Optimal Policy with Informal Child Care: An Efficiency Case for Sliding Scale Child Care Subsidies

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Abstract

In this paper, we extend an optimal tax problem *à* la Mirrlees to allow for unobserved informal child care activities. In our baseline model, individuals have unobserved heterogeneous disutility from effort spent on market work and informal child care activities. We show that there is an efficiency case for child care subsidies on formal child care cost. We discuss an implementation that allows for a Pareto improvement from a situation where the government cannot observe disutility from effort nor informal child care activities, to a situation where it is *as though* the government could observe informal child care activities. The optimal subsidies follow a sliding scale where higher child care subsidies are paid to lower income earners.

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1 Introduction

The past few decades have witnessed several labour market policies and welfare-to-work reforms encouraging work and promoting self-sufficiency among low income families. The introduction of the Earned Income Tax Credit in the United States and of the Working Families Tax Credit in the United Kingdom are well known examples of such policies. Similarly, legislations such as the 1996 Personal Responsibilities and Work Opportunities Reconciliation Act (PRWORA) in the United States aimed at getting low income families off welfare and into work via a combination of strict work requirements, time limits on welfare benefits, and child care subsidies among other reform components.

In this paper, we design an optimal policy within an asymmetric information framework where the government cannot observe agents' disutility from effort. We extend an optimal tax problem *à* la Mirrlees by also allowing agents to engage in unobserved informal child care activities. In our baseline model, individuals have unobserved heterogeneous disutility from effort spent on market work and informal child care activities. We show that there is an efficiency case for child care subsidies on formal child care cost. We discuss an implementation that allows for a Pareto improvement from a situation where the government cannot observe disutility from effort nor informal child care activities, to a situation where it is *as though* the government could observe informal child care activities. We show that the optimal subsidies follow a sliding scale where higher child care subsidies are paid to lower income earners.

Our paper contributes to two branches of the literature: (i) optimal policy and (ii) child care subsidies. Several studies have explored optimal policy in the presence of non-market activities such as unobserved home production or unobserved utility from leisure. In the optimal income tax literature, Beaudry, Blackorby, and Szalay (2009) analyse optimal taxes and employment subsidies in a multi-dimensional screening framework where agents have different unobserved values for both market and non-market activities. They find that there is an optimal cut-off wage where those above the cut-off wage are taxed and those below are subsidised while those without employment receive zero income transfer. Chone and Laroque (2008) argue for subsidisation of low skilled workers in a discrete labour supply framework where individuals have different productivities and work opportunity costs while Boone and Bovenberg (2006) find that an earned income tax credit may be optimal in a setting with unobserved job search effort. Alvarez-Parra and S*á*nchez (2009) analyse optimal unemployment insurance in the presence of a hidden labour market. They prescribe positive unemployment benefits which decline with the duration of unemployment and drop to zero at the point where unemployed agents participate in the hidden labour market.

We abstract from the multi-dimensional screening dimensions of Beaudry, Blackorby and Szalay (2009) and Chone and Laroque (2008) by considering a framework where agents have unobserved disutility from effort. Thus, disutility from effort exercised on the market yield similar disutility from effort exercised at home. Abstracting from the multi-dimensional screening issue allows us to focus our attention on the presence unobserved informal child care activities although it would be interesting to explore optimal policy in the presence of unobserved informal child care activities in such a setting in the future.

Since we focus on informal child care activities, we allow the government to use additional policy instruments such as child care subsidies on formal child care to indirectly tax or subsidise home activities. This is different from previous optimal policy studies in the sense that home activities can be indirectly subsidised as opposed to imposing a negative marginal tax rate based only on earned income of low skilled workers[1]. The option of engaging in unobserved informal child care activities encourage agents to jointly deviate by pretending to be of a different type and engaging in unobserved informal child care activities. It is therefore optimal to provide child care subsidies to low income earners in order to deter the higher income earners from mimicking the low income earners and engage in unobserved informal child care activities. In our implementation discussion, we argue that such child care subsidies may allow for a Pareto improvement from a situation where the government could not observe informal child care activities to a situation where it is *as though* the government could observe informal child care activities.

Our paper also contributes to the literature on child care subsidies. Blau (2003) surveys the literature on child care subsidy programs and documents three main arguments in support of child care subsidies. (i) Promotion of economic self-sufficiency where subsidies are given to low income families to get them off welfare and into work. The argument is that this will lead to human capital accumulation thereby leading to future cost savings for the government. (ii) Market imperfections where parents are uninformed about quality care providers. In this case, the subsidies are targeted towards high quality child care providers. (iii) Distributional issues where quality child care is viewed as a merit good. In this case, families do not

[1]Kleven, Richter, and Sorensen (2000) analyse optimal commodity taxation in the presence of household production and find that it may be optimal to impose a lower tax on market services which may be complementary to leisure. Their framework is however different from ours since they focus on Ramsey style optimal commodity taxation where the government aims at raising a given level of revenue from homogeneous individuals in the least distortionary way possible, as opposed to the Mirrleesian asymmetric information framework where taxation is used to incentivise heterogeneous individuals to work.

consider the positive externalities of quality care and underinvest in child care. Ho (2010) presents a case for child care subsidies in the context of optimal disability insurance within an intergenerational family framework where the grandparent generation can engage in unobserved informal grandchild care activities.

In this paper, we present an efficiency case for child care subsidies in an optimal policy framework where agents have heterogeneous disutility from effort and can engage in unobserved informal child care activities. Our argument for child care subsidies is in addition to the already existing arguments in the child care subsidies literature. We further show that it is optimal for child care subsidies to follow a sliding scale where lower income earners receive higher child care subsidies and discuss implementation of the optimal policy.

We present our main framework with heterogeneous disutility of effort in section 2 and optimal policy results in section 3. We discuss future model extensions in section 4.

2 Heterogeneous Disutility from Effort

2.1 Framework

2.1.1 Agents

Consider an economy with a continuum of agents who derive heterogeneous disutility from effort. Agents can be of type $\theta_n \in \{\theta_1, ..., \theta_N\}$ where disutility from effort is decreasing in type so that higher type agents have lower disutility from effort. Agents are of type θ_n with probability $p_n \in \{0, 1\}$ with $\sum_{n=1}^{N} p_n = 1$.

Agents can allocate effort to market work or to informal child care activities. An agent of type θ_n who devotes effort level l_n on the market produces $y_n = l_n$ of market goods so that we have a linear production function. Each agent has child care needs amounting to 1 unit of effort. An agent of type θ_n can also engage in informal child care activities and devotes effort level h_n towards child care. The remaining amount of child care, $1 - h_n$, is covered by purchasing child care from the formal child care market at cost ω per unit. We assume that the price of formal child care is lower than the price of market production so that $\omega < 1$.

Agents' utility function is separable in consumption c and effort e:

$$u(c) - v(e;\theta)$$

where $u_c > 0$, $u_{cc} < 0$ $\forall c \geq 0$ and $lim_{c \to 0}\ u_c = \infty$. The strict concavity of $u(c)$ implies that agents are risk-averse and therefore want to smooth consumption. We also have $v_e > 0$, $v_{ee} \geq 0$ $\forall e \geq 0$. The convexity of $v(e;\theta)$ in effort level implies that agents get non-decreasing disutility from effort. On the other hand, agents get no disutility from effort if they exert zero effort level so that $v(0;\theta) = 0$. Also, $v_{e\theta} < 0$ $\forall e \geq 0$ so that higher type agents have lower disutility from effort.

2.1.2 Government

We have a risk neutral government who provides insurance to agents. The government does not observe disutility from effort but knows the probability distribution of the different types of agents among the population. Moreover, the government can observe whether an agent is working on the market and market output produced by a working agent. On the other hand, the government does not observe agents' involvement in informal child care activities.

The government also has to satisfy its budget constraint such that aggregate net taxes are zero: $\sum_n p_n \tau_n = 0$. For the sake of convenience, we model the problem as though the government takes all production and distributes benefits to the agents. The government allocates benefits b_n according to the reported type of the agent. By the revelation principle, we can restrict ourselves to such direct mechanism. We therefore rewrite the government budget constraint as

$$\sum_n p_n b_n = \sum_n p_n y_n \tag{1}$$

Note that modeling the problem as though the government takes all production and distributes benefits is equivalent to imposing net taxes $\tau_n = y_n - b_n$ on agents of type θ_n $\forall \theta_n$. Note also that satisfying the government budget constraint implies that the economy's feasibility constraint would automatically be satisfied. Feasibility requires that aggregate consumption and spending on formal child care be equal to aggregate production in the economy: $\sum_n p_n [c_n + \omega(1 - h_n)] = \sum_n p_n y_n$ where consumption is made up of government benefit net of spending on formal child care: $c_n = b_n - \omega(1-h_n)$.

2.2 Second Best World

In this section, we consider the optimal policy in a situation where the government cannot observe agents' types but can observe informal child care activities. In other words, we consider the standard Mirrleesian case where we only have one level of asymmetric information between the government and agents in terms of unobserved disutility from effort. We call this framework the "second best" world.

Since the government cannot observe agents' types, agents will only reveal their true types if government policy is such that utility from revealing the truth is higher than utility from pretending to be a different type:

$$\forall \theta_n, \theta_k \qquad u\left(b_n - \omega\left(1-h_n\right)\right) - v\left(y_n + h_n; \theta_n\right) \geq u\left(b_k - \omega\left(1-h_k\right)\right) - v\left(y_k + h_k; \theta_n\right) \tag{2}$$

The government chooses benefits b_n, output y_n and recommends informal child care h_n for all types θ_n by maximising expected utility:

$$max_{\{b_n, y_n, h_n\}_{n=1}^{N}} \sum_n p_n\left[u\left(b_n - \omega\left(1-h_n\right)\right) - v\left(y_n + h_n; \theta_n\right)\right] \tag{3}$$

subject to government budget constraint (1) and to incentive compatibility constraints (2).

2.3 Third Best World

In this section, we consider the optimal policy in a situation where the government can observe neither agents' types nor informal child care activities. In other words, we extend the standard Mirrleesian case to allow for unobserved informal child care activities. We call this framework the "third best" world so as to differentiate from the "second best" world discussed above.

Since the government cannot observe informal child care activities, agents can choose their level of informal child care so as to maximise their private utility. The incentive compatibility constraints for truthful reporting of type becomes

$$\forall \theta_n, \theta_k \qquad u\left(b_n - \omega\left(1-\hat{h}_n^n\right)\right) - v\left(y_n + \hat{h}_n^n; \theta_n\right) \geq u\left(b_k - \omega\left(1-\hat{h}_n^k\right)\right) - v\left(y_k + \hat{h}_n^k; \theta_n\right) \tag{4}$$

where $\hat{h}_n^k = argmax_{\{h\}} \; u\left(b_k - \omega\left(1-h\right)\right) - v\left(y_k + h; \theta_n\right)$ is unobserved informal child care activities undertaken by an agent of type θ_n who reports to be of type θ_k.

The government chooses benefits b_n and output y_n for all types θ_n by maximising expected utility

$$max_{\{b_n, y_n\}_{n=1}^N} \sum_n p_n \left[u\left(b_n - \omega\left(1 - \hat{h}_n^n\right)\right) - v\left(y_n + \hat{h}_n^n; \theta_n\right)\right] \tag{5}$$

subject to government budget constraint (1) and incentive compatibility constraints (4).

3 Optimal Policy and Implementation

In this section, we characterise the optimal policy and discuss its implementation. We firstly show in Proposition 1 that in our second best world, no one would engage in informal child care activities, and in Proposition 2 that production and benefits are non-decreasing in types. We then proceed to show in Proposition 3 that there is an efficiency case for child care subsidies should the government not observe informal child care activities as in our third best world and in Proposition 4 that the case for strictly positive child care subsidies is very strong when the cost of formal child care is high relative to the price of market production. Finally, we show that there is a case for sliding scale child care subsidies in Proposition 5 and discuss an implementation where it is *as though* the government could observe informal child care activities, thereby bringing us back to the second best world.

3.1 Optimal Policy

Proposition 1: If the government cannot observe agents' types but can observe informal child care activities, optimal policy recommends $h_n^* = 0$ and $y_n^* \geq 0 \;\; \forall \theta_n$.

Proof: The proof follows from first order conditions from the government's problem. See Appendix A.

The intuition behind this result stems from the fact that cost of formal child care is lower than agents' market productivity. Thus, should the government require an agent to exert any effort, it would rather have the agent work on the market than at home. Following this result, we now show that the implications for optimal policy in our second best world are in line with the standard optimal policy literature with production and benefits being non-decreasing in types.

Proposition 2: If the government cannot observe agents' types but can observe informal child care activities, y_n^* and b_n^* are non-decreasing in types θ_n.

Proof: The proof follows from incentive compatibility constraints. See Appendix A.

Thus, higher type agents, i.e. those who have lower disutility from exerting effort, are expected to produce and work more. In order to incentivate the higher type agents to work more, higher benefits and therefore higher consumption need to be allocated to them. We now turn to analysing the optimal policy in our third best world and show that there is an efficiency case for child care subsidies on formal child care cost.

Proposition 3: If the government can observe neither agents' types nor informal child care activities, the second best optimal policy may not be optimal anymore. Agents' private condition for engaging in informal child care activities:

$$\forall \theta_n, \theta_k \qquad u_c\left(b_k^* - \omega\left(1 - \hat{h}_n^k\right)\right)\omega - v_e\left(y_k^* + \hat{h}_n^k; \theta_n\right) \leq 0 \quad , \quad \hat{h}_n^k \geq 0$$

where $\hat{h}_n^k$ is unobserved informal child care activities undertaken by an agent of type θ_n who reports to be of type θ_k.

Proof: We first consider the agent's private problem to show that at the second best optimal allocations, the agents may have an incentive to deviate from the government's recommended level of informal child care of $h_n^* = 0$ and engage in unobserved informal child care activities. See Appendix A.

The intuition behind this result is that the opportunity to engage in unobserved informal child care activities exacerbates the incentive compatibility constraints for truthful reporting of types. In mathematical terms, consider the incentive compatibility constraints in our second best world (2) and in our third best world (4). Now suppose that the government fixes benefits and output at the second best optimal allocations $\{b_n^*, y_n^*\}_{\theta_1}^{\theta_N}$. From Proposition 2, we have $b_n^* \geq b_{n-1}^*$ and $y_n^* \geq y_{n-1}^*$ $\forall \theta_n$. By the concavity of $u(.)$ and convexity of $v(.)$, we therefore expect that the level of unobserved informal child care of type θ_n agent will be lower when reporting her true type as opposed to pretending to be of lower type θ_{n-1}: $\hat{h}_n^n \leq \hat{h}_n^{n-1}$. Thus, at the second best allocations, both the left and right hand sides of the incentive compatibility constraints when the government does not observe informal child care (4) will be higher than when the government observes informal child care (2). By the concavity of the agent's problem, we would also expect the right hand side of equation (4). to increase by an amount at least as high as the increase in the left hand side of equation (4) thereby making the incentive compatibility constraint even harder to satisfy due to joint deviation.

The option of engaging in unobserved informal child care activities therefore allows agents to have potentially higher utility when deviating from the truth. Thus, the possibility of joint deviation provides a case for child care subsidies on formal child care cost. Such subsidies would lower the cost of formal child care and therefore discourage agents from engaging in informal child care activities so that the incentive for joint deviation would be dampened. We show in Proposition 4 below that the incentive to engage in unobserved informal child care activities is very high when the cost of formal child care is high relative to the price of market production.

Proposition 4: If the government can observe neither agents' types nor informal child care activities, agents will deviate from the second best optimal recommended level of child care activities and engage in a strictly positive level of unobserved informal child care activities if the cost of formal child care is high relative to the price of market production:

$$lim_{\omega\to 1} \qquad u_c\left(b_n^* - \omega\right)\omega > v_e\left(y_n^*;\theta_n\right) \quad at \quad h_n^* = 0 \qquad \forall \theta_n$$

Proof: We firstly show that only the local downward incentive compatibility constraints are relevant. We subsequently re-analyse our first order conditions from the government's second best problem to show that if the cost of formal child care is high relative to the price of market production agents will engage in strictly positive unobserved informal child care activities. See Appendix A.

The higher the cost of formal child care, the higher the incentives of agent to engage in strictly positive unobserved informal child care activities and therefore the harder it is to satisfy the incentive constraint for truthful reporting of types. We therefore have a case for strictly positive child care subsidies when the cost of formal child care is high relative to the price of market production. We show in Proposition 5 below that sliding scale child care subsidies where higher child care subsidies are paid to lower types may implement the second best optimum.

3.2 Implementation

Proposition 5: If the government can observe neither agents' types nor informal child care activities, a sliding scale child care subsidies scheme implements the optimum *as though* the government could observe informal child care activities:

$$\forall \theta_n \quad s_n = 1 - \frac{1}{\omega} \frac{v_e\left(y_n^*; \theta_N\right)}{u_c\left(c_n^*\right)} \,,\; b_n^{\dagger} = c_n^* + \left(1 - s_n\right)\omega$$

where s_n are child care subsidies on formal child care cost, $b_n^{\dagger}$ are adjusted benefits offered to agents of type θ_n and c_n^* is the second best optimal level of consumption. This is conditional on cost of formal child care being verifyable.

Proof: We first show that if an agent of type θ_m has no incentive to provide informal child care when reporting to be of type θ_n, then a lower type θ_{m-p} agent also has no incentive to engage in informal child care when reporting to be of type θ_n. This implies that child care subsidies designed so that the highest type θ_N agent does not engage in informal child care activities ensures that all agents will also not engage in informal child care activities. We characterise the optimal sliding scale child care subsidies scheme where higher type agents receive lower child care subsidies. We show that such a scheme can implement the second best optimum by showing that sliding scale child care subsidies s_n complemented by a reduction in benefits b_n can yield the same expected utility level as in the second best optimum while at the same time satisfying the government budget constraint and incentive compatibility constraints. See Appendix A.

When the government does not observe informal child care activities, the incentive compatibility constraint (4) is harder to satisfy. Agents can jointly deviate by misreporting their type and engaging in informal child care activities. On the other hand, child care subsidies help reduce the cost of formal child care and thereby discourage involvement in informal child care activities.

We can implement the optimum *as though* the government could observe informal child care activities by (i) designing child care subsidies on formal child care cost such that the highest type agent will not engage in any informal child care activities and (ii) adjusting benefits such that agents get the same consumption levels as in our second best optimum.

Child care subsidies designed such that the highest type agent's optimal private choice of informal child care is always zero will ensure that all other agents' optimal private choice of informal child care is also always zero. This result stem from the fact that disutility from effort is decreasing in types. Thus, if the highest type agent has high enough marginal disutility from informal child care when claiming to be of a certain type θ_n so that the highest type agent chooses zero informal child care, then lower type agents will have even higher marginal disutilities from informal child care when claiming to be of type θ_n and therefore also choose zero informal child care. Since an agent's private problem is concave, such child care subsidies will prevent joint deviation by agents, i.e., agents pretending to be of a different type than their own will also not engage in unobserved informal child care activities.

From Proposition 2, we have shown that optimal second best benefits and therefore consumption levels are non decreasing in types. We thus have a sliding scale child care subsidy scheme on formal child care cost where higher subsidies are paid to the lower type agents. The child care subsidies are conditional on cost of formal child care being verifyable[2] and vary with the level of output produced by the agents. At the same time, benefits are adjusted such that agents receive the same consumption level as in second best. This yields the same expected utility level as in second best while satisfying the government budget constraint and incentive compatibility constraints.

4 Future Extensions

We have shown from our baseline model with heterogeneous disutility from effort that unobserved informal child care activities exacerbates agents work incentives. We show that there is an efficiency case for child care subsidies on formal child care cost and that sliding scale child care subsidies help implement the optimum *as though* the government could observe informal child care activities.

In our model extensions, we consider a special case of the model where agents have heterogeneous market productivities but homogeneous home productivities. Preliminary results indicate that in this particular case, there exists a threshold productivity level below which it is optimal for agents to stay at home and engage in child care activities and above which it is optimal for agents to work in the labour market. The efficiency case for child care subsidies still holds for agents who are working on the labour

[2] For example by presenting receipts from child care providers.

market. On the other hand, agents who do not work on the labour market get constant benefits. We also consider models incorporating endogeneous fertility decisions and general equilibrium effects from the child care market.

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A Appendix

A.1 Proof of Proposition 1

In our second best world, the government does not observe agents' types but can observe informal child care activities. The government maximises expected utility subject to government budget constraint and incentive compatibility constraints for truthful reporting of type:

$$max_{\{b_n,y_n,h_n\}_{n=1}^N} \sum_n p_n \left[u\left(b_n - \omega\left(1-h_n\right)\right) - v\left(y_n + h_n;\theta_n\right)\right]$$

s.t.

$$\sum_n p_n b_n = \sum_n p_n y_n$$

$$\forall \theta_n, \theta_k \quad u\left(b_n - \omega\left(1-h_n\right)\right) - v\left(y_n + h_n;\theta_n\right) \geq u\left(b_k - \omega\left(1-h_k\right)\right) - v\left(y_k + h_k;\theta_n\right)$$

First order conditions:

$$\begin{array}{lclcl}
b_n & : & p_n u'\left(b_n - \omega\left(1-h_n\right)\right) - \lambda p_n + \sum_k \gamma_{n,k} u'\left(b_n - \omega\left(1-h_n\right)\right) - \sum_k \gamma_{k,n} u'\left(b_n - \omega\left(1-h_n\right)\right) & = & 0 \\
y_n & : & -p_n v'\left(y_n + h_n;\theta_n\right) + \lambda p_n - \sum_k \gamma_{n,k} v'\left(y_n + h_n;\theta_n\right) + \sum_k \gamma_{k,n} v'\left(y_n + h_n;\theta_k\right) & \leq & 0 \\
h_n & : & p_n \left[u'\left(b_n - \omega\left(1-h_n\right)\right)\omega - v'\left(y_n + h_n;\theta_n\right)\right] + \sum_k \gamma_{n,k} \left[u'\left(b_n - \omega\left(1-h_n\right)\right)\omega - v'\left(y_n + h_n;\theta_n\right)\right] & & \\
 & & - \sum_k \gamma_{k,n} \left[u'\left(b_n - \omega\left(1-h_n\right)\right)\omega - v'\left(y_n + h_n;\theta_k\right)\right] & \leq & 0
\end{array}$$

where λ is the Lagrange multiplier for the government budget constraint and $\gamma_{n,k}$ is the Lagrange multiplier corresponding to the incentive compatibility constraint of an agent of type θ_n pretending to be of type θ_k.

Substituting the first order condition for b_n into the first order condition for h_n and rearranging:

$$\begin{array}{lclcl}
h_n & : & \lambda p_n \omega - p_n v'\left(y_n + h_n;\theta_n\right) - \sum_k \gamma_{n,k} v'\left(y_n + h_n;\theta_n\right) + \sum_k \gamma_{k,n} v'\left(y_n + h_n;\theta_k\right) & \leq & 0 \\
y_n & : & \lambda p_n - p_n v'\left(y_n + h_n;\theta_n\right) - \sum_k \gamma_{n,k} v'\left(y_n + h_n;\theta_n\right) + \sum_k \gamma_{k,n} v'\left(y_n + h_n;\theta_k\right) & \leq & 0
\end{array}$$

Since $\omega < 1$, the left-hand side of the first order condition for h_n is always strictly smaller than the left-hand side of the first order condition for y_n. Thus, it must be that $h_n^* = 0$ and $y_n^* \geq 0$ $\forall \theta_n$.

A.2 Proof of Proposition 2

Using the fact that from Proposition 1 $h_n^* = 0 \; \forall \theta_n$, we can write the respective incentive compatibility constraint for type θ_n pretending to be type θ_k and vice versa:

$$\begin{aligned} u(b_n^* - \omega) - v(y_n^*; \theta_n) &\geq u(b_k^* - \omega) - v(y_k^*; \theta_n) \\ u(b_k^* - \omega) - v(y_k^*; \theta_k) &\geq u(b_n^* - \omega) - v(y_n^*; \theta_k) \end{aligned}$$

Adding the two incentive constraints together and rearranging:

$$v(y_k^*; \theta_n) - v(y_n^*; \theta_n) \geq v(y_k^*; \theta_k) - v(y_n^*; \theta_k)$$

If $y_k \geq y_n$, we can rewrite the inequality:

$$\int_{y_n^*}^{y_k^*} v_e(y; \theta_n) \; dy \geq \int_{y_n^*}^{y_k^*} v_e(y; \theta_k) \; dy$$

which holds only if $\theta_k > \theta_n$ since $v_{e\theta}(y; \theta_n) < 0$ i.e., when θ_n increases, disutility from higher effort decreases. Thus, it must be that if $\theta_k > \theta_n$, then $y_k^* \geq y_n^*$. Moreover, from the incentive compatibility constraint for type θ_k pretending to be type θ_n, we must also have $b_k^* \geq b_n^*$.

A.3 Proof of Proposition 3

We showed in Proposition 1 above that in our second best world, the government recommends $h_n^* = 0$. Now suppose that the government cannot observe informal child care activities but still offers the second best optimal policy $\{b_n^*, y_n^*, h_n^*\}_{\theta_1}^{\theta_N}$. The private problem of an agent of type θ_n pretending to be of type θ_k:

$$max_{\{h\}} \; u(b_k^* - \omega(1-h)) - v(y_k^* + h; \theta_n)$$

The first order condition with respect to informal child care is

$$u_c\left(b_k^* - \omega\left(1 - \hat{h}_n^k\right)\right)\omega - v_e\left(y_k^* + \hat{h}_n^k; \theta_n\right) \leq 0$$

where $\hat{h}_n^k$ is the agent's private optimal level of unobserved informal child care. The agent will engage in unobserved informal child care activities so long that the private net return from informal child care is positive

$$u_c\left(b_k^* - \omega\right)\omega - v_e\left(y_k^*; \theta_n\right) > 0 \quad at \quad \hat{h}_n^k = 0$$

A.4 Proof of Proposition 4

We first show in Lemma 1 that in our second best world, only the local downward incentive compatibility constraints are relevant among the downward incentive constraints. We then show in Lemma 2 that the same holds true among upward incentive compatibility constraints and in Lemma 3, we show that only the local downward incentive compatibility constraints are relevant.

Lemma 1: In our second best world, if an agent of type θ_n weakly prefers truth telling and declaring to be of type θ_n rather than declaring to be type θ_{n-k}, then a higher type θ_{n+p} agent will also weakly prefer declaring to be of type θ_n rather than declaring to be of type θ_{n-k}. This implies that only the local downward incentive compatibility constraints are relevant among the downward incentive constraints:

$$\forall\theta_n \qquad u\left(c_n^*\right) + v\left(y_n^*; \theta_n\right) \geq u\left(c_{n-1}^*\right) + v\left(y_{n-1}^*; \theta_n\right)$$

where $c_n^* = b_n^* - \omega \ \forall\theta_n$.

Proof: The lemma states that if

$$u\left(c_n^*\right) - v\left(y_n^*; \theta_n\right) \geq u\left(c_{n-k}^*\right) - v\left(y_{n-k}^*; \theta_n\right)$$

then

$$u\left(c_n^*\right) - v\left(y_n^*; \theta_{n+p}\right) \geq u\left(c_{n-k}^*\right) - v\left(y_{n-k}^*; \theta_{n+p}\right)$$

In other words, if

$$\Delta u\left(c_n^*\right) \geq \Delta v\left(y_n^*; \theta_n\right)$$

then

$$\Delta u\left(c_n^*\right) \geq \Delta v\left(y_n^*, ; \theta_{n+p}\right)$$

For this statement to hold, it must be that

$$\Delta v\left(y_n^*;\theta_n\right) \geq \Delta v\left(y_n^*;\theta_{n+p}\right)$$

which is true under the assumption that $v_{e\theta} < 0$ and convexity of $v(.)$. Thus, one can eliminate non local downward incentive compatibility constraints by sequential reasoning so that only local downward incentive compatibility constraints are relevant among downward incentive constraints. QED.

Lemma 2: In our second best world, if an agent of type θ_n weakly prefers truth telling and declaring to be of type θ_n rather than declaring to be of type θ_{n+k}, then a lower type θ_{n-p} agent would also weakly prefer declaring to be of type θ_n rather than declaring to be of type θ_{n+k}. This implies that only the local upward incentive compatibility constraints are relevant among the upward incentive constraints:

$$\forall\theta_n \qquad u\left(c_n^*\right) + v\left(y_n^*;\theta_n\right) \geq u\left(c_{n+1}^*\right) + v\left(y_{n+1}^*;\theta_n\right)$$

where $c_n^* = b_n^* - \omega \; \forall\theta_n$.

Proof: The lemma states that if

$$u\left(c_n^*\right) - v\left(y_n^*;\theta_n\right) \geq u\left(c_{n+k}^*\right) - v\left(y_{n+k}^*;\theta_n\right)$$

then

$$u\left(c_n^*\right) - v\left(y_n^*;\theta_{n-p}\right) \geq u\left(c_{n+k}^*\right) - v\left(y_{n+k}^*;\theta_{n-p}\right)$$

In other words, if

$$\Delta u\left(c_{n+k}^*\right) \leq \Delta v\left(y_{n+k}^*;\theta_n\right)$$

then

$$\Delta u\left(c_{n+k}^*\right) \leq \Delta v\left(y_{n+k}^*,;\theta_{n-p}\right)$$

For this statement to hold, it must be that

$$\Delta v\left(y_{n+k}^*;\theta_n\right) \leq \Delta v\left(y_{n+k}^*,;\theta_{n-p}\right)$$

which is true under the assumption that $v_{e\theta} < 0$ and convexity of $v(.)$. Thus, one can eliminate non local upward incentive compatibility constraints by sequential reasoning so that only local upward incentive compatibility constraints are relevant among upward incentive constraints. QED.

Lemma 3: Only the local downward incentive compatibility constraints bind:

$$\forall\theta_n \qquad u\left(b_n^* - \omega\right) + v\left(y_n^*; \theta_n\right) = u\left(b_{n-1}^* - \omega\right) + v\left(y_{n-1}^*; \theta_n\right)$$

Proof: Consider the set of incentive compatibility constraints

$$\begin{array}{rcl} u\left(b_N^* - \omega\right) - v\left(y_N^*; \theta_N\right) & \geq & u\left(b_{N-1}^* - \omega\right) - v\left(y_{N-1}^*; \theta_N\right) \\ u\left(b_{N-1}^* - \omega\right) - v\left(y_{N-1}^*; \theta_{N-1}\right) & \geq & u\left(b_N^* - \omega\right) - v\left(y_N^*; \theta_{N-1}\right) \\ u\left(b_{N-1}^* - \omega\right) - v\left(y_{N-1}^*; \theta_{N-1}\right) & \geq & u\left(b_{N-2}^* - \omega\right) - v\left(y_{N-2}^*; \theta_{N-1}\right) \\ u\left(b_{N-2}^* - \omega\right) - v\left(y_{N-2}^*; \theta_{N-2}\right) & \geq & u\left(b_{N-1}^* - \omega\right) - v\left(y_{N-1}^*; \theta_{N-2}\right) \\ \vdots & \vdots & \vdots \end{array}$$

It must be that the incentive compatibility constraint of an agent of type θ_N pretending to be of type θ_{N-1} is binding or else, one can decrease b_N^* slightly and the government will enjoy some cost savings which can be reallocated to lower type agents[3] while incentives are still preserved.

Now, consider the incentive compatibility constraint for an agent of type θ_N pretending to be of type θ_{N-1} and vice versa

$$\begin{array}{rcl} u\left(b_N^* - \omega\right) - v\left(y_N^*; \theta_N\right) & = & u\left(b_{N-1}^* - \omega\right) - v\left(y_{N-1}^*; \theta_N\right) \\ u\left(b_{N-1}^* - \omega\right) - v\left(y_{N-1}^*; \theta_{N-1}\right) & \geq & u\left(b_N^* - \omega\right) - v\left(y_N^*; \theta_{N-1}\right) \end{array}$$

That is

$$\begin{array}{rcl} \Delta u\left(b_N^* - \omega\right) & = & \Delta v\left(y_N^*, \theta_N\right) \\ \Delta u\left(b_N^* - \omega\right) & \leq & \Delta v\left(y_N^*, \theta_{N-1}\right) \end{array}$$

[3] By the concavity of $u(.)$ and the fact that lower type agents receive lower benefits from Proposition 2, such reallocation will lead to an overall increase in welfare.

By convexity of $v(.)$ and the fact that $v_{e\theta} < 0$, we know that

$$\Delta v\left(y_N^*, \theta_N\right) < \Delta v\left(y_N^*, \theta_{N-1}\right)$$

Thus, it must be that

$$\Delta u\left(b_N^* - \omega\right) < \Delta v\left(y_N^*, \theta_{N-1}\right)$$

implying that the incentive compatibility constraint for an agent of type θ_{N-1} pretending to be of type θ_N is not binding.

The set of relevant incentive compatibility constraints now become

$$\begin{array}{rcl} u\left(b_N^* - \omega\right) - v\left(y_N^*; \theta_N\right) & = & u\left(b_{N-1}^* - \omega\right) - v\left(y_{N-1}^*; \theta_N\right) \\ u\left(b_{N-1}^* - \omega\right) - v\left(y_{N-1}^*; \theta_{N-1}\right) & \geq & u\left(b_{N-2}^* - \omega\right) - v\left(y_{N-2}^*; \theta_{N-1}\right) \\ u\left(b_{N-2}^* - \omega\right) - v\left(y_{N-2}^*; \theta_{N-2}\right) & \geq & u\left(b_{N-1}^* - \omega\right) - v\left(y_{N-1}^*; \theta_{N-2}\right) \\ \vdots & \vdots & \vdots \end{array}$$

By the same line of argument, we can argue that incentive compatibility constraint for an agent of type θ_{N-1} pretending to be of type θ_{N-2} must be binding or else the planner can decrease b_{N-1}^* and b_N^* and reallocate the cost savings to lower type agents while preserving incentives. Then, by convexity and the fact that $v_{e\theta} < 0$, if the local downward incentive compatibility constraint is binding, the local upward incentive compatibility constraint cannot be binding. Thus, one can eliminate local upward incentive compatibility constraints by sequential reasoning so that only local downward incentive compatibility constraints remain relevant. QED.

We now proceed to show that should the government not observe informal child care activities as in our third best world, agents will deviate from the second best optimal recommended level of child care activities and engage in strictly positive level of unobserved informal child care activities if the cost of formal child care is high relative to the price of market production.

Consider the first order conditions in our second best world:

$$\begin{aligned} b_n &: \left[1 + \tfrac{\gamma_{n-1}}{p_n} - \tfrac{\gamma_{n+1}}{p_n}\right] u_c'\left(b_n^* - \omega\right) &=& \quad \lambda \\ y_n &: \left[1 + \tfrac{\gamma_{n-1}}{p_n} - \tfrac{\gamma_{n+1}}{p_n}\right] v_e\left(y_n^*; \theta_n\right) &=& \quad \lambda + \tfrac{\gamma_{n+1}}{p_n}\left[v_e\left(y_n^*; \theta_{n+1}\right) - v_e\left(y_n^*; \theta_n\right)\right] \end{aligned}$$

Since $v_e\left(y_n^*; \theta_{n+1}\right) - v_e\left(y_n^*; \theta_n\right) < 0$ by the fact that $v_{e\theta} < 0$, we can see that

$$u_c\left(b_n^* - \omega\right) > v_e\left(y_n^*; \theta_n\right)$$

which indicates that y_n^* is distorted downwards as compared to a full information setting. If the price of formal child care is high relative to the price of market production such that $\omega \backsimeq 1$, then we also have

$$u_c\left(b_n^* - \omega\right)\omega > v_e\left(y_n^*; \theta_n\right)$$

so that agents have an incentive to engage in a strictly positive level of unobserved informal child care activities.

A.5 Proof of Proposition 5

We first show in Lemma 4 that if an agent of type θ_m has no incentive to provide informal child care when reporting to be of type θ_n, then a lower type θ_{m-p} agent also has no incentive to engage in informal child care when reporting to be of type θ_n. This lemma implies that child care subsidies designed so that the highest type agent θ_N does not engage in informal child care activities ensures that all agents will also not engage in informal child care activities. We then proceed to characterise the optimal child care subsidies and discuss an implementation where it is as though the government could observe informal child care activities.

Lemma 4: If $\hat{h}_m^n = 0$, then $\hat{h}_{m-p}^n = 0$.

Proof: Consider the private first order conditions with respect to informal child care of an agent of type θ_m claiming to be of type θ_n and of an agent of type θ_{m-p} claiming to be of type θ_n respectively:

$$\begin{array}{rcl} u_c\left(b_n - \omega\left(1-\hat{h}^n_m\right)\right)\omega & \leq & v_e\left(y_n + \hat{h}^n_m;\theta_m\right) \quad , \quad \hat{h}^n_m \geq 0 \\ u_c\left(b_n - \omega\left(1-\hat{h}^n_{m-p}\right)\right)\omega & \leq & v_e\left(y_n + \hat{h}^n_{m-p};\theta_{m-p}\right) \quad , \quad \hat{h}^n_{m-p} \geq 0 \end{array}$$

The lemma states that if

$$u_c\left(b_n - \omega\right)\omega < v_e\left(y_n;\theta_m\right) \ , \ \hat{h}^n_m = 0$$

then

$$u_c\left(b_n - \omega\right)\omega < v_e\left(y_n;\theta_{m-p}\right) \ , \ \hat{h}^n_{m-p} = 0$$

For this statement to hold, it must be that

$$v_e\left(y_n;\theta_m\right) < v_e\left(y_n;\theta_{m-p}\right)$$

which is true since $v\left(.\right)$ is convex and $v_{e\theta} < 0$. QED

Thus, if one were to design child care subsidies such that the highest type θ_N agent has no incentive to engage in informal child care activities when declaring to be of type $\theta_n \in \{\theta_1, ..., \theta_N\}$, all other type agents will also have no incentive to engage in informal child care activities. Child care subsidies can therefore be designed such that the highest type θ_N agent's first order condition at zero informal child care is satisfied with strict equality:

$$u_c\left(b_n - \left(1-s_n\right)\omega\right)\left(1-s_n\right)\omega = v_e\left(y_n;\theta_N\right) \quad , \quad \hat{h}^n_N = 0$$

where s_n are child care subsidies on formal child care cost given to an agent who reports to be of type θ_n. Such subsidies will also at the same time ensure that for lower type agents $\theta_n \in \{\theta_1, ..., \theta_{N-1}\}$, their first order condition at zero informal child care is satisfied with strict inequality since $v\left(.\right)$ is convex and $v_{e\theta} < 0$:

$$u_c\left(b_n - \left(1-s_n\right)\omega\right)\left(1-s_n\right)\omega < v_e\left(y_n;\theta_n\right) \quad , \quad \hat{h}^n_n = 0$$

Since agents' private utility maximisation problem is concave, such subsidies also ensure that there will be no joint deviation anymore, i.e., agents pretending to be of a different type than their own will also not engage in unobserved informal child care activities. Thus, using the highest type agent's private first order condition with respect to informal child care, we can characterise the optimal child care subsidy scheme that induces agents to provide the second best optimal level of informal child care $\hat{h}_n^k = h_n^* = 0 \quad \forall \theta_n, \theta_k$. At the second best optimal consumption and output levels $\{c_n^*, y_n^*\}_{\theta_1}^{\theta_N}$, we can therefore get a child care subsidies scheme:

$$\forall \theta_n \quad s_n = 1 - \frac{1}{\omega} \frac{v_e\left(y_n^*; \theta_N\right)}{u_c\left(c_n^*\right)} \ , \ b_n^{\dagger} = c_n^* + \left(1 - s_n\right)\omega$$

where s_n are child care subsidies on formal child care cost, $b_n^{\dagger}$ are adjusted benefits offered to agents of type θ_n and c_n^* is the second best optimal level of consumption. The subsidies are sliding scale since from Proposition 2, we know that b_n^* and y_n^* are non decreasing in types. It follows therefore that $c_n^* = b_n^* - \omega$ is also non decreasing in types. By the concavity of $u\left(.\right)$ and convexity of $v\left(.\right)$, s_n is non increasing in types. Thus, we have sliding scale child care subsidies on formal child care cost such that lower type agents get higher child care subsidies.

We now show that such a subsidy scheme yield the same expected utility level as in the second best optimum while at the same time satisfying the government budget constraint and incentive compatibility constraints.

Families get the same expected utility since they get the same consumption levels as in second best:

$$\underbrace{\sum_n p_n \left[u\left(b_n^{\dagger} - \left(1 - s_n\right)\omega\right) - v\left(y_n^*; \theta_n\right)\right]}_{\textit{Sliding Scale Subsidy Scheme}} = \underbrace{\sum_n p_n \left[u\left(b_n^* - \omega\right) - v\left(y_n^*; \theta_n\right)\right]}_{\textit{Second Best Optimal Policy}}$$

where $c_n^* = b_n^{\dagger} - \left(1 - s_n\right)\omega = b_n^* - \omega \ \forall \theta_n$.

The government budget constraint is also satisfied:

$$\underbrace{\sum_n p_n \left(b_n^{\dagger} + s_n \omega\right) = \sum_n p_n y_n^*}_{\textit{Sliding Scale Subsidy Scheme}} \iff \underbrace{\sum_n p_n b_n^* = \sum_n p_n y_n^*}_{\textit{Second Best Optimal Policy}}$$

Since the child care subsidies ensure that agents do not engage in unobserved informal child care activities, incentive compatibility constraints for truthful reporting of type is also satisfied:

$$\forall \theta_n, \theta_k \qquad \begin{array}{c} \overbrace{u\left(b_n^{\dagger} - (1 - s_n)\,\omega\right) - v\left(y_n^*; \theta_n\right) \geq u\left(b_k^{\dagger} - (1 - s_k)\,\omega\right) - v\left(y_k^*; \theta_n\right)}^{\textit{Sliding Scale Subsidy Scheme}} \\ \Longleftrightarrow \\ \underbrace{u\left(b_n^* - \omega\right) - v\left(y_n^*; \theta_n\right) \geq u\left(b_k^* - \omega\right) - v\left(y_k^*; \theta_n\right)}_{\textit{Second Best Optimal Policy}} \end{array}$$