The pitfalls of speed-limit interest rate rules at the zero lower bound

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Abstract

We show that interest rate rules that feed back on growth rates (of output, or asset prices) may induce very severe recessions in the presence of a zero lower bound, through a purely self-fulfilling dynamic. This pathology is illustrated in a small New Keynesian model with interest rates responding to the growth rate of output; and in the Iacoviello (2005) model with interest rates responding to the growth rate of house prices. Our results provide a cautionary note, contrasting with previous work which has suggested several desirable properties of speed limit rules, namely that they are devices enabling the policymaker (i) to side-step uncertainty about natural rates (ii) to counter booms and busts in asset prices or (iii) to implement optimal commitment policies.

1 Introduction[1]

Following the financial crisis there has been a rapid expansion in the literature investigating the policy implications of the zero lower bound on the nominal interest rate. A number of recent papers, including DelNegro, Eggertson, Ferrero, and Kiyotaki (2010) and Corsetti, Kuester, Meier, and Muller (2010), consider the implications of this zero bound in economies characterised by financial frictions. As in the important analysis by Woodford (2011) and Christiano, Eichenbaum, and Rebelo (2009) (both of which focus on the effects of fiscal policy when interest rates are constrained), the approach of these papers is to treat the zero bound as a restriction on fluctuations that occur *in the neighbourhood of a steady state that involves a non-zero policy rate.* The problem that is considered is one of shocks having occurred of sufficient magnitude that in a fully linear system, absent any inequality constraints, the policy response would be to set a negative nominal interest rate. The focus of the analysis is to

[1] We thank out discussant, Stephen Murchinson, and participants at the Bank of Canada macro-modelling conference as well as Wouter den Haan for comments. All errors are our own. The views expressed here are those of the authors and not of the Bank of England or the Monetary Policy Committee.

establish outcomes in the event that this is not possible. The methodological approach commonly used owes much to the work of Eggertsson and Woodford (2003), who first considered the problem of setting optimal policy when the zero lower bound occasionally constrained outcomes.

An alternative approach to analysing the zero lower bound derives from the contribution of Benhabib, Schmitt-Grohe;, and Uribe (2002). This branch of the literature emphasises the distinction between local and global inflation determinacy when the policymaker follows a Taylor rule or similar deterministic policy strategy.[2] The key point is that the Taylorian feedback strategy inducing determinacy in the region of the 'desirable' steady state can no longer be pursued once it implies a negative policy rate. This fact induces a kink in the policymaker's reaction function, which in turn permits a second, 'undesirable' steady to persist, characterised by a fixed zero nominal rate, deflation and (usually) sub-optimally low output levels. Because the interest rate is fixed at zero in the neighbourhood of this steady state, the dynamics there will in general be indeterminate – in accordance with the well-known logic of Sargent and Wallace (1975). The analysis suggests that economies run the risk of becoming stuck in a deflationary steady state simply through a self-fulfilling emergence of deflationary expectations. Mertens and Ravn (2011) have extended this 'global' approach to studying the zero lower bound to a model with financial frictions, due originally to Iacoviello (2005). They suppose that outcomes in this economy are affected by random self-fulfilling bouts of pessimism, which cause the zero bound to bind, and the economy to move to the neighbourhood of the undesirable steady state – where it remains so long as the relevant non-fundamental ('sunspot') variable continues to give a pessimistic reading. They show that debt deflation and falls in collateral value, which occur in their model when pessimism arrives, magnify the falls in output associated with this 'liquidity trap' roughly fourfold relative to a model with no financial frictions.

Our analysis can be seen as bridging the gap between these two approaches. Like Benhabib et al., and Mertens and Ravn subsequently, one of our main findings is that large falls in output *can* be driven by self-fulfilling dynamics alone. But like Woodford, Christiano et al., Del Negro et al. and the majority of other recent studies of the zero bound, we focus only on fluctuations in the region of the 'desirable' steady state (one that satisfies the usual Blanchard-Kahn conditions for local determinacy). This allows us to illustrate an important theoretical possibility that has not been given much (if any) attention in the literature: namely that poorly-chosen policy rules may, in the presence of a zero bound, permit a finite number of initial values for the economy's choice variables to be consistent with convergence to the *same* steady state in a rational expectations equilibrium. This seems an important possibility to highlight, since it can occur without the co-ordinated *expectational* shift upon which the Benhabib et al. and Mertens and Ravn results depend. That is, we do not require that public expectations over economic outcomes in subsequent time periods should be

[2] We use 'determinacy' in the weak sense: 'unique non-explosiveness' here, neglecting the possibility of self-fulfilling hyperinflations. See Woodford (2003) for a more detailed analysis of these.

modelled as stochastic variables;[3] even with perfect foresight our self-fulfilling dynamics can be supported, and are consistent with convergence back to the 'desirable' steady state. Thus short-term outcomes can be volatile even whilst longer-term expectations are fully 'anchored' – fixed functions of the relevant vector of state variables, with no sunspots.

Perhaps surprisingly, we are able to generate these self-fulfilling dynamics in a 'plain vanilla' version of the familiar dynamic New Keynesian model, absent any financial frictions, capital, adjustment costs, or other complicating factors. All that is required is that the chosen policy rule – determining the extent to which the nominal interest rate feeds back on contemporary realisations of endogenous variables – should include certain terms that are conducive to sustaining the dynamic. In particular, we show that if the policymaker places a sufficiently high weight on controlling the growth rate of real output in this environment, substantial recessions can be induced without any exogenous shock process.

The logic behind our key result is the following. Suppose that away from the zero bound the policymaker is placing a large weight on the contemporaneous growth rate in real output, as well as feeding back on the rate of inflation in a manner that satisfies the Taylor principle. Suppose we conjecture that it is expected that the levels of output and inflation fall by substantial amounts – sufficient to ensure that the zero bound binds. Can such expectations in the short term be a rational expectations equilibrium? If the policymaker is sufficiently concerned about output *growth* then the very fact that output has fallen by so much implies a reluctance to see a rapid rebound in output subsequently – which in turn constrains inflation expectations, since low inflation is a by-product of any contractionary policy stance in the next time period. Such low inflation expectations imply higher real rates today, and that itself implies low output today, relative to steady state, validating the conjectured fall in expected output.

Whilst this analysis is of potential interest *per se* from a policy perspective, the model is a very simple one – so it is important to focus on the lessons that might be generalisable. The feature of the policy rule that causes problems is the feedback on the growth rate of a variable that co-moves with inflation as aggregate demand fluctuates. This implies that when the zero lower bound is binding, and aggregate demand is thus lower than an unconstrained policymaker would desire, agents have a reason to believe the policymaker will adopt a relatively tight policy stance in subsequent time periods. This in turn implies low inflation expectations, endorsing the initial low demand.

An alternative variable to output that is likely to co-move with inflation in this manner might be some measure of asset prices – and here too there have often been arguments made that monetary policy should 'prick' asset price bubbles by feeding back on asset price growth. To test the potential for similar pitfalls to arise when this is done, we follow Mertens and Ravn (2011) and

[3] Mertens and Ravn (2010) do just this, with a Markov process to determine whether expectations are 'pessimistic' or 'optimistic'.

consider a richer model due to Iacoviello (2005). This is a variant on the dynamic New Keynesian setup that introduces a real estate sector, with residential and commercial (productive) uses for real estate and (in the spirit of Kiyotaki and Moore (1997)) a collateral requirement limiting leveraged investment in commercial real estate. And indeed, we find that when the policy feedback coefficient on real estate price growth is high enough the possibility of self-fulfilling output (and house price) collapses returns. In this case, the very expectation that asset price growth will be resisted in the future can contribute to falling asset prices in the present. We also find similar dynamics to be possible in the event that output growth is targeted in this richer model. The magnitudes and co-movements of the variables in the 'crisis' episodes that we observe are very similar to those described in the work of Mertens and Ravn, but unlike those authors we do not need to justify deflationary expectations by a sufficiently high level of 'pessimism' obtaining, switching agents' focus to the liquidity trap steady state; rather, it is the *known* future policy response to contemporary outcomes that keeps real interest rates high.

The main policy lesson of this paper is that simple rules may have unanticipated consequences when interacted with the zero bound, particularly when they include terms in the growth rates of target variables. These results provide a counterweight to previous work that has suggested that interest rate rules that feed back from rates of changes may have desirable properties. In this regard, we recall first that it has been shown that interest rate rules that feed back from rates of change of certain variables can deliver outcomes equivalent to those under the optimal commitment policy. McCallum and Nelson (2004) and Stracca (2007) show this for rules involving terms in the change in the output gap. Giannoni and Woodford (2003) establish the same result, though for the case in which the objective function for policymakers is taken to include a term in the change in interest rates. Leduc and Natal (2011) show that a rule that feeds back from the rate of change in asset prices approximates the outcomes under optimal policy in a model with time-varying spreads.

Second, Orphanides and Williams (2002) showed that interest rate rules involving terms in the change in real quantities provided immunity against mismeasuring natural rate concepts that were required to operationalise conventional Taylor rules that are informed by gaps between the level of output/unemployment relative to their natural rates. They further emphasise the benefits of such rules by noting that in the absence of knowledge about just how much uncertainty there is, it is better to err on the side of assuming that there is more.

Third, there is evidence that speed-limit rules can characterise central bank policy at times. Walsh (2003) quotes the FOMC minutes in 2000, citing evidence in support of decisions to raise rates, thus: "*The [Federal Open Market] Committee remains concerned that over time, increases in demand will continue to exceed the growth in potential supply*" [Feb 2000]; and in May that year "*Increases in demand have remained in excess of even the rapid pace of productivity-driven gains in potential supply*". One piece of econometric evidence is Paez-Farrell (2009), who shows that speed-limit rules do a reasonable job of explaining past behaviour in central bank interest rates.

2 A simple model

2.1 Setup

We start by presenting the analysis in one of the simplest environments possible: a dynamic New Keynesian model with no capital, and no financial frictions, adjustment costs or the like. A representative consumer at time t maximises the objective:

$$U_t = E_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{c_{t+s}^{1-\sigma} - 1}{1-\sigma} - \frac{\left(l_{t+s}^s\right)^{1+\varphi}}{1+\varphi} \right\} \tag{1}$$

where l_t^s denotes labour supply at t and c_t a Dixit-Stiglitz aggregate across a continuum of consumption goods:

$$c_t = \left[\int_0^1 c_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{2}$$

If the money price of good j is $p_t(j)$, the minimum expenditure required to obtain a unit of c_t, P_t, is given by:

$$P_t = \left[\int_0^1 p_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}} \tag{3}$$

We denote by π_t the gross rate of inflation at t, $\frac{P_t}{P_{t-1}}$. The consumer's period-by-period budget constraint is then:

$$w_t l_t^s + T_t + \frac{R_{t-1}}{\pi_t} s_{t-1} = c_t + s_t \tag{4}$$

with w_t the real wage, s_t the real quantity of saving (in nominal bonds, paying gross interest R_t), and T_t a collection of lump-sum transfers to and from profit-making firms and the government. This constraint is coupled with a usual transversality/'no-Ponzi' restriction.

Labour demand comes from a large number of perfectly competitive firms producing homogenous intermediate goods according to a linear production function:

$$y_t = a l_t \tag{5}$$

for some fixed technology parameter a. These intermediate goods are purchased by final goods firms at a price p_t^I (expressed in units of consumption aggregate), who convert them one-for-one into their respective products. These firms are in turn owned by consumers, so the generic firm j looks to maximise the objective:

$$\Pi_t(j) = E_t \sum_{s=0}^{\infty} \beta^s \frac{c_{t+s}^{-\sigma}}{c_t^{-\sigma}} \left[\left((1+\tau) p_{t+s}(j) - P_{t+s} p_{t+s}^I \right) y_{t+s}(j) \right] \tag{6}$$

where $p_{t+s}(j)$ is the price at which firm j sells its goods in period $t+s$, y_{t+s} is its output level that period, and τ is a proportional revenue subsidy that is paid by the government to eliminate the steady-state losses due to market power. As in Calvo (1983), these firms are permitted to reset their prices only infrequently, with a probability of resetting equal to $(1-\theta)$ each period. We denote by y_t^f the aggregate final goods output level of relevance to consumers:

$$y_t^f \equiv \left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{7}$$

Policy is set according to a feedback rule of the following form:

$$R_t = \max \left\{ \beta^{-1} \pi_t^{\alpha_\pi} \left(\frac{y_t^f}{y_{ss}^f} \right)^{\alpha_y} \left(\frac{y_t^f}{y_{t-1}^f} \right)^{\alpha_{\Delta y}}, 1 \right\} \tag{8}$$

where y_{ss}^f is the steady-state aggregate level of final goods output. Note that β^{-1} is the steady-state gross nominal interest rate consistent with zero inflation. This setup nests a conventional Taylor rule (where $\alpha_\pi = 1.5$, $\alpha_y = 0.5$ and $\alpha_{\Delta y} = 0$), but also the possibility that the policymaker might wish to feed back on the growth rate of real output. We have explicitly incorporated the zero lower bound through this formulation: the gross nominal rate cannot fall below 1.

The model is sufficiently familiar that we jump immediately to a set of linear equations describing its movements in the vicinity of the steady state. Applying 'hats' to denote log deviations from the zero-inflation steady state, and reducing to a system of just three equations, we have:

$$\widehat{y}_t = E_t \widehat{y}_{t+1} - \frac{1}{\sigma} \left(\widehat{R}_t - E_t \widehat{\pi}_{t+1} \right) \tag{9}$$

$$\widehat{\pi}_t = \frac{(1-\theta)(1-\beta\theta)(\sigma+\varphi)}{\theta} \widehat{y}_t + \beta E_t \widehat{\pi}_{t+1} \tag{10}$$

$$\widehat{R}_t = \max \left\{ \alpha_\pi \widehat{\pi}_t + \alpha_y \widehat{y}_t + \alpha_{\Delta y} \left(\widehat{y}_t - \widehat{y}_{t-1} \right), \beta - 1 \right\} \tag{11}$$

(Note that $\beta - 1$ is, to first order, the log deviation in R_t required to reduce it from its steady state value of β^{-1} to the lower bound of 1.)

2.2 Results

The system described by the previous three equations is piecewise linear. We want to check whether particular calibrations of the policy parameters might permit self-fulfilling dynamics to take hold within it, quite aside from the possibility that we may shift to an alternative system in the neighbourhood of a liquidity trap steady state (it was this latter possibility that was first highlighted by Benhabib, Schmitt-Grohe;, and Uribe (2002)). That is, is it possible to see self-fulfilling output collapses in the initial period of a perfect-foresight dynamic equilibrium path that nonetheless converges on the locally 'determinate' steady

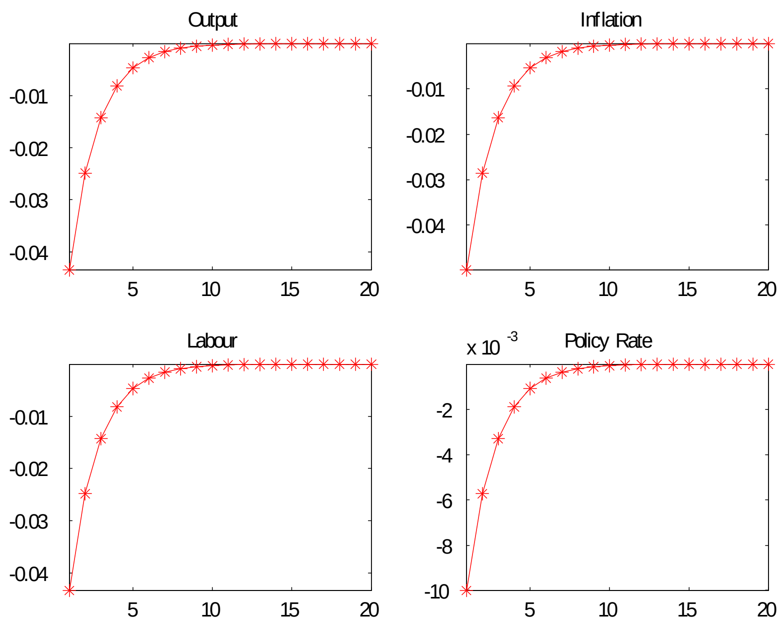

Figure 1: Self-fulfilling crisis: a simple New Keynesian model

state about which we have linearised? To test this we choose conventional values for the non-policy parameters: $\sigma = 1$, $\beta = 0.99$, $\theta = 0.67$ and $\varphi = 2$. For arbitrary values of the triple $(\alpha_\pi, \alpha_y, \alpha_{\Delta y})$ we can use standard methods to generate dynamics with the restriction $\widehat{R}_t = \beta - 1$ imposed in the initial time period alone, subsequently reverting to the unconstrained policy feedback rule. If these dynamics are such that the zero lower bound is satisfied at all future horizons *and* the initial responses of output and inflation are consistent with it indeed binding in the first period, then we have a self-fulfilling equilibrium of the desired form.

Our main result is that such combinations do indeed exist. For any pair of values that one chooses for α_π and α_y, a sufficiently high choice of $\alpha_{\Delta y}$ will permit 'crisis' episodes to occur. When $\alpha_y = 0$ is imposed, the key threshold (for our chosen calibration) is that $\alpha_{\Delta y} > \alpha_\pi$. Since the case in which $\alpha_{\Delta y} = \alpha_\pi$ corresponds to nominal GDP targeting, the implication is that we can come arbitrarily close to this strategy and admit self-fulfilling crises as a possibility. Figure 1 shows the dynamics associated with the values $\alpha_y = 0$, $\alpha_\pi = 1.5$ and $\alpha_{\Delta y} = 2$.

The linear model that obtains when the zero lower bound is not binding contains just one stable root, so the dynamics cannot be expected to vary much across the different variables. Clearly the potential magnitude of the crisis is

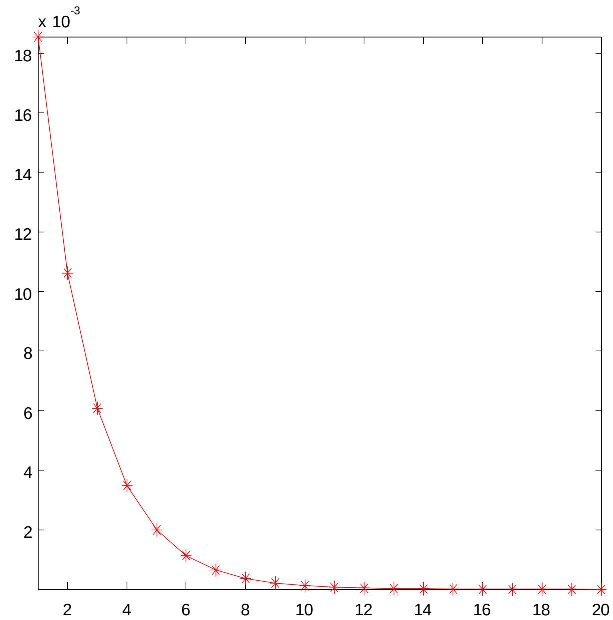

Figure 2: The real interest rate during the crisis episode

substantial, with output falling by over four per cent from its steady-state value. The key factor behind these movements is the evolution of the real interest rate, charted in Figure 2. This rises substantially in the initial period, as one might expect when the policymaker is constrained by the zero lower bound – but crucially it is also more than a percentage point above steady state in the second period, despite the zero bound having ceased to bind by then. This is central to the dynamics being possible: it is only because of this high *second-period* real rate that inflation expectations are so low in the first period – and thus the first-period real rate so high. The reason why policy is deliberately restrictive in the second time period is, in turn, that first-period output was so low that the 'natural' recovery from it is resisted by a policymaker feeding back on growth.

Alternative values for the policy parameters yield similar dynamics qualitatively, though the magnitudes become less severe the greater is $\alpha_{\Delta y}$ – since less of a deviation in output from steady state is then required to engineer a given policy contration in the second time period (and subsequently). The cost is greater persistence, as only small amounts of growth back to steady state are permitted for large $\alpha_{\Delta y}$.

3 A richer model

Whilst it is perhaps surprising that self-fulfilling equilibria of this form can be observed in a simple dynamic New Keynesian setup, that model is a restrictive and fairly unrealistic one. It is particularly restrictive for our purposes in the sense that very few variables are ultimately of relevance to outcomes, and thus the set of policy rules that could be assessed for their capacity to generate crises is limited. Given the relevance to contemporary policy discussions of models with imperfect financial markets, and the fact that these models can be used to assess the benefits of feeding back on asset price growth in particular, we now make use of one such model – due to Iacoviello (2005). This model contains four distinct classes of agent: households, entrepreneurs, final goods firms and a monetary policymaker. We discuss their problems in turn.

3.1 Households

The model works with a measure $\omega \in [0,1]$ of households and $(1-\omega)$ of entrepreneurs. At time t households maximise the objective function:

$$U_t^h = E_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{c_{t+s}^{1-\sigma} - 1}{1-\sigma} - \frac{\left(l_{t+s}^s\right)^{1+\varphi}}{1+\varphi} + \vartheta \ln\left(h_{t+s}\right) \right\} \tag{12}$$

where l_t^s is the household's labour supply, h_t the quantity of housing it owns and c_t is the usual Dixit-Stiglitz sub-utility function across the unit-measure continuum of differential goods produced:

$$c_t = \left[\int_0^1 c_t\left(j\right)^{\frac{\varepsilon_t - 1}{\varepsilon_t}} dj \right]^{\frac{\varepsilon_t}{\varepsilon_t - 1}} \tag{13}$$

where we allow the possibility that the elasticity of substitution across goods, ε_t, is time-varying. If the money price of good j is $p_t\left(j\right)$, the minimum expenditure required to obtain a unit of c_t, P_t, is given by:

$$P_t = \left[\int_0^1 p_t\left(j\right)^{1-\varepsilon_t} dj \right]^{\frac{1}{1-\varepsilon_t}} \tag{14}$$

The gross rate of consumer price inflation, $\frac{P_t}{P_{t-1}}$, is denoted π_t in what follows.

Households optimise subject to the period-by-period budget constraint (expressed in real terms):

$$w_t l_t^s + T_t + \frac{R_{t-1}}{\pi_t} s_{t-1} = c_t + p_t^h \left[\left(1+\tau^h\right) h_t - h_{t-1} \right] + s_t \tag{15}$$

with w_t the real wage, p_t^h the real price of housing, s_t the real quantity of saving (in nominal bonds, paying gross interest R_t), T_t a collection of lump-sum transfers to and from profit-making firms and the government and τ^h a housing tax introduced to ensure steady-state efficiency (of use if one wishes to contrast outcomes here with stabilisation policy determined by the optimisation of a welfare-based objective). This constraint is coupled with a usual transversality/'no-Ponzi' restriction.

3.2 Entrepreneurs

Entrepreneurs employ workers and make use of commercial real estate to produce intermediate goods, y_t, which are sold in a perfectly competitive market at price p^I to final goods firms. These entrepreneurs maximise a utility function expressed over consumption goods alone:

$$U_t^e = E_t \sum_{s=0}^{\infty} \left(\beta^e\right)^s \frac{\left(c_{t+s}^e\right)^{1-\sigma} - 1}{1-\sigma} \tag{16}$$

where the superscript e distinguishes their consumption of final goods from the household's, and $\beta^e < \beta$ holds.[4] This is subject to the period-by-period budget constraint:

$$b_t + (1-\tau^e)\left(p_t^I y_t - w_t l_t^d\right) = c_t^e + p_t^h\left(h_t^e - h_{t-1}^e\right) + \frac{R_{t-1}}{\pi_t} b_{t-1} \tag{17}$$

where l_t^d denotes labour demand, h_t^e commercial real estate (whose real price is also p_t^h) and b_t real borrowing, for which entrepreneurs are charged gross nominal rate R_t. τ^e is a tax on the proceeds of investment, also introduced to ensure steady-state efficiency. This is combined with an associated transversality/'no-Ponzi' condition, along with the collateral constraint:

$$b_t \leqslant m_t E_t \frac{\pi_{t+1}}{R_t} p_{t+1}^h h_t^e \tag{18}$$

with m_t the fraction of the monetary value of next period's commercial real estate that the entrepreneur is permitted to commit to the repayment of loans (this is subject to random fluctuations about a steady-state value denoted m), and the production function:

$$y_t = a_t \left(l_t^d\right)^{1-v} \left(h_{t-1}^e\right)^v \tag{19}$$

where the level of TFP, a_t, may likewise contain a stochastic component.

So long as the expected returns available to entrepreneurs from holding an extra unit of commercial real estate exceed the borrowing rate, the collateral constraint must hold with equality.[5] In this event entrepreneurs make their intertemporal choices as if faced with a single 'composite' asset, obtained by

[4] This condition is necessary to ensure a steady state in which financial leverage is observed, since the returns to leveraged investment must in general be higher than the real interest rate available to households.

[5] If it did not then the entrepreneur could always take on an extra ε units of commercial real estate, at real price $p_t^{h^e}$, and borrow an extra $\varepsilon p_t^{h^e}$ (for ε sufficiently small). Given the rate of return differential this would deliver an expected welfare gain at time $t+1$.

purchasing a unit of commercial real estate that they then leverage to the maximum possible extent. Thus they face an effective ex-post real rate of return on their savings, say RR^e_{t+1}, given by:

$$RR^e_{t+1} = \frac{\left(1-\tau^e\right) \upsilon p^I_{t+1} \frac{y_{t+1}}{h^e_t} + p^h_{t+1} - \frac{m_t E_t \pi_{t+1} p^h_{t+1}}{\pi_{t+1}}}{\left[p^h_t - m_t E_t \left(\frac{\pi_{t+1}}{R_t} p^h_{t+1}\right)\right]} \tag{20}$$

Many of the equilibrium consequences of fluctuations in the permitted leverage ratio m_t are best understood via their effects on this effective rate of return. In steady state it must equal the inverse of the entrepreneurial discount factor, β^e. Notice that it is only by barring households from investing in commercial real estate that we can provide the distinct rates of return that are necessary to guarantee stationary equilibrium consumption profiles for both entrepreneurs and households – despite the relative impatience of the former.[6]

3.3 Final goods firms

Final goods producers are monopolistically-competitive price setters owned by households, free to reset their prices only at stochastically-determined intervals – as in Calvo (1983). Each firm has access to a linear technology, converting intermediate goods one-for-one into final goods. The period-t profit level of firm j, $\Pi_t(j)$, thus satisfies:

$$\Pi_t(j) = \left(\left(1+\tau\right) p_t(j) - P_t p^I_t\right) y_t(j) \tag{21}$$

where τ is a production subsidy used to eliminate steady-state underemployment due to market power. Prices are then chosen to maximise the net present value (to households) of the firm's future stream of profits, assuming a fixed probability of resetting prices equal to θ each period.

3.4 Policy

The policymaker's only role is to set the (gross) nominal interest rate R_t. Again, we suppose R_t is set according to some 'simple rule', linking its value to contemporary realisations of the endogenous variables. Specifically, we consider rules of the following form:

$$R_t = \max\left\{\beta^{-1}\pi_t^{\alpha_\pi}\left(\frac{y^f_t}{y^f_{ss}}\right)^{\alpha_y}\left(\frac{y^f_t}{y^f_{t-1}}\right)^{\alpha_{\Delta y}}\left(\frac{p^h_t}{p^h_{t-1}}\right)^{\alpha_{\Delta p}}, 1\right\} \tag{22}$$

The inverse of the household discount factor features here as the steady-state nominal interest rate because the steady-state value of inflation is set to zero.

[6] With complete markets any general equilibrium would be Pareto efficient, and thus involve improvements upon the stationary outcome in the direction of allowing households to accumulate more and more wealth as time progresses.

The variables y_t^f and y_{ss}^f denote the contemporaneous and steady-state aggregate values of final output respectively. Again we allow for the possibility of feedback on the growth rate of real output, and the final term in the (unconstrained) policy rule now also allows for dependence on the growth rate of house prices – capturing a possible desire to 'prick' asset price bubbles by allowing them to feed into higher rates. The zero lower bound is translated here into a bound on the gross nominal rate at 1.

4 Model solution

We proceed to derive the basic model's key equations, considering optimal choices for households, entrepreneurs and firms in turn, followed by market clearing conditions and any further definitions.

4.1 Household optimality

Optimal dynamic choice for households implies a conventional consumption Euler equation:

$$c_t^{-\sigma} = \beta E_t c_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}} \tag{23}$$

The optimal choice of housing similarly implies:

$$\left(1+\tau^h\right) c_t^{-\sigma} p_t^h = \beta E_t c_{t+1}^{-\sigma} p_{t+1}^h + \vartheta h_t^{-1}. \tag{24}$$

Note that housing services provide a utility flow whose value to the consumer in equilibrium must exactly compensate for the fact that no financial return is paid (aside from any capital gain or loss).

Optimal intratemporal consumption-labour choice implies:

$$w_t c_t^{-\sigma} = (l_t^s)^{\varphi} \tag{25}$$

4.2 Entrepreneur optimality

The relevant Euler condition for entrepreneurs is:

$$(c_t^e)^{-\sigma} = \beta E_t \left(c_{t+1}^e\right)^{-\sigma} RR_{t+1}^e \tag{26}$$

We assume that the borrowing constraint binds with equality (this conjecture can then be verified *ex post*):

$$b_t = m_t E_t \frac{\pi_{t+1}}{R_t} p_{t+1}^h h_t^e \tag{27}$$

In their role as directors of intermediate goods firms, entrepreneurs must also ensure an optimal level of hiring:

$$w_t = (1-v) \frac{y_t}{l_t^d} p_t^I \tag{28}$$

Note that the right-hand-side of this equation is the marginal product of labour, converted into units of the numeraire (the final good).

4.3 Firm optimality

Optimal price-setting for Calvo-constrained firms (where $\widetilde{P}_t$ is the price chosen by firms resetting at t) gives:

$$E_t \sum_{s=0}^{\infty} (\beta\theta)^t \frac{c_{t+s}^{-\sigma}}{c_t^{-\sigma}} \left\{ (1+\tau) \frac{\widetilde{P}_t}{P_{t+s}} - \frac{\varepsilon_t}{\varepsilon_t - 1} p_{t+s}^I \right\} P_{t+s}^{\varepsilon_t} y_{t+s}^f = 0 \tag{29}$$

The composite discount factor applied to future marginal returns here reflects the firms' ownership by households. The firm is implicitly obliged to sell as many units as are demanded at the fixed price it posts, and with a single input it has no further choice variables.

4.4 Market clearing and further definitions

Labour market clearing gives:

$$(1-\omega)\, l_t^d = \omega l_t^s \tag{30}$$

Goods market clearing gives:

$$\omega c_t + (1-\omega)\, c_t^e = y_t^f \tag{31}$$

Real estate market clearing gives:

$$\omega h_t + (1-\omega)\, h_t^e = \overline{h} \tag{32}$$

where $\overline{h}$ is the aggregate stock of real estate, which is held fixed for simplicity.

It is easy to show that the aggregate level of final goods output will be related to intermediate goods output according to the equation:

$$\Delta_t y_t^f = (1-\omega)\, y_t \tag{33}$$

where the price dispersion index Δ_t is defined by:

$$\Delta_t \equiv \int_0^1 \left(\frac{P_t(j)}{P_t} \right)^{-\varepsilon_t} dj \tag{34}$$

The consumer price index then evolves in accordance with the Calvo pricing structure:

$$P_t = \left[\theta P_{t-1}^{1-\varepsilon_t} + (1-\theta)\, \widetilde{P}_t^{1-\varepsilon_t} \right]^{\frac{1}{1-\varepsilon_t}} \tag{35}$$

These equations are sufficient to complete the characterisation of the non-linear model.

We solve for the steady state of the model through standard techniques, and take linear approximations to the structural equations to analyse fluctuations in the neighbourhood of this. The associated linear equations are listed in Appendix 1.

5 Self-fulfilling crises: risks associated with simple rules

Our first set of simulations deals with the case in which policy is set according to a simple rule, consistent with equation 22. We choose the values for the structural parameters given in the table: these values are all carried over directly from Iacoviello (2005). For the policy rule we additionally assume $\alpha_\pi = 1.5$ and $\alpha_y = \alpha_{\Delta y} = 0$, neglecting feedback on the output gap or growth rate for simplicity. The parameter $\alpha_{\Delta p}$, giving the extent of feedback on house price growth, we set to 1.5, making this symmetric with the feedback parameter on goods price growth.[7] We can then proceed as in the simple New Keynesian model: by fixing the value of the nominal interest rate at the zero bound in the first time period alone, solving for the values of the endogenous variables in that and subsequent periods (under the assumption that the zero bound does not bind beyond the first period), and asking whether these realisations would indeed support zero rates at the start under the feedback rule 22.

β	household discount factor	0.993
β^e	entreprener discount factor	0.95
σ	inverse elasticity of intertemporal substitution	1
ϑ	weight on housing utility	0.11
φ	inverse Frisch elasticity of labour supply	2
ε	elasticity of substitution across final goods	6
ω	measure of household sector	0.979
v	elasticity of output with respect to CRE	0.05
θ	Calvo hazard rate	0.67
m	steady state permitted collateral ratio	0.85

The dynamics that we obtain are indeed consistent with a self-fulfilling crisis – that is, *given* the initial values for inflation, output and house prices that are implied by the model solution, a policymaker following rule 22 would wish to set nominal interest rates to zero. Figure 3 charts the dynamics for output, household consumption, inflation and house prices during this self-fulfilling crisis. As can be seen, all four fall by a substantial amount in the first period: output contracts by more than three percentage points, accompanied by a house price decline of around five per cent. The magnitudes are perhaps sufficiently large to challenge the validity of the linear approximation, though slightly less dramatic (as well as substantially greater) falls are possible when different values for the policy feedback parameters are chosen.[8]

The dynamics of this crisis are very similar to those of the simple New Keynesian case: in the first period a high real interest rate obtains because

[7] For the particular choices that we have made for other parameters, any value above roughly 0.97 will support similar self-fulfilling dynamics.

[8] Once again, when feedback on the 'growth' variable is increased the initial magnitude of the crisis is reduced – since lower future mean reversion is needed for the same policy contraction – but the persistence of the recession again increases.

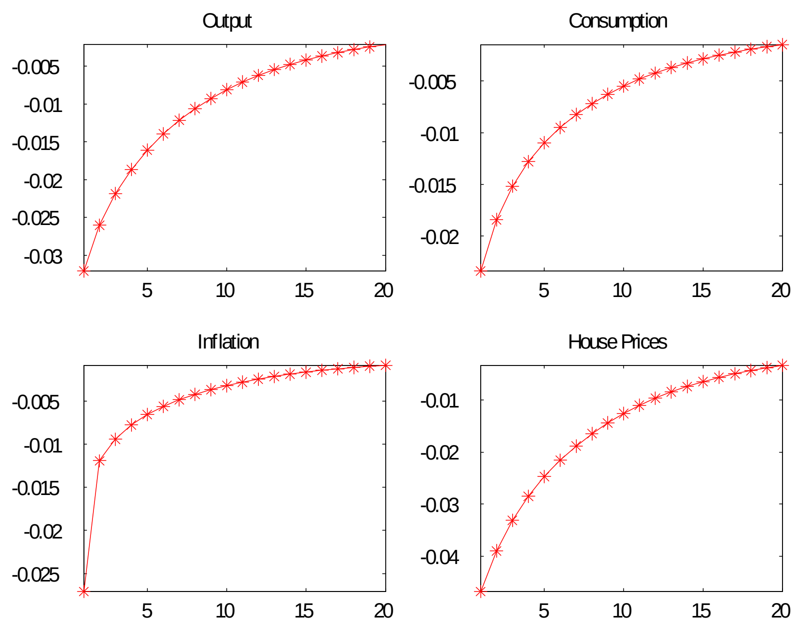

Figure 3: A self-fulfilling crisis

of deflationary expectations and the zero lower bound preventing more expansionary monetary policy. This makes entrepreneurs reluctant to take leveraged positions in commercial real estate because of the high real cost of servicing the associated debt – a factor compounded by the negative wealth effect that these agents suffer through a combination of real estate price falls and low profits in the intermediate goods sector (the later a product of stagnant consumption). If the policymaker were not concerned for asset price growth this contraction in commercial real estate would undermine the putative equilibrium, since it tends to put upwards pressure on future marginal costs in the intermediate goods sector, and hence on future inflation (reducing real interest rates). But when $\alpha_{\Delta p}$ is large enough the anticipated policy drive to prevent asset prices from returning too swiftly to their original levels counteracts this inflationary effect, causing expected future marginal costs to be below their steady state value and expected inflation to be substantially negative. That in turn generates the high real interest rates that we asserted from the start.

5.1 Discussion: multiple equilibria about a 'determinate' steady state

The dynamics here share a lot of the features (and magnitudes) associated with the self-fulfilling equilibria discussed by Mertens and Ravn (2011), but there is a crucial technical difference that it is worth reiterating. Those authors focus on the risk of a 'liquidity trap', whereby beliefs about the steady state to which the economy is converging switch at random from the 'desirable' state, whose local determinacy is guaranteed by satisfaction of the Taylor principle, to the alternative and locally indeterminate state (whose presence in the event of a zero lower bound was first highlighted by Benhabib, Schmitt-Grohe;, and Uribe (2002)). Deflationary expectations set in, and are expected to persist to the next time period with a sufficiently large probability that the policymaker is forced to accept stagnation – unable to cut real interest rates because of the zero bound. Because of the indeterminacy of this steady state there will generally exist a continuum of paths consistent with convergence on it. The crisis that we highlight can instead occur even when expectations are invariant functions of the fundamental variables in the economy, and thus operates even under a 'perfect foresight' assumption that the economy will converge on the desirable steady state with certainty. As discussed, it does require that deflation is anticipated the period after any initial house price fall, but instead of being the product of a fairly arbitrary bout of collective pessimism these deflationary expectations are now *derived from* the relatively contractionary policy that is known to follow in periods after a house price collapse – whilst the long-term focus of agents in the economy remains the locally 'determinate' steady state.

The clear implication is that (for this particular policy rule) the steady state in question is in fact associated with a form of *local* indeterminacy (so long as zero nominal rates may be considered 'local' to it), since more than one set of initial values for the control variables in the system can obtain, and be consistent with convergence to that steady state, absent any exogenous shocks. Unlike a linear system with an insufficient number of explosive roots, this indeterminacy does not admit a continuum of solutions – in fact, it admits just two: either a crisis can start, with nominal rates dropping to zero and the dynamics above obtaining, or the economy can remain in rest at the desired steady state.[9]

5.2 Sensitivity

The elasticity of intertemporal substitution is potentially a very important parameter in the results above. To see this, consider again for a moment the simple New Keynesian model, and suppose we were in the extreme situation of zero intertemporal elasticity (the limiting case where $\sigma \to \infty$). Then no matter how far the real interest rate were to deviate from steady state, output would remain constant through time – and thus no future policy contraction could be expected in response to its growth back. This appears (based on numerical simulations) to translate into a general rule in that model that self-fulfilling dynamics are only possible if $\alpha_{\Delta y} > \sigma\alpha_{\pi}$ (assuming $\alpha_y = 0$ still). Thus our assumption that $\sigma = 1$ both there and in our richer model might appear to be tipping the balance in favour of crises as a possibility – microeconometric evidence generally suggesting far lower elasticity, in the region of one fifth or lower.

When house price growth is the additional target variable this turns out not to be true. Figure 4 shows the dynamics associated with setting $\sigma = 5$ our version of the Iacoviello model – again with $\alpha_y = 0$ and $\alpha_{\Delta p} = \alpha_{\pi} = 1.5$. Again, the zero bound is set to bind in the first time period only, moving freely thereafter in accordance with rule 22; and again, outcomes in the initial period are sufficient to ensure the zero bound should indeed be binding. Noteworthy here are the relative responses of output and house prices: lower elasticity of intertemporal substitution has checked the former (as expected), but this does not stop the latter from moving by roughly the same amount as previously on impact – thus ensuring a relatively high real interest rate as prices gradually return to their original levels. So whilst it is certainly harder to generate crises as σ increases when policy is based on *output* growth targeting (in both models, according to our simulations), this doesn't carry over to policy rules that feed back on *house price* growth. Indeed, the set of values for $\alpha_{\Delta p}$ for which crisis dynamics are possible is now expanded relative to the $\sigma = 1$ case, with any value in excess of around 0.41 possible (as opposed to 0.97 previously). This is important in itself, as it implies even relatively modest feedback on house price growth could make the policymaker susceptible to the sorts of problems we highlight.

6 Conclusions

This paper higlights an important channel by which certain monetary policy feedback rules could permit self-fulfilling 'crisis' dynamics in the presence of a

[9] Strictly the point is that we have just two *perfect foresight* paths (as opposed to a continuum), since random reversion to the start of a crisis dynamic could potentially be introduced.

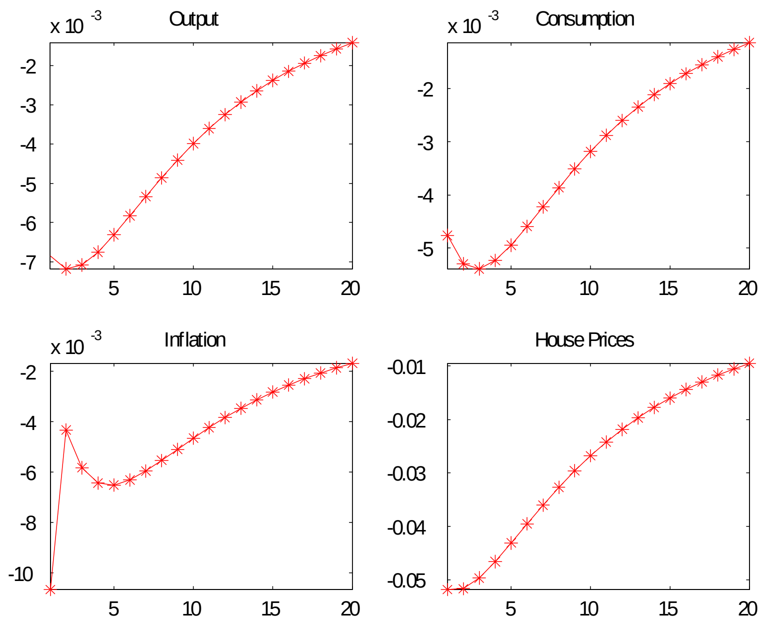

Figure 4: Dynamics with $\sigma = 5$

zero lower bound on nominal interest rates. Unlike similar penomena already highlighted in the literature, these crises are both self-fulfilling and consistent with convergence back to a zero-inflation steady state under perfect foresight. If central banks commit to interest rate rules that feed back on the change in the output gap, or on changes in asset prices, and so so sufficiently strongly, and the expectation arises that demand will be sufficiently weak to push interest rates to the zero bound, the expectation that thereafter interest rate policy will be tighter, having a concern not to induce a bounce back in demand (or asset prices) on account of the concern for the rate of change of demand, will be sufficient to mean that the initial expectation of weak demand will be self-fulfilling.

Our results provide a counterpoint to other work that has stressed that speed limit rules of the sort we have studied can have some desirable properties: namely, that they can be devices to mimic the optimal policy under commitment (see Giannoni and Woodford (2003), Stracca (2007), McCallum and Nelson (2004), Leduc and Natal (2011)); or that they are means to insulate the economy from the consequences of mismeasuring the levels of natural rate concepts needed as imputs to conventional monetary policy rules (see Orphanides and Williams (2002)). Since there is some evidence that these rules capture actual central bank behaviour (see, for example, Paez-Farrell (2009)), our cautionary tale is of more than academic interest.

We have abstracted from the possibility that the authorities might have at their disposal other instruments to substitute for monetary policy, (for example, fiscal tools, or unconventional monetary policy tools) and it might be expected that appropriate use of such instruments could rule out self-fulfilling attacks of the sort we have illustrated, at least for some regions of the parameter space for which such attacks were otherwise possible. That said, it would also very likely be the case that if those instruments were also governed by speed limit concerns, perhaps for the same reason that the interest rate tool were set in this way, the possibility for self-fulfilling recessions would remain. For now we leave these questions for future work.

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Appendix 1

In the region of its steady state, the model used in Section 3 onwards is described by the following linear (or piecewise linear) equations:

1. $\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \frac{(1-\beta\theta)(1-\theta)}{\theta} \widehat{p}_t^I$
2. $\widehat{c}_t = \widehat{c}_{t+1} - \frac{1}{\sigma}\left(\widehat{R}_t - E_t \widehat{\pi}_{t+1}\right)$
3. $\widehat{c}_t^e = \widehat{c}_{t+1}^e - \frac{1}{\sigma}\widehat{RR}_t$
4. $\widehat{RR}_t = \frac{\beta^e}{p_{ss}^h(1-m\beta)}\left[(1-\tau^e)\, v \frac{y_{ss}}{h_{ss}^e} E_t\left(\widehat{p}_{t+1}^I + \widehat{y}_{t+1} - \widehat{h}_t^e\right) + p_{ss}^h E_t \widehat{p}_{t+1}^h - \beta^{-1}\frac{b_{ss}}{h_{ss}^e} E_t\left(\widehat{R}_t - \widehat{\pi}_{t+1} + \widehat{b}_t - \widehat{h}_t^e\right)\right] - \frac{p_{ss}^h}{p_{ss}^h(1-m\beta)}\left[\widehat{p}_t^h - \beta m E_t\left(m_t + \widehat{\pi}_{t+1} - \widehat{R}_t + \widehat{p}_{t+1}^h\right)\right]$
5. $\left(1+\tau^h\right) p_{ss}^h c_{ss}^{-\sigma}\left(\widehat{p}_t^h - \sigma\widehat{c}_t\right) = \beta p_{ss}^h c_{ss}^{-\sigma} E_t\left(\widehat{p}_{t+1}^h - \sigma\widehat{c}_{t+1}\right) - \frac{\vartheta}{h_{ss}}\widehat{h}_t$
6. $\varphi \widehat{l}_t^s + \sigma\widehat{c}_t = \widehat{p}_t^I + \widehat{y}_t - \widehat{l}_t^s$
7. $\widehat{y}_t = (1-v)\,\widehat{l}_t^s + v\widehat{h}_{t-1}^e$
8. $\omega\widehat{c}_t + (1-\omega)\,\widehat{c}_t^e = \widehat{y}_t$
9. $\widehat{h}_t = \frac{\omega-1}{\omega}\frac{h_{ss}^e}{h_{ss}}\widehat{h}_t^e$
10. $\widehat{b}_t = E_t\left(\widehat{\pi}_{t+1} + \widehat{p}_{t+1}^h\right) - \widehat{R}_t + \widehat{h}_t^e$
11. $m\beta p_{ss}^h h_{ss}^e E_t\left(\widehat{\pi}_{t+1} + \widehat{p}_{t+1}^h - \widehat{R}_t + \widehat{h}_t^e\right) + (1-\tau^e)\, v y_{ss}\left(\widehat{y}_t + \widehat{p}_t^I\right) = c_{ss}\widehat{c}_t^e + p_{ss}^h h_{ss}^e\left(\widehat{h}_t^e - \widehat{h}_{t-1}^e\right) + \beta^{-1} b_{ss}\left(\widehat{R}_{t-1} - \widehat{\pi}_t + \widehat{b}_{t-1}\right)$
12. $\widehat{R}_t = \max\left\{\alpha_\pi\widehat{\pi}_t + \alpha_y\widehat{y}_t + \alpha_{\Delta y}\left(\widehat{y}_t - \widehat{y}_{t-1}\right) + \alpha_{\Delta p}\left(\widehat{p}_t^h - \widehat{p}_{t-1}^h\right), \beta - 1\right\}$

The subscript *ss* is used to denote steady-state values of the relevant variables, and 'hats' to denote log deviations from steady state.