

Parental education and fertility : the role of mortality in the demographic transition

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Abstract

This paper examines a novel mechanism between adult life expectancy and fertility. The paper advances the theory that the number of new production techniques a worker experiences during a lifetime increases the benefits from formal schooling by increasing workers' adaptability to new production techniques. For a given amount of yearly changes in production techniques, even slow improvements in life expectancy may ultimately trigger parents to acquire formal schooling. Since schooling increases the number of inventions of new techniques of production, which further increases life expectancy, a virtuous circle is established. As more time devoted to schooling leaves less time for children, the model suggests, consistent with recent empirical evidence, that parental education is one of the main causal factors of the fertility decline during the demographic transition.

Keywords: life expectancy; infant mortality, schooling; demographic transition

JEL Classification: J10; J13; N30; O10; O40

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1 Introduction

"Children have to be educated, but they have also to be left to educate themselves."

Ernest Dimmet

One of the most robust correlations is that between the education level of parents and their fertility: better educated parents have fewer children. What also seems to be consistently supported by data is that the economies moving from from stagnation to growth also experiences a significant drop in fertility, i.e. a demographic transition. This means that the answer to why parents decide to become educated must contribute significantly to obtain a better understanding of the factors that trigger an economy to transition from a stage of stagnation – where income per capita is close to subsistence level – to a stage of sustained economic growth.

This paper studies the role played by ongoing changes in mortality on the evolution of schooling, fertility and ultimately growth in income per worker. It advances the idea that a higher number of new production techniques experienced during a lifetime diminishes the value of learning specific production skills. In contrast, formal schooling, characterized by acquiring general applicable knowledge, helps individuals to adapt to new production techniques. This creates a crucial role for life expectancy: a longer life will, for a given annual rate of change in production techniques, increase the number of new techniques experienced during a lifetime creating an incentive to acquire formal schooling. If schooling itself contributes to the introduction of new production techniques a virtuous circle is created between life expectancy, schooling and the rate of change in production techniques.

The main message of the model is that the effect of life expectancy on growth in income per worker depends on the stage of development. The model suggests that the causal effect of lower infant mortality is to *decrease* growth in income per worker. However, these two variables may be positively associated if parents go to school, since schooling indirectly causes lower infant mortality and higher growth in income per capita. As seen in Figure 1 adult life expectancy increased significantly during the years of the demographic transition. The proposed theory suggest that such gains in adult life expectancy may both be harmful or beneficial for growth in income per worker due to counteracting aggregation and incentive effects. An increase in adult life expectancy means that more people survive, the population increases, and wages decreases. However, as adult life expectancy becomes sufficiently high, further improvements induces more schooling causing higher productivity of workers due to accumulation of human capital.

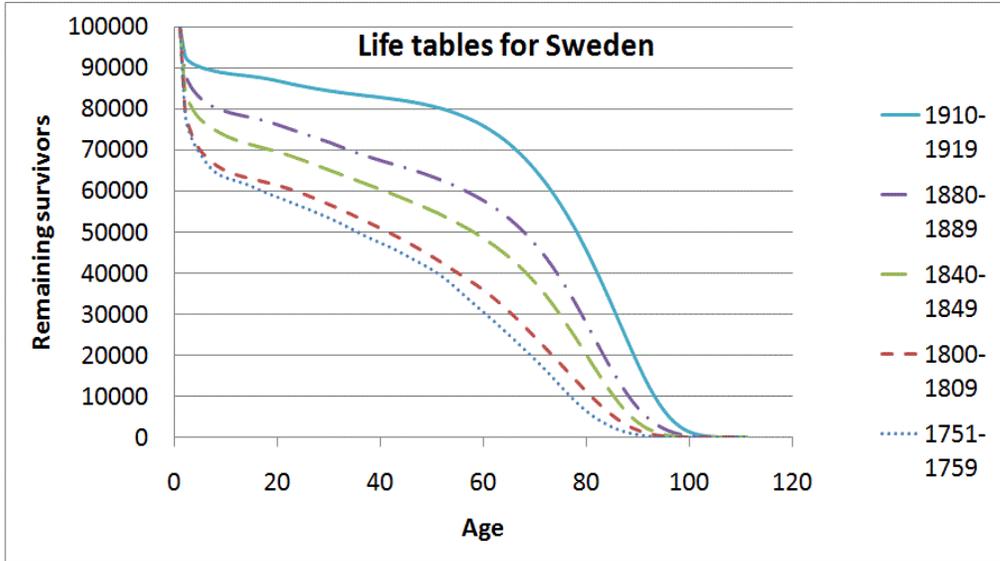


Figure 1: Life tables for Sweden in intervals of ten years

This boosts growth in income per worker which implies further increases in the time spend on schooling. Besides its direct effect on income per capita from its positive effect on human capital accumulation, more time spend on schooling leaves less time for children. Thus adult life expectancy, by triggering parents to go to school, causes a significant increase in the productivity of workers and a remarkable drop in fertility. These results are consistent with recent empirical studies by Hansen (2011) and Cervellati and Sunde (2011) showing that the causal effect of life expectancy on income per capita growth is small and negative prior to the demographic transition but highly positive thereafter.

The model follows the approach used in the literature explaining the decline in fertility with parents substituting quantity for quality of children, see among others Becker et al. (1990) and Galor and Weil (2000). However, this paper makes a clear distinction between quality provided to children and formal schooling, which in this model is only acquired by parents. Even though both quality and schooling time increases the accumulation of human capital, which is the engine of growth, I use the term quality to capture intergenerational transmission of specific skills that are mostly useful for applying existing techniques in the production process. In contrast, formal schooling acquired by parents is characterized by knowledge which is more generally applicable and therefore enable workers to cope with changes in production techniques. Since parents can only increase the human capital of their children by learning them specific skills, the distinction between skills and knowledge incorporates the idea that the schooling

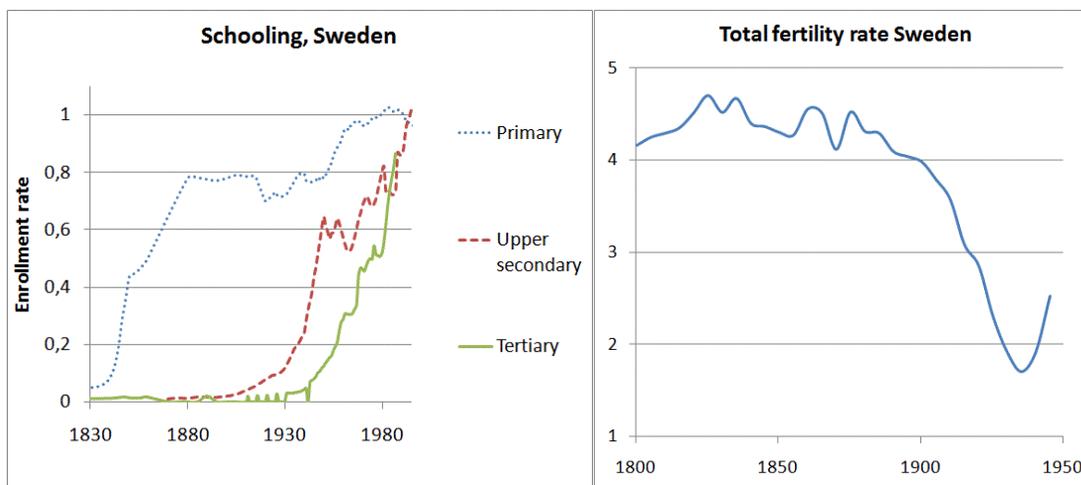


Figure 2: Evolution of education and fertility in Sweden.

Sources: Education data is from de la Croix et al. (2008); fertility data is from Keyfitz and Flieger (1968)

decision is made separately by each individual.¹ Thus, the model explains the demographic transition by increasing life expectancy that ultimately triggered *parents* to spend time in school and thus to spend less time on children.² This effect shows up in aggregate data by a significant increase in primary schooling before the drop in fertility and net fertility which took place around the same time where secondary schooling increased rapidly (see Figure 2).³ Consequently, the theory suggests that the first generations acquiring formal schooling should not be the smallest but giving birth to smaller generations. A line of studies, e.g. Cochrane (1979), Osili and Long (2008), Lehr (2009) and the recent study by Becker et al. (2011) finding evidence for a causal negative relation between parental education and fertility lending credence to the theory.

The paper proceeds as follows. Section 2 reports a summary of the related literature together with the evolution in data of the key variables of interests. In Section 3 the model is described and its main

¹The distinction between skills provided by parents and knowledge acquired by schooling makes it hard to justify that parents devote time to provide formal education to children, especially for the first generations acquiring schooling whose parents most likely were illiterate. In this period, parents typically faced a trade-off between sending their children to school or using them as labor on the farm. If parents care about the well being of each child they send the child to school when beneficial for the child. Therefore, the schooling decision is an individual decision and not part of the quantity-quality trade-off in this model.

²In this respect, the argument is related to Galor and Weil (1996) who explains the increase in the opportunity cost of children by capital accumulation which is relatively more complementary to women's than men's productivity.

³I thank David de la Croix for sharing the data on Swedish education.

predictions are shown and discussed. Section 4 provides numerical simulations of the model. Section 5 concludes.

2 Summary of related literature

There exist many competing theories of the determinants of the demographic transition.⁴ Within this literature, this paper is closest related to the literature that studies the effect of mortality on fertility.

One strand of this literature studies how the mortality of children affects the incentives of their parents (see, among others, Kalemli-Ozcan, 2002, 2008; Strulik, 2003, 2004; Boucekinne et al. 2003). For example, Kalemli-Ozcan (2008) argues that parents are induced to invest more in their children's schooling when child mortality decreases.⁵ Instead of focusing on any causal effect on fertility from infant mortality, via the incentives of parents, the present paper highlights the fundamental role of the rise in parental schooling as explanation of the observed negative association between the survivability of infants and (net) fertility during the demographic transition.

Related to this study is papers by Blackburn and Cipriani (2002), Hazan and Zoabi (2006) and Soares (2005) analyzing how exogenous changes to adult life expectancy affects fertility. Hazan and Zoabi (2006) find that an increase in children's longevity increases parental utility from quantity and quality equally. In contrast, this paper separates the schooling decision from the quantity-quality choice showing that increasing life-expectancy may cause more schooling depending on the rate of technological change.

Further, this paper is related to the growing body of literature within the field of Unified Growth Theory, initiated by the seminal papers of Galor and Weil (2000) and Galor and Moav (2002). In particular, a key argument founding the present model is also put forward in a similar context by Galor and Weil (2000). They argue that technological change is the prime force behind the increasing demand for human capital making parents substitute from quantity towards quality of children causing declining fertility. As technologies change more rapidly, parents decide to invest more in education of their children to make them acquire knowledge with more general applicability rather than skills related to specific production techniques that quickly can become outdated. The argument made here is complementary in

⁴See Galor (2011a, 2011b) for a comprehensive overview of the different theories.

⁵While the structure of the demand side of the present model is close to that used Kalemli-Ozcan (2008) the focus is different here. In particular, the present paper only focuses on budgetary effects of *infant* mortality in the quantity-quality trade-off. For a given mix between quantity and quality, I assume that more surviving children increases total time devoted to quality time (child rearing) but leaves total quantity time (child bearing) unchanged.

two dimensions: i) the present paper focuses on the incentives of parents to acquire schooling themselves and ii) that the eroding effect from technological change is magnified by increasing life expectancy.

Within this strand of literature the present paper is closest related to the contributions of Cervellati and Sunde (2005), Cervellati and Sunde (2008) and de la Croix and Licandro (2011). The model in de la Croix and Licandro (2011) also have parents deciding their own amount of schooling together with a tradeoff between quantity and quality of children where quality is adult life expectancy of the offspring. The authors explain the reversal of the relation between life expectancy and fertility by individuals escaping a subsistence consumption constraint. Cervellati and Sunde (2005) also provides a theory for the positive effects of longevity on human capital formation and thereby economic development, according to a positive feedback mechanism between the share of educated workers and longevity. This analysis is extended in Cervellati and Sunde (2008), where parents decide both children's and own education together with the number of children.⁶

The present paper adds to this literature by i) showing a non-monotonic effect of infant mortality and adult life expectancy on growth in income per worker, ii) predicting a non-monotonic evolution of net fertility, and thus the growth rate of the population, during the demographic transition explained solely by the rise in parental schooling induced by increasing life expectancy⁷ and iii) employing the idea that the amount of changes in production techniques experienced during a lifetime is the crucial determinant for triggering parents to acquire formal schooling and thereby causing a demographic transition.

3 The model

Consider an overlapping generations model where time is discrete and indexed by $t \in \{0, 1, 2, 3, \dots\}$. The life of each individual spans three periods. In the first period of life, infancy, individuals are reared by their parents. In the second period of life, the reproductive period, individuals go to school, rear children and consume the income generated from supplying labor to the market. In the last period of life, the mature period, individuals consume the wage income from their inelastic supply of human capital to the labor market.⁸

⁶Additional contributions of Unified Growth models includes Boucekine et al. (2002, 2007), Doepke (2004), Galor and Moav (2002), Galor and Mountford (2008), Lagerlöf (2003), Strulik and Weisdorf (2008, 2010) and Weisdorf (2004).

⁷Strulik and Weisdorf (2010) explains this non-monotonic evolution of net fertility by the decline in the relative price of food.

⁸I use the phrasing "mature" to make a distinction between the standard Diamond model and the present model. Thus, the last period of life should not be thought of as a period where individuals become old and unproductive but rather as

3.1 Demographics

Reproduction is asexual and each individual living in the reproductive period in period t gives birth to $b_t \geq 0$ children.⁹ Due to infant mortality only a fraction $\pi_t \in (0, 1)$ of the children survives to the reproductive period. Thus, net fertility, $n_t \equiv b_t \pi_t$, is the number of surviving children per parent.¹⁰ The number of individuals in the reproductive age in period t , L_t , is referred to as generation t . The workforce (the adult population) in period t is given by:

$$P_t \equiv L_t \left[1 + \frac{\phi_t}{n_{t-1}} \right] \quad (1)$$

where $\phi_{t+1} \in (0, 1]$ is the average length of life of t reproductive individuals in the mature period in period $t + 1$ where the length of life in the reproductive period is normalized to unity. It follows that the growth factor of the workforce is given by:

$$\lambda_t \equiv \frac{P_{t+1}}{P_t} = n_{t-1} \frac{n_t + \phi_{t+1}}{n_{t-1} + \phi_t}. \quad (2)$$

3.2 Technologies

Each individual, belonging to generation t , has $h_{1,t}$ units of human capital that can be supplied to the labor market. The size of parental human capital, $h_{1,t}$ is determined by the following technology:

$$h_{1,t} = h_1 (g_{t-1}, q_t, h_{1,t-1}) = [g_{t-1} q_t]^\varepsilon h_{1,t-1}, \quad (3)$$

where q_t is the share of the parental time endowment spend on child quality per surviving child, determined in period $t - 1$ by the individuals constituting generation $t - 1$. This gives rise to endogenous growth, where the growth factor of human capital is given by:

$$\gamma_t \equiv \frac{h_{1,t}}{h_{1,t-1}} = Z_1 [g_{t-1} q_t]^\varepsilon, \quad (4)$$

the years in the labor force after having went to school and given birth to children.

⁹Although the timing of events within a period is a "black box" in discrete time models, I assume that individuals give birth some time after entering the reproductive age making the childhood period shorter than the subsequent two periods of life. Suppose that the reproductive period of life amounts to T years and agents give birth to children after λT years in the period, the length of the childhood period is given by $[1 - \lambda]T$. For example, if $T = 20$ years and $\lambda = \frac{1}{4}$ then the childhood period amounts to 15 years (and individuals give birth at age $T = 20$.)

¹⁰Since individuals are only mortal after living through the reproductive period, $b_t \pi_t$ also denotes the net reproduction rate.

where $\varepsilon \in (0, 1)$ is the constant elasticity of parental human capital to its inputs of production. The impact of formal schooling on the accumulation of parental human capital is taken into account by:

$$g_{t-1} = g(s_{t-1}) \quad (5)$$

$$\text{where } g_s(s_{t-1}) > 0, g_{ss}(s_{t-1}) < 0 \text{ and } g_s(0) > 0$$

where s_{t-1} is the average units of schooling of generation $t - 2$. This is a "standing on the shoulder of giants" effect capturing the positive implications of formal schooling, e.g. accumulation of knowledge leading to scientific progress, on the level of parental human capital of generation t . Since this effect of schooling works at the aggregative level, individuals do not take this into account when choosing how much time to spend on schooling. Thus, parents of generation t decide their amount of schooling, s_{t+1} , only with the purpose of raising their own level of human capital in the mature period $h_{2,t+1}$. This is done subject to the following technology:

$$h_{2,t+1} = h_2(\kappa_{t+1}, s_{t+1}, h_{1,t}) = [\kappa_{t+1} + s_{t+1}]^\mu h_{1,t}, \quad (6)$$

where $\mu \in (0, 1)$. Due the specification in eq. (6), individuals may have positive mature human capital, and thereby positive earnings in the mature period, without schooling. Since quality time provides children with necessary skills for survival, while formal schooling improves the human capital above the necessary, the assumptions in (3) and (6) seems plausible. The possibility of a corner solution for s_t is central for the argument of this paper and the role of $\kappa_{t+1} > 0$ is therefore studied in detail in the next subsection.

There exists a single homogeneous good which is produced with the following technology:

$$Y_t = [X_t]^\alpha [H_{1,t} + H_{2,t}]^{1-\alpha}$$

where Y_t is aggregate output, $H_{1,t} \equiv h_{1,t}L_t l_t$ is total parental human capital supplied (where $l_t \equiv [1 - [\tau + \pi_t q_{t+1}] b_t - \sigma s_{t+1}]$ is the share of the unit time endowment that parents spend in the labor market in period t) which is a perfect substitute to the total mature human capital $H_{2,t} \equiv h_{2,t}L_{t-1}\phi_{t-1}$ supplied in period t . The variable X_t measures the level of quality units land, which takes the following functional form:

$$X_t = xL_{t-1} \tag{7}$$

where x is the units of raw land per worker.¹¹ This specification assures income per worker is independent of the size of the population but that the growth rate of income per worker is affected by the growth rate of the workforce.¹² The relation in (7) can be given a Boseropian foundation, i.e. population pressure in period $t-1$ urges individuals to search for new better land or use better cultivation techniques, e.g. a plow and shifting cultivation, which increases the quality units of land in period t (Boserup, 1981).

There is no property right over X_t .¹³ Hence, each efficiency unit of labor earns its average product:

$$\frac{Y_t}{H_{1,t} + H_{2,t}} \equiv w_t = \left[\frac{x}{n_{t-1}h_{1,t}l_t + \phi_{t-1}h_{2,t}} \right]^\alpha, \tag{8}$$

which reveals that, for given levels of human capital, higher life expectancy, either due to lower infant mortality or higher adult life expectancy, decreases income per worker. The total income in period t is divided between income per worker in the reproductive age:

$$w_t h_{1,t} l_t, \tag{9}$$

and income per worker in old age:

$$w_t h_{2,t}. \tag{10}$$

3.3 Preferences, budget constraints and optimization

The preferences of each individual in generation t are represented by the following expected utility function:

$$u^t = [1 - \chi] [\ln c_{1,t} + \phi_t \ln c_{2,t+1}] + \chi \ln n_t h_{1,t+1} \tag{11}$$

¹¹I follow the standard approach in the related literature by abstracting from physical capital as a factor of production.

¹²This assumption is not crucial for the main results of the model since, as will become apparent below, the evolution of the economy is independent of income per worker. However, the assumption simplifies matters by making growth in income per worker on the balanced growth path equal to the endogenous growth rate of parental human capital (see below).

¹³This is a common assumption in the literature. The issue of landownership has little importance here since I focus on the demographic transition where the income share to land diminishes rapidly. See Goldsmith, (1952).

where $c_{1,t}$ denotes consumption in the reproductive period, ϕ_t is the expected length of the mature period,¹⁴ $c_{2,t+1}$ denotes the flow of consumption during the mature period, n_t is number of surviving children and $h_{1,t+1}$ is the level of human capital embodied in each child.¹⁵ The fact that parents only care about their children's human capital ($h_{1,t+1}$) in the first period of the children's adult life (the children's parental period), is the mirror image of the assumption that the schooling decision is made by each individual separately when reach the reproductive period. This formulation of preferences can be given a evolutionary foundation since it is only higher parental human capital that improves the chances of future reproductive success because individuals are not fertile in the mature period of life. The parameter χ measures the taste for having children relative to consumption, which is allocated over two periods of life.¹⁶ To account for the life-cycle effect of changes in adult life expectancy on schooling it is assumed that individuals cannot (dis)save.¹⁷ Together this implies that the preferences comprises three tradeoffs: between own consumption and children, between quantity and quality of children and, finally, between present and future consumption.

Each individual in generation t faces the following budget constraint in the reproductive period:

$$c_{1,t} \leq w_t h_{1,t} [1 - [\tau + \pi_t [\rho + q_{t+1}]] b_t - \sigma s_{t+1}], \quad (12)$$

where $\tau \in (0, 1)$ is the fraction of the unit time endowment a parent spends on child bearing, $\rho \in (0, 1)$ is the fraction of total time each surviving child requires and q_{t+1} is the amount of time spend on quality of each surviving child. The budget states that parents only spend time on raising the children surviving infancy determined by the survival rate of infants π_t .¹⁸ Hence, all children surviving infancy will survive

¹⁴Relative to the length of the reproductive period which is normalized to unity. I assume that the size of each generation is large enough to apply the law of large number such that ϕ denotes both the average and expected length of life in the mature period of life.

¹⁵The interpretation of first period variables may either be as flow or stock variables since the length of the period is normalized to unity. Hence, the time constraint could be seen as a flow constraint holding at each instant or, alternatively, as I will do for simplicity, as the stock of time in the whole period.

¹⁶Using a homothetic utility function follows the standard approach in the literature.

¹⁷If individuals had access to capital markets they would likely be dissaving due to higher income in the mature period compared to the parental period. One may introduce an additional old-age period with zero earnings from labor income to incorporate a motive for life-cycle savings. However, this would complicate matters and is not the objective of this paper. For a theoretical study concerning mortality, fertility, and saving, see Boldrin and Jones (2002).

¹⁸The paper follows the related literature by abstracting from modeling children as a discrete number. Hence, both fertility and net fertility are continuous variables implying that π_t is both the survival rate and survival probability of infants.

the first two periods of life with certainty. The time spend on schooling is given by σs_{t+1} where $\sigma \in (0, 1)$ is the fraction of the time endowment spend per unit of schooling, s_{t+1} . The residual time is devoted to the labor market generating the income consumed.

In period $t + 1$ each individual of generation t , if still alive, supplies human capital inelastically to the labor market and may consume the entire earnings.

$$c_{2,t+1} \leq w_{t+1} h_{2,t+1}, \quad (13)$$

where $w_{t+1} h_{2,t+1}$ is the flow of income during the mature period of life. Substituting (12) and (13) together with (3) and (6) into (11) the problem that a member of generation t solves can be written as :

$$\begin{aligned} \{b_t, q_{t+1}, s_{t+1}\} = \arg \max & [1 - \chi] \ln (w_t h_{1,t} [1 - [\tau + \pi_t [\rho + q_{t+1}]] b_t - \tau s_{t+1}]) \\ & + [1 - \chi] \phi_{t+1} \mu \ln (w_{t+1} [\kappa_{t+1} + s_{t+1}]^\mu h_{1,t}) + \chi \ln (b_t \pi_t [g_t \pi_t q_{t+1}]^\varepsilon). \end{aligned}$$

Each individual in generation t takes $w_t h_{1,t}$, π_t , ϕ_{t+1} , κ_{t+1} and g_t as given when maximizing u^t by choosing non-negative numbers of b_t , q_{t+1} and s_{t+1} . The first order conditions for the problem writes:

$$\frac{\partial U}{\partial b_t} = 0 \implies \frac{[1 - \chi] [\tau + \pi_t [\rho + q_{t+1}]]}{1 - [\tau + \pi_t [\rho + q_{t+1}]] b_t - \sigma s_{t+1}} = \frac{\chi}{b_t} \quad (14)$$

$$\frac{\partial U}{\partial q_{t+1}} = 0 \implies \frac{[1 - \chi] \pi_t b_t}{1 - [\tau + \pi_t [\rho + q_{t+1}]] b_t - \sigma s_{t+1}} = \frac{\chi \varepsilon}{q_{t+1}} \quad (15)$$

$$\frac{\partial U}{\partial s_{t+1}} = 0 \implies \frac{[1 - \chi] \sigma}{1 - [\tau + \pi_t [\rho + q_{t+1}]] b_t - \sigma s_{t+1}} = \frac{[1 - \chi] \phi_{t+1} \mu}{\kappa_{t+1} + s_{t+1}}. \quad (16)$$

An interior (positive) solution for b_t and q_{t+1} is guaranteed by the logarithmic utility function and the functional form of the parental human capital technology. However, the specification of the technology for mature human capital may imply that zero schooling is optimal.¹⁹ Consequently, the demand functions are stated both for a corner and interior solution for s_{t+1} :

$$\tau \pi_t b_t = \begin{cases} \chi [1 - \varepsilon] & \text{if } \kappa_{t+1} \geq \hat{\kappa}_{t+1} \\ \chi [1 - \varepsilon] \Delta_{t+1} & \text{if } \kappa_{t+1} \leq \hat{\kappa}_{t+1} \end{cases} \quad (17)$$

$$\pi_t q_{t+1} = \frac{\varepsilon \tau}{1 - \varepsilon} \quad (18)$$

¹⁹Note, that this is not caused by any unusual assumptions about the preferences of the individual but is solely due to the specification in (6).

$$s_{t+1} = \begin{cases} 0 & \text{if } \kappa_{t+1} \geq \hat{\kappa}_{t+1} \\ \frac{\frac{1-\chi}{\sigma} \phi_t \mu - \kappa_t}{1 + \phi_t \mu [1-\chi]} & \text{if } \kappa_{t+1} < \hat{\kappa}_{t+1} \end{cases} \quad (19)$$

where $\hat{\kappa}_{t+1} \equiv \frac{1-\chi}{\sigma} \phi_{t+1} \mu$ and $\Delta_{t+1} \equiv \frac{1+\sigma \kappa_{t+1}}{1+\phi_{t+1} \mu [1-\chi]}$. It follows that $\Delta_{t+1} < 1 \iff s_{t+1} > 0$. Hence, fertility is lower for parents spending time in school compared to parents who did not go to school.²⁰ For an interior solution, it is evident that higher adult life expectancy has a positive effect on schooling. This is only due to the well known effect dating back to Ben-Porath (1967): higher life expectancy increases the expected benefits from schooling. However, life expectancy also induces individuals to acquire schooling through an additional effect captured by κ_{t+1} .

The presence of κ_{t+1} in the mature human capital technology formalizes the idea that individuals do not become completely obsolete if they do not acquire formal schooling, i.e. the skills learned in childhood generates positive earnings in mature life without schooling. Hence, κ_{t+1} may be interpreted as an innate capability to learn new production techniques. Suppose that it becomes increasingly hard for a mature worker, when only using innate capabilities, to maintain productivity the number of new techniques applied in production increases. This is in line with the findings in Aiyar et. al (2008) and Bar and Leukhina (2010) showing that the transmission of skills embodied in humans is the main source of improvements in production techniques before the demographic transition.

New production techniques can only be implemented in production process if workers are able to manage these new techniques. Thus, growth in parental human capital has an adverse effect on the level of mature human capital, which can be seen as "creative destruction".²¹ Since a longer life will increase the expected number of new techniques a mature worker is exposed to, a higher adult life expectancy will reinforce the adverse effect from growth in parental human capital on the level of mature human capital. This materializes into the following properties of κ_{t+1} :²²

$$\kappa_{t+1} = \kappa(\gamma_{t+1}, \phi_{t+1}), \quad (20)$$

²⁰This result is in line with the finding in Lehr (2009), reporting a negative impact on fertility of higher demand for secondary education.

²¹Although not as in the original Schumpeterian sense since the destruction in the present model is an intergenerational externality involving only the productivity of workers.

²²Of course, the number of new inventions that an individuals will expect to be exposes to, depends both in the distribution of the new inventions and the survival curve within the mature period. I will make no further assumptions about these figures and see $\kappa(\gamma, \phi)$ as a closed form representation capturing the mentioned effects.

where

$$\begin{aligned} \kappa_\gamma (\gamma_{t+1}, \phi_{t+1}) &< 0, \kappa_\phi (\gamma_{t+1}, \phi_{t+1}) < 0, \\ \kappa_{\gamma\gamma} (\gamma_{t+1}, \phi_{t+1}) &> 0, \kappa_{\phi\phi} (\gamma_{t+1}, \phi_{t+1}) > 0 \text{ and } \kappa_{\gamma\phi} (\gamma_{t+1}, \phi_{t+1}) > 0. \end{aligned} \tag{21}$$

Further, by using (19), this implies that:

$$\frac{\partial [\kappa (\gamma_{t+1}, \phi_{t+1}) + s (\gamma_{t+1}, \phi_{t+1})]^\mu h_{1,t}}{\partial \gamma_{t+1}} \begin{cases} < 0 \text{ if } \kappa_t \geq \hat{\kappa}_t \\ > 0 \text{ if } \kappa_t < \hat{\kappa}_t \end{cases}.$$

Hence, whether higher growth in parental human capital has a positive or negative net effect on the level of mature human capital depends on whether schoolings time is positive or zero.²³

The solution of the individual problem shows that if parents spend more time on schooling this only affects the time spend on children by decreasing the number of births whereas quality time per surviving child remains unchanged. This is because the total amount of time spend on quality is proportional to fertility but not vice versa. Thus, quality time is only affected by the relative gains and costs between quality and fertility. While the marginal gain (in utility) from giving birth is independent of infant mortality, the marginal costs increases due to time spend on child rearing. This means that fertility is increasing in infant mortality, i.e. parents give birth to fewer children when more of them survive. Since the cost increases less than proportionally with the survival rate of infants, the causal effect from an exogenous increase in the survival rate of infants is to increase net fertility. These results are consistent with the empirical evidence reported in Doepke (2004) and Murphy (2009).²⁴ Finally, it can be seen that parents *decrease* quality time in response to lower infant mortality such that the total time spend on child quality per child surviving infancy ($\pi_t q_{t+1}$) is unaffected by infant mortality.²⁵ This result is due to the increasing marginal cost of child quality if more infants survive. Consequently, if more children survive infancy, each child receives less quality time and vice versa. Since this result is only capturing

²³A summary of the derivatives of the human capital function is provided in the Appendix.

²⁴However, there is no consensus on whether infant mortality has a causal effect on fertility in the empirical literature studying this issue. See Galor (2011a, 2011b) and Lee (2003) for an overview.

²⁵This should not be confused with improvements in child mortality which may have positive incentive effects as argued by Kalemli-Ozcan (2008). In fact, this prediction of the model that higher infant mortality has a growth promoting effect is supported by empirical evidence reported in Ruff, Trinkaus and Holliday (1997). They calculate body mass of early modern humans and show that those exposed to a more harsh environment, measured by humans living above 30° N latitude, were significantly larger than those from under 30° N latitude. For a theoretical study of the implications of the harshness of the ecogeographical environment of the development of early humans, see Grall (2011). Further, the finding in Westendorp and Kirkwood (1998) lends credence to the present theory.

how changes in *infant* mortality affect the transmission of specific skills it illustrates a key argument deduced from the model: contrary to incentive effects that originate from changes in child mortality and adult life expectancy, there is no changes in incentives caused by infant mortality.²⁶ Thus the theory does not reject positive effects of provision of more "quality" (e.g. formal education, better nutrition etc.) for children through improvements in life expectancy conditional on surviving infancy. However, in the present model, such incentive effects are not captured by the quantity-quality tradeoff but rather by improvements in adult life expectancy.

The following proposition summarizes the effect of exogenous changes in infant mortality on a parent's demand for schooling, children and the quality time devoted to each surviving child:

Proposition 1 *An exogenous rise in the survival rate of infants:*

decreases fertility, i.e. $\frac{\partial b_t}{\partial \pi_t} < 0$,

has no effect on schooling time, i.e. $\frac{\partial s_{t+1}}{\partial \pi_t} = 0$,

decreases the time devoted to quality of each surviving child, i.e. $\frac{\partial q_{t+1}}{\partial \pi_t} = 0$.

Proof: Follows from differentiation of (17), (18) and (19).

The effect of adult life expectancy on parental choices is summarized as follows:

Proposition 2 *The time parents devote to quality per child is unaffected by adult life expectancy: $\frac{\partial q_{t+1}}{\partial \phi_{t+1}} =$*

0.

If parents do not spend time in school, fertility is unaffected by adult life expectancy:

$\frac{\partial b_t}{\partial \phi_{t+1}} = 0$, if $\kappa_{t+1} \geq \hat{\kappa}_{t+1}$.

If parents spend time in school, higher adult life expectancy lowers fertility and increases schooling time:

$\frac{\partial b_t}{\partial \phi_{t+1}} < 0$ and $\frac{\partial s_{t+1}}{\partial \phi_{t+1}} > 0$ if $\kappa_{t+1} < \hat{\kappa}_{t+1}$.

Proof: Follows from differentiation of (17), (18) and (19).

3.4 The dynamical system

This section describes the evolution of endogenous variables over time.

²⁶In the model, this is of course only true because the utility function is homothetic. Considering sequential fertility, Doepke (2005) shows that lower child mortality decreases fertility due to a lower "replacement effect" and also finds that net fertility increases.

3.4.1 Evolution of mortality

There exist different views about the determinants of mortality. Some scholars argue, in line with the finding in Preston (1975), that income and life expectancy is closely associated.²⁷ This line of thinking goes all the way back to Malthus (1826) and the equilibrating mechanism on population growth from low income causing higher death rates (the positive check) which is more recently restated in Fogel (1994). Other scholars argue that education, scientific progress and invention of medicine is the main determinant of mortality, see e.g. Cutler et al. (2006) and Soares (2007) for recent contributions. Of course, these explanations need not to be mutually exclusive: each view of the determinant of mortality may be more relevant during different stages of the development of an economy.²⁸ Since the level of parental human capital comprises accumulation of skills, formal schooling and growth in income per worker relating it to life expectancy does not exclude any of the factors suggested as being relevant for life expectancy.²⁹ Thus, the following relations are assumed:

$$\pi_{t+1} = \bar{\pi} + \pi(h_{1,t}), \quad (22)$$

with

$$\partial \pi_s(h_{1,t}) > 0, \pi(0) = 0 \text{ and } \lim_{\pi_t \rightarrow 1} \pi_s(h_{1,t}) = 0, \quad (23)$$

$$\phi_{t+1} = \bar{\phi} + \phi(h_{1,t}), \quad (24)$$

with:

$$\phi_{h_1}(h_{1,t}) > 0, \phi(0) = 0 \text{ and } \lim_{\phi_{t+1} \rightarrow 1} \phi_{h_1}(h_{1,t}) = 0. \quad (25)$$

meaning that the probability of surviving infancy is bounded from below by an exogenous level, $\bar{\pi} \in (0, 1]$. In addition, the specification in (23) implies that it is the level of parental human capital (determined by accumulation of skills and possibly formal schooling) that improves the parents' own adult life expectancy and their children's infant mortality rate.

²⁷It is still a dispute which way the causality is stronger. In that debate, see Dalgaard and Strulik (2010) for a recent contribution.

²⁸For a theoretical study of the long run evolution of life expectancy, see Galor and Moav (2005).

²⁹In addition, the intergenerational transmission of skills may also comprise the accumulation of "medical" experiences that have shown to be beneficial for health.

3.4.2 Evolution of income per worker on the balanced growth path

The level of parental human capital in period $t + 1$ is given by:

$$h_{1,t+1} = \gamma_{t+1} h_{1,t}, \quad (26)$$

with $h_{1,0} = h_1^0$ given. Thus, parental income per time unit supplied grows as follows:³⁰

$$h_{1,t+1} w_{t+1} = \gamma_{t+1} \omega_{t+1} w_t h_{1,t}, \quad (27)$$

where

$$\omega_{t+1} \equiv \frac{w_{t+1}}{w_t} = \left[\frac{n_{t-1} l_t + \phi_t \frac{[\kappa_t + s_t]^\mu}{\gamma_t}}{\gamma_{t+1} \left[n_t l_{t+1} + \phi_{t+1} \frac{[\kappa_{t+1} + s_{t+1}]^\mu}{\gamma_{t+1}} \right]} \right]^\alpha. \quad (28)$$

Clearly, increasing net fertility and increasing adult life expectancy lowers the growth in wages. Inspecting the growth in income per worker on the balanced growth path is instructive for the role played by ω .

Definition 1 *The balanced growth path of the economy is characterized by $\gamma_t = \gamma$, $\kappa_t = \kappa$, $\hat{\kappa}_t = \hat{\kappa}$, $\omega_t = \omega$, $b_t = b$, $q_t = q$, $s_t = s$, $\pi_t = \pi$ and $\phi_t = \phi$ for all t .*

By inserting (8) into (27) the growth factor of income per worker (both parents and mature workers) on the balanced growth path is given by:

$$\gamma \omega = \gamma^{1-\alpha}, \quad (29)$$

A stronger Malthusian effect, i.e. a higher α , decreases growth in income per worker since human capital is embodied in workers. Further, (29) shows that neither the level nor the growth in population has any direct effect on growth in income per worker in the long run. However, substituting (17), (18) and (19) into (3) and then use it in (29) yields:

$$\gamma \omega = \left[g(s) \frac{\varepsilon [\tau + \pi \rho]}{[1 - \varepsilon] \pi} \right]^{\varepsilon [1-\alpha]}. \quad (30)$$

showing that higher survival probability of infants, entailing higher net fertility, results in lower growth in income per worker. The effect of mortality on the growth factor of income per worker on the balanced growth path is summarized on the following proposition:

³⁰ Alternatively, one may call this full income, i.e. if parents use the total time endowment to work. However, full time work would affect w_t at the aggregate level which is why is stick to the frasing of $w_t h_{1,t}$.

Proposition 3 *An exogenous increase in infant mortality increases growth in income per worker on the balanced growth path: $-\frac{\partial \gamma \omega}{\partial \pi} > 0$ if $\kappa < \hat{\kappa}$.*

If parents do not spend time on schooling, there is no effect of an exogenous increase in adult life expectancy on the growth in income per worker on the balanced growth path: $\frac{\partial \gamma \omega}{\partial \phi} = 0$ if $\kappa > \hat{\kappa}$

If parents spend time on schooling an exogenous increase in adult life expectancy increases schooling on the balanced growth path: $\frac{\partial \gamma \omega}{\partial \phi} > 0$ if $\kappa < \hat{\kappa}$.

Proof: Follows from differentiation of (30) and using (23) and (25).

Remembering (19) and that $\kappa = \kappa(\gamma, \phi)$ it is evident that life expectancy will be growth promoting only for a sufficient high level of life expectancy where individuals find it optimal to acquire schooling. Due to (18), it is seen in (29) that higher infant mortality improves the balanced growth of income per worker. This finding is in line with Voigtländer and Voth (2009, 2010) arguing for positive effects from the wars and the plague. These phenomena are characterized by causing death with little or no time for people to react to the changes in the mortality environment, i.e. there are no incentive effects linked to such changes in mortality and therefore only positive effects due to a lower Malthusian pressure. Exactly the same feature holds for infant mortality since little can be done by the parents if a child die in infancy. Here, however, besides the Malthusian effect on the aggregate level, the mechanism originates inside the family: as more infants survive each surviving child will obtain fewer skills which lowers growth in income per worker.

To sum up, the impact of mortality on growth in income per worker is not clear cut. Exogenous improvement in survivability of infants will dampen growth, but these two variables may still be positively associated due to schooling which increases both the survival probability of infants and growth in income per worker. The effect of an exogenous increase in adult life expectancy depends, among other things, on the level of life expectancy. If higher life expectancy triggers parents to acquire schooling, growth in income per worker will increase due to higher growth factor of parental human capital. Together, these finding may help explain the mixed empirical findings of the causal effect of life expectancy on growth in income per capita (see Acemoglu and Johnson, 2008; and Lorentzen et al, 2008). Further, the model provides a theoretical foundation for the empirical studies by Hansen (2011) and Cervellati and Sunde (2011) reporting negative effects from lower mortality on growth in income per capita for countries that have not gone through the demographic transition and a large positive effect for post-transitional countries.

3.5 Evolution of the economy

The level of parental human capital evolves as follows:

$$h_{1,t+1} = \gamma_{t+1} h_{1,t} \quad (31)$$

Inserting the expression for quality (17) into the parental human capital technology (3) implies the growth factor of parental human capital evolves according to:

$$\gamma_{t+1} \equiv \begin{cases} \gamma^I(\pi(h_{1,t})) & \text{if } \kappa_t \geq \hat{\kappa}_t \\ \gamma^{II}(s_t, \pi(h_{1,t})) & \text{if } \kappa_t < \hat{\kappa}_t \end{cases} \quad (32)$$

where

$$\begin{aligned} \gamma^I &\equiv \left[g(0) \frac{\varepsilon[\tau + \pi_t \rho]}{[1 - \varepsilon] \pi_t} \right]^\varepsilon \text{ and} \\ \gamma^{II} &\equiv \left[g(s_t) \frac{\varepsilon[\tau + \pi_t \rho]}{[1 - \varepsilon] \pi_t} \right]^\varepsilon \end{aligned} \quad (33)$$

Finally, the schooling time is given by:

$$s_{t+1} \equiv \begin{cases} s^I & \text{if } \kappa_{t+1} \geq \hat{\kappa}_{t+1} \\ s^{II}(\phi(h_{1,t}), \gamma_{t+1}) & \text{if } \kappa_{t+1} < \hat{\kappa}_{t+1} \end{cases} \quad (34)$$

where

$$\begin{aligned} s^I &= 0 \\ s^{II} &= \frac{\frac{1-\chi}{\sigma} \phi_{t+1} \mu - \kappa(\phi_{t+1}, \gamma_{t+1})}{1 + \phi_{t+1} \mu [1 - \chi]} \end{aligned}$$

The initial conditions are:

$$s_0 = s^0 \text{ and } h_{1,0} = h_1^0.$$

The equations in (31), (32) and (34) constitutes a three-dimensional first order nonlinear system of difference equations that determines the evolution of the whole economy. Hence, the evolution of the level and growth in human capital, and thereby the evolution of infant mortality and adult life expectancy and schooling, is independent of the size of the population and the level of income per worker.

3.5.1 The dynamics of adult life expectancy and growth in parental human capital

For illustrating the main result of the paper it is unveiling to consider the following system:³¹

³¹The fact that ϕ_{t+1} is a function of γ_t can be seen in (24) using (3).

$$\begin{aligned}\gamma_{t+1} &\equiv \begin{cases} \gamma^I(\pi(h_1)) & \text{if } \kappa_t \geq \hat{\kappa}_t \\ \gamma^{II}(s(\phi_t), h_1) & \text{if } \kappa_t < \hat{\kappa}_t \end{cases} \\ \phi_{t+1} &= \begin{cases} \bar{\phi} + \phi^I(h_1) & \text{if } \kappa_t \geq \hat{\kappa}_t \\ \bar{\phi} + \phi^{II}(\gamma_t, h_1) & \text{if } \kappa_t < \hat{\kappa}_t \end{cases}\end{aligned}\quad (35)$$

which governs the dynamics of the growth factor and adult life expectancy for a given level of parental human capital, h_1 . Now define the set of all pairs (γ_{t+1}, ϕ_t) where $s_{t+1} = 0$:

Definition 2 $\mathcal{Z} \equiv \{(\gamma_{t+1}, \phi_{t+1}) : \kappa_{t+1} \geq \hat{\kappa}_{t+1}\}$ and the boundary $\mathcal{B}(\mathcal{Z}) \equiv \{(\gamma_{t+1}, \phi_{t+1}) : \kappa_{t+1} = \hat{\kappa}_{t+1}\}$

Using the properties given in (21) implies:

Lemma 1 For all pairs $(\gamma_{t+1}, \phi_{t+1}) \in \mathcal{B}(\mathcal{Z})$, ϕ_{t+1} is a decreasing convex function of γ_{t+1} .

Proof: Provided in the Appendix

The properties of ϕ_{t+1} is given in (22) and the properties of γ_{t+1} can be summarized as:

Lemma 2 γ_{t+1} is a decreasing convex function of π_t : $(\gamma_{t+1})_{\pi} < 0$, $(\gamma_{t+1})_{\pi\pi} < 0$

γ_{t+1} is unaffected by s_t for a corner solution for s_t and a increasing convex function of s_t for an interior solution of s_t :

$(\gamma_{t+1})_s = 0$ if $\kappa_t \geq \hat{\kappa}_t$ and $(\gamma_{t+1})_s > 0$ and $(\gamma_{t+1})_{ss} < 0$ if $\kappa_t < \hat{\kappa}_t$.

Proof: Follows from (33), (23) and (5).

Any steady state of the conditional system in (35) is given by the intersection of the loci of the system in (35):

Definition 3 A steady state equilibrium of the system (10) is a vector $\mathcal{E}(h_1) = \{\gamma^{\mathcal{E}}, \phi^{\mathcal{E}}\}$ where $\mathcal{E}(h_1) \in \{[0, \infty], (0, 1]\}$. There exist two types of equilibria $\mathcal{E}^I(h_1) \equiv \{\gamma^I(h_1), \phi^I(h_1)\}$ and $\mathcal{E}^{II}(h_1) = \{\gamma^{II}(s(\phi^{II}), h_1), \bar{\phi} + \phi^{II}(\gamma)$

Figure 3 illustrates that the conditional system has at least one steady state and at most three depending on the fixed level of parental human capital, h_1 and the convexity of function γ^{II} and ϕ^{II} . The following Proposition summarizes the main properties of the system:

Proposition 4 The system in (35) has at least one steady state equilibrium which may be either type $\mathcal{E}^I(h_1)$ or type $\mathcal{E}^{II}(h_1)$. If it is unique it is globally stable. The system has at most three equilibria. If there are more multiple steady state equilibria then $\mathcal{E}^{II}(h_1) \gg \mathcal{E}^I(h_1)$.

Proof: Provided in the Appendix.

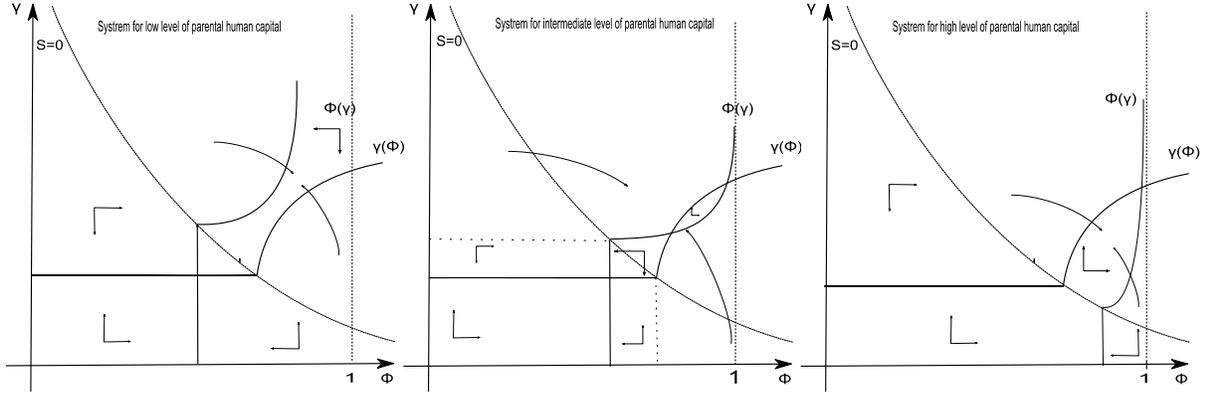


Figure 3: Evolution of the conditional dynamical system

As seen in Figure 3 the $\gamma_{t+1}(\phi_t)$ locus shifts to the right and the $\phi_{t+1}(\gamma_t)$ locus shifts downwards if h_1 . Consequently, as parental human capital increases, it has a positive effect on adult life expectancy and a negative effect on growth in parental human capital by decreasing infant mortality. If the first effect is stronger than the latter, as depicted in Figure 3, then increasing levels of parental human capital will eventually make the steady state equilibrium of type $\mathcal{E}^I(h_1)$ vanish. This implies a boost the growth rate of parental human capital (and thus income per worker) and adult life expectancy. The slow progress in the level of parental human capital is the main mechanism of the model that may take the economy from a stage of low economic growth an low life expectancy to a stage of sustained economic growth and low fertility.

3.6 Simulation

Having illustrated the main dynamics leading to the a economic takeoff, I now study the implications for the evolution of the whole economy with particular interest in the models capability to generate a demographic transition. This is done by simulating the model.³²

Suppose the economy starts on a path where growth rates of parental human capital are small but positive. As the level of parental human capital increases, i.e. skills that are embodied in workers increases, more infants survive which decreases quality time per children and thereby growth. If this effect is not severe the economy remains on a path where growth in parental human capital is positive which implies slow improvement in adult life expectancy and lower infant mortality. Then a race begins

³²This is not a calibration exercise as in Lagerlöf (2006), but just an example of how well the model can reproduce a demographic transition. The functional forms and paramter values used for the simulation is reported in the appendix.

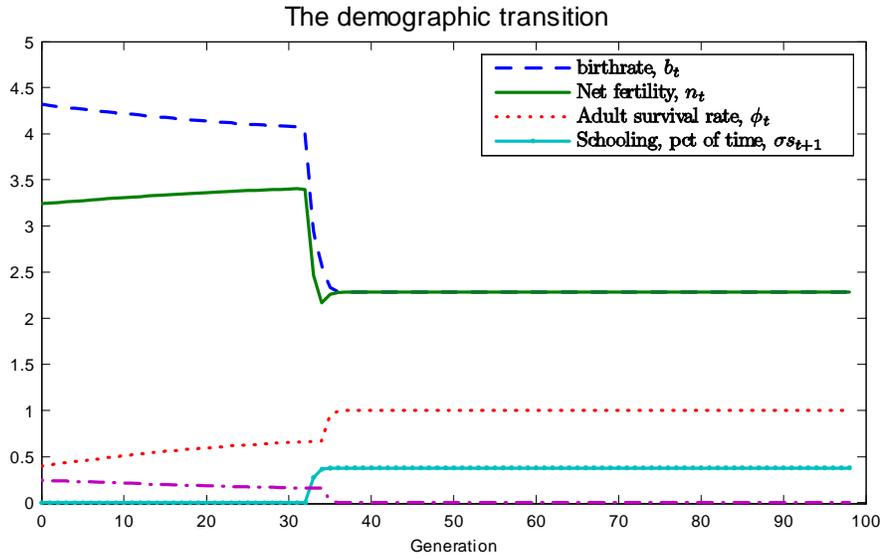


Figure 4

between the effect on the growth rate and the effect on life expectancy, which determines the timing of the demographic transition. If the increase in life expectancy is relatively slow the growth rate will decrease over time and the growth rate may become negative and the economy may stay on a path with low growth. On the contrary, and as can be seen in Figure 4, if higher levels of parental human capital have a sufficiently strong positive impact on life expectancy this will, eventually, trigger parents to start spending time on schooling implying a massive drop in fertility. This decline in fertility is caused by a remarkable increase in parental schooling due to i) a significant drop in infant mortality since schooling boosts growth in parental human capital and ii) less time is left to spend on children when more time is devoted to schooling. Due to the last effect, net fertility also decreases after a long period with an increasing trend (due to small improvements in the survival rate of infants.) This implies a non-monotonic evolution of net fertility (and thereby net reproduction rates) over time during the demographic transition, which matches the empirical evidence (see e.g. Reher, 2004).

The speed of adjust of the drop in fertility depends mainly on the how rapid schooling increases which depends on the strength of the chain of between the level of parental human capital leading to increased life expectancy inducing more schooling causing higher growth in parental human capital. In other word, if the virtuous circle between, adult life expectancy, schooling and growth in parental human

capital is stronger the transition is faster.³³ These implications is perfectly consistent with the evidence found in Lehr (2009) showing lower fertility in respond to increasing productivity at advanced stages of development. Figure 4 shows that the transition is ended after only two generations which is in line with the data shown in Figure 2.³⁴

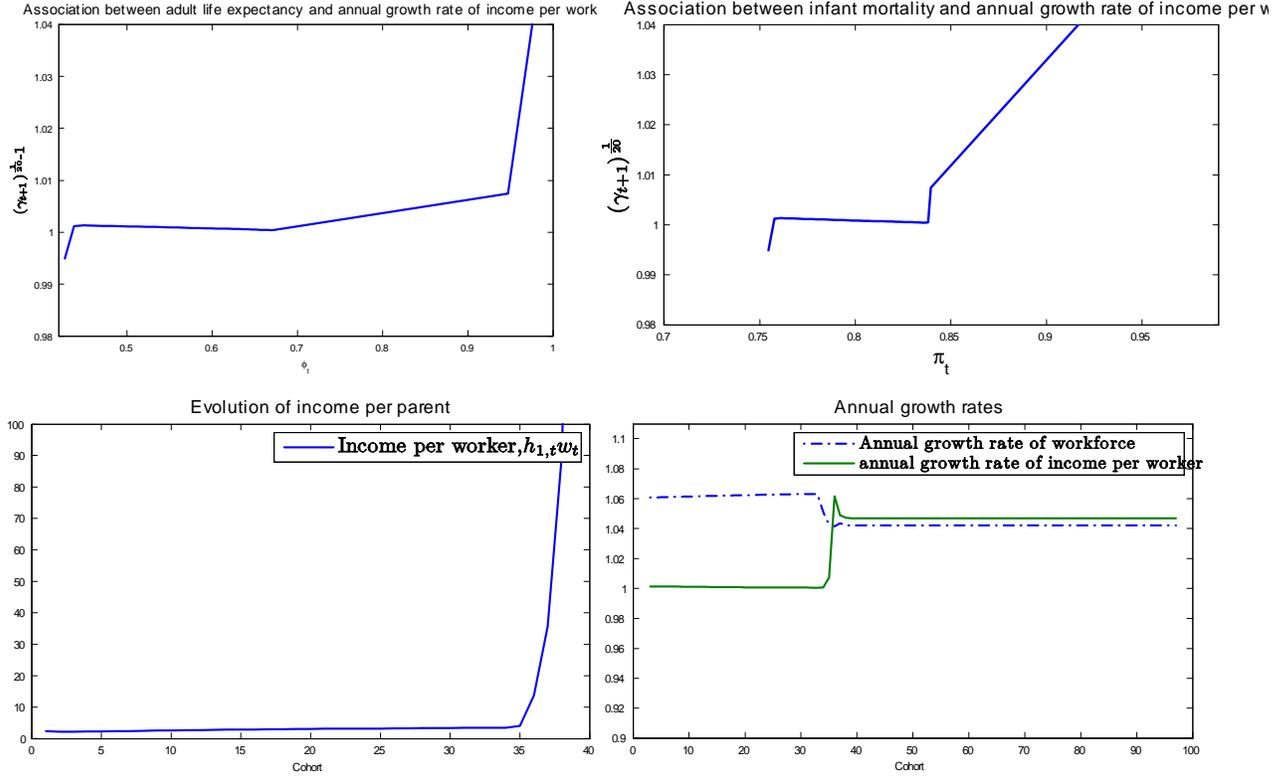


Figure 5

The relation between growth in income per capita and life expectancy is provided in the top panel of Figure 5. This confirms the intuition that life expectancy is not positively correlated with income growth for all stages of development. Clearly, after the takeoff, the relation is strong and positive, but prior to the transition the relation is weaker and may be negative.

³³ Additional factors may of course affect the timing and the speed of the transition, e.g. child labor. See Doepke (2004); Doepke and Zilibotti (2005); Hazan and Berdugo (2002) for studies in this field.

³⁴ This is not a calibration exercise as in Lagerlöf (2006), but just an example of how well the model can reproduce a demographic transition. The functional forms and parameter values used for the simulation is reported in the appendix.

3.7 Concluding remarks

This paper offers a theoretical foundation for the recent empirical evidence showing that higher life expectancy does not have a uniform impact on economic growth. Indeed, the paper suggest that lower infant mortality will dampen growth while increasing life expectancy may have a negative or positive effect depending on which stage of the economic development. Further, the theory suggests that the timing of the transition depends crucially on accumulation of human capital and that this accumulation can be sustained: if infant mortality increases rapidly with the level of human capital this would lead to more surviving children and less amount of skills are transmitted to each child. This dampens growth and may leave the economy on a low growth track. However, if the economy can sustain even just small growth rates in human capital then the implied improvements in adult life expectancy will, ultimately, induce parents to acquire schooling and the demographic transition begins initiating a takeoff to sustained economic growth.

3.8 Appendix

Using these properties of $\kappa(\gamma_{t+1}\phi_{t+1})$ implies the following derivatives of the mature human capital function in (6):

$$\begin{aligned}
 h_{2_s}(\kappa_{t+1}, s_{t+1}, h_{1,t}) &> 0, \quad h_{2_{ss}}(\kappa_{t+1}, s_{t+1}, h_{1,t}) < 0 \\
 h_{2_\gamma}(\kappa_{t+1}, s_{t+1}, h_{1,t}) &< 0, \quad h_{2_\phi}(\kappa_{t+1}, s_{t+1}, h_{1,t}) < 0, \quad h_{\gamma\phi}(\kappa_{t+1}, s_{t+1}, h_{1,t}) > 0 \\
 h_{2_{\phi\phi}}(\kappa_{t+1}, s_{t+1}, h_{1,t}) &> 0, \quad h_{2_{\gamma\gamma}}(\kappa_{t+1}, s_{t+1}, h_{1,t}) > 0 \\
 h_{2_{s\phi}}(\kappa_{t+1}, s_{t+1}, h_{1,t}) &> 0, \quad h_{s\gamma}(\kappa_{t+1}, s_{t+1}, h_{1,t}) > 0,
 \end{aligned}$$

which follows from simple differentiation of (6) and using (21). How schooling affect the reinforcing effect of life expectancy on the erosion effect of the growth rate on parental human capital is indeterminate for the functional form used here. Hence, $h_{s\gamma\phi}(\kappa_{t+1}, s_{t+1}, h_{1,t}) \stackrel{\leq}{\geq} 0$.

Hence, a higher growth factor of parental human capital, γ , erodes mature human capital with an increasing impact in the growth factor itself and from higher life expectancy. The same intuition goes for the impact of life expectancy. Moreover, schooling and life expectancy complement each other in the formation of mature human capital, which also holds for schooling and growth in parental human capital.

Proof of Lemma 2

It is to be shown that for all pairs $(\gamma_{t+1}, \phi_{t+1}) \in \mathcal{B}(\mathcal{Z})$ it holds that ϕ_{t+1} is a decreasing convex function of γ_{t+1} . From the definition of $\mathcal{B}(\mathcal{Z})$ we know that γ_{t+1} and ϕ_{t+1} is implicitly related. Define this functional relation by (where time indices are drop for simplicity) $G \equiv \kappa(\gamma, \phi) - \frac{1-\chi}{\sigma}\phi\mu = 0$. It follows from the *Implicit Function Theorem* that $\phi_\gamma = \frac{\kappa_\gamma}{\frac{1-\chi}{\sigma}\mu - \kappa_\phi}$ and $\phi_{\gamma\gamma} = \frac{\kappa_{\gamma\gamma} + \kappa_{\gamma\phi}\phi_\gamma + [\kappa_{\phi\gamma} + \kappa_{\phi\phi}\phi_\gamma]\phi_\gamma}{\frac{1-\chi}{\sigma}\mu - \kappa_\phi}$ where $\frac{1-\chi}{\sigma}\mu - \kappa_\phi > 0$ holds from (19). Hence, it follows from the properties of $\kappa(\gamma_{t+1}, \phi_{t+1})$ in (21) that $\phi_\gamma > 0$ and $\phi_{\gamma\gamma} < 0$.

Proof of Proposition 4

First define $\bar{h}_1 \equiv \left\{0 < h_1 : \left(\gamma^I(\pi(h_1)), \bar{\phi} + \phi^I(h_1)\right) \in \mathcal{Z}\right\}$. Then it follows from the definition of steady state equilibria that for all $h_1 \notin \bar{h}_1$ the set of equilibria is given by $\mathcal{E}^{II}(h_1)$. Case 1, $s_t \geq 0$ for all t : For any $h_1 \in \bar{h}_1$ we know that $\mathcal{E}^I(h_1)$ is a steady state equilibrium where $s_t = 0$, $\gamma_{t+1} = \gamma^I(\pi(h_1))$ and $\phi_{t+1} = \bar{\phi} + \phi^I(h_1)$ for all t . If this equilibrium is unique it is therefore globally stable. In the general case $s_{t+1} \geq 0$ for all t two case can distinguished. Case 2, $s_{t+1} > 0$ for all t : First, since $s_{t+1} > 0$ we know that $h_1 \notin \bar{h}_1$. Hence, $\mathcal{E}^I(h_1)$ is not at steady state equilibrium. Second, we know from Lemma 2 that γ_{t+1} is an increasing and convex function of ϕ_{t+1} and from (25) that ϕ_{t+1} is increasing in γ_t but unaffected in the limit where ϕ_{t+1} approaches 1 from below. Hence, from any initial value, the system diverges away from $\mathcal{E}^I(h_1)$ and since ϕ_{t+1} and γ_{t+1} is increasing over time, at a diminishing rate, it will reach the equilibrium $\mathcal{E}^{II}(h_1)$ which is therefore unique and globally stable.

Case 3, $s_t \geq 0$ for all t : This case comprises both case 1 and case 2. If $h_1 \in \bar{h}_1$ we now that $\mathcal{E}^I(h_1)$ exists, and in addition, depending on the value of h_1 and the degree of convexity of γ^{II} and ϕ^{II} there may be zero, one or two steady state equilibria of the type $\mathcal{E}^{II}(h_1)$ as depicted in Figure 3. It follows directly from (33) and (25) that $\mathcal{E}^{II}(h_1) \gg \mathcal{E}^I(h_1)$.

The functions are parameterized in the following way:

$$\begin{aligned}\kappa_{t+1} &= \frac{v}{\gamma_{t+1}\phi_{t+1}}, \quad g(s_{t+1}) = Z[\eta + \delta_1[\delta_2 s_{t+1}]^\varphi], \\ \phi_{t+1} &= \bar{\phi} + [1 + \bar{\phi}] \frac{\xi_1 h_1}{\xi_2 + \xi_1 h_1}, \quad \pi_{t+1} = \bar{\pi} + [1 - \bar{\pi}] \frac{\zeta_1 h_1}{\zeta_2 + \zeta_1 h_1}\end{aligned}$$

The following parameter values and initial conditions are used for the simulation in the paper:

$\chi = 0.27; \tau = 0.01; \rho = 0.015; \sigma = 0.27; \eta = 3.9; \varepsilon = 0.66; \mu = 0.9$
 $\alpha = 0.6; v = 1.1; \eta = 3.9; Z = 5; \delta_1 = 40; \delta_2 = 0.01; L_0 = 5; X = 4;$
 $\xi_1 = 0.1; \xi_2 = 0.3; \bar{\phi} = 0.2; \zeta_1 = 0.2; \zeta_2 = 1; \bar{\pi} = 0.7; s_0 = 0; h_{1,0} = 1$

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