Labelling contests with endogenous precision*

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Abstract

We propose a simple theory of labelling for credence goods of differing quality. We model the competition for the first, second, etc. labels as a rank order tournament in which firms can jointly control the ranking precision through individual information emission. This information may be interpreted as endogenously established (input for) labelling agencies, experts or regulatory bodies. While the labels can be seen as a public good guiding the consumers' purchasing decisions, individual firms have incentives to free ride on the competitors' information emission. The theory seems to be applicable to many industries including advertising, investment rating, the production (and pirating) of computer software, movies or music, etc. (JEL *C7, D7, H4, M3.* Keywords: *Labelling, Contests, Advertising.*)

1 Introduction

We employ a tournament to provide a ranking of goods or services with a credence aspect to the uninformed public.[1] In the applications we have in mind, different qualities of some good cannot be ranked by the consumers. An ordinal ranking of these goods—typically provided by experts on the basis of information provided by the producers of the goods—is, however, perfectly useful to inform consumers' consumption decisions. Applications of our theory include the labelling of organic produce (Soil Association, Demeter, Which?), medical drugs (GPs), general practitioners and doctors (Medicare, NHS), academic departments (research evaluations), novels (Pulitzer, Man Booker Prize), politicians and cars (media), software, games and movies (critics), financial products and firms (rating agencies), or political issues of conscience (activists).

The industry we use as the leading example to motivate our setup is the marketing and distribution of software, movies, e-books, or music. These clearly have a credence aspect: one needs to

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[1] "A *credence good* is a good (or service) whose utility is difficult or impossible to ascertain for the consumer. In contrast to *experience goods*, this utility is difficult to gauge even after consumption." (Wikipedia)

read a book or watch a movie to know if it is any good. Likewise, one needs substantial expertise to be able to evaluate new software. The deployment discussion for Microsoft's Windows 7, for instance, is extensive and wholly impenetrable to the uninitiated.[2] Moreover, even after mastering one product, one still has no means of comparison with other products. For this industry, rankings are provided among others by the Internet Movie Database `imdb.com` or VCD quality `vcdq.com` the role of which is comparable to film review magazines. Another type of (commented) ranking combined with download statistics is provided through piracy sites such as The Pirate Bay (`thepiratebay.org`) or Demonoid (`demonoid.me`).[3] Posting (links to) stolen software for download creates a loss in potential profits to the producer. But the download statistics generated by the (mostly illegal) downloads also create an informational externality (comparable to advertising) which makes choice based on other users' perceived qualities possible. There is, therefore, a social benefit to the activities of the above mentioned facilitators of illegal downloading since they provide a public service instrumental for informed choice. This service is genuine—since the pirates are seen to violate the commercial interests of the ranked copyright owners, their service is credible.[4] The producers themselves cannot credibly provide relative rankings and there is no alternative ranking which is comparably unbiased.

The contribution of this paper is twofold. On the theoretical side we provide the first analysis of a contest with endogenously chosen ranking precision. This makes applications to (partial) credence goods markets possible where complex product descriptions and evaluations are translated into a simple ordinal ranking. Secondly, in these stylised applications, we integrate both the consumer and producer side in a novel way such that consumer demand reacts endogenously to the firms' choice of qualities and information dissemination policies.

1.1 Related literature

The idea that in many circumstances efficiency can be induced through a rank order tournament is due to Lazear and Rosen (1981). The idea has found numerous applications and extensions, for instance in the work of Green and Stokey (1983), Nalebuff and Stiglitz (1983), Dixit (1987), Moldovanu and Sela (2001), or Siegel (2009). To our knowledge, however, there is no prior contribution which allows contestants to endogenously control the ranking precision strategically. For a detailed survey of the contests literature see the comprehensive Konrad (2008).

Research on credence goods was initiated by Pitchik and Schotter (1987) and found path breaking applications in, for instance, Taylor (1995), Emons (1997), Feddersen and Gilligan (2001), and

[2] See, for example, `http://technet.microsoft.com/en-us/deployment/default.aspx`.

[3] The rankings are real numbers in $[0, 10]$ at `imdb.com` and $\star$, $\star\star$, $\star\star\star$, $\star\star\star\star$, $\star\star\star\star\star$ at `vcdq.com`. The Pirate Bay and Demonoid are public BitTorrent trackers providing detailed user comments on the items they make available for download. What matters most for evaluation purposes are the number of downloads ('seeders' and 'leechers'). The Pirate Bay has more than 5 million registered users and is "one of the world's largest facilitators of illegal downloading" and "the most visible member of a burgeoning international anti-copyright or pro-piracy movement" (Los Angeles Times, 29-Apr-07). According to its web site, The Pirate Bay indexed 3,578.781 torrents and served "about 16.888,498.602,639.360 bytes of multi media action" on its eighth birthday (15-Sep-2011).

[4] There are examples of the copyright owners using these rankings strategically. See, for instance, `http://www.wired.com/threatlevel/tag/nude-nuns-with-big-guns/`.

Dulleck, Kerschbamer, and Sutter (2009). The more distant case of experience goods is analysed in Milgrom and Roberts (1986); there is an intermediate attempt by Hahn (2004). Recent surveys are presented by McCluskey (2000) and Dulleck and Kerschbamer (2006) which allow us to keep our review of this literature minimal.

The literature on labelling is young but already well developed. See for instance Lerner and Tirole (2006), Baksi and Bose (2007), Roe and Sheldon (2007), Lerner, Farhi, and Tirole (2010) or Harbaugh, Maxwell, and Roussillon (2011) and the papers cited therein. As our contest setup is a novel approach to the labelling problem, there seem to be no directly applicable papers.

There is a large industrial organisation literature on vertical discrimination in oligopoly markets (see, for instance, Jaskold Gabszewicz, Shaked, Sutton, and Thisse (1981), Shaked and Sutton (1983) or, more recently, Bonnisseau and Lahmandi-Ayed (2007)). As far as we are aware, though, there exist no attempts to integrate quality signals generated by a ranking into the vertical differentiation literature.

Barigozzi, Garella, and Peitz (2009) employ a contest to develop a (comparative) advertising model. They do not, however, consider ranking precisions, credence goods or model the demand side. In general, the advertising literature does not seem to have developed the idea of contests. For a recent review of this literature see Bagwell (2007).

2 The model

2.1 Supply side

There are two risk-neutral firms $\mathcal{N} = \{1, 2\}$ each of whom produces a good of quality θ_i, $i \in \mathcal{N}$ distributed according to $\theta_i \sim F_{[0,\infty)}$ with positive density $f(\theta_i)$.[5] Throughout the analysis we only make use of the ratio of these qualities $x_i = \theta_i/\theta_j$, $i = \{1, 2\}$ and $j = 3 - i$, and write $x = \theta_1/\theta_2$ if there is no danger of confusion. Without loss of generality we re-index firms such that $\theta_1 \geq \theta_2$. We assume that these qualities are mutually known among firms but that only the distribution of qualities F is known to the consumers. In the basic model, production and distribution of the goods are assumed to be costless. Moreover, once the good is produced, there is nothing a firm can do to alter its quality.[6]

Firm i, however, can choose to release information $\varepsilon_i \in \mathbb{R}$ on its product. Together with the other firm's emitted information ε_j, this information determines the precision that is used (by some labelling agency) to rank the products. In particular, we assume that the ranking precision is determined by the sum of available information $r = \varepsilon_1 + \varepsilon_2$. This information r affects the level of product differentiation in the consumer market and thus the firm's expected profit represented by the winner's and loser's prizes $P^1(r)$, $P^2(r)$ in the contest. For the parts of the analysis where we deal with exogenous prizes, we assume that $P^1(r) > P^2(r) \geq 0$ for $r > 0$. More precisely, we

[5] Although we generalise some of our results in later sections of this paper, we lay out the model in the main body of the paper for two firms only. With respect to the obtained intuition, this is without loss of generality.

[6] We extend the model to allow for the sequential choice of quality and precision in section 5.

assume that firm $i \in \mathcal{N}$ maximises

$$\max_{\varepsilon_i} u_i(\theta, \varepsilon) = q(x_i, r)P^1(r) + (1 - q(x_i, r))P^2(r) - c(|\varepsilon_i|) \tag{1}$$

where the information cost $c(|\varepsilon_i|)$ is strictly convex with $c(0) = 0$ and $q(x_i, r)$ is player i's probability of being ranked first. We assume that this noisy ranking of the firms' qualities $q(\theta, \varepsilon)$ is both observable and verifiable, that $q_i(\cdot)$ is strictly increasing in θ_i, strictly decreasing in θ_j, equal to $^1/_2$ for identical qualities, twice continuously differentiable, and zero for $\theta_i = 0$ with $\theta_{j\neq i} > 0$, $j = 3-i$. Moreover, we assume that $q_i(\cdot)$ is strictly increasing in $r = \varepsilon_i + \varepsilon_j$ if $\theta_i > \theta_j$.

2.2 Demand side

Consumers are represented through a distribution of preferences $\mu \sim G_{[0,s]}$ with continuous and strictly positive density $g(\mu) > 0$. Apart from these private preferences, the main element informing individual demand is an expert ranking of the qualities θ_1, and θ_2 arising spontaneously through the firms' dissemination of information ε.[7] In the absence of a ranking and because we are in a credence market, we assume that consumers cannot distinguish between products. Products of identical expected qualities are assumed to be sold at the same price with the firms sharing expected profits equally. Consumers do not know the realisation of product qualities but can form expectations of these based on the commonly known distribution F. If the firms disseminate information then consumers observe the total amount of information in the market (i.e., the ranking precision) r, but not the individually emitted components ε. The utility of a type-μ consumer is assumed to be quasi-linear with

$$v(\mu, \theta) = \mu\tilde{\theta} - \tilde{p} \geq 0 \tag{2}$$

where $\tilde{p}$ is the price paid for a product of quality $\tilde{\theta}$.

In this setup, consumer demand only depends on expectations, that is, on the order statistics $f_{(k:n)}(\theta)$ giving the probabilities of the random variable θ coming k^{th}, $1 \leq k \leq n$, among $n = 2$ independent random draws from the distribution of qualities $F(\theta)$. We denote the expectation of the k-ranked product quality by

$$\mathbb{E}[\Theta_{(k:n)}] = \int_0^1 \theta f_{(k:n)}(\theta) d\theta. \tag{3}$$

In our labelling contest, a firm's prize for coming k^{th} is the expected consumer demand captured by the k^{th}-labelled product given the observed ranking q. To determine this demand, we use a vertical product differentiation model where the expected quality is determined through the product rank.[8] Given their mutually known qualities, firms decide on both their optimal, rank-dependent prices and the amount of information to emit. Thus, there are two steps, first firm $i = 1, 2$ chooses and

[7] This strategically emitted information may be interpreted as the number of endogenously generated non-strategic experts performing a ranking of products. In this interpretation, this is a model where experts give advice to consumers in a credence market.

[8] The classic references are Jaskold Gabszewicz, Shaked, Sutton, and Thisse (1981) and Shaked and Sutton (1983).

announces prices (p_i^1, p_i^2) conditional on each possible rank k.[9] This defines the prizes P^1, and P^2 for the ranking tournament in which the firms subsequently choose ε_i in the second step.

It is useful to define the consumers' expectation of the k^{th}-ranked product quality for an observed ranking. For rank $1 \leq k \leq n = 2$, this expectation is given by

$$\Lambda^k = \sum_{k=1}^{2} \tilde{q}_i^k(\tilde{x}, r)\, \mathbb{E}[\Theta_{(i:n)}] \tag{4}$$

where we denote the consumers' rank-dependent winning expectations over q by

$$\tilde{q}^1 = (\tilde{q}_1^1(\tilde{x}, r), \tilde{q}_2^1(\tilde{x}, r)) = \left(\frac{1}{1 + \tilde{x}^{-r}}, \frac{1}{1 + \tilde{x}^{r}} \right), \text{ and } \tilde{x} = \frac{\mathbb{E}[\Theta_{(1:2)}]}{\mathbb{E}[\Theta_{(2:2)}]}. \tag{5}$$

(The rank-dependent expectation of losing $\tilde{q}^2$ is defined accordingly.) Given an observed ranking q and announced prices p, a marginal consumer of valuation $\hat{\mu}_{k+1}^k$ is indifferent between buying the k^{th} and $k + 1^{\text{st}}$ ranked products iff

$$\mu \Lambda^k - p_k = \mu \Lambda^{k+1} - p_{k+1} \tag{6}$$

resulting in the vector of cutoffs

$$\hat{\mu} = (\hat{\mu}_1^0 = s,\ \hat{\mu}_2^1 = \frac{p_1 - p_2}{\Lambda^1 - \Lambda^2},\ \hat{\mu}_3^2 = \frac{p_2}{\Lambda^2}) \tag{7}$$

Given these cutoffs, the first and second ranked firms maximise their profits by choosing p_1^* and $p2^*$, respectively, such as to

$$\begin{aligned} \arg\max_{p_1} P^1(r) &= p_1 \int_{\hat{\mu}_2^1(r)}^{\hat{\mu}_1^0 = s} g(\mu) d\mu = p_1 \left(G(\hat{\mu}_1^0) - G(\hat{\mu}_2^1) \right), \\ \arg\max_{p_2} P^2(r) &= p_2 \int_{\hat{\mu}_3^2(r)}^{\hat{\mu}_2^1(r)} g(\mu) d\mu = p_2 \left(G(\hat{\mu}_2^1) - G(\hat{\mu}_3^2) \right). \end{aligned} \tag{8}$$

We write the full vectors of prizes $(P^1(r), P^2(r))$ and prices $(p_1(r), p_2(r))$ as $P(r)$ and $p(r)$, respectively. Both are used in the firms' maximisation problem of choosing ε.

Proposition 1. *For any number of firms $n = |\mathcal{N}|$ and any distribution G with strictly positive and weakly concave density g of consumers' tastes μ satisfying the condition that*

$$\frac{g(\mu)}{\int_\mu^\infty g(x) dx} \text{ is strictly increasing,} \tag{9}$$

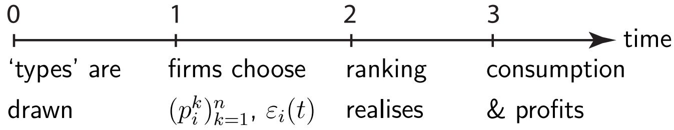
there exists an equilibrium vector of announced prices $p_1 < p_2 < \cdots < p_n$.[10]

[9] Since firms commonly know product qualities, each firm faces the same optimisation problem for choosing $p_i(k : n)$ and thus selects the same vector of conditional product prices.

[10] The proof is derived from Jaskold Gabszewicz, Shaked, Sutton, and Thisse (1981). Since their working paper seems to be hard to obtain and our setup is slightly different, we nevertheless state the full proof in the appendix.

2.3 Timing and information

Both prizes P and prices p are functions of the available information, i.e., the ranking precision $r = \varepsilon_1 + \varepsilon_2$. We are looking for (subgame perfect) asymmetric, pure strategy equilibria where each firm chooses pairs $((p_i^k)_{k=1}^2, \varepsilon_i)_{i=1}^2$. The complete timing of the interaction is shown below.



Consumers cannot observe the firms' individual ε, but they can observe the amount of overall available information r. Consumers use this information in determining the precision with which the ranking of product labels is correct, i.e. corresponding to the true order of qualities. The yardstick we use to measure the effects of labelling is the allocation a benevolent social planner would strive to implement. After an illustrative example we discuss the formal structure of the game giving rise to an endogenous ranking precision and characterise the firm's equilibrium behaviour. We extend the model in section 5 to allow for sequential choice of quality and precision and derive results under both symmetric and asymmetric market structures. We finish with an efficiency and welfare analysis and discuss whether the introduction of labelling is socially beneficial. Generalisations, proofs and details can be found in the appendix.

3 Example

Consider the following simple example with two firms of uniform quality and uniform preferences $\theta, \mu \sim U_{[0,s]}$, $s \in \mathbb{N}$. Qualities θ_i are unobservable to the consumers but known among competitors. Thus, to the consumer, the expected quality of a product is given conditional on its rank. A firm chooses i) the price of her product conditional on the product's rank and the distribution of consumer preferences and ii) the resources ε_i it wishes to expend on influencing the overall ranking precision and demand.

We start by modelling the demand side. Expected qualities are obtained through summing up the k^{th} order statistic among n independent draws (3) and denoted by $\mathbb{E}[\Theta_{(k:n)}]$. For the Uniform distribution on the s-scaled unit interval these expectations are just $\mathbb{E}[\Theta_{(k:n)}] = \frac{sk}{n+1}$. Given an observed ranking, the consumers assess the expected qualities of the k^{th}-ranked products as

$$\Lambda^1 = \frac{1}{1+\tilde{x}^{-r}}\,\mathbb{E}[\Theta_{(1:2)}] + \frac{1}{1+\tilde{x}^{r}}\,\mathbb{E}[\Theta_{(2:2)}],\ \Lambda^2 = \frac{1}{1+\tilde{x}^{r}}\,\mathbb{E}[\Theta_{(1:2)}] + \frac{1}{1+\tilde{x}^{-r}}\,\mathbb{E}[\Theta_{(2:2)}],\ \tilde{x} = \frac{\mathbb{E}[\Theta_{(1:2)}]}{\mathbb{E}[\Theta_{(2:2)}]}.$$

A 'type'-μ consumer is indifferent between the first- and second ranked products iff $\mu\Lambda^1 - p_1 = \mu\Lambda^2 - p_2$, i.e., we obtain the market cutoffs[11]

$$\hat{\mu} = \left(\hat{\mu}_1^0 = s,\ \hat{\mu}_2^1 = \frac{p_1 - p_2}{\Lambda^1 - \Lambda^2} = \left(3 + \frac{6}{2^r - 1}\right)(p_1 - p_2),\ \hat{\mu}_2^3 = p_2/\Lambda^2\right). \tag{10}$$

[11] Notice that, because $\hat{\mu}_2^3 > 0$, the market will not be fully served.

Given these cutoffs, the first ranked firm maximises her profit by choosing p_1

$$\max_{p_1} P^1 = p_1 \int_{\hat{\mu}_2^1}^{\hat{\mu}_1^0} g(\mu) d\mu = p_1(\hat{\mu}_1^0 - \hat{\mu}_2^1) \tag{11}$$

and the second ranked firm maximises her profit by choosing p_2

$$\max_{p_2} P^2 = p_2 \int_{\hat{\mu}_3^2}^{\hat{\mu}_2^1} g(\mu) d\mu = p_2(\hat{\mu}_2^1 - \hat{\mu}_3^2). \tag{12}$$

Maximisation wrt p_i and solving gives

$$p_1^*(r) = s^2 \frac{2\left(2^r - 1\right)\left(2^{r+1} + 1\right)}{3\left(2 + 9 \times 2^r + 7 \times 4^r\right)}, \; p_2^*(r) = s^2 \frac{2^r + 4^r - 2}{3\left(2 + 9 \times 2^r + 7 \times 4^r\right)} \tag{13}$$

resulting in the equilibrium cutoffs

$$\hat{\mu}_1^0 = s, \; \hat{\mu}_2^1 = \frac{3 \times 2^r s}{2 + 7 \times 2^r}, \; \hat{\mu}_3^2 = \frac{(2^r - 1)s}{2 + 7 \times 2^r} > 0 \tag{14}$$

giving, in turn, rank dependent prizes as functions of the available information of

$$\begin{aligned} P^1(r) = (\hat{\mu}_1^0 - \hat{\mu}_2^1)p_1^* = s^3 \frac{4\left(2^r - 1\right)\left(1 + 2^{r+1}\right)^2}{3\left(1 + 2^r\right)\left(2 + 7 \times 2^r\right)^2}, \\ P^2(r) = (\hat{\mu}_2^1 - \hat{\mu}_3^2)p_2^* = s^3 \frac{\left(2^r - 1\right)\left(2 + 2^r\right)\left(1 + 2^{r+1}\right)}{3\left(1 + 2^r\right)\left(2 + 7 \times 2^r\right)^2}. \end{aligned} \tag{15}$$

How these prizes result from consumer demand is illustrated in figure 1.

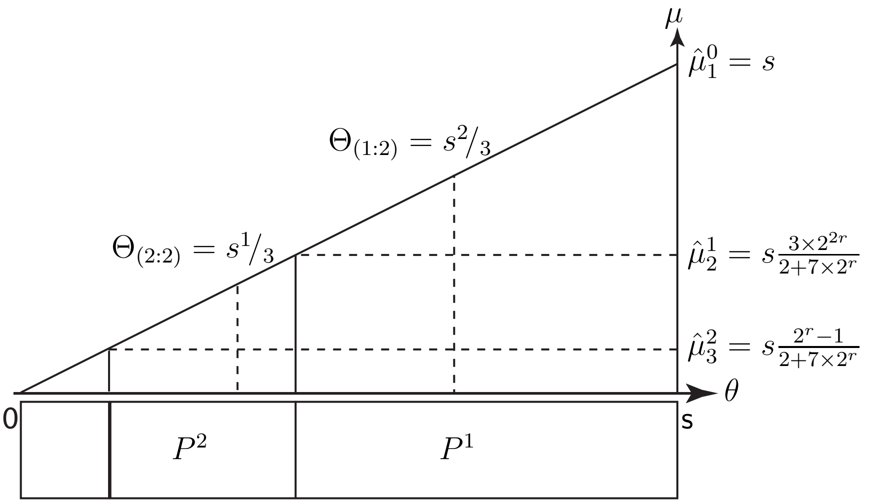


Figure 1: The labelled example market.

Given the prizes (15), we turn to the supply side and state firm $i = 1, 2$'s maximisation problem (under mutually known $x_i = \theta_i/\theta_j$) as

$$\max_{\varepsilon_i} \frac{1}{1+x_i^{-r}} P^1(r) + \frac{1}{1+x_i^r} P^2(r) - \frac{\varepsilon_i^2}{2}, \text{ where } r = \varepsilon_i + \varepsilon_j \text{ and } j = i - 3. \tag{16}$$

In order to get numerical values, we set we set $\theta_1 = {}^3/_4$, $\theta_2 = {}^1/_4$ (i.e., $x = 3$), and $s = 4$. After taking derivatives wrt ε_i, we find the asymmetric equilibrium candidate as ($\varepsilon_1^* = 1.847, \varepsilon_2^* = -0.088$), implying that $r^* = 1.759$. Figure 2 verifies this candidate as the unique pure strategy equilibrium.

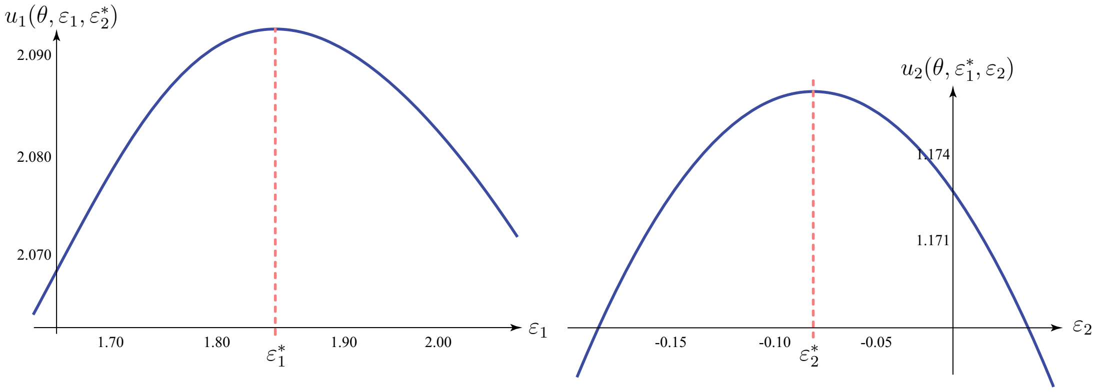


Figure 2: The two players' optimal choice of ε_i in asymmetric equilibrium.

We will show in section 6 that, in this simple example, labelling a single product (and thus creating a full ranking) improves consumer welfare over the unlabelled case. The next section derives more general insights into the machinations of precision contests and derives the general analytic form of the firms' information dissemination policy functions.

4 Analysis

We begin the analysis by stating a simple but useful general property of this type of contest.

Lemma 1. *Consider player $i \in \mathcal{N}$ with objective* (1). *A necessary condition for a maximum is*

$$c'(|\varepsilon_1|) + c'(|\varepsilon_2|) = P'^1(r) + P'^2(r). \tag{17}$$

Equipped with this Lemma, we now develop our intuition step by step, starting with a simple case leading up to the full application we study. Before we analyse the contest, however, let us quickly remark that (17) cannot be socially optimal because a planner maximising benefits over costs solves the problem

$$\max_{\varepsilon_1,\varepsilon_2} q(x,r)\left(P^1(r) + P^2(r)\right) + (1 - q(x,r))\left(P^1(r) + P^2(r)\right) - c(|\varepsilon_1|) - c(|\varepsilon_2|) \tag{18}$$

which results in the foc

$$c'(|\varepsilon_1|) + c'(|\varepsilon_2|) = 2P'^1(r) + 2P'^2(r). \tag{19}$$

The precision of the privately provided product ranking is therefore necessarily inefficient.

From now on we assume that the probability of being ranked first $q(x)$ is given by the generalised Tullock success function $q(x_i, r) = \frac{1}{1+x_i^{-r}}$, $i = \{1,2\}$ and $j = 3 - i$.[12] Moreover, individual information emission cost is assumed to be quadratic $c_i(|\varepsilon_i|) = \varepsilon^2/2$. Notice that for these particular quadratic cost functions Lemma 1 implies the equilibrium identity

$$r^* = \varepsilon_1^* + \varepsilon_2^* = P'^1(r) + P'^2(r). \tag{20}$$

In the first version of the precision contest, we set $P^2 = 0$ and study the players' behaviour under a single, constant prize P^1. Player i's problem is

$$\max_{\varepsilon_i} u_i(\theta, \varepsilon) = q(x_i, r)P^1 - c(|\varepsilon_i|) = \frac{1}{1 + x_i^{-r}} P^1 - \frac{\varepsilon_i^2}{2} \tag{21}$$

with focs

$$\varepsilon_1 = P^1 \frac{x^r \log(x)}{(1 + x^r)^2} = -\varepsilon_2. \tag{22}$$

Inserting $r = 0$ from lemma 1 results in the bidding functions

$$\varepsilon_1^* = P^1 \frac{\log(x)}{4} = -\varepsilon_2^*. \tag{23}$$

Proposition 2. *Provided that $x = \theta_1/\theta_2 \leq e^{\sqrt{\frac{2}{P^1}}\, 3^{3/4}}$, the contest* (21) *with exogenous prize P^1 has the unique pure strategy equilibrium* (23).[13]

This result confirms the intuition that, with identical (marginal) cost, whatever the higher quality firm invests in increased ranking precision is undone by the low quality firm. Players thus engage in a pure chance draw and obtain symmetric utility.[14] Equilibrium uniqueness depends on the contest not to be too asymmetric. If x increases further beyond the above sufficient condition then equilibrium existence becomes problematic and eventually fails. Extremely asymmetric contests do therefore not exhibit a pure strategy equilibrium.

In the next version of our precision contest, we leave $P^2 = 0$ and study the players' behaviour when the single prize $P^1(r)$ reacts to the amount of information produced in a linear fashion $ur = u(\varepsilon_1 + \varepsilon_2)$, $u > 0$. The players' problem is to

$$\max_{\varepsilon_i} u_i(\theta, \varepsilon) = q(x_i, r)P^1(r) - c(|\varepsilon_i|) = \frac{1}{1 + x_i^{-r}} ur - \frac{\varepsilon_i^2}{2}. \tag{25}$$

[12] We define $q(0/0, r) = 1/2$ and $q(\theta./0, r) = 1$ for $\theta. > 0$ for completeness.

[13] Throughout the paper, e stands for (Euler's) exponential constant.

[14] For a variant where information policies do not cancel out consider, for instance,

$$\max_{\varepsilon_i} u_i(\theta, \varepsilon) = \frac{1}{1 + x_i^{-r}} P - \frac{\varepsilon_i^2}{2x_i} \tag{24}$$

which leads to the necessary condition $\varepsilon_i^* = -x_i^2 \varepsilon_j^*$. Already this simple variant seems analytically intractable.

Taking the derivative wrt ε_i gives player i's foc implicitly as

$$\varepsilon_i = \frac{u x_i^r \left(1 + x_i^r + r \log(x_i)\right)}{\left(1 + x_i^r\right)^2}, \;\; r = \varepsilon_1 + \varepsilon_2, \tag{26}$$

which, by applying the insight from Lemma 1, transforms into the mutual best response functions

$$\varepsilon_1^* = \frac{u x^u (1 + x^u + u \log(x))}{(1 + x^u)^2}, \;\; \varepsilon_2^* = \frac{u(1 + x^u - u x^u \log(x))}{(1 + x^u)^2} \tag{27}$$

which are independent of r. Below figure 4 graphs these as functions of x for $u = 2$. The following

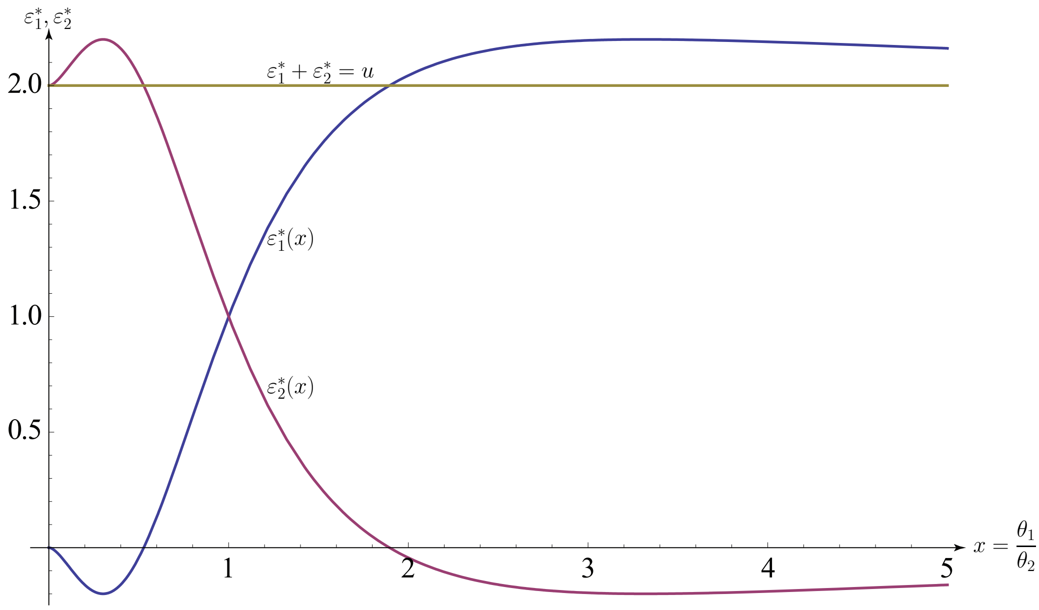


Figure 3: Firms' information dissemination policies as functions of their quality ratio θ_1/θ_2.

proposition shows that this derivation does not depend on the linearity of prize $P^1(r)$.

Proposition 3. *Consider contest* (25)*. For any positive, weakly concave and twice continuously differentiable prize* $P^1(r)$*,* $r = \varepsilon_1 + \varepsilon_2$*, we obtain the equilibrium bidding functions* (27) *where* $u = r$ *solves* $r = P'^1(r)$*.*

Notice that concavity in the above proposition is only a sufficient condition. There are many examples of convex functions which have derivatives with a fixed point. It can be easily verified that the limits of the best response functions in (27) are given by

$$\varepsilon_1^*(0) = 0, \;\; \varepsilon_2^*(0) = u, \;\; \lim_{x \to \infty} \varepsilon_1^*(x) = u, \;\; \lim_{x \to \infty} \varepsilon_2^*(x) = 0. \tag{28}$$

Notice that, apart for the limits, for $x \notin [\text{ProductLog}(\frac{1}{e})^{1/u}, \text{ProductLog}(\frac{1}{e})^{-1/u}] (\approx [0.53, 1.89]$ for $u = 2)$, we find that $\varepsilon_i^*(x) \notin [0, 1]$ although $\varepsilon_1 + \varepsilon_2 \equiv u$ everywhere.[15] For given x, the functions (27) turn out to be non-monotonic in u.

[15] The ProductLog (also known as Lambert-W or Omega) function gives the principal solution for w in $z = we^w$.

Let us now consider the two-prize extension of this simple contest. As for the first prize $P^1(r)$, assume that the second prize is linear in r and given by $P^2(r) = tr$. A player then faces the problem

$$\max_{\varepsilon_i} u_i(\theta, \varepsilon) = q(x_i, r)P^1(r) + (1 - q(x_i, r))P^2(r) - c(|\varepsilon_i|) = \frac{1}{1 + x_i^{-r}} ur + \frac{1}{1 + x_i^r} tr - \frac{\varepsilon_i^2}{2} \quad (29)$$

where $u > t \geq 0$. Taking derivatives gives the focs which, by substituting $r = u + t$ in equilibrium, transform into the mutual best response functions

$$\begin{aligned} \varepsilon_1^* &= \frac{t + ux^{2(u+t)} + (u+t)x^{u+t}(1 + (u-t)\log(x))}{(1 + x^{u+t})^2}, \\ \varepsilon_2^* &= \frac{u + tx^{2(u+t)} + (u+t)x^{u+t}(1 + (t-u)\log(x))}{(1 + x^{u+t})^2}. \end{aligned} \quad (30)$$

Corollary 1. *Consider contest* (29)*. Given equilibrium existence, for any positive, weakly concave and twice continuously differentiable prize sum* $P(r) \equiv P^1(r) + P^2(r)$*,* $r = \varepsilon_1 + \varepsilon_2$*, we obtain the equilibrium bidding functions* (30) *where* $u + t = r$ *solves* $r = P'(r)$*.*

The argument follows directly from the proof of proposition 3.

4.1 The full application

In our application, the two prizes derived from the consumer side are non-linear in r and given by

$$P^1(r) = \frac{4\,(2^r - 1)\,(1 + 2^{r+1})^2\, s^3}{3\,(1 + 2^r)\,(2 + 7 \times 2^r)^2},\ P^2(r) = \frac{(2^r - 1)\,(2 + 2^r)\,(1 + 2^{r+1})\, s^3}{3\,(1 + 2^r)\,(2 + 7 \times 2^r)^2}. \quad (31)$$

This gives rise to the players' problem

$$\max_{\varepsilon_i} \frac{1}{1 + x_i^{-r}} P^1(r) + \frac{1}{1 + x_i^r} P^2(r) - \frac{\varepsilon_i^2}{2} \quad (32)$$

with foc

$$\varepsilon_i = \frac{x^r \log(x)(P^1(r) - P^2(r)) + (1 + x^r)\,(x^r P'^1(r) + P'^2(r))}{(1 + x^r)^2} \quad (33)$$

which generalises (30). As there, we transform the foc into bidding functions for the equilibrium r^* given by Lemma 1 as solution to the fixed point problem[16]

$$r = P'^1(r) + P'^2(r) = \frac{2^{6+r}\,(22 + 81 \times 2^r + 27 \times 4^{r+1} + 59 \times 8^r)\log(2)}{(1 + 2^r)^2\,(2 + 7 \times 2^r)^3} \approx 1.7593. \quad (34)$$

[16] Since both $P'^1(r)$ and $P'^2(r)$ are convex and downward sloping for $r > 0$, their sum is convex and downward sloping, too. Since $P'^1(0) + P'^2(0) > 0$, there is a unique fixed point giving a unique equilibrium contest precision level.

Inserting this r^* into the foc (33) we obtain the equilibrium information policies

$$\begin{aligned}\varepsilon_1^* &= \frac{P'^2(r^*) + P'^1(r^*)x^{2r^*} + x^{r^*}(P'^1(r^*) + P'^2(r^*) + (P^1(r^*) - P^2(r^*))\log(x))}{(1+x^{r^*})^2}, \\ \varepsilon_2^* &= \frac{P'^1(r^*) + P'^2(r^*)x^{2r^*} + x^{r^*}(P'^1(r^*) + P'^2(r^*) + (P^2(r^*) - P^1(r^*))\log(x))}{(1+x^{r^*})^2}.\end{aligned} \tag{35}$$

Using example parameters $s = 4$ and $x = 3$, these reproduce the previously obtained values of

$$\varepsilon_1^* = 1.847, \varepsilon_2^* = -0.088. \tag{36}$$

Notice that, since our analysis is independent of the particular functional form of prizes in (31), we can also model different markets such as the downloading of software or advertising without any change to the main analysis above. All that changes are (31) and their derivatives. To give an example of both markets, consider the simple variant of prizes (31)

$$B^1(r) = r^\gamma P^1(r),\ B^2(r) = r^\gamma P^2(r),\ \text{where } \gamma \in \{-\frac{1}{2}, 0, +\frac{1}{2}\} \tag{37}$$

in order to model market demand derived from the same model as above but with an increasing, constant, and decreasing demand aspect of the information provided.[17] The corresponding information output for each parameter value (again for $s = 4$ and $x = 3$) is

$$\begin{aligned} -{}^1\!/_2: &\quad \varepsilon_1^* = 1.157, \quad \varepsilon_2^* = -0.177, \quad r^* = 0.980, \\ 0: &\quad \varepsilon_1^* = 1.847, \quad \varepsilon_2^* = -0.088, \quad r^* = 1.759, \\ {}^1\!/_2: &\quad \varepsilon_1^* = 3.118, \quad \varepsilon_2^* = 0.089, \quad r^* = 3.207. \end{aligned}$$

Quite intuitively, there is still competition for the larger prize in the shrinking market but the lower quality firm prefers to obfuscate the ranking (e.g., by hiring lawyers to take down ranking sites in the downloading example rather than promoting the high quality product through freely available software). The second line corresponds to our initial example without direct demand impact of information. In the growing market, both firms choose to 'advertise' and expand demand although the higher quality firm has much more incentives to do so than the lower quality firm.

Finally, we would like to point at another consequence of lemma 1. The interaction we model is a dynamic game where consumers potentially react by updating their prior beliefs on the distribution of qualities F after observing the firms' equilibrium information policy r^*. Since total r^* turns out to be constant in equilibrium for any ratio of qualities x, this updating step is trivial and consumers leave their priors unchanged.[18]

[17] The possible parameter set for γ is chosen such as not to impinge on equilibrium existence in our example.
[18] We assume that priors remain unchanged following out of equilibrium firm behaviour.

5 Sequential choice of quality and precision

We now modify the timing from section 2.3 to allow for the sequential choice of quality and precision. This analysis is computationally difficult in the full setup of section 2. In order not to cloud the intuition behind sequential competition along both dimensions overly with technical problems, we analyse two simple model variants. The first is symmetric with a single prize (and has applications in its own right) while the second analyses the two-prizes version of the model.

5.1 Symmetric firms

In this model variant, ex-ante symmetric firms choose first qualities and then, after observing and learning mutual qualities, they choose the ranking precision. More precisely, we assume that firms $i \in \mathcal{N}$ maximise

$$\max_{\theta_i,\varepsilon_i} \frac{1}{1+x_i^{-r}} P^1(r) - \frac{\theta_i^2}{2} - \delta\frac{\varepsilon_i^2}{2},\ r = \varepsilon_i + \varepsilon_j,\ j = i-3 \tag{38}$$

where the prize $P^1(r)$ satisfies the assumptions from proposition 3. Assume that the timing is such that qualities $\theta_i \in (0, \infty]$ are simultaneously chosen at the first stage and precisions $\varepsilon_i \in \mathbb{R}$, depreciated at rate $\delta \in (0, 1)$ wrt the first stage, are chosen simultaneously at the second stage. In order to solve this problem, we apply backward induction and lemma 1 to solve for $r^* = P'^1(r^*)$, set $u = P'^1(r^*)$ and adopt the optimal second stage choice of ε^*, (27) from proposition 3, to obtain the equilibrium information policies

$$\varepsilon_1^* = \frac{ux^u\left(1 + x^u + u\log(x)\right)}{\left(1 + x^u\right)^2},\ \varepsilon_2^* = \frac{u\left(1 + x^u - ux^u\log(x)\right)}{\left(1 + x^u\right)^2}. \tag{39}$$

Given these second stage equilibrium choices—which obviously depend on x and thus first period choices of θ—we proceed to the first stage, take derivatives of (38) wrt θ_i, simplify for the symmetric $x = 1$ and directly obtain the first stage equilibrium choice of qualities

$$\theta^* = \theta_1^* = \theta_2^* = \sqrt{\frac{u^3(1-\delta)}{4}}. \tag{40}$$

Figure 4 verifies this candidate as the unique pure strategy equilibrium $(\theta^*, \varepsilon^*)$ for parameters $u = 2$ and $\delta = 3/4$.[19] As usual, pure strategy equilibrium existence in the quality contest is problematic for large u.

5.2 Asymmetric firms

In this model variant, asymmetric firms first choose qualities and then, after observing and learning mutual qualities, they choose the ranking precision. Assume that firms $i \in \mathcal{N}$ maximise the following

[19] The symmetric equilibrium values for the graphed example of figure 4 are $\theta^* = 0.707$ and $\varepsilon^* = 1$.

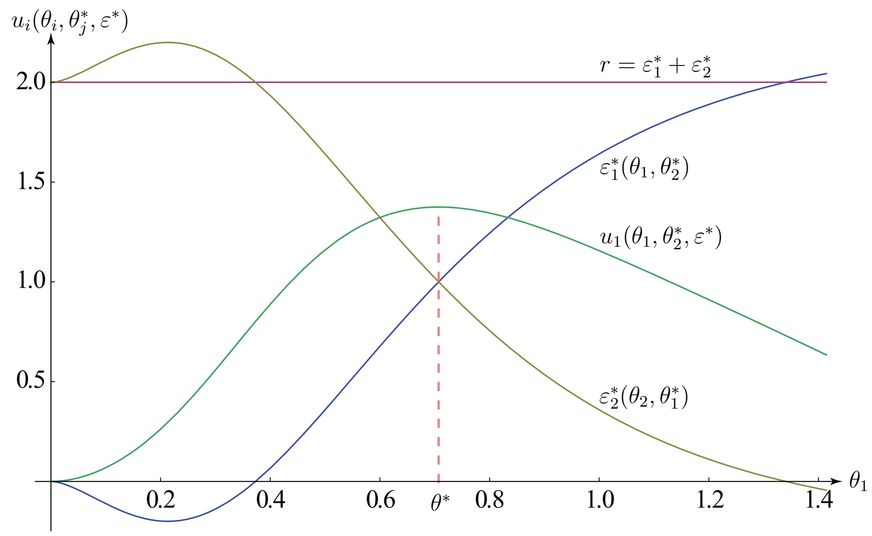


Figure 4: The optimal choice of qualities θ_i in symmetric pure strategy, subgame perfect equilibrium.

variant of (1)

$$\max_{\theta_i, \varepsilon_i} \frac{1}{1+x_i^{-r}} P^1(r) + \frac{1}{1+x_i^{r}} P^2(r) - \frac{\theta_i^d}{d\gamma_i} - \delta \frac{\varepsilon_i^2}{2}, \ \gamma_1 = 1, \ \gamma_2 \in (0, \infty), \ r = \varepsilon_i + \varepsilon_j, \ j = i - 3 \quad (41)$$

where prizes $P^1(r)$ and $P^2(r)$ satisfy the assumptions from corollary 1, $\delta \in (0, 1)$ is and $d > 0$ is the quality cost curvature. Assume that the timing is such that qualities $\theta_i \in (0, \infty]$ are simultaneously chosen at the first stage and precisions $\varepsilon_i \in \mathbb{R}$ are chosen simultaneously at the second stage. In order to solve this problem, we apply backward induction and lemma 1 to solve for the fixed point $r^* = P'^1(r) + P'^2(r)$, set $u = P'^1(r^*)$, $t = P'^2(r^*)$, and $\Delta P = P^1(r^*) - P^2(r^*)$ and adopt the optimal second stage choice of ε^* from (33) to obtain equilibrium information policies

$$\varepsilon_1^* = \frac{t + ux^{2(u+t)} + x^{u+t}(u + t + \Delta P \log(x))}{(1 + x^{u+t})^2}, \varepsilon_2^* = \frac{u + tx^{2(u+t)} + x^{u+t}(u + t - \Delta P \log(x))}{(1 + x^{u+t})^2}.$$

We proceed to the first stage and take derivatives of (41) wrt θ_i to obtain the necessary conditions

$$\begin{aligned} (\theta_1)^d &= \alpha_1 \left(\left(\alpha_2 - 2t\delta - 2u\delta x^{t+u} \right) \alpha_5^2 + \alpha_4 \left(\alpha_5 \left(3(t-u)x^{t+u} + ux^{2(t+u)-t} \right) - \alpha_3 \right) \right), \\ (\theta_2)^d &= \gamma\alpha_1 \left(\left(\alpha_2 - 2u\delta - 2t\delta x^{t+u} \right) \alpha_5^2 + \alpha_4 \left(\alpha_5 \left(3(u-t)x^{t+u} + tx^{2(t+u)} - u \right) + \alpha_3 \right) \right) \end{aligned} \quad (42)$$

where, to simplify the expression, we set

$$\alpha_1 = \frac{(t+u)(u-t)x^{t+u}}{(1+x^{t+u})^5}, \ \alpha_2 = t + u + x^{t+u}(t+u), \ \alpha_3 = (t-u)(t+u)x^{t+u}\left(x^{t+u} - 1\right)\log(x),$$
$$\alpha_4 = (t+u)\delta\log(x), \text{ and } \alpha_5 = 1 + x^{t+u}.$$

As (42) are rather unwieldy, we evaluate numerically for the first stage equilibrium qualities (θ_1^*, θ_2^*). Figure 5 verifies this candidate as the unique pure strategy equilibrium $((\theta_1^*, \theta_2^*), (\varepsilon_1^*, \varepsilon_2^*))$ for parameters $u = 4/3$, $t = 2/3$, $d = 2$ and $\gamma_2 = 2/3$.[20] As usual, pure strategy equilibrium existence in the quality contest is problematic for large $u + t$.

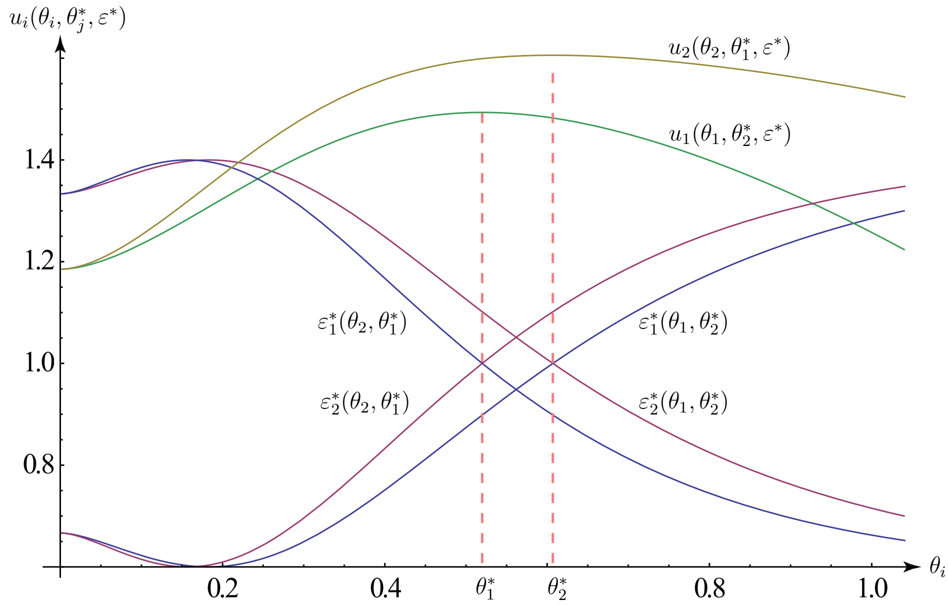


Figure 5: The optimal choice of qualities θ_i in asymmetric pure strategy, subgame perfect equilibrium.

6 Efficiency and welfare analysis

On the issue of competition in this market we would like to point out that even if two firms are in full price competition in the unlabelled credence market, the introduction of a single label will create two separated monopoly markets. For the purposes of comparison, we make the following assumption.

Assumption. *If the number of products given the same label is higher than one, then whatever level of competition governs the originally unlabelled credence market also applies to this 'overlabelled' market (segment).*

Our measure of inefficiency in this type of market corresponds to the mass of consumers who are excluded from consumption although they value the good higher than its marginal production cost of zero. As labelling gives consumers the ability to choose among differentiated products, we have two consumer welfare effects pulling in different directions: consumer welfare is improved through the increased differentiation but decreased through the firms' ability to charge monopoly prices. Producer welfare is unambiguously improved through the introduction of labelling. In order to discuss the social repercussions of introducing a ranking quantitatively, this section formalises a welfare function. The discussion takes the ex-ante point of view, that is, we evaluate consumers' and producer's expectations before any private information becomes available.

[20] The equilibrium values for the graphed example of figure 5 are $\theta^* = (0.520, 0.607)$ and $\varepsilon^* = (0.898, 1.102)$.

6.1 Benchmark case: unlabelled market

For a pure credence good and in the absence of labelling, consumers expect a pooled quality of $\mathbb{E}[\Theta]$ to which cartelised firms optimally set a pooled price of $\bar{p} = p_1 = p_2$.[21] This expected pooled quality type is given by

$$\mathbb{E}[\Theta] = \int_0^s f(\theta)d(\theta) = \mathbb{E}[\Theta_{(1,1)}]. \tag{43}$$

Then the indifferent, lowest participating consumer 'type' $\underline{\mu}$ is identified through

$$v(\cdot) = \underline{\mu}\,\mathbb{E}[\Theta] - \bar{p} = 0 \Leftrightarrow \underline{\mu} = \frac{\bar{p}}{\mathbb{E}[\Theta]} > 0. \tag{44}$$

Thus, a monopolist cartel chooses $\bar{p}$ to maximise its surplus

$$\bar{\mathsf{PS}}_m = \bar{p}\int_{\underline{\mu}}^s g(\mu)d\mu \;\Leftrightarrow \bar{p} = \mathbb{E}[\Theta]\frac{1-G(\underline{\mu})}{g(\underline{\mu})} > 0 \tag{45}$$

while consumers enjoy

$$\bar{\mathsf{CS}}_m = \int_{\underline{\mu}}^s \left(\mu\,\mathbb{E}[\Theta] - \bar{p}\right) g(\mu)d\mu. \tag{46}$$

Total welfare under fully colluding firms is then given by $\bar{W}_m = \bar{\mathsf{PS}}_m + \bar{\mathsf{CS}}_m$. Competitive firms, however, have incentives to decrease their price under $\bar{p}$ in order to capture the full demand. This Bertrand undercutting strategy leads to $\bar{p} = \underline{\mu} = 0$ in (45), implying zero profits and resulting in consumers obtaining the full surplus

$$\bar{\mathsf{CS}}_b = \mathbb{E}[\Theta]\int_{\underline{\mu}}^s \mu g(\mu)d\mu = \bar{\mathsf{W}}_b \text{ and } \bar{\mathsf{PS}}_b = 0. \tag{47}$$

The actually incurred welfare in the unlabelled credence market lies in between these limiting cases and depends on the industry's degree of competitiveness.[22]

6.2 The fully labelled market

We define a fully labelled market as one awarding $n-1$ distinct labels 1st, 2nd, 3rd... to n products. In a fully labelled 2-product market, ex-ante expected producer surplus is symmetric and given by

$$\mathsf{PS}_{\#1} + \mathsf{PS}_{\#2} = P^1(r) + P^2(r) - c(\tilde{x}, r^*),\; c(\tilde{x}, r^*) = \frac{(\varepsilon_1^*(\tilde{x}_1))^2}{2} + \frac{(\varepsilon_2^*(\tilde{x}_2))^2}{2} \tag{48}$$

for expected ratios $\tilde{x}_1 = \mathbb{E}[\Theta_{(1:2)}]/\,\mathbb{E}[\Theta_{(2:2)}]$ and $\tilde{x}_2 = \mathbb{E}[\Theta_{(2:2)}]/\,\mathbb{E}[\Theta_{(1:2)}]$, prizes $P(r)$ come from (8), $r^* = P'^1(r^*) + P'^2(r^*)$ given by (20), and ε_i^* from (35). Recall that consumers expect ranked qualities Λ defined in (4) and winning probabilities (5). Similarly, expected consumer surplus is the

[21] In the present analysis we assume that this is true. It is not hard, however, to underpin this with a simple symmetric model with just that equilibrium behaviour.

[22] Nearly all industries mentioned in the introduction exhibit some form of price setting behaviour. For books and DVDs, there are pre-printed recommended retail prices, (UK) university fees are capped at some typically charged amount, doctors have standard fees for their services etc.

sum of

$$\mathsf{CS}_{\#1} = \int_{\hat\mu_2^1}^{\hat\mu_1^0} \left(\mu\Lambda^1 - p_1\right) g(\mu)d\mu,\ \mathsf{CS}_{\#2} = \int_{\hat\mu_3^2}^{\hat\mu_2^1} \left(\mu\Lambda^2 - p_2\right) g(\mu)d\mu \tag{49}$$

with equilibrium cutoffs $\hat\mu$ and (monopoly) prices p defined in (7) and (8). Notice that—although the welfare components (48) and (49) may be involved—the combined welfare for each labelled market segment is given by the simple measure

$$\mathsf{W}_{\#} = \mathsf{CS}_{\#} + \mathsf{PS}_{\#} = \int_{\hat\mu_2^1}^{\hat\mu_1^0} \mu\,\mathbb{E}[\Theta_{(1:2)}]g(\mu)d\mu + \int_{\hat\mu_3^2}^{\hat\mu_2^1} \mu\,\mathbb{E}[\Theta_{(2:2)}]g(\mu)d\mu - c(r^*). \tag{50}$$

Under the uniform distribution and for $\tilde q^2 = \left(\frac{1}{1+x^r}, \frac{1}{1+x^{-r}}\right)$ defined in (5), we obtain

$$0 < \hat\mu_3^2 = \frac{p_2^*}{\Lambda^2} = \frac{p_2}{\tilde q_1^2\,\mathbb{E}[\Theta_{(1:2)}] + \tilde q_2^2\,\mathbb{E}[\Theta_{(2:2)}]} < \frac{\bar p}{\mathbb{E}[\Theta]} = \underline{\mu}, \tag{51}$$

because, in equilibrium, $\underline{\mu} = \frac{s}{2}$ and $\hat\mu_3^2 = \frac{\hat\mu_2^1}{2}$ with $\hat\mu_2^1 < s$.[23] Thus, the lowest cutoff in the labelled market is unambiguously smaller than the cutoff in the unlabelled market $\underline{\mu}$ from (44) and the mass of consumers excluded from consumption in the labelled market is strictly smaller than in the unlabelled market. This property generalises and allows us to prove the following result on market efficiency.

Proposition 4. *For any number of firms $n = |\mathcal{N}|$, if the distribution of consumer tastes $G(\mu)$ satisfies the condition*

$$\frac{G(\hat\mu_n^{n-1}) - G(\hat\mu_{n+1}^n)}{g(\hat\mu_{n+1}^n)} < \frac{1 - G(\underline{\mu})}{g(\underline{\mu})} \tag{52}$$

on the lowest equilibrium cutoff $\hat\mu_{n+1}^n$, then the fully labelled and cartelised market is strictly more efficient than its unlabelled equivalent under equilibrium cutoff $\underline{\mu}$.

Corollary 2. *Since the lhs of* (52) *is decreasing in n while the rhs is constant, there exists a number of distinctly labelled competitors $\tilde n$ such that, for any $n > \tilde n$,* (52) *holds for any distribution of underlying consumer tastes $G(\cdot)$ satisfying condition* (9)*.*

We now turn to the comparison of social welfare in the two market types. In the unlabelled market, the welfare in the monopoly and fully competitive Bertrand equilibrium cases are given by

$$\bar{\mathsf{W}}_m = \int_{\underline{\mu}_m}^{\hat\mu_1^0} \mathbb{E}[\Theta]\mu g(\mu)d\mu \quad \text{and} \quad \bar{\mathsf{W}}_b = \int_{\underline{\mu}_b}^{\hat\mu_1^0} \mathbb{E}[\Theta]\mu g(\mu)d\mu \tag{53}$$

where (as in the previous subsection) we use subscripts m, b to discriminate between the monopoly pricing and Bertrand undercutting policies, respectively. Note that $\underline{\mu}_m > \underline{\mu}_b = 0$ which lets us transform the above into

$$\bar{\mathsf{W}}_m = \mathbb{E}[\Theta](\mathbb{E}[\mu] - \int_{\underline{\mu}_b=0}^{\underline{\mu}_m} \mu g(\mu)d\mu) \quad \text{and} \quad \bar{\mathsf{W}}_b = \mathbb{E}[\Theta]\,\mathbb{E}[\mu]. \tag{54}$$

[23] Notice that this follows from the fact that stochastic orders are equally spaced for the uniform distribution.

Thus, little surprisingly, the unlabelled Bertrand welfare is strictly above the welfare under cartelised prices $\bar{\mathsf{W}}_b > \bar{\mathsf{W}}_m$. Using the Chasles relation, we rewrite the unlabelled Bertrand welfare $\bar{\mathsf{W}}_b$ as

$$\bar{\mathsf{W}}_b = \int_{\underline{\mu}_b}^{\hat{\mu}_3^2} \mathbb{E}[\Theta]\mu g(\mu)d\mu + \int_{\hat{\mu}_3^2}^{\hat{\mu}_2^1} \mathbb{E}[\Theta]\mu g(\mu)d\mu + \int_{\hat{\mu}_2^1}^{\hat{\mu}_1^0} \mathbb{E}[\Theta]\mu g(\mu)d\mu. \tag{55}$$

We subtract the preceding expression from the simplified form of expected welfare $\mathsf{W}_\#$ in the labelled market (50), and obtain

$$\mathsf{W}_\# - \bar{\mathsf{W}}_b = \int_{\hat{\mu}_1^2}^{\hat{\mu}_1^0} \mu(\mathbb{E}[\Theta_{(1:2)}] - \mathbb{E}[\Theta])g(\mu)d\mu + \int_{\hat{\mu}_3^2}^{\hat{\mu}_2^1} \mu(\mathbb{E}[\Theta_{(2:2)}] - \mathbb{E}[\Theta])g(\mu)d\mu - \int_{\underline{\mu}_b}^{\hat{\mu}_3^2} \mathbb{E}[\Theta]\mu g(\mu)d\mu - c(r^*)$$

After some manipulation (see proof for details), we arrive at the following proposition.

Proposition 5. *Total welfare in the fully labelled market is strictly higher than in the unlabelled market if the following condition is respected*

$$\frac{\mathbb{E}[\Theta_{(1:2)}] - \mathbb{E}[\Theta]}{\mathbb{E}[\Theta]} > \frac{\mathbb{E}[\mu|\hat{\mu}_3^2 > \mu > \underline{\mu}_b] + \frac{c(r)}{\mathbb{E}[\Theta]}}{(\mathbb{E}[\mu|\hat{\mu}_1^0 > \mu > \hat{\mu}_2^1] - \mathbb{E}[\mu|\hat{\mu}_2^1 > \mu > \hat{\mu}_3^2])} \tag{56}$$

We now illustrate this result in the setup of our example from section 3. There, the (monopoly) prices which firms charge give a consumer surplus of

$$\mathsf{CS}_{\#m} = \frac{(14 + 13 \times 2^r)\left(1 + 2^{r+1}\right)^2 s^3}{6\left(2^r + 1\right)\left(2 + 7 \times 2^r\right)^2} \tag{57}$$

for which we can derive the following numerical relationships

$$\underbrace{\mathsf{CS}_{Pm}}_{=s^3\frac{1}{16}} < \underbrace{\lim_{r\to\infty} \mathsf{CS}_{\#m}}_{=s^3\frac{26}{147}} < \mathsf{CS}_{\#m} < \underbrace{\mathsf{CS}_{Pb}}_{=s^3\frac{1}{4}}. \tag{58}$$

Figure 6.2 shows the relation between these for different levels of industry cartelisation. As above, subscripts $\{b, m\}$ denote the cases of Bertrand and monopolistic competition, respectively. Subscript $\#$ denotes a surplus under the ranking while subscript P denotes fully pooled quality expectations.

Numerical results for $n > 2$ products confirm these relationships.

7 Concluding remarks

We present the first analysis of a contest model dealing with endogenous ranking (or labelling) precisions. As this precision is endogenously generated, we effectively allow players to choose the ranking technology for their market. We obtain a tractable analytical description of the firms' information dissemination functions together with first and tentative efficiency and welfare results. This, however, is not the main contribution of this paper: its main strength lies in unifying the consumer and producer side in one simple model parameterised in all strategic choices by the endogenous

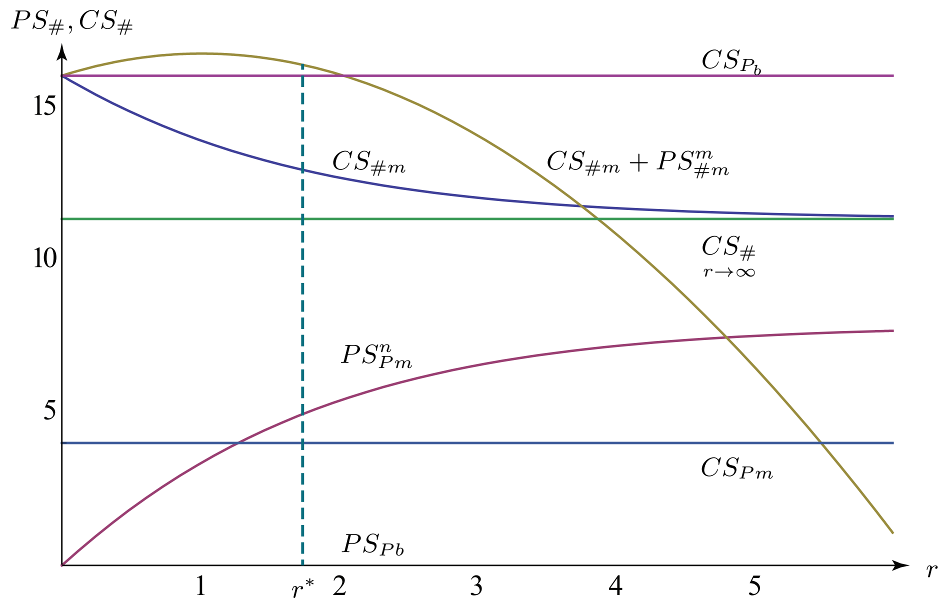


Figure 6: The introduction of labelling is welfare improving unless the industry is fully Bertrand-competitive.

ranking precision. We hope that our simple model is capable of illuminating the workings of some of the many credence markets we discuss in the introduction.

Appendix

Proof of proposition 1. We wish to establish that, for any given vector of prices $(p_1, \ldots, p_n)$ for competing products, the revenue function of firm k, $1 \leq k \leq n$, is a continuous single peaked (quasi-concave) function of its price p_k. The existence of an equilibrium then follows immediately from standard arguments. First see that the first derivative equals to:

$$\frac{\partial P^k}{\partial p_k} = \int_{\hat{\mu}^k_{k+1}}^{\hat{\mu}^{k-1}_k} g(\mu) d\mu - \frac{p_k \left(g(\hat{\mu}^{k-1}_k) + g(\hat{\mu}^k_{k+1}) \right)}{D_k} = 0. \tag{59}$$

Underlying that:

$$p^*_k = \frac{D_k \left(\int_{\hat{\mu}^k_{k+1}}^{\hat{\mu}^{k-1}_k} g(\mu) d\mu \right)}{\left(g(\hat{\mu}^{k-1}_k) + g(\hat{\mu}^k_{k+1}) \right)}. \tag{60}$$

The second derivatives leads to the following equations

$$\frac{\partial^2 P^k}{\partial^2 p_k} = -\frac{2D_k \left(g(\hat{\mu}^{k-1}_k) + g(\hat{\mu}^k_{k+1}) \right) + p_k \left(-g'(\hat{\mu}^{k-1}_k) + g'(\hat{\mu}^k_{k+1}) \right)}{D^2_k}. \tag{61}$$

Moreover, note that a sufficient condition for the function P^k to be concave is that $g(\mu)$ is weakly concave. Indeed if $g(\mu)$ is weakly concave, $-g'(\hat{\mu}^{k-1}_k) + g'(\hat{\mu}^k_{k+1}) \geq 0$ as $\hat{\mu}^{k-1}_k > \hat{\mu}^k_{k+1}$ and $g'(\cdot)$ weakly decreasing. Substituting by p_k and assuming that the distribution satisfies the condition that

$\forall \mu \in [0, b]$ with $b \in [0, s[$ we have

$$g(\mu)^2 - g'(\mu) \int_{\mu}^{b} g(x)dx > 0, \tag{62}$$

which implies (9), we can prove that P^k is a concave function at that point which is therefore a global maximum

$$\begin{aligned}
2D_k \left(g(\hat{\mu}_k^{k-1}) + g(\hat{\mu}_{k+1}^k)\right) + p_k \left(-g'(\hat{\mu}_k^{k-1}) + g'(\hat{\mu}_{k+1}^k)\right) > 0 \\
2D_k \left(g(\hat{\mu}_k^{k-1}) + g(\hat{\mu}_{k+1}^k)\right) > p_k \left(g'(\hat{\mu}_k^{k-1}) - g'(\hat{\mu}_{k+1}^k)\right) \\
2D_k \left(g(\hat{\mu}_k^{k-1}) + g(\hat{\mu}_{k+1}^k)\right) > \frac{D_k \left(G(\hat{\mu}_k^{k-1}) - G(\hat{\mu}_{k+1}^k)\right)}{\left(g(\hat{\mu}_k^{k-1}) + g(\hat{\mu}_{k+1}^k)\right)} \left(g'(\hat{\mu}_k^{k-1}) - g'(\hat{\mu}_{k+1}^k)\right) \\
2\left(g(\hat{\mu}_k^{k-1}) + g(\hat{\mu}_{k+1}^k)\right)^2 > \left(G(\hat{\mu}_k^{k-1}) - G(\hat{\mu}_{k+1}^k)\right)\left(g'(\hat{\mu}_k^{k-1}) - g'(\hat{\mu}_{k+1}^k)\right) \\
2\left(g(\hat{\mu}_k^{k-1}) + g(\hat{\mu}_{k+1}^k)\right)^2 - \left(G(\hat{\mu}_k^{k-1}) - G(\hat{\mu}_{k+1}^k)\right)\left(g'(\hat{\mu}_k^{k-1}) - g'(\hat{\mu}_{k+1}^k)\right) > 0 \\
2\left(g(\hat{\mu}_k^{k-1}) + g(\hat{\mu}_{k+1}^k)\right)^2 - g'(\hat{\mu}_k^{k-1})\left(G(\hat{\mu}_k^{k-1}) - G(\hat{\mu}_{k+1}^k)\right) + g'(\hat{\mu}_{k+1}^k)\left(G(\hat{\mu}_k^{k-1}) - G(\hat{\mu}_{k+1}^k)\right) > 0 \\
2\left(2g(\hat{\mu}_{k+1}^k)g(\hat{\mu}_k^{k-1}) + g(\hat{\mu}_{k+1}^k)^2\right)^2 + g'(\hat{\mu}_{k+1}^k)\left(G(\hat{\mu}_k^{k-1}) - G(\hat{\mu}_{k+1}^k)\right) \\
+ \underbrace{g(\hat{\mu}_k^{k-1})^2 - \left(G(\hat{\mu}_k^{k-1}) - G(\hat{\mu}_{k+1}^k)\right) g'(\hat{\mu}_k^{k-1})}_{>0} > 0.
\end{aligned}$$

Thus any distribution G which satisfies (9) leads to the claimed equilibrium behaviour. □

Proof of lemma 1. Given the objective (1), the two focs wrt ε_i, $i = 1, 2$ are

$$c'(|\varepsilon_1|) - P'^2(r) = x, \ -c'(|\varepsilon_2|) + P'^1(r) = x \tag{63}$$

where

$$x = q'(r)(P^1(r) - P^2(r)) + q(r)(P'^1(r) - P'^2(r)) \tag{64}$$

The first pair of equations readily transforms into our claim (17). □

Proof of proposition 2. Consider the equilibrium candidate (23). Notice that the positive branch of $q(x, r) = \frac{1}{1+x_i^{-r}}$ is not concave for sufficiently large x and thus the optimisation problem is not well-behaved. We pursue a sufficient condition for $q(x, r)$ to be weakly concave. Given equilibrium behaviour of the opponent, i.e., $\varepsilon_2^* = -\frac{P^1 \log(x)}{4}$, firm 1's objective has a curvature of

$$\frac{P^1 x^{\varepsilon_1 + \frac{P^1 \log(x)}{4}} \left(x^{\frac{P^1 \log(x)}{4}} - x^{\varepsilon_1}\right) \log(x)^2 - \left(x^{\varepsilon_1} + x^{\frac{P^1 \log(x)}{4}}\right)^3}{\left(x^{\varepsilon_1} + x^{\frac{P^1 \log(x)}{4}}\right)^3} \tag{65}$$

which we want to ensure to be negative. We therefore rearrange and consider the problem of finding

x satisfying

$$x^{\varepsilon_1+\frac{P^1\log(x)}{2}}-x^{2\varepsilon_1+\frac{P^1\log(x)}{4}}-\frac{\left(x^{\varepsilon_1}+x^{\frac{P^1\log(x)}{4}}\right)^3}{P^1\log(x)^2}<0. \tag{66}$$

Inspection of the first two terms reveals that the only candidate region for a maximum is $\varepsilon_1 < \frac{P^1\log(x)}{4}$. We take the derivative of the lhs wrt to ε_1 to obtain the critical point

$$x^{\varepsilon_1+\frac{P^1\log(x)}{2}}\log(x)-\frac{3x^{\varepsilon_1}\left(x^{\varepsilon_1}+x^{\frac{P^1\log(x)}{4}}\right)^2}{P^1\log(x)}-2x^{2\varepsilon_1+\frac{P^1\log(x)}{4}}\log(x)=0. \tag{67}$$

This is solved by

$$\tilde{\varepsilon}_1=\frac{\frac{P^1}{4}\log(x)^2+\log\left(\frac{1}{3}\left(\sqrt{P^1}\sqrt{\log(x)^2\left(9+P^1\log(x)^2\right)}-3-P^1\log(x)^2\right)\right)}{\log(x)}. \tag{68}$$

Plugging this critical $\tilde{\varepsilon}_1$ back into (66) gives the possible solutions

$$\tilde{x}=\left\{e^{\sqrt{\frac{2}{P^1}}\ 3^{3/4}},\ e^{-\sqrt{\frac{2}{P^1}}\ 3^{3/4}}\right\} \tag{69}$$

only the first of which is compatible with (23). Thus the objective is concave for $x \le \tilde{x} = e^{\sqrt{\frac{2}{P^1}}\ 3^{3/4}}$. Since $q(x,0)>0$ and $\lim_{r\to\infty} q(x,r)=1$, this implies a unique intersection with strictly convex costs satisfying $c(0)=0$. (The argument for player 2 is symmetric.) □

Proof of proposition 3. Given concavity of $P^1(r)$, the term $q(x,r)P^1(r)$ is an increasing concave transformation of the equivalent term in proposition 2. Since any nondecreasing concave transformation of a concave function is concave, the same sufficient condition applies as in proposition 2. Since P^1 can be replaced by the concave $P^1(r)-P^2(r)$, the same proof also applies to the two-prize version of the contest. □

Proof of proposition 4. We define total welfare as $\mathsf{W}_\# = \sum_{k=1}^{n} \mathsf{W}_\#^k$ over

$$\mathsf{W}_\#^k=\mathsf{CS}_\#^k+\mathsf{PS}_\#^k=\int_{\hat{\mu}_{k+1}^k}^{\hat{\mu}_k^{k-1}}\mu\,\mathbb{E}[\Theta_{(k:n)}]g(\mu)d\mu-\frac{\varepsilon^*(\tilde{x}_k)^2}{2} \tag{70}$$

where the ratio vector $\tilde{x}_k=\mathbb{E}[\Theta_{(k:n)}]/\,\mathbb{E}[\Theta_{(-k:n)}]$, with $-k$ specifying $(1,\dots,k-1,k+1,\dots,n)$, denotes the expectation of the n-player generalisation of $x=\theta_1/\theta_2$ consisting of the $(n-1)$ ratios of $\mathbb{E}[\Theta_{(k:n)}]$ over the other order statistics. Similar to (51) we generalise $\hat{\mu}=(\hat{\mu}_1^0=1,\dots,\hat{\mu}_{n+1}^n)$ with

$$\hat{\mu}_{k+1}^k=\frac{p_k-p_{k+1}}{\Lambda^k-\Lambda^{k+1}},\ \hat{\mu}_{n+1}^n=\frac{p_n}{\Lambda^n}=\frac{p_n^*}{\sum_{i=1}^{n}\tilde{q}_i^n\,\mathbb{E}[\Theta_{(i:n)}]}>0 \tag{71}$$

where we define the n-players extension of $\tilde{q}_i^k$ (5) by the vector of ratio-dependent probabilities

$$\tilde{q}^1 = (\tilde{q}_1^1(\tilde{x}_1^1, \tilde{x}_2^1, \dots, \tilde{x}_n^1, r), \tilde{q}_2^1(\tilde{x}_1^2, \tilde{x}_2^2, \dots, \tilde{x}_n^2, r), \dots, \tilde{q}_n^1(\tilde{x}_1^n, \tilde{x}_2^n, \dots, \tilde{x}_n^n, r)) \tag{72}$$

with $\tilde{x}_k^i = \dfrac{\mathbb{E}[\Theta_{(i:n)}]}{\mathbb{E}[\Theta_{(k:n)}]}$. We assume the usual properties of these probabilities; for details see, for instance, Giebe and Schweinzer (2011, section 5.1). We know from (20) that $r^* > 0$ in equilibrium. Therefore, the lowest quality expectation is strictly below the unlabelled market expectation

$$\Lambda^n = \sum_{i=1}^{n} \tilde{q}_i^n \, \mathbb{E}[\Theta_{(i:n)}] < \mathbb{E}[\Theta]. \tag{73}$$

We know moreover that the choice of the optimal (monopoly) price p_n^* for the lowest-labelled market segment

$$\arg\max_{p_n} p_n \int_{\hat{\mu}_{n+1}^n}^{\hat{\mu}_n^{n-1}} g(\mu) d\mu = p_n \left(G(\hat{\mu}_n^{n-1}) - G(\hat{\mu}_{n+1}^n) \right) \tag{74}$$

leads to an equilibrium price

$$p_n^* = \Lambda^n \frac{G(\hat{\mu}_n^{n-1}) - G(\hat{\mu}_{n+1}^n)}{g(\hat{\mu}_{n+1}^n)} \tag{75}$$

and therefore the cutoff

$$\hat{\mu}_{n+1}^n = \frac{G(\hat{\mu}_n^{n-1}) - G(\hat{\mu}_{n+1}^n)}{g(\hat{\mu}_{n+1}^n)}. \tag{76}$$

Comparing this to the unlabelled cutoff (44), we obtain (52) under which the labelled firms serve more consumers than the unlabelled firms. □

Proof of proposition 5. Applying the Chasles relation, welfare in the unlabelled market and undercutting strategy $\bar{\mathsf{W}}_b$ can be rewritten as

$$\bar{\mathsf{W}}_b = \int_{\underline{\mu}_b}^{\hat{\mu}_3^2} \mathbb{E}[\Theta]\mu g(\mu) d\mu + \int_{\hat{\mu}_3^2}^{\hat{\mu}_2^1} \mathbb{E}[\Theta]\mu g(\mu) d\mu + \int_{\hat{\mu}_2^1}^{\hat{\mu}_1^0} \mathbb{E}[\Theta]\mu g(\mu) d\mu. \tag{77}$$

We subtract the preceding expression from the simplified form of expected welfare $\mathsf{W}_\#$ in the labelled market (50) and denote $c(r) = \frac{(\varepsilon_1^*(\tilde{x}_1))^2}{2} + \frac{(\varepsilon_2^*(\tilde{x}_2))^2}{2}$ to obtain

$$\mathsf{W}_\# - \bar{\mathsf{W}}_b = \int_{\hat{\mu}_2^1}^{\hat{\mu}_1^0} \mu(\mathbb{E}[\Theta_{(1:2)}] - \mathbb{E}[\Theta]) g(\mu) d\mu + \int_{\hat{\mu}_3^2}^{\hat{\mu}_2^1} \mu(\mathbb{E}[\Theta_{(2:2)}] - \mathbb{E}[\Theta]) g(\mu) d\mu - \int_{\underline{\mu}_b}^{\hat{\mu}_3^2} \mathbb{E}[\Theta]\mu g(\mu) d\mu - c(r).$$

From the definition of order statistics we obtain that

$$(\mathbb{E}[\Theta_{(1:2)}] - \mathbb{E}[\Theta]) = -(\mathbb{E}[\Theta_{(2:2)}] - \mathbb{E}[\Theta]) \text{ and } (\mathbb{E}[\Theta_{(2:2)}] < \mathbb{E}[\Theta]). \tag{78}$$

Indeed, the probability density function of the k^{th} highest order statistic among n draws can be written as

$$f_{(k:n)}(x) = n \begin{pmatrix} n-1 \\ k-1 \end{pmatrix} F(x)^{n-k} (1 - F(x))^{k-1} f(x). \tag{79}$$

Applying this pdf function to our situation, where $\mathbb{E}[\Theta_{(1:2)}]$ is the expected value of the draw ranked first of the two draws and $\mathbb{E}[\Theta_{(2:2)}]$ is the expected value of the draw ranked second of the two draws. Finally $\mathbb{E}[\Theta]$ can be understood as the draw ranked first of only one draw, i.e. $\mathbb{E}[\Theta_{(1:1)}] = \mathbb{E}[\Theta]$. This gives us the following equation

$$\begin{gathered}
\mathbb{E}[\Theta_{(1:2)}] - \mathbb{E}[\Theta] = \int_0^s 2F(x)f(x)xdx - \int_0^s f(x)xdx \\
-(\mathbb{E}[\Theta_{(2:2)}] - \mathbb{E}[\Theta]) = -\int_0^s 2(1-F(x))f(x)xdx + \int_0^s f(x)xdx \\
\mathbb{E}[\Theta_{(1:2)}] - \mathbb{E}[\Theta] + \mathbb{E}[\Theta_{(2:2)}] - \mathbb{E}[\Theta] = 0 \\
\int_0^s 2F(x)f(x)xdx - \int_0^s f(x)xdx + \int_0^s 2(1-F(x))f(x)xdx - \int_0^s f(x)xdx \\
\int_0^s (2(F(x)) + 2(1-F(x)))f(x)xdx - 2\int_0^s f(x)xdx \\
\int_0^s 2f(x)xdx - 2\int_0^s f(x)xdx = 0
\end{gathered} \tag{80}$$

which proves (78). Then

$$\begin{aligned}
\mathsf{W}_{\#} - \bar{\mathsf{W}}_b &= (\mathbb{E}[\Theta_{(1:2)}] - \mathbb{E}[\Theta]) \left(\int_{\hat{\mu}_1^2}^{\hat{\mu}_1^0} \mu g(\mu) d\mu - \int_{\hat{\mu}_3^2}^{\hat{\mu}_2^1} \mu g(\mu) d\mu \right) - \int_{\underline{\mu}_b}^{\hat{\mu}_3^2} \mathbb{E}[\Theta] \mu g(\mu) d\mu - c(r) \\
&= (\mathbb{E}[\Theta_{(1:2)}] - \mathbb{E}[\Theta]) \left(\mathbb{E}[\mu | \hat{\mu}_1^0 > \mu > \hat{\mu}_1^2] - \mathbb{E}[\mu | \hat{\mu}_2^1 > \mu > \hat{\mu}_3^2] \right) - \mathbb{E}[\Theta]\, \mathbb{E}[\mu | \hat{\mu}_3^2 > \mu > \underline{\mu}_b]
\end{aligned}$$

which is strictly positive as long as (56) is respected. □

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