A General Equilibrium Model of Environmental Option Values

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Abstract

In this paper we consider the option value of the environment employing a general equilibrium growth model with a stochastic technology. In our model, as in existing studies, because of irreversibility, the environment has significant real option value. However, unlike the existing literature in which the uncertainty of the value of the environment is given exogenously, the value of the environment is endogenously determined. In our model, the elasticity of substitution η between the environment and consumption plays a crucial role. We show that the option value, and hence, the optimal decision are both affected by η not only quantitatively but also qualitatively.

KEYWORDS: Real option values, environment, general equilibrium, elasticity of substitution.

JEL CLASSIFICATION: G13, Q31

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1 Introduction

The application of option value theory in the environmental economics literature is now well established (eg, Arrow and Fisher, 1974, Conrad, 1997, Ulph and Ulph, 1997, Bulte et al., 2002, Kassar and Lasserre, 2004, Morgan et al., 2008, and Leroux et al., 2009) Within the environmental economics literature reference is frequently made to quasi-option value (QOV). Fisher (2000) argues the QOV and what Dixit and Pindyck (1994) refer to as real option value are the same. In this paper we employ option value throughout. Option values have been applied to various environmental and resource issues including forestry, biodiversity, land use decisions and climate change. The reason for this breadth of applications is because many decisions relating to environmental resource use can result in actions that yield irreversible outcomes, such that the net benefits of the action are uncertain. The extent of uncertainty facing environmental economists, as a result of the interaction between economy and environment, has lead Pindyck (2007) to observe that uncertainty in economics is at its greatest in the environmental context.

In this paper, to make our discussion concrete, we focus on a simple land use problem, though the model can be applied to many other country or regional level decision making issues. We consider how an economy should divide land use between agriculture and the environment. As such, this paper can be broadly regarded as an extension of Conrad (1997), Bulte et al. (2002) and Leroux et al. (2009), who all consider essentially the same problem, the identification of the optimal timing and amount of land conversion from an environmental use (R) (ie, old growth forest) to farmland (A). This literature considers the situation, in which once R is converted into A, it cannot be converted back to R again, and as such a decision on land conversion involves the evaluation of the option value of R. As is well known, real option theory tells us that, under irreversibility and uncertainty, the optimal decision is slower land conversion than the land conversion implied by a simple present value comparison. This is intuitive as people are more cautious in making decisions that they cannot take back later.

Within this literature the application of real option theory has typically been conducted by employing a partial equilibrium framework, in which the value of the environment is assumed to be given by an exogenous stochastic process. In this paper, we take a different approach and assume the value of the environment to be endogenous within a general equilibrium model. We consider this to be an appropriate means by which to examine the environment, as in many cases resource use problems are framed at the macro and sector wide level of the economy. More importantly, the general equilibrium framework developed in this paper makes the value of the environment endogenous. Consider an area of old growth forest. What we see today is physically almost the same old growth forest that would have existed many years ago, which implies that the change in the value (more precisely the shadow price) of R is not because of the change in the quality of R but the change in the valuation of the environment This is very different from other goods produced, such as personal computers. We believe that identifying the mechanism behind the changes in the value of the environment is a significant contribution to the literature.

Our model postulates that the value of R changes because, as people become richer, they demand more R mainly because of the income effect. This income effect on R is nothing more than that on general consumption goods (C). However, while the supply of C increases as technology improves, the supply of R is limited, and hence, as R becomes more scarce relative to C, the (shadow) price of R increases in equilibrium. Figure 1 captures this idea with a fixed supply of R.[1]

[Figure 1: Indifference Curve around here]

In Figure 1, as society become richer, the chosen mix moves up to the upper-right and the optimum point moves up along the vertical line at the fixed level of R. The shadow price of R relative to the price of C, represented by the slope of the indifference curve at the optimum, becomes higher. This explains one prominent observation that people in rich countries tend to be more eager to conserve the environment than those in poor countries, such that eagerness is a proxy for the shadow price of R.

In addition to the endogenous value of R, an advantage of employing a general equilibrium model is that we can use standard economic variables such as total factor productivity (TFP) to estimate the exogenous technology process. In the existing literature the value of the environment is normally assumed to follow an exogenous stochastic process, but an important problem is that the value of the environment is not readily observable or estimable. This has lead researchers to employ various proxies to estimate key model parameters. For example, Conrad (1997) assumes that amenity value for a stand of old growth forest follows geometric Brownian motion (GBM), and his trend and volatility

[1] Figure 1 only captures the fixed supply of R (ie, R cannot increase or decrease), while what we consider in this article is the irreversibility of R (ie, R cannot increase but can decrease). In addition, the figure ignores the production side. We however believe that Figure 1 conveys the key intuition of the endogenous change in the shadow price of R.

parameters are estimated by time series data on the numbers of visitors to the forest. Bulte et al. (2002) also employ visitor numbers, in this case to Costa Rica. As noted by Forsyth (2000), this approach can be valid as long as benefits are broadly defined (ie, all non-timber benefits). However, there are also many reasons to think that tourist numbers are a poor proxy; eg, the residents of an area may limit the number of tourists to conserve their precious environment.

Unlike the existing partial equilibrium models, the source of uncertainty in our model is a technology shock which we assume follows GBM. One of the key advantages of our approach is that the parameters for this technology process can be estimated by using readily available data. In this paper, we use TFP to estimate the technology process. More specifically, we use agricultural TFP (agri-TFP) and a GDP based TFP (GDP-TFP); the former is chosen because farmland is the most important reason to convert reserved forest in many developing countries, while the latter is used because we can interpret A as general land use other than the reserved environment, which is perhaps more relevant to developed countries. As noted above, although we label A as agricultural land in our theoretical model we do not need to interpret A literally as farmland in our empirical application. We show results for both TFP measures and it turns out that our results are quite sensitive to this choice. In fact, environmental applications of the option value model are in general very likely to be sensitive to the parameters of the exogenous shock process. As such our model is no exception in the sense that the numerical results are considerably different between these two parameter assumptions of the stochastic technology process. This lack of robustness is commonly observed in this literature. However, one of the merits of our model is that, given the clear parameter derivation process in our method, we can discuss which assumption is the most suitable for any empirical application.

Previewing our results, the elasticity of substitution η between general consumption goods and the service flow from the environment plays a key role not only quantitatively but also qualitatively. More specifically, if R is elastic ($\eta > 1$), as people become richer, the optimal R decreases, while if $\eta < 1$ it increases; ie, there is a threshold at unit elasticity $\eta = 1$. Note that η represents how the environment is irreplaceable in people's mind. In our model, the production of C requires A as an input and its productivity grows stochastically. While the improvement of farmland productivity directly makes the conversion of R into farmland A more attractive, it also makes people richer, such that the shadow price of R increases. Intuitively, if R is elastic, the reduction of the service flow from R is not very painful, since it can easily be compensated by additional C, and hence, society will prefer the production of more C by converting R into A. At first glance, the case with $\eta < 1$ may seem to be counterintuitive; as the productivity of A improves, the demand for A decreases. This seemingly paradoxical behavior is caused by the same mechanism that generates Baumol's curse (Baumol, 1967), in which the key intuition is the cost reduction effect of technological improvement. When the elasticity is low ($\eta < 1$), the optimal combination of R and C does not change very much on the demand side (literally inelastic), as more productive A implies that less A is necessary to produce a given level of C on the supply side. Note that we find that this property holds even without imposing the irreversibility of R.

The importance of η is well understood in the environmental literature. For example, Heal (2009) notes that the elasticity of substitution in production is likely less than one because of basic technological limitations. However, in the case of consumption, although there are reasons similar to Heal (2009) to assume that $\eta < 1$, it is less clear if it indeed is less than one on the demand side, than on the supply side. As a result, we examine the behaviour of our model for both η less than and greater than one.

Quantitatively we find that the effects of the option value of R is negligible, if the technological parameters are taken from GDP-TFP. This is mainly because, compared to the trend GDP growth rate, its volatility is relatively small; indeed, it is quite rare to see a negative GDP growth rate in developed countries. In contrast, if these parameters are based on agri-TFP, which is more volatile, then we find significant effects for the option value R; that is, if the option value is ignored, and society mistakenly converts too much R into A, a mistake can be very large. We find that under reasonable parameter assumptions, the resultant R is smaller than the optimal R by 5 to 30%; in this sense, it is important to take into account the option value of R. At the same time, however, we also find that, even with agri-TFP, the option value of R itself is very small, and, as a result, society's welfare loss due to the ignorance of the option value is also very small. This result has previously been noted in the literature by Bulte et al. (2002) and others. In this sense, the option value is not important. These seemingly contradictory results coexist because, for our utility function, the value function F is very flat in the neighborhood of the optimal point. That is, a flat value function implies that a large change in R (ΔR which is a mistake in this case) leads to only a small change in F (ΔF which is a welfare loss in this case), such that $\Delta F/\Delta R$ is small in absolute terms. In a similar vein, losing a small option value causes a small welfare loss (ΔF, since the value function is the sum of the non-option and option values), which is associated with a large mistake in land use (ΔR). This finding implies that it is important to choose a proper measure to evaluate the effects of a land conversion in practical policy debates.

The structure of this paper is as follows. Section 2 introduces myopic and dynamic models and solve them. The myopic model is effectively a static general equilibrium model without the irreversibility constraint. It offers a benchmark to be compared to the dynamic model in which we explicitly analyze the option value of the environment. Also, the myopic model illuminates how the value (shadow price) of the environment is endogenously determined and how the elasticity of substitution between consumption and the environment affects optimal land allocation. Section 3 presents the numerical results of the dynamic model. Section 4 provides further discussions of the model and results and Section 5 concludes.

2 Model

2.1 Model Setup

We consider the problem of land conversion in a continuous time dynamic general equilibrium setting. Our model economy is characterized by the following objective function (1) and constraints (2).

$$\text{Value Function} \quad : \quad F(R, W) = \max \mathrm{E}_0 \int_0^{\infty} e^{-\rho t} U(C, R)\, \mathrm{d}t \tag{1a}$$

where

$$\text{Flow Utility} \quad : \quad U(C, R) = \frac{C^{1-1/\eta}}{1-1/\eta} + \phi \frac{R^{1-1/\eta}}{1-1/\eta} \tag{1b}$$

subject to

$$\text{Total Land:} \quad 1 = A + R \tag{2a}$$

$$\text{Agricultural Production:} \quad Y = WA \tag{2b}$$

$$\text{Resource Constraint:} \quad Y = C + W\kappa v \tag{2c}$$

$$\text{Technological Growth:} \quad \mathrm{d}W = \tilde{\alpha}W\mathrm{d}t + \tilde{\sigma}W\mathrm{d}w \tag{2d}$$

$$\text{Land Conversion Rate:} \quad \mathrm{d}A = -\mathrm{d}R = v\mathrm{d}t \geq 0 \tag{2e}$$

We assume that the government maximizes the value function F, which is the present value (PV) of society's flow utility U with discount rate ρ, by choosing the optimal land conversion rate ν (1). We assume that U exhibits constant elasticity of substitution (CES), and it is increasing in both general consumption goods C and the service flow from the environment R. The first term of (1b) implies (i) constant elasticity of intertemporal substitution ($1/\eta$) and (ii) constant relative risk aversion (η), while the first and second terms imply (iii) constant elasticity of substitution (η) between C and R. Note, ϕ is the relative importance of the service flow from R, and it also absorbs the difference in measurement units. This functional form is chosen mainly because of its tractability and popularity, but it may be subject to the criticism that three elasticities are all governed by single parameter η.

For ease of exposition, the total land mass is normalized to be one (2a). There are two possible uses of land; as agricultural land A or as the reserved environment R (eg, old growth forest, conservation reserves, etc.). Output Y is produced by a linear technology (2b), in which the only production factor is A, and its technology level W follows a GBM (2d). The only source of uncertainty in our model is W. Output is consumed or used as land conversion cost $W\kappa v$. We assume that the land conversion cost is a linear function of conversion rate ν, and the marginal cost of land conversion $W\kappa$ is proportional to W, where $\kappa > 0$ is a parameter.

We assume that land conversion is an irreversible decision (2e). $-\mathrm{d}R \geq 0$ implies that the government can reduce R but cannot increase it. It is known that, since there is no upper bound for the conversion speed, optimal land conversion is either (i) convert no land ($\nu = 0$) if $R < R^*(W)$ or (ii) jump[2] immediately to optimal R^* otherwise. Note that optimal R^* is a function of the technology

[2] To be precise, R does not jump, if we define jump as a non-continuous movement. R moves continuously but non-differentiable. Its movement is non-differentiable because it is so zig-zag. In a sense, it exhibits successive "twitches".

level $R^* = R^*(W)$, and hence, is changing over time. Since curve $R^*(W)$ demarcates the state space between conversion and non-conversion regions and the optimal locus "closely" follows $R^*(W)$, this type of optimal control problem is referred to as barrier control.[3]

2.1.1 Short Form

To simplify our exposition, we substitute out several variables. The following short form notation is exactly the same as the above model setup.

$$\begin{aligned}
\text{Value Function} \quad &: \quad F(R,Z) = \max_{\nu} \mathrm{E}_0 \int_0^{\infty} e^{-\rho t} U(R,Z,v)\,\mathrm{d}t && \text{(3a)} \\
\text{Flow Utility} \quad &: \quad U(R,Z,v) = Z\frac{(1-R-\kappa v)^{1-1/\eta}}{1-1/\eta} + \phi\frac{R^{1-1/\eta}}{1-1/\eta} && \text{(3b)} \\
\text{Technological Growth} \quad &: \quad \frac{\mathrm{d}Z}{Z} = \alpha \mathrm{d}t + \sigma \mathrm{d}w && \text{(3c)} \\
\text{Land Conversion Rate} \quad &: \quad \mathrm{d}A = -\mathrm{d}R = v\mathrm{d}t \geq 0 && \text{(3d)}
\end{aligned}$$

where $\alpha = (1-1/\eta)\left(\tilde{\alpha} - \tilde{\sigma}^2/2\eta\right)$, $\sigma^2 = (1-1/\eta)^2\tilde{\sigma}^2$ and $Z = W^{1-1/\eta}$. We always assume that $\alpha < \rho$; otherwise the value function F explodes. In the following derivation, we use the short form version of the model. In this short form, α, σ, ρ, η, κ and ϕ are parameters, and the other variables (uppercase letters and ν) evolve over time, of which the state variables are R and Z (or W), while the only choice variable is v. In non-algebraic expositions, both Z and W are referred to as technology.

2.2 Myopic Version

Before we examine the dynamic version of the model, we consider a myopic model. The key feature of the myopic version of the model is that it does not include the irreversibility constraint and we assume there are no conversion costs (i.e., $\kappa = 0$). Within these assumptions, the myopic choice is rational. The myopic version of the model provides us with a useful benchmark in evaluating the option

This fact comes from our assumption that the stochastic term in GBM is a Wiener process. Knowing this, we still (ab)use the word "jump" to depict such a continuous but sudden change in R.

[3] Bulte et al. (2002) also studies a barrier control model (2e) in a partial equilibrium setting. Note that (2e) is equivalent to $\nu \geq 0$. If, on the other hand, there is an upper limit of the land conversion rate (ie, $\bar{\nu} \geq \nu \geq 0$), it is known that the optimal land conversion rate is either (i) convert no land ($\nu = 0$) or (ii) convert as quickly as possible ($\nu = \bar{\nu}$). This type of optimal control problem is called "bang-bang" model and Leroux et al. (2009) analyzed this again in a partial equilibrium setup. Finally, the land allocation is limited to $R = 1$ or $R = 0$ in Conrad (1997); in this model, effectively the optimization problem is equivalent to choosing optimal timing of land conversion (ie, there is no need to consider optimal $R^*(Z)$ by assumption).

value of R, and the dynamic version inherits many of the properties from it. Importantly, the myopic version alone shows that the shadow price of R is higher for countries with a higher technology level (and hence with higher income), and the elasticity η between C and R plays a key role, implying these results hold even without option value considerations.

Given W, the myopic version is effectively a static model (or a sequence of static models over time), and the optimization problem reduces to the maximization of the flow utility U.

$$\text{Myopic Optimization} \quad : \quad \max_{R} \frac{C^{1-1/\eta}}{1-1/\eta} + \phi \frac{R^{1-1/\eta}}{1-1/\eta} \tag{4a}$$

$$\text{s.t. } C = W(1-R) \tag{4b}$$

where the solution is given by

$$R^{\dagger} = \frac{\phi^{\eta}}{W^{\eta-1}+\phi^{\eta}} \tag{5a}$$

$$P = \phi \left(\frac{R^{\dagger}}{C}\right)^{\frac{-1}{\eta}} = \phi \left(\frac{R^{\dagger}}{W(1-R^{\dagger})}\right)^{\frac{-1}{\eta}} = W \tag{5b}$$

In the myopic version, $R^{\dagger}$ is labelled as the optimal choice. P is the shadow price of R which is equal to the slope of the indifference curve at the optimum quantity of reserved land ($R = R^{\dagger}$). Regardless of the parameter values, CES utility implies that P is increasing in the technology level W; indeed, $P = W$ in this simple version. On the other hand, the optimal level of $R^{\dagger}$ can be both increasing and decreasing in W. That is, if $\eta < 1$ (which implies that the environment is inelastic), the optimal level of $R^{\dagger}$ increases as technology improves, and vice versa (see Figure 5).

Also, figure 2 shows the shape of the flow utility function (4a), which is equivalent to (3b) with $\kappa = 0$, for several values of W. The value function (3a) of the dynamic model inherits many properties from the flow utility function (4a). The point that we want to focus on here is that the flow utility U is quite flat for intermediate values of R, and so is the dynamic value function F. We will discuss the importance of this subsequently.

[Figure 2: Period Utility around here]

2.3 Solution to the Fundamental Partial Differential Equation

In this and the next subsections, we solve the dynamic version with the irreversibility constraint (3). We follow the standard derivation strategy, in which we first derive the fundamental partial differential equation (PDE) and then solve it separately in the conversion and non-conversion regions (see Figure 5 to preview the shapes of regions). Note that, for each level of Z, we can find the optimal $R^* = R(Z^*)$. If $R < R^*$, due to the irreversibility condition, the best thing that the government can do is just staying at the current R, while, if $R > R^*$, then the land is converted immediately so that $R = R^*$. In this way, the optimal $R^* = R(Z)$ demarcates the two regions.

2.3.1 Fundamental PDE

Following the standard option value methodology (see Dixit and Pindyck, 1994), we rewrite (3a) in differential form.

$$\rho F(R, Z)\,\mathrm{d}t = \max\{U(R, Z, v)\,\mathrm{d}t + E_0[\mathrm{d}F(R, Z)]\} \tag{6}$$

To find $\mathrm{d}F$, we apply Ito's Lemma to F.

$$\mathrm{d}F(R, Z) = F_R \mathrm{d}R + \left(F_Z \alpha Z + F_{ZZ}\frac{\sigma^2}{2}Z^2\right)\mathrm{d}t + F_Z \sigma Z \mathrm{d}w \tag{7}$$

where $F_Z = \partial F/\partial Z$ and so on. The expected value of the last term on the right hand side of (6) is zero; $E_0[\mathrm{d}w] = 0$. Hence, by substituting $\mathrm{d}F$ out of (6) and dividing all terms by $\mathrm{d}t$, we obtain the following fundamental partial differential equation (PDE) that governs the value function F.

$$\rho F(R, Z) = Z\frac{(1 - R - \kappa v^*)^{1-1/\eta}}{1 - 1/\eta} + \phi\frac{R^{1-1/\eta}}{1 - 1/\eta} + F_Z \alpha Z + F_{ZZ}\frac{\sigma^2}{2}Z^2 - F_R v^* \tag{8}$$

We now solve this function for the two regions.

2.3.2 Non-Conversion Region ($v^* = 0$)

In the region with $R < R^*$, the government does not convert the land; $v^* = 0$. In this region, the PDE (8) is not really a PDE but is merely an ordinary differential equation (ODE). Using the superscript 0 on F^0 to indicate the value function that is defined in the non-conversion region, we can now re-express equation (8) as:

$$\rho F^{0}(R,Z)=Z\frac{(1-R)^{1-1/\eta}}{1-1/\eta}+\phi\frac{R^{1-1/\eta}}{1-1/\eta}+F_{Z}^{0}\alpha Z+F_{ZZ}^{0}\frac{\sigma^{2}}{2}Z^{2} \tag{9}$$

The analytical solution for this type of ODE is available.[4]

$$\begin{aligned} F^{0}(R,Z) &= \frac{Z}{\rho-\alpha}\frac{(1-R)^{1-1/\eta}}{1-1/\eta}+\frac{\phi}{\rho}\frac{R^{1-1/\eta}}{1-1/\eta}+B(R)Z^{\beta} \\ \beta &= a_1+a_2>1 \text{ where } a_1=\frac{1}{2}-\frac{\alpha}{\sigma^2} \text{ and } a_2=\sqrt{a_1^2+\frac{2}{\sigma^2}\rho} \end{aligned} \tag{10}$$

Term $B(R)$ is the integration constant with respect to Z, and hence it may not be (and indeed *is not* in this case) a constant with respect to R. We will determine the exact functional form of $B(R)$ by using free boundary conditions which are introduced shortly. We also note that we have already used a boundary condition to eliminate the term with $\beta_1<0$ to derive (10), where β_1 is the negative root of the characteristic polynomial of (9). Since, in general, it is the boundary conditions that characterize the economic properties of a model, we discuss these in detail altogether. In fact, many real option problems share the same PDE as (8), and the differences among problems typically come from the boundary conditions.

An examination of (10) reveals that the first two terms are the present value of the two terms of the flow utility (3b).[5]

$$\int_0^\infty e^{-\rho t}\left\{\frac{Z(t)(1-R)^{1-1/\eta}}{1-1/\eta}\right\}\mathrm{d}t = \frac{1}{\rho-\alpha}\frac{Z(1-R)^{1-1/\eta}}{1-1/\eta}=\frac{1}{\rho-\alpha}\frac{C^{1-1/\eta}}{1-1/\eta} \tag{11a}$$

$$\int_0^\infty e^{-\rho t}\left\{\phi\frac{R^{1-1/\eta}}{1-1/\eta}\right\}\mathrm{d}t = \frac{\phi}{\rho}\frac{R^{1-1/\eta}}{1-1/\eta} \tag{11b}$$

[4] We omit the derivation of this, since it can be found in many places, including Dixit and Pindyck (1994) and Bulte et al (2002).

[5] This is the so-called discounted dividend model (DDM) in accounting. The present value of an eternal constant dividend flow D is

$$\int_0^\infty e^{-\rho t}D\mathrm{d}t=\frac{D}{\rho}$$

If the dividend grows at the rate of α, $D(t)=e^{\alpha t}D$ where $D=D(0)$ is the dividend at $t=0$. Then, it follows that

$$\int_0^\infty e^{-\rho t}D(t)\,\mathrm{d}t=\int_0^\infty e^{-\rho t}e^{\alpha t}D\mathrm{d}t=\int_0^\infty e^{-(\rho-\alpha)t}D\mathrm{d}t=\frac{D}{\rho-\alpha}$$

It is important to note that the effective discount rate for the consumption term is $\rho - \alpha > 0$. This is different from the subjective discount rate ρ, because of the technological growth. The drift term in (3c) implies that the redefined technology $Z(t)$ grows at rate α on average. In contrast, since there is no such growth component in the environment term, its effective discount rate is simply ρ. Note, these expressions implicitly assume that R is constant at the current level. That is, these terms represent the value of total land (A and R) if there is no possibility of land conversion in the future. Hence, the remaining term $B(R) Z^{\beta} > 0$ in (10) represents an additional value that emerges from the possibility of future land conversion, ie, the option value.

Typically, option values are explained as follows. If a decision is irreversible and the situation is uncertain, even though the net present value of an irreversible decision is positive, it may still be better to wait for the arrival of new information, rather than making such an irreversible decision right now. This can be understood as follows in our case. Suppose for a while there are only two possible choices R_1 and R_2. If $R_1 < R_2$, choosing R_2 is keeping an option in the sense that it is still possible to move to R_1 at some point in the future, while choosing R_1 is exercising an option in the sense that it is no longer possible to return back to R_2. In this case, even if the present value of having R_1 forever is higher than that of having R_2 forever (ie, $diff > 0$ below), choosing R_1 may still not be optimal.

$$diff = \left\{ \frac{Z}{\rho - \alpha} \frac{(1 - R_1)^{1-1/\eta}}{1 - 1/\eta} + \frac{\phi}{\rho} \frac{R_1^{1-1/\eta}}{1 - 1/\eta} \right\} - \left\{ \frac{Z}{\rho - \alpha} \frac{(1 - R_2)^{1-1/\eta}}{1 - 1/\eta} + \frac{\phi}{\rho} \frac{R_2^{1-1/\eta}}{1 - 1/\eta} \right\}$$

This is precisely because of the difference in option values; $B(R_1) Z^{\beta} < B(R_2) Z^{\beta}$. That is, since Z is stochastic, the government wants to wait for the arrival of new information about Z until the $diff$ becomes large enough to make sure that the optimal choice is moving to R_1.

2.3.3 Conversion Region ($\nu^* > 0$)

In the conversion region ($R > R^*$), since R jumps to R^* immediately, the total value of land is the value of the optimal land allocation at R^* minus the conversion cost (measured in utility terms). By letting F^1 be the value function for the conversion region, we have as follows

$$\begin{aligned} F^1(R,Z) &= F^0(R^*,Z) - \text{Conversion Cost} \\ &= F^0(R^*,Z) - \kappa Z \int_{R^*(Z)}^{R} (1 - \mathrm{d}R')^{\frac{-1}{\eta}} \\ &= F^0(R^*,Z) + \kappa Z \left\{ \frac{(1-R)^{1-1/\eta}}{1-1/\eta} - \frac{(1-R^*)^{1-1/\eta}}{1-1/\eta} \right\} \end{aligned} \tag{12}$$

where $\kappa Z (1-R)^{\frac{-1}{\eta}}$ is the marginal disutility of land conversion, and the total conversion cost is the integral of it from R to R^*. Note that the integration is with respect to R', and a prime is added simply in order to distinguish R' from its end value R.

2.4 Boundary Conditions

As previously noted, in general, boundary conditions play a key role in real option problems. In this model, we have five boundary conditions, of which the main boundary conditions that characterize our model are three free boundary conditions. These free boundary conditions relate to the barrier curve. The adjective "free" implies that the boundary condition can move freely; more precisely the position of the boundary is determined endogenously. We now explain the three free boundary conditions as well as the two other boundary conditions employed in our model.

[Figure 3: Boundary Conditions around here]

2.4.1 Free Boundary Conditions

The free boundary is the demarcation curve between the two regions, which is algebraically represented by optimal $R^*(Z)$ as a function of Z. The free boundary is also referred to as a barrier curve. There are three conditions imposed on the free boundary:

◊ *Boundary Condition 1:* The level matching condition (LM) implies that the values of F^0 and F^1 must be the same on the barrier curve. That is,

$$F^0(R^*,Z) = F^1(R^*,Z) \tag{LM}$$

We have already used this to derive F^1; obviously (12) implies that the (LM) holds when $R = R^*$.

$\Diamond$ *Boundary Condition 2:* Following Dixit and Stiglitz (1994), we use the value matching (VM) condition to signify the first order optimality condition (with respect to ν): $F_R^0(R^*, Z) = u_v(R^*, Z, v)$. Or equivalently,

$$\frac{-Z}{\rho - \alpha}(1 - R^*)^{-1/\eta} + \frac{\phi}{\rho} R^{*-1/\eta} + B'(R^*) Z^\beta = \kappa Z (1 - R^*)^{-1/\eta} \tag{VM}$$

Intuitively, (VM) is obtained by taking the first derivative of the RHS of (12) with respect to R and setting it equal to zero. The left hand side of (VM) shows the marginal gain by converting one more unit of R to A and the right hand side shows the marginal cost of land conversion. At the optimum (ie, evaluated at $R = R^*$), the marginal benefit must be equated to the marginal cost. If the left hand side is greater than the right hand side, then more land should be converted, and vice versa.

$\Diamond$ *Boundary Condition 3:* The smooth pasting condition (SP) must also hold: $F_{RZ}^0(R^*, Z) = u_{vZ}(R^*, Z, v)$, or

$$\frac{-1}{\rho - \alpha}(1 - R^*)^{-1/\eta} + \beta B'(R^*) Z^{\beta - 1} = \kappa (1 - R^*)^{-1/\eta} \tag{SP}$$

This is the first derivative of (VM) with respect to Z. This is perhaps the most difficult condition to explain intuitively. Essentially, if this condition does not hold, then $E_t[F_R^0(R^*, Z) \text{ at } t + \mathrm{d}t] \neq \mathrm{E}_t[u_v(R^*, Z, v) \text{ at } t + \mathrm{d}t]$, which means that the "first order optimality condition" in the next moment does not hold. Since $\mathrm{d}t$ is an arbitrary short time duration, if, say, $E_t[F_R^0(R^*, Z) \text{ at } t + \mathrm{d}t] > \mathrm{E}_t[u_v(R^*, Z, v) \text{ at } t + \mathrm{d}t]$, then converting more R is optimal. See Dixit and Stiglitz (1994) for more details on smooth pasting conditions.

Optimal $R^*(Z)$ Given these three free boundary conditions, we now obtain R^* as a function of Z (or equivalently W), by eliminating the unknown function $B'(R^*)$ from (VM) and (SP). This yields

$$\begin{aligned} R^* &= R^*(Z) = \frac{\phi_2^\eta}{Z^\eta + \phi_2^\eta} \qquad \left(= \frac{\phi_2^\eta}{W^{\eta-1} + \phi_2^\eta}\right) \\ \text{where } \phi_2 &= \frac{\beta}{\beta - 1} \frac{\phi}{\rho\left(\frac{1}{\rho - \alpha} + \kappa\right)} \qquad \left(= \frac{\beta}{\beta - 1} \frac{\rho - \alpha}{\rho} \phi \text{ if } \kappa = 0\right) \end{aligned} \tag{13}$$

Note that $\phi_2 > \phi$ for κ small enough. For simplicity we assume $\kappa = 0$ in the following discussion. Comparing this result with myopic solution (5a), we find that the functional forms are the exactly same except for the difference between ϕ and ϕ_2. More precisely, since $\phi_2 > \phi$, $R^* > R^\dagger$ for any Z (see Figure 5). That is, if the irreversible nature of R is ignored, the government mistakenly converts too much R into A, and such a mistake $R^* - R^\dagger$ is larger when the ratio $\phi_2/\phi > 1$ is larger. It is easy to show that, if $\kappa = 0$, ϕ_2/ϕ reduces to a function of only α/σ^2 and ρ/σ^2, where remember that α and σ are trend and volatility parameters for Z (see (3c)). Figure 4 plots ϕ_2/ϕ as a function of α/σ^2 for several ρ/σ^2. It is obvious from the figure that ϕ_2/ϕ is larger if volatility σ is larger relative to α and ρ. This is quite intuitive because in general the option value is larger when the environment is more uncertain.

[Figure 4: ϕ_2/ϕ around here]

Option Value To find the option value $B(R^*)$, we first eliminate $Z(R^*)$ from (VM) and (SP) to obtain $B'(R^*)$ (14). Next, integrate $B'(R^*)$ to recover $B(R^*)$.

$$B'(R^*) = \phi_3 \frac{R^{*\frac{\beta-1}{\eta}}}{(1-R^*)^{\frac{\beta}{\eta}}} \tag{14}$$

$$B(R) = \int_0^R B'(R')\,\mathrm{d}R' = \phi_3 \int_0^R \frac{R'^{\frac{\beta-1}{\eta}}}{(1-R')^{\frac{\beta}{\eta}}}\mathrm{d}R' \tag{15}$$

$$\text{where } \phi_3 = \frac{\phi}{\phi_2}\left(\kappa + \frac{1}{\rho-\alpha} - \frac{1}{\rho}\right) \tag{16}$$

Note, that $Z(R^*)$ is an inverse function of $R^*(Z)$. Such an inverse function exists because the relationship between R^* and Z is monotonic (see Figure 5). Also, again the integration is with respect to R', and a prime is added to R' to distinguish R' from its end value R.

2.4.2 Additional Boundary Conditions

The two remaining boundary conditions are imposed along lines $Z = 0$ and $R = 0$.

◊ *Boundary Condition 4:* This condition appears in most real option problems. It states that the value function must be non explosive at $Z = 0$. Remember that, since (9) is a second order ODE, its characteristic polynomial has two roots ($\beta > 1$ and $\beta_1 < 0$) and there are two elementary solutions corresponding to these roots. If we have term $B_1(R)\,Z^{\beta_1}$, $F^0(R, Z)$ explodes at $Z = 0$, meaning that $F^0(R, 0)$ can be either $+\infty$ or $-\infty$. Also, it can be shown that $F^0(R, Z) \geq 0$ for any Z if $\eta > 1$, and hence the possible minimum value of F^0 is bounded by 0. All in all, having term $B_1(R)Z^{\beta_1}$ of the negative root, $F^0(R,0) = +\infty$. Our intuition, however, tells us that, regardless of the land allocation, a very bad technology must generate a very low value function (note that if $\eta > 1$, $Z = 0$ implies $W = 0$), which contradicts to the existence of term $B_1(R)Z^{\beta_1}$. For $\eta < 1$, a similar argument leads us to eliminate the term with $\beta_1 < 0$; ie, we determine integration constant $B_1(R)$ to be zero.

◊ *Boundary Condition 5:* At $R = 0$, we impose the condition that the option value is zero. This is simply because, without land to be converted, there is no chance to exercise an option in the future. This condition is used in (15), in which integration is between 0 and R. The starting value 0 of the integration (i.e., lower subscript on $\int$) is determined by this zero option value condition.

3 Numerical Results

To illustrate the optimal land conversion problem, we undertake a numerical exercise.[6]

3.1 Parameter Selection

We first need to provide a number of parameter values. Our choice of parameter values are summarized in Table 1.

[Table 1: Parameters around here]

Our choice of value for the discount rate, 4%, is guided by the macroeconomic literature. It must be close to the long-run average of the risk-free short-term bond rate. Interestingly, our choice is significantly less than the value, frequently 7%, employed in previous studies such as Bulte et al. (2002) and Leroux et al. (2009). While our discount rate is a primitive parameter, theirs needs to be effective discount rates to reflect the partial equilibrium nature of their model setup. We will discuss this important distinction further in Section 4.2, in relation to environmental discounting.

For the trend growth rate of technology and its volatility, as discussed in the Introduction, we experiment with two sets of parameters. The first one is agricultural TFP (agri-TFP), motivated by the fact that the leading alternative land use to the reserved environment is farmland especially in

[6] The Matlab code for our numerical results is available at:
http://www.kent.ac.uk/economics/research/papers/2011/1107.html

developing countries. Employing the data set of Huffman and Evenson (1993) which has TFP for US agriculture from 1950 to 1990 we find that $\tilde{\alpha}$ is equal to approximately 0.0211 (2.11%) and $\tilde{\sigma}$ is 0.0604. We also note from the agricultural productivity literature that many countries have annual TFP growth rates of approximately 2% (eg, Heady et al, 2010), although volatility is not typicaly reported. The second choice is TFP for GDP (GDP-TFP), which may be suitable for developed countries, because we do not need to interpret A literary but can understand A as any land use alternative to the environment. Based on our own estimates of Cobb-Douglas production functions for Japan, UK and US, we find that the trend growth rate of the Solow residual and its volatility are much lower than those of agri-TFP; the average values are $\tilde{\alpha} = 0.0117$ and $\tilde{\sigma} = 0.0112$.

In terms of the elasticity of substitution η between R and C, as we have already noted, the choice of this parameter alters model behaviour not only quantitatively but also qualitatively. As a result, we do not employ a single value for η. Instead, we have chosen two values 5.0 and 0.7, because they illustrate the importance of this parameter on the behaviour of our model. At this point, it is also worth noting that in the partial equilibrium models of Bulte et al. (2002) and Leroux et al. (2009) it is the agricultural returns to scale parameter that plays a similar role. In these models, it is necessary to impose the restriction of decreasing returns to scale on the agriculture sector for their models to generate meaningful results. However, since our model needs to be consistent with stylised macro growth behaviour, it is difficult to deviate from constant returns to scale (CRS) production function as this is the standard assumption in growth models. Also, there is little guidance to help identify appropriate values for ϕ (ie, the relative importance of the service flow from R in the utility). Here, we simply assume that it is equal to 1.0; fifty-fifty weights on C and R. Note that this choice of ϕ is innocuous in the sense that its value depends on the choice of the measurement units of variables, specifically, the choice of the measurement unit of Z (and hence that of W). Hence, changing ϕ simply changes the scale unit of the W axis in the following figures (but in a non-linear way). In Section 4.3, we discuss how to estimate ϕ and η.

Finally, we simply assume κ (coefficient on the marginal cost of land conversion) is zero, mainly because we want to keep the comparability with the myopic case. This assumption is in keeping with Bulte et al. (2002).

[Table 2: Key Numbers around here]

3.2 Barrier Curve

Given our choice of parameter values, we are now able to plot the barrier curve. As discussed, the effective weight of ϕ_2 on R is larger than the myopic weight ϕ (see Table 2 for ϕ_2/ϕ), and as such the optimal level of the reserved environment R^* is larger than the myopic level of $R^\dagger$, as shown Figure 5. The *horizontal* gap between the solid and broken lines shows the mistake that the government or society would make if it ignores the option value of R. In other words, if the government ignores the irreversible nature of R, it converts too much R into agricultural land A. This gap tends to be larger when $\eta > 1$, because the option value tends to be larger when $\eta > 1$, which we discuss subsequently. Also, the gap is larger with agri-TFP, because the volatility of TFP growth is more significant relative to the TFP trend growth rate; ie, $\tilde{\alpha}/\tilde{\sigma}^2$ is lower for agri-TFP than GDP-TFP. For agri-TFP, if $\eta = 5$, the gap can be almost 30% of the optimal size of R, which is a terrible mistake.

[Figure 5: Barrier Curves around here]

If we evaluate the size of mistakes by examining the *vertical* gap between the solid and broken lines, we obtain a further insight into the model. Consider the case with agri-TFP with $\eta = 0.7$, for example. Suppose $R = 0.5$ and the current level of technology W is around 0.60. In this case, we can numerically show that the land use is optimal; ie, $0.5 = R^*(0.60)$. Similarly, we can show that $R = 0.5$ is myopically optimal if $W = 1.00$; ie, $0.5 = R^\dagger(1.00)$. The vertical gap along line $R = 5.0$, ie, the difference between 0.60 and 1.00, is very large, reflecting the steep slope of the barrier curve. On average, it takes 25.2 years for technology W to grow from 0.60 to 1.00.[7] Hence, if the government mistakenly chooses $R = 0.5$ when $W = 1$, then we can say that such a choice was optimal more than 25 years ago. Similarly, for agri-TFP with $\eta = 5.0$, we can numerically show $0.5 = R^*(1.08)$ and $0.5 = R^\dagger(1.00)$, and the technology gap between 1.00 and 1.08 corresponds to 3.9 years, which is not large because the barrier curve is flat in this case. Hence, if the government mistakenly chooses $R = 0.5$ when $W = 1.00$, then it will be optimal in 4 years. Though we do not discuss the vertical gap any further, this gives a hint to the question that "why does the government convert the land to $R^*(W)$ knowing that, if $\eta < 1$, such a R^* will result in too much land conversion in the future?"; it

[7] Ignoring the volatility term $\tilde{\sigma}dw$ in (2d), to evaluate a vertical gap in terms of time, solve $W_T = W_0 e^{\tilde{\alpha}T}$ for T; ie, $T = \ln(W_T/W_0)/\tilde{\alpha}$, where W_0 and W_T are initial and end technology levels, respectively.

is because, even if the government chooses $R > R^* (W)$, it will be optimal only in the very distant future.

3.3 Option Value

Figure 6 shows the option value of R. Remember that the option value is given by $B(Z) Z^{\beta}$ in (10), and the option value is zero in the conversion region. This is because, in the conversion region, the option is exercised immediately (ie, R is converted into A), which means that the remaining option value is zero. Note that the spikes for $\eta = 0.7$ and the jagged edges for $\eta = 5.0$ are as a result of the quality of the graphic resolution[8]; in reality, they have smooth edges in all four panels. For all cases, the option value tends to be higher near the barrier curve, because future conversion is more likely.[9] Qualitatively, the shapes do not differ between agri-TFP and GDP-TFP, but quantitatively agri-TFP generates larger option values because the size of its volatility $\tilde{\sigma}$ relative to its trend growth rate $\tilde{\alpha}$ is larger. Also, the option value is higher with $\eta > 1$ compared to the cases with $\eta < 1$. Indeed, in the two right panels, it is almost zero and it takes non-negligible value only in the very narrow region near the barrier curve. This is because, since W is improving on average, for $\eta < 1$, an upward-sloping barrier curve implies that it is unlikely to move into the conversion region from the non-conversion region in the near future.[10] We will discuss this further in Section 4.1. However, even for $\eta > 1$, the option value is small relative to the magnitude of the value function (compare the units of z-axes with those in Figure 7).

[Figure 6: Option Value around here]

[8]Under GDP-TFP with $\eta = 0.7$, β takes very large values, which makes the computation of (15) unstable. More specifically, under such a parameter assumption, $\beta/\eta = 624.5$ and $(\beta - 1)/\eta = 623.1$ are both large numbers; hence the inside of the integration is almost "L" shaped, and so is $B(R)$. This not only makes the numerical integration in (15) unstable but also deteriorates the graphical representation of the option value. Since we compute the option value on equidistant grid points, some grids capture the points very close to the barrier curve and give relatively large option values, which appear as spikes in the lower-right panel of Figure 6, while other grids are a bit remote from the curve and lead to small values, which are dwarfed by the spikes and hence are indistinguishable from zero. Our gird points are chosen mainly because proper Matlab 3D graphic commands require equidistant grids.

[9]For $\eta > 1$, the option value reaches its peak near the inflection point of the barrier curve, while, for $\eta < 1$, it takes the maximum value at the origin.

[10]In addition, when the absolute value of the slope of the barrier curve is smaller (i.e., flatter), the option value tends to be larger. The intuition behind this is in the same vein as the discussion on the upward- or downward-sloping barrier curve; it is more likely to cross over the barrier curve when it is flatter. That is, the option value is not monotonic in η. It is decreasing in η for $\eta < 1$ and increasing in η for $\eta > 1$; its minimum value is 0 at $\eta = 1$.

3.4 Value Function

Figure 7 shows the shape of the value function over both conversion and non-conversion regions.[11] Given W (ie, along the direction of the R axis), the value function is quite flat. This property is inherited from the flow utility, which is also very flat with respect to R.

[Figure 7: Value Function around here]

This flat shape along the R axis explains why a small option value can cause a significant gap between R^* and $R^\dagger$. That is, the flat value function implies that a large change in R leads to a small change in F (value function), which, in turn, of course, a small loss in F due to the ignorance of the option value causes a large change in R. However, this flatness also implies that a large gap (or mistake) in R is not very painful, in the sense that the loss of value function, due to the missing option value, is not very large. This finding is in keeping with those previously reported by Alders et al. (1996) and Bulte et al. (2002).

More formally, Figure 8 shows the consumption equivalence of the welfare loss. Suppose a country has mistakenly converted too much land and has $R^\dagger$. This country's welfare (ie, value) is $F\left(R^\dagger, W\right)$, which is typically[12] lower than the optimal value $F\left(R^*, W\right)$. Suppose that an international organization or the like is going to give this country some additional consumption to compensate this welfare gap; then, the consumption equivalence means the amount of this additional consumption good.

[Figure 8: Equivalent Consumption Loss around here]

From Figure 8, we find the following three key observations. First, though the qualitative properties are the same between agri-TFP and GDP-TFP, the magnitudes tend to be larger for agri-TFP, which is simply because agri-TFP is more volatile than GDP-TFP. For example, with agri-TFP, the gap between optimal and myopic R can be near 30% of optimal R^* if $\eta = 5.0$. This means that, if the

[11] In general, a CES functional form assumption implies that both flow utility (3b or equivalently 4a) and value function (3a) take negative values for $\eta < 1$. Assuming $Z = W^{1-1/\eta}$, the technology Z is decreasing in W for $\eta < 1$. In this case, Z must be decreasing, because an increase in U is represented by the shrinking negative term; $Z\frac{(1-R)^{1-1/\eta}}{1-1/\eta} < 0$.

[12] Actually, if $\kappa \neq 0$, non-optimal R' can give higher value $F\left(R', W\right)$ than $F\left(R^*, W\right)$. This is because the conversion cost $W\kappa\nu$ is a sunk cost, which is not included in $F\left(R', W\right)$. Note that the optimality of $F\left(R^*, W\right)$ means

$$F\left(R^*, W\right) \geq F\left(R', W\right) - \kappa Z \int_{R^*(Z)}^{R} \left(1 - \mathrm{d}R'\right)^{\frac{-1}{\eta}} \text{ for any } R'$$

where the second term shows the land conversion cost $W\kappa\nu$ evalueted in utility term (see 12).

irreversibility of R is ignored, significantly too much R may be converted into A in developing countries, in which the leading alternative use of R is farmland. However, for GDP-TFP, which is much more stable than agri-TFP, the gap between R^* and $R^\dagger$ is much smaller.

Second, the consumption equivalent welfare loss tends to be larger for $\eta < 1$, mainly because a low elasticity of substitution literally means that people are reluctant to substitute R for C. In a sense, a low η means that preferences are not flexible and, hence, the welfare loss tends to be large. In contrast, the effect of η on the gap between R^* and $R^\dagger$ is mixed and complicated. With GDP-TFP, although the option value is larger for $\eta > 1$, the horizontal gap between R^* and $R^\dagger$ is smaller for $\eta > 1$,[13] which may seem to be strange. However, since the gap is also affected by the flexibility of preferences, the size of the option value is not monotonically translated into the size of the gap, especially when we compare two η, one of which exceeds 1 and the other does not.

Finally, the consumption equivalent welfare loss is not large for all four parameter sets. That is, the welfare loss of ignoring the option value is very small even with agri-TFP. The cohabitation of the possible large gap in R and the small welfare loss may seems to be paradoxical, but this is the consequence of the flat value function, as discussed above. A flat value function implies that a large change in R leads to a small change in the value function, which in turn means that a small loss in the value function is related to a large effect on R. In terms of policy implication, as long as we believe that maximizing the economic welfare is the ultimate goal, this result leads us to conclude that it is not very harmful to ignore the option value of unconverted land R.

4 Further Discussions

4.1 Initial State Dependence

It is interesting to investigate in more detail the dynamics of the barrier control model we have developed. Consider the left panel of Figure 9. Suppose that an economy starts from somewhere in the conversion region (north east of the barrier). Then, no sooner does the government find that it is located in the conversion region, it jumps to R^* horizontally. This also means that if a positive shock hits this economy, then it again jumps to R^* horizontally. However, if a negative technology shock

[13] However, if we measure the gap between R^* and $R^\dagger$ based on the *vertical* distance, we can say the gap is always larger for $\eta < 1$.

hits, it will be pushed into the non-conversion region, and so no government action will result. That is, a negative shock leads to no action, while a positive shock yields a horizontal jump. This results in a zigzag movement along the barrier curve, which is the reason why this type of dynamics is called barrier control. Note that, since, whatever the starting point is, all economies will be on the barrier curve in the end, there is little initial state dependence.

[Figure 9: Barrier Control Dynamics around here]

However, a cumbersome outcome occurs for $\eta < 1$. In this case, once an economy is pushed a long way into the non-conversion region, since a *technology is increasing on average*, it is likely to stay in that region forever. Of course, it is also possible that the economy is hit by a large negative shock and moves to left, but such a possibility is low. To be more precise, it is known that the drift term $\tilde{\alpha}\mathrm{d}t$ dominates the stochastic term $\tilde{\sigma}\mathrm{d}w$ in (2d) in the long run. Hence, once an economy is a long way inside the non-conversion region, it will not come back to the conversion region again. This dominance of the drift term over the stochastic term has previously been noted in the environmental economic option value literature (eg, Bulte et al., 2002), albeit in relation to environmental values as opposed to technology shocks.

One important implication of this behaviour is that an upward sloping barrier curve exhibits strong initial state dependence, which is often regarded as something theoretically undesirable. For example, with initial state dependence for any starting point at the initial date, we can always ask ourselves what happened one day before the initial date. This type of question is not important if the dynamics are not dependent on the initial state. This is somewhat troublesome because, on the one hand, we find support for a low elasticity, but, on the other hand, having $\eta < 1$ causes this initial value dependence problem.

Also, it is worth considering the main mechanism by which the elasticity of substitution affects the option value quantitatively. Remember the following two observations: (i) technology is improving on average (ie, an economy tends to move upward in Figure 9); and (ii) the option value is high when the possibility of exercising the real option is high in the near future. So consider an economy which rests just to the left of the barrier curve. Since it is in the non-conversion region, it has a positive option value. If $\eta > 1$, the barrier curve is downward sloping and hence, a positive shock, which takes place more often, pushes the economy to the conversion region (and hence the real option is likely to be exercised). However, if $\eta < 1$, the barrier is upward sloping and hence a negative shock, which takes place less often, sends the economy into the conversion region, meaning that the possibility of exercising the real option (ie, land conversion) in the near future is less likely. As a result the option value is smaller. A smaller option value for $\eta < 1$ implies that the myopic mistake is smaller.

4.2 Environmental Discounting

As already noted (10) exhibits different (effective) discount rates for C and R. That is, the effective discount rate for C is $\rho - \alpha$, while that for R is ρ in our model. Note that since α can be negative or positive, $\rho - \alpha$ can be larger or smaller than ρ. Indeed, as our model is grounded in principle in relation the land use, it naturally generates the theoretical prediction that is asserted as environmental discounting; note that the irreversibility constraint is not essential to generate these dual discounting rates. On the one hand, in our model, we assume that *physically* the quality of the environment does not change over time, while the production cost of C decreases as technology improves. Hence, only the effective discount rate for C is affected by α. On the other hand, typically the literature of on environmental discounting assumes that the speed of the (exogenous) growth in the value of the environment is faster than that of the technological growth. So, because of the similar logic, the discount rate for the environment is lower than ρ. Of course, it is, at least in principle, possible to extend our model so that the production function includes R in addition to A, so that as the quality of R increases exogenously (though we still believe that assuming exogenous increases in the value of R is not really satisfactory). Essentially, the differences in the effective discount rates arises without adding any special assumptions, if the model is fairly micro-founded such as that employed here.

As such, the difference in the value of the discount rate between ours and Leroux et al. (2009) and Bulte et al. (2002) can be explained by environmental discounting. On the one hand, our discount rate ρ is a *primitive parameter*, which is something in people's mind and is given to the economic model, since we model both demand and supply sides explicitly. On the other hand, both Leroux et al. (2009) and Bulte et al. (2002) only consider the production side. Hence, the proper discount rate for them is not a deep parameter; instead, they need to choose *effective discount* rates for their own context that implicitly take into account the factors omitted.

4.3 Empirical Implication

In the parameter selection, instead of pinning down the best guess of η, given its importance, we employed two possible values for η, greater and smaller than 1. Like many other researchers we have emphasized the importance of the elasticity of substitution between reproductive goods and the environment. However, most existing research discusses the elasticity of substitution as it relates to the production side of the problem. In the way in which we have developed our model the elasticity emerges in different way. As a result our theoretical model offers two (at least potentially) estimable equations for η. The first equation is given by (5b).

$$\text{Myopic: } \ln P = \ln \phi - \frac{1}{\eta}(\ln R - \ln C) \tag{17}$$

This equation holds in the context that the irreversibility is, if not irrelevant, negligible. Regressing the log of the shadow price of R on $\ln R - \ln C$, we can obtain the estimates of $\ln \phi$ and $1/\eta$. In this equation, we define the shadow price P of the environment as the value of consumption compensation required to accept for the temporary reduction of (the service flow from) environment R. It is for the temporary reduction of R, because R is assumed to be reversible. Remember that (17) is the optimal level within the assumption that the irreversibility constraint is absent. Also, remember that we do not worry about the difference in measurement units, since ϕ absorbs it.

The second means by which we might estimate η is derived mainly from the value function (10) and optimality condition (13). See Appendix A.2 for the derivation and some additional remarks. It is useful, when the irreversibility constraint is under consideration.

$$\text{Dynamic: } \ln P = \left(\ln \phi + \ln \frac{1}{\rho} + \ln \frac{\beta}{\beta - 1}\right) - \frac{1}{\eta}(\ln R - \ln C) \tag{18}$$

In this case, the constant term is the mixture of many parameters, but, since we can calculate β from η, $\tilde{\alpha}$ and $\tilde{\sigma}$, all parameters are identifiable, provided that $\tilde{\alpha}$, $\tilde{\sigma}$ and ρ are already estimated or known. For (18), we define the shadow price as the value of a one-off consumption compensation to accept a one unit of permanent reduction in R. It is for the permanent reduction of R, because R is assumed be irreversible.

Interestingly, the coefficient on $\ln R - \ln C$ is the same for the both, and the difference concentrates on the constant term. Intuitively, $\ln 1/\rho$ represents the difference between temporary and permanent reductions. Since we assume $\rho = 0.04$, the present value of one unit permanent reduction of R is larger than that of temporary reduction by a factor of $1/\rho = 25$.[14] In addition, the option value of R increases the shadow price of R by $\beta/(\beta - 1) > 1$. As is clear in Table 2, $1/\rho$ accounts the most of the difference between (17) and (18). For agri-TFP with $\eta = 5.0$, however, the option value increases the shadow price P by 81.9%, which is significant, although its effect is negligible for $\eta = 0.7$.

The obvious problem with this approach is that shadow price P is usually unobservable. However, we could employ non-market valuation methods to evaluate P. If such evaluation is available, say, by conducting a stated preference survey, then we can estimate either (17) or (18), depending on the context. For example, if individual level data in one region is available over a number of years, assuming that individual characteristics appear only in ϕ (ie, assuming that all individuals share a common η), the following panel estimation may be implementable.

$$\begin{aligned} \ln P_{it} &= \ln \phi_{it} - \frac{1}{\eta} \left(\ln R_t - \ln C_{it} \right) \\ \phi_{it} &= \phi \left(X_{it} \right) \text{ where } X_{it} \text{ includes any individual characteristics, etc.} \end{aligned}$$

A problem with this proposal is that there are very few stated preferences studies that explicitly take account of temporal changes in value. In a survey of the literature Skourtos et al. (2010) identify one which examines the temporal reliability of stated preference estimates. However, in principle with the development of stated preference databases there is potential for the use of some form of meta analysis to reveal the shadow price P. Another caveat of this approach is that both (17) and (18) are derived at the societal and government level and not for individuals (see Appendix A.2 for details). However, we believe these equations are good proxies for individual environmental pricing. Setting aside the option value of the environment, as many researchers discuss, the elasticity of substitution between reproductive goods and environmental goods plays a key role in many areas of environmental economics. Thus, the empirical approaches identified here, that emerge from our model, may well be worth pursuing as a means to estimate η in the future.

[14] See footnote 5.

5 Summary and Conclusions

In this paper, we consider the option value of the environment based on a simple general equilibrium growth model with stochastic technology shock. In our model, as in existing environmental option value studies, because of irreversibility, the environment can in principle have a significant real option value. However, unlike the existing literature in which the uncertainty of the value of the environment is given exogenously, the value of the environment is endogenously determined. Since we have assumed that the environment is in finite supply, as society becomes richer, the relative (shadow) price of the environment increases in our model, which at least potentially explains why people living in rich countries are more eager to conserving the environment than those in developing countries.

The most crucial parameter in our model is the elasticity of substitution η between consumption and the service flow from the environment; the value of the environment is mainly dependent on how easily the environment can be substituted by consumption. We have shown that by changing η we can significantly alter model behaviour not only quantitatively but also qualitatively. Thus, when $\eta < 1$, the optimal amount of the environment is increasing as people becomes richer, and vice versa. This result is not dependent on the irreversibility of the environment and hence apears to hold in a wide class of economic models.

With the irreversibility constraint on the land conversion imposed, as anticipated, a huge proportion of the environment can be mistakenly converted, if the option value of the reserved environment is ignored. For example, if we assume that the main alternative use of the environment is farmland, given large uncertainty in agricultural TFP, the area of mistakenly converted reserved land can be near 30% of the optimal reserved land if $\eta = 5.0$, and 6% to 8% if $\eta = 0.7$. Such a mistake is much smaller, if the parameters are taken from GDP based TFP; since the alternative land use is more general in developed countries, this parameter choice is proper for them. In this sense, the option value of the environment is important in developing countries, in which the leading reason for land conversion is to obtain farmland, but not very important in developed countries. Interestingly, at the same time, the same numerical experiments suggest that such a huge mistake in land allocation does not lead to a large welfare loss even with agri-TFP, in the sense that the loss of the value function due to the ignorance of the option value can be compensated by a negligible amount of additional consumption. This seemingly paradoxical coexistence of a possible huge mistake in land allocation and small welfare loss is because of the flat value function in the neighborhood of the optimal point.

In addition to our parametric assumptions, obviously, these quantitative results crucially depend on our functional assumptions as well. We have chosen the CES utility function because it is standard in the macroeconomic literature and because its constant η makes the problem tractable. Investigating the quantitative and qualitative model behavior with non-CES utility function may well be worth pursuing in future research.

Setting aside the option value discussions, our analysis has also revealed some interesting insights into environmental economics more generally. First, under our general equilibrium approach, the trend growth rate of the shock process and its volatility are easily estimated. In many cases, quantitative results are quite sensitive to these two parameters. One of the challenges of the partial equilibrium modelling approach is that they rely on the assumption that, say, the value of the environment follows an exogenous stochastic process, but such a process is often hard to estimate. Second, our explicit modelling of both production and household sides makes clear that the dual-rates of discounting for reproductive goods and the environment arise even without adding any special assumptions (even our irreversibility constraint plays little role in the emergence of dual discount rates), which means that the theory of environmental discounting holds in a wide range of models as long as they have a sound economic rationale. Finally, from our model two equations emerge which are at least potentially estimable, provided that data on environmental prices are available by, say, some non-market valuation studies. Since, it is widely recognized that η plays a significant role in many branches of environmental economics, we believe that estimating η and testing if it is larger than or less than unity is a very worth challenge.

A Appendix

A.1 Estimation of TFP

While we employ the numbers from Huffman and Evenson (1993) for agricultural TFP, because we cannot find the growth rate of GDP-TFP and its volatility from the literature, we have estimated TFP for Japan, U.K. and U.S. employing the Solow residuals approach. Assuming a Cobb-Douglas aggregate production function, we can estimate TFP as something remaining after attributing the GDP growth rate to the contributions of capital and labour .

$$\frac{\mathrm{d}W}{W} = \frac{\mathrm{d}Y}{Y} - \phi_K \frac{\mathrm{d}K}{K} - \phi_L \frac{\mathrm{d}L}{L}$$

where W, K, L and Y are TFP, capital, labour and output, respectively. Parameters ϕ_K and ϕ_L show capital and labour shares. Table 3 shows the estimates of TFP based on (a) OLS estimates of ϕ_K and ϕ_L and (b) fixed ϕ_K and ϕ_L. In OLS estimates, constant returns to scale (CRS) is not satisfied for Japan and U.S., but imposing CRS as a restriction reduces estimation performance. We limit ourselves to this simple estimation equation, because using more elaborated methods is beyond our scope. Fixed factor shares are chosen so that they are consistent with the macroeconomic literature. The average of the estimated trend growth rate of TFP $\tilde{\alpha}$ and its volatility $\tilde{\sigma}$ are 0.0117 and 0.0112 for OLS estimates and 0.0131 and 0.0129 for fixed factor shares calculation. We adopt the average OLS estimates, but our model results change little if we employ the fixed factor shares assumption (mainly because $\tilde{\alpha}/\tilde{\sigma}^2$ are very close to each other in the both cases).

[Table 3: TFP around here]

A.2 Derivation of (18)

We define the price P of the environment as the amount of a one-time consumption increase (ΔC_0) to compensate the decrease in R forever (this decrease is permanent due to the irreversibility assumption). Rewrite (10) by using (11a) and $C = W(1-R)$.

$$\begin{aligned}
F^0(R,Z) &= \frac{Z}{\rho-\alpha}\frac{(1-R)^{1-1/\eta}}{1-1/\eta} + \frac{\phi}{\rho}\frac{R^{1-1/\eta}}{1-1/\eta} + B(R)Z^\beta \\
&= \int_0^\infty e^{-\rho t}\frac{C_t^{1-1/\eta}}{1-1/\eta}\mathrm{d}t + \frac{\phi}{\rho}\frac{R^{1-1/\eta}}{1-1/\eta} + B(R)Z^\beta \\
\frac{\partial F^0}{\partial R} &= \frac{\phi}{\rho}R^{-1/\eta} + B'(R)Z^\beta \quad \text{given } C_t \text{ for } 0 \le t \\
\frac{\partial F^0}{\partial C_0} &= C_0^{-1/\eta} = (W(1-R))^{-1/\eta} = Z^{\frac{-1}{\eta-1}}(1-R)^{-1/\eta}
\end{aligned}$$

We put time subscript on C_t to discriminate current consumption C_0 from future consumption in the integral. Note that all variables other than C_t show the current values; e.g., $R = R_0$. From (SP), at optimum (i.e., $R = R^*$),

$$B'(R)Z^\beta = \frac{W}{\beta(\rho-\alpha)}(W(1-R))^{-1/\eta} = \frac{WC^{-1/\eta}}{\beta(\rho-\alpha)}$$

By the implicit function theorem, for a fixed level of the value function $\bar{F}^0$

$$\begin{aligned}
P &= -\frac{\partial \bar{F}^0/\partial R}{\partial \bar{F}^0/\partial C_0} = \frac{\phi}{\rho}\left(\frac{R}{C_0}\right)^{-1/\eta} + \frac{W}{\beta(\rho-\alpha)} \\
&= \frac{\phi}{\rho}\left(\frac{R}{W(1-R)}\right)^{-1/\eta} + \frac{1/\beta}{\rho-\alpha}W \\
&= \left(\frac{\phi/\phi_2}{\rho} + \frac{1/\beta}{\rho-\alpha}\right)W
\end{aligned}$$

where (13) is used to derive the second line. Hence, eliminating W from the first line by using the third line, we obtain:

$$\begin{aligned}
P &= \frac{\phi}{\rho}\left(\frac{R}{C_0}\right)^{-1/\eta} + \frac{\frac{1/\beta}{\rho-\alpha}}{\frac{\phi/\phi_2}{\rho} + \frac{1/\beta}{\rho-\alpha}}P = \left(1 + \frac{\frac{1/\beta}{\rho-\alpha}}{\frac{\phi/\phi_2}{\rho}}\right)\frac{\phi}{\rho}\left(\frac{R}{C_0}\right)^{-1/\eta} \\
&= \left(\frac{\phi}{\rho} + \frac{\phi_2/\beta}{\rho-\alpha}\right)\left(\frac{R}{C_0}\right)^{-1/\eta} = \left(1 + \frac{1}{\beta-1}\right)\frac{\phi}{\rho}\left(\frac{R}{C_0}\right)^{-1/\eta} = \frac{\beta}{\beta-1}\frac{\phi}{\rho}\left(\frac{R}{C_0}\right)^{-1/\eta}
\end{aligned}$$

Or equivalently, omitting time subscript on C,

$$P = \frac{\phi}{\rho}\left(\frac{R}{C}\right)^{-1/\eta} + \frac{1}{\beta-1}\frac{\phi}{\rho}\left(\frac{R}{C}\right)^{-1/\eta}$$

where the first and second terms show the decrease in the value function due to the permanent loss of R and due to losing option value of R, respectively. Comparing to (17), the effect of the permanent loss of R is larger than that of the temporary loss of R by $1/\rho$. Also, it takes into account the option value as well. Again, taking the logarithm, we obtain (18):

$$\ln P = \ln \phi + \ln \frac{\beta/\rho}{\beta - 1} - \frac{1}{\eta} (\ln R - \ln C)$$

Our model suggests that the log of environmental price in this case must be higher than (17) by $\ln \frac{\beta/\rho}{\beta-1} = \ln (1/\rho) + \ln (\beta/ (\beta - 1)) > 0$.

The price of R in terms of C derived in this subsection is a concept very close to the equivalent consumption measure of the myopic loss in Figure 8. Unlike Figure 8, however, setting aside some rather technical differences, here we do not assume that lost R is used as a production factor. This assumption is chosen to study the environmental pricing by individuals, because presumably they do not take into account the benefits from converted land. However, in Figure 8, even if society mistakenly converts too much R, such converted land is used for production, which mitigates the loss in the value function. As a result, the price increases due to the option value ($\beta/ (\beta - 1)$ in Table 2 are large especially for $\eta = 5$, while the equivalent consumption measures for ignoring it are negligible in Figure 8.

Finally, note that there is one caveat in using (18) for individual level data. That is, (18) is derived under the assumption that $R = R^*$ for society, which may not be true for all individuals in society. In other words, (18) is true only under the existence of aggregate utility, which, in general, conflicts with the variations observed in individual level data. In this sense, (18) is most suitable to time series data or cross country data, but obtaining such data is usually very costly.

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References

Albers, H.J., Fisher, A.C. and Hanemann, W.M. (1996). Valuation and Management of Tropical Forests: Implications of Uncertainty and Irreversibility, *Environmental and Resource Economics*, 8(1): 39-61.

Arrow, K.J. and Fisher, A.C. (1974). Environmental Preservation, Uncertainty, and Irreversibility, *Quarterly Journal of Economics*, LXXXVIII(2): 312-319.

Baumol, W.J. (1967) Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis, *American Economic Review*, 57(3): 415-426.

Bulte, E., van Soest, D.P., van Kooten, G.C. and Schipper, R.A. (2002). Forest Conservation in Costa Rica when Nonuse Benefits are Uncertain but Rising, *American Journal of Agricultural Economics*, 84(1): 150-160.

Buttle, J. and Rondeau, D. (2004). An Incremental Analysis of the Value of Expanding a Wilderness Area, *Canadian Journal of Economics*, 37(1): 189-198.

Conrad, J.M. (1997). On the Option Value of Old-Growth Forest, *Ecological Economics*, 22: 97-102.

Dixit, A.K. and Pindyck, R.S. (1994). *Investment Under Uncertainty*, Princeton University Press, Princeton NJ.

Fisher, A.C. (2000). Investment Under Uncertainty and Option Value in Environmental Economics, *Resource and Energy Economics*, 22(3): 197-204.

Forsyth, M. (2000). On Estimating the Option Value of Preserving a Wilderness Area, *Canadian Journal of Economics*, 33(2): 413-434.

Gerlagh, R. and van der Zwaan, B.C.C. (2002). Long-Term Substitutability Between Environmental and Man-Made Goods, *Journal of Environmental Economics and Management*, 44(2): 329-345

Heady, D., Alauddin, M. and Prasada Rao, D.S. (2010). Explaining Agricultural Productivity Growth: An International Perspective. *Agricultural Economics*, 41: 1-14.

Heal, G. (2009). Climate Economics: A Meta Review and Some Suggestions for Future Research, *Review of Environmental Economics and Policy*, 3(1): 4-21.

Huffman, W.E. and Evenson, R.E. (1993). *Science for Agriculture*, Ames: ISU Press.

Kassar, I. and Lasserre, P. (2004). Species Preservation and Biodiversity Value: A Real Options Approach, *Journal of Environmental Economics and Management*, 48(2): 857-879.

Leroux, A.D., Martin, V.L. and Goeschl, T. (2009). Optimal Conservation, Extinction Debt, and the Augmented Quasi-Option value, *Journal of Environmental Economics and Management*, 58(1): 43-57.

Pindyck, R.S. (2007) Uncertainty in Environmental Economics, *Review of Environmental Economics and Policy* 1(1), 45-65.

Reed, W.J. and Clarke, H.R. (1990). Harvest Decisions and Asset Valuation for Biological Resources Exhibiting Size-Dependent Stochastic Growth, International Economic Review, 31(1): 147-169.

Skourtos, M., Kontogianni, A. and Harrison, P.A. (2010). Reviewing the Dynamics of Economic Values and Preferences for Ecosystem Goods and Services, *Biodiversity Conservation*, 19(10): 2855-2872.

Ulph, A. and Ulph, D. (1997). Global Warming, Irreversibility and Learning, *Economic Journal*, 107(442): 636-650.

B Tables and Figures

Table 1: Parameters

Symbol	Agri-TFP	GDP-TFP	Meaning
ρ	0.04	0.04	Discount Rate
$\tilde{\alpha}$	0.0211	0.0117	Trend Growth Rate of W
$\tilde{\sigma}$	0.0604	0.0112	Volatility of W Growth
η	5.0/0.7	5.0/0.7	Elasticity of Substitution Between C and R
ϕ	1.00	1.00	Relative Importance of Service Flow of R
k	0.00	0.00	Marginal Cost of of Land Conversion

Table 2: Key Numbers in Each Simulation

		α	σ	β	ϕ_2/ϕ	$\ln \frac{\beta/\rho}{\beta-1}$	$1/\rho$	$\frac{\beta}{\beta-1}$
Agri TFP	$\eta = 5.0$	0.017	0.048	2.221	1.064	3.817	25.00	1.819
($\tilde{\alpha} = 0.0211$, $\tilde{\sigma} = 0.0604$)	$\eta = 0.7$	-0.008	0.026	28.80	1.241	3.254	25.00	1.036
GDP TFP	$\eta = 5.0$	0.009	0.009	4.255	1.013	3.487	25.00	1.307
($\tilde{\alpha} = 0.0117$, $\tilde{\sigma} = 0.0112$)	$\eta = 0.7$	-0.005	0.005	437.2	1.534	3.221	25.00	1.022

Table 3: Estimation of Solow Residuals

		OLS			Fixed Factor Shares		
		JPN	UK	US	JPN	UK	US
Constant	A	0.0073	0.0144	0.0131	0.0134	0.0141	0.0114
std er		0.004	0.011	0.008	-	-	-
Capital		0.4024	0.3284	0.1444	0.35	0.35	0.35
std er		0.123	0.492	0.333	-	-	-
Labour		1.1818	0.7160	0.9653	0.65	0.65	0.65
std er		0.184	0.153	0.094	-	-	-
SD(e)	B	0.0118	0.0130	0.0088	0.0153	0.0130	0.0104
R2 adj		0.792	0.516	0.822	-	-	-
Period		1980-09	1972-09	1970-09	1980-09	1972-09	1970-09
$\tilde{\alpha}$		0.0074	0.0145	0.0132	0.0135	0.0142	0.0115
$\tilde{\sigma}$		0.0118	0.0130	0.0088	0.0153	0.0130	0.0104

Notes: Data frequency is annual. "SD(e)" shows the standard deviation of residuals.
Our estimates are: $\tilde{\alpha} = A + B^2/2$ and $\tilde{\sigma} = B$. See Reed and Clarke (1990).

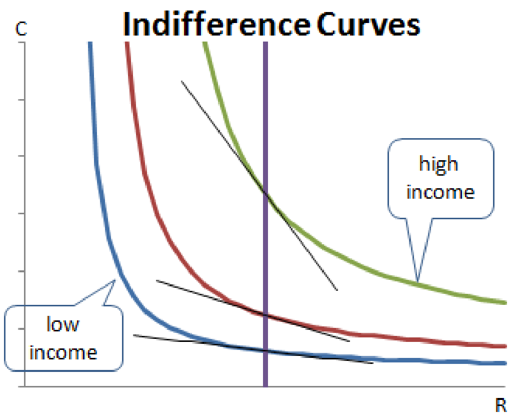

Figure 1: Indifference Curves with Fixed Supply of R.

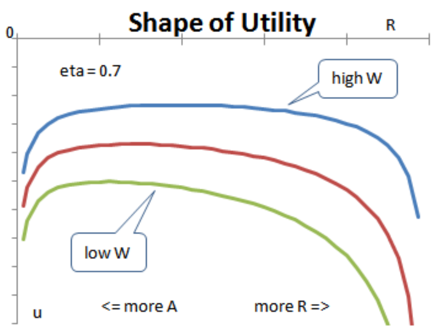

Figure 2: Shape of Flow Utility as a Function of R with $\eta < 1$.

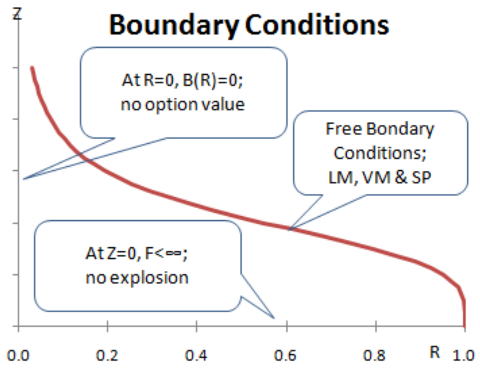

Figure 3: Boundary Conditions.

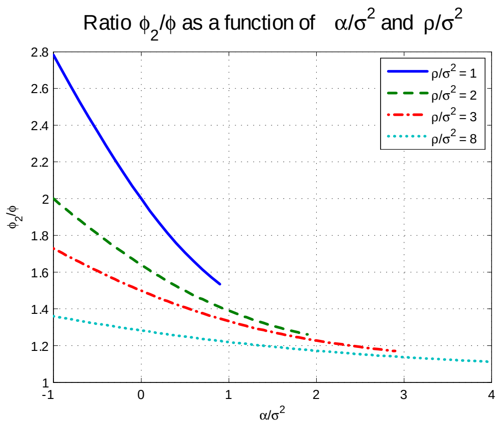

Figure 4: Ratio ϕ_2/ϕ as a function of ρ/σ^2 and α/σ^2. Note that $\alpha < \rho$ and α can be negative.

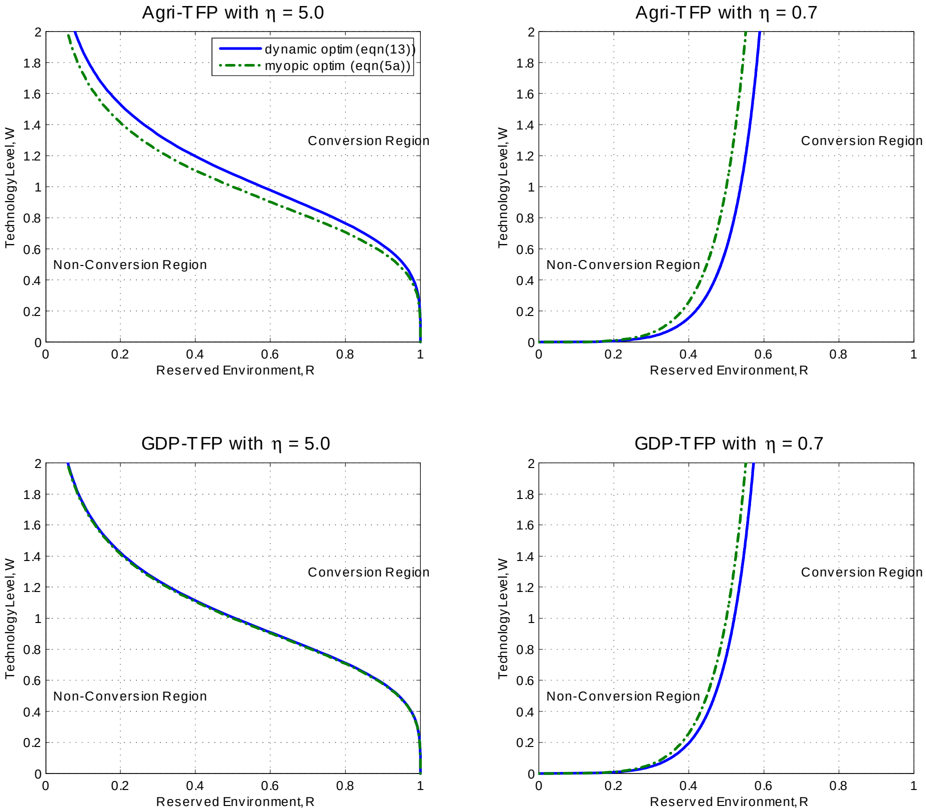

Figure 5: Myopic R and Optimal R.

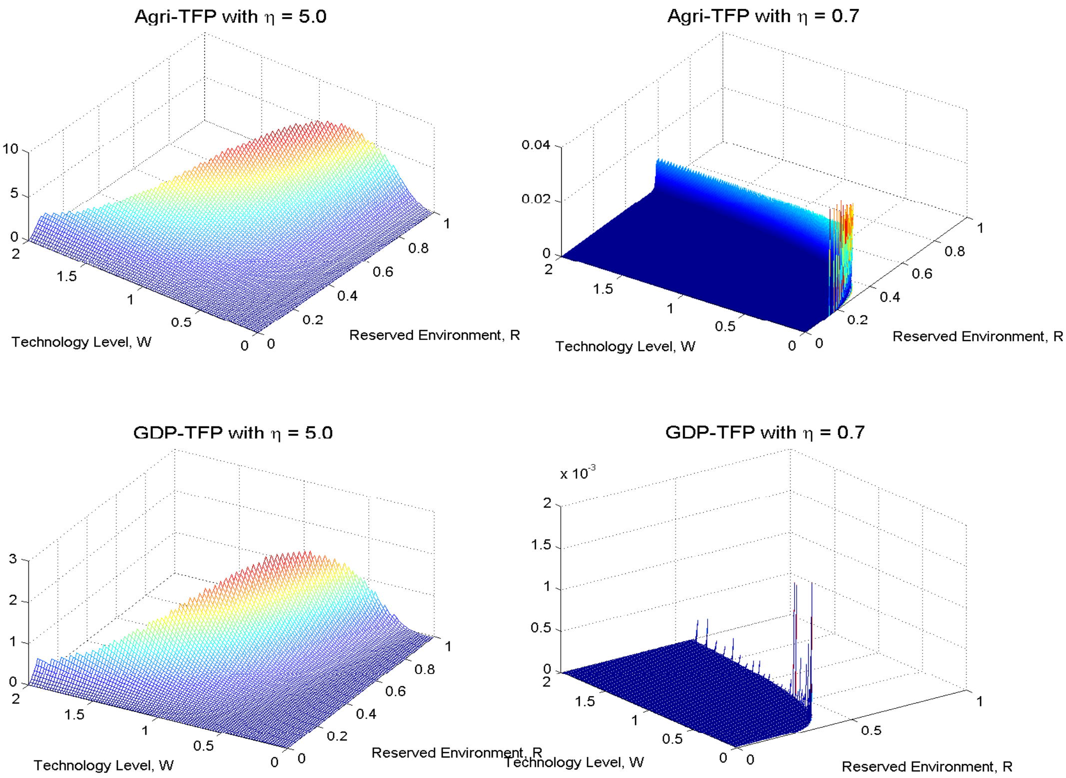

Figure 6: Option value $B(R)\,Z^{\beta}$.

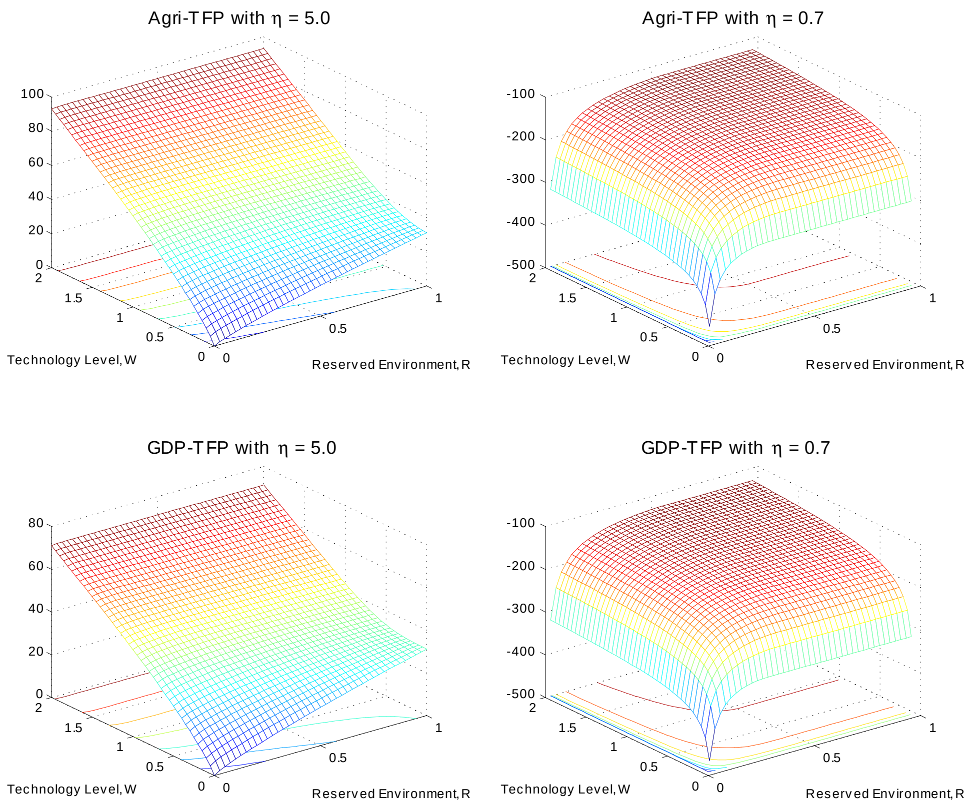

Figure 7: Value Function

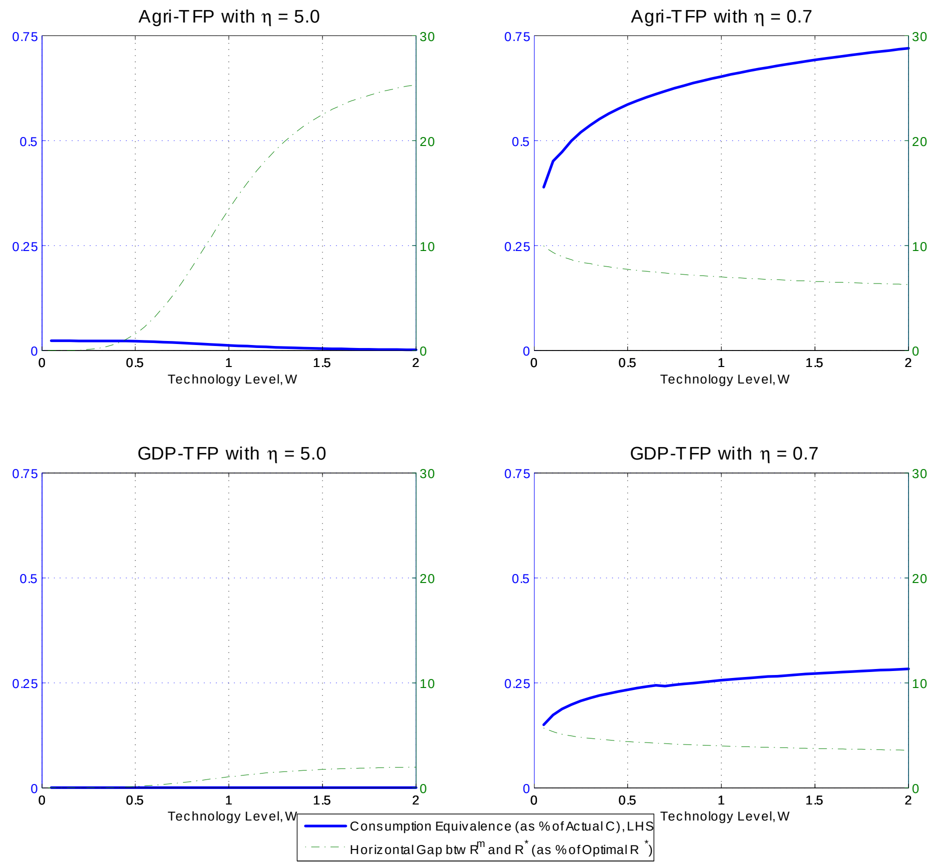

Figure 8: Equivalent consumption measure and the gap between R^* and $R^\dagger$.

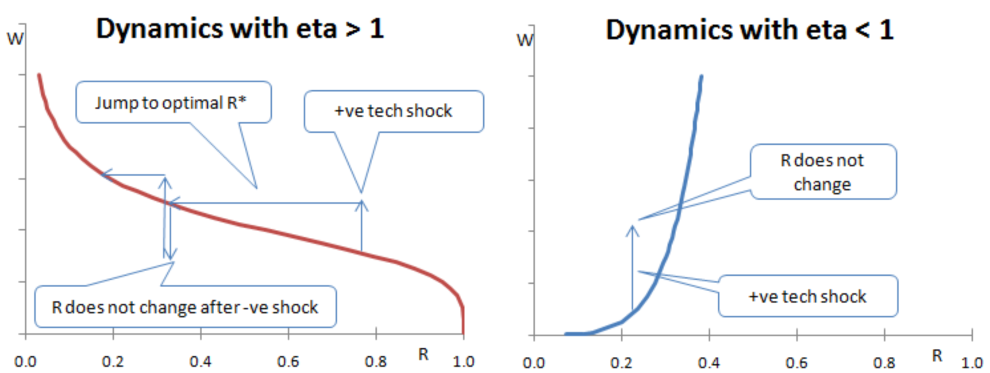

Figure 9: Conceptual diagram to show the dynamics of barrier control model.