A General Equilibrium Model of Environmental Option Values

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1 Introduction

1.1 Key Features ★

Real Option

- **Irreversibility**:
  - Once you convert old forest to farmland, then you cannot recover it anymore.
- **Uncertainty**:
  - The importance of the environment was not fully understood a couple of decades ago, implying the uncertainty in the value of the environment.

General Equilibrium

1. **Little has done** for RO in GE.
2. It helps **to identify some key parameters**.
3. Often a decision on the environment is made **at country/region level**.
4. It reveals the source of the environment's value (**endogenous environment value**).

1.2 Literature

The model classification here is based on **how to convert reserved land to farmland**.

◊ **All or Nothing** Model:

- Reserve all land or develop all land.

◊ **Bang-Bang** Model:

- Conversion rate v must be positive but the maximum speed is limited: $0 \leq v \leq v^*$.
- In this case, the optimal choice is $v = 0$ or v^*.

◊ **Barrier Control** Model:

- Conversion rate v must be positive and its speed is unlimited: $0 \leq v$.

◊ Research Area Matrix (cited papers are not exhaustive)

	DP(1994)	Partial Equim	General Equim
All or Nothing	Ch.5	e.g., Conrad (1996)	
Bang-Bang	Ch.10	e.g., Leroux et al. (2009)	
Barrier Control	Ch.11	e.g., Bulte (2002)	We are here!

NB: DP is *the* textbook by Dixit and Pindyck (1994) "investment under uncertainty".

2 Previewing the Model

2.1 Previewing the Model: Constraints ★

total land:	$1 = A + R$
production:	$Y = WA$
resource const:	$Y = C + W\kappa v$
tech growth:	$\mathrm{d}W = \alpha' W \mathrm{d}t + \sigma' W \mathrm{d}w$
conversion rate:	$\mathrm{d}A = -\mathrm{d}R = v\mathrm{d}t \geq 0$

◊ Total land supply is normalized to be 1 and it can be used as farmland A or reserved environment R.
◊ The only production factor is A. W is technology level, which follows a GBM.
◊ Output Y is consumed or used as the conversion cost of land from $W\kappa v$. Marginal cost of land conversion $W\kappa$ is proportional to W.
◊ Irreversibility comes from non-negative conversion rate v.

2.2 Utility and Value Function

$$\text{value function}: F(R,W) = \max E_0 \int_0^\infty e^{-\rho t} u(R,W,v)\,\mathrm{d}t$$

◊ **Value function** F is the **expected PV of flow utility** u with discount rate ρ.
◊ The state variables are **reserved environment** R and **technology** W in production.
◊ The only choice variable is the **speed of land conversion** v.

$$\text{flow utility}: \; u(R,W,v) = \frac{C^{1-1/\eta}}{1-1/\eta} + \phi \frac{(R - R_{\min})^{1-1/\eta}}{1-1/\eta}$$

◊ 1st term implies (i) constant elasticity of intertemporal substitution ($1/\eta$) and (ii) constant relative risk aversion (η).
◊ 2nd term implies (iii) constant elasticity of substitution (η) between consumption C and reserved land R.
◊ ϕ is the relative importance of the service flow from R, and it also absorbs the problem of different measurement units.
◊ Assume that $R_{\min} = 0$; $R_{\min}$ only shifts the lower bound of R.

2.3 The Model in Short Form

◊ Change of variable: $Z := W^{1-1/\eta}$ with $\alpha := (1 - 1/\eta)\left(\alpha' - \sigma'^2/2\eta\right)$ and $\sigma := (1 - 1/\eta)\,\sigma'$

◊ In short form,

$$\begin{aligned} F(R,Z) &= \max E_0 \int_0^\infty e^{-\rho t} u(R,Z,v)\,\mathrm{d}t \\ u(R,Z,v) &= Z\frac{(1-R-\kappa v)^{1-1/\eta}}{1-1/\eta} + \phi\frac{R^{1-1/\eta}}{1-1/\eta} \\ \mathrm{d}Z &= \alpha Z \mathrm{d}t + \sigma Z \mathrm{d}w \\ \mathrm{d}A &= -\mathrm{d}R = v\mathrm{d}t \geq 0 \end{aligned}$$

- Technically It’s quite standard.

2.4 Intuition behind the Model ★★

◊ We claim:

- An old forest now has been the same old forest since several 100yrs ago.
  This is totally different from, say, a PC, which is much faster than 10yrs ago.
- The value of an old forest has increased, because **we** have changed.
- In most existing works, the value of R follows an exogenous stochastic process, which we disagree with. **The environment value should be determined endogenously**.

◊ In our mode:

- The value of R increases, because (i) it is non-reproductive and (ii) the productivity of alternative use A increases.
- As W increases, people become richer. Hence, due to income effect, the demand for the service flow from R increases.
- The increase in W also implies the cost reduction in producing Y, meaning that, **given demand level, higher W implies lower demand for A**.
- This idea is essentially the same as **Baumol's curse**.

2.5 Myopic Version

◊ Value function F inherits several features of flow utility u.
◊ Myopic version only can explain much of the intuition above.

◊ Assume no dynamics or no irreversibility constraint with $\kappa = 0$.

$$\max u(R, W, v) = W^{1-1/\eta}\frac{(1-R)^{1-1/\eta}}{1-1/\eta} + \phi\frac{R^{1-1/\eta}}{1-1/\eta}$$

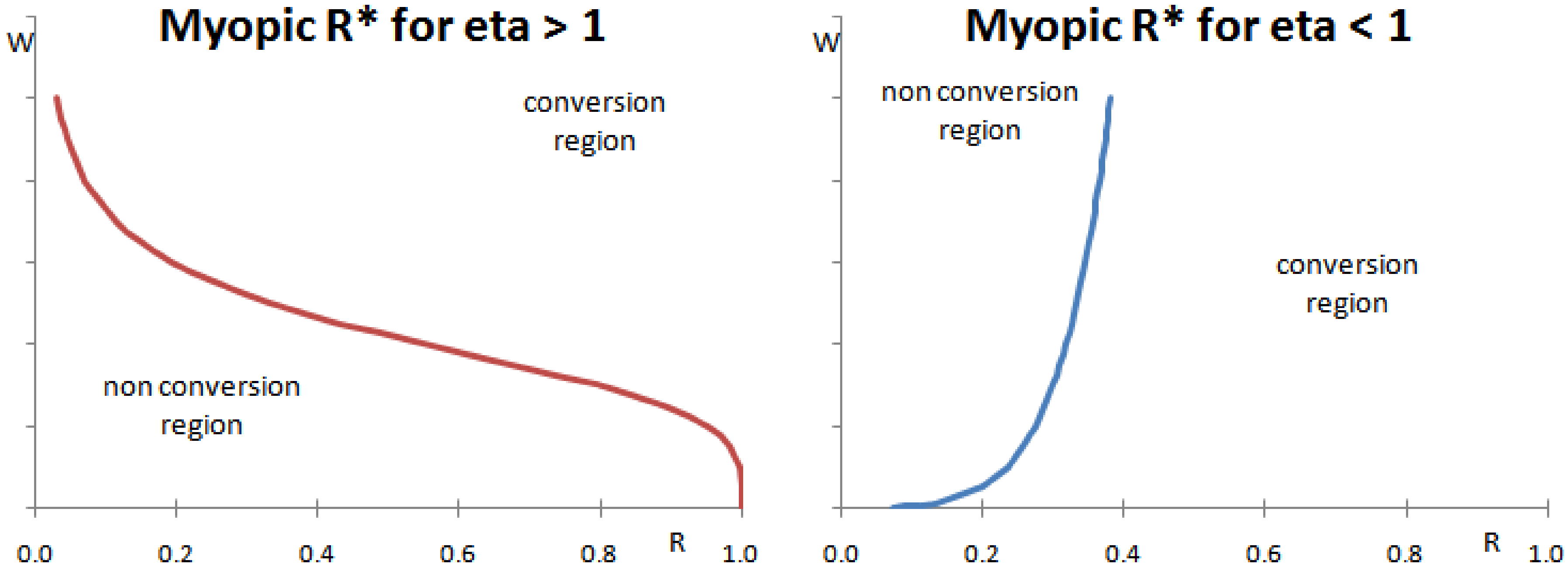
◊ Rational Optimality Conditions:

myopic reserve land: $$R^m = \frac{1}{\phi^{-\eta}W^{\eta-1}+1}$$

shadow price of *R*: $$P = \phi\left(\frac{R}{WA}\right)^{\frac{-1}{\eta}} = \phi\left(\frac{R}{C}\right)^{\frac{-1}{\eta}} = W$$

- As W increases, the shadow price of reserved environment R increases
- As W increases, myopic R increases if $\eta < 1$, and vise versa; i.e., if R is irreplaceable, the demand for R increases as people become richer.
- Obviously, η **plays an important role** in the following.

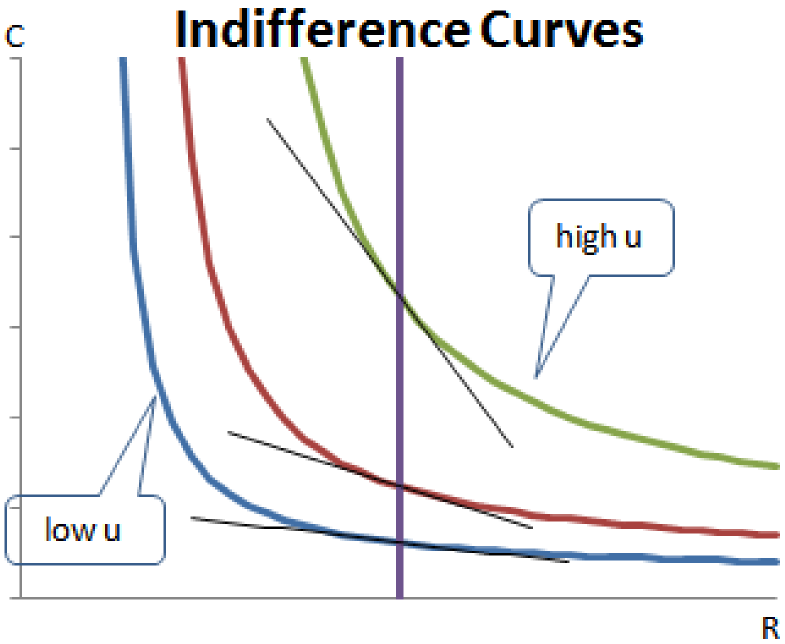
◊ Importance of η on $R\bigstar$

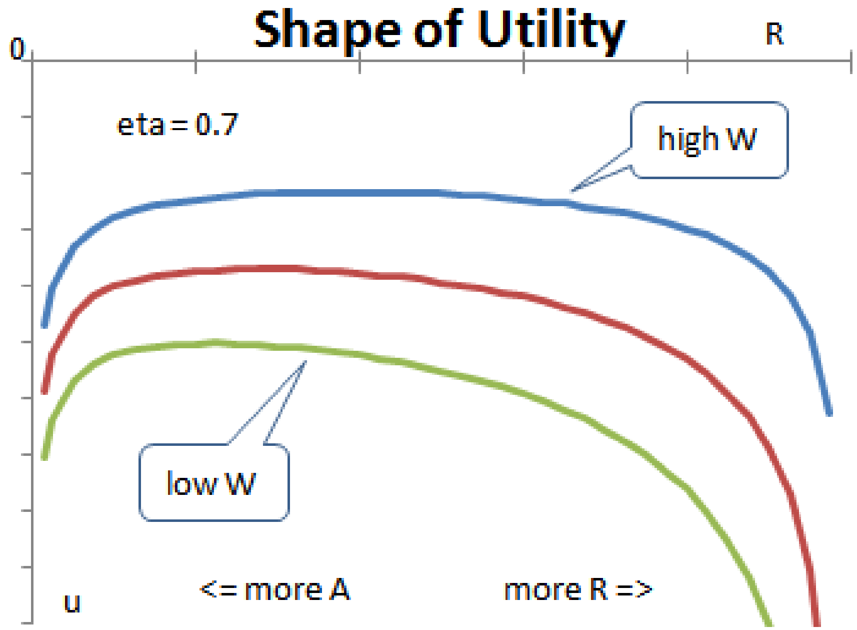


- These graphs show the main reason why η is important.

- Even this myopic version captures the fact that the people in developed countries want to preserve environment more than those in developing countries, despite the fact that they destroyed the environment in the past.

◊ Non-Reproductive Nature of R ★





- If the supply of R is limited, the shadow price (slope of the indifference curve at the optimum) increases as people become richer.
- The flow utility is quite flat for the middle value of R.

3 Solution

◊ Value Function:

$$F(R,Z) = \max E_0 \int_0^\infty e^{-\rho t} \pi(R,Z,v)\,\mathrm{d}t$$

In recursive form,

$$F(R,Z) = \max \left\{ u(R,Z,v)\,\mathrm{d}t + e^{-\rho \mathrm{d}t} E_0 \left[F(R - v\mathrm{d}t, Z + \mathrm{d}Z) \right] \right\}$$

In differential form,

$$\rho F(R,Z) = \max \left\{ u(R,Z,v) + \frac{E_0 [\mathrm{d}F(R,Z)]}{\mathrm{d}t} \right\}$$

◊ Ito's Lemma for $\mathrm{d}F$:

$$\mathrm{d}F(R,Z) = F_R \mathrm{d}R + \left(F_Z \alpha Z + F_{ZZ} \frac{\sigma^2}{2} Z^2 \right) \mathrm{d}t + F_Z \sigma Z \mathrm{d}w$$

where the last term in RHS disappears in expectation.

Using $\mathrm{d}R/\mathrm{d}t = -v$,

$$\frac{E_0\left[\mathrm{d}F\left(R,Z\right)\right]}{\mathrm{d}t} = F_Z\alpha Z + F_{ZZ}\frac{\sigma^2}{2}Z^2 - F_R v$$

3.1 Value Function with Optimal Conversion Rate v^*

Hence, the following PDE governs the value function.

$$\rho F\left(R,Z\right) = Z\frac{\left(1-R-\kappa v^*\right)^{1-1/\eta}}{1-1/\eta} + \phi\frac{R^{1-1/\eta}}{1-1/\eta} + F_Z\alpha Z + F_{ZZ}\frac{\sigma^2}{2}Z^2 - F_R v^*$$

- Use Z, rather than W, for derivation.

3.2 Non-Conversion Region ($v = 0$)

In the region with $v = 0$, the PDE is not really a PDE but is merely an ODE.

$$\rho F^0 \left(R, Z \right) = Z \frac{\left(1 - R \right)^{1-1/\eta}}{1 - 1/\eta} + \phi \frac{R^{1-1/\eta}}{1 - 1/\eta} + F_Z^0 \alpha Z + F_{ZZ}^0 \frac{\sigma^2}{2} Z^2$$

The analytical solution is available in this region.

$$F^0 = \frac{Z}{\rho - \alpha} \frac{\left(1 - R \right)^{1-1/\eta}}{1 - 1/\eta} + \frac{\phi}{\rho} \frac{R^{1-1/\eta}}{1 - 1/\eta} + B \left(R \right) Z^\beta \quad (1)$$

$$\beta = a_1 - a_2 > 1 \text{ where } a_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} \text{ and } a_2 = \sqrt{a_1^2 + \frac{2}{\sigma^2} \rho}$$

- $B \left(R \right)$ is an integration constant wrt Z, which is determined by the free boundary conditions.
- The first 2 terms show the value if you fix R at the current level forever; the same formula as DDM.
- The last term shows the value of the possibility that you can change R in the future, we call this **option value**.
- Option value takes place only in non-conversion region.

3.3 Conversion Region ($v > 0$)

For the conversion region, since R jumps to R^* immediately, the value is the value of optimal land allocation at R^* minus conversion cost.

$$\begin{aligned} F^1(R,Z) &= F^0(R^*(Z),Z) - \text{conversion cost} \\ &= F^0(R^*(Z),Z) - \kappa Z \int_{R^*(Z)}^{R} (1-\mathrm{d}R')^{\frac{-1}{\eta}} \\ &= F^0(R^*(Z),Z) + \kappa Z \left\{ \frac{(1-R)^{1-1/\eta}}{1-1/\eta} - \frac{(1-R^*(Z))^{1-1/\eta}}{1-1/\eta} \right\} \end{aligned} \quad (2)$$

3.4 Free Boundary Conditions

The conditions that must be satisfied on the boarder.

1. Level matching: Used already for Conversion Region.

2. Slope matching: **The marginal benefit of conversion is equal to the marginal cost of conversion**.

$$\begin{aligned} F_R^0(R^*, Z) &= u_v(R^*, Z, v) \\ \frac{-Z}{\rho - \alpha}(1 - R^*)^{-1/\eta} + \frac{\phi}{\rho} R^{*-1/\eta} + B'(R^*) Z^\beta &= \kappa Z (1 - R^*)^{-1/\eta} \end{aligned} \quad (3)$$

3. Smooth pasting:

$$\begin{aligned} F_{RZ}^0(R^*, Z) &= u_{vZ}(R^*, Z, v) \\ \frac{-1}{\rho - \alpha}(1 - R^*)^{-1/\eta} + \beta B'(R^*) Z^{\beta - 1} &= k (1 - R^*)^{-1/\eta} \end{aligned} \quad (4)$$

◊ Computation:

step1: From (3) and (4), we can find:

optimal R as func of $Z\,(W)$: $R^*\,(Z(W)) = \dfrac{1}{\phi_2^{-\eta} Z^{\eta} + 1} = \dfrac{1}{\phi_2^{-\eta} W^{\eta-1} + 1}$

integral const in F^0 : $B\,(R) = \phi_3 \displaystyle\int_0^R \frac{x^{\frac{\beta-1}{\eta}}}{(1-x)^{\frac{\beta}{\eta}}} \mathrm{d}x$

where $\phi_2 = \dfrac{\beta}{\beta - 1} \dfrac{\phi}{\rho(1/(\rho-\alpha)+\kappa)}$ and $\phi_3 = \phi_2^{-1}\left(\kappa\phi + \dfrac{\phi}{\rho-\alpha} - \dfrac{\phi}{\rho}\right)$

step2: From (1), we can find F^0 for non-conversion region.

step3: From (2), we can find F^1 for conversion region.

4 Numerical Examples

◊ Parameters:

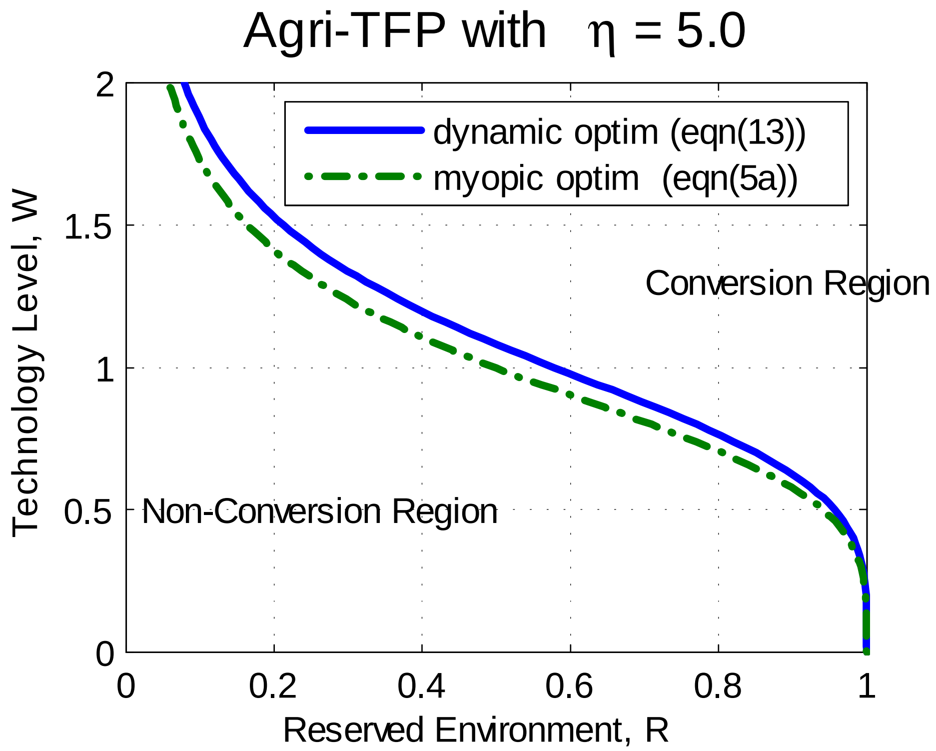
	Agri-TFP	GDP-TFP	description	
ρ	0.04	0.04	discount rate	
α'	0.0211	0.0117	trend growth rate of W	$\alpha := (1 - 1/\eta)\left(\alpha' - \sigma'^2/2\eta\right)$
σ'	0.0604	0.0112	variance of W growth	$\sigma := (1 - 1/\eta)\,\sigma'$
η	$5.0/0.7$	$5.0/0.7$	elast. of subs. btw C and R.	
ϕ	1.0	1.0	relative importance of R in π	$\phi_2 = \frac{\beta}{\beta-1}\frac{\phi}{\rho(1/(\rho-\alpha)+\kappa)}$
κ	0.0	0.0	$W\kappa$ is MC of land conversion	

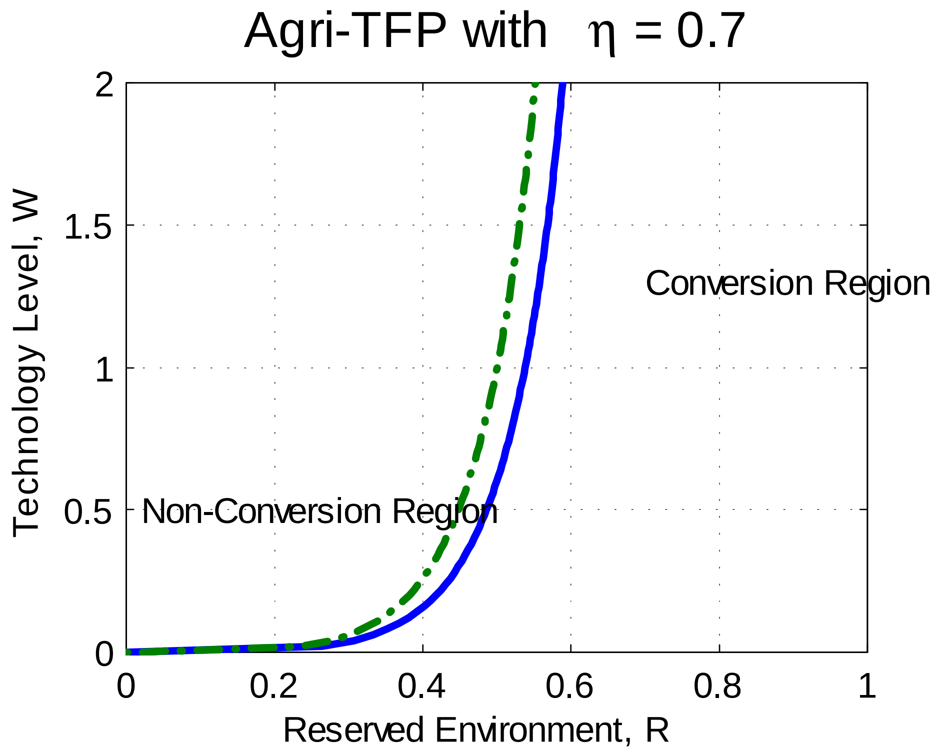
- Agri-TFP is obtained from the data of Huffman and Evenson (1993).
- GDP-TFP is estimated by the data of Japan, UK and US.

=> In the following, the results with Agri-TFP are shown.

=> One of the advantages of GE modelling is that parameter extraction is easy.

◊ Myopic choice leads to too little reserved environment R. ★

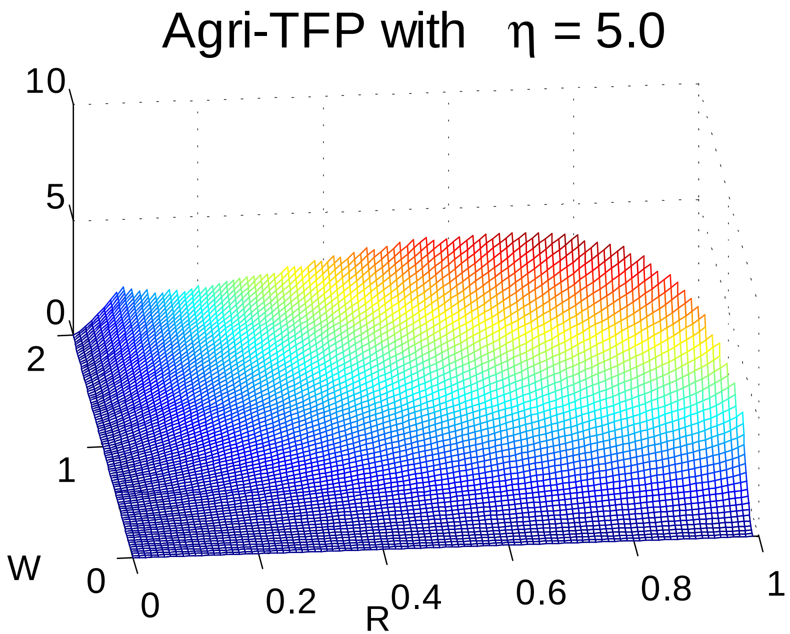


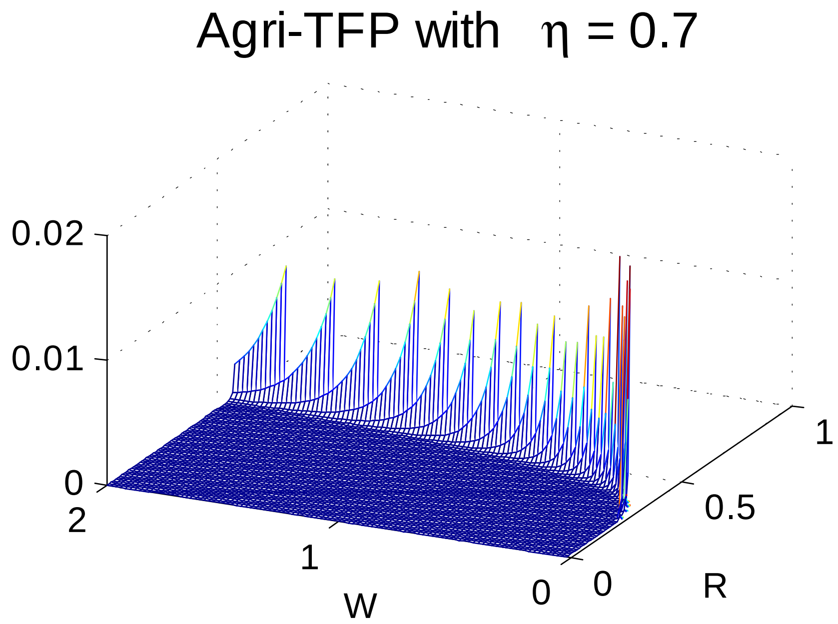


- The mistake when $\eta = 5.0$ is larger than that when $\eta = 0.7$.

4.1 Option Value ★

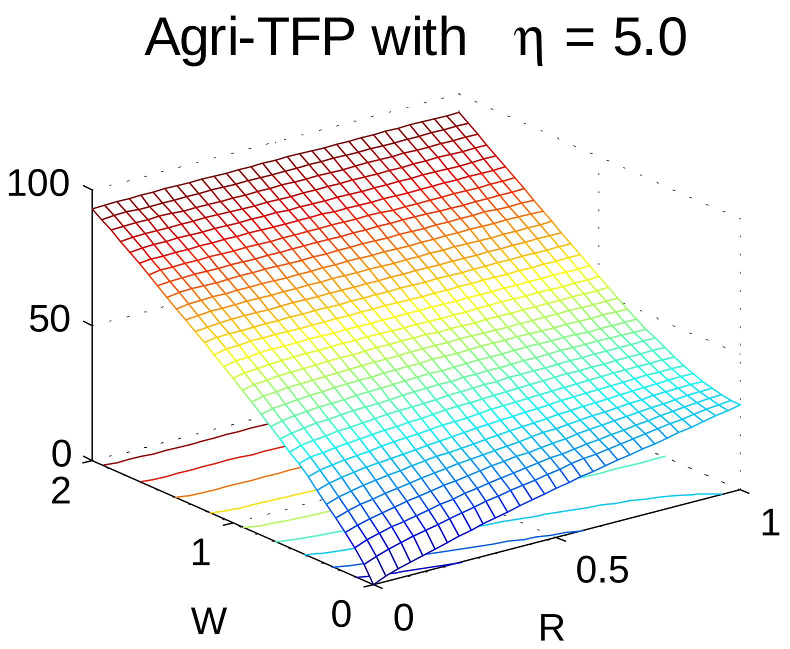
◊ Option value is zero in conversion region; conversion is exercising an option.
◊ Option value is high near the barrier, since future conversion is more likely.
◊ Option value is high when $\eta > 1$. Since W is improving over time, a downward-sloping barrier implies crossing the barrier in near future is more likely.
◊ Option value is small. Have a look at the unit of vertical axis.
◊ The jaggy edge for $\eta = 0.7$ is due to the resolution; it actually has a smooth edge.

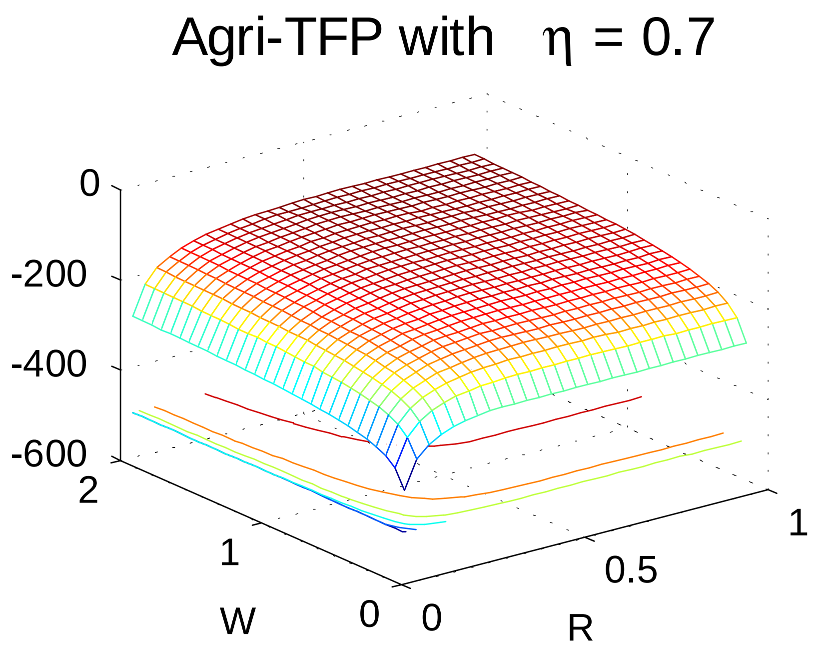




4.2 Value Function in Both Regions ★

◊ Given W, along R axis, the value function is flat.



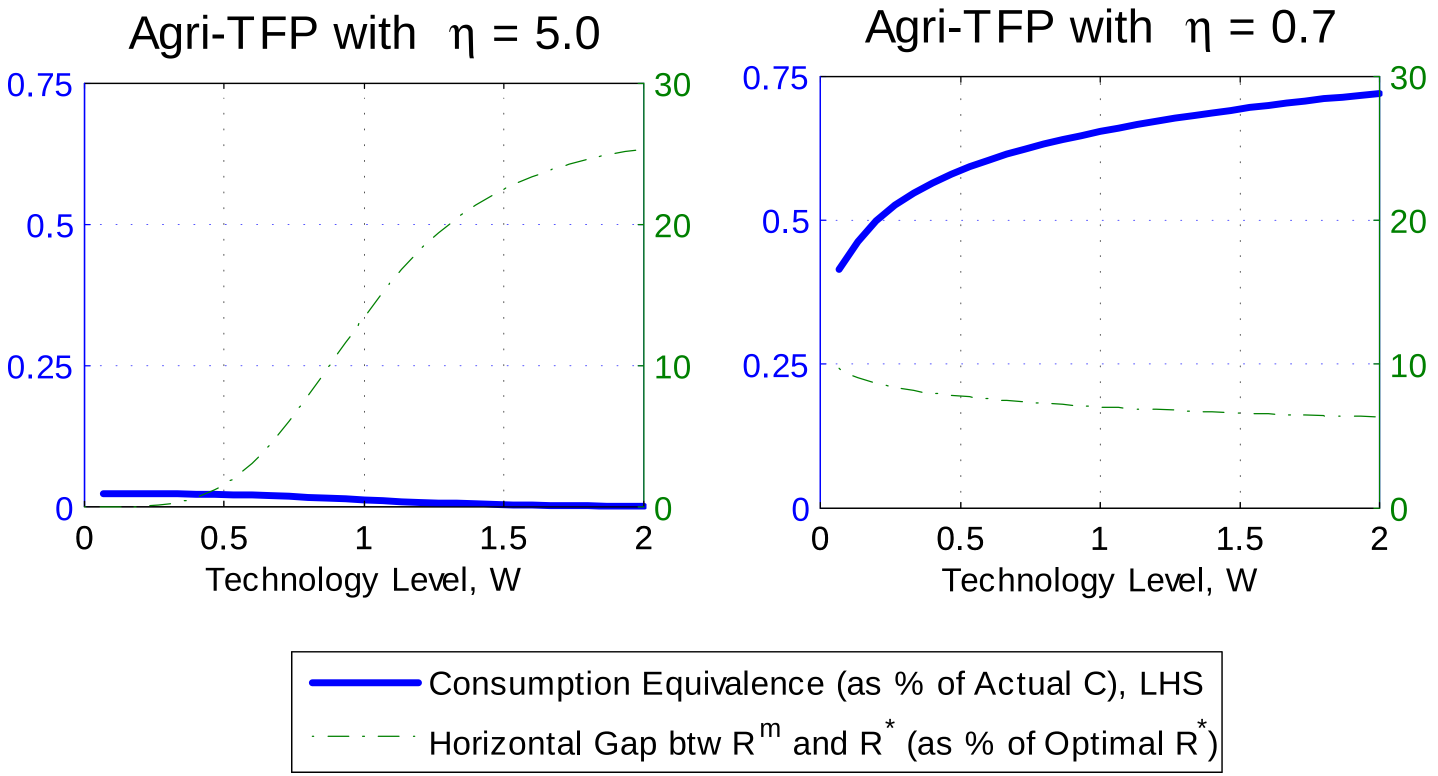


- This flat shape is largely inherited from the flow utility (or myopic model).

4.3 Implication ★★

◊ **A small option value can cause a large gap btw R^m and R^*.**
Why? => **Because the value function is flat**.

◊ However, **A flat value function implies that a large mistake is not very painful.**



$\Diamond$ Dynamic Property:
If $\eta > 1$, there is a common growth path along the barrier.
If $\eta < 1$, there is a strong initial value dependence.
... This is because of the direction of tech growth.

- If $\eta > 1$, typically an economy crosses the boarder from non-conversion region to conversion region. Every time W goes up, the economy shifts to the left, and hence it moves along the barrier curve.
- If $\eta < 1$, typically an economy crosses the barrier curve from conversion region to non-conversion region. Since W is increasing on average, once it enters into non-conversion region, it tends to stay in the same region forever.

Hence, although $\eta < 1$ seems to be economically more plausible, it generates a bit strange model behavior.

5 Empirical Implication ★★

The barrier curve implies the following two relationships. If the irreversibility is not important, use (Myopic), and if the irreversibility is significant, use (Dynamic).

$$\text{Myopic :} \quad \ln P = \ln \phi - \frac{1}{\eta}(\ln R - \ln C) \tag{5}$$

$$\text{Dynamic:} \quad \ln P = \left(\ln \phi + \ln \frac{1}{\rho} + \ln \frac{\beta}{\beta - 1}\right) - \frac{1}{\eta}(\ln R - \ln C) \tag{6}$$

Obviously P is not observable. But, if a stated preference survey or the like is available, the both of above equations are estimable. For example, if individual level data in one region is available over a number of years, assuming that individual characteristics appear only in ϕ (ie, assuming that all individuals share a common η), the following panel estimation may be implementable.

$$\begin{aligned} \ln P_{it} &= \ln \phi_{it} - \frac{1}{\eta}(\ln R_t - \ln C_{it}) \\ \phi_{it} &= \phi(X_{it}) \text{ where } X_{it} \text{ includes any individual characteristics, etc.} \end{aligned}$$

The importance of η is frequently reported in general environment literature.

6 Conclusion ★★

◊ We conducted the **real option** exercise for **reserved environments** in a very simple **general equilibrium** framework.

◊ In our general equilibrium setup, **the value of the environment is determined endogenously**.

◊ **Elasticity of substitution** η **btw** C **and** R **plays an important role**. Literally, η means how environments are irreplaceable. If environments are really irreplaceable (inelastic), then the demand for R increases as people becomes richer.

◊ **The value function is flat** under our functional and parametric assumptions. Hence,
(a) If the option value is ignored, the mistake can be huge. However,
(b) It also implies that the mistake is not really painful.

◊ We offer **two empirically testable equations**.