Do credit market imperfections justify a central bank's response to asset price fluctuations?

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Abstract

Do credit market imperfections justify a central bank's response to asset price fluctuations? This paper addresses this question from the perspective of equilibrium determinacy. In the model used in this paper, the prices are sticky and the working capital of firms is subject to asset values because of a lack of commitment. If credit market imperfections exist to a small degree, the Taylor principle is a necessary and sufficient condition for equilibrium determinacy, and monetary policy response to asset price fluctuations is good from the perspective of equilibrium determinacy. However, if credit market imperfections exist to a large degree such that the collateral constraint is binding, then the Taylor principle no longer guarantees equilibrium determinacy, and monetary policy response to asset price fluctuations becomes a source of equilibrium indeterminacy. We find that the existence of credit market imperfections denies the propriety of monetary policy response to asset price fluctuations. We also find that improvements in credit market imperfections reduce the determinacy region of the model parameters.

Keywords: asset prices; credit market imperfections; collateral constraints; equilibrium indeterminacy; monetary policy; sticky prices; Taylor principle

JEL classification: C62; E32; E44; E52

1 Introduction

One of the classical topics in the context of monetary policy is a central bank's response to asset price fluctuations. Japan's boom period during the late 1980s and the long stagnation during the 1990s and the recent boom and bust of the U.S. economy seem to imply that a central bank should respond to asset price fluctuations. It is often said that credit market imperfections play an important role in the boom-bust periods. In this scenario, should a central bank respond to asset price fluctuations and do credit market imperfections justify such a central bank's response?

In this paper, we address this question from the perspective of equilibrium determinacy. Following the standard Calvo-type setting, the prices are sticky in our model. We also assume that the working capital of firms is subject to asset values because of a lack of commitment.

In cases where the collateral constraint never binds, the Taylor principle guarantees equilibrium determinacy. In this case, a positive response of monetary policy to asset price fluctuations increases the determinacy region of the parameters.

If the collateral constraint binds deeply, the properties of the determinacy regions completely differ from when the collateral constraint never binds. We find that the Taylor principle no longer guarantees equilibrium determinacy and that monetary policy response to asset price fluctuations is a source of equilibrium indeterminacy. We show that both the sufficiently positive and negative sensitivity of monetary policy are sources of equilibrium indeterminacy, while a slightly positive or negative response of monetary policy to asset price fluctuations might increase the determinacy region.

Our results imply that the existence of credit market imperfections denies the propriety of monetary policy response to asset price fluctuations.

We also investigate the relationship between credit market imperfections and the determinacy region. We find that improvements in credit market imperfections can sometimes reduce the determinacy region of model parameters. While it is intuitive that improvements in credit market imperfections have positive effects, our result shows that this simple intuition is not correct from the perspective of equilibrium determinacy.

Many papers on monetary policy and asset prices deal with these topics from the welfare perspective. Bernanke and Gertler (2001) and Gilchrist and Leahy (2002) find that responding to asset price fluctuations is not important. Iacoviello (2005) shows that monetary policy response to asset price fluctuations generates welfare gain. Faia and Monacelli (2007) find that monetary policy should negatively respond to asset price fluctuations. In this paper, however, we discuss this question from the perspective of equilibrium indeterminacy.

This paper is closely related to the study by Carlstrom and Fuerst (2007), who also focus on equilibrium indeterminacy. Carlstrom and Fuerst (2007) show that equilibrium indeterminacy arises if monetary policy responds to share prices in a standard sticky-price economy. However, in their model, there is no credit market imperfection. We show that in our model, monetary policy response to asset price fluctuations is a source of equilibrium indeterminacy in an economy with credit market imperfections, while it is a source of equilibrium determinacy if there is no credit market imperfection.

Collateral constraints are often employed to account for the observed facts of business cycles in modern macroeconomics. Bernanke, Gertler, and Gilchrist (1999), Kiyotaki and Moore (1997), Liu, Wand, and Zha (2009) show that collateral constraints amplify shock effects. Carlstrom and Fuerst (1997, 1998) show that collateral constraints generate hump-shaped responses to shocks. Kobayashi, Nakajima, and Inaba (2010) and Kobayashi and Nutahara (2007) show that a model with collateral constraints generates comovements of output, consumption, labor, and investment to news shocks. Monacelli (2009) shows that a model with collateral constraints accounts for sectoral comovements to monetary policy shocks. Given this information, analyses of a model with credit market imperfections would be important.

This paper is also related to Harrison and Weder's (2009) study. They investigate equilibrium indeterminacy in a real model with collateral constraints and increasing returns to scale. In this paper, we consider equilibrium indeterminacy in a monetary model with collateral constraints and constant returns to scale.

The rest of this paper is organized as follows. In Section 2, we introduce our model. In our model, prices are sticky and the working capital is subject to asset values because of a lack of commitment. In Section 3, we investigate the equilibrium determinacy of the model and present the main results. In Section 4, we verify the robustness of our results. Finally, in Section 5, we draw conclusions.

2 The model

Our model is based on one employed by Carlstrom and Fuerst (2007). One departure from their model is that our model contains capital stock[1] and there is a collateral constraint on working capital. In order to allow for a collateral constraint, our economy deviates slightly from that in study of Carlstrom and Fuerst (2007). However, the equilibrium system is identical to the system of Carlstrom and Fuerst (2007) that includes capital if collateral constraints never bind.

2.1 Households: workers and managers

In our model, we consider households that consist of workers and managers. The household begins period t with M_t cash balances, B_{t-1} one-period nominal bonds that pay R_{t-1} gross interest rate, N_{t-1} shares of the stock of retailers that sell at price Q_t and pay dividend D_t.

The utility function is

[1]For simplicity of the analyses, we assume that the total supply of capital is fixed. Dupor (2001) and Carlstrom and Fuerst (2004) analyze the equilibrium determinacy of sticky-price models with variable capital stock. In their models, the credit markets are perfect.

$$U(C_t, H_t, M_{t+1}/P_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \zeta \frac{H_t^{1+\gamma}}{1+\gamma} + V(M_{t+1}/P_t), \tag{1}$$

where $\sigma > 0$, $\gamma > 0$, $\zeta > 0$, V is increasing and concave, C_t denotes consumption, H_t denotes labor supply, and M_{t+1}/P_t denotes real cash balances at the beginning of period t.

At the beginning of each period, a household is divided into a worker and a manager. A worker supplies labor H_t and earns wage income $P_t W_t H_t$, where P_t denotes the aggregate price level. A manager produces homogenous goods. The production function of managers is

$$Y_t = K_t^{\alpha} L_t^{1-\alpha}, \tag{2}$$

where $0 < \alpha < 1$, Y_t denotes output, K_t denotes capital stock, and L_t denotes labor demand. We assume that managers have to pay wages to workers in advance and they borrow working capital from banks. We also assume that their borrowing for working capital is subject to a collateral constraint:

$$P_t W_t L_t \leq \varphi P_t Q_t K_t. \tag{3}$$

where $0 < \varphi \leq 1$ since there is a lack of commitment among managers.[2] In order to consider a collateral constraint, we assume that a worker cannot supply to a manager from the same agent.

The agent's budget constraint is

$$\begin{aligned} &P_t C_t + M_{t+1} + P_t Q_t K_{t+1} + B_{t+1} + P_t W_t L_t \\ &\quad \leq P_t Z_t Y_t + P_t W_t H_t + M_t + P_t Q_t K_t + R_{t-1} B_t + X_t, \end{aligned} \tag{4}$$

[2]A similar setting of credit market imperfections is employed by Harrison and Weder (2009), Kobayashi, Nakajima, and Inaba (2010), Kobayashi and Nutahara (2007), and Mendoza (2010). We assume that money does not affect collateral constraints, since there is also a lack of commitment among workers, as considered by Kobayashi (2011).

where B_t denotes the bond holding and Z_t denotes the relative price of goods produced by managers[3] and X_t denotes monetary injection.

The first-order conditions are as follows:

$$\zeta C_t^{\sigma} H_t^{\gamma} = W_t, \tag{5}$$

$$C_t^{-\sigma} = \beta C_{t+1}^{-\sigma} \frac{R_t}{\Pi_{t+1}}, \tag{6}$$

$$C_t^{-\sigma} Q_t = \beta C_{t+1}^{-\sigma} \left[Q_{t+1}(1 + \varphi \Theta_{t+1}) + \alpha Z_{t+1} \frac{Y_{t+1}}{K_{t+1}} \right], \tag{7}$$

$$W_t(1 + \Theta_t) = (1 - \alpha) Z_t \frac{Y_t}{L_t}, \tag{8}$$

$$(W_t L_t - \varphi Q_t K_t)\Theta_t = 0, \qquad \Theta_t \geq 0, \tag{9}$$

where $\Pi_{t+1} \equiv P_{t+1}/P_t$ and Θ_t denote the ratio of the Lagrange multiplier of the collateral constraint to that of the budget constraint, and it can be interpreted as the inefficiency of the collateral constraint. Equation (5) is the intratemporal optimization condition; (6) is the Euler equation of bond; (7) is the Euler equation of capital stock; (8) is the marginal productivity condition of labor; and (9) is the collateral constraint.

Equation (7) can be rewritten in the form of familiar equations on asset prices:

$$Q_t = \left[Q_{t+1}(1 + \varphi \Theta_{t+1}) + \alpha Z_{t+1} \frac{Y_{t+1}}{K_{t+1}} \right] \frac{\Pi_{t+1}}{R_t}. \tag{10}$$

2.2 Retailers

We assume the existence of monopolistically competitive retailers, as in the study of Bernanke, Gertler, and Gilchrist (1999).

Retailers buy goods at price $P_t Z_t$ from managers, produce differentiated goods using a linear technology, and set prices. The price can be re-optimized at period t only with probability $1 - \kappa$. Under this standard Calvo-type sticky-price setting, as shown by Yun

[3] Z_t is also interpreted as the real marginal cost.

(1996), the New Keynesian Phillips curve is

$$\pi_t = \lambda z_t + \beta \pi_{t+1}, \tag{11}$$

where

$$\lambda \equiv \frac{(1-\kappa)(1-\kappa\beta)}{\kappa}, \tag{12}$$

and the lowercased letters denote log deviations from the steady state.

2.3 Monetary policy

We assume that the monetary authority follows a modified Taylor rule:

$$r_t = \tau_\pi \pi_t + \tau_k q_t, \tag{13}$$

where r_t denotes the log-deviation of the nominal interest rate, R_t, from a steady state. The parameters τ_π and τ_k denote the central bank's response to inflation and asset price fluctuations.

In this paper, we focus on the case of $\tau_\pi > 0$.

2.4 Market clearing conditions

For simplicity of the analyses, we assume that the total supply of capital stock is 1,

$$K_t = 1, \tag{14}$$

and the clearing condition of the goods market is

$$Y_t = C_t. \tag{15}$$

We focus on a symmetric equilibrium. The labor market clearing condition is

$$H_t = L_t. \tag{16}$$

2.5 Equilibrium

The equilibrium system of this economy is

$$\zeta C_t^{\sigma} L_t^{\gamma} = W_t, \tag{17}$$

$$C_t^{-\sigma} = \beta C_{t+1}^{-\sigma} \frac{R_t}{\Pi_{t+1}}, \tag{18}$$

$$Q_t = \left[Q_{t+1}(1 + \varphi \Theta_{t+1}) + \alpha Z_{t+1} Y_{t+1} \right] \frac{\Pi_{t+1}}{R_t}, \tag{19}$$

$$W_t(1 + \Theta_t) = (1 - \alpha) Z_t \frac{Y_t}{L_t}, \tag{20}$$

$$(W_t L_t - \varphi Q_t)\Theta_t = 0, \qquad \Theta_t \geq 0, \tag{21}$$

$$Y_t = L_t^{1-\alpha}, \tag{22}$$

$$Y_t = C_t, \tag{23}$$

$$\pi_t = \lambda z_t + \beta \pi_{t+1}, \tag{24}$$

$$r_t = \tau_\pi \pi_t + \tau_q q_t. \tag{25}$$

Before proceeding to the main analysis, we investigate the condition for the binding collateral constraint. The following condition is necessary and sufficient for a binding collateral constraint at a steady state.

Proposition 1. *A collateral constraint (3) is binding at a steady state if and only if*

$$\varphi < \varphi^{max} \equiv \frac{(1-\alpha)(1-\beta)}{\alpha\beta}. \tag{26}$$

Proof. By the steady-state equilibrium system, we obtain

$$W = \zeta L^{\sigma(1-\alpha)+\gamma},$$

$$L = \frac{1}{\zeta} \left[\frac{1-\alpha}{1+\Theta} Z \right]^{1/(\sigma(1-\alpha)+\gamma+\alpha)},$$

$$Q = \frac{\alpha Z}{\frac{1}{\beta} - (1 + \varphi\Theta)} \left[\frac{1-\alpha}{1+\Theta} Z \right]^{(1-\alpha)/(\sigma(1-\alpha)+\gamma+\alpha)}.$$

Inserting these into a collateral constraint, $WL = \varphi Q$, yields

$$\Theta = \frac{(1-\alpha)(1-\beta)}{\beta\varphi} - \alpha,$$

where Θ is greater than 0 if and only if $\varphi < \varphi^{\max}$. □

3 Main results

3.1 A sticky-price economy where the collateral constraint never binds:

First, consider a case where the collateral constraint never binds. In this case, the following proposition holds.

Proposition 2. *Assume that (i)* $\beta > \frac{1+\gamma}{\sigma(1-\alpha)+1+\gamma}$ *and (ii) the collateral constraint never binds. Then, a necessary and sufficient condition for equilibrium determinacy is*

$$(\tau_\pi - 1)\lambda + \tau_k B(1-\beta) > 0.$$

where

$$B \equiv \frac{\sigma(1-\alpha) + 1 + \gamma}{\sigma(1-\alpha) + \alpha + \gamma} > 0.$$

Proof. See the proof of Proposition 1 by Nutahara (2010). The model where the collateral constraint never binds is the same as the model of Nutahara (2010) with $\tau_q = 0$. □

Nutahara (2010) shows that the Taylor principle–a permanent increase in the inflation rate leads to a more than proportionate increase in the inflation rate–is a necessary and sufficient condition for equilibrium determinacy in this model. Proposition 2 implies that a positive response of monetary policy to asset price fluctuations is a source of equilibrium determinacy, as follows:

Remark 1. *Even if $\tau_\pi < 1$, equilibrium determinacy is guaranteed if*

$$\tau_k > \frac{(1-\tau_\pi)\lambda}{B(1-\beta)}$$

in a sticky-price economy where the collateral constraint never binds.

An increase in inflation implies a high real marginal cost through the Phillips curve, and this implies a high rental rate of capital and high capital prices. Then, a positive monetary policy response to capital prices implicitly strengthens the overall response to inflation, and this is a source of equilibrium determinacy.

3.2 A sticky-price economy with the binding collateral constraint:

Here, we focus on a case where a collateral constraint is deeply binding. It is convenient to log-linearize our equilibrium system for the analyses. The linearized system with a binding collateral constraint is as follows:

$$\sigma c_t + \gamma \ell_t = w_t, \tag{27}$$

$$\sigma(c_{t+1} - c_t) = r_t - \pi_{t+1}, \tag{28}$$

$$q_t = \beta(1+\varphi\Theta)\left[q_{t+1} + \frac{\varphi\Theta}{1+\varphi\Theta}\theta_{t+1}\right] + [1-\beta(1+\varphi\Theta)](z_{t+1} + c_{t+1}) + (\pi_{t+1} - r_t), \tag{29}$$

$$c_t = (1-\alpha)\ell_t, \tag{30}$$

$$w_t + \ell_t = q_t, \tag{31}$$

$$z_t + c_t - \ell_t = w_t + \frac{\Theta}{1+\Theta}\theta_t, \tag{32}$$

$$\pi_t = \beta\pi_{t+1} + \lambda z_t, \tag{33}$$

$$r_t = \tau_\pi \pi_t + \tau_k q_t, \tag{34}$$

where the lowercased letters denote log deviations from the steady state. This system is reduced to the following matrix form:

$$
\begin{bmatrix} 1 & 0 & \Phi_1 \\ 1 & 1-\beta(1-\varphi) & \Phi_2 \\ \beta & 0 & 0 \end{bmatrix}
\begin{bmatrix} \pi_{t+1} \\ z_{t+1} \\ q_{t+1} \end{bmatrix}
=
\begin{bmatrix} \tau_\pi & 0 & \Phi_1+\tau_k \\ \tau_\pi & 0 & 1+\tau_k \\ 1 & -\lambda & 0 \end{bmatrix}
\begin{bmatrix} \pi_t \\ z_t \\ q_t \end{bmatrix}, \qquad (35)
$$

where

$$
\Phi_1 \equiv \frac{\sigma}{\sigma + \frac{1+\gamma}{1-\alpha}},
$$

$$
\Phi_2 \equiv \frac{[\sigma + \frac{1+\gamma}{1-\alpha} - 1]\beta(1-\varphi) + 1}{\sigma + \frac{1+\gamma}{1-\alpha}}.
$$

The first equation is the Euler equation of consumption (28). The second one is the Euler equation of asset price (29). The last one is the New Keynesian Phillips curve (33). Note that this system is closed by only first and second equations with π_t and q_t.

It is obvious that $0 < \Phi_1 < 1$ and $\Phi_2 > 0$. It is shown that $\Phi_2 < 1$ since

$$
1 - \Phi_2 = [1-\beta(1-\varphi)]\left(1 - \frac{1}{\sigma + \frac{1+\gamma}{1-\alpha}}\right) > 0. \qquad (36)
$$

This system can be rewritten as

$$
\begin{bmatrix} \pi_t \\ z_t \\ q_t \end{bmatrix}
=
\underbrace{
\begin{bmatrix} \tau_\pi & 0 & \Phi_1+\tau_k \\ \tau_\pi & 0 & 1+\tau_k \\ 1 & -\lambda & 0 \end{bmatrix}^{-1}
\begin{bmatrix} 1 & 0 & \Phi_1 \\ 1 & 1-\beta(1-\varphi) & \Phi_2 \\ \beta & 0 & 0 \end{bmatrix}
}_{\boldsymbol{G}}
\begin{bmatrix} \pi_{t+1} \\ z_{t+1} \\ q_{t+1} \end{bmatrix}, \qquad (37)
$$

Let T, M, and D denote the trace, the sum of the principal minors of order 2, and the determinant of $\boldsymbol{G}$, and they are given by

$$
T = \frac{1}{\tau_\pi \lambda(\Phi_1 - 1)}\Big\{\tau_\pi \lambda(\Phi_1 - \Phi_2) + \lambda(\Phi_1 - 1) + (\tau_k + \Phi_1)[1-\beta(1-\varphi)]\Big\}, \qquad (38)
$$

$$
M = \frac{1}{\tau_\pi \lambda(\Phi_1 - 1)}\Big\{\lambda(\Phi_1 - \Phi_2) + \Phi_1(1+\beta)[1-\beta(1-\varphi)] + \tau_k \beta[1-\beta(1-\varphi)]\Big\}, \qquad (39)
$$

$$
D = \frac{1}{\tau_\pi \lambda(\Phi_1 - 1)}\Big\{\beta\Phi_1[1-\beta(1-\varphi)]\Big\}. \qquad (40)
$$

The characteristic equation is

$$F(x) = -x^3 + Tx^2 - Mx + D, \tag{41}$$

and all roots should be within the unit circle for equilibrium determinacy.

A necessary and sufficient condition for equilibrium determinacy of this three-dimensional system is as follows.

Proposition 3. *Assume that $\tau_\pi > 0$, (26), and that the collateral constraint is always binding. A necessary and sufficient condition for equilibrium determinacy is*

$$(A) \quad \tau_\pi > \tau_\pi^{min} \equiv \frac{\beta\Phi_1[1-\beta(1-\varphi)]}{\lambda(1-\Phi_1)},$$
$$(B) \quad \tau_k^{min} < \tau_k < \tau_k^{max},$$
$$(C) \quad D^2 - TD + M < 1,$$

where

$$\tau_k^{min} \equiv -(\tau_\pi - 1)\frac{\lambda(1-\Phi_2)}{(1-\beta)[1-\beta(1-\varphi)]} < 0,$$
$$\tau_k^{max} \equiv (\tau_\pi + 1)\frac{\lambda(1-2\Phi_1+\Phi_2)}{1-\beta^2+\beta\varphi(1+\beta)} - 2\Phi_2.$$

Proof. Brooks (2004) shows that the necessary and sufficient conditions of the first-order three-dimensional discrete system are $|D| < 1$, $|T+D| < M+1$, and $D^2 - TD + M < 1$. The condition $|D| < 1$ is equivalent with (A). The condition $|T+D| < M+1$ is equivalent with (B). If $T+D > 0$, $|T+D| < M+1$ implies that $\tau_k^{\text{min}} < \tau_k < \tau_k^{\text{thres}}$ and that $\tau_k^{\text{thres}} \leq \tau_k < \tau_k^{\text{max}}$ if $T+D \leq 0$, where

$$\tau_k^{\text{thres}} \equiv \frac{\lambda(\Phi_2-\Phi_1)}{1-\beta(1-\varphi)}\tau_\pi + \frac{1}{1-\beta(1-\varphi)}\left\{\lambda(1-\Phi_1) + \Phi_1\left[1-\beta^2+\beta\varphi(1+\varphi)\right]\right\}.$$

□

Taylor principle and equilibrium indeterminacy: Under some conditions, the threshold $\tau_\pi^{\min}$ in Proposition 3 is greater than 1. For example, if $\sigma = 1$, $\gamma = 0$, $\alpha = 0.3$, $\beta = .99$, $\varphi = 0.02$, $\lambda = 0.019$, then $\tau_\pi^{\min}$ is about 1.09. If $\tau_k = 0$, the Taylor principle implies that $\tau_\pi > 1$. However, if $1 < \tau_\pi < \tau_\pi^{\min}$, then equilibrium indeterminacy arises. Therefore, the Taylor principle does not guarantee equilibrium determinacy.

Remark 2. *The Taylor principle does not guarantee equilibrium determinacy in a sticky-price economy with a binding collateral constraint.*

Monetary policy and equilibrium indeterminacy: Next, consider the relationship between equilibrium indeterminacy and the monetary policy response to asset price fluctuations. The following condition of τ_q, the central bank's response to asset price fluctuations, is necessary for equilibrium determinacy.

Remark 3. *If $\tau_k < \tau_k^{min}$ or $\tau_k > \tau_k^{max}$, equilibrium indeterminacy arises.*

Remark 3 implies that if a central bank is sufficiently sensitive to asset price fluctuations, equilibrium indeterminacy arises. Note that both the positive and negative sensitivity of monetary policy are sources of equilibrium indeterminacy.

To investigate the determinacy region of the model, we conduct numerical analyses. We employ the following parameter values along the lines of those used in the literature. The discount factor β is 0.99. The utility function is made up of log consumption and indivisible labor ($\sigma = 1$ and $\gamma = 0$). The cost weight of capital in the production function, α, is 0.3. The parameter of the New Keynesian Phillips curve, λ, is 0.019. The steady-state real marginal cost, Z, is 0.85. The parameter of credit market imperfections, φ, is 0.02 for the deeply binding collateral constraint at a steady state. Under these parameter values, we calculate the eigenvalues of $\boldsymbol{G}$ in equation (37) to check equilibrium determinacy.

Figure 1 shows the determinacy region in the (τ_k, τ_π) plane.

[Insert Figure 1]

In the region with red diamonds, equilibrium is determinate. Equilibrium indeterminacy arises in other regions. The vertical axis denotes the central bank's response to inflation τ_π. The horizontal axis denotes the central bank's response to the asset price τ_k.

As in Remark 3, both the sufficiently positive and negative sensitivity of monetary policy are sources of equilibrium indeterminacy. In Figure 1, the positive slope of the determinacy region reflects $\tau_k^{\max}$, and the negative slope reflects (C) in Proposition 3. Note that the determinacy region of τ_π is the largest when τ_k is slightly negative. However, the sign of the determinacy region's peak τ_k might be positive. Further, Figure 2 is similar to Figure 1 if $\gamma = 3$.

[Insert Figure 2]

Figure 2 shows that the determinacy region is shaped by three lines: the positive slope line of $\tau_k^{\max}$, the negative slope line of $\tau_k^{\min}$, and (C) in Proposition 3. The determinacy region of τ_π is the largest when τ_k is positive.

Finally, we find that both the sufficiently positive and negative sensitivity of monetary policy are sources of equilibrium indeterminacy, while a slightly positive or negative response might increase equilibrium determinacy. This result implies that the existence of credit market imperfections denies the propriety of monetary policy response to asset price fluctuations.

Credit market imperfections and equilibrium indeterminacy: How do credit market imperfections affect equilibrium determinacy? To address this question, we employ numerical analyses.

Figure 3 shows the determinacy region in the (τ_π, φ) plane.

[Insert Figure 3]

As in Figure 1, the region with red diamonds denotes equilibrium determinacy and the others indicate equilibrium indeterminacy. The vertical axis denotes the collateral constraint parameter φ. The horizontal axis denotes the central bank's response to inflation τ_π. We set the parameter of the central bank's response to asset prices, τ_k, as 0, and the others are the same as those in Figure 1. We consider cases where $\varphi \in [\varphi^{\text{max}}/2, \varphi^{\text{max}}]$.

Figure 3 shows that improvements in credit market imperfections, or increases in φ, have nonlinear effects on the determinacy region. If φ is larger than 0.018, an increase in φ reduces the determinacy region of τ_π, and higher values of τ_π are necessary for equilibrium determinacy. This is because τ_π^{min} in Proposition 3 is increasing in φ. If φ is smaller than about 0.017, an increase in φ does not affect the determinacy region of τ_π. The reason for this is as follows. As shown in the proof of Proposition **??**, a necessary condition for equilibrium determinacy is $F(-1) > 0$. If τ_k, a necessary and sufficient condition for $F(-1) > 0$ is $\tau_\pi > 1$. When φ is smaller than about 0.017, the threshold value τ_π^{min} is less than 1, and the condition $\tau_\pi > 1$ is binding.

If φ is large enough so that the collateral constraint never binds, equilibrium is determinate where $\tau_\pi > 1$. Then, sufficient improvements in credit market imperfections might increase the determinacy region. Generally, however, improvements in credit market imperfections can reduce the determinacy regions of τ_π, as shown in Figure 3.

Figure 4 shows the determinacy region in the (τ_k, φ) plane.

[Insert Figure 4]

The vertical axis is the collateral constraint parameter φ. The horizontal axis is the central bank's response to the asset price τ_k. We set the parameter of the central bank's response to inflation, τ_π, as 1.5, and the others are the same as those in the previous analysis.

Figure 4 shows that improvements in credit market imperfections reduce the determinacy region of τ_k. This is because τ_k^{min} in Proposition **??** increases in φ and τ_k^{max} decreases in φ.

Finally, we find that improvements in credit market imperfections can reduce the determinacy region of τ_π and τ_k. While it is intuitive that improvements in credit market imperfections have positive effects, our result shows that this simple intuition is not correct from the perspective of equilibrium determinacy.

4 Robustness

4.1 Capital prices versus share prices

In the previous section, we considered a modified Taylor rule that responds to capital price q_t. Here, we consider a Taylor rule that responds to share prices Q_t^S, as considered by Carlstrom and Fuerst (2004):

$$r_t = \tau_\pi \pi_t + \tau_{qs} q_t^s, \tag{42}$$

where q_t^s denotes the log-deviation of Q_t^S from a steady state.[4]

If we introduce the share, our model differs from the one discussed in Section 2, as follows. The budget constraint of household (4) becomes

$$\begin{aligned} & P_t C_t + M_{t+1} + P_t Q_t K_{t+1} + B_{t+1} + P_t Q_t^S S_{t+1} + P_t W_t L_t \\ & \quad \leq P_t Z_t Y_t + P_t W_t H_t + M_t + P_t Q_t K_t + R_{t-1} B_t + P_t (Q_t^S + D_t) S_t + X_t, \end{aligned} \tag{43}$$

where S_t denotes shares and D_t denotes share dividends. Under this setting, the asset price equation of the share is

$$Q_t^S = \left[Q_{t+1}^S + D_{t+1} \right] \frac{\Pi_{t+1}}{R_t}. \tag{44}$$

The dividend is the reflected profit of monopolistic retailers:

$$D_t = (1 - Z_t) Y_t. \tag{45}$$

[4]Nutahara (2010) shows that the type of asset, capital, or share matters for equilibrium determinacy in a sticky-price model with a perfect credit market.

In this case, the equilibrium system consists of four jump variables and complex variables. Then, we investigate the determinacy region by numerical analyses.

Figure 5 is similar to Figure 1.

[Insert Figure 5]

In the region with red diamonds, equilibrium is determinate; equilibrium indeterminacy arises in other regions; the horizontal axis denotes the central bank's response to the share price τ_{qs}; and the parameter values are the same as in Figure 1. As in the case of monetary policy responding to the capital price, both the sufficiently positive and negative sensitivity of monetary policy to the share prices are sources of equilibrium indeterminacy.

Figure 6 is similar to Figure 4.

[Insert Figure 6]

The horizontal axis denotes the central bank's response to the share price τ_{qs}. The parameter values are the same as in Figure 4. Similar to Figure 4, we find that improvements in credit market imperfections reduce the determinacy regions of τ_{qs}.

Then, our results in Section 3 are robust when monetary policy responds to the share prices.

4.2 Variation of capital stock

For simplicity of the analyses, we have so far assumed that the total supply of capital stock is fixed. In this subsection, however, we consider a case where capital evolves over time.

As a result, the households' budget constraint (4) becomes

$$\begin{aligned} & P_tC_t + P_tI_t + M_{t+1} + B_{t+1} + P_tQ_tK_{t+1} \\ & \qquad \leq P_tW_tH_t + M_t + R_{t-1}B_t + (1-\delta)P_tQ_tK_t + X_t, \end{aligned} \tag{46}$$

where I_t is the investment and $\delta \in (0, 1)$ is the depreciation rate of capital.

We assume that the capital price varies since there is an adjustment cost of investment.

The evolution of capital stock is

$$K_{t+1} = (1 - \delta)K_t + H(I_t), \tag{47}$$

where, following Carlstrom and Fuerst (2005), $H(\cdot)$ is increasing and concave with $H(0) = 0$. We specify the functional form of $H(I_t)$ as

$$H(I_t) \equiv bI_t^{\eta}, \tag{48}$$

where η is between 0 and 1. The first-order condition for investment is

$$Q_t = \frac{1}{H'(I_t)}. \tag{49}$$

The parameter b is chosen so that $Q_t = 1$ in the steady state, its value in the economy with no adjustment cost. Then, $b = I^{1-\eta}/\eta$, where I denotes a steady-state investment.

The resource constraint (15) is

$$C_t + I_t = Y_t. \tag{50}$$

The following is a condition for the binding collateral constraint in this case.

Proposition 4. *In an economy with a variation of capital stock, a collateral constraint (3) is binding at a steady state if and only if*

$$\varphi < \varphi_K^{max} \equiv \frac{(1-\alpha)\left[\frac{1}{\beta} - 1 + \delta\right]}{\alpha}.$$

Proof. See Appendix. □

In this case, the equilibrium system consists of three jump variables and one state variable. We investigate the determinacy region by conducting numerical analyses. The depreciation rate of capital, δ, is 0.025, and the adjustment cost parameter, η, is 0.5. The others are the same as those employed in Section 3.

Figure 7 is similar to Figure 1.

[Insert Figure 7]

In the region with red diamonds, equilibrium is determinate; equilibrium indeterminacy arises in other regions; and the horizontal axis denotes the central bank's response to the share price τ_k. As in Figure 1, both the sufficiently positive and negative sensitivity of monetary policy to the capital prices are sources of equilibrium indeterminacy.

Figure 8 is similar to Figure 3.

[Insert Figure 8]

The horizontal axis denotes the parameter of credit market imperfections, φ. We consider cases where $\varphi \in [\varphi_k^{\max}/5, \varphi_k^{\max}]$. The parameter values are the same as in Figure 3.

Figure 9 is similar to Figure 4.

[Insert Figure 9]

The parameter values are the same as in Figure 4.

Similar to Figures 4 and 5, Figures 8 and 9 indicate that improvements in credit market imperfections can reduce the determinacy regions of τ_k and τ_k. Finally, our results in Section 3 are robust when capital stock varies over time.

5 Concluding remarks

In this paper, we investigated monetary policy response to asset price fluctuations in a monetary economy with credit market imperfections. This study was carried out from the perspective of equilibrium determinacy. In our model, the prices are sticky and the working capital of firms is subject to asset values because of a lack of commitment problem. We found that the Taylor principle does not guarantee equilibrium determinacy. We also found that monetary policy response to asset price fluctuations is a source of equilibrium indeterminacy and that improvements in credit market imperfections can reduce the determinacy region of model parameters in some cases.

Our results imply that a central bank should not respond to asset price fluctuations if the credit market is imperfect and that improvements in credit market imperfections might not be good from the perspective of equilibrium determinacy.

Appendix: Proof of Proposition 4

Proof. The equilibrium system of the economy with a variation of capital stock is

$$\zeta C_t^{\sigma} L_t^{\gamma} = W_t,$$

$$C_t^{-\sigma} = \beta C_{t+1}^{-\sigma} \frac{R_t}{\Pi_{t+1}},$$

$$Q_t = \left[(1-\delta+\varphi\Theta_{t+1})Q_{t+1} + \alpha Z_{t+1}\frac{Y_{t+1}}{K_{t+1}}\right]\frac{\Pi_{t+1}}{R_t}.$$

$$W_t(1+\Theta_t) = (1-\alpha)Z_t\frac{Y_t}{L_t},$$

$$(W_t L_t - \varphi Q_t K_t)\Theta_t = 0, \qquad \Theta_t \geq 0,$$

$$Y_t = K_t^{\alpha} L_t^{1-\alpha},$$

$$Q_t\left[\frac{I_t}{I_{ss}}\right]^{1-\gamma}.$$

$$Y_t = C_t + I_t,$$

$$K_{t+1} = (1-\delta)K_t + H(I_t),$$

$$\pi_t = \lambda z_t + \beta\pi_{t+1},$$

$$r_t = \tau_\pi \pi_t + \tau_q q_t.$$

At a steady state, with $\Pi = L = 1$ and $H(I_t) \equiv bI_t^{\eta}$, this system becomes

$$R = \frac{1}{\beta},$$
$$1 = \beta\left[(1 - \delta + \varphi\Theta) + \alpha Z\frac{Y}{K}\right].$$
$$W(1 + \Theta) = (1 - \alpha)ZY,$$
$$(W - \varphi K)\Theta = 0, \qquad \Theta \geq 0,$$
$$Y = K^{\alpha},$$
$$Q = 1.$$
$$Y = C + \eta\delta K.$$

Since

$$W = \frac{1-\alpha}{1+\Theta}ZK^{\alpha}, \qquad \text{and} \qquad K = \left[\frac{\alpha Z}{\frac{1}{\beta} - 1 + \delta - \varphi\Theta}\right]^{\frac{1}{1-\alpha}},$$

the binding collateral constraint is

$$\frac{1-\alpha}{1+\Theta} = \varphi \cdot \frac{\alpha}{\frac{1}{\beta} - 1 + \delta - \varphi\Theta}.$$

Then, the Lagrange multiplier of the collateral constraint Θ is

$$\Theta = \frac{(1-\alpha)\left[\frac{1}{\beta} - 1 + \delta\right]}{\varphi} - \alpha.$$

Therefore, the collateral constraint is binding at a steady state if and only if

$$\varphi < \frac{(1-\alpha)\left[\frac{1}{\beta} - 1 + \delta\right]}{\alpha}.$$

□

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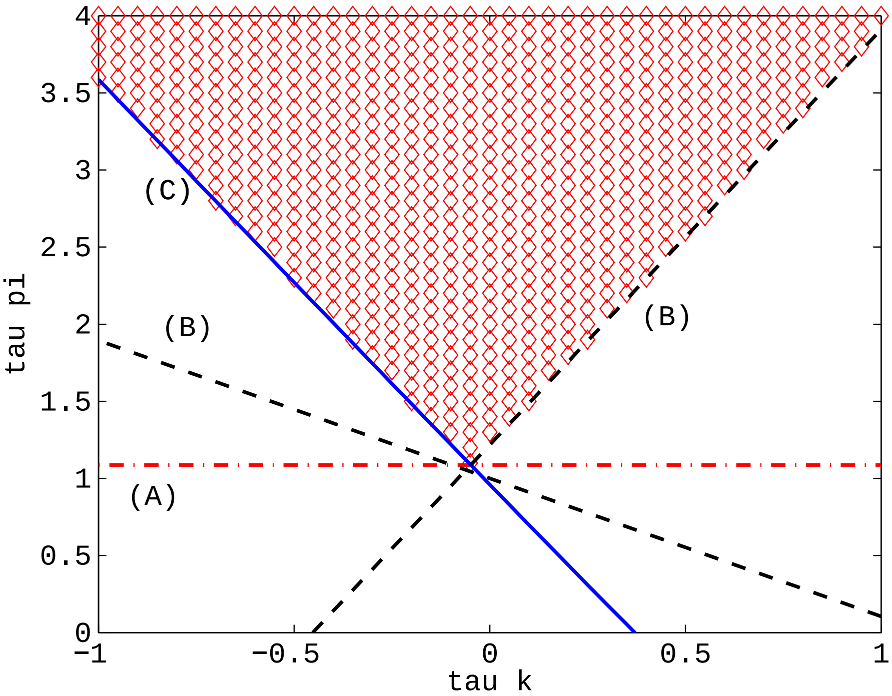
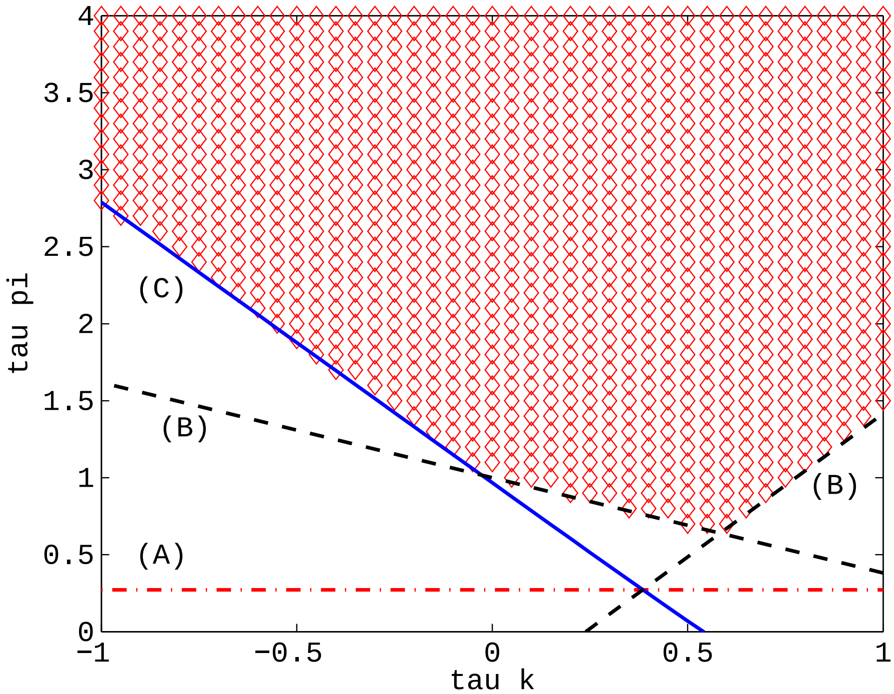
Figure 1: Effect of monetary policy on the determinacy region (1)



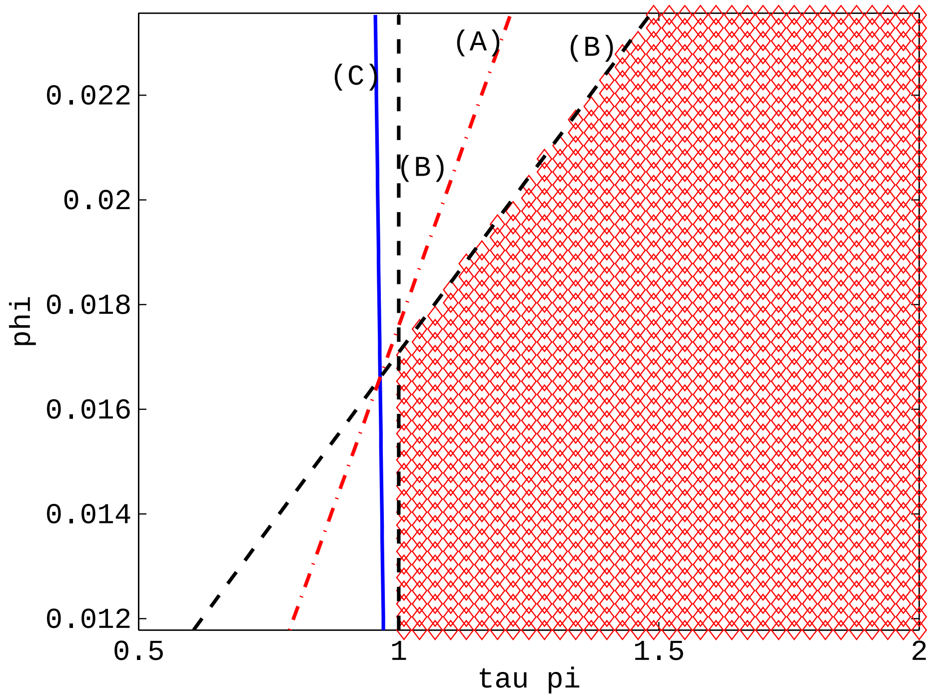
Notes: In the region with red diamonds, equilibrium is determinate, and in other regions, equilibrium is indeterminate. The vertical axis denotes the central bank's response to inflation τ_π. The horizontal axis denotes the central bank's response to the capital price τ_k. Other parameters are $\sigma = 1$, $\gamma = 0$, $\alpha = 0.3$, $\beta = .99$, $\lambda = 0.019$, $Z = 0.85$, and $\varphi = 0.02$.

Figure 2: Effect of monetary policy on the determinacy region (2)



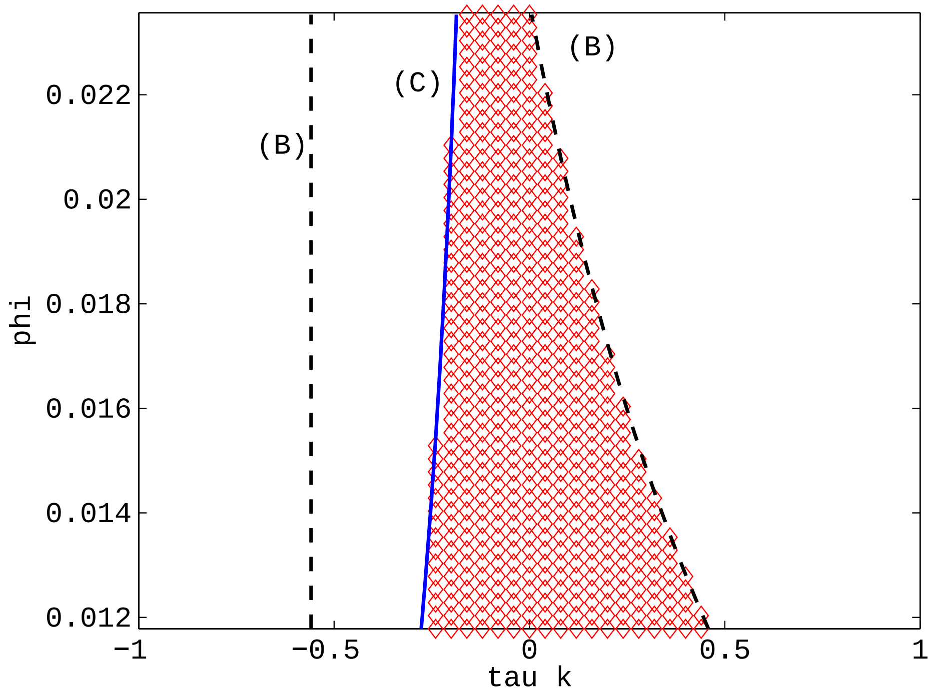
Notes: In the region with red diamonds, equilibrium is determinate, and in others, equilibrium is indeterminate. The vertical axis denotes the central bank's response to inflation τ_π. The horizontal axis denotes the central bank's response to the capital price τ_k. Other parameters are $\sigma = 1$, $\gamma = 3$, $\alpha = 0.3$, $\beta = .99$, $\lambda = 0.019$, $Z = 0.85$, and $\varphi = 0.02$.

Figure 3: Effect of credit market imperfections on the determinacy region (1)



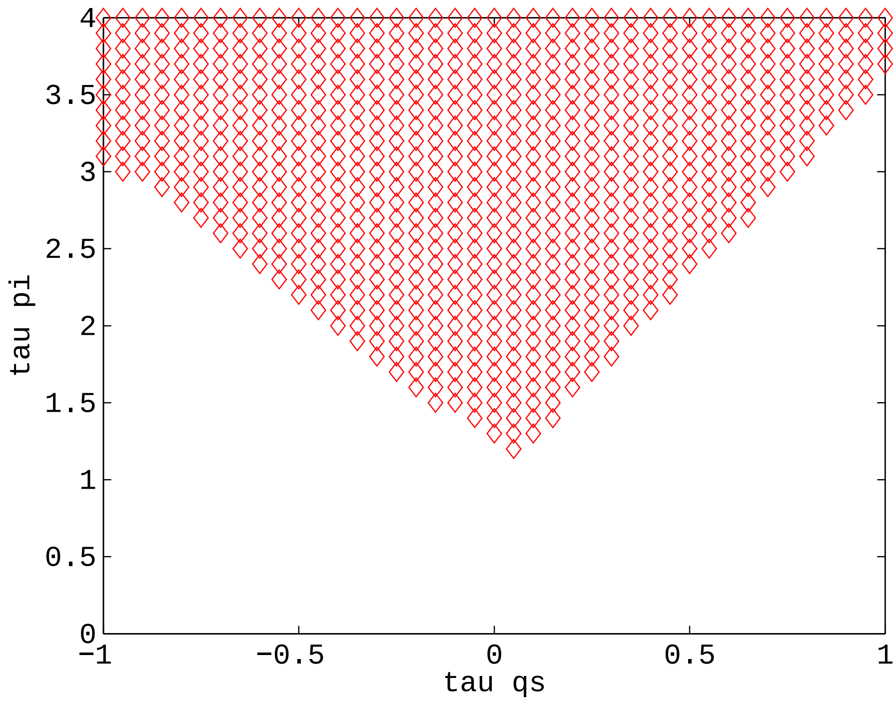
Notes: In the region with red diamonds, equilibrium is determinate, and in other regions, equilibrium is indeterminate. The vertical axis denotes the collateral constraint parameter φ. The horizontal axis denotes the central bank's response to inflation τ_π. Other parameters are $\sigma = 1$, $\gamma = 0$, $\alpha = 0.3$, $\beta = .99$, $\lambda = 0.019$, $Z = 0.85$, and $\tau_k = 0$.

Figure 4: Effect of credit market imperfections on the determinacy region (2)



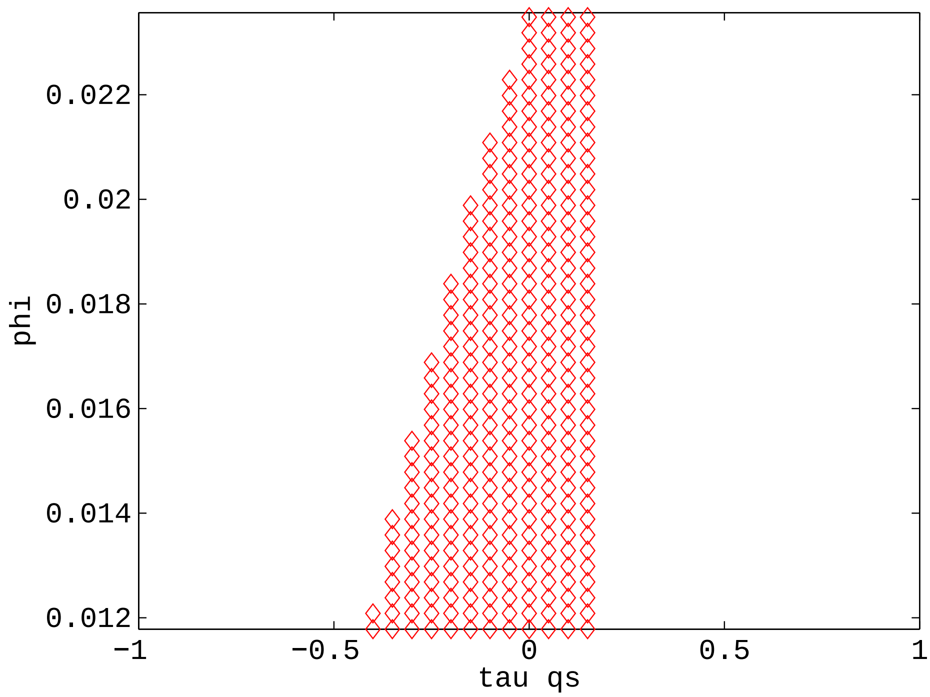
Notes: In the region with red diamonds, equilibrium is determinate, and in other regions, equilibrium is indeterminate. The vertical axis denotes the collateral constraint parameter φ. The horizontal axis denotes the central bank's response to the capital price τ_k. Other parameters are $\sigma = 1$, $\gamma = 0$, $\alpha = 0.3$, $\beta = .99$, $\lambda = 0.019$, $Z = 0.85$, and $\tau_\pi = 1.5$.

Figure 5: Effect of monetary policy on the determinacy region (3): Share prices



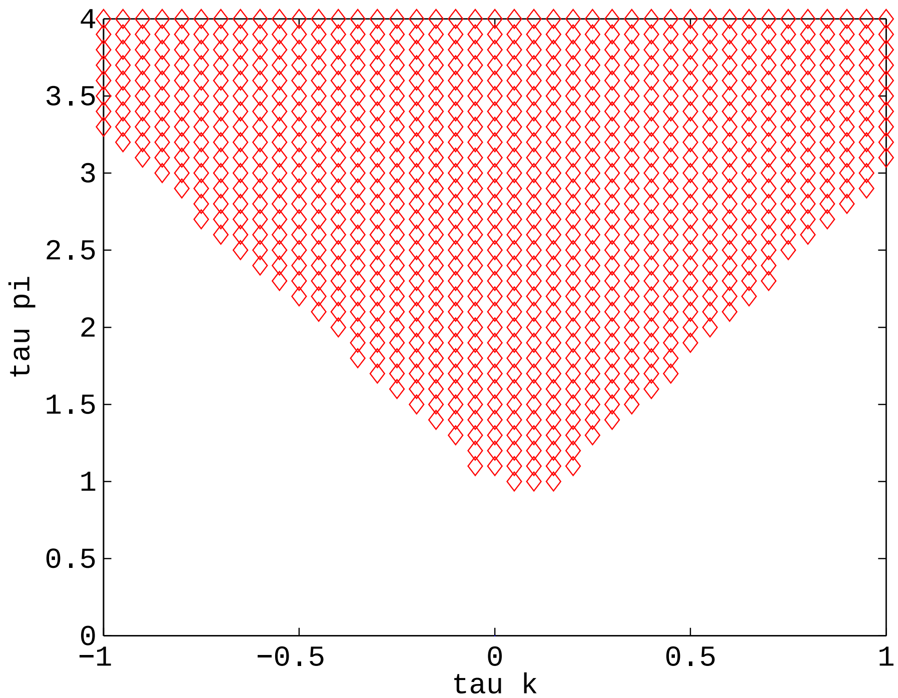
Notes: In the region with red diamonds, equilibrium is determinate, and in other regions, equilibrium is indeterminate. The vertical axis denotes the central bank's response to inflation τ_π. The horizontal axis denotes the central bank's response to the share price τ_{qs}. Other parameters are $\sigma = 1$, $\gamma = 0$, $\alpha = 0.3$, $\beta = .99$, $\lambda = 0.019$, $Z = 0.85$, and $\varphi = 0.02$.

Figure 6: Effect of credit market imperfections on the determinacy region (3): Share prices



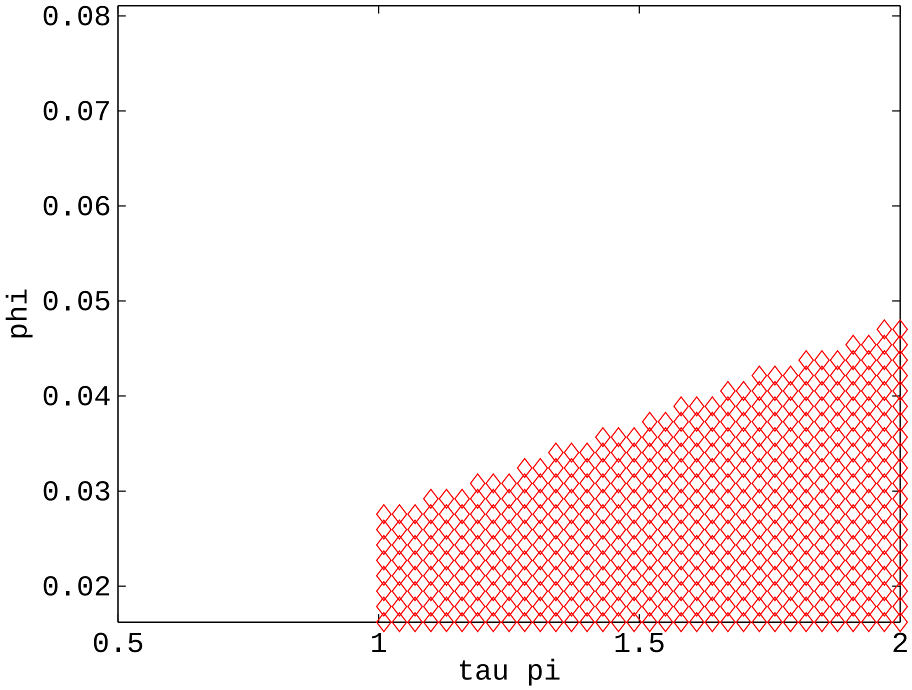
Notes: In the region with red diamonds, equilibrium is determinate, and in other regions, equilibrium is indeterminate. The vertical axis denotes the collateral constraint parameter φ. The horizontal axis denotes the central bank's response to the share price τ_{qs}. Other parameters are $\sigma = 1$, $\gamma = 0$, $\alpha = 0.3$, $\beta = .99$, $\lambda = 0.019$, $Z = 0.85$, and $\tau_{\pi} = 1.5$.

Figure 7: Effect of monetary policy on the determinacy region (4): Variation of capital stock



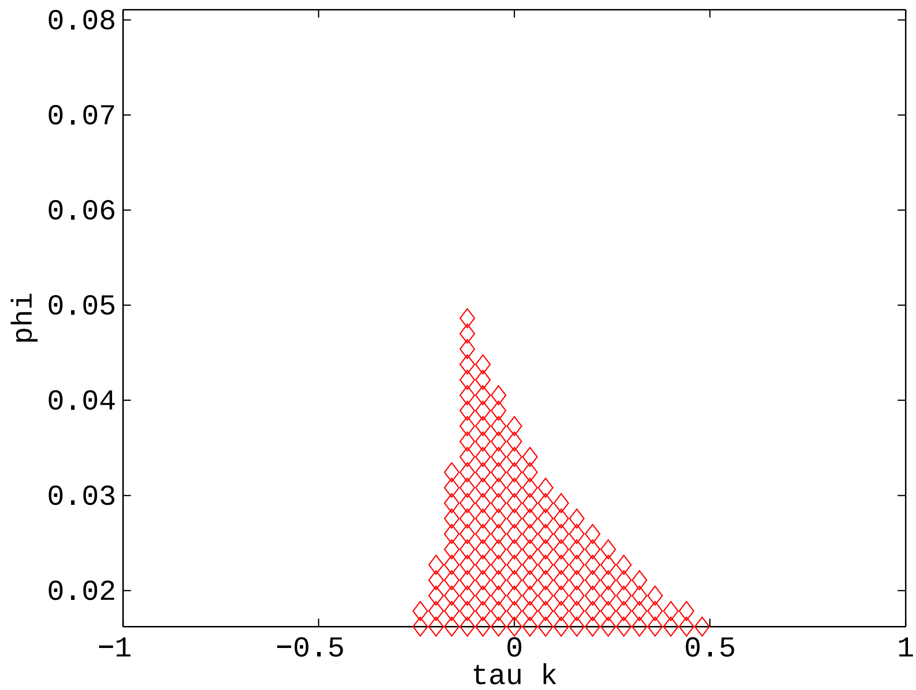
Notes: In the region with red diamonds, equilibrium is determinate, and in other regions, equilibrium is indeterminate. The vertical axis denotes the central bank's response to inflation τ_π. The horizontal axis denotes the central bank's response to the capital price τ_k. Other parameters are $\sigma = 1$, $\gamma = 0$, $\alpha = 0.3$, $\beta = .99$, $\lambda = 0.019$, $Z = 0.85$, $\delta = 0.025$, $\eta = 0.5$, and $\varphi = 0.02$.

Figure 8: Effect of credit market imperfections on the determinacy region (4): Variation of capital stock



Notes: In the region with red diamonds, equilibrium is determinate, and in other regions, equilibrium is indeterminate. The vertical axis denotes the collateral constraint parameter φ. The horizontal axis denotes the central bank's response to inflation τ_π. Other parameters are $\sigma = 1$, $\gamma = 0$, $\alpha = 0.3$, $\beta = .99$, $\lambda = 0.019$, $Z = 0.85$, $\delta = 0.025$, $\eta = 0.5$, and $\tau_k = 0$.

Figure 9: Effect of credit market imperfections on the determinacy region (5): Variation of capital stock



Notes: In the region with red diamonds, equilibrium is determinate, and in other regions, equilibrium is indeterminate. The vertical axis denotes the collateral constraint parameter φ. The horizontal axis denotes the central bank's response to the capital price τ_k. Other parameters are $\sigma = 1$, $\gamma = 0$, $\alpha = 0.3$, $\beta = .99$, $\lambda = 0.019$, $Z = 0.85$, $\delta = 0.025$, $\eta = 0.5$, and $\tau_\pi = 1.5$.