

# Fiscal Sustainability, Default Risk and Euro Area Sovereign Bond Spreads\*

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## Abstract

This paper develops an arbitrage-free affine term structure model of potentially defaultable sovereign bonds to model a cross-section of eight euro area government bond yield curves since January 1999. The existence of a common monetary policy under European Monetary Union determines the short end of the yield curves, whereas decentralized debt policies drive expected default probabilities and thereby spreads towards Germany, assumed to be free of default risk. The pricing factors are three observable area-wide macroeconomic variables and measures of national fiscal sustainability, which we proxy by expected changes in debt/GDP ratios of the respective countries. Our model explains spreads both before and during the crisis to an impressive extent. The deterioration in public finances was the major driver of the widening in spreads since 2008 through both heightened compensations for default risk and increases in risk premia. We also present perceived probabilities of sovereign default at any maturity and assess their elasticity to shifts in expected changes in debt/GDP ratios.

*JEL classification:* E6, H6.

*Keywords:* Government debt, affine term structure models, default risk, yield spreads, fiscal projections.

# 1 Introduction

The question of the effects of fiscal policies on interest rates has for decades been a central aspect of macroeconomic debates about the effects of fiscal policies on resource allocation more broadly. Since the onset of the sovereign debt crisis in the euro area in late 2009, the nexus between fiscal sustainability and interest rates has moved beyond the academic debates to the center of financial market attention and subsequently of macroeconomic policy in Europe as well. Whereas the first wave of widening yield spreads of euro area government bonds vis-à-vis German bonds following the Lehman event was largely synchronous (even though of different magnitude) across most euro area countries, the evolution of spreads over the past two years (as of this writing) was substantially differentiated, as financial markets have been rife with speculation about the possibility of a sovereign default by one or more member countries of the euro area. At the heart of these events are investors' doubts whether fiscal policy makers can deliver policies, including the stabilization of their financial sectors, that are consistent with government solvency. In light of this dramatic break in yield spreads since 2008, a key question is to what extent this can be explained by changes in investors' assessment of sustainability of member countries' national fiscal policies, as opposed to a change in the pricing behaviour of bond-market investors in the wake of the financial crisis. The answer to this question is important because of its implications for the predicted effects on the yield curve of member countries' current efforts to restrain the expected path of government debt.

In this paper, we use recent advances in term structure modeling to explain the evolution of euro area sovereign yield spreads with the goal of understanding the role of macroeconomic variables and especially of fiscal policies in determining yield spreads across countries and maturities, both before and since the onset of the financial crisis.<sup>1</sup> Specifically, we jointly model the zero-coupon yield curve of government bonds of Germany plus yield spreads of government bond yields of seven other euro area countries within

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<sup>1</sup>From the perspective of academic research, the events discussed above play out against the backdrop of intense efforts over the past decade to arrive at a better understanding of the macroeconomic determinants of asset prices in general, and of the linkages between the term structure of interest rates and the macroeconomy in particular. Gürkaynak and Wright (2011) provide an up-to-date survey of research on the term structure from a macroeconomic perspective.

an affine term structure model, assuming that bonds of the other euro area countries are potentially defaultable, and using only interpretable macroeconomic variables as factors. The probabilities of default perceived by investors are linked to the same macroeconomic fundamentals that drive yields. We estimate this model using a data set of government bond yields of those eight countries covering the period from the beginning of stage three of European Monetary Union (EMU) in January 1999 to June 2011. For illustration, the yield spreads at the 5-year maturity are shown in Figure 1.

Our key findings are: (i) A small set of macroeconomic variables can fit the term structures of all eight countries remarkably well, without assuming any break in the pricing behaviour of investors; (ii) model-implied expectations of future yield spreads and future changes in debt/GDP coincide closely with survey-based measures, thus suggesting that both historical and risk-neutral factor dynamics are accurately estimated; (iii) yield spreads vis-à-vis German yields since the onset of the crisis are overwhelmingly explained by the expected change in a country's debt/GDP ratio; the common area-wide factors explain little of the variation in spreads; and (iv) when we decompose spreads into contributions from risk premia, i.e. extra returns demanded by risk-averse investors for term and default risk, and contributions from default risk compensation that would be demanded by risk-neutral investors, the former contributions are moderately larger than the latter, underlining the importance of modelling risk premia in a term-structure framework.

Although current events lend urgency to a better understanding of the determinants of euro area sovereign yield spreads, these spreads have already been studied extensively; we discuss the related literature in the following subsection. Euro area sovereign yield curves are particularly interesting because they allow us to study the macroeconomic determinants of credit spreads. Under the assumption (maintained throughout our study) that the probability of a country leaving the euro is considered nil, expectations of future short-term interest rates are identical across countries and exchange-rate risk is not priced. Hence our data set allows us to focus on the pricing of credit risk in relation to the common monetary policy on the one hand and country-specific fiscal policy on the other.

The literature in this area has mostly focused on regressions of yield spreads of other euro area members vis-à-vis Germany at a specific maturity on country-specific variables such as fiscal variables, proxies for liquidity (such as size of the outstanding debt), proxies

for time-varying risk aversion (as captured e.g. by private credit spreads) etc. We share with this literature the focus on fiscal variables (in addition to macroeconomic determinants of the common short-term interest rate) as explanatory variables. We depart from these earlier studies by estimating a multi-country affine term structure model, thereby using the entire cross-sectional information in the term structure by imposing the restrictions implied by ruling out arbitrage across maturities and borrowers, and by allowing for the interaction between macroeconomic variables and prices of risk. We thereby arrive at a much richer interpretation of the determination of yield spreads than could be obtained by regression-based methods.

Within the finance literature on the term structure, our paper is the first application of an affine term structure model of defaultable bonds to euro area yield curves that uses macroeconomic variables as factors.<sup>2</sup> Linking the term structures to macroeconomic variables is challenging for several reasons. First, given the high persistence of yields, our sample is very short, spanning only two interest rate cycles. Estimates of the historical dynamics of the short-term interest rate and other macro variables would thus be expected to be very imprecise. We address this problem by using a range of survey data on expectations in estimation.

The still short sample period since 1999 is also problematic because we are interested in the influence of fiscal variables that are usually published only once a quarter, and even then are often based in part on interpolation of annual information. In order to have enough observations for the estimation of the term structure model, we rely on monthly data, which is a compromise between the higher-frequency yield data and the lower-frequency macro data. Using national accounts data interpolated to monthly frequency is problematic because of econometric concerns about smoothing. We therefore rely in our estimation on the Kalman filter to extract our national fiscal variables from

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<sup>2</sup>This use of macroeconomic variables as factors is not uncontroversial. Duffee (2009) and Joslin et al. (2010) document the importance of unspanned macro risks, i.e. that current macroeconomic variables cannot be recovered from current yields, but that macroeconomic variables nonetheless can affect future yield curves through their impact on expected future short-term interest rates. We nonetheless prefer to use macroeconomic variables directly as factors rather than the method proposed by Joslin et al. (2010), because the former facilitates the interpretation of the fitted yields, spreads and premia in terms of contributions from the macroeconomic variables rather than from latent pricing factors.

the relatively scarce observed fiscal information, constraining the estimated variables to be close to their observed counterpart whenever available.<sup>3</sup> Besides, since we estimate the complete model in one step by maximum likelihood methods, we can make use of the information contained in spreads when extracting the fiscal factors. Our national fiscal variables are then neither purely latent pricing factors nor purely observable fiscal variables. For this reason, we label them “pseudo-observable” factors and interpret them as reflecting the perception by investors of the countries’ fiscal sustainability. We present evidence that these estimated factors correlate in many cases closely with publicly available projections for the same variables.

A final problem is related to two fundamental assumptions within the class of affine term structure models. First is the assumption that the factors are exogenous to the yields. In our case, this precludes both a direct feedback from yields to the expected change in government debt (the “snowball effect”) and more fundamentally the possibility of multiple equilibria (as analyzed e.g. by Calvo, 1988). Second, the model also assumes that each issuer is a risk by itself and unrelated to the default events of other borrowers, except insofar as the default probabilities may be driven by common factors or correlations among shocks to the fiscal factors. This assumption precludes contagion effects in the form of one country’s default raising the perceived default probability of others. It also may have become questionable due to the introduction of the EFSF in May 2010 as well as the effects of the ECB’s Securities Markets Programme. For these reasons, we present results for a sample which excludes Greek, Irish and Portuguese yields after the announcement of an EFSF program for the respective country.<sup>4</sup>

We first conduct some preliminary regression analysis to check whether our selected set of macro variables effectively spans the intra-EMU spread curves. The term structure model of multiple defaultable issuers that we then estimate using these variables as pricing factors appears to fit yields and spreads for all countries and a wide range of maturities impressively well, despite the fact that we do not use pure latent factors in our term

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<sup>3</sup>Allowing for some discrepancy may be justified notably by the fact that our observed data are revised data that differ from the information set available in real time.

<sup>4</sup>A further issue is that it is not clear to what extent trade took place at some of the most extreme price quotes for these three countries.

structure model and restrict ourselves to three euro area-wide macro factors and, for each country, one fiscal factor. Having an affine term structure model of euro area spreads allows us then to analyze these spreads in several interesting directions. We notably make use of this model to derive estimates of perceived sovereign default probabilities for each country over the period of study and provide estimates of the sensitivity of spreads at different maturities and of default probabilities at different horizons to the expected future fiscal policy.

In the following section we review related literature. In section 3 we present some exploratory results using OLS regressions. Section 4 describes the affine term structure model. Section 5 presents our results. Section 6 concludes. Details on the data and the model specification are in appendices.

## 2 Relation to the literature

As mentioned earlier, a large empirical literature has studied government bond spreads in the euro area since the beginning of European monetary union in 1999 with the goal of identifying the determinants of sovereign spreads in the absence of exchange-rate risk. Many of these studies rely on regressions of yield spreads at certain maturities on candidate explanatory variables. A common finding in this literature, beginning with Codogno et al. (2003) and Bernoth et al. (2006) and including more recent studies such as Manganelli and Wolswijk (2009), Haugh et al. (2009) and Schuknecht et al. (2010), is that euro area sovereign yield spreads seem to strongly comove. Principal component analysis regularly reveals that the first principal component accounts for more than 80% in the total variation of yield spreads. This finding suggests that a common factor, frequently interpreted as time-varying risk aversion of international investors that affects all yield spreads through the repricing of given country-specific risk characteristics, is the dominant force, making it difficult *a priori* to identify the role of country-specific variables such as fiscal policies in the determination of spreads. Laubach (2011), however, presents evidence that the strength of comovement among yield spreads varies substantially over time and has weakened since 2009.

Because the aforementioned studies do not use term structure models to model risk

premia, yield spreads can only be explained as compensation for either liquidity risk or default risk (under the maintained assumption that investors assign zero probability to the event of a member country leaving the euro). How to distinguish between these two interpretations of spreads has been a source of disagreement in the literature. Although since the eruption of the Greek fiscal crisis in November 2009 it seems plausible that default risk has been the dominant market concern, the relative importance of liquidity versus default risk is less clear during the first ten years of EMU. In their early study based on four years of monthly data, Codogno et al. (2003) concluded that “the risk of default is a small but important component of yield differentials” while liquidity factors seemed to be of lesser importance (see also Monfort and Renne, 2011).

Several recent studies conclude that the importance of liquidity risk seems to vary over time with proxies of international investor risk aversion. Beber et al. (2009), using intraday European bond quotes from the period April 2003 to December 2004, find that differences in credit quality among countries play a major role, but that “in times of market stress, investors chase liquidity, not credit quality.” By contrast, Favero et al. (2010) conclude that the interaction between liquidity demand and risk is *negative*. They attribute the difference between their results and those of Beber et al. to the fact that Beber et al. “control for country-specific risk but do not consider aggregate risk factors.” In pooled regressions of quarterly spread data for ten euro area countries including an interaction term between a proxy for risk aversion and the volume of bonds outstanding (as proxy for liquidity) as well as various fiscal variables to account for credit risk, Haugh et al. (2009) find a significant role for liquidity in line with the sign of Beber et al., with liquidity (or lack thereof) making a large contribution to the spreads of Irish and Finnish government bonds in late 2008 and early 2009.<sup>5</sup> While we do not deny that liquidity risk may in some instances and for some countries (those with small size of debt outstanding relative to euro area sovereign debt overall) have a sizeable role to play, we interpret the results from this literature as pointing more consistently to an important role of credit risk factors emanating from public finances, and therefore concentrate on those. This view is furthermore vindicated by the fact that we mainly focus on countries whose debt

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<sup>5</sup> Aßmann and Boysen-Hogrefe (2010) model time-varying risk aversion as a latent variable and conclude, similar to Beber et al. (2009) and Haugh et al. (2009), that liquidity matters in times of stress.



markets are generally considered large and liquid, as it is the case for the biggest four euro-area countries or for a largely indebted but nevertheless “core” EMU-country like Belgium. Admittedly, this claim is weaker concerning Greece, Portugal and Ireland, but we think that it would be difficult to argue that concerns about fiscal sustainability in these countries were not the key driver of the surge in their bond yields over the past two years.

We depart from the literature discussed so far by using a no-arbitrage term structure model so as to exploit the information contained in the entire maturity spectrum of yield spreads. Not only can we multiply the number of observations used in the analysis, we can also sharpen the conclusions regarding the determinants of yield spreads by estimating their effects on bonds of different maturities. The “essentially affine” class of term structure models that we use was first proposed by Duffee (2002) as a special case of affine term structure models. Beginning with the work of Ang and Piazzesi (2003), a growing literature has explored the role of macroeconomic variables as factors. A recent application to European data is Lemke (2008), who estimates a model with only observable macroeconomic factors for German bond yields. For U.S. data, Dai and Philippon (2006) and Laubach (2011) include fiscal variables among the factors.

In order to study the role of default risk in determining yield *spreads*, we employ the extension of affine term structure models to defaultable bonds proposed by Duffie and Singleton (1999). Geyer et al. (2004) provide an early application of such a model to euro area spreads, without, however, including macroeconomic variables as factors.<sup>6</sup> More recently, Monfort and Renne (2011) generalize this model to account for regime switching and both default risk and liquidity factors and apply this framework to jointly model a swap yield curve, ten sovereign yield curves and a German agency yield curve, using latent factors. Unlike our study, they allow for a wide range of possible parameter shifts, including changes in factor volatilities. By contrast, we are interested in exploring the possibility that the behavior of spreads can be explained by macroeconomic fundamentals without having to resort to parameter shifts.

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<sup>6</sup> Amato and Luisi (2006) is to our knowledge the first use of an affine term structure model of defaultable bonds with macroeconomic variables as factors, but applied to U.S. corporate bond spreads.

### 3 Fiscal sustainability and euro-area sovereign bond yields: preliminary evidence

Affine term structure models rely on the assumption that linear relations hold between bond yields and the factors that drive the yield curve. When the factors are observable macro variables, simple OLS regressions can provide useful insights about the set of variables that are likely to span the curve of each country, as shown in Dai and Philippon (2006). Following these authors, we present in this section the results of regressions of bond spreads against Germany of several maturities on measures of the fiscal stance and area-wide macroeconomic controls for each of the seven “potentially defaultable” countries in our sample. The idea is that if, for any given country, a given fiscal variable fails to explain significantly bond spreads of different maturities in simple reduced-form regressions, then there is no point including this variable as a factor in our more sophisticated (and heavily constrained) no-arbitrage multicountry term structure model.

For each of the seven countries, we use monthly observations of spreads at 1, 5 and 10 years maturities, that we regress on the 1-month risk free short term rate, a measure of global volatility in financial markets, a monthly indicator for the position in the euro-area business cycle, and a measure of national fiscal balances. Appendix A details the sources and methodology for computing the zero-coupon yields used throughout this study. The short term rate is measured using prices of 1-month OIS swaps rather than euro area interbank rates, which have been obviously comprising a certain amount of premia for credit and liquidity risks since the start of the financial crisis in August 2007.<sup>7</sup> We take the (log of) the Chicago Board VIX to gauge global financial volatility, while Eurostat’s business confidence indicator, ESI, is used as a measure of the euro area business cycle.

The appropriate choice of the most relevant measure of fiscal balances at the national level is a matter of debate. Previous studies frequently consider the deficit to GDP or the debt to GDP ratios, or forecasts thereof (see e.g. Codogno et al., 2003, for the euro area and Laubach, 2009, Dai and Philippon, 2006, for the US). Bernoth et al. (2006) argue that debt service (defined as the ratio of gross interest payments to current government revenue)

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<sup>7</sup>See for instance Eisenschmidt and Tapking (2009) for an analysis of the related premia. See also Taylor and Williams (2009) for an analysis of the US case.

is more appropriate when trying to assess the impact of fiscal balances on euro area bond yields, if only because governments have less incentive to manipulate it than other measures that are used officially to monitor whether national fiscal positions meet the obligations set out by the Stability and Growth Pact. Furthermore, Haugh et al. (2009) find that both fiscal deficit and debt service help to explain a substantial part of cross-sectional variations in euro area bond yields during the early stages of the financial crisis. Either one of these three measures suffers potentially from an endogeneity problem. In practice, as long as the average maturity of countries' debt is not too short, so that the share of total debt that needs to be refinanced each period is contained, the contemporaneous effect of changes in interest rates on either the deficit/GDP ratio or the debt service ratio is rather modest.<sup>8</sup>

In our study we decide to use the change expected in the debt/GDP ratio over the next 12 months as our measure of fiscal sustainability. We need a measure that is both forward-looking and sufficiently persistent to capture the persistent dynamics of bond yields. The debt/GDP ratio itself might be problematic due to very persistent downward trends during the pre-crisis part of the sample in some of the countries. Insofar as investors anticipated these trends in highly indebted countries as part of a convergence process under EMU, the initially high debt levels in these countries may not have been perceived as signalling an unsustainable fiscal position. This assessment would, however, be reflected in the *change* of the debt/GDP ratio. We focus on the expected change over the next 12 months to take account of the forward-looking behavior of investors and to smooth through high-frequency noise. That said, we freely admit that in the context of the financial crisis, it is particularly difficult to decide which aspect of fiscal balances is the most relevant in investors' assessment of sustainability.<sup>9</sup> We take the data on actual (as opposed to

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<sup>8</sup>Gross debt issuance in 2010 ranged from between 8 and 10 percent of GDP for Germany, France and Spain, to nearly 17 percent for Italy and Greece. An increase in the spread of 100 bps for a country that needs to refinance debt in the amount of 10 percent of GDP would add in the same year at most 0.1 percent of GDP to the deficit. The endogeneity problem has become quantitatively important for Greece, Ireland and Portugal towards the end of the sample, which is one reason why we exclude these countries from the time of the announcement of their respective EFSF programs.

<sup>9</sup>This would be especially true for countries perceived to have large contingent liabilities towards their

expected) debt/GDP from the OECD Economic Outlook database, which is published semi-annually and provides data at quarterly frequency. For the needs of the preliminary regressions conducted in this section, we interpolated these quarterly series using simple cubic splines. However, in the subsequent estimation of our affine term structure model, the monthly fiscal variables are extracted from available quarterly information in a more satisfying manner using the Kalman filter, as detailed in section 4.1 below.

Tables 1 and 2 present the results of these preliminary regressions of spreads on our set of macro variables. First, the annual change in the debt/GDP ratio has a significant positive impact on spreads for most countries, notably at longer maturities. Using this variable to fit our pseudo-observable fiscal factors in the fully-fledged model is thus vindicated. Second, global financial uncertainty also impacts spreads positively, as intuition would suggest. Finally, short term rates are negatively correlated with spreads, while the area-wide business cycle is positively correlated, at least for four countries and at some maturities. Overall, based on these preliminary results, we decided to use (model-implied forecasts of) the annual change in the debt/GDP ratio as our best measure of fiscal balances in the following.<sup>10</sup>

## 4 An affine term structure model of defaultable bonds

### 4.1 Dynamics of the pricing factors under the historical measure

Let  $r$  denote the number of countries, and  $n_y$  the number of maturities of zero-coupon yields per country that we try to match. Let  $n_x$  denote the number of observable factors, and  $n_f$  the number of latent factors that explain the  $n_y \cdot r$  yields.

Time is discrete and is measured in months. The vector of observable factors  $x_t$  consists of three variables ( $n_x = 3$ ),

$$x_t = [y_t^1, v_t, z_t]$$

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financial sectors.

<sup>10</sup>Note that since these model-implied forecasts of the trend in the debt/GDP ratios are proportional to the current values due to the restrictions in the historical VAR below, the conclusions from the preliminary regressions remain valid.

where  $y_t^1$  denotes the short (one-period) riskfree nominal interest rate that is common to all  $r$  countries (specifically the one-month EONIA overnight index swap rate),  $v_t$  the Chicago Board's Options Exchange Market Index (henceforth VIX) measuring implied volatility of S&P 500 index options, and  $z_t$  the European Commission's Economic Sentiment Indicator (henceforth ESI) for the euro area. In addition to the observed factors, there is one unobserved factor for each country except Germany (so  $n_f = r - 1$ ). As we explain below, these latent factors are filtered in such a way that they can be interpreted as the country's expected change in the ratio of gross general government debt to GDP over the 12 months to come. Let  $f_t$  denote the  $(r - 1) \times 1$  vector  $[f_{2,t}, \dots, f_{r,t}]$  of unobserved fiscal factors (where country 1 is set to be Germany).

In general, the dynamics of the observed and the latent factors could be described by an unrestricted VAR with  $p$  lags. However, since the factor dynamics have to be estimated under both the historical and the risk-neutral measures, an excessively large number of parameters to be estimated is a major concern.<sup>11</sup> For reasons of parsimony, we therefore assume that the latent factors follow AR(1) processes, and that the observed factors follow a joint VAR(1). Let  $X_t = [f_t', x_t']'$  denote the vector of factors,

$$X_t = \begin{bmatrix} \mu_f \\ \mu_x \end{bmatrix} + \begin{bmatrix} \Phi_{ff} & 0 \\ 0 & \Phi_{xx} \end{bmatrix} X_{t-1} + \begin{bmatrix} \Sigma_f & 0 \\ 0 & \Sigma_x \end{bmatrix} \begin{bmatrix} \varepsilon_t^f \\ \varepsilon_t^x \end{bmatrix} \quad (1)$$

$$= \mu + \Phi X_{t-1} + \Sigma \varepsilon_t \quad (2)$$

where  $\Phi_{ff}$  is a diagonal matrix and the vectors  $\varepsilon_t^f$  and  $\varepsilon_t^x$  are i.i.d.  $N(0, I)$ . We also assume that the matrix  $\Sigma_x$  is diagonal, but we allow for non-zero off-diagonal terms in the covariance matrix of innovations to the latent fiscal factors,  $\Sigma_f$ , in order to accommodate the simultaneous surge in debt/GDP ratios across euro area economies during the crisis. Note that the lower-left block in  $\Phi$  is assumed to be zero, which amounts to assuming that there is no feedback from the national fiscal factors on the euro area business cycle, an assumption made for the sake of parsimony in an otherwise already highly parameterized setup. More crucial is the assumption that the fiscal factors do not affect the short-rate dynamics, but affect spreads only through their implications for default intensities, as

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<sup>11</sup>Even with only one lag ( $p = 1$ ), estimating the completely unrestricted model would amount to estimating roughly 300 parameters.

described below. Finally, we demean the three common macro factors before estimation, and assume that all constants are zero under the historical dynamics (i.e.  $\mu = \mathbf{0}$ ), but not under the risk-neutral dynamics. This implies that we impose a zero unconditional mean on the fiscal factors  $f$ . Since they are identified as the expected *change* in the debt/GDP ratios, this assumption is consistent with the view that the debt/GDP ratio must be stationary.

## 4.2 Dynamics of the factors under the risk-neutral measure

It is well-known that the existence of a positive stochastic discount factor is equivalent to the absence of arbitrage opportunities (see, e.g., Hansen and Richard, 1987). Following, amongst many others, Ang and Piazzesi (2003), we postulate the following form for the stochastic discount factor  $m_{t,t+1}$ :

$$m_{t,t+1} = \exp(-r_t) \frac{\xi_{t+1}}{\xi_t}$$

where  $\xi_t$  follows a log-normal process defined by

$$\xi_t = \xi_{t-1} \exp\left(-\frac{1}{2}\lambda'_{t-1}\lambda_{t-1} - \lambda'_{t-1}\varepsilon_t\right)$$

with  $\lambda_t = \lambda_0 + \lambda_1 X_t$ . Under these assumptions, it can be shown that the dynamics of the pricing factors under the risk-neutral measure  $\mathbb{Q}$  are given by

$$X_t = \mu^* + \Phi^* X_{t-1} + \Sigma \varepsilon_t^* \tag{3}$$

where the vector  $\varepsilon_t^*$  is i.i.d.  $N^{\mathbb{Q}}(0, I)$  and

$$\begin{aligned} \mu^* &= \mu - \lambda_0 \Sigma \\ \Phi^* &= \Phi - \lambda_1 \Sigma. \end{aligned}$$

One possibility is to directly estimate the elements of the vector  $\lambda_0$  and the matrix  $\lambda_1$ . Note, however, that without further restrictions, this would imply estimating 90 free parameters. Instead of imposing restrictions directly on the parameters of the market price of risk, we directly estimate the vector  $\mu^*$  and the matrix  $\Phi^*$  and we impose the same restrictions on  $\Phi^*$  as we did earlier on  $\Phi$ . Hence we estimate the  $n_x^2 = 9$  elements of  $\Phi_{xx}^*$ ,

the  $r = 8$  diagonal elements of  $\Phi_{ff}^*$  and the  $r + n_x = 10$  elements of  $\mu^*$ , which makes 28 parameters for the risk-neutral dynamics alone.

Note that for our choices of  $n_x = 3$ ,  $r = 8$  and  $p = 1$ , there are in total 106 parameters to estimate, demonstrating the need for introducing the large number of parameter restrictions adopted here.

### 4.3 Bond pricing

Let us denote by  $P(t, h)$  the price at time  $t$  of a risk-free zero-coupon bond of residual maturity  $h$ . This price is given by:

$$P(t, h) = E(m_{t,t+1} \dots m_{t,t+h}) \quad \text{or} \quad P(t, h) = E^{\mathbb{Q}}(\exp(-r_t - r_{t+1} \dots - r_{t+h-1})).$$

To price bonds subject to credit risk, we introduce default intensities – or hazard rates – for each country. The default intensity of country  $j$ , denoted by  $s_{j,t}$ , reflects credit risk embedded in the bonds issued by this country. If recovery rates were nil, the default intensity at time  $t$  would be the default probability of the considered debtor at that period. However, recovery rates are strictly positive processes. Therefore, the hazard rates  $s_{j,t}$  should be more rigorously termed as “recovery-adjusted default intensities” (see, e.g. Monfort and Renne, 2011).<sup>12</sup> Duffie and Singleton (1999) show that defaultable bonds can be priced using the same formulas as for risk-free bonds by simply replacing the short-term risk-free rate  $r_t$  by the default-adjusted short-term rate  $r_t + s_{j,t+1}$ . Formally, denoting by  $P_j(t, h)$  the price at time  $t$  of a bond of residual maturity  $h$  issued by country  $j$ , we have:

$$P_j(t, h) = E^{\mathbb{Q}}(\exp[-(r_t + s_{j,t+1}) - \dots - (r_{t+h-1} + s_{j,t+h})]). \quad (4)$$

Appendix C shows that bond prices are exponential affine in the factors  $X_t$  when the hazard rates are affine in the same factors, i.e. when country  $j$ 's hazard rates is given by

$$s_{j,t} = \gamma_{j,0} + \gamma_{j,1} X_t \quad (5)$$

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<sup>12</sup>Intuitively, with a constant recovery rate of  $R$ , the recovery-adjusted default intensity  $s_{j,t}$  would be approximately equal to  $(1 - R)\tilde{s}_{j,t}$  where  $\tilde{s}_{j,t}$  is the default probability of country  $j$  at time  $t$ .

Stack the  $\gamma_{j,0}$  in the  $(r-1) \times 1$  vector  $\gamma_0$  and the row vectors  $\gamma_{j,1}$  in the  $(r-1) \times (r-1+n_x)$  matrix  $\Gamma_1$ . Because data on credit default swaps suggest that German sovereign default risk is not literally assessed to be nil, the default intensities for the remaining countries should be interpreted as intensities *relative* to that of Germany. To conserve on the number of parameters characterizing the hazard rates that need to be estimated, we assume that the vector  $\gamma_{j,1}$  loads on the latent factor  $f_{j,t}$  of country  $j$  as well as on the three observed factors, but not on latent factors of countries  $i \neq j$ . With these assumptions, the structure of the matrix  $\Gamma_1$  is

$$\Gamma_1 = \begin{bmatrix} \gamma_{2,f} & \cdots & 0 & & \\ \vdots & \ddots & \vdots & & \\ 0 & \cdots & \gamma_{r,f} & & \\ & & & \Gamma_x & \end{bmatrix} \quad (6)$$

where the matrix  $\Gamma_x$  is of dimension  $(r-1) \times n_x$ . Let  $\gamma_f$  denote the  $(r-1) \times 1$  vector of diagonal elements of the left block of  $\Gamma_1$ .

For the functional form of the default intensities assumed above, bond prices are given by

$$P_j(t, h) = \exp(A_{j,h} + B_{j,h}X_t)$$

where the matrices  $A_{j,h}$  and  $B_{j,h}$  are obtained as functions of  $\mu^*$ ,  $\Phi^*$ ,  $\Sigma$ ,  $\gamma_0$  and  $\Gamma_1$  by applying recursive formulas. The continuously compounded yields, denoted by  $y_{j,t}^h$  and defined by  $-\log(P_j(t, h))/h$ , are given by:

$$y_{j,t}^h = \bar{A}_{j,h} + \bar{B}_{j,h}X_t \quad (7)$$

with  $\bar{A}_{j,h} = -A_{j,h}/h$  and  $\bar{B}_{j,h} = -B_{j,h}/h$ .

## 5 Estimation and results

### 5.1 Estimation

In this section we describe how we estimate the historical and the risk-neutral dynamics jointly. Because of the presence of latent factors, we write the model in state-space form and estimate it by maximum likelihood using the Kalman filter. We thus estimate the parameters under both dynamics jointly so as to allow information contained in the yields



to affect the estimates of the latent factors.<sup>13</sup> We are able to do so thanks to an efficient algorithm for the computation of the pricing matrices  $A_{j,h}$  and  $B_{j,h}$  described in Appendix C.<sup>14</sup>

In the measurement equation, we use observations on the  $n_x$  observed factors  $x_t$ , which are by definition observed without error. The  $n_y \cdot r$  yields are assumed to be observed with measurement error to avoid the problem of stochastic singularity, as is common in the literature. To conserve on parameters, we restrict for each country the standard deviation of the measurement errors for the three maturities used in estimation to be the same. The measurement error standard deviations are commonly used as the measure of fit of the model.

In addition, we use two further sets of variables that are assumed to be observed with error. First, as mentioned before, we would like to interpret the latent factors as representing the influence of fiscal policy on yields. The challenge in this regard is that it is difficult to measure the arrival of new information about fiscal policies that affect bond yields. National account measures of fiscal variables arrive only quarterly, and are even then in part based on interpolated annual data. To address these problems at least in part, we define the true fiscal variable  $f_j$  of country  $j$  as latent, but assume that it represents the expected change in country  $j$ 's debt/GDP ratio over the next 12 months. To implement this, let  $d_t$  denote the  $r \times 1$  vector of the 12-months change in the countries' debt/GDP ratios, which we take to be observed every third month, when a new end-of-quarter debt level is being published. Due to our assumption that the fiscal variables follow

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<sup>13</sup>In a different modeling framework, Ejsing et al. (2011) are similarly using information contained in yields to derive real-time estimates of fiscal variables. However, their model does not take advantage of the information contained in the whole yield curve, nor of the panel dimension of data, while ours does.

<sup>14</sup>The estimation of the parameters requires, for each draw of  $(\Theta, \Theta^*)'$ , the computation of the pricing matrices  $A_{i,h}$ ,  $B_{i,h}$  with  $h = 1, \dots, 120$  and  $i = 1, \dots, 8$  for each evaluation of the log-likelihood. Using standard recursive procedures would here be particularly computationally intensive. The intuition of the algorithm developed here rests on two simple ideas: To concatenate the country-specific pricing matrices so as to compute them for all countries simultaneously, and to select nested observed bond maturities (e.g. 1 month, 1, 5, 10 years) so that one can switch iteration steps from one month to one year while computing some intermediary matrices. We are thereby able to reduce estimation time by a factor 10 at monthly frequency. At weekly frequency, computation time would have been reduced by a factor of roughly 100.

univariate AR(1) processes, the model-implied forecast of the debt/GDP ratio would have to be  $f_t \equiv \widehat{E}_t[d_{t+12}]$ , where  $\widehat{E}_t$  is the model-implied expectation given by  $\Phi_{ff}^{12}d_t$  if  $d_t$  was included in the VAR. In those months when a new observation of  $d_t$  becomes available, we include this information in the measurement equation by assuming that

$$f_t = \Phi_{ff}^{12}d_t + \varepsilon_{m,t}, \quad E[\varepsilon_{m,t}\varepsilon'_{m,t}] = \Sigma_m$$

where  $\varepsilon_{m,t}$  denotes the vector of measurement errors and  $\Sigma_m$  is a diagonal matrix. We calibrate the standard deviations of these measurement errors by assuming that they are proportional to the empirical standard deviations  $\Sigma_d$  of the observed series of the change in the debt/GDP ratios.

The second additional source of information used in our estimation are Consensus Forecasts of euro area short-term interest rates, as well as long-term yields for some euro area countries. The use of this information is motivated by the concern that, given the high degree of persistence in yields, the historical dynamics of the yields are difficult to estimate with precision. Kim and Orphanides (2005) show that the use of survey expectations of interest rates (which are interpreted as forecasts of yields under the historical measure) in the estimation of affine term structure models leads to estimates that are more plausible along several dimensions. Monthly Consensus Forecasts at the 12-month horizon are available for the 3-month Euribor and for 10-year government bond yields of four of our countries: Germany, France, Italy and Spain.

Because the short rate  $y_t^1$  is the one-month EONIA OIS rate, we interpret the Euribor forecasts as equivalent to three-month averages of one-month EONIA OIS forecasts. We use the Euribor forecasts only through July 2007, because thereafter the previously trivial spread between Euribor and EONIA OIS rates at comparable maturities widened sharply. The corresponding measurement equation is

$$y_{t+12|t}^{cf,3} = \frac{1}{3} \sum_{k=12}^{14} \widehat{E}_t[y_{t+k}^1] + \varepsilon_t^{cf,3} \quad (8)$$

where  $y_{t+k|t}^{cf,h}$  denotes the Consensus Forecast  $k$  periods ahead of the yield of maturity  $h$ , and the expectation  $E$  is taken under the historical dynamics. We use the available forecasts for 10-year government bond yields for the entire sample, leading to measurement

equations of the form

$$y_{j,t+12|t}^{cf,120} = \widehat{E}_t[y_{j,t+12}^{120}] + \varepsilon_{j,t}^{cf,120} \quad (9)$$

The standard deviations  $\sigma_{cf,3}$  and  $\sigma_{cf,120,j}$ ,  $j = 1, \dots, 4$  are estimated; smaller values for these parameters imply that the historical dynamics are to a larger extent chosen so as to closely reproduce the Consensus Forecasts.

Let  $Y_t$  denote the  $(n_y \cdot r) \times 1$  vector of yields included in the estimation, and let  $Z_t$  denote the vectors of observable variables. In periods in which the fiscal variables are observed and in which the Consensus Forecasts for Euribor are used as observable variable (i.e. prior to August 2007), the vector  $Z_t$  is given by

$$Z_t = [d_t', x_t', Y_t', y_{t+12|t}^{cf,3}, y_{t+12|t}^{cf,120}]'$$

In periods in which either the fiscal variables are unobserved or the Consensus Forecasts for Euribor are not used,  $Z_t$  is correspondingly shortened.

While in principle one could work with the definition of the state vector  $X$  given above and the transition equation given by (2), it proves more convenient to rewrite the state-space model in a slightly different form described in Appendix B. The full vector of parameters to be estimated is also listed there.

## 5.2 Estimation results

Tables 3 and 4 present the results for the estimated parameters of both the historical and risk-neutral or pricing dynamics of the factors, Table 5 shows the parameters of the covariance matrix of the innovations to the factors, Table 6 the parameters of the default intensities and Table 7 the standard deviations of measurement errors on yields, spreads and surveys. As is evident from Tables 3 and 4, the persistence of the factors under the risk-neutral dynamics is estimated to be higher than under the historical measure.

Figure 2 plots the observed yields of German bonds with maturities of 1, 5 and 10 years against the yields simulated with our model. Similarly, Figures 3-4 show the observed and fitted yield spreads against Germany for the remaining seven euro area countries at the 5-year maturity. The standard deviations of the measurement errors on German yields and the other countries' spreads are shown in the upper row of Table 7 (recall that for each country we impose that measurement error standard deviations are constant across

maturities). By the standards of macro-factor term structure models, the fit of the German yields is reasonably good, with a measurement error standard deviation of 45 basis points. By comparison, Lemke (2008) obtains measurement error standard deviations of 29 basis points, but in a model in which the market price of risk parameters are chosen to only fit German yields and not in addition spreads of seven countries at three maturities. For fitting the spreads, unsurprisingly the measurement errors are largest for Greece and Portugal, but for Ireland the standard deviation has already dropped to 22 basis points, and for the remaining four countries it is a remarkably small 15 basis points or less.<sup>15</sup>

Figures 3-4 also show the contribution of the observed euro area common macro factors to the fitted spreads. Interestingly, these macro factors are instrumental in fitting the spreads before 2008, but play a limited role thereafter. Conversely, the country-specific fiscal factors stand out as major drivers of the widening in spreads observed since then. We will analyze the contributions of the fiscal factors and their implications in greater detail below.

As discussed above, we use survey-based forecasts for both the short rate and for the 10-year yields of several countries to refine the estimation of the historical dynamics of the pricing factors. Figures 5-6 compare these survey-based forecasts with the forecasts implied by our model. Importantly, our model-implied forecast of the short rate one year ahead closely tracks the Consensus forecasts, which suggests that the model-implied historical short-rate dynamics are similar to those held by investor, thereby resolving the well-known persistence bias that usually plagues the estimated dynamics of the short rate in ATS models of the yield curve.<sup>16</sup> Although the model-implied 12-month-ahead expectation of the German 10-year yield does not track the Consensus Forecast as closely (the standard error, shown in Table 7, is 54 basis points), the model-implied expected spreads for France, Italy and Spain match the expected spreads computed from the Consensus Forecasts up to errors with a standard deviation of 10 to 15 basis points.

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<sup>15</sup>This is all the more remarkable because we restrict the state space of factors spanning the eight yield curves to observable or “pseudo observable” factors only, while most studies in this literature also incorporate purely latent factors in their models (see e.g. Ang and Piazzesi, 2003, Dai and Philippon, 2006, Rudebusch and Wu, 2008).

<sup>16</sup>See notably Jardet et al. (2009) and Bauer et al. (2011).

### 5.3 Fiscal sustainability, risk premia, and default risk compensation

As shown in Figures 3-4, the fiscal factors play the dominant role in explaining the evolution of yield spreads since 2008. To substantiate our claim that these fiscal factors can be interpreted as market expectations of the change in debt/GDP ratios over the next 12 months, Figures 7 and 8 plot the factors against  $\Phi_{ff}^{12}d_t$  in those months in which we use published past 12-month changes in debt/GDP ratios as observable variables. The measurement error standard deviations are calibrated from the historical innovations to debt/GDP ratios, which means that we allow larger fitting errors for those countries (notably Ireland and Greece) that have more volatile changes in debt/GDP ratios during our sample.<sup>17</sup> Nonetheless, the figures show that these latent factors in fact closely approximate the historical data. Moreover, since our one-step estimation procedure takes advantage of the information contained in the yields as well as the information in the observed  $d_{i,t}$  to estimate the national fiscal pricing factors, these factors can be viewed as reflecting the *perception by investors* of the future trend in the national debt-to-GDP ratios. As shown by the diamonds in these figures, the implied expected changes in debt/GDP are also broadly similar to OECD projections of these variables published at the time, lending further credence to our interpretation.

The no-arbitrage assumption underlying our ATS model allows to decompose the spreads at any maturity into the contribution of risk premia and compensations for default. While risk premia reflect the risk aversion of investors, i.e. their sensitivity to the variance of pricing factors, the compensation for default risk corresponds to the extra yield that risk-neutral investors require to hold a bond that may default before maturity. Technically, risk premia are obtained by subtracting spreads computed under the historical dynamics of the factors (i.e. the dynamics that would matter if investors were neutral to risk) from fitted spreads (i.e. computed under the pricing or risk-neutral dynamics). In contrast, compensations for default are obtained by subtracting from the spreads computed under the historical dynamics of the factors the simulated spreads one would get

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<sup>17</sup>In practice, the corresponding measurement equation reads:  $d_t = \Phi_{ff}^{-12}f_t + \tilde{\varepsilon}_{m,t}$ ,  $E[\tilde{\varepsilon}_{m,t}\tilde{\varepsilon}'_{m,t}] = \tilde{\Sigma}_m$ , and we set  $\tilde{\Sigma}_m = 0.25 \Sigma_d$ . The resulting standard deviations are 0.92% for Belgium, 0.78% for France, 1.42% for Greece, 2.80% for Ireland, 0.86% for Italy, 0.88% for Portugal and 1.24% for Spain.

if there were no probability of default (i.e. if the parameters  $\gamma_{j,0}$  and  $\gamma_{j,1}$  in the default intensities  $s_{j,t}$  were all set to zero). Figures 9-10 show the observed 5-year spreads plotted against both the risk premia and the compensations for default. For three of the countries (Belgium, Ireland, and Portugal), risk premia and compensation are almost of the same size and comove closely together. For the remaining four (France, Greece, Italy and Spain) the contribution to spreads from risk premia is larger than that from compensation for default.<sup>18</sup>

Because the compensations for default are the product of the probability of default and the loss in the event of default (or “loss given default”, LGD), it is possible to derive the perceived probabilities of default (PD) that are associated with these compensations for default only if one is willing to make an assumption about LGD (see appendix D for details). We assume here that the LGD rate is constant and equal to 50% of the market price in the absence of default. Figures 11-12 show the time series of the estimated probabilities of default for all countries since 1999 at the five-year horizon, together with 90% confidence intervals. First, note that the point estimates of these probabilities (as well as the associated compensations) remain in positive territory in general, although this constraint was not explicitly imposed on the model. Consistently with the tiny intra euro area spreads at that time, default probabilities were close to zero before 2008. They remained contained (below 2% for all countries except Ireland and Portugal) until the onset of the Greek sovereign debt crisis in November 2009; since then they have shot up for all countries except France.

Whereas Figures 11-12 provide the time series of PDs at a fixed maturity horizon, the model also allows to retrieve the whole term structure of PDs at any date. Figures 13-14 show term structures (together with 90% confidence intervals) for all seven countries as of end August 2010, i.e. just before the inception of both the EFSF and the Securities Market Program of the ECB as an answer to the Greek crisis. PD curves look generally upward-slopping and concave, which reflects higher perceived probabilities of default over the short-medium term than in the long term.

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<sup>18</sup>Interestingly, for France, Italy and Spain, the three countries for which survey expectations of future long-term yields are available, the comovement between risk premia and default compensation is the lowest, indicating that the survey information may help to disentangle these two components.

Finally, our model enables us to quantify, again at any horizon, the sensitivity of both spreads and perceived probabilities of default to the fiscal factors that we interpret as the expected change in debt/GDP ratios over the next 12 months. Table 8 presents these sensitivities for all countries at the 2, 5 and 10 year horizons. All the estimated coefficients are highly significant except in the case of France.<sup>19</sup> For example, a one percent expected increase in the debt/GDP ratio of Greece over the next year should translate into an increase in its 5-year spread against Germany by 37 bp and in its probability of default by some 2%.<sup>20</sup> The sensitivities of spreads to fiscal balances are about half as large for Spain (about 20 bp) and between Greek and Spanish figures for Italy and Portugal (about 30 bp). Finally, we find that the sensitivities of spreads to fiscal factors are generally slightly increasing with maturity, except in the case of Greece. Meanwhile, the sensitivities of the PD are consistently increasing with maturities for all countries.<sup>21</sup>

In view of the prospective reduction in debt/GDP ratios over the coming years, as expressed for example in the debt reduction programmes that EMU member countries must submit to the European Commission, a natural question to ask is what are the predictions of our model under alternative scenarios of debt reduction. The results presented above show that conditions similar to those prevailing in the early 2000s would lead to non-negative spreads and non-negative (or at least not statistically significant) default probabilities. More drastic debt reduction would cause our model to predict negative spreads and default probabilities, which in our view points to two limitations of our results. At a mechanical level, the linear structure of our model implies that there is no positivity constraint for either spreads or default probabilities. We do not know of any way to incorporate such a constraint without losing the ability to compute closed-form

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<sup>19</sup>The sensitivities of spreads are simply components of the  $B_{i,h}$  matrices above. The sensitivities of PD are taken from similar matrices but adjusted for the LGD. We get the standard deviations of these sensitivity coefficients from the standard deviations of the estimated parameters using the delta method.

<sup>20</sup>Note, however, that sensitivities of the PD to the changes in debt/GDP ratios are valid for changes around zero only because of the first order approximation used to derive the PD from the estimated default densities.

<sup>21</sup>Sensitivities of risk premia to the fiscal factors can explain diverging maturity patterns of spreads vs PD sensitivities.

asset pricing formulas.

The second, more fundamental limitation is that, with our choice of the expected change in the debt/GDP ratio, we are focussing on a particular measure of fiscal sustainability which we have shown to be useful for understanding the behavior of spreads in the period before and during the crisis. In our view it would be entirely plausible if, under different circumstances, market participants' attention shifted to a different measure of fiscal sustainability, such as the *level* of debt. With a long enough sample and several crisis episodes, one could assess this hypothesis by e.g. estimating a model with multiple fiscal indicators, in which the weights assigned to these indicators shifted between regimes. Given that we observe a short sample with only one transition from a non-crisis to a crisis state, however, estimation of the transition probabilities between regimes is infeasible. If investors perceive the possibility of such regime shifts, it is also conceivable that, even if economic and fiscal conditions were to return to their state of the early 2000s, spreads might nonetheless not return to their pre-crisis levels due to a lasting change in the perception of transition probabilities between crisis and non-crisis states. For all these reasons, unfortunately, we would caution against using the model estimated in this paper as a tool for prediction.

## 6 Conclusion

At the time of this writing, the explosion in EMU government bond yield spreads is posing a major threat not only to individual countries' solvency, but to financial stability in Europe and worldwide. Understanding the causes of this explosion is therefore of first-order importance. One long-standing strand of thinking in this respect are self-fulfilling expectations, i.e. spontaneous runs on government debt. In this paper, we have sought to examine an alternative view, that spreads reflect underlying economic fundamentals, notably investors' assessment of fiscal sustainability. Without wanting to deny a role for self-fulfilling expectations – in fact, for high enough debt levels these runs are near certain to happen – it seems important to examine the role that economic fundamentals and economic policies might have played in setting off the dramatic increase in yield spreads.

To this end we have estimated a multi-country affine term structure model of poten-



tially defaultable bonds, in which the factors are exclusively interpretable macroeconomic factors. We have shown that this model matches yields and spreads for eight EMU countries impressively well both before and during the crisis, given the large number of constraints we impose on the model. Our results suggest two main conclusions: (i) Fiscal factors, in particular the sharp increase in government debt, explains the bulk of the increase in spreads since 2008. This finding points to limited room for fiscal stabilization policies (including government-financed financial-sector stabilization) that entail sharp increases in government debt, perhaps because investors are pessimistic about the ability for subsequent debt reduction. (ii) The increase in spreads was to a large extent driven by increases in risk premia, implying that the deterioration of the economic and fiscal outlook during the crisis led to an *endogenous* increase in the market price of risk, thereby magnifying the effects on yield spreads much beyond pure compensation for perceived higher default risk.

For the reasons discussed before, we would caution against using the model estimated in this paper for predictive purposes during an extended period of fiscal consolidation. Nonetheless, the results are entirely supportive of the view that enacting policies that improve the prospects for fiscal sustainability will have large beneficial effects, through a virtuous cycle of reduced spreads which in turn facilitate the reduction of debt burdens.

As more data become available, EMU government bond yield spreads will continue to provide an excellent laboratory to test different views about the determinants of credit risk and its pricing. With sufficient data at hand, exploring the role of regime shifts in the determination of spreads seems a particularly promising direction. We leave this for future work.

# Appendices

## A Data

In this section, we detail how we constructed the data for zero-coupon government bond yields at different maturities.

The estimation of the model indeed requires zero-coupon yields. However, governments usually issue coupon-bearing bonds. In order to have the most comparable data across countries, we estimate the zero-coupon yield curves using the same methodology for seven countries: Germany France, Ireland, Italy, Portugal, Spain and Greece. For some maturities and some dates, Belgian yields obtained with this methodology present some unsatisfactory level in the early 2000s (with Belgian long-term yields slightly lower than the German ones). Hence, for Belgium, we use zero-coupon yields computed by the National Bank of Belgium. The series of Greek zero-coupon yields start in mid-2001, a few months after Greece joined the euro-area (leaving aside convergence effects in early 2001).

As Gurkaynak, Sack and Wright (2005) , we resort to a parametric approach (see BIS, 2005, for an overview of zero-coupon estimation methods). We choose the parametric form originally proposed by Nelson and Siegel (1987). Specifically, the yield of a zero-coupon bond with a time to maturity  $m$  for a point in time  $t$  is given by:<sup>22</sup>

$$y_t^m(\theta) = \beta_0 + \beta_1 \left(-\frac{\tau_1}{m}\right) \left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right) + \beta_2 \left[\left(\frac{\tau_1}{m}\right) \left(1 - \exp\left(-\frac{m}{\tau_1}\right)\right) - \exp\left(-\frac{m}{\tau_1}\right)\right]$$

where  $\theta$  is the vector of parameters  $[\beta_0, \beta_1, \beta_2, \beta_3, \tau_1, \tau_2]'$ . Assume that, for a given country and a given date  $t$ , we dispose of observed prices of  $N$  coupon-bearing bonds (with fixed coupon), denoted by  $P_{1,t}, P_{2,t}, \dots, P_{N,t}$ . Let us denote by  $CF_{k,i,t}$  the  $i^{th}$  (on  $n_k$ ) cash flows

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<sup>22</sup>We use the Nelson-Siegel form rather than the extended version of Svensson (1994) because the latter requires more data to be estimated properly (and for some countries and some dates, we have too small a number of coupon-bond prices).

that will be paid by the  $k^{th}$  bond at the date  $\tau_{k,i}$ . We can use the zero-coupon yields  $\{y_t^m(\theta)\}_{m \geq 0}$  to compute a modeled (dirty) price  $\hat{P}_{k,t}$  for this  $k^{th}$  bond:

$$\hat{P}_{k,t}(\theta) = \sum_{i=1}^{n_k} CF_{k,i,t} \exp\left(-\tau_{k,i} y_t^{\tau_{k,i}-t}(\theta)\right).$$

The approach then consists in looking for the vector  $\theta$  that minimizes the distance between the  $N$  observed prices and modeled bond prices. Specifically, the vector  $\theta_t$  is given by:

$$\theta_t = \arg \min_{\theta} \sum_{k=1}^N \omega_k (P_{k,t} - \hat{P}_{k,t}(\theta))^2$$

where the  $\omega_k$ 's are some weights that are chosen with respect to the preferences that one may have regarding the fit of different parts of the yield curve. Intuitively, taking the same value for all the  $\omega_k$ 's would lead to large yield errors for financial instruments with relatively short remaining time to maturity. This is linked to the concept of duration (i.e. the elasticity of the price with respect to one plus the yield): a given change in the yield corresponds to a small/large change in the price of a bond with a short/long term to maturity or duration. Since we do not want to favour a particular segment of the yield-curve fit, we weight the price error of each bond by the inverse of the remaining time to maturity.<sup>23</sup>

Coupon-bond prices come from Datastream.<sup>24</sup> In the same spirit as Gurkaynak et al. (2005), different filters are applied in order to remove those prices that would obviously bias the obtained yields. In particular, the prices of bonds that were issued before 1990 or that have atypical coupons (below 1% or above 10%) are excluded. In addition, the prices of bonds that have a time to maturity lower than 1 month are excluded.<sup>25</sup>

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<sup>23</sup>Using remaining time to maturity instead of duration has not a large effect on estimated yields as long as we are not concerned with the very long end of the yield curve.

<sup>24</sup>Naturally, the number of bonds used differ among the countries (from 19 bonds for the Netherlands to 175 bonds for Germany).

<sup>25</sup>The trading volume of a bond usually decreases considerably when it approaches its maturity date.

## B State space model representation

The expression (7) for the log yield to maturity of the  $h$ -period bond reads (see Appendix C and section 4.3 for details):

$$y_{j,t}^h = \bar{A}_{j,h} + \bar{B}_{j,h} X_t$$

Stacking the equations for the  $n_y \cdot r$  yields  $Y_t$ , we can write these as

$$Y_t = \bar{A} + \bar{B} X_t = \bar{A} + [\bar{B}_f \ \bar{B}_x] \begin{bmatrix} f_t \\ x_t \end{bmatrix}$$

Recall the definition of the vectors and matrices  $\mu$ ,  $\Phi$ ,  $\Sigma$  in (2). With these definitions, the model-implied expectations on the right-hand side of (8) and (9) are given by

$$E_t[y_{t+k}^1] = \zeta_3^0 + \zeta_3^1 X_t, \quad E_t[y_{j,t+12}^{120}] = \zeta_{j,120}^0 + \zeta_{j,120}^1 X_t$$

where

$$\zeta_3^0 = \iota_{y^1} (I - \Phi)^{-1} \left[ I - \left( \sum_{k=12}^{14} \Phi^k \right) / 3 \right] \mu, \quad \zeta_3^1 = \iota_{y^1} \left( \sum_{k=12}^{14} \Phi^k \right) / 3, \quad \zeta_{j,120}^0 = \iota_{j,120} A, \quad \zeta_{j,120}^1 = \iota_{j,120} B \Phi^{12}$$

and  $\iota_{y^1}$  and  $\iota_{j,120}$  are selection vectors that select  $y^1$  in  $X_t$  and the 120-month yield of country  $j$  in  $Y_t$ , respectively. Let  $\zeta_3^f$ ,  $\zeta_3^x$  and  $\zeta_{j,120}^f$ ,  $\zeta_{j,120}^x$  denote the partitions of  $\zeta_3^1$ ,  $\zeta_{j,120}^1$  conforming to  $f_t$ ,  $x_t$ . Finally, let  $\zeta_{120}^f$ ,  $\zeta_{120}^x$  denote the  $4 \times 1$  vectors of coefficients for the four countries for which Consensus Forecasts of 10-year government bond yields are available (Germany, France, Italy and Spain).

As discussed above, the vector of observables is given by

$$Z_t = [d'_t, x'_t, Y'_t, y_{t+12|t}^{cf,3}, y_{t+12|t}^{cf,120}]'$$

Define the vector  $S_t \equiv [f'_t, f'_{t-1}]'$  of current and lagged latent factors, and  $\tilde{X} \equiv [1, x'_t, x'_{t-1}]'$ . Recall the partitioning of the transition equation for  $X_t$ , equation (1), repeated here for convenience:

$$X_t = \begin{bmatrix} \mu_f \\ \mu_x \end{bmatrix} + \begin{bmatrix} \Phi_{ff} & 0 \\ \Phi_{xf} & \Phi_{xx} \end{bmatrix} X_{t-1} + \begin{bmatrix} \Sigma_f & 0 \\ 0 & \Sigma_x \end{bmatrix} \begin{bmatrix} \varepsilon_t^f \\ \varepsilon_t^x \end{bmatrix}$$

With this notation, the state-space model can be written as

$$S_t = \begin{bmatrix} \mu_f \\ 0 \end{bmatrix} + \begin{bmatrix} \Phi_{ff} & 0 \\ I & 0 \end{bmatrix} S_{t-1} + \begin{bmatrix} \Sigma_f & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_t^f \\ 0 \end{bmatrix} \quad (10)$$

$$Z_t = C \tilde{X}_t + G S_t + U_t \quad (11)$$

$$\Leftrightarrow \begin{bmatrix} d_t \\ x_t \\ Y_t \\ y_{t+12|t}^{cf,3} \\ y_{t+12|t}^{cf,120} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ \mu_x & 0 & \Phi_{xx} \\ \bar{A} & \bar{B}_x & 0 \\ \zeta_3^0 & \zeta_3^x & 0 \\ \zeta_{120}^0 & \zeta_{120}^x & 0 \end{bmatrix} \begin{bmatrix} 1 \\ x_t \\ x_{t-1} \end{bmatrix} + \begin{bmatrix} I & 0 \\ 0 & \Phi_{xf} \\ \bar{B}_f & 0 \\ \zeta_3^f & 0 \\ \zeta_{120}^f & 0 \end{bmatrix} \begin{bmatrix} f_t \\ f_{t-1} \end{bmatrix} + \begin{bmatrix} \Sigma_{u,f} u_t^f \\ \Sigma_x \varepsilon_t^x \\ \Sigma_y u_t^y \\ \sigma_{u,3} u_t^{cf,3} \\ \Sigma_{u,120} u_t^{cf,120} \end{bmatrix}$$

The parameter vector  $\theta$  to be estimated consists of the parameters of the historical factor dynamics  $\text{diag}(\Phi_{ff})$ ,  $\text{vec}(\Phi_{xx})$ ,  $\text{diag}(\Sigma)$  in (2), those of the risk-neutral dynamics  $\mu^*$ ,  $\text{diag}(\Phi_{ff}^*)$ ,  $\text{vec}(\Phi_{xx}^*)$  in (3),  $\gamma_0$ ,  $\gamma_f$ ,  $\text{vec}(\Gamma_x)$  in (5)-(6), and the standard deviations of measurement errors, i.e. the diagonal elements of the matrices  $\Sigma_y$  multiplying the yield measurement errors  $u_t^y$ ,  $\sigma_{u,3}$  and  $\Sigma_{u,120}$ . As discussed in the main text, the diagonal matrix of measurement errors  $\Sigma_{u,f}$  is calibrated as 0.25 times the historical standard deviations of 12-month changes of the individual countries' debt/GDP ratios.

## C Bond-pricing algorithm

In this appendix, we present an algorithm that we have developed in order to fasten the computation of bonds' prices and yields (compared with standard techniques). In a first part, we recall the general formula of the price of a defaultable zero-coupon bond, considering the general case of non-zero recovery rate. In a second part, we detail the algorithm that we use in the numerical implementation of these formulas.

### C.1 The price of a defaultable zero-coupon bond

Let us consider the price  $P_i(t, h)$  at time  $t$  of a defaultable zero-coupon bond issued by debtor  $i$  and with a residual time to maturity of  $h$ . Assuming that the recovery pay-off is settled as soon as the debtor has defaulted (there is no delay between default and payment), the bond is worthless at time  $t$  if debtor  $i$  has defaulted before  $t - 1$ . If the

debtor defaults at date  $t$ , then the bond price is equal to the recovery pay-off, that is assumed to be equal to a fraction  $\zeta$  of what the bond would have been worth conditional on no default (this is the so-called “recovery of market value” assumption introduced by Duffie and Singleton, 1999). The latter (virtual) price is denoted by  $\tilde{P}_i(t, h)$ . Formally, we have:

$$P_i(t, h) = \begin{cases} 0 & \text{if } D_{i,t-1} = 1 \\ \tilde{P}_i(t, h) & \text{if } D_{i,t} = 0 \\ \zeta \tilde{P}_i(t, h) & \text{if } D_{i,t} = 1 \text{ and } D_{i,t-1} = 0 \end{cases}$$

where  $D_{i,t}$  is a default-indicator variable ( $D_{i,t} = 1$  if debtor  $i$  has defaulted at, or before, time  $t$ ,  $D_{i,t} = 0$  otherwise). Let us consider the case where debtor  $i$  has not defaulted at –or before–  $t$ . In that case, we have  $P_i(t, h) = \tilde{P}_i(t, h) = E_t^{\mathbb{Q}}[e^{-r_t} P_i(t+1, h-1)]$ . Using the law of iterated expectations (and noting that  $\tilde{P}_i(t+1, h-1)$  does not depend on  $D_{i,t+1}$ ), we have:

$$\begin{aligned} P_i(t, h) = \tilde{P}_i(t, h) &= E_t^{\mathbb{Q}} \left[ e^{-r_t} E_t^{\mathbb{Q}} \left( \left( \mathbb{I}_{\{D_{i,t+1}=0\}} + \xi_i \mathbb{I}_{\{D_{i,t+1}=1\}} \right) \tilde{P}_i(t+1, h-1) \middle| X_{t+1} \right) \right] \\ &= E_t^{\mathbb{Q}} \left[ e^{-r_t} \tilde{P}_i(t+1, h-1) E_t^{\mathbb{Q}} \left( \mathbb{I}_{\{D_{i,t+1}=0\}} + \xi_i \mathbb{I}_{\{D_{i,t+1}=1\}} \middle| X_{t+1} \right) \right] \\ &= E_t^{\mathbb{Q}} \left[ e^{-r_t} \tilde{P}_i(t+1, h-1) (e^{-\tilde{s}_{i,t+1}} + \zeta(1 - e^{-\tilde{s}_{i,t+1}})) \right]. \end{aligned}$$

where  $\tilde{s}_{i,t}$  denotes the default probability of issuer  $i$  at time  $t$ , conditional on  $X_t$ . Defining a recovery-adjusted default intensity  $s_{i,t+1}$  by  $e^{-s_{i,t+1}} = e^{-\tilde{s}_{i,t+1}} + \zeta(1 - e^{-\tilde{s}_{i,t+1}})$ , we obtain

$$\tilde{P}_i(t, h) = E_t^{\mathbb{Q}} \left[ e^{-r_t - s_{i,t+1}} \tilde{P}_i(t+1, h-1) \right]$$

and, by iterating, we get equation (4).

$$\tilde{P}_i(t, h) = E_t^{\mathbb{Q}} \left[ e^{-r_t - s_{i,t+1} \dots - r_{t+h-1} - s_{i,t+h}} \right] \quad (12)$$

$\tilde{P}_i(t, h)$  is a multi-horizon Laplace-transform of  $r_t$  and  $s_{i,t+1}$ . Inasmuch as both  $r_t$  and  $s_{i,t+1}$  are affine functions of the pricing factors  $X_t$ , which follow Gaussian processes, then  $\tilde{P}_i(t, h)$  is an exponential affine function of  $X_t$ .

## C.2 Computation of equation (12)

In their seminal paper, Ang and Piazzesi (2003) show that there exist matrices  $A_h$  and  $B_h$  that are such that  $E(\exp(\delta' X_{t+1} + \dots + \delta' X_{t+h})) = \exp(A_h + B_h F_t)$  when  $X_t$  follows

a vector auto-regressive process. Besides, they provide formulas to compute recursively these matrices  $A_h$  and  $B_h$ . Ang and Piazzesi use these formulas to price risk-free zero-coupon bonds, but such formulas are readily usable to solve equation (12), provided that both  $r_t$  and the  $s_{i,t}$ 's are affine in  $X_t$ . While this algorithm may be appropriate when the frequency of the data is relatively low and when only one yield curve (one debtor) is considered, it may considerably slow the estimation process in alternative cases. Here, we propose an algorithm that is particularly efficient when the frequency of the data is relatively high (higher than quarterly, say) and/or when different issuers are considered. Recall that (under the risk-neutral measure) the risk factors  $X_t$  follow the vector auto-regressive process:

$$X_t = \mu^* + \Phi^* X_{t-1} + \Sigma \varepsilon_t^*, \quad \varepsilon_t^* \sim \mathcal{N}(0, I).$$

Without loss of generality, we can assume that the vector  $X_{t+1}$  contains predetermined entries such that the risk-free short-term rate of the previous period ( $r_t$ ) is affine in  $X_{t+1}$  (but known at time  $t$ ):  $r_t = \delta_0 + \delta_1' X_{t+1}$ . Then, if the default intensities are also affine in  $X_{t+1}$  ( $s_{i,t+1} = \gamma_{i,0} + \gamma_{i,1}' X_{t+1}$ ), equation (12) becomes:

$$\tilde{P}_i(t, h) = \exp(-h\tilde{\gamma}_{i,0}) E_t^{\mathbb{Q}} [\exp(-\tilde{\gamma}_{i,1}' (X_{t+1} + \dots X_{t+h}))]$$

where  $\tilde{\gamma}_{i,0} = \delta_0 + \gamma_{i,0}$  and  $\tilde{\gamma}_{i,1} = \delta_1 + \gamma_{i,1}$ . Let us denote  $X_{t+1} + \dots X_{t+h}$  by  $F_{t+h,h}$ . It is important to note that  $F_{t+h,h}$  is a Gaussian random variable under the risk-neutral measure (conditional on information available at time  $t$ ), since it means that it is immediate to compute bond prices issued by any debtor as soon as one knows the mean and variance of  $F_{t+h,h}$ . In other words, if one is given the law of  $F_{t+h,h}$ , computing matrices  $A_{i,h}$  and  $B_{i,h}$  for  $N$  debtors ( $i \in \{1, \dots, N\}$ ) takes virtually no more time than to compute those associated with the risk-free yield curve alone. Specifically, if

$$F_{t+h,h} \sim \mathcal{N}^{\mathbb{Q}}(\chi_{0,h} + \chi_{1,h} X_t, \Omega_h),$$

then,  $\tilde{P}_i(t, h) = \exp(A_{i,h} + B_{i,h} X_t)$ , where:

$$\begin{cases} A_{i,h} &= -h\tilde{\gamma}_{i,0} - \tilde{\gamma}_{i,1}' \chi_{0,h} + \frac{1}{2} \tilde{\gamma}_{i,1}' \Omega_h \tilde{\gamma}_{i,1} \\ B_{i,h} &= -\tilde{\gamma}_{i,1}' \chi_{1,h}. \end{cases}$$

It remains to compute  $\chi_{0,h}$ ,  $\chi_{1,h}$  and  $\Omega_h$ . We have:

$$\begin{aligned} F_{t+h,h} &= \left( hI + (h-1)\Phi^* + \dots + \Phi^{*h-1} \right) \mu^* + \\ &\quad \left( \Phi^* + \Phi^{*2} \dots + \Phi^{*h} \right) X_t + \\ &\quad \left( I + \dots + \Phi^{*h-1} \right) \varepsilon_{t+1}^* + \left( I + \dots + \Phi^{*h-2} \right) \varepsilon_{t+2}^* + \dots + \varepsilon_{t+h}^*, \end{aligned}$$

implying:

$$\begin{cases} \chi_{1,h} &= \Phi^* (\Phi^{*h} - I) (\Phi^* - I)^{-1} \\ \chi_{0,h} &= [\chi_{1,h} - hI] (\Phi^* - I)^{-1} \mu^* \end{cases}$$

and:

$$\begin{aligned} \Omega_h &= \text{Var} \left( (I + \dots + \Phi^{*h-1}) \varepsilon_{t+1}^* + (I + \dots + \Phi^{*h-2}) \varepsilon_{t+2}^* + \dots + \varepsilon_{t+h}^* \right) \\ &= (\Phi^* - I)^{-1} \left[ (\Phi^{*h} - I) \Sigma \Sigma' (\Phi^{*h} - I)' + \dots \right. \\ &\quad \left. + (\Phi^* - I) \Sigma \Sigma' (\Phi^* - I)' \right] (\Phi^* - I)^{-1} \\ &= (\Phi^* - I)^{-1} [(h-1)\Sigma \Sigma' - \Lambda_h \Sigma \Sigma' - \Sigma \Sigma' \Lambda_h' + \Pi(h, \Phi^*, \Sigma)] (\Phi^* - I)^{-1} \end{aligned}$$

where  $\Lambda_h = \Phi^* (\Phi^{*h} - I) (\Phi^* - I)^{-1}$  and  $\Pi : (h, \Phi^*, \Sigma) \rightarrow (\Phi^{*h}) \Sigma \Sigma' (\Phi^{*h})' + \dots + (\Phi^*) \Sigma \Sigma' (\Phi^*)' + \Sigma \Sigma'$ . Instead of using a brute-force approach (based on  $h$  loops) to compute  $\Pi(h, \Phi^*, \Sigma)$ , we exploit the fact that  $\Pi(kh, \Phi^*, \Sigma) = \Pi(k, \Phi^{*k}, \Pi(h, \Phi^*, \Sigma))$ . This substantially reduces the computation time of  $\Pi(h, \Phi^*, \Sigma)$  (Obviously, the higher the frequency of the model and/or the longer the considered maturities, the higher the gain in computation time is.)

## D Derivation of the default probabilities

The probability that country  $i$  defaults during the next  $h$  periods is given by:

$$\begin{aligned} PD_i(t, h) &= E_t(\mathbb{I}(d_{i,t+h} = 1)) \\ &= 1 - E_t(\mathbb{I}(d_{i,t+h} = 0)) \end{aligned}$$

It is straightforward to show that

$$E_t(\mathbb{I}(d_{i,t+h} = 0)) = E_t(\exp[-\tilde{s}_{i,t+1} - \dots - \tilde{s}_{i,t+h}]) \quad (13)$$



where the  $\tilde{s}_{i,t+j}$ 's are the default intensities. Note however that the intensities that are estimated in our model (the  $s_{t+j}$ 's) are adjusted for the LGD and thus not equivalent to these  $\tilde{s}_{t+j}$ 's because the latter are not corrected for the potential recovery pay-offs. In order to compute estimates of the default probabilities, we need to rest on the so-called "recovery proportional to market value" assumption (see previous appendix).

Hence, to the extent that the  $s_{t+j}$ 's are small, we get as an approximation

$$\tilde{s}_{i,t} \approx \frac{s_{i,t}}{1-\zeta} = \frac{1}{1-\zeta} (\gamma_{i,0} + \gamma_{i,1}X_t).$$

Then, it appears that the right-hand side of equation 13 is approximately exponential affine in  $(X_{t+1}, \dots, X_{t+h})$ , and can therefore be computed by using the same kind of formulas as those used to compute bond yields (see Section 4.3).

## References

- [1] Amato, Jeffery and Maurizio Luisi, 2006. “Macro factors in the term structure of credit spreads.” BIS working paper no. 203.
- [2] Ang, Andrew and Monika Piazzesi, 2003. “A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables.” *Journal of Monetary Economics* 50 (4), 745-787.
- [3] Aßmann, Christian, and Jens Boysen-Hogrefe, 2009. “Determinants of government bond spreads in the euro area – in good times as in bad.” Kiel Institute for the World Economy, working paper 1548.
- [4] Bauer, Michael, Rudebush, Glenn and Cynthia Wu, 2011. “Unbiased estimation of dynamic term structure models.” FRB of San Francisco Working Paper No. 2011-12.
- [5] Beber, Alessandro, Michael Brandt, and Kenneth Kavajecz, 2009. “Flight-to-quality or flight-to-liquidity? Evidence from the euro-area bond market.” *Review of Financial Studies* 22 (3), 925-957.
- [6] Bernoth, Kerstin, Jürgen von Hagen and Ludger Schuknecht, 2006. “Sovereign risk premiums in the European government bond market.” Available at <http://www.sfbtr15.de/dipa/151.pdf>.
- [7] Calvo, Guillermo, 1988. “Servicing the public debt: The role of expectations.” *American Economic Review* 78 (4), 647-661.
- [8] Codogno, Lorenzo, Carlo Favero and Alessandro Missale, 2003. “Yield spreads on EMU government bonds.” *Economic Policy* Vol. 18, Issue 37, 503-532.
- [9] Dai, Qiang, and Thomas Philippon, 2006. “Fiscal policy and the term structure of interest rates.” Working paper, November.
- [10] Duffee, Gregory, 2002. “Term premia and interest rate forecasts in affine models.” *Journal of Finance* 57, 405-443.

- [11] Duffee, Gregory, 2009. "Information in (and not in) the term structure." Manuscript, Johns Hopkins University, August.
- [12] Duffie, Darrell and Kenneth Singleton, 1999. "Modeling term structures of defaultable bonds." *Review of Financial Studies* 12 (4), 687-720.
- [13] Eisenschmidt, Jens and Jens Tapking, 2009. "Liquidity risk premia in unsecured interbank money markets." ECB Working Paper No. 1025.
- [14] Ejsing, Jacob, Wolfgang Lemke and Emil Margaritov, 2010. "Fiscal expectations and euro area sovereign bond spreads," manuscript, European Central Bank.
- [15] Favero, Carlo, Marco Pagano, and Ernst-Ludwig von Thadden, 2010. "How does liquidity affect government bond yields?" *Journal of Financial and Quantitative Analysis* 45 (1), 107-134.
- [16] Geyer, Alois, Stephan Kossmeier, and Stefan Pichler, 2004. "Measuring systematic risk in EMU government yield spreads." *Review of Finance* 8 (2), 171-197.
- [17] Gürkaynak, Refet and Jonathan Wright, 2011. "Macroeconomics and the term structure." *Journal of Economic Literature*, forthcoming.
- [18] Gürkaynak, Refet, Brian Sack and Jonathan Wright, 2007. "The U.S. treasury yield curve: 1961 to the present." *Journal of Monetary Economics* 54 (8), 2291-2304.
- [19] Hansen, Lars Peter and Scott Richard, 1987. "The role of conditioning information in deducing testable restrictions implied by dynamic asset pricing models." *Econometrica* 50 (3), 587-614.
- [20] Haugh, David, Patrice Ollivaud and David Turner, 2009. "What drives sovereign risk premiums? An analysis of recent evidence from the euro area." OECD Economics Department, working paper no. 718, July.
- [21] Jardet, Caroline, Monfort, Alain and Fulvio Pegoraro, 2009. "No-arbitrage near-cointegrated VAR(p) term structure models, term premia and GDP growth." Banque de France Working Paper No. 234.

- [22] Joslin, Scott, Marcel Pribsch and Kenneth Singleton, 2010. “Risk premiums in dynamic term structure models with unspanned macro risks.” Manuscript, Stanford University, September.
- [23] Kim, Don H. and Athanasios Orphanides, 2005. “Term structure estimation with survey data on interest rate forecasts.” Federal Reserve Board, Finance and Economics Discussion Series 2005-48.
- [24] Laubach, Thomas, 2009. “New evidence on the interest rate effects of budget deficits and debt.” *Journal of the European Economic Association* 7, 858-885.
- [25] Laubach, Thomas, 2011. “Fiscal policy and interest rates: The role of sovereign default risk.” In Richard Clarida and Francesco Giavazzi (eds.), *NBER International Seminar on Macroeconomics* 2010, 7-29.
- [26] Lemke, Wolfgang, 2008. “An affine macro-finance term structure model for the euro area.” *North American Journal of Economics and Finance* 19, 41-69.
- [27] Manganelli, Simone, and Guido Wolswijk, 2009. “What drives spreads in the euro area government bond market?” *Economic Policy* Vol. 24, Issue 58, 191-240.
- [28] Monfort, Alain, and Jean-Paul Renne, 2011. “Credit and liquidity risks in euro-area sovereign yield curves.” Working paper, Banque de France, February.
- [29] Nelson, C. and A. Siegel, 1987. “Parsimonious modeling of yield curves.” *Journal of Business* 60, 473-489.
- [30] Rudebusch, Glenn and Tao Wu, 2008. “A macro-finance model of the term structure, monetary policy, and the economy.” *Economic Journal* 118, 906-926.
- [31] Schuknecht, Ludger, Jürgen von Hagen and Guido Wolswijk, 2010. “Government bond risk premiums in the EU revisited: The impact of the financial crisis.” European Central Bank, working paper 1152, February.
- [32] Sgherri, Silvia, and Edda Zoli, 2009. “Euro area sovereign risk during the crisis.” IMF working paper WP/09/222.

- [33] Svensson, Lars, 1994. "Estimating and interpreting forward rates: Sweden 1992-4."  
NBER working paper 4871.
- [34] Taylor, John and John C. Williams, 2009. "A black swan in the money market."  
*American Economic Journal: Macroeconomics* 1 (1), 58-83.

Table 1: Preliminary regressions of spreads on euro area macro and national fiscal variables.

	Belgium			France			Greece			Ireland		
	1-year	5-year	10-year	1-year	5-year	10-year	1-year	5-year	10-year	1-year	5-year	10-year
$r_t$	-0,026 (-0,74)	-0,068 (-1,37)	-0,062 (-1,43)	-0,0036 (-0,18)	-0,017 (-0,51)	-0,019 (-0,42)	-0,28* (-1,89)	-0,21* (-1,79)	-0,13* (-1,75)	-0,077** (-2,1)	-0,18* (-1,89)	-0,16* (-1,75)
$LVI X_t$	0,11* (1,65)	0,39*** (4,11)	0,38*** (4,18)	0,054 (1,53)	0,15*** (2,74)	0,16** (2,28)	0,83*** (2,89)	0,81*** (3,44)	0,57*** (3,75)	0,36*** (4,7)	0,68*** (5,3)	0,62*** (5,19)
$ESI_t$	0,068*** (2,75)	0,025 (0,49)	0,091* (1,95)	0,0054 (0,51)	-0,02 (-1,04)	0,0094 (0,29)	0,15 (0,91)	-0,11 (-0,79)	-0,16* (-1,7)	0,049 (0,82)	0,15 (1,29)	0,16 (1,64)
$f_{i,t}$	0,012 (1,04)	0,026* (1,72)	0,033** (2,48)	-0,0054 (-0,97)	0,0072 (0,68)	0,011 (0,8)	0,085*** (3,79)	0,096*** (5,36)	0,085*** (7,44)	0,022*** (3,47)	0,055*** (4,4)	0,06*** (5,34)
$R^2$	0,18	0,54	0,57	0,079	0,57	0,37	0,5	0,71	0,8	0,63	0,77	0,83
$DW$	0,41	0,32	0,46	1,18	0,43	0,64	0,42	0,31	0,37	1,07	0,52	0,44

Table 2: Preliminary regressions of spreads on euro area macro and national fiscal variables (cont'd).

	Italy			Portugal			Spain		
	1-year	5-year	10-year	1-year	5-year	10-year	1-year	5-year	10-year
$r_t$	-0,14 (-1,5)	-0,13 (-1,08)	-0,1 (-0,86)	-0,19** (-2,25)	-0,29*** (-2,78)	-0,27*** (-2,92)	-0,15 (-1,33)	-0,16 (-1,28)	-0,13 (-1,16)
$LVIX_t$	0,51*** (3,76)	0,71*** (4,33)	0,69*** (4,1)	0,84*** (3,93)	1,2*** (4,48)	1*** (4,46)	0,38 (1,4)	0,56* (1,9)	0,49* (1,89)
$ESI_t$	0,089** (2,44)	0,075 (1,27)	0,11 (1,51)	0,28*** (2,67)	0,33** (2,22)	0,28** (2,12)	0,14** (2,13)	0,13 (1,64)	0,14** (1,97)
$f_{i,t}$	0,019 (0,81)	0,045 (1,51)	0,063** (1,98)	0,099*** (2,89)	0,14*** (3,05)	0,12*** (2,82)	0,031 (1,26)	0,048* (1,75)	0,056** (2,19)
$R^2$	0,49	0,57	0,58	0,53	0,65	0,64	0,47	0,56	0,62
$DW$	0,47	0,32	0,29	0,96	0,33	0,26	0,51	0,37	0,26

Table 3: Estimated parameters of the historical dynamics of the pricing factors.

			$\Phi_{xx,r}$	$\Phi_{xx,LVIX}$	$\Phi_{xx,ESI}$
$\Phi_{ff,BE}$	0,993*** (0,0049)	$\Phi_{xx,r}$	0,978***	-0,0095	0,0433
$\Phi_{ff,FR}$	0,976*** (0,0069)	$\Phi_{xx,LVIX}$	(0,022)	(0,063)	(0,031)
$\Phi_{ff,GR}$	0,968*** (0,034)	$\Phi_{xx,ESI}$	0,044*	0,915***	-0,0126
$\Phi_{ff,IR}$	0,968*** (0,025)		(0,026)	(0,08)	(0,028)
$\Phi_{ff,IT}$	0,979*** (0,029)		-0,0404	-0,00753	0,971***
$\Phi_{ff,PO}$	0,954*** (0,018)		(0,035)	(0,1)	(0,039)
$\Phi_{ff,SP}$	0,979*** (0,025)				



Table 4: Estimated parameters of the risk-neutral dynamics of the pricing factors.

$\Phi_{ff,BE}^*$	1,01***		$\Phi_{xx,r}^*$	$\Phi_{xx,LVIX}^*$	$\Phi_{xx,ESI}^*$
	(0,0073)	$\Phi_{xx,r}^*$	0,981***	0,00726	-0
$\Phi_{ff,FR}^*$	1,01***		(0,0091)	(0,01)	(0,0062)
	(0,008)	$\Phi_{xx,LVIX}^*$	-0,000359	1***	-0
$\Phi_{ff,GR}^*$	0,99***		(0,031)	(0,011)	(0,0081)
	(0,0099)	$\Phi_{xx,ESI}^*$	-0,00644	0	0,999***
$\Phi_{ff,IR}^*$	1,01***		(0,056)	(0,022)	(0,014)
	(0,0038)				
$\Phi_{ff,IT}^*$	1***				
	(0,0048)				
$\Phi_{ff,PO}^*$	1***				
	(0,0093)				
$\Phi_{ff,SP}^*$	1***				
	(0,0033)				

Table 5: Estimated parameters of the covariance matrix of innovations to the pricing factors.

	$\Sigma_{ff,BE}$	$\Sigma_{ff,FR}$	$\Sigma_{ff,GR}$	$\Sigma_{ff,IR}$	$\Sigma_{ff,IT}$	$\Sigma_{ff,PO}$	$\Sigma_{ff,SP}$
$\Sigma_{ff,BE}$	0,82*** (0,31)						
$\Sigma_{ff,FR}$	0,19* (0,11)	0,38* (0,21)					
$\Sigma_{ff,GR}$	0,64*** (0,19)	0,64*** (0,19)	1,27*** (0,37)				
$\Sigma_{ff,IR}$	0,94** (0,46)	0,94** (0,46)	0,94** (0,46)	1,88** (0,92)			
$\Sigma_{ff,IT}$	0,12*** (0,03)	0,12*** (0,03)	0,12*** (0,03)	0,12*** (0,03)	0,24*** (0,06)		
$\Sigma_{ff,PO}$	0,31*** (0,01)	0,31*** (0,01)	0,31*** (0,01)	0,31*** (0,01)	0,31*** (0,01)	0,62*** (0,01)	
$\Sigma_{ff,SP}$	0,22*** (0,07)	0,22*** (0,07)	0,22*** (0,07)	0,22*** (0,07)	0,22*** (0,07)	0,22*** (0,07)	0,43*** (0,14)

	$\Sigma_{xx,r}$	$\Sigma_{xx,LVIX}$	$\Sigma_{xx,ESI}$
$\Sigma_{xx,r}$	0,17*** (0,02)		
$\Sigma_{xx,LVIX}$	-0,04 (0,03)	0,17*** (0,03)	
$\Sigma_{xx,ESI}$	0,09*** (0,03)	-0,02 (0,03)	0,15*** (0,02)

Table 6: Estimated parameters of the default sensitivities.

	$\gamma_0$	$\gamma_{1,f}$	$\gamma_{1,r}$	$\gamma_{1,LVIX}$	$\gamma_{1,ESI}$
<i>BE</i>	1,4*	0,614***	-0,0547	2,73*	1,69
	(0,79)	(0,2)	(0,99)	(1,4)	(1,1)
<i>FR</i>	0	0,223	-0,0102	0,616	0,389
	(0,21)	(0,15)	(0,31)	(0,48)	(0,25)
<i>GR</i>	0,132	4,1***	-3,17	21,6***	11,4***
	(2,8)	(0,84)	(2,4)	(1,1)	(1,6)
<i>IR</i>	1,24	0,733***	-0,245	6,42**	4,06***
	(1,9)	(0,13)	(1,8)	(2,9)	(1,4)
<i>IT</i>	0,552	2,27***	-0,121	3,75***	2,25***
	(0,56)	(0,7)	(0,75)	(1,3)	(0,69)
<i>PO</i>	0	2,27***	-0,656	6,82**	4,89***
	(1,9)	(0,57)	(1,4)	(3)	(1,3)
<i>SP</i>	1,43	1,54***	-0,15	3,8*	2,41**
	(1,2)	(0,46)	(0,97)	(2,1)	(1,2)

Table 7: Estimated standard deviations of measurement errors associated with current yields (first row) and survey-based forecasts of yields or spreads (second row).

	<i>r</i>	<i>GE</i>	<i>BE</i>	<i>FR</i>	<i>GR</i>	<i>IR</i>	<i>IT</i>	<i>PO</i>	<i>SP</i>
$\sigma_R$		0,447***	0,139***	0,0759***	0,514***	0,216***	0,119***	0,353***	0,147***
		(0,042)	(0,029)	(0,007)	(0,12)	(0,041)	(0,035)	(0,077)	(0,016)
$\sigma_{forec}$	0,319***	0,537***		0,0979***			0,157***		0,148***
	(0,048)	(0,19)		(0,016)			(0,029)		(0,038)

Table 8: Estimated elasticities of spreads and PDs to the national fiscal factors. Due to constraints embedded in the model, the fiscal factors can be interpreted as the expected change in the Debt/GDP ratio at a one-year horizon.

	Spreads (in bp/pp)			PD (in pp/pp)		
	2-year	5-year	10-year	2-year	5-year	10-year
Belgium	8 (2,6)	9 (3,3)	11,1 (6,2)	0,22 (0,07)	0,38 (0,13)	0,47 (0,17)
France	3,10 (1,9)	3,80 (2,2)	5,70 (3,6)	0,07 (0,04)	0,12 (0,08)	0,13 (0,11)
Greece	43,4 (9,2)	36,6 (11,2)	28,2 (14)	1,35 (0,44)	2,14 (1,22)	2,44 (1,85)
Ireland	9,4 (1,8)	10,4 (2,5)	12,3 (4,2)	0,27 (0,09)	0,49 (0,33)	0,63 (0,67)
Italy	28,9 (8,8)	31,5 (10,2)	36,6 (15,2)	0,64 (0,19)	0,89 (0,33)	0,94 (0,39)
Portugal	28 (7,3)	29,1 (10,5)	31,1 (19,5)	0,84 (0,25)	1,51 (0,85)	1,92 (1,69)
Spain	19,1 (5,5)	20,1 (5,7)	21,8 (6,7)	0,5 (0,24)	0,78 (0,53)	0,89 (0,73)

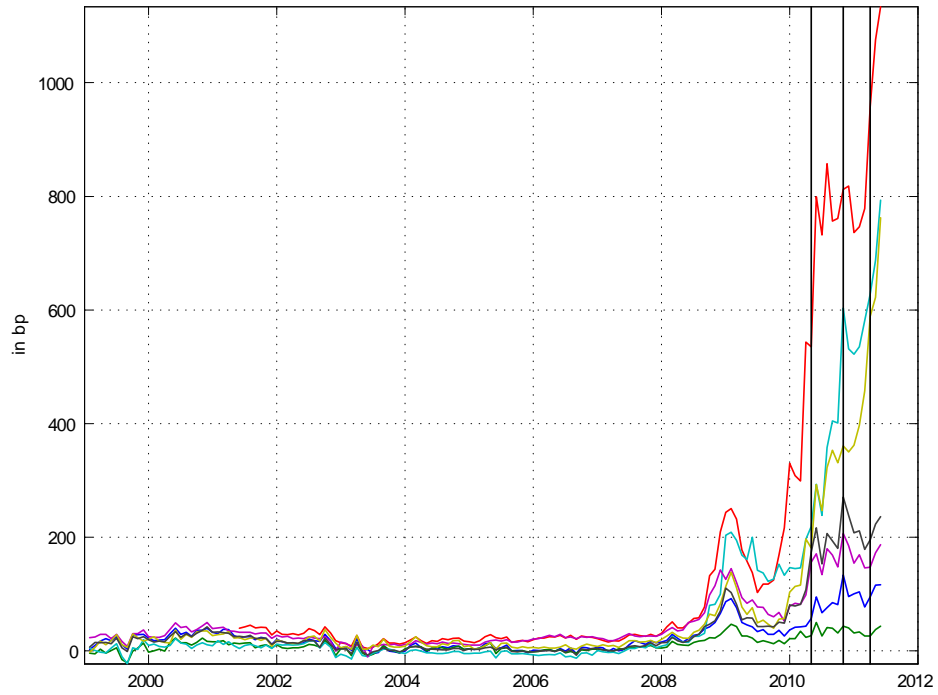


Figure 1: 10-year sovereign bond spreads of selected euro area countries against Germany. Vertical lines indicate months when Greece, Ireland and Portugal asked for assistance to the EFSF (in May 2010, November 2010 and April 2011 resp.)

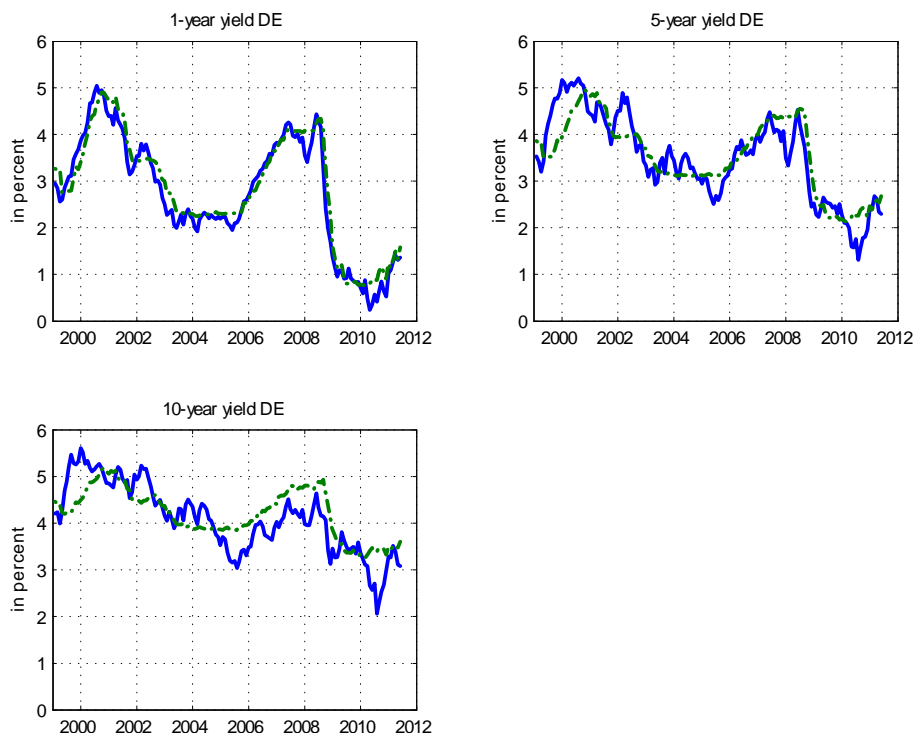


Figure 2: Fitted (dashed) and observed (solid) German yields

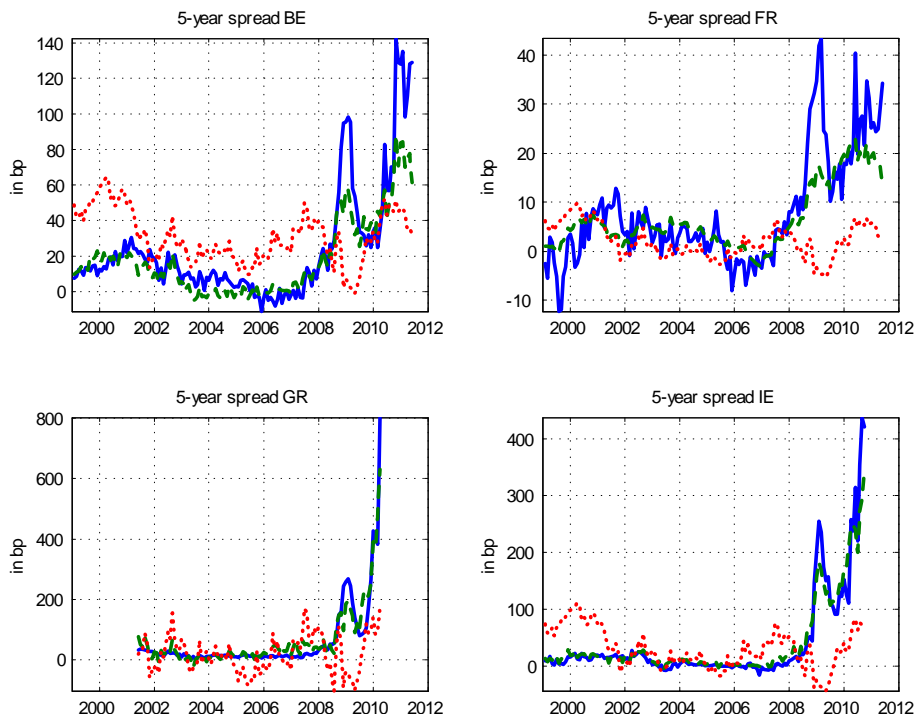


Figure 3: Fitted (dashed) and observed (solid) spreads, together with the contribution of the three euro area observed factors (dotted lines).

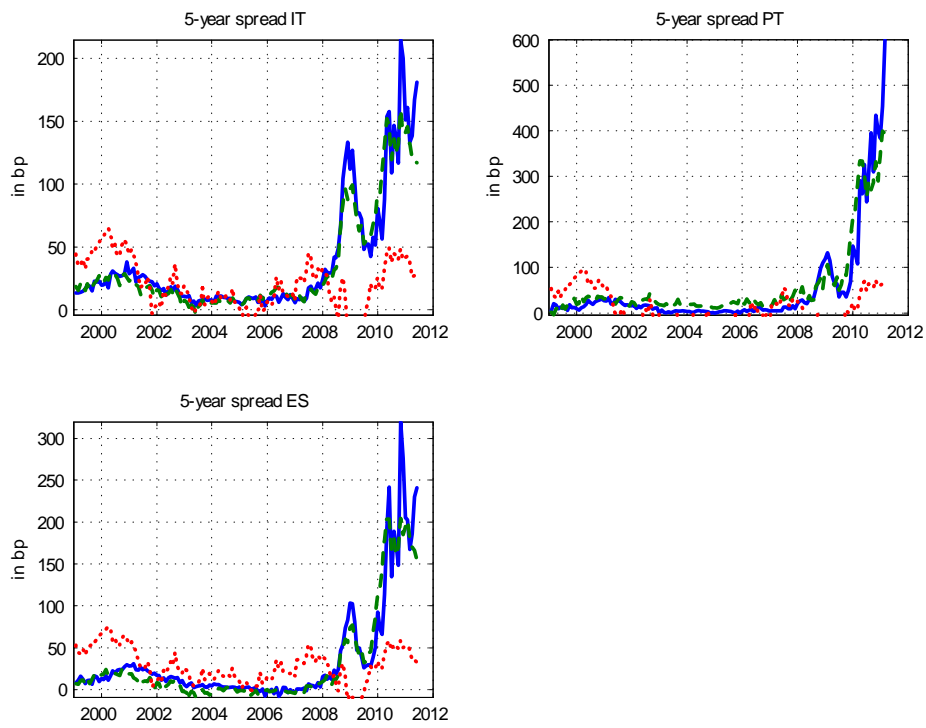


Figure 4: Fitted (dashed) and observed (solid) spreads, together with the contribution of the three euro area observed factors (dotted lines).





Figure 5: Survey-based (solid) and model-implied (dashed) forecasts of 3-month EURIBOR.

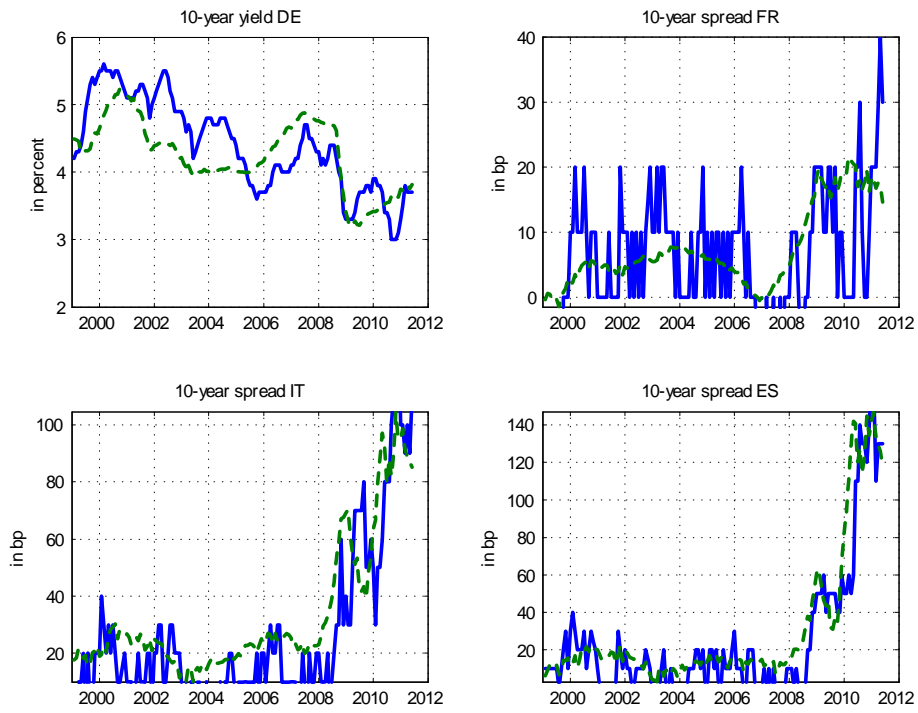


Figure 6: Survey-based (solid) and model-implied (dashed) forecasts of 10-year German yields and French, Italian and Spanish 10-year spreads.

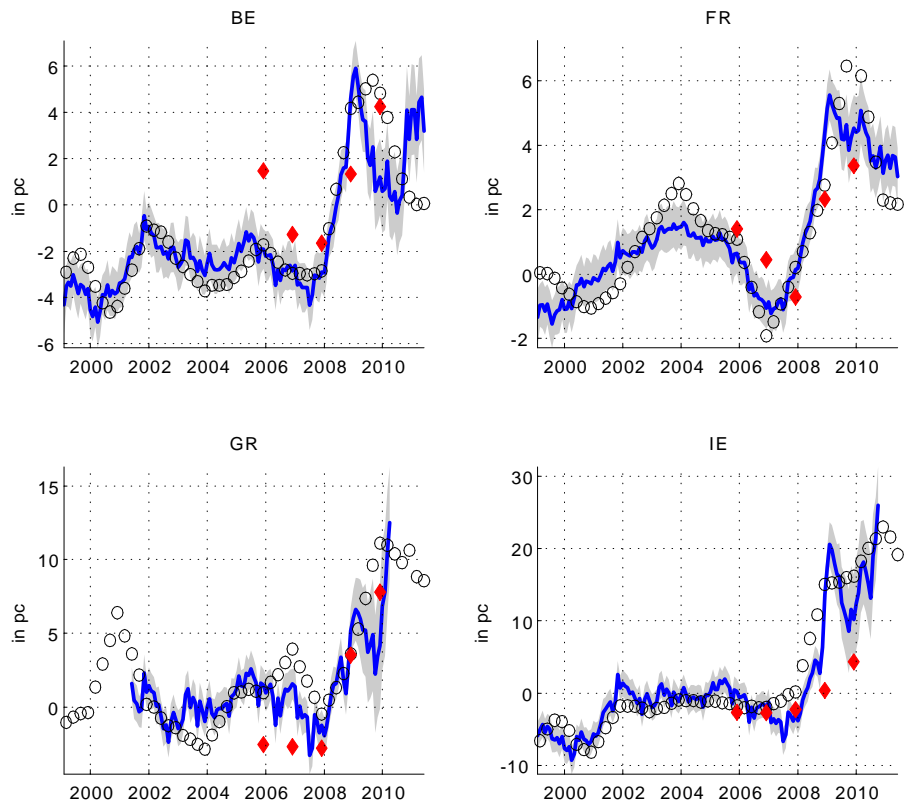


Figure 7: Model-implied "observed" (circles) and estimated (solid) expected one-year-ahead change in the Debt/GDP ratio, together with real-time OECD forecasts (diamonds). The grey-shaded areas correspond to 95% confidence interval (reflecting the Kalman-filtering errors).

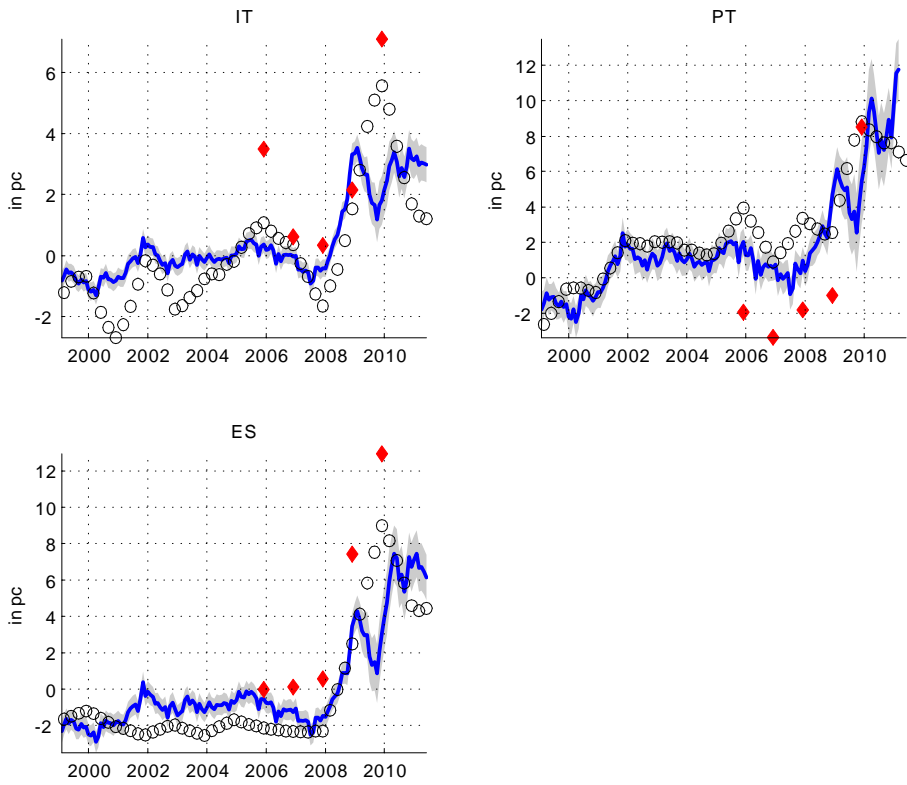


Figure 8: Model-implied "observed" (circles) and estimated (solid) expected one-year-ahead change in the Debt/GDP ratio, together with real-time OECD forecasts (diamonds) (cont'd). The grey-shaded areas correspond to 95% confidence interval (reflecting the Kalman-filtering errors).

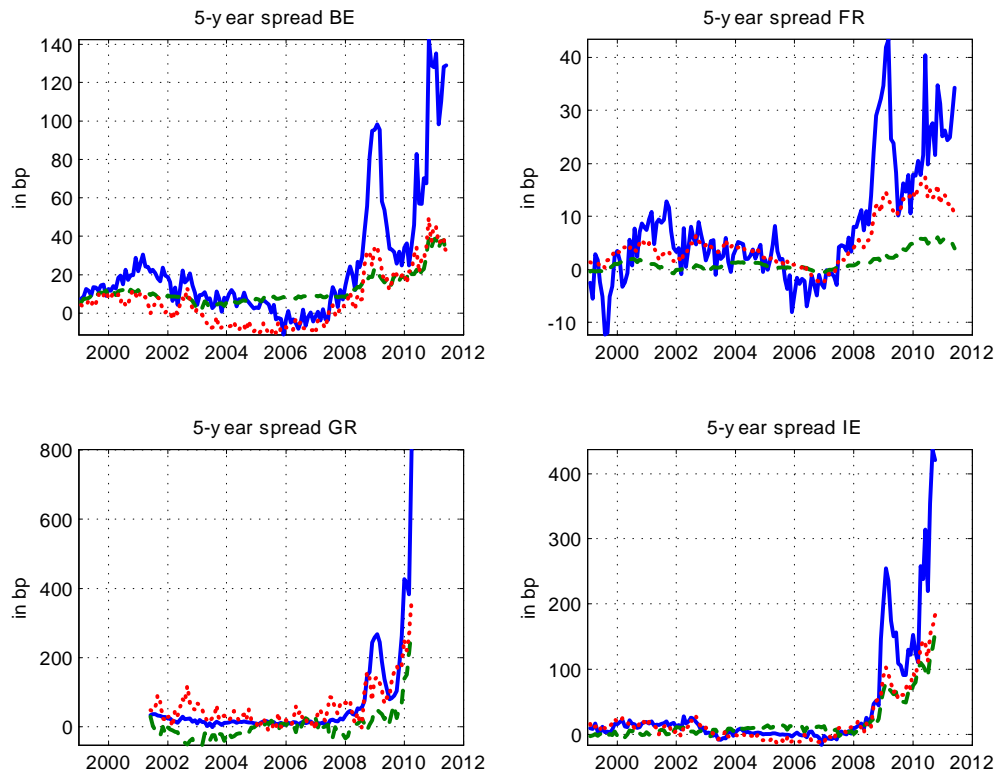


Figure 9: 5-year spreads (solid) vs term premia (dotted) and compensations for default (dashed)

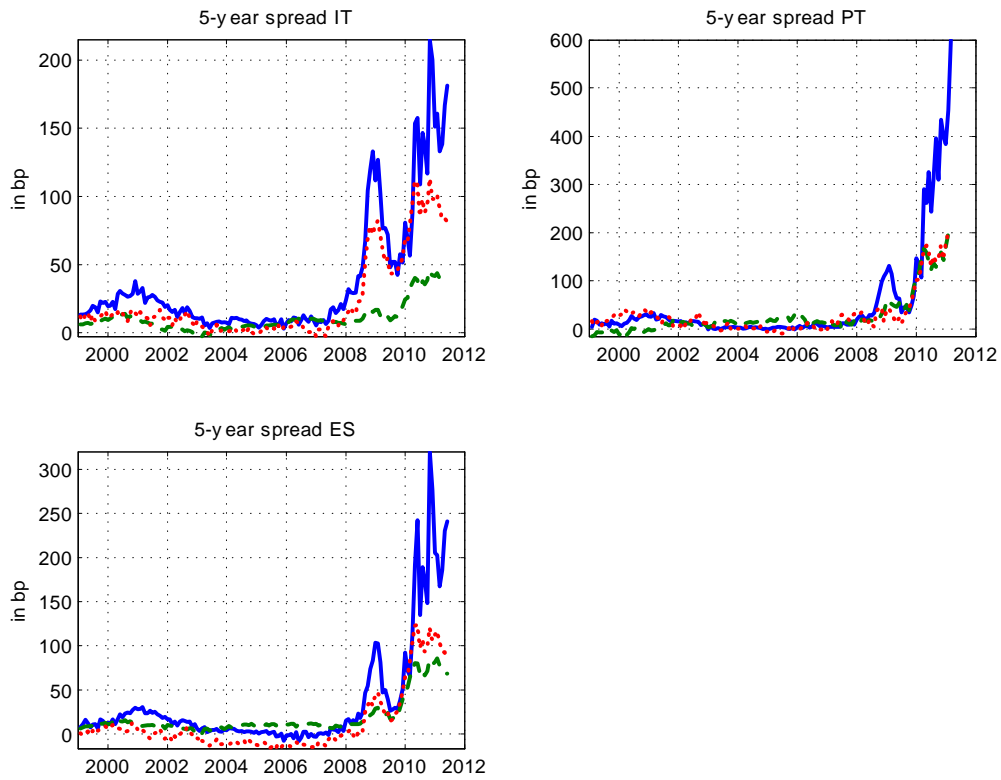


Figure 10: 5-year spreads (solid) vs term premia (dotted) and compensations for default (dashed)

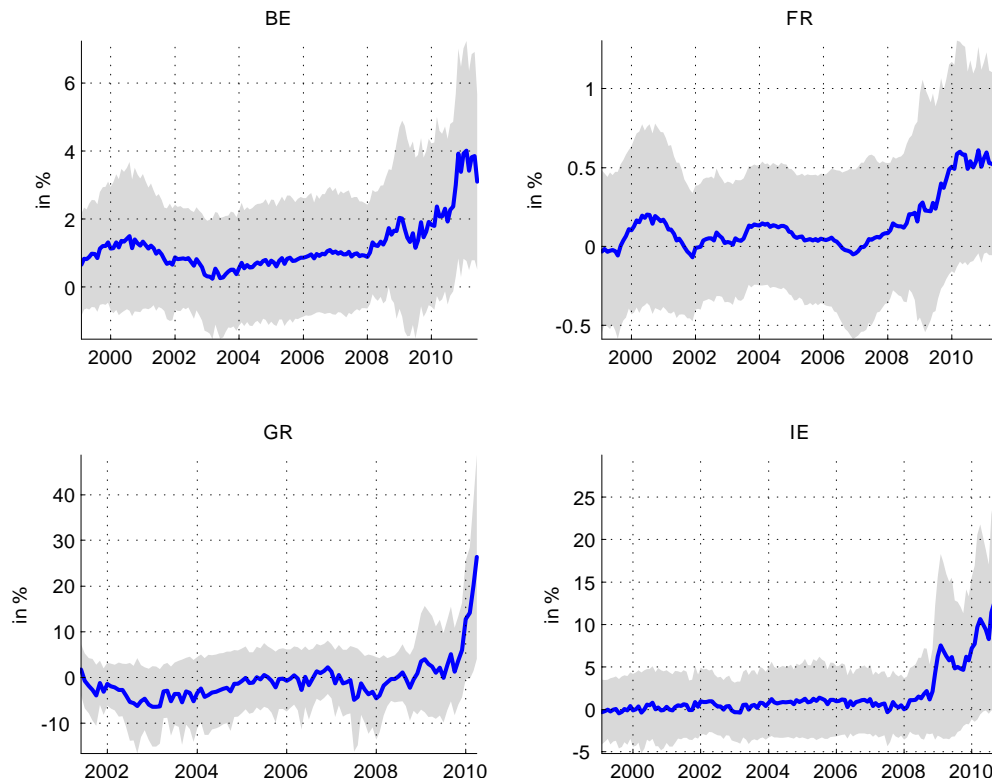


Figure 11: Expected default probabilities at 5 years horizon (assuming LGD of 50%).

The grey-shaded areas correspond to 90% confidence intervals.

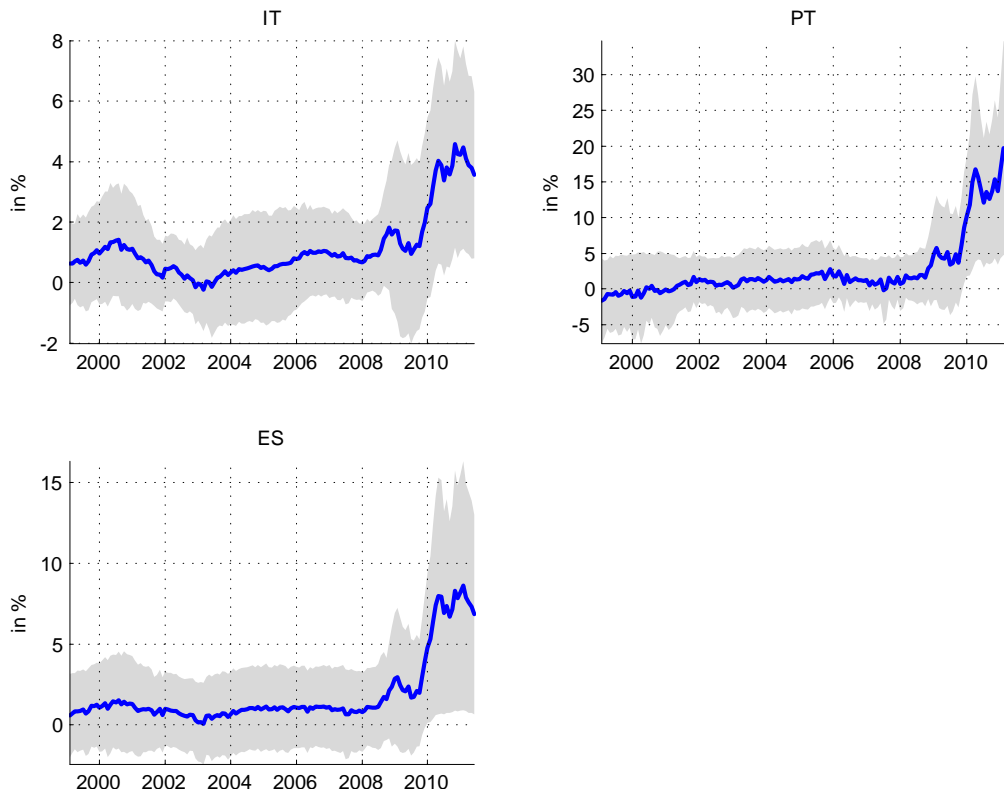


Figure 12: Expected default probabilities at 5 years horizon (assuming LGD of 50%).  
 (cont'd). The grey-shaded areas correspond to 90% confidence intervals.



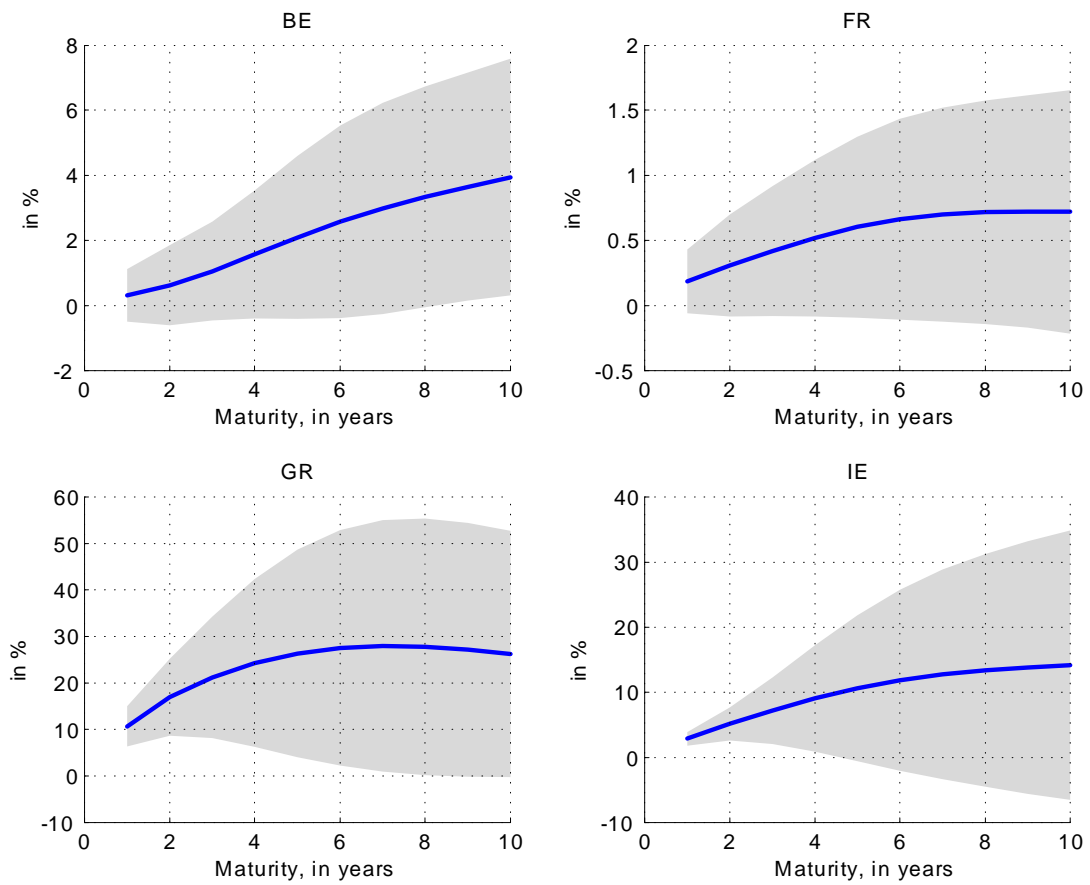


Figure 13: Term structure of perceived default probabilities as of end April 2010. The grey-shaded areas correspond to 90% confidence intervals.

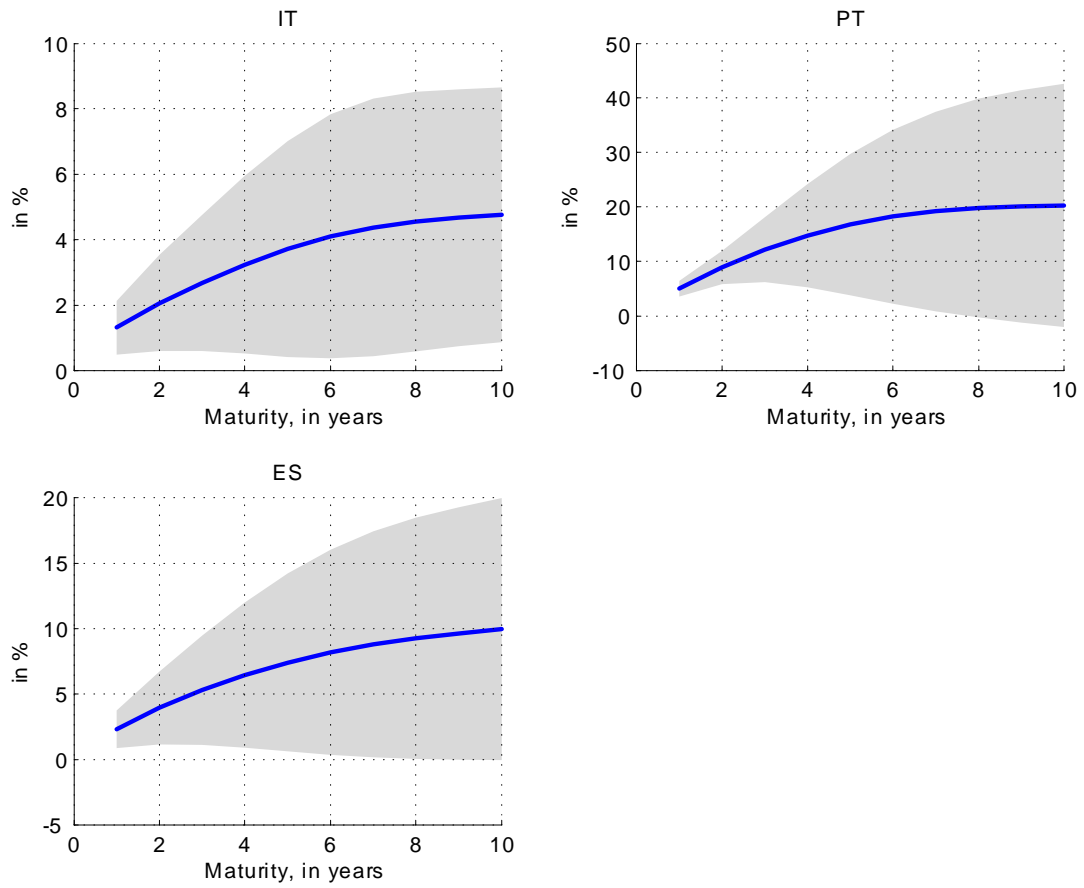


Figure 14: Term structure of perceived default probabilities as of end April 2010. (cont'd). The grey-shaded areas correspond to 90% confidence intervals.