{\mathbb{E}\left[\theta\right]_t} \in [1; \infty) \tag{7}$$

When all sectors benefit from the whole stock of knowledge (*i.e.* when $\mathbb{E}\left[\theta\right]_t = N_t$), this technological distance is equal to one. It is infinite when sectors cannot share knowledge (*i.e.* when $\mathbb{E}\left[\theta\right]_t = 0$).

Introducing the fact that innovations are differentiated with respect to their scope of diffusion generalizes the standard innovation-based endogenous growth theory. Firstly, this formalization potentially allows to consider that innovations with different scopes of diffusion can coexist within a single model. Moreover, depending on the choice of the set of parameters $(p_0, p_n, p_W, \underline{\theta}, \overline{\theta}_t)$, one can obtain a large collection of models. In particular, two basic particular cases evoked above in Subsection 2.1 appear to be polar cases of our framework. Let us present formally under which assumptions these polar cases can be obtained and what are their main characteristics regarding knowledge diffusion. Growth models with global knowledge spillovers can be obtained setting $p_W = 1$ (*i.e.* $p_0 = p_n = 0$) and $\overline{\theta}_t = N_t$. Accordingly, one gets:

$$\overline{\Omega_t^\omega} = \left[\omega - \frac{N_t}{2}; \omega + \frac{N_t}{2}\right] = \Omega_t,\ \mathcal{P}_t^\omega = \int_{\Omega_t} \chi_{ht}\, dh = \mathcal{K}_t, \forall \omega \in \Omega_t,\ \mathbb{E}\left[\theta\right]_t = N_t, \text{ and } \mathcal{D}_t = 1$$

Growth models with no inter-sectorial knowledge spillovers (only intra-sectorial knowledge spillovers) can be obtained setting $p_0 = 1$ (*i.e.* $p_n = p_W = 0$). Accordingly, one gets:

$$\mathcal{P}_t^\omega = \chi_{\omega t}, \forall \omega \in \Omega_t,\ \mathbb{E}\left[\theta\right]_t = 0, \text{ and } \mathcal{D}_t = \infty$$

In the former polar case, all innovations diffuse to the whole set of sectors: the scope of diffusion of any innovation is the measure of the whole circle Ω_t (*i.e.* N_t). Therefore, the pool used in each sector is the whole disposable knowledge in the economy. The intensity of inter-sectorial knowledge spillovers is maximal and the technological distance is minimal. In the latter, there is no inter-sectorial diffusion of knowledge: the scope of diffusion of any innovation is nil. Therefore, the pool used in each sector consists only in the specific knowledge previously created in this sector. The intensity of inter-sectorial knowledge spillovers is nil and the technological distance is maximal. We will get back on those two polar cases in more detail in Subsection 3.1.

2.3 The Environment

Now that we have presented the way knowledge accumulates, let us complete the presentation of the economy. We denote by g_{z_t} the rate of growth, $\dot{z}_t/z_t$, of any variable z_t.

Population, L_t, grows at constant rate $g_{L_t} = n$, $n \geqslant 0$. Each household is modelled as a dynastic family which maximizes the discounted utility[11] $\mathcal{U} = \int_0^\infty L_t u(c_t) e^{-\rho t} dt$, where $\rho > n$ is the common subjective discount rate and $u(c_t)$ is the individual instantaneous utility at time t, which is given by[12] $u(c_t) = \ln(c_t)$. The initial size of the population, L_0, is normalized to unity. Then, the population of workers in the economy at time t is $L_t = e^{nt}$. Intertemporal preferences of the representative household are thus given by:

$$\mathcal{U} = \int_0^\infty \ln(c_t) e^{(n-\rho)t} dt \tag{8}$$

At each date t, each of the L_t identical households is endowed with one unit of labor that is supplied inelastically. The total quantity of labor, L_t, is used to produce the final good and in R&D activities. Thus, the labor constraint is:

$$L_t = L_t^Y + \int_{\Omega_t} l_{\omega t}\, d\omega \tag{9}$$

Besides labor, the production of the final good requires the use of all available intermediate goods, each of which is associated with its own level of knowledge. The final good production technology is:

$$Y_t = (L_t^Y)^{1-\alpha} \int_{\Omega_t} \chi_{\omega t} (x_{\omega t})^\alpha d\omega\ ,\ 0 < \alpha < 1, \tag{10}$$

The final good has two competing uses. Firstly, it is used in the production of intermediate goods along with:

$$x_{\omega t} = \frac{y_{\omega t}}{\chi_{\omega t}}\ ,\ \omega \in \Omega_t\,, \tag{11}$$

[11] Barro & Sala-i-Martin (1995, Ch. 2) provide more details on this formulation of the households behavior within the context of the Ramsey model of growth. See also Segerstrom (1998).

[12] The results are robust if one considers a more general C.E.S. instantaneous utility function of parameter $\varepsilon \in]0;1[$, $u(c_t) = \frac{c_t^{1-\varepsilon}}{1-\varepsilon}$.

where $y_{\omega t}$ is the quantity of final good used to produce $x_{\omega t}$ units of intermediate good ω. This technology illustrates the increasing complexity in the production of intermediate goods: as the quality of a given intermediate good increases, its production requires more resources. Secondly, it is consumed by the representative household in quantity c_t. One gets the following constraint on the final good market:

$$Y_t = L_t c_t + \int_{\Omega_t} y_{\omega t} d\omega \tag{12}$$

In the following sections, we study a decentralized economy which is in direct line with the analysis conducted by Aghion & Howitt (1992), in which R&D activities are funded by monopoly profits on the sale of intermediate goods embodying the knowledge. We normalize the price of final good to one, and denote respectively by w_t, r_t and $q_{\omega t}$ ($\omega \in \Omega_t$) the wage, the interest rate and the price of intermediate good ω at date t. The final good market, the labor market and the financial market are perfectly competitive. Once invented, an intermediate good can be modified, improved as the result of several steps of innovations. Regarding their markets, we consider Schumpeter's creative destruction mechanism. It involves that, in a given intermediate sector, the firm that succeeds in innovating is granted a patent and can monopolize the intermediate good production and sale until replaced by the next innovator.

There is potentially a divergence between the equilibrium allocation and the first-best optimal one. Indeed, as usual, there are two sources of inefficiency. The first one results from the presence of monopolies on the production and sale of intermediate goods. This distortion can be mitigated by an *ad valorem* subsidy ψ, $\psi \in [0; 1]$, on each intermediate good demand. The second externality is triggered by the fact that there is no market for knowledge. It can be corrected by a public tool φ, $\varphi \in (-1; \infty)$, which can consist in a subsidy or in a tax on the profits of R&D activities, depending on whether the R&D effort is sub-optimal or over-optimal. We characterize the set of equilibria as functions of the public tools vector (ψ, φ): at each vector (ψ, φ) is associated a particular equilibrium. Formally, an *equilibrium* is represented as time paths of set of prices $\left\{\left(\{q_{\omega t}\}_{\omega \in \Omega_t}, w_t, r_t\right)\right\}_{t=0}^{\infty}$ and of quantities $\left\{\left(c_t, Y_t, \{x_{\omega t}\}_{\omega \in \Omega_t}, L_t^Y, \{l_{\omega t}\}_{\omega \in \Omega_t}, \{\chi_{\omega t}\}_{\omega \in \Omega_t}\right)\right\}_{t=0}^{\infty}$, such that: the final good market, the labor market and the financial market clear; on each intermediate good market, the incumbent monopolizes the production and sale until replaced by the next innovator; there is free entry on each R&D activity (*i.e.* the zero profit condition holds for each R&D activity); firms maximize their profits; and the representative household maximizes her utility.

In the next sections, in order to shed a new light on the issue of scale effects, we use the growth rate of per capita consumption as a function of (ψ, φ); we denote it by $g_t(\psi, \varphi)$. All computations as well as the complete characterization of the decentralized economy are given in Appendix (Section 6). In particular, Lemma A1 (Subsection 6.2.1) and Lemma A2 (Subsection 6.2.2) provide the time paths of set of prices and of quantities, in the case of no population growth and of constant population growth, respectively.

3 Growth Theory with and without Scale Effects: The Standard Literature

In this section, we show that this model generalizes the standard fully endogenous growth literature. In particular, we recover the main results regarding the scale effects property. Moreover, this unifying framework enables us to shed a new light on the way the standard literature removes this non desirable property. We proceed in two steps. Firstly, we assume that the size of the population and the measure of the set of intermediate goods sectors are constant, and show that the resulting model exhibits scale effects, as the seminal models of the literature. Secondly, we introduce population growth, examine how the literature has removed scale effects, and show that the commonly shared assumption introduced by several authors is equivalent to undermine the role of knowledge diffusion.

3.1 Growth Theory with Scale Effects

Let us first assume that the size of the population and the measure of the set of intermediate goods sectors are constant: $L_t = L$ (*i.e.* $n = 0$) and $N_t = N$, for all t. For consistency, we assume that the maximal diffusion of knowledge (*i.e.* the scope of diffusion of wide innovations) is constant: $\overline{\theta}_t = \overline{\theta}$, for all t. Hence, from (5), $\mathbb{E}[\theta] = p_n\underline{\theta} + p_W\overline{\theta}$ is independent of time. The growth rate, expressed as a function of any vector of public tools (ψ, φ) is[13]:

$$g(\psi,\varphi) = \frac{\lambda\sigma L}{N}\left(\frac{(1+\varphi)\alpha - (1-\psi)\frac{\rho N}{\lambda L}}{(1+\varphi)\alpha + (1-\psi)}\right)(p_0 + \mathbb{E}[\theta]) \tag{13}$$

As seen in (13), both economic policy tools, ψ and φ, have a positive impact on the decentralized equilibrium growth rate of per capita consumption. Moreover, this general class of models exhibits scale effects, since the rate of growth, $g(\psi,\varphi)$, is increasing in L. Those two results are the ones exhibited in the seminal endogenous growth models. Finally, let us underline that the growth rate is increasing in the intensity of inter-sectorial knowledge spillovers, $\mathbb{E}[\theta]$. The model presented in this section thus generalizes the existing literature by explicitly introducing the role of knowledge diffusion. In order to illustrate the general nature of this framework, let us get back to the two polar cases described in Subsection 2.2 above. The growth rate of the economy in models with global knowledge spillovers (obtained from (13) in which $\mathbb{E}[\theta] = N$) is:

$$g(\psi,\varphi) = \lambda\sigma L\left(\frac{(1+\varphi)\alpha - (1-\psi)\frac{\rho N}{\lambda L}}{(1+\varphi)\alpha + (1-\psi)}\right)$$

The growth rate in models with no inter-sectorial knowledge spillovers (obtained from (13) in which $\mathbb{E}[\theta] = 0$) is:

$$g(\psi,\varphi) = \frac{\lambda\sigma L}{N}\left(\frac{(1+\varphi)\alpha - (1-\psi)\frac{\rho N}{\lambda L}}{(1+\varphi)\alpha + (1-\psi)}\right)$$

As stated above, both of these types of model exhibit scale effects at any equilibrium. Proposition 3 below summarizes the basic result of the subsection.

Proposition 3. *Assume $L_t = L$ and $N_t = N$. The model exhibits the scale effects property for any $\mathbb{E}[\theta]$.*

The issue of the presence of scale effects has been tackled in various ways, both empirically and theoretically. In particular, two major methodologies have been developed to remove this unwanted property, giving birth to two branches of growth models: semi-endogenous and fully endogenous. Successive reviews have given rise to a debate regarding the respective relevancy of using one or the other type of frameworks. Li (2000), for instance, argues that semi-endogenous growth theory is more general than endogenous growth theory. Ha & Howitt (2007) maintain that fully endogenous growth is more accurate. Like Madsen (2008), they argue that empirical evidences are more supportive of fully endogenous Schumpeterian growth theory than they are of semi-endogenous growth theory. As mentioned above, in the present paper, we focus on fully endogenous Schumpeterian growth models. In the next subsection, we investigate how the theory has cancelled scale effects from this range of models while preserving their endogenous feature.

3.2 Fully Endogenous Growth Theory without Scale Effects

Let us now consider population growth (*i.e.* $g_{L_t} = n > 0$) and focus on how the models of endogenous growth without scale-effects allow for constant population growth while maintaining the effects of public policies. As underlined by Jones (1999), Laincz & Peretto (2006), Dinopoulos & Sener (2007), or Aghion & Howitt (2009, Ch. 4), these models all have in common to suppose that the number of sectors is proportional to employment, *i.e.* $N_t = \gamma L_t$, where γ is a positive constant. In other words, the number of sectors increases as the population level does so. As stated by Laincz & Peretto, this relation "might induce one to conclude that this class of models requires another "knife-edge" condition in that one needs to assume that the number of firms is exactly proportional to population". They add that there are several ways to justify this first assumption. Their main argument in favor of this relation is empirical: according to their data, the number

[13]Computations of this growth rate are provided in Appendix.

of establishments is proportional to employment[14]. Here, the scale effects property is removed through a "variety expansion mechanism". Two types of model can a priori be distinguished in the existing literature.

A first type considers that the pool of knowledge used in each sector is solely the knowledge produced in this sector (see, for example, the model proposed by Peretto, 1999). Under this specification in which there is no inter-sectorial diffusion of knowledge, the condition $N_t = \gamma L_t$ is sufficient to remove the scale effects property. Indeed, let us assume $\mathbb{E}[\theta]_t = 0$. Accordingly, the growth rate is[15]:

$$g(\psi, \varphi) = \frac{\lambda\sigma}{\gamma}\left(\frac{(1+\varphi)\alpha - (1-\psi)\frac{\rho\gamma}{\lambda}}{(1+\varphi)\alpha + 1 - \psi}\right) + n \tag{14}$$

As seen in this expression, there are no scale effects in this particular case of our framework[16], and the effects of public policies are maintained. These results are summarized in Proposition 4 below.

Proposition 4. *Assume $g_{L_t} = n > 0$ and $N_t = \gamma L_t$. If $\mathbb{E}[\theta]_t = 0, \forall t$, (i.e. if there is no inter-sectorial diffusion of knowledge), both policy tools, ψ and φ, have a positive impact on the decentralized equilibrium growth rate and the model does not exhibit the scale effects property.*

A second type of scale-invariant fully endogenous growth model can be identified. As emphasized in the overviews provided, among others, by Jones (1999), Laincz & Peretto (2006), Dinopoulos & Sener (2007), or Ha & Howitt (2007), in most models of the endogenous growth without scale effects theory, the analysis focuses on the sector level rather than on the economy level. In order to remove the scale effects property while maintaining the effects of public policies, in addition to the condition of proportionality between the number of sectors and the level of the population detailed hereinbefore, it is assumed that new knowledge is produced using the average knowledge across all sectors. A defense for this specification echoes to Young (1998)'s insight that, as population grows, the proliferation of the intermediate sectors reduces the efficiency of R&D activities in improving the quality of an existing product because the R&D effort is diluted in more sectors. More precisely, those formalizations lean on a normalization of the pool of disposable knowledge by the number of sectors, and hence equivalently by the level of the population. The argument commonly put forward to justify this normalization by the size of the economy is as follows. Given that the number of sectors is a measure of R&D difficultly, dividing the pools of knowledge by the size of the economy is a way to account for the fact that higher R&D difficulty implies that the same amount of R&D resources generates a lower production of knowledge. The rest of this subsection tackles this normalization matter.

In the light of these surveys, we argue that this formalization boils down to considering that the pool of knowledge in which each sector draws from to produce new knowledge is reduced to the knowledge which is specific to this sector. In other words, as sketched above in Subsection 2.1 (see Particular case 2), models originally considering some type of interaction between sectors come down to models in which there is only intra-sectorial diffusion of knowledge. In order to illustrate

[14] The linear relationship $N_t = \gamma L_t$ can also be justified theoretically. For instance, it is introduced endogenously in Howitt (1999), or in Segerstrom (2000), among others. It has also been set up in a more straightforward scheme as exposed, for instance, in Aghion & Howitt (2009, Ch. 4, Section 4.4). As an illustration, let us adapt the methodology they propose within our framework. Assume that, at date t, the probability of inventing a new intermediate good is a linear function of the population size, L_t. Note that, unlike in Howitt (1999), there are no R&D expenditure here. Moreover, suppose that an exogenous fraction ξ of intermediate goods becomes obsolete and vanishes at each date t. Thus, the variation of the number of sectors at each date t is given by $\dot{N}_t = \kappa L_t - \xi N_t$, where κ and ξ are positive parameters, and where $L_t = e^{nt}$. The solution of this non-homogeneous first-order linear differential equation is:

$$N_t = \frac{\kappa}{n+\xi}\left(e^{nt} - e^{-\xi t}\right), \forall t \Leftrightarrow \frac{N_t}{L_t} = \frac{\kappa}{n+\xi}\frac{e^{nt} - e^{-\xi t}}{e^{nt}} = \frac{\kappa}{n+\xi}\left(1 - e^{-(\xi+n)t}\right), \forall t$$

Consequently, the ratio length of the list of intermediate sectors over population level will eventually stabilize at a steady-state value, $(N_t/L_t)^{ss} = \kappa/(n+\xi) \equiv \gamma$. Indeed, (N_t/L_t) converges to $\kappa/(n+\xi)$ as $t \to \infty$, because $\xi + n > 0$.

[15] This growth rate is obtained setting $\mathbb{E}[\theta] = 0$ in the expression (37) of the growth rate given in Lemma A2 provided in Appendix (see Subsection 6.2.2).

[16] The model exhibits weak scale effects; however, the growth rate of the economy is strictly positive even if the population growth rate is nil. For more details on weak scale effects, see, for instance, Jones (2005).

this matter, let us consider the knowledge production function presented in (4), which, as stated above, is the one considered in these overviews, making use of our notations. In Proposition 5, we show that this assumption of normalization is equivalent to wiping out knowledge diffusion.

Proposition 5. *The normalization introduced in the standard endogenous growth without scale-effects is equivalent to assuming* $\mathbb{E}[\theta]_t = 0, \forall t$ *(or, equivalently,* $p_0 = 1$*), in the present framework.*

Proof. Using (1) and (4), one easily sees that the expression of the pools of knowledge usually considered in the literature is $\mathcal{P}_t^\omega = \frac{1}{N_t}\int_{\Omega_t} \chi_{ht}\, dh = \frac{\mathcal{K}_t}{N_t}$. Now, consider the standard assumption of symmetry, in which $\chi_{\omega t} = \chi_t, \forall \omega \in \Omega_t$. Accordingly, on has $\mathcal{K}_t = N_t \chi_t$. Finally, these pools of knowledge reduce to the following expression:

$$\mathcal{P}_t^\omega = \chi_t, \forall \omega \in \Omega_t \tag{15}$$

In the framework we propose, the pools are given by (6); in the symmetric case, this expression reduces to:

$$\mathcal{P}_t^\omega = (p_0 + \mathbb{E}[\theta]_t)\, \chi_t\ ,\ \forall \omega \in \Omega_t \tag{16}$$

Identifying (15) and (16) yields $p_0 + \mathbb{E}[\theta]_t = 1 \Leftrightarrow p_0 + p_n\underline{\theta} + p_W\overline{\theta}_t = 1 \Leftrightarrow p_n(\underline{\theta} - 1) + p_W(\overline{\theta}_t - 1) = 0$. Since $1 < \underline{\theta} < \overline{\theta}_t$, the latter equality holds if and only if $p_n = p_W = 0 \Leftrightarrow \mathbb{E}[\theta]_t = 0, \forall t$. □

The scale-invariant models of fully endogenous growth theory as surveyed by Jones (1999), Laincz & Peretto (2006), Dinopoulos & Sener (2007), or Ha & Howitt (2007), implicitly assume either that knowledge spillovers are only intra-sectorial, or that they depend on average knowledge. As it has been argued in Subsection 2.1, and shown in Propositions 5, those two cases are closely related: in both, there are no inter-sectorial spillovers (*i.e.* $\mathbb{E}[\theta]_t = 0$), or, equivalently, the technological distance, $\mathcal{D}_t$, is infinite (see equation (7)).

Concluding this section, one has to emphasize that, in order to remove the scale effects property, the standard endogenous growth without scale effects theory implicitly eliminates knowledge diffusion across sectors. One can however think that this comes down to ignoring the fundamental property of non rivalry of knowledge, whereas the common view on how knowledge accumulates emphasizes the significance of this non rivalry. Indeed, it is generally agreed that new pieces of knowledge "diffuse gradually, through a process in which one sector gets ideas from the research and experience of others"[17].

Is it still possible to suppress the non desirable property of scale effects while maintaining the effects of public policies, but without removing knowledge diffusion? We tackle this question in the next section.

4 Knowledge Diffusion, Technological Distance, and Scale Effects

In this section, we study the consequences of various assumptions on the scope of diffusion of knowledge on three issues: i) the effects of public policies, ii) the presence of scale effects, iii) the existence of a non explosive growth rate.

Throughout the section, the size of the economy, measured by the level of population, L_t, increases at the constant rate $n > 0$. Moreover, we keep the commonly shared assumption that the number of sectors is proportional to employment, $N_t = \gamma L_t$.

We study the property of scale effects, that is the impact of the size of an economy on its growth rate. More precisely, we investigate the link between knowledge diffusion and the presence of scale effects. Recall that, within the model, knowledge diffusion is characterized by its scope, $\mathbb{E}[\theta]_t = p_n\underline{\theta} + p_W\overline{\theta}_t, \forall t$. Moreover, since $\overline{\theta}_t \in (\underline{\theta}, N_t]$, this scope of diffusion is influenced by the size of the economy.

In line with the standard Schumpeterian approach, we focus on the usually studied symmetric case in which $\chi_{\omega t} = \chi_t$ and $l_{\omega t} = l_t$, $\forall \omega \in \Omega_t$. Hence, in each sector, the resulting pools and laws

[17] Aghion & Howitt (1998, Ch. 3). Similar statements can be found, for instance, in Scotchmer (1991), or in Chantrel, Grimaud & Tournemaine (2010).

of accumulation of knowledge are $\mathcal{P}_t^{\omega} = \mathcal{P}_t = (p_0 + \mathbb{E}[\theta]_t)\,\chi_t$ and $\dot{\chi}_{\omega t} = \dot{\chi}_t = \lambda\sigma l_t\,(p_0 + \mathbb{E}[\theta]_t)\,\chi_t$, $\forall \omega \in \Omega_t$, respectively. Finally, given a vector of public policies (ψ, φ), the equilibrium in the decentralized economy is characterized by the following growth rate of per capita consumption[18]:

$$g_t(\psi, \varphi) = \frac{\lambda\sigma}{\gamma}\left(\frac{(1+\varphi)\alpha - (1-\psi)\frac{\rho\gamma}{\lambda}}{(1+\varphi)\alpha + 1 - \psi}\right)(p_0 + \mathbb{E}[\theta]_t) + n \tag{17}$$

From (17), we deduce the two following Lemmas.

Lemma 1. *Both policy tools, ψ and φ, have a positive impact on the equilibrium growth rate, $g_t(\psi, \varphi)$, for any $\mathbb{E}[\theta]_t$.*

Lemma 2. *Knowledge diffusion, measured by $\mathbb{E}[\theta]_t$, has a positive impact on the rate of growth, $g_t(\psi, \varphi)$, for any vector of public tools (ψ, φ).*

The model potentially exhibits the scale effects property since the growth rate depends positively on $\mathbb{E}[\theta]_t$, which itself depends on the size of the economy, measured equivalently by L_t or N_t. In order to apprehend with more accuracy the impact of knowledge diffusion, let us introduce a measure of scale effects:

$$\mathcal{S}_t = \frac{\partial g_t(\psi, \varphi)}{\partial L_t} = \frac{\lambda\sigma}{\gamma}\left(\frac{(1+\varphi)\alpha - (1-\psi)\frac{\rho\gamma}{\lambda}}{(1+\varphi)\alpha + 1 - \psi}\right) p_W \frac{\partial \overline{\theta}_t}{\partial L_t} \tag{18}$$

Recall that, as seen in Proposition 4 above, in the case in which inter-sectorial knowledge diffusion is removed (*i.e.* when $\mathbb{E}[\theta]_t = 0, \forall t$), the scale effects property is eliminated. This result is revisited here. Indeed, since in this polar case one assumes that $p_0 = 1$, and thus that $p_W = 0$, the measure of scale effects, $\mathcal{S}_t$, is nil.

From now on, we assume that $\mathbb{E}[\theta]_t \neq 0, \forall t$. We proceed in three steps, each of which consisting in making an assumption on the scope of diffusion of wide innovations, $\overline{\theta}_t$.

We start by considering a simple case in which the scope of diffusion of wide innovations is assumed to be independent of time: $\overline{\theta}_t = \overline{\theta}, \forall t$. This enables us to propose a scale-invariant growth model, which explicitly takes into account inter-sectorial knowledge diffusion. Then, we consider that, as the economy expands, $\overline{\theta}_t$ increases, but less quickly. A particular case of this set up consists in assuming that $\overline{\theta}_t$ is bounded above. This framework restores scale effects; however, they are limited. Finally, we show that, even when one considers the possibility of global diffusion of knowledge ($\overline{\theta}_t = N_t$), scale effects are tenuous if the probability of innovations to diffuse globally (p_W), is low. The main results are summarized in Propositions 6, 7, and 8, below.

4.1 Non Extending Knowledge Diffusion: $\overline{\theta}_t = \overline{\theta}$

Let us first consider the simple case in which $\overline{\theta}_t$ is independent of time. Consequently, the scope of diffusion of knowledge is also time-invariant: $\mathbb{E}[\theta]_t = \mathbb{E}[\theta] = p_n\underline{\theta} + p_W\overline{\theta}, \forall t$. Accordingly, as seen in (17), there are no scale effects in this set up. In other words, their measure, $\mathcal{S}_t$, is nil, as seen in (18). Moreover, contrary to the cases summarized in Propositions 4 and 5 above, this model does not eliminate the assumption of inter-sectorial knowledge diffusion. Therefore, it is possible to completely eliminate scale effects while maintaining the effects of public policies on the growth rate, and without wiping out inter-sectorial knowledge diffusion. These results are summarized in Proposition 6 below.

Proposition 6. *Assume $g_{L_t} = n > 0$ and $N_t = \gamma L_t$. If $\overline{\theta}_t = \overline{\theta}, \forall t$, the model does not exhibit the scale effects property (i.e. $\mathcal{S}_t = 0$) for any $\mathbb{E}[\theta]$. Moreover, the rate of growth, $g(\psi, \varphi)$, is constant over time.*

[18]Computations of this growth rate are provided in Appendix.

Here, unlike in most fully endogenous growth models in the standard theory, scale effects are removed while preserving inter-sectorial knowledge spillovers. We thus provide a scale-invariant endogenous growth model which allows for population growth and for knowledge diffusion across sectors. However, the scope of diffusion of wide innovations, $\overline{\theta}_t$, and thus, the intensity of knowledge spillovers, $\mathbb{E}[\theta]_t$, have been assumed to be time-invariant. Due to the non rivalry property of knowledge, it perhaps seems unreasonable to suppose that, as the economy expands, the scope of knowledge diffusion remains constant. In the next subsection, we relax this assumption: we introduce the fact that, as the size of the economy (measured equivalently by L_t or N_t) increases, the scope of diffusion of wide innovations spreads.

4.2 Extending Knowledge Diffusion: $\overline{\theta}_t = \overline{\theta}(N_t)$

Because knowledge is a non rival good, it appears natural to think that the scope of knowledge diffusion increases with the size of the economy. In this subsection, we consider that the scope of diffusion of wide innovations, $\overline{\theta}_t$, increases over time. Since $N_t = \gamma L_t = \gamma e^{nt}, \forall t$, this is equivalent to assume that $\overline{\theta}_t$ is an increasing function of N_t: $\overline{\theta}_t = \overline{\theta}(N_t)$, with $\overline{\theta}^{'}(N_t) > 0$. Accordingly, the intensity of knowledge spillovers, $\mathbb{E}[\theta]_t = p_n\underline{\theta} + p_W\overline{\theta}(N_t)$, increases with the size of the economy. As seen in (17), scale effects are restored. Therefore, we potentially face two problems. On the one hand, the presence of the scale effects property has been proved to be strongly at odds with empirics. On the other, this property involves a theoretical problem: if population grows at a positive and constant rate, the economy's growth rate increases exponentially over time and becomes infinite.

In what follows, we show that, under reasonable assumptions, none of these two issues is relevant within our framework. In this respect, we examine two cases. In the first one, the scope of diffusion of wide innovations, $\overline{\theta}_t$, grows, but not as fast as the size of the economy. A particular case of this framework would be to consider that $\overline{\theta}_t$ is bounded from above. In the second one, we focus on the limit case in which wide innovations are global, that is, diffuse to the whole economy.

4.2.1 Slowly Extending Knowledge Diffusion: $\overline{\theta}_t = \overline{\theta}(N_t)$, $\overline{\theta}^{'}(N_t) > 0$ and $\overline{\theta}^{''}(N_t) < 0$

We study the case in which $\overline{\theta}_t$ spreads less quickly than the economy. Formally, we assume that $\overline{\theta}_t = \overline{\theta}(N_t)$, where $\overline{\theta}(.)$ is an increasing and strictly concave differentiable function of class C^2. The concavity assumption can be justified by arguing that the expansion of the economy goes along with an increasing complexity which curbs down the speed at which the scope of diffusion of knowledge spreads. Somehow, we come across the seminal idea at the source of the endogenous growth without scale effects theory again. The results obtained under those assumptions are presented in Proposition 7 below.

Proposition 7. *Assume $g_{L_t} = n > 0$, $N_t = \gamma L_t$, and $\overline{\theta}_t = \overline{\theta}(N_t)$, where $\overline{\theta}(.) \in C^2$, with $\frac{d\overline{\theta}_t}{dN_t} = \overline{\theta}^{'}(N_t) > 0$ and $\frac{d^2\overline{\theta}_t}{dN_t^2} = \overline{\theta}^{''}(N_t) < 0$.*

i) *For any $p_W > 0$, the model exhibits the scale effects property:*

$$\mathcal{S}_t = \lambda\sigma\left(\frac{(1+\varphi)\alpha - (1-\psi)\frac{\rho\gamma}{\lambda}}{(1+\varphi)\alpha + 1 - \psi}\right)p_W\overline{\theta}^{'}(N_t) > 0$$

ii) *For any $p_W > 0$, the measure of scale effects is decreasing over time: $\dot{\mathcal{S}}_t < 0, \forall t$.*

iii) *Moreover, if $\lim_{N_t\to\infty}\overline{\theta}^{'}(N_t) = 0$, scale effects asymptotically vanish: $\lim_{t\to\infty} S_t = 0$.*

Corollary. *Without any extra assumption on $\overline{\theta}(.)$, the intensity of knowledge spillovers, $\mathbb{E}[\theta]_t$, and thus the growth rate, $g_t(\psi,\varphi)$, increase over time and can potentially become infinite. However, if $\overline{\theta}(.)$ is bounded above (i.e. if, under the assumptions of Proposition 7, $\lim_{N_t\to\infty}\overline{\theta}(N_t) = \overline{\theta}$, where $\overline{\theta}$ is finite), $g_t(\psi,\varphi)$ is increasing over time but bounded above.*

Despite the fact that scale effects are restored, the model is not really inconsistent with empirics. Indeed, as shown in Proposition 7, as the size of the economy increases, scale effects progressively lessen and eventually disappear. As regards the theoretical problem involved by the presence of scale effects, if the extra assumption of bounded scope of diffusion is added, the economy's growth rate increases but does not become infinite.

4.2.2 Global Diffusion: $\overline{\theta}_t = N_t$

Let us now take a look at the case in which global diffusion of knowledge is introduced. We allow some innovations to reach all the sectors in the economy. In other words, we consider the possible arrival of a type of innovations that are generally qualified as "general purpose technologies (GPT)" such as electricity or ICT (see, for instance, Helpman, 1998).

Formally, we assume that the scope of diffusion of wide innovations (which occur with probability p_W) is $\overline{\theta}_t = N_t$. Accordingly, the intensity of inter-sectorial knowledge spillovers is $\mathbb{E}[\theta]_t = p_n\underline{\theta} + p_W N_t = p_n\underline{\theta} + p_W\gamma L_t$. The main result is summarized in Proposition 8 below.

Proposition 8. *Assume $g_{L_t} = n > 0$, $N_t = \gamma L_t$, and $\overline{\theta}_t = N_t$. For any $p_W > 0$, the measure of scale effects is constant over time:*

$$\mathcal{S}_t = \lambda\sigma\left(\frac{(1+\varphi)\alpha - (1-\psi)\frac{\rho\gamma}{\lambda}}{(1+\varphi)\alpha + 1 - \psi}\right)p_W$$

This measure is decreasing when p_W decreases, and $\lim_{P_W\to 0}\mathcal{S}_t = 0$.

The presence of global innovations implies scale effects. In this case, their significance clearly depends on the probability of emergence of these global innovations, p_W. It is empirically reasonable to assume that GPT are quite rare in the large mass of discoveries. In the present framework, this would correspond to assuming that p_W is small with respect to the probabilities of specific and narrow innovations, $p_0 + p_n$. To sum up, even in the case in which there are scale effects due to the presence of global diffusion of knowledge, their impact on growth can be considered as limited since the probability of occurrence of general purpose technologies is low.

4.3 Technological Distance and Scale Effects

Let us now use the concept of technological distance defined in (7) to go back over the results presented in this section. In Proposition 9, we examine again the three cases developed in Subsections 4.1, 4.2.1, and 4.2.2.

Proposition 9. *Assume $g_{L_t} = n > 0$ and $N_t = \gamma L_t$.*

i) *If $\overline{\theta}_t = \overline{\theta}, \forall t$, then $\mathbb{E}[\theta]_t = \mathbb{E}[\theta] = p_n\underline{\theta} + p_W\overline{\theta}, \forall t$. Thus, $\mathcal{D}_t$ is an increasing function of time and $\lim_{t\to\infty}\mathcal{D}_t = \infty$.*

ii) *If $\overline{\theta}_t = \overline{\theta}(N_t)$, then $\mathbb{E}[\theta]_t = p_n\underline{\theta} + p_W\overline{\theta}(N_t)$. If, moreover, $\overline{\theta}(.) \in C^2$, with $\overline{\theta}'(N_t) > 0$ and $\overline{\theta}''(N_t) < 0$, $\mathcal{D}_t$ is an increasing function of time. Finally, if $\lim_{N_t\to\infty}\overline{\theta}'(N_t) = 0$ (or if $\lim_{N_t\to\infty}\overline{\theta}(N_t) = \overline{\theta}$, where $\overline{\theta}$ is finite), then $\lim_{t\to\infty}\mathcal{D}_t = \infty$.*

iii) *If $\overline{\theta}_t = N_t$, then $\mathbb{E}[\theta]_t = p_n\underline{\theta} + p_W N_t$. Thus, $\mathcal{D}_t$ is an increasing function of time and $\lim_{t\to\infty}\mathcal{D}_t = 1/p_W$. Furthermore, $\lim_{p_W\to 0}(\lim_{t\to\infty}\mathcal{D}_t) = \infty$.*

It appears in this proposition that, under several assumptions, in each of the three cases investigated, the technological distance becomes infinite. Under exactly the same assumptions, we have shown in Propositions 6, 7, and 8, that scale effects disappear. More precisely, as seen in Propositions 7 and 9-ii), we have shown that, asymptotically, scale effects dissipate and the technological distance becomes infinite, respectively. Here, as in Peretto & Smulders (2002), "dilution of public knowledge causes the scale effect to vanish asymptotically. The mechanism behind this result is increasing technological distance." Moreover, these authors suggest that "general purpose technologies might [thus] decrease technological distance". As seen in Proposition 9-iii), we propose a model which exhibits this result: the larger the probability of occurrence of GPT, p_W, the smaller the technological distance. Moreover, as stated above in Proposition 8, we emphasize the link between the presence of scale effects and the occurrence of GPT.

Consequently, one could think that the absence of scale effects is directly linked with the fact that the technological distance is infinite. In fact, this is not completely accurate. Indeed, as shown in Proposition 6, it is possible to have simultaneously a positive scope of diffusion, a finite technological distance, and no scale effects.

5 Conclusion

Knowledge diffusion within and across sectors shapes the pools of knowledge in which R&D activities draw from to innovate. Accordingly, it plays a crucial role in determining the level of disposable knowledge in an economy, and thus in the process of knowledge accumulation. The present paper developed a fully endogenous Schumpeterian growth model which explicitly takes into account that knowledge accumulation and knowledge diffusion are intrinsically linked, to reassess the already well debated issue of scale effects.

Firstly, we derived a general and very tractable law of accumulation of knowledge. The resulting framework appears to encompass most models of the related literature, whether one considers fully endogenous growth (both with and without scale effects), or semi-endogenous growth. Indeed, it turns out that most models considered so far in the standard literature can be obtained from the formalization introduced in this paper by making specific assumptions related to the accumulation of knowledge.

Secondly, introducing explicitly knowledge diffusion allowed us to explain how the pools of knowledge arise. In order to consider that each innovation can diffuse with more or less magnitude within the economy and, therefore, influence a more or less significant range of R&D activities, we combine the formalization of a circular product differentiation model of Salop (1979) with the main insights of the standard models of endogenous growth based on innovation.

This rather general model, which enables to consider a large variety of cases, depending on the specifications used, allowed us to shed a new light on the formalization used in the standard fully endogenous growth models to remove the non desirable scale effects property. In particular, we showed that the frameworks generally developed to avoid the presence of scale effects amount to considering that there is no inter-sectorial diffusion of knowledge (in other words, the pool used in each sector is limited to the knowledge which is specific to this sector).

Besides giving an alternative view on the endogenous growth theory without scale effects, the crux of this paper tackles the issue of determining whether it is possible to eliminate scale effects while maintaining the effects of public policies, but without suppressing knowledge diffusion. We gave three answers to this question. Firstly, we started by providing a simple framework for which the answer is positive. More precisely, we proposed a fully endogenous Schumpeterian model with inter-sectorial knowledge diffusion, which does not exhibit scale effects if the maximum diffusion of knowledge is assumed to be independent of the size of the economy. This latter assumption can be relaxed, considering that the maximum diffusion is extending as the number of sectors increases. In this more reasonable setting, two sub-cases have been investigated. In the first one, the maximum diffusion spreads more slowly than the economy. Scale effects recur; however, their impact on growth decreases and vanishes asymptotically. In the second one, we considered the limit case in which we allowed some innovations to diffuse to the whole economy (*i.e.* we allowed for global diffusion of knowledge); then, the maximum diffusion spreads at the same speed as the economy. Scale effects remain, but their impact is tenuous. Indeed, we showed that the significance of scale effects depends positively on the probability of occurrence of global innovations, which can be assumed to be rather low since such general purpose technologies are quite rare in the whole mass of discoveries.

6 Appendix: The Decentralized Economy

The Appendix provides the detailed analysis of the decentralised economy. In particular, we fully characterize the set of equilibria as functions of the public tools vector (ψ, φ) in two cases. In Lemmas A1 and A2, we give the time paths of set of prices and of quantities, in the case of constant population and of constant population growth, respectively.

6.1 Agents Behavior

The representative household maximizes her intertemporal utility given by (8) subject to her budget constraint: $\dot{b}_t = w_t + r_t b_t - c_t - n b_t - T_t/L_t$, where b_t denotes the per capita financial asset and T_t is a lump-sum tax charged by the government in order to finance public policies. This yields the usual Keynes-Ramsey condition:

$$r_t = g_{c_t} + \rho \tag{19}$$

In the final sector, the competitive firm maximizes its profit given by:

$$\pi_t^Y = (L_t^Y)^{1-\alpha} \int_{\Omega_t} \chi_{\omega t}(x_{\omega t})^{\alpha} d\omega - w_t L_t^Y - \int_{\Omega_t} (1-\psi) q_{\omega t} x_{\omega t} d\omega$$

The first-order conditions yield:

$$w_t = (1-\alpha)\frac{Y_t}{L_t^Y} \quad \text{and} \quad q_{\omega t} = \frac{\alpha (L_t^Y)^{(1-\alpha)} \chi_{\omega t}(x_{\omega t})^{\alpha-1}}{1-\psi} \;, \; \forall \omega \in \Omega_t \tag{20}$$

Given the technology of intermediate goods production (11), the incumbent monopoly in each intermediate good sector ω, $\omega \in \Omega_t$, maximizes the following instantaneous profit:

$$\pi_t^{x_\omega} = q_{\omega t} x_{\omega t} - y_{\omega t} = (q_{\omega t} - \chi_{\omega t}) x_{\omega t},$$

where the demand for intermediate good ω, $x_{\omega t}$, is given by (20). After maximization, one obtains the usual symmetric use of intermediate goods in the final good production and mark-up on the price of intermediate goods:

$$x_{\omega t} = x_t = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{1}{1-\alpha}} L_t^Y \quad \text{and} \quad q_{\omega t} = \frac{\chi_{\omega t}}{\alpha} \;, \; \forall \omega \in \Omega_t \tag{21}$$

Together with the definition of the whole disposable knowledge in the economy (1), (21) allows us to rewrite the final good production function (10), the wage expression given in (20), and the instantaneous monopoly profit on the sale of each intermediate good ω, respectively as:

$$Y_t = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} L_t^Y \mathcal{K}_t \;, \; w_t = (1-\alpha)\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} \mathcal{K}_t \;\text{ and}$$

$$\pi_t^{x_\omega} = \frac{1-\alpha}{\alpha}\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{1}{1-\alpha}} L_t^Y \chi_{\omega t} \;, \; \forall \omega \in \Omega_t \tag{22}$$

Log-differentiating with respect to time the expression of the final good production function given in (22), gives:

$$g_{Y_t} = g_{L_t^Y} + g_{\mathcal{K}_t} \tag{23}$$

Furthermore, using (1), (11), and (21), the final good resource constraint (12) can be rewritten as $Y_t = L_t c_t + \left[\alpha^2/(1-\psi)\right]^{\frac{1}{1-\alpha}} L_t^Y \mathcal{K}_t$. Dividing both sides by Y_t and using the expression of Y_t given in (22), one gets $L_t c_t / Y_t = 1 - \alpha^2/(1-\psi)$. Log-differentiating this expression gives:

$$g_{Y_t} = g_{c_t} + n \tag{24}$$

Let us now consider any R&D activity ω, $\omega \in \Omega_t$, and derive the innovators' arbitrage condition. Given the governmental intervention on behalf of research activities, the incumbent innovator, having successfully innovated at date t, receives, at any date $\tau > t$, the net profit

$\tilde{\pi}_{\tau}^{x_\omega} = (1+\varphi)\pi_{\tau}^{x_\omega}$ with probability $e^{-\int_t^\tau \lambda l_{\omega u} du}$ (*i.e.* provided that there is no innovation upgrading intermediate good ω between t and τ). The sum of the present values of the incumbent's expected net profits on the sale of intermediate good ω, at date t, is therefore:

$$\tilde{\Pi}_t^{x_\omega} = \int_t^\infty \tilde{\pi}_\tau^{x_\omega} e^{-\int_t^\tau (r_u + \lambda l_{\omega u})du} d\tau,$$

Differentiating this expression with respect to time gives the standard arbitrage condition in each R&D activity ω:

$$r_t + \lambda l_{\omega t} = \frac{\dot{\tilde{\Pi}}_t^{x_\omega}}{\tilde{\Pi}_t^{x_\omega}} + \frac{\tilde{\pi}_t^{x_\omega}}{\tilde{\Pi}_t^{x_\omega}} \ , \ \forall \omega \in \Omega_t \tag{25}$$

The free-entry condition[19] in each R&D activity ω is $w_t = \lambda \tilde{\Pi}_t^{x_\omega}$, where $\lambda \tilde{\Pi}_t^{x_\omega}$ is the expected revenue when one unit of labor is invested in R&D[20], and w_t is the cost of one unit of labor, which is given in (22). This condition gives $\tilde{\Pi}_t^{x_\omega} = \tilde{\Pi}_t^{x} = (1-\alpha)\left[\alpha^2/(1-\psi)\right]^{\frac{\alpha}{1-\alpha}} \mathcal{K}_t/\lambda \ , \ \forall \omega \in \Omega_t$ Consequently, one has $\dot{\tilde{\Pi}}_t^{x_\omega}/\tilde{\Pi}_t^{x_\omega} = g_{\mathcal{K}_t}$ and $\tilde{\pi}_t^{x_\omega}/\tilde{\Pi}_t^{x_\omega} = \frac{(1+\varphi)\lambda\alpha\chi_{\omega t} L_t^Y}{(1-\psi)\mathcal{K}_t}$, $\forall \omega \in \Omega_t$. Replacing in (25), one can rewrite the arbitrage condition as follows:

$$r_t + \lambda l_{\omega t} = g_{\mathcal{K}_t} + \frac{(1+\varphi)\lambda\alpha L_t^Y \chi_{\omega t}}{(1-\psi)\mathcal{K}_t} \ , \ \forall \omega \in \Omega_t \tag{26}$$

6.2 Symmetric Equilibrium

As in the standard literature, in order to keep the model tractable, we make the usual symmetry assumption, in which $l_{\omega t} = l_t$ and $\chi_{\omega t} = \chi_t$, $\forall \omega \in \Omega_t$. Consequently, one has $\mathcal{K}_t = N_t \chi_t$. Then, the growth rate of the whole disposable knowledge is given by:

$$g_{\mathcal{K}_t} = g_{\chi_t} + n \tag{27}$$

Furthermore, the pools of knowledge and the laws of accumulation of knowledge in each sector ω are respectively:

$$\mathcal{P}_t^\omega = \mathcal{P}_t = \left(p_0 + p_n \underline{\theta} + p_W \overline{\theta}_t\right)\chi_t = \left(p_0 + \mathbb{E}\left[\theta\right]_t\right)\chi_t \text{ and } \dot{\chi}_{\omega t} = \dot{\chi}_t = \lambda\sigma l_t \left(p_0 + \mathbb{E}\left[\theta\right]_t\right)\chi_t$$

Therefore, one has the following knowledge growth rates in each sector ω:

$$g_{\chi_{\omega t}} = g_{\chi_t} = \lambda\sigma\left(p_0 + \mathbb{E}\left[\theta\right]_t\right) l_t \ , \ \forall \omega \in \Omega_t \tag{28}$$

Finally, we can rewrite (26), the arbitrage condition in any R&D activity ω, $\omega \in \Omega_t$, as:

$$r_t + \lambda l_t = \lambda\sigma\left(p_0 + \mathbb{E}\left[\theta\right]_t\right) l_t + n + \frac{(1+\varphi)\lambda\alpha L_t^Y}{(1-\psi)N_t} \tag{29}$$

The equilibrium quantities, growth rates and prices are characterized by equations (9), (19), (21), (22), (23), (24), (27), (28) and (29):

$$\begin{cases} L_t = L_t^Y + N_t l_t & (l1) \\ r_t = g_{c_t} + \rho & (l2) \\ x_{\omega t} = x_t = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{1}{1-\alpha}} L_t^Y \quad \text{and} \quad q_{\omega t} = \frac{\chi_{\omega t}}{\alpha} \ , \ \forall \omega \in \Omega_t & (l3) \\ Y_t = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} L_t^Y \mathcal{K}_t \ , \ w_t = (1-\alpha)\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} \mathcal{K}_t & (l4) \\ g_{Y_t} = g_{c_t} + n & (l5) \\ g_{Y_t} = g_{L_t^Y} + g_{\mathcal{K}_t} & (l6) \\ g_{\mathcal{K}_t} = g_{\chi_t} + n & (l7) \\ g_{\chi_{\omega t}} = g_{\chi_t} = \lambda\sigma\left(p_0 + \mathbb{E}\left[\theta\right]_t\right) l_t \ , \ \forall \omega \in \Omega_t & (l8) \\ r_t + \lambda l_t = \lambda\sigma\left(p_0 + \mathbb{E}\left[\theta\right]_t\right) l_t + n + \frac{(1+\varphi)\lambda\alpha L_t^Y}{(1-\psi)N_t} & (l9) \end{cases} \tag{30}$$

[19] Equivalently, one could consider here the zero profit condition.

[20] Indeed, innovations in sector ω are assumed to occur along with a Poisson arrival rate of $\lambda l_{\omega t}$: for one unit of labor is invested in R&D activity ω, the probability to obtain one innovation at date t is thus λ. Moreover, its value, taking into account the R&D public policy, is $\tilde{\Pi}_t^{x_\omega}$.

Using ($l2$) and ($l9$), one gets:

$$g_{c_t} + \rho + \lambda l_t = \lambda\sigma\left(p_0 + \mathbb{E}\left[\theta\right]_t\right) l_t + n + \frac{(1+\varphi)\lambda\alpha L_t^Y}{(1-\psi)N_t} \tag{31}$$

Moreover, from ($l5$), ($l6$), ($l7$) and ($l8$), one has:

$$g_{c_t} + n = g_{L_t^Y} + g_{X_t} + n \Leftrightarrow g_{c_t} = g_{L_t^Y} + \lambda\sigma\left(p_0 + \mathbb{E}\left[\theta\right]_t\right) l_t \tag{32}$$

Combining (31) and (32) gives:

$$g_{L_t^Y} + \rho + \lambda l_t = n + \frac{(1+\varphi)\lambda\alpha L_t^Y}{(1-\psi)N_t}$$

Finally, using the labor constraint ($l1$), and rearranging the terms, one gets the following differential equation in L_t^Y:

$$g_{L_t^Y} - \frac{\lambda}{N_t}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right] L_t^Y = n - \rho - \lambda\frac{L_t}{N_t} \tag{33}$$

Let us now successively consider two cases:

Case 1 - No Population Growth: $L_t = L$ (*i.e.* $n = 0$), $N_t = N$, and $\overline{\theta}_t = \overline{\theta}, \forall t$.

Case 2 - Population Growth: $g_{L_t} = n > 0$, $N_t = \gamma L_t$, and $\overline{\theta}_t = \overline{\theta}\left(N_t\right)$, $\forall t$. We will consider several assumptions regarding the function $\overline{\theta}\left(.\right)$.

6.2.1 No Population Growth

In this first case, we consider the seminal framework in which the population level and the number of sectors are assumed to be constant. For consistency, we also suppose that the maximum diffusion of knowledge is independent of time. Formally, we assume that $L_t = L$ (*i.e.* $n = 0$), $N_t = N$, and $\overline{\theta}_t = \overline{\theta}, \forall t$. Accordingly, one has $\Omega_t = \Omega, \forall t$, and (33) writes:

$$g_{L_t^Y} - \frac{\lambda}{N}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right] L_t^Y = -\left(\lambda\frac{L}{N} + \rho\right) \tag{34}$$

In order to solve this differential equation, we use a variable substitution: let $X_t = 1/L_t^Y$. Log-differentiation with respect to time writes $g_{X_t} = -g_{L_t^Y}$. Substituting into (34) gives the following first-order linear differential equation:

$$-g_{X_t} - \frac{\lambda}{N}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]\frac{1}{X_t} = -\left(\lambda\frac{L}{N} + \rho\right) \Leftrightarrow \dot{X}_t - \left(\lambda\frac{L}{N} + \rho\right) X_t = -\frac{\lambda}{N}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]$$

Its solution is:

$$X_t = e^{\left(\lambda\frac{L}{N} + \rho\right)t}\left(X_0 - \frac{N}{\lambda L + \rho N}\frac{\lambda}{N}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]\right) + \frac{N}{\lambda L + \rho N}\frac{\lambda}{N}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]$$

$$\Leftrightarrow X_t = e^{\left(\lambda\frac{L}{N} + \rho\right)t}\left(X_0 - \frac{\lambda}{\lambda L + \rho N}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]\right) + \frac{\lambda}{\lambda L + \rho N}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]$$

Accordingly, one gets:

$$L_t^Y = \frac{\lambda L + \rho N}{e^{\left(\lambda\frac{L}{N} + \rho\right)t}\left(\frac{\lambda L + \rho N}{L_0^Y} - \lambda\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]\right) + \lambda\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]}$$

Using the transversality condition in the program of the representative household, we can show that L_t^Y immediately jumps to its steady-state level: $L^Y = \left(\lambda L + \rho N\right)/\lambda\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]$. The transversality condition is only satisfied when $L_t^Y = L_0^Y = \frac{\lambda L + \rho N}{\lambda\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]} = \frac{(1-\psi)(\lambda L + \rho N)}{\lambda[(1+\varphi)\alpha + 1 - \psi]}, \forall t$. Thus, one has $g_{L_t^Y} = 0$. Therefore, substituting into the system (30), one gets Lemma A1 below, in which we provide the complete characterization of the decentralized equilibrium. Note that, since $\overline{\theta}_t$ is independent of time, $\mathbb{E}\left[\theta\right]_t = \mathbb{E}\left[\theta\right], \forall t$.

Lemma A1. *Assume $L_t = L$ (i.e. $n = 0$), $N_t = N$, and $\overline{\theta}_t = \overline{\theta}, \forall t$. One has $\Omega_t = \Omega$, and $\mathbb{E}[\theta]_t = \mathbb{E}[\theta] = p_n\underline{\theta} + p_W\overline{\theta}, \forall t$.*

The repartition of labor at equilibrium is:

$$L_t^Y = L^Y = \frac{(1-\psi)(\lambda L + \rho N)}{\lambda\left[1-\psi+(1+\varphi)\alpha\right]}\ ,\ \text{and}\ l_{\omega t} = l = \frac{L}{N}\frac{(1+\varphi)\alpha - (1-\psi)\frac{\rho N}{\lambda L}}{(1+\varphi)\alpha + 1 - \psi}, \forall \omega \in \Omega$$

The quantity of each intermediate good is:

$$x_{\omega t} = x = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{1}{1-\alpha}} L^Y,\ \forall \omega \in \Omega$$

Given a vector of public policies (ψ, φ), the equilibrium growth rates are:

$$g_{c_t} = g_{\mathcal{K}_t} = g_{X_t} = g_{Y_t} = g(\psi, \varphi) = \frac{\lambda \sigma L}{N}\left(\frac{(1+\varphi)\alpha - (1-\psi)\frac{\rho N}{\lambda L}}{(1+\varphi)\alpha + 1 - \psi}\right)(p_0 + \mathbb{E}[\theta])$$

The prices are:

$$r = g(\psi, \varphi) + \rho,\ w_t = (1-\alpha)\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} \mathcal{K}_t,\ \text{and}\ q_{\omega t} = q_t = \frac{\mathcal{K}_t}{\alpha N},\ \forall \omega \in \Omega$$

6.2.2 Population Growth

In this subsection, we consider constant population growth ($g_{L_t} = n > 0$). In order to allow for population growth, we introduce the commonly shared assumption of proportionality between the number of sectors and the population level ($N_t = \gamma L_t$). Under those assumptions, (33), becomes:

$$g_{L_t^Y} - \frac{\lambda}{\gamma L_t}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right] L_t^Y = n - \rho - \frac{\lambda}{\gamma} \tag{35}$$

As previously, in order to solve (35), let $X_t = 1/L_t^Y$ (which implies $g_{X_t} = -g_{L_t^Y}$). This gives the following first-order linear differential equation in X_t:

$$-g_{X_t} - \frac{\lambda}{\gamma L_t}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right]\frac{1}{X_t} = n - \rho - \frac{\lambda}{\gamma} \Leftrightarrow \dot{X}_t - \left(\frac{\lambda}{\gamma} + \rho - n\right) X_t = -\frac{\lambda}{\gamma}\left[1 + \frac{1+\varphi}{1-\psi}\alpha\right] e^{-nt}$$

Its solution is:

$$X_t = e^{\left(\frac{\lambda}{\gamma}+\rho-n\right)t}\left[X_0 - \frac{1}{\left(\frac{\lambda}{\gamma}+\rho-n\right) - (-n)}\frac{\lambda}{\gamma}\left(1 + \frac{1+\varphi}{1-\psi}\alpha\right)\right] + \frac{1}{\left(\frac{\lambda}{\gamma}+\rho-n\right) - (-n)}\frac{\lambda}{\gamma}\left(1 + \frac{1+\varphi}{1-\psi}\alpha\right) e^{-nt}$$

$$\Leftrightarrow X_t = e^{\left(\frac{\lambda}{\gamma}+\rho-n\right)t}\left[X_0 - \frac{\lambda}{\lambda+\gamma\rho}\left(1 + \frac{1+\varphi}{1-\psi}\alpha\right)\right] + \frac{\lambda}{\lambda+\gamma\rho}\left(1 + \frac{1+\varphi}{1-\psi}\alpha\right) e^{-nt}$$

Accordingly, one gets:

$$L_t^Y = \frac{\lambda + \gamma\rho}{e^{\left(\frac{\lambda}{\gamma}+\rho-n\right)t}\left[\frac{\lambda+\gamma\rho}{L_0^Y} - \lambda\left(1 + \frac{1+\varphi}{1-\psi}\alpha\right)\right] + \lambda\left(1 + \frac{1+\varphi}{1-\psi}\alpha\right) e^{-nt}}$$

Using the transversality condition in the program of the representative household, one obtains:

$$L_t^Y = \frac{1 + \frac{\rho\gamma}{\lambda}}{1 + \frac{1+\varphi}{1-\psi}\alpha} L_t = \frac{(1-\psi)\left(1 + \frac{\rho\gamma}{\lambda}\right)}{(1+\varphi)\alpha + 1 - \psi} L_t \text{ and } g^e_{L_t^Y} = n \tag{36}$$

Plugging (36) in the system (30) gives the characterization of the decentralized equilibrium, given in Lemma A2 below. Note that, since the maximum diffusion of knowledge, $\overline{\theta}_t$, is increasing in N_t, so is $\mathbb{E}[\theta]_t$.

Lemma A2. *Assume $g_{L_t} = n > 0$, $N_t = \gamma L_t$, and $\overline{\theta}_t = \overline{\theta}(N_t)$, $\forall t$. The resulting expression of the expected scope of diffusion of innovations is $\mathbb{E}[\theta]_t = p_n \underline{\theta} + p_W \overline{\theta}(\gamma L_t)$.*

The repartition of labor at equilibrium is:

$$L_t^Y = \frac{(1-\psi)\left(1+\frac{\rho\gamma}{\lambda}\right)}{(1+\varphi)\alpha+1-\psi} L_t \ , \text{ and } l_{\omega t} = l = \frac{(1+\varphi)\alpha-(1-\psi)\frac{\rho\gamma}{\lambda}}{\left[(1+\varphi)\alpha+1-\psi\right]\gamma}, \forall \omega \in \Omega_t$$

The quantity of each intermediate good is:

$$x_{\omega t} = x_t = \left(\frac{\alpha^2}{1-\psi}\right)^{\frac{1}{1-\alpha}} L_t^Y, \ \forall \omega \in \Omega_t$$

Given a vector of public policies (ψ, φ), the equilibrium growth rates are:

$$g_{\chi_{\omega t}} = g_{\chi_t} = \lambda\sigma\left(p_0 + \mathbb{E}[\theta]_t\right) l, \ \forall \omega \in \Omega_t$$

$$g_{c_t} = g_{Y_t} - n = g_{\mathcal{K}_t} = g_{\chi_t} + n = g_t(\psi, \varphi) = \frac{\lambda\sigma}{\gamma}\left(\frac{(1+\varphi)\alpha-(1-\psi)\frac{\rho\gamma}{\lambda}}{(1+\varphi)\alpha+1-\psi}\right)\left(p_0 + \mathbb{E}[\theta]_t\right) + n \quad (37)$$

The prices are:

$$r_t = g_t(\psi, \varphi) + \rho, \ w_t = (1-\alpha)\left(\frac{\alpha^2}{1-\psi}\right)^{\frac{\alpha}{1-\alpha}} \mathcal{K}_t, \text{ and } q_{\omega t} = q_t = \frac{\mathcal{K}_t}{\alpha\gamma L_t}, \ \forall \omega \in \Omega_t$$

References

[1] Acemoglu D. (2009), *Modern Economic Growth*, Princeton NJ, Princeton University Press.

[2] Aghion P., Howitt P. (1992), A Model of Growth through Creative Destruction, *Econometrica*, 60, pp. 323-351.

[3] Aghion P., Howitt P. (1998), *Endogenous Growth Theory*, Cambridge MA, MIT Press.

[4] Aghion P., Howitt P. (2009), *The Economics of Growth*, Cambridge MA, MIT Press.

[5] Backus D., Kehoe P., Kehoe T. (1992), In search of scale effects in trade and growth, *Journal of Economic Theory*, 58(2), pp. 377-409.

[6] Barro R., Sala-i-Martin X. (1995), *Economic Growth*, New York, McGraw-Hill.

[7] Barro R., Sala-i-Martin X. (2003), *Economic Growth*, second edition, Cambridge MA, MIT Press.

[8] Chantrel E., Grimaud A., Tournemaine F. (2010), Pricing Knowledge and Funding Research of New Technology Sectors in a Growth Model, Forthcoming in *Journal of Public Economic Theory*.

[9] Chari V., Hopenhayn H. (1991), Vintage Human Capital, Growth, and the Diffusion of New Technology, *Journal of Political Economy*, 99(6), pp. 1142-1165.

[10] Cozzi G., Giordani P.E., Zamparelli L. (2007), The Refoundation of the Symmetric Equilibrium in Schumpeterian Growth Models, *Journal of Economic Theory*, 136(1), September, pp. 788-797.

[11] Dinopoulos E., Sener F. (2007), New Directions in Schumpeterian Growth Theory, in Hanusch H. and Pyka A. (eds.), *Elgar Companion to Neo-Schumpeterian Economics*, Edward Elgar.

[12] Dinopoulos E., Thompson P. (1998), Schumpeterian Growth Without Scale Effects, *Journal of Economic Growth*, 3, pp. 313-335.

[13] Eicher T., Turnovsky S. (1999), Non-Scale Models of Economic Growth, *Economic Journal*, 109, pp. 394-415.

[14] Garner P. (2010), A note on endogenous growth and scale effects, *Economics Letters*, 106, pp. 98-100.

[15] Griliches Z. (1992), The search for R&D spillovers, *Scandinavian Journal of Economics*, 94 (supplement), pp. 29-47.

[16] Griliches Z. (1995), R&D and Productivity: Econometric Results and Measurement Issues, in Stoneman P. (ed.), *Handbook of the Economics of Innovation and Technical Change*, Blackwell Handbooks in Economics.

[17] Grossman G., Helpman E. (1991), Quality Ladders in the Theory of Growth, *Review of Economic Studies*, 58, pp. 43-61.

[18] Ha J., Howitt P. (2007), Accounting for Trends in Productivity and R&D: A Schumpeterian Critique of Semi-Endogenous Growth Theory, *Journal of Money, Credit and Banking*, 39(4), pp. 733-774.

[19] Hall B. (2004), Innovation and Diffusion, in Fagerberg J., Mowery D.C., and Nelson R.R., (eds.), *Handbook on Innovation*, Oxford, Oxford University Press.

[20] Hall B., Mairesse J., Mohnen P. (2010), Measuring the returns to R&D, in Hall B. and Rosenberg N. (eds.), *Handbook of the Economics of Innovation*, Elsevier.

[21] Helpman E. (ed.) (1998), *General Purpose Technologies and Economic Growth*, Cambridge MA, MIT Press.

[22] Howitt P. (1999), Steady Endogenous Growth with Population and R&D Inputs Growing, *Journal of Political Economy*, 107, pp. 715-730.

[23] Jones C. (1995a), Time Series Tests of Endogenous Growth Models, *Quarterly Journal of Economics*, 110, pp. 495-525.

[24] Jones C. (1995b), R&D-Based Models of Economic Growth, *Journal of Political Economy*, 103, pp. 759-784.

[25] Jones C. (1999), Growth: With or Without Scale Effects?, *American Economic Review, Papers and Proceedings*, 89, pp. 139-144.

[26] Jones C. (2005), Growth and Ideas, in Aghion P. and Durlauf S. (eds.), *Handbook of Economic Growth*, Elsevier Volume 1B, pp. 1063-1111.

[27] Kortum S. (1997), Research, patenting, and technological change, *Econometrica*, 65, pp. 1389-1419.

[28] Laincz C., Peretto P. (2006), Scale effects in endogenous growth theory: an error of aggregation not specification, *Journal of Economic Growth*, 11, pp. 263-288.

[29] Li C.W. (2000), Endogenous vs. semi-endogenous growth in a two-R&D-sector model, *Economic Journal*, 110, pp. 109-122.

[30] Li C.W. (2002), Growth and Scale Effects: the Role of Knowledge Spillovers, *Economics Letters*, 74, pp. 177-185.

[31] Li C.W. (2003), Endogenous Growth Without Scale Effects: A Comment, *American Economic Review*, 93(3), pp. 1009-1017.

[32] Madsen J. (2008), Semi-Endogenous versus Schumpeterian Growth Models: Testing the Knowledge Production Function using International Data, *Journal of Economic Growth*, 13(1), pp. 1-26.

[33] Peretto P. (1998), Technological Change and Population Growth, *Journal of Economic Growth*, 3(4), pp. 283-311.

[34] Peretto P. (1999), Cost reduction, entry, and the interdependence of market structure and economic growth, *Journal of Monetary Economics*, 43, pp. 173-195.

[35] Peretto P. (2007), Corporate taxes, growth and welfare in a Schumpeterian economy, *Journal of Economic Theory*, 137, pp. 353-382.

[36] Peretto P., Smulders S. (2002), Technological Distance, Growth and Scale Effects, *Economic Journal*, 112, pp. 603-624.

[37] Romer P. (1990), Endogenous Technological Change, *Journal of Political Economy*, 98 (5), pp. 71-102.

[38] Salop S. (1979), Monopolistic Competition with Outside Goods, *Bell Journal of Economics*, 10, pp. 141-156.

[39] Schulstad P. (1993), Knowledge diffusion in an endogenous growth model, *Economics Letters*, 42, pp. 275-278.

[40] Scotchmer S. (1991), Standing on the Shoulders of Giants: Cumulative Research and the Patent Law, *Journal of Economic Perspective*, 5 (1), pp. 29-41.

[41] Segerstrom P. (1998), Endogenous Growth Without Scale Effects, *American Economic Review*, 88, pp. 1290-1310.

[42] Segerstrom P. (2000), The Long-Run Growth Effects of R&D Subsidies, *Journal of Economic Growth*, Springer, 5(3), pp. 277-305.

[43] Smulders S., van de Klundert T. (1995), Imperfect competition, concentration and growth with firm-specific R&D, *European Economic Review*, 39, pp. 139-160.

[44] Young A. (1998), Growth Without Scale Effects, *Journal of Political Economy*, 106, pp. 41-63.