What do frictions mean for Q-theory?

by Maria Cecilia Bustamante

London School of Economics

LSE

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Good Q, Bad Q

- The empirical evidence on neoclassical investment models is **controversial**

- When applied to investment, **the Q-theory performs poorly** in the aggregate and the cross section (Caballero 1997, 2000)
    - Hayashi (1982) predicts that average Q as a sufficient statistic for investment
    - Yet the coefficient on Q is usually low or insignificant
    - Cash flows are significant (Fazzari et al, 1989)

- When applied to stock returns, **investment factor models** perform well in the aggregate and the cross section (Cochrane 1991, 1996)

Finance and the Q-theory

- Asset pricers rely heavily on the Q-theory to explain anomalies (Liu, Whited and Zhang, 2009)
- While the Q-theory does not work in the data, corporate finance relies on practices that date back from Hayashi (1982)
  - Average Q enters linearly on investment equations
  - Average Q as a good proxy for the growth opportunities of the firm
- This paper reviews such practices using a Q-model with frictions
  - **Alternative approach** to test investment equations
  - **Strong empirical support** for all predictions of the model

Identifying marginal q

- Investment policies depend on the **unobservable** marginal product of capital q
- In Hayashi (1982), **marginal q equals the observable average Q in a frictionless environment**

$$q = Q$$

- Real and financing frictions create a **wedge between q and Q**

$$q - Q = -\left(\begin{array}{c} \text{shadow cost of} \\ \text{frictions on equity} \end{array} \right)$$

- Several papers argue that there is **measurement error** in Q (Erickson and Whited, 2000)

Real vs Financing Frictions

- The corporate/macro literature on Q-theory usually test the role of real and financing frictions **separatedly**.

- The macro literature studies the impact of **fixed costs of investment for all equity financed firms**.
    - Abel and Eberly (1994) predict non-linear investment policies due to fixed costs.
    - Barnet and Sakellaris (1998) test such prediction assuming all equity financed firms.

- The finance literature focuses on **financing frictions instead.**
    - Whited (1992) and Bond and Meghir (1994) test the Euler equation of investment with financing frictions
    - Hennessy, Levy and Whited (2006) test investment policies subject to financing frictions

Main contribution

This paper.explores the empirical implications of a Q-model with **real and financing frictions,.**and...

1. provides a rationale to the poor performance of standard empirical tests;
2. shows theoretically and empirically that investment and external financing are concave in average Q and convex in cashflows;
3. derives a log likelihood function to test investment equations in reduced or structural form;
4. structurally estimates firms' frictions for multiple sample splits, including industries.

Why do standard tests underperform?

OLS regressions are subject to multiple biases

- The standard OLS regression of investment on average Q **overlooks** how frictions affect corporate policies.
- **Measurement error**
  - Average Q is a noisy proxy of q for multiple reasons
- **Omitted variables**
  - When firms rely on external financing, investment depends on q and cashflows.
  - When firms rely on external financing, investment is concave in q
  - When firms rely on external financing, investment is convex in cashflows
- **Truncation bias**
  - Due to their fixed costs, firms invest only if q is sufficiently high

Non linear corporate policies

Investment and external financing are weakly concave in Q and cashflows

- Higher investment rates require higher reliance on external financing, and hence
  - ...investment is less sensitive to average Q for higher levels of investment.
  - ...investment is more sensitive to cashflows for higher levels of investment.
- When subject to fixed costs, firms only invest if average Q is sufficiently high
  - More financially constrained firms have larger inertia regions
- Rationale for the **wide range of estimates for the curvature of adjustment costs** in the empirical macro literature
- External financing is also weakly concave in Q and weakly convex in cashflows when firms invest

An alternative empirical approach

The log likelihood function of investment policies

- Many models in the literature highlight that investment policies are non linear
  - ...yet few papers elaborate on how to test them
  - With investment as the dependent variable, **non linearities and measurement error** are tough problems to address

- This paper provides a tractable approach to test **non linear investment policies subject to measurement error**
  - Several difficulties are overcome when Q is the dependent variable
  - **Log likelihood function** of investment policies that can be tested in reduced or structural form

Empirical findings

Reduced form and structural results

Reduced form findings

- Investment is concave in Q and weakly convex in cashflows
    - The coefficient on Q increases when we add Q^2 in investment regressions
- External financing issues are concave in Q and weakly convex in cashflows

Structural results

- All firms on average: adjustment costs of 7.6%; external financing costs 9.4%; and fixed costs of 38%
- More **financially constrained** firms subject to higher marginal costs of financing
- Large differences in frictions in the **cross section of industries**
    - How do firms' production technologies affect their ability to invest/get financed?

Closest papers in the literature

- ***Q*-theory and investment frictions:** Hayashi (1982), Abel and Eberly (1994), Caballero and Leahy (1996).
- ***Q*-theory and financing frictions:** Gomes (2001), Hennessy, Levy and Whited (2006), Bolton, Chen and Wang (2010)
- **Empirical tests on *Q*-theory:** Hennessy, Levy and Whited (2006), Barnett and Sakellaris (1998) and Schennach and Hu (2011)
- **Structural estimation:** Cooper and Haltinwanger (2006), Hennessy and Whited (2005), and Li and Liu (2011).

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Contracting Framework: Real side

- **Investors are risk neutral** and discount at a constant rate $r > 0$
- The firm is subject to shocks ϵ_t that follow a Wiener process
- **The firm is a price taker** and operating profit $\pi(K_t, \epsilon_t)$ linear in capital
- The **manager maximizes shareholder value**

- **The law of motion of capital** is given by

$$dK_t = \left[I_t - \delta_k K_t\right] dt, \quad K_0 > 0$$

- Investment is subject to **quadratic adjustment costs and fixed costs** (Abel and Eberly, 1994)

$$C(I_t, K_t) = \tfrac{\alpha}{2} K_t \left(\frac{I_t}{K_t} - \delta\right)^2 + f K_t \mathbf{1}_t^I$$

Contracting Framework: Financing side

- Firm has senior debt outstanding with a consol bond b which is defaultable

- The **external financing requirement** of the firm is given by

$$X_t = I_t + C\left(I_t, K_t\right) + b - \pi\left(K_t, \epsilon_t\right)$$

- Firms issue securities to fund investment if $X_t > 0$ and distribute dividends $D_t > 0$ otherwise, such that $D_t \equiv -X_t$

- Denote $\widetilde{X}_t = X_t - b$
- Firms are subject to **convex costs of external financing**

$$H(X_t, K_t) = X_t + \frac{\theta}{2} K_t \left(\frac{\widetilde{X}_t}{K_t}\right)^2 \mathbf{1}_t^X$$

The Problem of the Firm

The manager **maximizes the value of existing shares** such that

$$S(K_t, \epsilon_t) = \max_{I_t, T} E_t \left[\int_0^T e^{-rs} \left[-H\left(X_s, K_s\right) \right] ds \right]$$

The corresponding **Bellman equation** is then

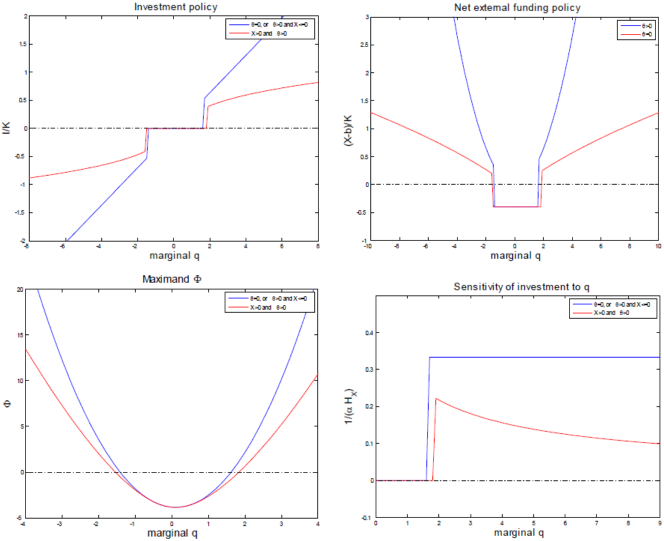
$$rS = -H\left(X, K\right) + \left(I - \delta K\right) S_K + \mu\left(\epsilon\right) S_\epsilon + \frac{\sigma(\epsilon)^2}{2} S_{\epsilon\epsilon}$$

s.t.

$$X_t = I_t + C\left(I_t, K_t\right) + b - \pi\left(K_t, \epsilon_t\right)$$

The model in pictures

Comparison with Abel and Eberly (1994)



Optimal Investment Policies

Both fixed costs and financing frictions affect investment rates

$$\frac{I}{K} = \begin{cases} \delta_k - \frac{1}{\alpha} + \frac{1}{\alpha H_X} q & \text{if } q \notin \Theta \\ 0 & \text{if } q \in \Theta \end{cases}$$

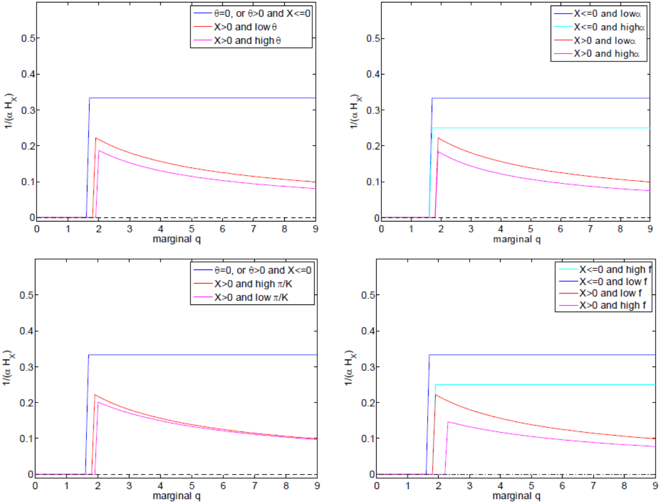
- $\frac{I}{K}$ is increasing in q, and also increasing in $\frac{\pi}{K}$ if $X > 0$;
- $\frac{I}{K}$ strictly concave in q if $X > 0$, and linear in q otherwise;
- $\frac{I}{K}$ is strictly convex in $\frac{\pi}{K}$ if $X > 0$;
- $\frac{\partial \frac{I}{K}}{\partial q}$ is decreasing in α $\forall X$, and decreasing in θ and f.if $X > 0$.

The Inertia Region of Investment

- The thresholds q_1 and q_2 are the limits of the inertia region $\Theta \equiv [q_1; q_2]$
- [Abel and Eberly, 1994] When $X \leq 0$, the thresholds q_1 and q_2 are constant, and increasing in α and f;
- [This paper] When $X > 0$, q_1 and q_2 are not constant
    - The thresholds q_1 and q_2 are increasing in α, f and θ, and decreasing in $\frac{\pi}{K}$.

Comparative statics on investment

How do frictions affect investment rates?



Optimal External Financing

Debt financing is lumpy and may depend on marginal q

$$\frac{\widetilde{X}}{K} = \begin{cases} \delta + f - \frac{1}{2\alpha} + \frac{1}{2\alpha}\left(\frac{q}{H_X}\right)^2 - \frac{\pi}{K} & \text{if } q \notin \Theta \\ \frac{\alpha\delta^2}{2} - \frac{\pi}{K} & \text{if } q \in \Theta \end{cases}$$

- $\frac{\widetilde{X}}{K}$ depends jointly on $\frac{\pi}{K}$ $\forall I$ and q if and only if $q \notin \Theta$;
- $\frac{\widetilde{X}}{K}$ is increasing in q $\forall q \notin \Theta$, concave in q if $X > 0$ and convex in q otherwise; and
- $\frac{\widetilde{X}}{K}$ depends mechanically on $\frac{\pi}{K}$ $\forall I$ and $\forall q$.

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Investment as an dependent variable

- In the model, average Q is a noisy proxy of marginal q due to overhang effects
  - More generally, there is a wedge between Q and q due to multiple reasons

- Hence the null hypothesis is such that

$$\begin{array}{ll} \text{Investment equation} & \frac{I}{K} = m\left(q; \frac{\pi}{K}\right) + u \\ \text{Average } Q \text{ equation} & Q = q + \upsilon \end{array}$$

where the wedge $\upsilon \sim N(\kappa, \sigma_\upsilon^2)$ is orthogonal to Q.

- The function $m(.)$ is non linear and may be proxied with a Taylor expansion
- Schennach and Hu (2011) provide a non parametric approach to test this model.

Investment as an explanatory variable

An alternative approach

- Several econometric difficulties are overcome when Q is the dependent variable

$$Q = \phi_0 + \phi_1 \tfrac{I}{K} + \phi_2 \left(\tfrac{I}{K}\right)^2 + \phi_3 \left(\tfrac{I}{K}\right)^3 + \phi_4 \tfrac{\pi}{K} + \phi_5 \tfrac{\pi}{K} \tfrac{I}{K} + \iota$$

where $\iota \equiv u + v$ and ϕ_i are **explicit functions** of α, δ, θ and f

- Yet **firms do not invest** all the time

$$\begin{array}{ll} \text{Probability of investing} & \Pr\left(q \geqslant q_2\right) = \Phi\left[\frac{\Delta^+ - Z'\zeta}{\sigma_\iota}\right] \\ \text{Probability of disinvesting} & \Pr\left(q \leq q_1\right) = 1 - \Phi\left[\frac{\Delta^- - Z'\zeta}{\sigma_\iota}\right] \end{array}$$

The log likelihood function of investment

A simple tool to test the model

- Two additional simplifying assumptions are
  - $f^{+} \neq f^{-}$, $f^{+} > 0$ and $f^{-} \longrightarrow \infty$
  - Δ^{+} is constant.

- Then the log likelihood with Q as the dependent variable is

$$\ln L\left(\zeta, \Delta^{+} \middle| Z\right) = \ln\left(\phi\left(\frac{Q - Z'\zeta}{\sigma_{\iota}}\right)\right) - \ln\left(\sigma_{\iota}\right) - \ln\left(1 - \Phi\left(\frac{\Delta^{+} - Z'\zeta}{\sigma_{\iota}}\right)\right)$$

- I use this log likelihood to test the model in
  - **Reduced form**: this is a truncated regression with an endogenous Δ^{+}
  - **Structural form**: using the constraints on the coefficients and the identity for X_t

External financing policies

Alternative empirical approaches

- $\widetilde{X}$ as the dependent variable implies

$$\begin{array}{ll} \text{External financing} & \frac{\widetilde{X}}{K} = f\left(q; \frac{\pi}{K}\right) + u \\ \text{Average } Q & Q = q + \upsilon \end{array}$$

- We can also have Q as a function of external financing and investment
- **Structural estimation:** log likelihood function on investment + constraints on coefficients + identity on external financing

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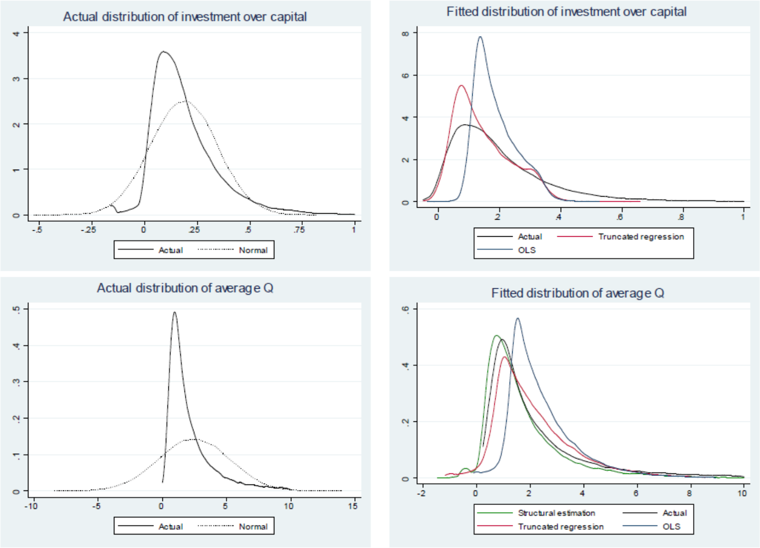
Working Database

- CRSP-COMPUSTAT files
- Time range: 1980 to 2008
- Keep firms with 5 consecutive years of data or more
- Sample filtered for missing data, negative assets, and non positive gross capital stock/sales
- Winsor to eliminate extreme values

- Overall: 53, 230 observations grouped into 4, 839 firms

Non-normal distribution of investment and Q

Comparing actual and fitted values



Actual distribution of investment over capital

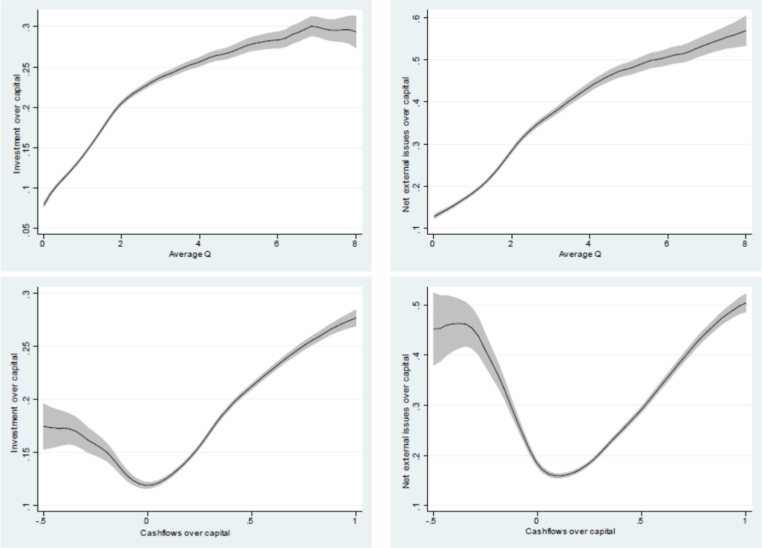
Fitted distribution of investment over capital

Actual distribution of average Q

Fitted distribution of average Q

Non parametric results

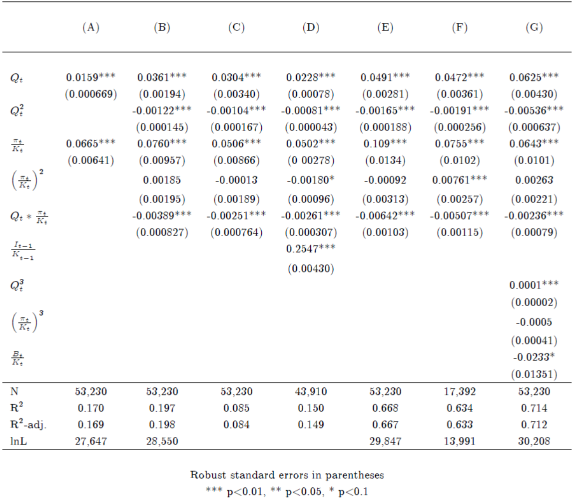
Investment and external issues are non linear in Q and cashflows



Investment as the explanatory variable

Investment is non linear in Q and cashflows

$$\frac{I}{K} \equiv \phi_0 + \phi_1 Q + \phi_2 Q^2 + \phi_3 \frac{\pi}{K} + \phi_4 \left(\frac{\pi}{K}\right)^2 + \phi_5 Q \frac{\pi}{K} + u$$



	(A)	(B)	(C)	(D)	(E)	(F)	(G)
Q_t	0.0159***	0.0361***	0.0304***	0.0228***	0.0491***	0.0472***	0.0625***
	(0.000669)	(0.00194)	(0.00340)	(0.00078)	(0.00281)	(0.00361)	(0.00430)
Q_t^2		-0.00122***	-0.00104***	-0.00081***	-0.00165***	-0.00191***	-0.00536***
		(0.000145)	(0.000167)	(0.000043)	(0.000188)	(0.000256)	(0.000637)
$\frac{\pi_t}{K_t}$	0.0665***	0.0760***	0.0506***	0.0502***	0.109***	0.0755***	0.0643***
	(0.00641)	(0.00957)	(0.00866)	(0 00278)	(0.0134)	(0.0102)	(0.0101)
$\left(\frac{\pi_t}{K_t}\right)^2$		0.00185	-0.00013	-0.00180*	-0.00092	0.00761***	0.00263
		(0.00195)	(0.00189)	(0.00096)	(0.00313)	(0.00257)	(0.00221)
$Q_t * \frac{\pi_t}{K_t}$		-0.00389***	-0.00251***	-0.00261***	-0.00642***	-0.00507***	-0.00236***
		(0.000827)	(0.000764)	(0.000307)	(0.00103)	(0.00115)	(0.00079)
$\frac{I_{t-1}}{K_{t-1}}$				0.2547***			
				(0.00430)			
Q_t^3							0.0001***
							(0.00002)
$\left(\frac{\pi_t}{K_t}\right)^3$							-0.0005
							(0.00041)
$\frac{B_t}{K_t}$							-0.0233*
							(0.01351)
N	53,230	53,230	53,230	43,910	53,230	17,392	53,230
R^2	0.170	0.197	0.085	0.150	0.668	0.634	0.714
R^2-adj.	0.169	0.198	0.084	0.149	0.667	0.633	0.712
lnL	27,647	28,550			29,847	13,991	30,208

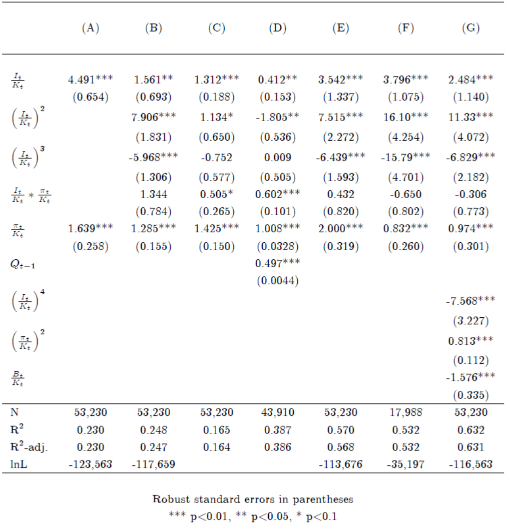
Robust standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

Investment as the dependent variable

Higher orders of investment are significant

$$Q = \phi_0 + \phi_1 \frac{I}{K} + \phi_2 \left(\frac{I}{K}\right)^2 + \phi_3 \left(\frac{I}{K}\right)^3 + \phi_4 \frac{\pi}{K} + \phi_5 \frac{\pi}{K}\frac{I}{K} + \iota$$



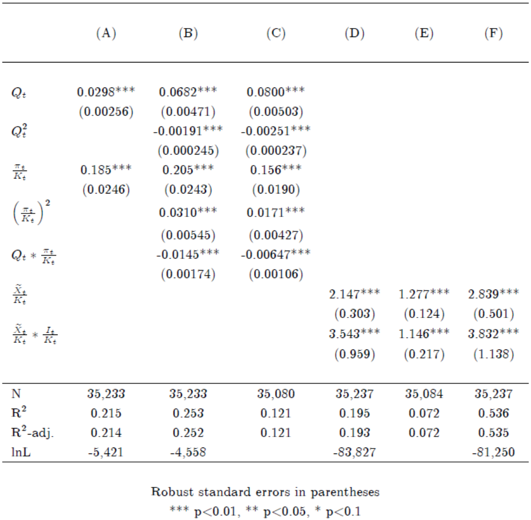
	(A)	(B)	(C)	(D)	(E)	(F)	(G)
$\frac{I_t}{K_t}$	4.491***	1.561**	1.312***	0.412**	3.542***	3.796***	2.484***
	(0.654)	(0.693)	(0.188)	(0.153)	(1.337)	(1.075)	(1.140)
$\left(\frac{I_t}{K_t}\right)^2$		7.906***	1.134*	-1.805**	7.515***	16.10***	11.33***
		(1.831)	(0.650)	(0.536)	(2.272)	(4.254)	(4.072)
$\left(\frac{I_t}{K_t}\right)^3$		-5.968***	-0.752	0.009	-6.439***	-15.79***	-6.829***
		(1.306)	(0.577)	(0.505)	(1.593)	(4.701)	(2.182)
$\frac{I_t}{K_t} * \frac{\pi_t}{K_t}$		1.344	0.505*	0.602***	0.432	-0.650	-0.306
		(0.784)	(0.265)	(0.101)	(0.820)	(0.802)	(0.773)
$\frac{\pi_t}{K_t}$	1.639***	1.285***	1.425***	1.008***	2.000***	0.832***	0.974***
	(0.258)	(0.155)	(0.150)	(0.0328)	(0.319)	(0.260)	(0.301)
Q_{t-1}				0.497***			
				(0.0044)			
$\left(\frac{I_t}{K_t}\right)^4$							-7.568***
							(3.227)
$\left(\frac{\pi_t}{K_t}\right)^2$							0.813***
							(0.112)
$\frac{B_t}{K_t}$							-1.576***
							(0.335)
N	53,230	53,230	53,230	43,910	53,230	17,988	53,230
R^2	0.230	0.248	0.165	0.387	0.570	0.532	0.632
R^2-adj.	0.230	0.247	0.164	0.386	0.568	0.532	0.631
lnL	-123,563	-117,659			-113,676	-35,197	-116,563

Robust standard errors in parentheses

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

The Q-theory of external financing

External financing is significantly related to average Q



	(A)	(B)	(C)	(D)	(E)	(F)
Q_t	0.0298***	0.0682***	0.0800***			
	(0.00256)	(0.00471)	(0.00503)			
Q_t^2		-0.00191***	-0.00251***			
		(0.000245)	(0.000237)			
$\frac{\pi_t}{K_t}$	0.185***	0.205***	0.156***			
	(0.0246)	(0.0243)	(0.0190)			
$\left(\frac{\pi_t}{K_t}\right)^2$		0.0310***	0.0171***			
		(0.00545)	(0.00427)			
$Q_t * \frac{\pi_t}{K_t}$		-0.0145***	-0.00647***			
		(0.00174)	(0.00106)			
$\frac{\tilde{X}_t}{K_t}$				2.147***	1.277***	2.839***
				(0.303)	(0.124)	(0.501)
$\frac{\tilde{X}_t}{K_t} * \frac{I_t}{K_t}$				3.543***	1.146***	3.832***
				(0.959)	(0.217)	(1.138)
N	35,233	35,233	35,080	35,237	35,084	35,237
R^2	0.215	0.253	0.121	0.195	0.072	0.536
R^2-adj.	0.214	0.252	0.121	0.193	0.072	0.535
lnL	-5,421	-4,558		-83,827		-81,250

Robust standard errors in parentheses
*** $p<0.01$, ** $p<0.05$, * $p<0.1$

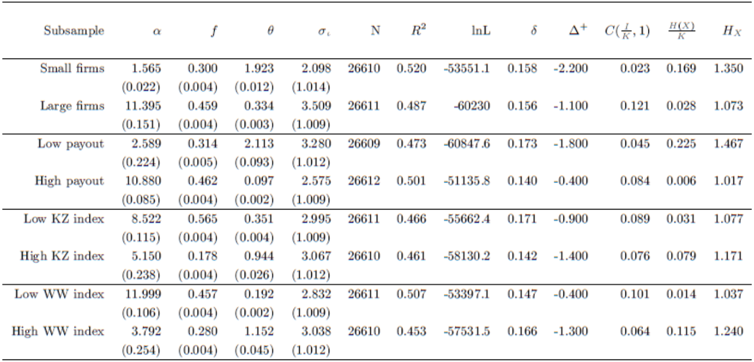
Structural estimation

Average costs of investment and external financing for all firms

α	f	θ	σ_ι	Δ^+
6.094	0.376	0.783	3.025	-1.680
(0.089)	(0.003)	(0.007)	(1.008)	
N	R^2	lnL	χ^2	δ
53230	0.482	-116997	0.001	0.157
$C(\frac{I}{K},1)$	$\frac{H(X,K)}{K}$	H_X	q	κ
0.076	0.094	1.221	1.451	1.017

Financially constrained vs. unconstrained firms

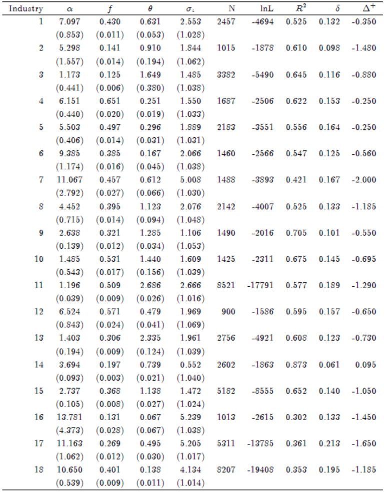
More financially constrained firms have higher costs of external financing



Subsample	α	f	θ	σ_ι	N	R^2	lnL	δ	Δ^+	$C(\frac{I}{K},1)$	$\frac{H(X)}{K}$	H_X
Small firms	1.565	0.300	1.923	2.098	26610	0.520	-53551.1	0.158	-2.200	0.023	0.169	1.350
	(0.022)	(0.004)	(0.012)	(1.014)								
Large firms	11.395	0.459	0.334	3.509	26611	0.487	-60230	0.156	-1.100	0.121	0.028	1.073
	(0.151)	(0.004)	(0.003)	(1.009)								
Low payout	2.589	0.314	2.113	3.280	26609	0.473	-60847.6	0.173	-1.800	0.045	0.225	1.467
	(0.224)	(0.005)	(0.093)	(1.012)								
High payout	10.880	0.462	0.097	2.575	26612	0.501	-51135.8	0.140	-0.400	0.084	0.006	1.017
	(0.085)	(0.004)	(0.002)	(1.009)								
Low KZ index	8.522	0.565	0.351	2.995	26611	0.466	-55662.4	0.171	-0.900	0.089	0.031	1.077
	(0.115)	(0.004)	(0.004)	(1.009)								
High KZ index	5.150	0.178	0.944	3.067	26610	0.461	-58130.2	0.142	-1.400	0.076	0.079	1.171
	(0.238)	(0.004)	(0.026)	(1.012)								
Low WW index	11.999	0.457	0.192	2.832	26611	0.507	-53397.1	0.147	-0.400	0.101	0.014	1.037
	(0.106)	(0.004)	(0.002)	(1.009)								
High WW index	3.792	0.280	1.152	3.038	26610	0.453	-57531.5	0.166	-1.300	0.064	0.115	1.240
	(0.254)	(0.004)	(0.045)	(1.012)								

Real and financing frictions by industry

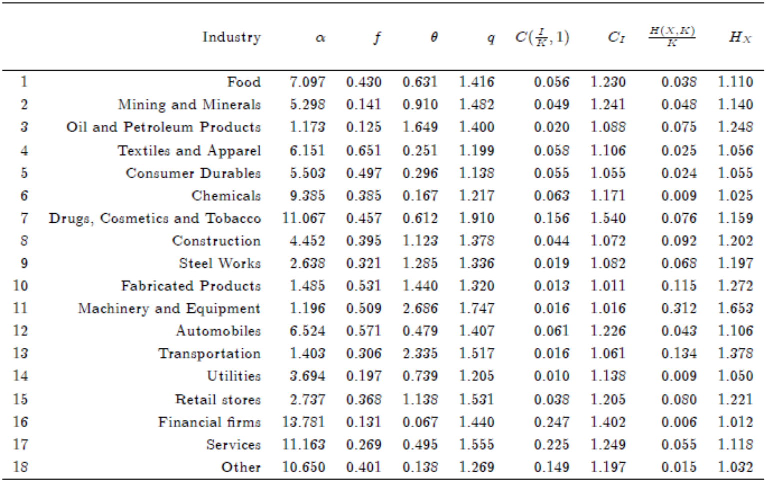
High cross sectional variation in frictions across industries



Industry	α	f	θ	σ_ι	N	lnL	R^2	δ	Δ^+
1	7.097	0.430	0.631	2.553	2457	-4694	0.525	0.132	-0.350
	(0.853)	(0.011)	(0.053)	(1.028)					
2	5.298	0.141	0.910	1.844	1015	-1878	0.610	0.098	-1.480
	(1.557)	(0.014)	(0.194)	(1.062)					
3	1.173	0.125	1.649	1.485	3382	-5490	0.645	0.116	-0.880
	(0.441)	(0.006)	(0.380)	(1.038)					
4	6.151	0.651	0.251	1.550	1687	-2506	0.622	0.153	-0.250
	(0.440)	(0.020)	(0.019)	(1.033)					
5	5.503	0.497	0.296	1.889	2183	-3551	0.556	0.164	-0.250
	(0.406)	(0.014)	(0.031)	(1.031)					
6	9.385	0.385	0.167	2.066	1460	-2566	0.547	0.125	-0.560
	(1.174)	(0.016)	(0.045)	(1.038)					
7	11.067	0.457	0.612	5.008	1488	-3893	0.421	0.167	-2.000
	(2.792)	(0.027)	(0.066)	(1.030)					
8	4.452	0.395	1.123	2.076	2142	-4007	0.525	0.133	-1.185
	(0.715)	(0.014)	(0.094)	(1.048)					
9	2.638	0.321	1.285	1.106	1490	-2016	0.705	0.101	-0.550
	(0.139)	(0.012)	(0.034)	(1.053)					
10	1.485	0.531	1.440	1.609	1425	-2311	0.675	0.145	-0.695
	(0.543)	(0.017)	(0.156)	(1.039)					
11	1.196	0.509	2.686	2.666	8521	-17791	0.577	0.189	-1.290
	(0.039)	(0.009)	(0.026)	(1.016)					
12	6.524	0.571	0.479	1.969	900	-1586	0.595	0.157	-0.650
	(0.843)	(0.024)	(0.041)	(1.069)					
13	1.403	0.306	2.335	1.961	2756	-4921	0.608	0.123	-0.730
	(0.194)	(0.009)	(0.124)	(1.039)					
14	3.694	0.197	0.739	0.552	2602	-1863	0.873	0.061	0.095
	(0.093)	(0.003)	(0.021)	(1.040)					
15	2.737	0.368	1.138	1.472	5182	-8555	0.652	0.140	-1.050
	(0.105)	(0.008)	(0.027)	(1.024)					
16	13.781	0.131	0.067	5.239	1013	-2615	0.302	0.133	-1.450
	(4.373)	(0.028)	(0.067)	(1.038)					
17	11.163	0.269	0.495	5.205	5311	-13785	0.361	0.213	-1.650
	(1.062)	(0.012)	(0.030)	(1.017)					
18	10.650	0.401	0.138	4.134	8207	-19408	0.353	0.195	-1.185
	(0.539)	(0.009)	(0.011)	(1.014)					

Real and financing frictions by industry

Average costs of investment and external financing as a % of capital



	Industry	α	f	θ	q	$C(\frac{I}{K},1)$	C_I	$\frac{H(X,K)}{K}$	H_X
1	Food	7.097	0.430	0.631	1.416	0.056	1.230	0.038	1.110
2	Mining and Minerals	5.298	0.141	0.910	1.482	0.049	1.241	0.048	1.140
3	Oil and Petroleum Products	1.173	0.125	1.649	1.400	0.020	1.088	0.075	1.248
4	Textiles and Apparel	6.151	0.651	0.251	1.199	0.058	1.106	0.025	1.056
5	Consumer Durables	5.503	0.497	0.296	1.138	0.055	1.055	0.024	1.055
6	Chemicals	9.385	0.385	0.167	1.217	0.063	1.171	0.009	1.025
7	Drugs, Cosmetics and Tobacco	11.067	0.457	0.612	1.910	0.156	1.540	0.076	1.159
8	Construction	4.452	0.395	1.123	1.378	0.044	1.072	0.092	1.202
9	Steel Works	2.638	0.321	1.285	1.336	0.019	1.082	0.068	1.197
10	Fabricated Products	1.485	0.531	1.440	1.320	0.013	1.011	0.115	1.272
11	Machinery and Equipment	1.196	0.509	2.686	1.747	0.016	1.016	0.312	1.653
12	Automobiles	6.524	0.571	0.479	1.407	0.061	1.226	0.043	1.106
13	Transportation	1.403	0.306	2.335	1.517	0.016	1.061	0.134	1.378
14	Utilities	3.694	0.197	0.739	1.205	0.010	1.138	0.009	1.050
15	Retail stores	2.737	0.368	1.138	1.531	0.038	1.205	0.080	1.221
16	Financial firms	13.781	0.131	0.067	1.440	0.247	1.402	0.006	1.012
17	Services	11.163	0.269	0.495	1.555	0.225	1.249	0.055	1.118
18	Other	10.650	0.401	0.138	1.269	0.149	1.197	0.015	1.032

Conclusions

- The estimation of investment as a function of Q is biased if real and financing frictions are neglected.
- Investment and external financing are weakly concave in Q and weakly convex in cashflows.
- The log likelihood function of a Q-theory model is a useful tool to test investment equations.
- There is high cross sectional variation in real and financing frictions across industries.