A Structural Model of Short-term Reversals

Matti Suominen and Kalle Rinne*

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We present a structural model of the stock market where a subset of investors is infrequently present in the market. We study the implications of this assumption on liquidity, return reversal patterns, illiquidity premium, costs of immediacy and volatility.

Our model predicts that the stock's return reversal pattern is exponential and that the amount of return reversal, the speed of return reversal and the transitory part of the stock's return volatility are all related to the stock's liquidity. Interestingly, in contrast to common perception, fast return reversal is typically a sign of inefficient, illiquid markets, thus not a sign of efficiency. Other key predictions are that the illiquidity premium is non-monotonic in several structural parameters of the model, such as the number of market makers, and that the costs of immediacy are also non-monotonic in several structural parameters, such as the number of market makers and the parameter of investors' risk aversion.

Empirically, our key findings are that, on average, 24% of NYSE and Amex traded stocks' excess returns revert within the first week, that the pattern of return reversal is exponential, and that nearly 20% of daily volatility is transitory. We also find that both the speed of return reversal and the amount of transitory volatility depend on the stock's liquidity, as predicted by the model. For illiquid stocks, return reversals are faster and a greater amount of the volatility, 27%, is transitory.

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* Both authors are from the Aalto University School of Economics (former Helsinki School of Economics). The contact is Matti Suominen. Address: P.O. Box 21210, 00076 Aalto, Helsinki Finland. E-mail: matti.suominen@hse.fi, tel.: +358-50-5245678, fax: +358-9-43138678. We thank Robin Greenwood, Petri Jylhä, Juhani Linnainmaa, Patrik Sandås, Mark Seasholes and the seminar participants at the Aalto University, Bank of Finland, EFMA in Århus, Asian Finance Association meetings in Hong Kong and the Econometric Society World Congress in Shanghai for their constructive comments.

1. Introduction

A striking empirical fact is that many investors trade very infrequently. Agnew, Balduzzi, and Sundén (2003), for instance, study US investors' 401(k) retirement accounts and find that, on average, investors re-balance their portfolios only once in 3.85 years. Similarly, Grinblatt and Keloharju (2009) study the trading behavior of ninety-five thousand Finnish investors, using data that contain these investors' holdings of all Finnish stocks. In their sample, they find that most individual investors either did not trade at all, or traded only once during their eight-year sample window. [1]

In this paper, we present a structural model of the stock market that is based on the assumption that a subset of investors is rarely present in the stock market. The investors who are continuously present in the stock market are called market makers, while the investors who are rarely present in the market are referred to as transitory investors. Although we call the first class of investors market makers, it should be thought that this class includes all investors who frequently monitor the stock market. In our model, short-term return reversals emerge, as in Grossman and Miller (1988), from transitory investors' endowment shocks due to imperfect risk bearing ability of market makers.[2]

We define liquidity as the expected price impact from investors' endowment shocks. Using our structural model, we study theoretically the effects of various structural parameters on the stock's

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[1] See Duffie (2010) and Investment Company Institute 2005 report for additional evidence on trading activity in the US.

[2] The idea that some investors are transitory (they arrive at the market only once to adjust their portfolios) or inattentive (they arrive at the market regularly but infrequently) is common in the literature, see, e.g., Duffie and Sun (1993) or Roşu (2009).

liquidity, return reversal pattern, illiquidity premium, volatility, the costs of immediacy to transitory investors and the market makers' returns from providing immediacy.

Our model predicts that stocks' return reversal pattern is exponential. This result is a direct consequence of the assumption of transitory investors. Second, in our model, the degree of return reversal and the speed of return reversal depend on the same structural parameters that determine liquidity. These parameters are the number of market makers and transitory investors, stocks' fundamental risk, the variability of transitory investors' endowment shocks and agents' degree of risk aversion. Interestingly, in contrast to common perception, fast return reversal is typically a sign of inefficient, illiquid markets, thus not a sign of efficiency. The reason for this is that in a liquid market the market makers carry large inventories, and as a result the transitory investors' endowment shocks have a long lasting effect on market prices.

One important feature of our model is that the market makers (modeled as myopic investors) care about the short-term price risk. In our model, the transitory investors' endowment shocks cause transitory short-term price volatility. In equilibrium, this affects the market makers' average stock investment, which affects the stock's expected return and leads to an illiquidity premium in stock returns, as in Acharya and Pedersen (2005). Interestingly, the illiquidity premium is non-monotonic in the number of market makers and transitory investors, and does not move one to one with the level of liquidity. We then study the portfolio returns obtained by the transitory investors and the market makers. We show that the transitory investors suffer costs of immediacy, as in Grossman and Miller (1988), as their endowment shocks cause them to trade at unfavorable prices. We show that the expected costs of immediacy to transitory investors are non-monotonic in the investors' parameter of risk aversion, the number of market makers and the number of transitory investors. Correspondingly, the market makers' returns from investing include, besides the illiquidity premium, additional returns from providing immediacy.

Finally, we use our model to discover several endogenous variables that can be expected, in the presence of transitory investors, to empirically proxy for liquidity. Such variables are first-order autocorrelation in returns, percentage of transitory volatility and the speed of mean reversion.

The second part of our paper is empirical. Our empirical estimates, using data at the daily frequency and using the full available history for stocks traded in NYSE and Amex, show that historically 24% of stocks' daily excess returns have reverted within a week. Our empirical estimates strongly support the prediction that the short-term return reversal pattern is exponential.[3] Also, many of our other empirical findings are consistent with the theory: We find, for instance, that in both cross-section and time series, the degree of mean reversion is negatively related to liquidity, thus confirming the empirical findings in Avramov, Chordia and Goyal (2006). In addition, the speed of mean reversion depends on liquidity, so that mean reversion is faster for illiquid stocks and during time periods with low liquidity. Finally, there is a large amount of transitory volatility, which also depends on the level of liquidity. For illiquid stocks, according to our estimates, 27% of volatility is transitory.[4] In addition, we show that the level of liquidity is closely related in cross-section and in time series with stocks' expected return

[3] Conrad and Kaul (1989) present evidence that stocks' return reversal pattern is exponential also at the weekly frequency. As we discuss below, recognizing that the return reversal pattern is exponential improves significantly an econometrician's ability to forecast short-term returns and volatility.
[4] Transitory volatility refers to short-term volatility that is due to temporary price shocks.

(illiquidity premium), the returns to contrarian trading strategies (which can be viewed as returns from providing immediacy) and trading volume.

Our model builds upon earlier models of short-term reversals, such as Grossman and Miller (1988). Our model is particularly closely related to Duffie (2010), which presents a multi-period model for asset price dynamics following supply and demand shocks in the market; and Hendershott and Menkveld (2010), which studies asset price dynamics caused by market makers' inventory considerations. In both of these papers, the assumption of asynchronously arriving investors leads to gradual price recovery from the price impact that follows from supply or demand shocks. Besides predicting a similar exponential pattern of return reversal, our model predicts an exponentially declining autocorrelation function. In addition, we extend our analysis in new dimensions. For instance, we endogenize the expected periodic supply of the asset and derive closed-form expressions for stocks' expected returns, illiquidity premium, costs of immediacy and unconditional return volatility. The closed-form expressions, in turn, allow us to study how these endogenous variables depend, often in a non-monotonic manner, on the structural parameters of the model. Other closely related theories are presented in Campbell, Grossman and Wang (1993), Chordia and Subrahmanyam (2004) and Weill (2007).[5] [6]

Empirically, many papers have documented short-term stock return reversals, and many of these papers, like ours, relate the phenomenon to liquidity provision. First, Duffie (2010) reviews

[5] Other related papers include Ho and Stoll (1983), Rubinstein and Wolinsky (1987), Campbell and Kyle (1992), Lynch (1996), Gabaix and Laibson (2001), Kyle and Xiong (2001), Gromb and Vayanos (2002), Goettler, Parlour and Rajan (2005), Foucault, Kadan and Kandel (2005), Duffie, Gârleanu and Pedersen (2005, 2007), Biais and Weill (2009), Huang and Wang (2009, 2010), and Bacchetta and van Wincoop (2010). Duffie (2010) and Gromb and Vayanos (2010) are excellent reviews of the related literature.

[6] Related are also Pagano (1989b), Vayanos and Wang (2007) and Johnson (2008), who study the relationship between trading volume and liquidity; and Pagano (1989a), Allen and Gale (1994) and Xiong (2001), who look at the effect of liquidity shocks on volatility. See also Aiyagari and Gertler (1999) and Attari and Mello (2006).

existing empirical evidence of short-term reversals that follows different types of large supply or demand shocks in the market. In addition, it is well documented that stock returns more systematically revert at short horizons. For instance, Jegadeesh (1990) and Lehmann (1990) find evidence from stock return time series data of significant return reversals at one-month and one-week horizons, respectively. These short-term return reversals are linked to market makers' inventory considerations in Jegadeesh and Titman (1993). Hendershott and Seasholes (2007), Comerton-Forde *et al*. (2010) and Hendershott and Menkveld (2010) present clear evidence that market makers' inventory considerations affect short-term reversals and liquidity (See also Andrade, Chang and Seasholes, 2008). Jylhä, Rinne and Suominen (2011), in turn, show that hedge fund flows affect liquidity, volatility and the amount of short-term return reversals; and, along with Aragon and Strahan (2011), that hedge funds provide immediacy in the stock market. Along the same lines, Brogaard (2011) presents evidence that high-frequency traders engage in reversal trades and reduce volatility. Finally, Foucault, Sraer and Thesmar (2011) present evidence that retail investors create noise in the stock market and that a reduction in the retail investors' trading activity improves liquidity and reduces short-term return reversals and volatility.[7]

Our paper is also closely related to the literature that looks at the relation between liquidity, asset prices and investors' portfolio returns, such as Pastor and Stambaugh (2003) and Acharya and Pedersen (2005), which model and document an illiquidity premium in stock returns. Also related is Brennan and Wang (2009), which looks at the asset pricing implications of short-term volatility due to mispricing. Several recent papers, including Khandani and Lo (2011), Nagel

[7] They find that this occurs despite the fact that most of the retail investors in their sample act as contrarian traders, consistent with the findings in Kaniel, Saar and Titman (2008).

(2011), Jylhä *et al*. (2011) and Rinne and Suominen (2010), in turn, empirically evaluate the returns from providing immediacy or the costs of immediacy.

On the theoretical side, our main contribution is to present a dynamic model of the stock market with transitory investors, which allows us to study the relations between liquidity, mean reversion patterns, illiquidity premium, costs of immediacy and volatility. Interestingly, we show that illiquidity premium and the costs of immediacy are non-monotonic in several structural parameters of the model, and do not move one to one with the level of liquidity.

Empirically, our main contributions are to document the exponential structure of the return reversal in excess returns, and to provide new evidence on the amount and the speed of return reversal, and how these depend on liquidity. Importantly, we also document the impact of liquidity on transitory volatility. Our result that liquidity and transitory volatility are negatively related complements the findings in Adrian and Rosenberg (2008).[8]

The rest of the paper is organized as follows: In Section 2, we present the model and characterize the equilibrium. In Section 3, we describe the model's implications for liquidity and in Section 4 the implications for the return reversal patterns. In Section 5, we study the model's implications for volatility and the stock's expected return. In Section 6, we look at the costs of immediacy and the returns from providing immediacy, while in Section 7 we propose new empirical proxies for liquidity. In Section 8, we present our empirical findings related to stocks' return reversal patterns and look at the empirical relationships between liquidity, return reversal patterns,

[8] Our empirical finding that transitory volatility and reversals are closely related in time series was originally presented in Rinne and Suominen (2010).

illiquidity premium, returns from providing immediacy and volatility. Section 9 concludes the paper.

2. Model and equilibrium

2.1 Model

In this section, we present our multi-period model of a stock market with two types of investors: transitory long-term investors and short-term market makers. The main difference to standard market microstructure models is an assumption that the transitory long-term investors are rarely present in the stock market. Our motivation for making this assumption is based on the notion that these investors have some (unmodeled) cost of being present in the market and thus arrive at the stock market only when they inherit portfolio imbalances. On the other hand, the market makers retain a continuous presence in the stock market and benefit from the trading opportunities that the transitory investors' portfolio imbalances create.[9]

More precisely, let us consider the following model: There are $2T+1$ periods, $t \in \{-T,\ T\}$, and two assets: a safe and a risky asset.[10] The safe asset is in elastic supply, while the risky asset is in strictly positive supply $S > 0$. The periodic gross return on the safe asset is $R \equiv 1+r$, where $r > 0$. The risky asset, stock, pays a periodic dividend,

$$\tilde{D}_t = v + \tilde{\varepsilon}_t \tag{1}$$

[9] There is currently a growing stream of literature on the economics of partially segmented financial markets (See Duffie, 2010, and Gromb and Vayanos, 2010, for surveys of this literature). Intertemporal market segmentation, similar to that in our setting, is present in, for example, Grossman and Miller (1988), Spiegel and Subrahmanyam (1995) and Roşu (2009).

[10] We later consider the limiting economy where $T \rightarrow \infty$.

in all periods $t \leq T$. Here, v is the expected periodic dividend, while $\tilde{\varepsilon}_t$ a normally distributed periodic shock to the dividend, that is independently and identically distributed in all periods t with mean zero and variance σ_{ε}^2. Here, as throughout the paper, we use a tilde to highlight a random variable. We assume that the period t dividend is observable, and paid out at the beginning of period t, and denote by $\tilde{P}_t$ the period t ex-dividend stock price. All stock trading in period t takes place at the ex-dividend stock price.

We now describe the two types of agents. First, there are overlapping generations of cohorts of $M \geq 0$ market makers, who live for two periods. They invest when they are young and consume when old. We take the number of market makers, M, as exogenously given and assume that all generations of market makers have zero endowment of the stock.[11]

Second, there are overlapping generations of transitory long-term investors. In each period t, a new cohort of $K > 0$ transitory investors is born and arrives at the market to trade. After trading, these K investors exit the stock market. Thus, in contrast to the market makers, the transitory investors are present in the stock market only in one period. We assume that the average stock endowment of the period t transitory investors, $\tilde{\omega}_t^e$, is normally distributed with mean $\overline{\omega} > 0$ and variance σ_{ω}^2, independent of the average endowment of the other generations of investors, stock

[11] Our assumption is equivalent to assuming M myopic market makers who live throughout the $2T+1$ periods and are continuously present in the market. Other papers that assume myopic market makers in dynamic asset pricing models include Campbell *et al*. (1993), Spiegel and Subrahmanyam (1995), Acharya and Pedersen (2005), Bacchetta and van Wincoop (2006), Duffie (2010) and Jylhä and Suominen (2011). Strategic reasons (outside the scope of our model) can be used to justify the assumption that even long-lived investors be myopic. As in Kondor (2009), investors acting as market makers may care about short-term profits to make sure that they have sufficient capital available in case better short-term trading opportunities emerge. The myopic behavior can also be thought of as a behavioral bias.

prices and dividend shocks. The transitory investors, who are absent from the stock market in period t, have access only to the safe asset. This implies that they invest all the dividends they receive from their previous stock investments (borrow money for all the dividends they pay in case they are short of the asset) at the risk-free rate r until period T, when they consume their wealth. Finally, in order to keep the number of investors finite even in the limit where $T \to \infty$, we assume that the transitory investors may die before they are able to consume. In other words, we assume that the transitory investors who are alive in period t survive to the next period only with probability $1\text{-}\rho \in (0,1)$.[12]

Now, it is straightforward to show that for a period t transitory investor, the expected additional period T wealth that comes from receiving one additional share of the risky asset ex-dividend in period t, conditional on this investor surviving to period T, is:

$$v_t = v\left(\frac{R^{T\text{-}t} - 1}{r}\right). \tag{2}$$

This is the time t expectation of the forward, time T value of the stream of dividends in periods $\tau \in [t+1,T]$, assuming that all dividends received are re-invested at the risk-free rate for periods $s \in [\tau,T]$. Similarly, for such an investor, the variance of the period T wealth, associated with a one-share investment in the risky asset in period t is:

[12] It makes no difference if we assume that also the market makers die with probability ρ before they can consume (and assume that the dying market makers' positions are inherited by the next generation of market makers).

$$\sigma_t^2 = \sigma_\varepsilon^2 \left(\frac{R^{T-t} - 1}{r} \right). \tag{3}$$

Next, we characterize the investors' and the market makers' problems. We assume that all investors and market makers behave competitively and in period t they select their stock holdings, taking as given the stock price, P_t. We also assume that both types of investors evaluate their terminal wealth using a CARA utility function. The transitory investors care about their period T wealth, $\tilde{W}_T$, while the period t market makers care about their period $t+1$ wealth, $\tilde{W}_{t+1}$. Let a denote the parameter of risk aversion for both types of investors and $E_t[\cdot]$ the expectation operator conditioned on time t public information set, Ω_t. We assume that Ω_t includes the current and past stock prices, the current and past transitory investors' endowments, and the period $t-1$ market makers' inventories at the beginning of period t (we refer to the old market makers' stock holdings at the beginning of the period as their inventories). At time t, a transitory investor, i, with an endowed wealth of $W_{i,t} = \omega_{i,t}^e (P_t + D_t)$, then selects his period t stock investment, $\omega_{i,t}^d$, to maximize

$$\max_{\omega_{i,t}^d} \; E_t - \exp\left(-a\tilde{W}_{i,T}\left(\omega_{i,t}^d\right)\right) \tag{4}$$

$\Leftrightarrow$

$$\max_{\omega_{i,t}^d} -exp\left(-a\left(R^{T-t}W_{i,t} + \omega_{i,t}^d\left(v_t - R^{T-t}P_t\right) \; - \frac{a}{2}\omega_{i,t}^{d\,2}\sigma_t^2\right)\right). \tag{5}$$

Denoting by $\mu_{j,t}^{d}$ a period *t* short-term trader's stock investment, a short-term trader *j* in period *t* maximizes

$$\max_{\mu_{j,t}^{d}} \; E_t - exp\left(-a\widetilde{W}_{j,t+1}\left(\mu_{j,t}^{d}\right)\right) \tag{6}$$

$$\Leftrightarrow$$

$$\max_{\mu_{j,t}^{d}} \; E_t - exp\left(-a\mu_{j,t}^{d}\left[\widetilde{P}_{t+1} + \widetilde{D}_{t+1} - RP_t\right]\right). \tag{7}$$

Equilibrium prevails when both investors' and market makers' actions maximize their objective functions (5) and (7), taking as given the strategies of the other market participants, and markets clear.

2.2. Equilibrium

The first-order condition to (5) implies that the period *t* transitory investors' optimal investment into stocks is:

$$\omega_{i,t}^{d} = \omega_{t}^{d} = \frac{v_t - R^{T-t}P_t}{a\sigma_t^2} = \frac{v}{a\sigma_\varepsilon^2} - \frac{r}{a\sigma_\varepsilon^2}\left(\frac{R^{T\text{-}t}}{R^{T\text{-}t} - 1}\right)P_t\,. \tag{8}$$

Let us now look at the market makers' behavior. Assuming that the next trading period's price is normally distributed, given Ω_t, with mean $E_t(P_{t+1})$ and a constant variance $\sigma_{P_{t+1}}^2$, as is confirmed in the Appendix, after taking expectations, the first-order condition to (7) gives that a period t market maker's demand for the risky security, $\mu_{j,t}^{d}$, equals:

$$\mu_{j,t}^{d} = \frac{E_t\left(\widetilde{P}_{t+1}\right) + v - RP_t}{a\left(\sigma_{P_{t+1}}^2 + \sigma_\varepsilon^2\right)}. \tag{9}$$

Having characterized the investors' and market makers' optimal investment choices, we use the market clearing condition to solve for the equilibrium stock price. Denote by μ_t^e for $t > -T$ a representative period *t-1* market makers' inventory at the beginning of period *t*, which equals his investment in period *t-1*, $\mu_t^e = \mu_{t-1}^d$, and let $\mu_{-T}^e = 0$. Using this notation, market clearing implies:

$$M\mu_t^d + K\omega_t^d = M\mu_t^e + K\omega_t^e. \tag{10}$$

We now present our first proposition. In Proposition 1, the critical endogenous variables are $\overline{P}$, $\overline{\mu}$ and *y*. Here, $\overline{P}$ is the expected stock price, $\overline{\mu}$ the market makers' expected inventory, while *y* can be thought of as a measure of illiquidity. All propositions are proved in the Appendix.

Proposition 1 (Equilibrium): *There exist* $\overline{P}$, $\overline{\mu} > 0$ *and* $y > 0$, *and an equilibrium market clearing price* P_t, *such that in the limit where* $T \to \infty$, P_t *approaches:*

$$P_t = \overline{P} - y\left[M\left(\mu_t^e - \overline{\mu}\right) + K\left(\omega_t^e - \overline{\omega}\right)\right]. \tag{11}$$

In the limit where $T \to \infty$, *the stock price* P_t *is normally distributed with mean* $\overline{P}$ *and variance* $\Phi = y^2\left(M^2\sigma_\mu^2 + K^2\sigma_\omega^2\right)$, *where* σ_μ^2 *is defined in the Appendix. Conditional on period t-1* *information set, Ω_{t-1}, the stock price P_t is normally distributed with mean* $\overline{P} - yM\left(\mu_{t-1}^{d} - \overline{\mu}\right)$ *and variance*

$$\sigma_P^2 = E_{t-1}\left[P_t - E_{t-1}\left(P_t\right)\right]^2 = y^2 K^2 \sigma_\omega^2. \tag{12}$$

Proposition 1 shows that there is a price impact from investors' unexpected endowment shocks. Consider, for instance, the case where the market makers' time t inventories are at their expected, long-term average level: $\mu_t^e = \overline{\mu}$. In this case, when $\omega_t^e > \overline{\omega}$, so that the period t transitory investors have an abnormally large positive endowment of the stock, and thus a selling need, the price P_t will, in equilibrium, decline below the long-term average price $\overline{P}$ in order to clear the market. In contrast, when the period t transitory investors have an abnormally small endowment of the stock, the price rises above the long-term average price. The price impact from the transitory investors' unexpected endowment shocks equals y times these investors' unexpected aggregate endowment, $y \cdot K\left(\omega_t^e - \overline{\omega}\right)$. The magnitude of the price impact that follows from the endowment shocks is thus entirely determined by y. Given this, y measures well what we commonly understand by illiquidity. From this point onwards, we refer to y as our measure of (il)liquidity.

3. Liquidity

In our structural model, liquidity, return reversal patterns and expected returns are co-determined, along with several other variables of interest, by the exogenously given structural parameters: the supply of the asset, the number of the two different types of investors, the variability of the transitory investors' endowment shocks, investors' risk aversion and variability of dividend shocks. First, let us look at (il)liquidity.

Proposition 2 (Liquidity): *Our measure of illiquidity, y, defined in Proposition 1, is decreasing in the number of market makers, M, and increasing in the parameter of investors' risk aversion, a, the variability of the dividends,* σ_ε^2*, and the variability of the transitory investors' average endowment,* σ_ω^2*. The effect of the number of transitory investors, K, on y is ambiguous. In the limit,* $y \rightarrow 0$ *when either* $M \rightarrow \infty$*,* $a \rightarrow 0$*, or* $\sigma_\varepsilon^2 \rightarrow 0$*. Illiquidity, y, is independent of the supply of the asset, S.*

Proposition 2 shows that market liquidity increases, i.e., *y* decreases, when the number of market makers increases or when investors' level of risk aversion, *a*, decreases. When the number of market makers is small, or they, along with the investors, are highly risk averse, the price impact from transitory investors' endowment shocks is large (markets are illiquid). In this case, the market makers' collective risk bearing ability is low, and a large price reaction is needed to induce the market makers and transitory investors to take the positions that clear the market. Market liquidity is also low when future dividends or transitory investors' endowment shocks are highly risky – as implied by the fact that *y* increases in the variances of dividend innovations and investors' endowments. This occurs as the increased variability in dividend innovations subjects market makers to information-related risks, while increased variability in investors' endowment shocks subjects market makers to short-term price risks. The effect of the number of transitory investors, *K*, on market liquidity and the price impact is ambiguous. On the one hand, a greater number of investors that arrive at the market in future periods, *K*, increases the stock price risk that a short-term trader faces, due to larger collective investors' future endowment shocks. This has an increasing effect on the price impact from investors' endowment shocks, i.e., y. On the other hand, the greater the number of investors that arrive at the market in every period, the easier it is for the market makers (as a group) to download their inventories to future generations of investors. Because of these two opposing forces, the effect of K on the price impact, y, is ambiguous in our model. [13] [14]

4. Model implication for return reversal

4.1 Expected price recovery

In this section, we show that the pattern of price reversal is exponential. This, as we show later, has significant implications for an econometrician trying to forecast future short-term stock returns using past returns.

First, in Proposition 3, we describe the expected future returns, given the full information set, Ω_t. A critical variable in this proposition, and in Propositions 4-8, is a variable B, which measures the persistence of a temporary price impact, that is to say, the persistence of the stock's price deviations from the long-term average stock price.

[13] The result that y may increase in K is similar to the finding in De Long *et al*. (1990) that noise trader risk reduces rational investors' willingness to enter into arbitrage.

[14] Proposition 2 shows the determinants of liquidity. These results suggest that even in a more elaborate setting where the parameters may change over time, we can expect that increases in risk aversion or uncertainty related to firms' fundamental value decrease liquidity. In addition, our results suggest that an increase in uncertainty over the transitory investors' excess endowments decreases liquidity (e.g., liquidations of large hedge funds or investment banks could create such uncertainty) and that an increase in the arrival rate of transitory investors at the market can either increase or decrease liquidity.

Proposition 3 (expected price recovery): *There exist $A > 0$ and $0 < B < 1$, such that when $M > 0$ the expected price change from date $t+\tau$ onwards, where $\tau \geq 0$, given the time t public information set, Ω_t, is:*

$$E_t\left[P_{t+\tau+1} - P_{t+\tau}\right] = A \cdot B^{\tau}\left(M\left(\mu_t^e - \overline{\mu}\right) + K\left(\omega_t^e - \overline{\omega}\right)\right). \tag{13}$$

Here, $\sum_{\tau=0}^{\infty} A \cdot B^{\tau} = y$, implying, given Proposition 1, that prices are expected to eventually revert fully to their long-term mean. In this sense, all price volatility in our model, $\sigma_P = \sqrt{\sigma_P^2}$, is transitory. The pattern of price mean reversion depends on the model parameters: In particular, A is decreasing and B increasing in the number of market makers, M, and A is increasing and B decreasing in the variability of investors' endowment, σ_ω^2, the variability of dividends, σ_ε^2, the number of transitory investors, K, and the parameter of investors' risk aversion, a. When $M = 0$, $E_t\left[P_{t+\tau+1} - P_{t+\tau}\right] = y\left(K\left(\omega_t^e - \overline{\omega}\right)\right)$, implying that the expected price recovery from investors' endowment shocks occurs in the very next period.

Equation (13) shows that there is mean reversion. Following a negative price impact from investors' positive unexpected endowment shock, $\omega_t^e > \overline{\omega}$, implied by Proposition 1, in future periods, prices revert. Proposition 3 shows that when $M > 0$, mean reversion is gradual in the sense that in this case the expected excess returns are positive for several future periods. Note that the pattern of price reversal takes an exponential form and thus the expected price reversal is the highest in the first few periods.[15]

[15] Our model's prediction that price reversal pattern is exponential is similar to that in Duffie (2010) and Hendershott and Menkveld (2010).

In our model, the gradual mean reversion is due to the gradual mean reversion in market makers' inventories. When the number of market makers is large, or they are not highly risk averse, prices are slow to converge to their fundamental value (as is implied by a large *B*). In this case, the market makers collectively carry large inventories and as a group download their inventory imbalances to transitory investors only gradually. The opposite occurs when the number of market makers is small or they are highly risk averse. In this case, from Proposition 1, we know that the transitory investors' portfolio imbalances have a large price impact. Nevertheless, prices quickly revert to their fundamental value. This occurs as the few (or highly risk averse) market makers then collectively hold only small inventories and thus can quickly download the inventories to future generations of transitory investors. In the extreme case where there are no market makers, $M = 0$, the expected mean reversion from price changes caused by investors' endowment shocks, is over in the very next day. As Proposition 3 shows, the price recovery pattern depends also on the level of fundamental dividend risk, σ_{ε}^{2}, the number of transitory investors, *K*, and the variability of the transitory investors' endowment shocks, σ_{ω}^{2}.

4.2. Expected pattern for return reversal

In each period, stock returns are affected by both dividend shocks and the transitory investors' endowment shocks. While the dividend shocks have no effect on future returns in our setting, the transitory investors' endowment shocks affect future expected returns as they affect market makers' inventories. Because the transitory investors' endowment shocks affect both the contemporaneous returns, given Proposition 1, and future price changes, given Proposition 3, and in opposite directions, our model predicts mean reversion in stock returns. Given that mean reversion is gradual (by Proposition 3), we should expect to observe long lasting negative autocorrelation in returns. Proposition 4 characterizes the return autocorrelation structure predicted by our model.

Proposition 4 (return autocorrelation): *When $M > 0$, the expected return autocorrelation between period $t+1$ excess returns, R_{t+1}, and past excess returns, R_{t-f}, where $f \geq 0$ is $\rho(1+f) = C \cdot B^{f}$, where $0 < B < 1$ and $C < 0$. Here, B, the persistence of price impact, defined in the proof of Proposition 3, is increasing in M and decreasing in a, K, σ_{ω}^{2} and σ_{ε}^{2}, while $C \equiv \rho(1)$ is decreasing in M and increasing in a, K, σ_{ω}^{2} and σ_{ε}^{2}.*

Proposition 4 shows the intuitive result that future returns have the highest negative correlation with the most recent past returns, i.e., returns where f is small. This is intuitive, given that in our model, the return predictability is due to market makers' inventories, and these revert back to their long-term mean over time, implying that returns several days previously correlate little with future returns. It is interesting to note that the same exponential structure, as in the case of the expected price mean reversion, prevails also in the autocorrelation function. In fact, in our model, the base of the exponential, B, is the same in both cases.

When M is small, the market makers' inventories revert particularly quickly to zero, implying that the returns several days previously have little predictive power on future returns (B is small and $\rho(1+f)$ approaches zero quickly as f increases). When M is large, on the other hand, the market makers collectively carry large excess inventories for a long time. In this case, even the returns from several days previously can affect the expected future returns of stocks (and $\rho(1+f)$ approaches zero more gradually as f increases). As proposition 4 shows, other model parameters, besides *M*, such as the variability of transitory investors' endowment, σ_ω^2, the variability of dividends, σ_ε^2, the number of transitory investors, *K*, and the parameter of risk aversion, *a*, affect the autocorrelation structure and hence the pattern of return reversals.

5. Transitory Price Volatility, Expected Returns and Illiquidity Premium

5.1. Volatility

Our model allows us to make predictions about unconditional volatility.

Proposition 5 (Unconditional volatility): *In the limit where $T \to \infty$, the unconditional standard deviation of Z days' return, $\sum_{\tau=0}^{Z-1}\left(P_{t-\tau} + D_{t-\tau} - P_{t-\tau-1}\right)$, is:*

$$\sigma_Z = \sqrt{Z\sigma_\varepsilon^2 + 2\left(\frac{1-B^Z}{1-B^2}\right)\sigma_P^2}\,. \tag{14}$$

Here, $B \in (0,1)$ is a parameter that determines the persistence of price impact (defined in the proof of Proposition 3) and σ_P^2 is the conditional price variability defined in Proposition 1. Estimates of standard deviations of returns, that are taken in D=1 and Z >1 day intervals, σ_D and σ_Z, satisfy $\sigma_Z < \sqrt{Z}\sigma_D$. In the limit where $y \to 0$ (i.e., when either $M \to \infty$, $a \to 0$ or $\sigma_\varepsilon^2 \to 0$), $\sigma_Z - \sqrt{Z}\sigma_D \to 0$. Furthermore, as $y \to 0$, σ_Z approaches $\sqrt{Z}\sigma_\varepsilon$ from above.

Proposition 5 implies that when estimating volatility using daily as opposed to monthly returns, the annualized estimates of volatility are higher when using the common square root rule to annualize volatility. The intuition for this result is the following. Daily returns are affected by both the dividend innovations and the price impact from transitory investors' endowment shocks. When returns are measured over longer horizons, the effect of the price impact from transitory investors' endowment shocks is smaller, as the price impact is negatively correlated with the short-term future expected returns, given Propositions 1 and 3. This implies that when volatility is measured over longer horizons, the volatility estimates reflect to a greater degree only the fundamental volatility that comes from the dividend shocks.

Proposition 5 implies that the return variance (volatility squared) perceived by myopic investors, who look only one day ahead, is the highest when compared to expected returns (recall that expected returns increase proportionally to time). In fact, the longer the investor's investment horizon, the smaller the perceived return variance compared to expected returns. If myopic investors form a large group of market makers, this, as we argue below, will have implications for stocks' expected returns.

5.2. Expected Returns and Illiquidity Premium

For the remainder of this section, we make an additional stationarity assumption that the transitory investors' expected endowment equals their expected investment.

Assumption A1: $\overline{\varpi} = E\left(\omega_t^d\right)$.

Given this assumption, we can make predictions about stocks' expected excess returns (in excess of the risk-free rate) and the illiquidity premium.

Proposition 6 (Expected returns): *Stocks' unconditional expected excess return equals*

$$E\left[(\widetilde{P}_{t+1}) + v - RP_t\right] = v - r\overline{P} = \frac{aS}{\left[\dfrac{M}{\sigma_P^2 + \sigma_\varepsilon^2} + \dfrac{K}{\rho\sigma_\varepsilon^2}\right]}. \qquad (15)$$

Stocks' expected excess return is increasing in the supply of the asset, S, the parameter of risk aversion, a, the variability of the dividends, σ_ε^2*, and the variability of the transitory investors' average endowment,* σ_ω^2*. The expected return is decreasing in the number of market makers, M, while the effect of the number of transitory investors, K, on expected returns is ambiguous.*

Equation (15) shows, among other things, that stocks' expected excess returns increase in the short-term price risk, σ_P^2. It is also interesting to note that the number of market makers, *M*, affects expected excess returns through two different channels. First, an increase in *M* leads to an increase in the number of investors, which leads each investor to hold in equilibrium less of the risky asset, thus reducing the required expected return. Second, an increase in *M* reduces the price volatility, and in this way increases each market maker's optimal holdings in the stock, leading to a further reduction in stock's expected excess return.

As σ_P^2 is positive only in illiquid markets, where $y > 0$, given Proposition 1, similarly as in the model presented in Acharya and Pedersen (2005), our model implies an illiquidity premium in a stock's expected returns. Illiquidity premium arises as the market makers (recall that our interpretation is that this class of investors includes all myopic investors) hold smaller positions in an illiquid security, as compared to a liquid security, due to the short-term price risk, σ_P^2. Thus, to clear the market, transitory investors must be induced to hold larger positions in an illiquid stock, pushing the stock's market clearing price down and its expected return up. We define the illiquidity premium, $PREM_{ILLIQ}$, as the additional expected return to the risky asset, compared to its expected return in the case where the number of investors is the same, but where the asset is perfectly liquid, i.e., $y = 0$, because there is no short-term endowment risk, $\sigma_\omega^2 = 0$. In Proposition 7, we characterize the underlying variables that affect the illiquidity premium.

Proposition 7 (Illiquidity premium): *Stocks' illiquidity premium is:*

$$PREM_{ILLIQ} = E\left(\widetilde{P}_{t+1} + v - RP_t\right) - E\left(\widetilde{P}_{t+1} + v - RP_t \middle| \sigma_\omega^2 = 0\right)$$

$$\frac{aS}{\left[\dfrac{M}{\sigma_P^2 + \sigma_\varepsilon^2} + \dfrac{K}{\rho\sigma_\varepsilon^2}\right]} - \frac{aS}{\left[\dfrac{M}{\sigma_\varepsilon^2} + \dfrac{K}{\rho\sigma_\varepsilon^2}\right]} = \frac{aS\sigma_\varepsilon^2\sigma_P^2}{\left[M\sigma_\varepsilon^2 + \left(\sigma_P^2 + \sigma_\varepsilon^2\right)\dfrac{K}{\rho}\right]\left[1 + \dfrac{K}{M\rho}\right]} > 0. \qquad (16)$$

Illiquidity premium is increasing in the supply of the asset, S, the parameter of risk aversion, a, the variability of dividends, σ_ε^2 *and the variability of investors' endowment shocks,* σ_ω^2. *The effect of the number of market makers, M, and the number of transitory investors, K, on illiquidity premium is ambiguous.*

Not surprisingly, apart from *K* and *M*, the determinants of illiquidity premium affect illiquidity premium and illiquidity in a similar direction. Also, in both cases, the effect of *K* is ambiguous.

Somewhat surprisingly, the effect of *M* on illiquidity premium is ambiguous. As long-term investors do not care about short-term price risk, there is no illiquidity premium in the limit where $M \to 0$. Similarly, in the limit where $M \to \infty$, there is no illiquidity premium as the markets are then perfectly liquid. For intermediate values of *M*, however, there is an illiquidity premium as the market makers hold smaller positions in an illiquid stock as compared to a liquid stock (due to the short-term price risk), which affects the equilibrium price through market clearing.[16]

6. Costs of immediacy and the returns from providing immediacy

In our model, investors typically earn higher returns in illiquid stocks, due to the illiquidity premium. It is interesting to note, however, that neither type of investors' expected returns equal the stocks' expected returns (15). This is due to the fact that long-term investors, the transitory investors, typically suffer a cost of immediacy when adjusting their positions, while the market makers typically earn excess returns from providing immediacy. The reason is the following. When transitory investors have large endowments, and thus a selling need, the price is typically below the expected price $\overline{P}$. In this case, the transitory investors sell stock below its long-term "fair price", while market makers make purchases at a favorable price. In turn, when the transitory investors have small endowments, and thus a buying need, the price is typically above the long-term "fair price", $\overline{P}$. In this case, the transitory investors buy stock above its "fair price", while market makers sell at this favorable price. As a result, the volume weighted average price that the transitory investors pay for their stock investments exceeds $\overline{P}$, and the volume weighted average price at which the transitory investors sell the stock is below $\overline{P}$, leading to costs of immediacy to those investors. In turn, the volume weighted average purchasing price of the market makers is below $\overline{P}$, and the average selling price of the market makers is above $\overline{P}$, leading to excess profits, which we call the returns from providing immediacy.

[16] Similar to this result, it is easy to show that holding the total number of investors $M + K/\rho$ fixed, the expected returns are a non-monotonic function of the proportion of myopic investors.

Let us define as the costs of immediacy, C_{IMM}, the losses that a period t transitory investor in expectation suffers from trading at unfavorable prices. At an aggregate level, these costs of immediacy correspond with the returns from providing immediacy to the market makers.

Proposition 8 (The costs of immediacy): *The expected periodic losses to a transitory investor from trading at a price different from* $\overline{P}$ *equal:*

$$C_{IMM} = \left| \frac{1}{K} E\left[M\left(\mu_t^d - \mu_t^e\right)\left(P_t - \overline{P}\right) \right] \right| = \left| -\frac{y}{K}\left(BK^2\sigma_\omega^2 - (1-B)M^2\sigma_{\mu_e}^2 \right) \right| \tag{17}$$

$$= \left| \frac{-yBK\sigma_\omega^2}{1+B} \right| > 0. \tag{18}$$

These expected costs of immediacy, C_{IMM}*, are increasing in the fundamental dividend risk,* σ_ε^2*, the variability of transitory investors' endowment shocks,* σ_ω^2*, and are non-monotonic in the parameter of risk aversion, a, the number of transitory investors, K, and the number of market makers, M. The costs of immediacy do not depend on the supply of the asset, S.*

We define a market maker's return from providing immediacy as $R_{IMM} = \frac{K}{M} C_{IMM}$.

Note that, from (17), the expected costs of immediacy to transitory investors comprise two components. The first component is the expected loss to a period t transitory investor due to the price impact that the period t transitory investors' collective endowment has. The second effect is, however, the opposite. It is a positive effect that comes from market makers' need to download their inventories to transitory investors at favorable prices. For instance, the transitory investors in period t are expected to gain if market makers in period t hold large inventories because in this case the large selling needs of the market makers, in expectation, offer the transitory investors an opportunity to make investments to the stock at a price which is below the stock's "fair value", $\overline{P}$. In this case, the roles are reversed and the transitory investors actually provide immediacy to the market makers.

Interestingly, the costs of immediacy are non-monotonic in the number of market makers, transitory investors and the parameter of risk aversion. First, note that the monetary costs of immediacy approach zero when $M \rightarrow 0$ or $M \rightarrow \infty$. In the limit where $M \rightarrow 0$, there is no trading between transitory investors and the market makers, while in the limit where $M \rightarrow \infty$, there are no costs of immediacy as the markets are perfectly liquid. Similarly, the *per capita* costs of immediacy approach zero when $K \rightarrow 0$ or $K \rightarrow \infty$ for the very same reasons: as $K \rightarrow 0$, markets become perfectly liquid, whereas when $K \rightarrow \infty$, the expected trading that each transitory investor conducts with the market makers approaches zero. Also, when a is small, markets are highly liquid, whereas when a increases, the trading volume between transitory investors and market makers eventually approaches zero. This is explained by the fact that for large parameters of risk aversion, the market makers become highly reluctant to take any positions due to the high level of short-term price risk.

Given Propositions 6-8, we have that the market makers' expected returns come from three different sources. First, a market maker earns on his average stock holdings the fair return from investing in a liquid stock, and, second, the illiquidity premium. In addition, he earns returns from providing immediacy to the transitory investors.

$$\text{Market maker's total return} = \left(\text{Fair return from a liquid stock} + \text{Illiquidity premium}\right) \times \text{Market maker's average stock holding} + \text{Returns from providing immediacy} \qquad (19)$$

7. Liquidity and the stock market performance

7.1. Liquidity, reversals and volatility

The main result of this section is that the speed of mean reversion and market liquidity are typically negatively related. Hence, in contrast to common perception, fast price recovery is likely to be a sign of inefficient, illiquid markets and not a sign of efficiency. [17]

Proposition 9 presents alternative measures for liquidity that are motivated by the previous propositions.

Proposition 9 (Empirical proxies for liquidity): *In the region where y is increasing in K, changes in all parameters that increase liquidity, i.e., decrease y, increase the persistence of price impact, B, the first-order autocorrelation of stock returns,* $\rho(1) < 0$*, and decrease the ratio of transitory to fundamental variance,* $\sigma_P^2 / \sigma_\varepsilon^2$.

[17] Our prediction that stocks' price reversal is faster for illiquid securities differs from the prediction in Duffie, Gârleanu and Pedersen (2007) related to over-the-counter traded securities. Their prediction is that in over-the-counter markets, reversals are more gradual for illiquid securities.

Proposition 9 is important as it may help us obtain better empirical measures of liquidity. It suggests that besides the three liquidity measures most commonly used (turnover, market capitalization and a measure of illiquidity presented in Amihud (2002)),[18] liquidity can potentially be estimated from the return autocorrelation, the speed of mean reversion and the ratio of transitory to fundamental volatility.[19]

7.2. Market segmentation and cross-sectional stock returns

Several recent papers, such as Gromb and Vayanos (2002), Vayanos and Vila (2008) and Greenwood and Vayanos (2010), argue that financial markets are partially segmented for various reasons. In our model, there is only a single risky asset. It is easy, however, to extend our model to the case of segmented financial markets and multiple securities. If we assume that the markets for all stocks are perfectly segmented, the equilibrium is the same as that described in Section 2 of this paper for every single security in the economy, with stock-specific structural parameters determining stock-specific levels of liquidity, expected return and return reversal patterns.

Under the assumption of perfect market segmentation, therefore, our model makes predictions that in cross-section we would observe differences in return reversal patterns, volatilities and stocks' expected returns so that, for instance, the expected returns are higher and return reversals are faster for illiquid stocks.

[18] According to Amihud (2002), a stock's liquidity can be estimated by comparing stocks' daily absolute returns with the corresponding days' trading volumes.

[19] It turns out that in our model, turnover and liquidity are not as closely connected as the variables mentioned in Proposition 9. For instance, it can be shown that while liquidity decreases in the variability of transitory investors' endowment shocks, expected trading volume may increase in this variable. Johnson (2008) shows also that liquidity and trading volume need not be positively related.

Although we do not develop such an extension of our model, it is plausible to think that in partially segmented markets, where the market makers observe all trading opportunities across assets, similar cross-sectional differences in reversal patterns, volatilities and expected returns would continue to exist, but to a lesser extent than in perfectly segmented markets.

8. Empirical findings

To provide empirical support for our theoretical model, we study how the amount and speed of return reversal, liquidity, returns from providing immediacy, illiquidity premium, transitory volatility and trading volume are related both in cross-section and in time series. As the focus of this paper is theoretical, we merely describe the variations in these endogenous variables across liquidity quartiles in cross-section and look at their correlation in time series. The estimation of the underlying structural parameters is left for future research.

8.1. Patterns of stock return reversal at the NYSE and Amex

Propositions 1 and 3 predict that there is mean reversion in stock returns, and that mean reversion takes an exponential shape. In addition, Proposition 4 suggests that several past days' returns affect expected future returns, but that the effect of more distant returns is smaller. In this section of the paper, we test these predictions, while interpreting the model so that one period corresponds to one day.[20]

Our sample includes all stocks listed in the daily CRSP file from the beginning of the database, 31st December 1925 to 31st December 2008, which fulfill the following requirements: 1) the

[20] We can motivate this by noting that this interval of time is sufficiently long for both new information and investors to arrive at the market in each period.

security is an ordinary common stock, 2) the company is incorporated in the US, 3) the stock is listed in the NYSE or the Amex, and 4) the company's SIC code is available and it is included in the Fama and French 48 industries, excluding the industry Other.

To study mean reversion patterns within a month, we perform regressions, where we regress stocks' Z-days' future excess returns, where $1 \leq Z \leq 20$, with each of its 20 past days' returns (there are roughly 20 trading days per month). We focus on mean reversion within a month, as most studies looking at short-term return reversals look at the reversals at either one-day, one-week or one-month horizons. More precisely, we perform for each day a cross-sectional regression, in which we regress the stocks' next Z-days' excess returns following day t, $RZ_{i,t}$, where i indexes firms on each of the stocks' past 20 days' excess returns, $R_{i,t-f}$, where $0 \leq f \leq 19$. The regression that we consider is thus:

$$RZ_{i,t} = \alpha_t + \sum_{f=0}^{19} \beta_{Z,f} R_{i,t-f} + \eta_{i,t} \tag{20}$$

where $\eta_{i,t}$ is the stock-specific error term.

In this paper, we calculate excess returns by deducting from stocks' returns the returns to a corresponding equal-weighted Fama and French 48 industry index. This method is chosen to minimize the noise in our estimates of short-term mean reversion.[21] [22]

[21] The definitions of the FF industries were obtained from Kenneth French's website: http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

[22] The approach is similar to that in Hameed and Huang (2010), who also define excess returns relative to industry indexes. Our results are highly similar if excess returns are calculated relative to a value-weighted industry index, or

Table 1A provides the average coefficients from the 21,985 daily cross-sectional regressions of (20), when Z varies between 1 and 20.[23]

[INSERT TABLE 1]

As Table 1 shows, all the coefficients from the regressions (20) are negative, indicating that there is mean reversion in our data. Furthermore, the coefficients are collectively both economically and statistically highly significant. The only coefficient that is not individually statistically significant at the 0.1% level is the coefficient for R_{t-19} in the regression for one-day future returns, $R1$. The cumulative 5-day mean reversion of past daily excess returns implied by the estimate for $\hat{\beta}_{5,0}$, as shown in Table 1, is 24%. Similarly, the cumulative 20-day mean reversion of past daily excess returns implied by Table 1 is 29%.[24] [25]

From Table 1, it can be seen that the return reversal pattern is close to an exponential pattern, as predicted in Proposition 3. This is also documented in Figure 1, which shows (using the estimates from Table 1) the percentage of 20-day reversal that occurs on each of the first ten days. As Figure 1 shows, on average, roughly 50% of the 20-day return reversal occurs on the

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an equal-weighted CRSP index as in Lehmann (1990), Lo and MacKinlay (1990), Khandani and Lo (2007, 2011) and Nagel (2011).

[23] These regressions include every stock in NYSE and Amex (which satisfy the previously stated criteria) where we observe both the 20-day return history and the 20-day future returns. In addition, we require that the stocks have not changed their Fama and French industry during the 40-day period.

[24] We take the observed mean reversion in Table 1 to be evidence of imperfect market liquidity, as our theory suggests, and likewise in Campbell *et al*. (1993), Pastor and Stambaugh (2003) and Nagel (2011), for example.

[25] In recent years, the level of mean reversion has been smaller. For instance, when using data from the last ten years only, the cumulative 5-day return reversal is only 11%.

first day in the future, and that another 34% of the reversal occurs gradually during the next four days, implying that 84% of the one-month return reversal occurs within the first week.[26]

[INSERT FIGURE 1]

Consistent with Proposition 4, as shown in Panel B of Figure 1, the autocorrelation estimates have also an exponentially declining pattern (in absolute value). Consistent with our theory, we thus find strong support for the exponential pattern of mean reversion. This is clear evidence that the interaction between short-term market makers and transitory investors is a major factor affecting the price formation in the stock market.

The observation that recent past returns have the greatest effect on future expected returns makes a great difference to an econometrician trying to forecast future returns. Our econometric model that explains monthly returns using all 20 past daily returns has an average adjusted R^2 of 6.7%. If we simply regress the next month's return on the past month's return, the average adjusted R^2 is only 1.6%. These findings are important when estimating the returns to short-term contrarian trading strategies, as we will do shortly.[27]

8.2 Defining other variables of interest

[26] Note that since we calculate our returns from closing prices, we view the bid-ask bounce to be part of the return reversal that is due to the price impact from trading.

[27] Using the past 20 daily returns instead of just one month's return to forecast stocks' future monthly return results in a statistically significant improvement of the model (at the 5% level) in 94% of the daily cross-sectional regressions.

Our model makes predictions related not only to short-term return reversal patterns but also to liquidity premium, transitory volatility and the returns from providing immediacy. We estimate these variables as follows:

First, one of our measures of illiquidity is the Amihud (2002) *ILLIQ*-measure. Second, following Acharya and Pedersen (2005), we use stock-specific estimates of *ILLIQ* to estimate the illiquidity premium, $PREM_{ILLIQ}$. To estimate the *percentage of transitory volatility*, we compare annualized volatility estimates based on daily and monthly returns (annualized using the standard square root rule).

Our first proxy for the available returns from providing immediacy, R_{IMM}, is the monthly returns to a zero-investment long-short trading strategy, where stocks are first daily ranked based on their one-week expected returns, and then, second, a long-short portfolio is formed with a long position (equally weighted) in the quartile of stocks with the highest expected 5-day returns and a short position (equally weighted) in the quartile of stocks with the smallest expected 5-day returns. After 5 days, such positions are closed. When forming the portfolios, the 5-day expected returns are evaluated using the regression coefficients from regression (20) – estimated with the past 6-months' data up to 6-days prior to taking positions – and the stocks' past 20-days' returns. In addition, we require that the stocks in the long and short portfolios have a positive trading volume on the day that positions are taken. We refer to this proxy for the returns from providing immediacy as $R_{IMM}(EqualW)$. When we study the returns from providing immediacy in cross-sectional subsamples, $R_{IMM}(EqualW)$ is calculated similarly, except that the long and the short portfolios now contain the top and the bottom four deciles of stocks ranked by their expected returns.[28]

We also consider a second proxy for the available returns for providing immediacy, where the strategy is otherwise similar to the previous one but – as in Khandani and Lo (2011) and Nagel (2011) – instead of equally weighting the top and bottom quartiles of stocks ranked by their expected returns, stocks' expected returns are used as portfolio weights when forming the long and the short portfolios. We refer to this second proxy for the returns from providing immediacy as $R_{IMM}(Exp.Ret.W)$.[29]

Finally, our annual estimates of B, the persistence of stocks' price deviations from the long-term average stock price, are obtained from the stocks' autocorrelation structure as follows. For any given sample of stocks for which we estimate B, we require that the stocks' annual estimates of B and C minimize the sum of the squared differences between the estimated annual average autocorrelation coefficients, $\hat{\rho}(1+f)$, and the predicted autocorrelation coefficients from Proposition 4, $\rho(1+f) = C \cdot B^{f}$, for all $0 \leq f \leq 19$.

8.3 Cross-sectional variations in liquidity and market performance

In Table 2, we document the cross-sectional variation in the time-series averages of several variables of interest across quartiles of stocks that are formed based on two measures of stocks'

[28] Otherwise, the number of stocks in the long and the short portfolios become very small in the early sample period.
[29] These approaches to estimating the returns from providing immediacy are highly similar to those used in Nagel (2011), Rinne and Suominen (2011) and Jylhä, Rinne and Suominen (2011). As all the above mentioned papers show, the monthly returns from this type of immediacy providing trading strategies are significant but time varying.

liquidity: firms' past year's market capitalization and their past year's average Amihud *ILLIQ*-measure.

[INSERT TABLE 2]

Clearly, the amount of mean reversion, as measured by the degree of first-order autocorrelation, or 5- or 20-day reversal, and the patterns of mean reversion differ across liquidity quartiles. In the case of more liquid stocks, the expected degree of return reversal is smaller and the speed of return reversal is more gradual. These results are consistent with our theory. The result that the return reversals are larger among illiquid stocks parallels the findings in Avramov, Chordia and Goyal (2006).

Table 2 also shows that the two measures of the returns from providing immediacy, $R_{IMM}(Exp.Ret.W)$ and $R_{IMM}(EqualW)$, as well as the raw stock returns, are higher for illiquid stocks. Our finding that the returns from providing immediacy are higher for small stocks parallels the findings in Khandani and Lo (2011), which studies the returns to a closely related contrarian trading strategy. The finding that the raw stock returns are higher for illiquid stocks is consistent with the illiquidity premium documented in Acharya and Pedersen (2005).

Finally, Table 2 shows how the average daily volatility and turnover depend on firms' market capitalization and the level of liquidity. The positive turnover difference between liquid and illiquid stocks is statistically significantly different from zero at the 1% level. In addition, both the stocks' estimated total volatility and the amount of transitory volatility decrease with stocks' liquidity. The differences between the various volatility estimates, between liquid and illiquid stocks, are statistically significant at the 0.1% level. The results related to transitory volatility match the predictions of our theory, under the assumption stated in Proposition 9. Note the significant amount of volatility that is transitory: According to our results, historically in illiquid stocks, 27% of volatility has been transitory.

To summarize, Table 2 shows that the degree of return reversal, mean reversion patterns, percentage of transitory volatility, the returns from providing immediacy and stocks' expected returns are all closely related cross-sectionally with the level of liquidity.

8.4 Time-series variation in liquidity and market performance

Table 3 shows the time-series correlations between the various liquidity measures suggested in Proposition 9, namely $\hat{\rho}(1)$, B and the percentage of transitory volatility, as well as the returns from providing immediacy, the amount of 5- and 20-day return reversal, the Amihud *ILLIQ*-measure, stocks' return volatility and turnover.

[INSERT TABLE 3]

Interestingly, all the measures of liquidity (or illiquidity) suggested in Proposition 9 are highly correlated with one another and the Amihud *ILLIQ*-measure (the correlation coefficients of these measures of liquidity and illiquidity are highlighted in Table 3).

There are several interesting findings in Table 3 related to our measures of liquidity (or illiquidity) and the Amihud *ILLIQ*-measure, however, which suggests that they measure somewhat different things. Note how the Amihud *ILLIQ*-measure is highly correlated with volatility, but that it has almost a zero-correlation with another common measure of liquidity, stock turnover. In contrast, our measures of liquidity correlate highly with both volatility and turnover. These findings are consistent with the idea that a large part of the time-series variation in the Amihud *ILLIQ*-measure comes from time variations in the amount of private information related trading in the market.[30] In contrast, our measures of liquidity are constructed using a theory where agents have no private information, suggesting that our measures of liquidity measure to a greater extent the non-information related time-variation in liquidity. The large correlations of our measures of liquidity with the average turnover suggest that non-information related liquidity is a major factor affecting the trading activity in the stock market. Finally, related to these findings, it is interesting to note that while volume and volatility are known to be positively correlated (See, e.g., Karpoff, 1987), the percentage of transitory volatility and volume have a strong negative correlation with each other. Note also that, according to Table 3, the contemporaneous correlation between illiquidity premium and the percentage of transitory volatility is negative. This finding may be related to the idea that, as argued by Acharya and Pedersen (2005), an increase in liquidity results in positive returns on a portfolio exposed to illiquidity premium as it decreases the future illiquidity premium.

Figure 2 presents graphically the time series of the key variables of interest. There is no clear time-trend in the illiquidity premium. In turn, the fact that the amounts of 1-, 5- and 20-day return reversal, the percentage of transitory volatility and the returns from providing immediacy

[30] Indeed, the Amihud-measure is commonly linked with Kyle's λ, as defined in Kyle (1985).

decline, and the persistence of the price impact increases over time all suggest that the level of liquidity has increased over time.

[INSERT FIGURE 2]

9. Conclusion

We presented a structural model of the stock market that is based on the assumption that some investors are rarely present in the stock market. Our model predicts an exponential pattern of return reversal, and that the degree of return reversal, the speed of return reversal and the stock's return volatility depend on the same factors that determine the stock's liquidity.

Our model generates predictions related to illiquidity premium in stock returns as well as the costs of immediacy (returns from providing immediacy). Interestingly, both of these are non-monotonic in several structural parameters of the model including, for instance, the number of market makers. Thus, they do not vary one to one with the level of liquidity. Finally, our theory proposes some new empirical proxies for liquidity. According to our results, liquidity is closely related to autocorrelation in returns, the speed of return reversals and the ratio of transitory to fundamental variance.

Our empirical estimates, using the fully available history for NYSE and Amex stocks, suggest that 24% of stocks' excess returns revert within a week. Our empirical estimates strongly support the prediction that return reversals follow an exponential pattern. We also show that, in accordance with our theory, for illiquid stocks the degree of return reversal is larger and the speed of return reversal is faster. Finally, we show that in line with our theory there is a great deal of transitory volatility, whose amount depends on the level of liquidity.

As the relations between many of the endogenous variables of interest, such as liquidity, returns from providing immediacy and illiquidity premium, need not be monotonic according to our model, we feel that there is a need for much more empirical analysis on the relationships between these variables. We hope that our theoretical model can guide such future empirical work as it is one of the few models where these phenomena co-exist and can be characterized explicitly. We believe that that it is crucial for the literature to keep developing models such as ours, where illiquidity premium and the returns from providing immediacy co-exist to support the voluminous empirical research on investment and trading returns related to liquidity.

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Appendix

Proof of Proposition 1: Equations (8), (9) and (10) imply that

$$M\left(\frac{E_t(\widetilde{P}_{t+1})+v-RP_t}{a\left(\sigma^2_{P_{t+1}}+\sigma^2_\varepsilon\right)}\right)+K\left(\frac{v}{a\sigma^2_\varepsilon}-\frac{r}{a\sigma^2_\varepsilon}\left(\frac{R^{T-t}}{R^{T-t}-1}\right)P_t\right)=M\mu^e_t+K\omega^e_t. \tag{A1}$$

Equation (A1), in turn, implies that the market clearing price in period t is:

$$P_t=\frac{\dfrac{M(E_t(\widetilde{P}_{t+1})+v)}{a\left(\sigma^2_{P_{t+1}}+\sigma^2_\varepsilon\right)}+\dfrac{Kv}{a\sigma^2_\varepsilon}-(M\mu^e_t+K\omega^e_t)}{\left[\dfrac{MR}{a\left(\sigma^2_{P_{t+1}}+\sigma^2_\varepsilon\right)}+\dfrac{Kr}{a\sigma^2_\varepsilon}\left(\dfrac{R^{T-t}}{R^{T-t}-1}\right)\right]}. \tag{A2}$$

Taking the limit of (A2), when $T \rightarrow \infty$, we obtain:

$$P_t=\frac{\dfrac{ME_t(\widetilde{P}_{t+1}+v)}{a\left(\sigma^2_{P_{t+1}}+\sigma^2_\varepsilon\right)}}{\left[\dfrac{MR}{a\left(\sigma^2_{P_{t+1}}+\sigma^2_\varepsilon\right)}+\dfrac{Kr}{a\sigma^2_\varepsilon}\right]}+\frac{\dfrac{Kv}{a\sigma^2_\varepsilon}}{\left[\dfrac{MR}{a\left(\sigma^2_{P_{t+1}}+\sigma^2_\varepsilon\right)}+\dfrac{Kr}{a\sigma^2_\varepsilon}\right]}-\frac{M\mu^e_t+K\omega^e_t}{\left[\dfrac{MR}{a\left(\sigma^2_{P_{t+1}}+\sigma^2_\varepsilon\right)}+\dfrac{Kr}{a\sigma^2_\varepsilon}\right]}. \tag{A3}$$

Next, we conjecture, and then verify, that there exists an equilibrium price, which satisfies (A3), that is linear in $\left(M\mu^e_t+K\omega^e_t\right)$, in particular:

$$P_t=\overline{P}_t+y_t\left(M\overline{\mu}+K\overline{\omega}\right)-y_t\left(M\mu^e_t+K\omega^e_t\right),\ \text{for } t\leq T\text{-}1, \tag{A4}$$

for some $\overline{P}_t$ and y_t, given $\overline{\mu}$ (which is a constant to be defined later). Note that in period *T-1*, $E_{T-1}(\widetilde{P}_T) = 0$, implying, given (A3), that P_{T-1} is linear in $M\mu^e_{T-1} + K\omega^e_{T-1}$. Thus, in period *T-2*, a linear price exists such that $E_{T-2}(\widetilde{P}_{T-1})$ and P_{T-2} are linear in $M\mu^e_{T-2} + K\omega^e_{T-2}$, since, given (9), in this case $E_{T-2}(M\mu^e_{T-1} + K\omega^e_{T-1}) = M\mu^d_{T-2} + K\overline{\omega}$ is linear in $M\mu^e_{T-2} + K\omega^e_{T-2}$. Similarly, for any arbitrary period $t < T\text{-}2$. This confirms our conjecture. In addition, using the same kind of arguments, we can confirm that the price P_{t+1} is normally distributed, given Ω_t, with a variance $\sigma^2_{P_{t+1}}$ that is independent of the price P_t.

We now conjecture, and verify below, that in the limit where $T \rightarrow \infty$, there exists $\overline{P}_t$ and y_t that are time independent. Furthermore, we define $\overline{\mu}$ as the market makers' expected stock holding in any given period t in this limiting economy. In this case, $\sigma^2_{P_{t+1}}$ is also time independent and can be denoted by σ^2_P. Let us now try to solve for such $\overline{P}$, y and $\overline{\mu}$. If we find such $\overline{P}$, y and $\overline{\mu}$, this verifies our conjecture. Equation (A4) can now be written as:

$$P_t = \overline{P} - y\left(M\left(\mu^e_t - \overline{\mu}\right) + K\left(\omega^e_t - \overline{\omega}\right)\right). \tag{A5}$$

Now, given (9),

$$\mu^e_{t+1} = \mu^d_t = \frac{E_t(\widetilde{P}_{t+1}) + v - RP_t}{a\left(\sigma^2_P + \sigma^2_\varepsilon\right)}, \tag{A6}$$

which implies, given (A5), that

$$E_t\tilde{P}_{t+1} = \overline{P} - yM\left(\frac{E_t(\tilde{P}_{t+1}) + v - RP_t}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)} - \overline{\mu}\right). \tag{A7}$$

When $\mu_t^e = \overline{\mu}$ and $\omega_t^e = \overline{\varpi}$ we should expect that $\mu_{t+1}^e = \overline{\mu}$, and that $E_t(\tilde{P}_{t+1}) = P_t = \overline{P}$. Under that assumption, (A6) implies that

$$\overline{\mu} = \frac{v - r\overline{P}}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)}. \tag{A8}$$

Second, given (8), in the limit where $T \to \infty$, the transitory investors' demand, given price $\overline{P}$, is:

$$\omega^d(\overline{P}) = \frac{v - r\overline{P}}{a\sigma_\varepsilon^2}. \tag{A9}$$

Market clearing (10) then gives for $\mu_t^e = \overline{\mu}$ and $\omega_t^e = \overline{\varpi}$:

$$\overline{P} = \frac{1}{r}\left(v - \frac{a\left(M\overline{\mu} + K\overline{\varpi}\right)}{\left[\frac{M}{\sigma_P^2 + \sigma_\varepsilon^2} + \frac{K}{\sigma_\varepsilon^2}\right]}\right). \tag{A10}$$

Let us assume (A10) holds. Using (A8) in (A7), we obtain:

$$E_t\widetilde{P}_{t+1} = \overline{P} - yM\left(\frac{E_t(\widetilde{P}_{t+1}) + v - RP_t}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)} - \frac{v - r\overline{P}}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)}\right) \tag{A11}$$

$$= \frac{\overline{P} + \dfrac{yM}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)}\left(RP_t - r\overline{P}\right)}{1 + \dfrac{yM}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)}} = \overline{P}\left[\frac{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right) - yMr}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right) + yM}\right] + \frac{yMRP_t}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right) + yM}. \tag{A12}$$

Next, using this in (A3), we obtain:

$$P_t\left[\frac{MR}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)} + \frac{Kr}{a\sigma_\varepsilon^2}\right] = \frac{ME_t(\widetilde{P}_{t+1} + v)}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)} + \frac{Kv}{a\sigma_\varepsilon^2} - \left(M\mu_t^e + K\omega_t^e\right) \tag{A13}$$

$$= \frac{M}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)}\left(\overline{P}\left[\frac{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right) - yMr}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right) + yM}\right] + \frac{yMRP_t}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right) + yM}\right) + \frac{Mv}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)} + \frac{Kv}{a\sigma_\varepsilon^2} - \left(M\mu_t^e + K\omega_t^e\right).$$

=>

$$P_t\left[\frac{MR}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right) + yM} + \frac{Kr}{a\sigma_\varepsilon^2}\right] =$$

$$= \left(\frac{M}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)}\overline{P}\left[\frac{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right) - yMr}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right) + yM}\right] + \frac{Mv}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)} + \frac{Kv}{a\sigma_\varepsilon^2} - M\overline{\mu} - K\overline{\omega}\right) - \left(M\left(\mu_t^e - \overline{\mu}\right) + K\left(\omega_t^e - \overline{\omega}\right)\right). \tag{A14}$$

Now, using (A8) and (A10), we obtain:

$$P_t = \overline{P} - \frac{\left(M\left(\mu_t^e - \overline{\mu}\right) + K\left(\omega_t^e - \overline{\varpi}\right)\right)}{\left[\frac{MR}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right) + yM} + \frac{Kr}{a\sigma_\varepsilon^2}\right]}. \tag{A15}$$

Now, we can verify our conjecture, as there exists a $y > 0$ that solves:

$$y = \frac{1}{\left[\frac{MR}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right) + yM} + \frac{Kr}{a\sigma_\varepsilon^2}\right]} = \frac{1}{\left[\frac{R}{\frac{a\left(y^2K^2\sigma_\omega^2 + \sigma_\varepsilon^2\right)}{M} + y} + \frac{Kr}{a\sigma_\varepsilon^2}\right]}. \tag{A16}$$

As then:

$$\sigma_P^2 = y^2K^2\sigma_\omega^2 \;. \tag{A17}$$

Other statements regarding conditional and unconditional means and variances follow directly from equations (A6), (A7), (A16) and (A9) defining $\sigma_\mu^2 = E\left(\mu_t^e - \overline{\mu}\right)^2$.

Proof of Proposition 2: The comparative statics results with respect to other variables besides K can now be confirmed from (A16). It is also easy to confirm that the effect of K on y is ambiguous (by considering the extreme cases where $\sigma_\omega^2 = 0$ or σ_ω^2 is very large). Other statements regarding conditional and unconditional means and variances follow directly from equations (A5), (A6) and (A7), defining $\sigma_\mu^2 = E\left(\mu_t^e - \overline{\mu}\right)^2$.

Proof of Proposition 3: Define

$$B = \frac{RMy}{My + a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)}, \tag{A18}$$

so that:

$$1 - B = \frac{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right) - rMy}{My + a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)}. \tag{A19}$$

Using (A12), (A18) and (A19), we have $E_t \widetilde{P}_{t+1} = \overline{P}(1 - B) + BP_t$, so that, given (A15) and (A16),

$$\begin{aligned} &E_t\left(\widetilde{P}_{t+1}\right) - P_t = (1 - B)\left(\overline{P} - P_t\right) \\ &= (1 - B)y\left[M\left(\mu_t^e - \overline{\mu}\right) + K\left(\omega_t^e - \overline{\omega}\right)\right] \end{aligned} \tag{A20}$$

Using (A16) and (A18), we can show that 0 < B < *1*. Next, using (A20), we obtain that for $\tau > 1$,

$$E_t\left[P_{t+\tau+1} - P_{t+\tau}\right] = (1 - B)yE_t M\left(\mu_{t+\tau}^e - \overline{\mu}\right). \tag{A21}$$

We thus need to estimate $E_t M\left(\mu_{t+\tau}^e - \overline{\mu}\right)$. Using (A6) and (A8) to estimate

$$\mu_{t+1}^e - \overline{\mu} = \frac{E_t(\widetilde{P}_{t+1}) - RP_t + r\overline{P}}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)} \tag{A22}$$

we obtain, using (A5), (A20) and (A22):

$$M\left(\mu_{t+1}^{e}-\overline{\mu}\right)=M\frac{E_t(\widetilde{P}_{t+1})-RP_t+r\overline{P}}{a\left(\sigma_P^2+\sigma_\varepsilon^2\right)}$$

$$=M\frac{(1-B)y\left[M\left(\mu_t^e-\overline{\mu}\right)+K\left(\omega_t^e-\overline{\omega}\right)\right]+r\left(\overline{P}-P_t\right)}{a\left(\sigma_P^2+\sigma_\varepsilon^2\right)}$$

$$=B\left[M\left(\mu_t^e-\overline{\mu}\right)+K\left(\omega_t^e-\overline{\omega}\right)\right]. \tag{A23}$$

Now,

$$E_tM\left(\mu_{t+2}^e-\overline{\mu}\right)=E_t\left(M\left(\mu_{t+2}^e-\overline{\mu}\right)\middle|\mu_{t+1}^e,\omega_t^e\right)$$
$$=E_t\left(B\left[M\left(\mu_{t+1}^e-\overline{\mu}\right)+K\left(\omega_{t+1}^e-\overline{\omega}\right)\right]\middle|\omega_t^e\right)=B^2\left[M\left(\mu_t^e-\overline{\mu}\right)+K\left(\omega_t^e-\overline{\omega}\right)\right] \tag{A24}$$

Similarly, for all $\tau \geq 2$,

$$E_tM\left(\mu_{t+\tau}^e-\overline{\mu}\right)=B^\tau\left[M\left(\mu_t^e-\overline{\mu}\right)+K\left(\omega_t^e-\overline{\omega}\right)\right] \tag{A25 }$$

We obtain:

$$E_t\left[P_{t+\tau+1}-P_{t+\tau}\right]=(1-B)yB^\tau\left[M\left(\mu_t^e-\overline{\mu}\right)+K\left(\omega_t^e-\overline{\omega}\right)\right], \tag{A26}$$

implying that A in Proposition 3 is given by $A=(1\text{-}B)y$.

Results regarding comparative statics with respect to M now follow from Proposition 1 and the comparative statistics of B. We can rewrite the expression for y, (A16), as:

$$1 - B = \frac{Kry}{a\sigma_\varepsilon^2}. \tag{A27}$$

First, note that given (A27) and our earlier comparative statics results for *y* in Proposition 1, *B* is increasing in *M*. Second, from (A27), we see that *B* is decreasing in *a* if *y/a* is increasing in *a*. This, in turn, holds as, given (A16):

$$(y/a) = \frac{1}{\left[\frac{MR}{\left(\sigma_P^2 + \sigma_\varepsilon^2\right) + M\left(y/a\right)} + \frac{Kr}{\sigma_\varepsilon^2}\right]}, \tag{A28}$$

as, given Proposition 1, $\sigma_P^2 = y^2 K^2 \sigma_\omega^2$ is increasing in *a.* Using the same line of proof, it is also easy to see that *B* is decreasing in σ_ε^2 and *K*. First, the result regarding σ_ε^2 follows as:

$$(y/\sigma_\varepsilon^2) = \frac{1}{\left[\frac{MR}{a\left(K^2 y\sigma_\omega^2 (y/\sigma_\varepsilon^2) + 1\right) + M\left(y/\sigma_\varepsilon^2\right)} + \frac{Kr}{a}\right]} \tag{A29}$$

is increasing in σ_ε^2 as *y* is increasing in σ_ε^2. The result related to *K*, in turn, follows as:

$$(Ky) = \frac{1}{\left[\frac{MR}{Ka\left(y^2 K^2 \sigma_\omega^2 + \sigma_\varepsilon^2\right) + M\left(Ky\right)} + \frac{r}{a\sigma_\varepsilon^2}\right]} \tag{A30}$$

is increasing in *K*. The result regarding σ_ω^2 follows from (A27) given the previous result that *y* is increasing in σ_ω^2.

Finally, note that:

$$\sum_{\tau=0}^{\infty}(1-B)(B)^{\tau}=1. \tag{A31}$$

Also, when $M = 0$, $E_t\left(M\mu_{t+1}^e\right)=0$, implying expected full price recovery in the very next period.

Proof of Proposition 4: Using (1), (11) and (A20), we can write:

$$(P_{t-f}+D_{t-f}-P_{t-f-1})=v+(1-B)y\left(M\left(\mu_{t-f-1}^e-\overline{\mu}\right)+K\left(\omega_{t-f-1}^e-\overline{\omega}\right)\right)+\varepsilon_{t-f}-yK\left(\omega_{t-f}^e-\overline{\omega}\right). \tag{A32}$$

Direct calculation, given (A32), and noting that future dividend and endowment shocks are not correlated with past shocks, now gives:

$$COV\left[(P_{t+1}+D_{t+1}-P_t),(P_{t-f}+D_{t-f}-P_{t-f-1})\right]= \tag{A33}$$

$$=(1-B)yCOV\left(M\left(\mu_t^e-\overline{\mu}\right)+K\left(\omega_t^e-\overline{\omega}\right),(1-B)y\left(M\left(\mu_{t-f-1}^e-\overline{\mu}\right)+K\left(\omega_{t-f-1}^e-\overline{\omega}\right)\right)-yK\left(\omega_{t-f}^e-\overline{\omega}\right)\right).$$

Now, using (A23), making successive substitutions, we obtain:

$$M\left(\mu_t^e-\overline{\mu}\right)+K\left(\omega_t^e-\overline{\omega}\right)=K\left(\omega_t^e-\overline{\omega}\right)+B\left(M\left(\mu_{t-1}^e-\overline{\mu}\right)+K\left(\omega_{t-1}^e-\overline{\omega}\right)\right)$$

$$=\sum_{j=0}^{f} B^{j}\left(K\left(\omega_{t-j}^{e}-\overline{\omega}\right)\right)+B^{f+1}\left(M\left(\mu_{t-f-1}^{e}-\overline{\mu}\right)+K\left(\omega_{t-f-1}^{e}-\overline{\omega}\right)\right), \tag{A34}$$

so that autocorrelation at lag (1+*f*) is:

$$\rho(1+f)=\frac{E_{t}COV\left[(P_{t+1}+D_{t+1}-P_{t}),(P_{t-f}+D_{t-f}-P_{t-f-1})\right]}{\sqrt{VAR(P_{t-f}+D_{t-f}-P_{t-f-1})}\sqrt{VAR(P_{t+1}+D_{t+1}-P_{t})}}$$

$$=(1-B)yB^{f}\,\frac{\left((1-B)yB\left(M^{2}\sigma_{\mu_e}^{2}+K^{2}\sigma_{\omega}^{2}\right)-yK^{2}\sigma_{\omega}^{2}\right)}{(1-B)^{2}y^{2}\left(M^{2}\sigma_{\mu_e}^{2}+K^{2}\sigma_{\omega}^{2}\right)+y^{2}K^{2}\sigma_{\omega}^{2}+\sigma_{\varepsilon}^{2}}, \tag{A35}$$

where, given (A34), letting $f=(T+t-1)$,

$$M^{2}\sigma_{\mu_e}^{2}=E(M^{2}\sigma_{\mu_e}^{2})=\sum_{\tau=-T}^{t-1}\left(B^{2}\right)^{t-\tau}K^{2}\sigma_{\omega}^{2}. \tag{A36}$$

In the limit where $T\rightarrow\infty$, this equals:

$$\lim_{T\rightarrow\infty} M^{2}\sigma_{\mu_e}^{2}=\left(\frac{B^{2}}{1-B^{2}}\right)K^{2}\sigma_{\omega}^{2}. \tag{A37}$$

Substituting (A37) into (A35) we find that *C* is negative as (A35) is negative when:

$$(1-B)B\left[\left(\frac{B^{2}}{1-B^{2}}\right)+1\right]<1, \tag{A38}$$

which holds for all $B < 1$. This, in turn, is implied by (A18) and (A16).

Using (A37) and the result that $\sigma_P^2 = y^2 K^2 \sigma_\omega^2$, one day return autocorrelation, in the limit where $T \to \infty$, is:

$$
\begin{aligned}
\rho(1) &= \frac{E_t COV\left[(P_{t+1} + D_{t+1} - P_t),(P_t + D_t - P_{t-1})\right]}{\sqrt{VAR(P_t + D_t - P_{t-1})}\sqrt{VAR(P_{t+1} + D_{t+1} - P_t)}} \\
&= y(1-B)\frac{\left(y(1-B)B\left(M^2\sigma_{\mu_e}^2 + K^2\sigma_\omega^2\right) - yK^2\sigma_\omega^2\right)}{\left(y(1-B)\right)^2\left(M^2\sigma_{\mu_e}^2 + K^2\sigma_\omega^2\right) + y^2K^2\sigma_\omega^2 + \sigma_\varepsilon^2} \\
&= \frac{\left(-\left(\frac{1-B}{1+B}\right)\sigma_P^2\right)}{\left(\frac{1-B}{1+B}\right)\sigma_P^2 + \sigma_P^2 + \sigma_\varepsilon^2} = \frac{-1}{1 + \frac{(1+B)}{(1-B)}\left(1 + \frac{\sigma_\varepsilon^2}{\sigma_P^2}\right)} \\
&= \frac{-1}{1 + \frac{(1+B)}{(1-B)}\left(1 + \left(\frac{1}{1-B}\right)^2 \frac{r^2}{a^2\sigma_\omega^2\sigma_\varepsilon^2}\right)} < 0. \qquad \text{(A39)}
\end{aligned}
$$

The comparative statics results can now be confirmed using the earlier comparative statics results for B.

Proof of Proposition 5: Using previous results, we have that the stock return is:

$$
\begin{aligned}
R_t &= P_t + v + \varepsilon_t - P_{t-1} \\
&= v + \varepsilon_t - y\left[M\left(\mu_t^e - \bar{\mu}\right) + K\left(\omega_t^e - \bar{\omega}\right)\right] + y\left[M\left(\mu_{t-1}^e - \bar{\mu}\right) + K\left(\omega_{t-1}^e - \bar{\omega}\right)\right]
\end{aligned} \qquad \text{(A40)}
$$

The volatility of daily returns is thus, given (A40) and (A34):

$$\sigma_D = \sqrt{\sigma_\varepsilon^2 + 2y^2\left(M^2\sigma_{\mu_e}^2 + K^2\sigma_\omega^2\right) - 2y^2\, cov\left(M\left(\mu_t^e - \overline{\mu}\right), M\left(\mu_{t-1}^e - \overline{\mu}\right) + K\left(\omega_{t-1}^e - \overline{\omega}\right)\right)}$$

$$= \sqrt{\sigma_\varepsilon^2 + 2(1-B)y^2\left(M^2\sigma_{\mu_e}^2 + K^2\sigma_\omega^2\right)} \qquad \text{(A41)}$$

$$= \sqrt{\sigma_\varepsilon^2 + \frac{2}{1+B}y^2K^2\sigma_\omega^2} = \sqrt{\sigma_\varepsilon^2 + \frac{2}{1+B}\sigma_P^2}$$

in the limit where T → ∞, given our previous results.

The sum of $Z \geq 1$ consecutive past days' returns is:

$$\sum_{\tau=0}^{Z-1}(P_{t-\tau} + D_{t-\tau} - P_{t-\tau-1}) = P_t + \sum_{\tau=0}^{Z-1} D_{t-\tau} - P_{t-Z}$$
$$= Zv + \sum_{\tau=0}^{Z-1}\varepsilon_{t-\tau} - y\left[M\left(\mu_t^e - \overline{\mu}\right) + K\left(\omega_t^e - \overline{\omega}\right)\right] + y\left[M\left(\mu_{t-Z}^e - \overline{\mu}\right) + K\left(\omega_{t-Z}^e - \overline{\omega}\right)\right]. \qquad \text{(A42)}$$

Hence, $Z > 1$ days' return volatility satisfies, given (A34):

$$\sigma_Z = \sqrt{Z\sigma_\varepsilon^2 + 2y^2\left(M^2\sigma_{\mu_e}^2 + K^2\sigma_\omega^2\right) - 2y^2\, cov\left[M\left(\mu_t^e - \mu\right), M\left(\mu_{t-Z}^e - \overline{\mu}\right) + K\left(\omega_{t-Z}^e - \overline{\omega}\right)\right]}$$

$$= \sqrt{Z\sigma_\varepsilon^2 + 2\left(1-B^Z\right)y^2\left(M^2\sigma_{\mu_e}^2 + K^2\sigma_\omega^2\right)} \qquad \text{(A43)}$$

$$= \sqrt{Z\sigma_\varepsilon^2 + \frac{2\left(1-B^Z\right)}{1-B^2}\sigma_P^2} < \sqrt{Z\sigma_\varepsilon^2 + 2Z\frac{1-B}{1-B^2}\sigma_P^2} = \sqrt{Z}\sigma_D^2,$$

implying that $\sigma_Z < \sqrt{Z}\sigma_D$ $\forall Z > 1$. In the limit as $y \rightarrow 0$, $\sigma_Z - \sqrt{Z}\sigma_D \rightarrow 0$ and σ_Z approaches $\sqrt{Z}\sigma_\varepsilon$ from above. This occurs as $y \rightarrow 0$ implies that $B \rightarrow 1$, as given (A27) we have $\left[B = 1 - \frac{Kry}{a\sigma_\varepsilon^2}\right]$, and:

$$\frac{\sigma_p^2}{1-B^2} = \frac{y^2K^2\sigma_\omega^2}{1-\left[\frac{R}{\frac{a\left(y^2K^2\sigma_\omega^2+\sigma_\varepsilon^2\right)}{My}+1}\right]^2} = \frac{K^2\sigma_\omega^2}{1/y^2 - \left[\frac{R}{\frac{a\left(y^2K^2\sigma_\omega^2+\sigma_\varepsilon^2\right)}{M}+y}\right]^2} \rightarrow 0 \cdot \tag{A44}$$

Proof of Proposition 6: Denote by s_t the periodic supply of the asset: $s_t = \left(M\mu_t^e + K\omega_t^e\right)$. Market clearing implies, using (A6) and the limit of (8) as $T \rightarrow \infty$, that

$$M\frac{E_t(\widetilde{P}_{t+1}) + v - RP_t}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)} + K\frac{v - rP_t}{a\sigma_\varepsilon^2} = s_t \tag{A45}$$

Taking expectations gives:

$$\left[v - r\overline{P}\right]\left(\frac{M}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)} + \frac{K}{a\sigma_\varepsilon^2}\right) = E(s_t). \tag{A46}$$

Now, expected periodic supply equals the total supply of the asset, minus the expected holdings of the transitory investors who are absent from the market. Under assumption (A1) this implies:

$$E(s_t) = S - \frac{K(1-\rho)}{\rho}\frac{\left[v - r\overline{P}\right]}{a\sigma_\varepsilon^2}. \tag{A47}$$

Using (A46) and (A47) we obtain:

$$\left[v - r\overline{P}\right]\left(\frac{M}{a\left(\sigma_P^2 + \sigma_\varepsilon^2\right)} + \frac{K}{\rho a \sigma_\varepsilon^2}\right) = S. \tag{A48}$$

which gives the result that

$$E\left[E_t(\widetilde{P}_{t+1}) + v - RP_t\right] = v - r\overline{P} = \frac{aS}{\left[\frac{M}{\sigma_P^2 + \sigma_\varepsilon^2} + \frac{K}{\rho\sigma_\varepsilon^2}\right]} = \frac{aS}{\left[\frac{M}{y^2K^2\sigma_\omega^2 + \sigma_\varepsilon^2} + \frac{K}{\rho\sigma_\varepsilon^2}\right]}. \tag{A49}$$

Given the results stated in Proposition 1, this is increasing in σ_ε^2, *a, S,* σ_ω^2. It is decreasing in *M*. It is easy to verify that the effect of *K* on expected returns is ambiguous (by considering the extreme cases where $\sigma_\omega^2 = 0$ or σ_ω^2 is very large).

Proof of Proposition 7: Using (A49) we obtain:

$$PREM_{ILLIQ} = \frac{aS}{\left[\frac{M}{\sigma_P^2+\sigma_\varepsilon^2}+\frac{K}{\rho\sigma_\varepsilon^2}\right]} - \frac{aS}{\left[\frac{M}{\sigma_\varepsilon^2}+\frac{K}{\rho\sigma_\varepsilon^2}\right]} = \frac{aS\sigma_\varepsilon^2\sigma_P^2}{\left[M\sigma_\varepsilon^2+\left(\sigma_P^2+\sigma_\varepsilon^2\right)\frac{K}{\rho}\right]\left[1+\frac{K}{M\rho}\right]}. \tag{A50}$$

Proposition 1 states that y and σ_P^2 are increasing in a, σ_ω^2 and σ_ε^2. Given this result, it is easy to verify that the liquidity premium is increasing in σ_ε^2, a, S and σ_ω^2. The fact that $PREM_{ILLIQ}$ is non-monotonic in M and K can be verified by noting that $PREM_{ILLIQ}$ approaches zero in the limit when either M or K approaches zero or infinity.

Proof of Proposition 8: The "fair trading price" in period t is $\overline{P}$. In the proof below, we use (A34) and (A37). The expected losses to a period t transitory investor from trading at unfair prices are defined as

$$\mathrm{E}\ \frac{1}{K}\left[M\left(\mu_t^d-\mu_t^e\right)\left(P_t-\overline{P}\right)\right]$$

$$=\frac{1}{K}E\left[M\left(\mu_{t+1}^e-\overline{\mu}\right)-M\left(\mu_t^e-\overline{\mu}\right)\right]\left(P_t-\overline{P}\right)$$

$$=\frac{1}{K}E\left[B\left(M\left(\mu_t^e-\overline{\mu}\right)+K\left(\omega_t^e-\overline{\omega}\right)\right)-M\left(\mu_t^e-\overline{\mu}\right)\right]\left(P_t-\overline{P}\right)$$

$$=\frac{1}{K}E-\left[BK\left(\omega_t^e-\overline{\omega}\right)-(1-B)M\left(\mu_t^e-\overline{\mu}\right)\right]y\left(M\left[\mu_t^e-\overline{\mu}\right]+K\left[\omega_t^e-\overline{\omega}\right]\right)$$

$$=-\frac{y}{K}\left(BK^2\sigma_\omega^2-(1-B)M^2\sigma_{\mu_e}^2\right)=\frac{-yBK\sigma_\omega^2}{1+B}=\frac{-\left(B-B^2\right)a\sigma_\varepsilon^2\sigma_\omega^2}{(1+B)r}<0. \tag{A51}$$

In the last step, we used (A27). The fact that this is non-monotonic in *M* and *K* can be confirmed by taking the limits of (A51) with respect to these variables as they approach zero and infinity (noting that given (A30) the limit of *yK* when *K* approaches infinity is $a\sigma_\varepsilon^2 / r$, implying given (A27) that in the limit as *K*→∞, *B*→*0*). The fact that this is increasing in σ_ε^2 can be seen given Proposition 3, by noting that

$$By = \frac{R}{1/y + \frac{a\left(y^2K^2\sigma_\omega^2 + \sigma_\varepsilon^2\right)}{My^2}} = \frac{R}{1/y + \frac{a\left(K^2\sigma_\omega^2\right)}{M} + \frac{a\left(\sigma_\varepsilon^2\right)}{My^2}}, \tag{A52}$$

is increasing in σ_ε^2 given Proposition 2 and (A29). Similarly

$$By\sigma_\omega^2 = \frac{R}{\frac{1}{y\sigma_\omega^2} + \frac{a\left(K^2\right)}{M} + \frac{a\left(\sigma_\varepsilon^2\right)}{My^2\sigma_\omega^2}}, \tag{A53}$$

is increasing in σ_ω^2. Finally note that the limit of

$$By = \frac{R}{\frac{1}{y} + \frac{a\left(K^2\sigma_\omega^2\right)}{M} + \frac{a\left(\sigma_\varepsilon^2\right)}{My^2}}, \tag{A54}$$

is zero either as *a* goes to zero or infinity. The intuition for this last result, that the costs of immediacy approach zero as *a*→∞, is that expected trading volume declines and eventually approaches zero as the parameter of risk aversion increases.

Proof of Proposition 9: The claim related to *B* follows immediately from Propositions 2 and 3. The claim related to the volatility ratio follows from Proposition 2, Equation (A29) and the fact that $\sigma_p^2/\sigma_\varepsilon^2 = y^2 K^2 \sigma_\omega^2 / \sigma_\varepsilon^2$. The result regarding the first order autocorrelation can be verified from A(39), given the comparative statics results related to *B* in Proposition 3.

Table 1: Patterns of mean reversion at NYSE and Amex

This table shows the average coefficients, $\hat{\beta}_{Z,f}$ from 21,985 daily cross-sectional regressions (regression (20) in the text) where stocks' *Z*-day future excess returns, *RZ*, where Z ≤ 20, are regressed on each of the stocks' past twenty days' excess returns, R_{t-f}, where $f \in \{0,1,2,3,\ldots,19\}$. The excess returns are calculated relative to the corresponding equal-weighted Fama-French 48 industry index returns. Statistical significance is calculated using Fama-Macbeth standard errors. The table is qualitatively similar if excess returns are calculated using value-weighted Fama-French 48 industry index returns or equal-weighted CRSP-index returns. All coefficients that are statistically significant at the 0.1% level are bolded (only one of the coefficients is not statistically significant).

A. Excess returns RZ, where 1 ≤ Z ≤ 10

Past returns R_{t-f}, where $0 \le f \le 19$

	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10
Rt	**-0.149**	**-0.182**	**-0.207**	**-0.225**	**-0.236**	**-0.248**	**-0.257**	**-0.264**	**-0.269**	**-0.273**
Rt-1	**-0.066**	**-0.097**	**-0.117**	**-0.130**	**-0.144**	**-0.154**	**-0.163**	**-0.168**	**-0.172**	**-0.176**
Rt-2	**-0.045**	**-0.067**	**-0.083**	**-0.097**	**-0.109**	**-0.117**	**-0.124**	**-0.128**	**-0.132**	**-0.136**
Rt-3	**-0.031**	**-0.048**	**-0.063**	**-0.076**	**-0.085**	**-0.092**	**-0.097**	**-0.102**	**-0.106**	**-0.110**
Rt-4	**-0.022**	**-0.039**	**-0.052**	**-0.062**	**-0.069**	**-0.075**	**-0.079**	**-0.084**	**-0.088**	**-0.090**
Rt-5	**-0.020**	**-0.035**	**-0.045**	**-0.053**	**-0.058**	**-0.063**	**-0.068**	**-0.072**	**-0.074**	**-0.077**
Rt-6	**-0.017**	**-0.029**	**-0.037**	**-0.043**	**-0.048**	**-0.053**	**-0.058**	**-0.060**	**-0.062**	**-0.065**
Rt-7	**-0.014**	**-0.022**	**-0.029**	**-0.035**	**-0.040**	**-0.044**	**-0.047**	**-0.050**	**-0.052**	**-0.055**
Rt-8	**-0.011**	**-0.018**	**-0.024**	**-0.029**	**-0.034**	**-0.037**	**-0.039**	**-0.042**	**-0.045**	**-0.048**
Rt-9	**-0.009**	**-0.015**	**-0.020**	**-0.026**	**-0.028**	**-0.031**	**-0.034**	**-0.037**	**-0.040**	**-0.041**
Rt-10	**-0.007**	**-0.013**	**-0.018**	**-0.021**	**-0.024**	**-0.027**	**-0.030**	**-0.033**	**-0.034**	**-0.035**
Rt-11	**-0.006**	**-0.012**	**-0.015**	**-0.018**	**-0.021**	**-0.024**	**-0.027**	**-0.028**	**-0.029**	**-0.031**
Rt-12	**-0.006**	**-0.009**	**-0.012**	**-0.015**	**-0.019**	**-0.021**	**-0.023**	**-0.024**	**-0.026**	**-0.027**
Rt-13	**-0.003**	**-0.007**	**-0.010**	**-0.014**	**-0.016**	**-0.018**	**-0.019**	**-0.020**	**-0.022**	**-0.023**
Rt-14	**-0.003**	**-0.007**	**-0.010**	**-0.013**	**-0.015**	**-0.016**	**-0.018**	**-0.019**	**-0.020**	**-0.020**
Rt-15	**-0.003**	**-0.007**	**-0.010**	**-0.012**	**-0.013**	**-0.014**	**-0.015**	**-0.017**	**-0.016**	**-0.017**
Rt-16	**-0.004**	**-0.006**	**-0.008**	**-0.009**	**-0.011**	**-0.012**	**-0.013**	**-0.013**	**-0.014**	**-0.015**
Rt-17	**-0.003**	**-0.004**	**-0.005**	**-0.007**	**-0.009**	**-0.010**	**-0.010**	**-0.010**	**-0.011**	**-0.012**
Rt-18	**-0.002**	**-0.003**	**-0.004**	**-0.006**	**-0.007**	**-0.006**	**-0.007**	**-0.008**	**-0.009**	**-0.010**
Rt-19	-0.001	**-0.002**	**-0.003**	**-0.004**	**-0.004**	**-0.004**	**-0.005**	**-0.007**	**-0.007**	**-0.008**
Mean R^2	0.11	0.11	0.11	0.10	0.10	0.10	0.10	0.10	0.09	0.09
Median R^2	0.08	0.08	0.08	0.07	0.07	0.07	0.07	0.07	0.07	0.07

B. Excess returns RZ, where 11 ≤ Z ≤ 20

Past returns R_{t-f}, where $0 \le f \le 19$

	R11	R12	R13	R14	R15	R16	R17	R18	R19	R20
Rt	**-0.276**	**-0.280**	**-0.283**	**-0.285**	**-0.287**	**-0.288**	**-0.291**	**-0.292**	**-0.293**	**-0.293**
Rt-1	**-0.180**	**-0.184**	**-0.186**	**-0.187**	**-0.189**	**-0.192**	**-0.194**	**-0.194**	**-0.195**	**-0.196**
Rt-2	**-0.140**	**-0.142**	**-0.144**	**-0.146**	**-0.149**	**-0.150**	**-0.151**	**-0.152**	**-0.153**	**-0.154**
Rt-3	**-0.111**	**-0.114**	**-0.116**	**-0.118**	**-0.120**	**-0.121**	**-0.122**	**-0.123**	**-0.124**	**-0.125**
Rt-4	**-0.092**	**-0.095**	**-0.097**	**-0.099**	**-0.100**	**-0.101**	**-0.102**	**-0.103**	**-0.104**	**-0.104**
Rt-5	**-0.079**	**-0.082**	**-0.084**	**-0.085**	**-0.086**	**-0.087**	**-0.088**	**-0.089**	**-0.089**	**-0.089**
Rt-6	**-0.068**	**-0.070**	**-0.071**	**-0.072**	**-0.074**	**-0.074**	**-0.075**	**-0.075**	**-0.075**	**-0.077**
Rt-7	**-0.058**	**-0.059**	**-0.060**	**-0.061**	**-0.062**	**-0.063**	**-0.063**	**-0.063**	**-0.065**	**-0.066**
Rt-8	**-0.049**	**-0.050**	**-0.052**	**-0.052**	**-0.053**	**-0.053**	**-0.054**	**-0.055**	**-0.056**	**-0.057**
Rt-9	**-0.042**	**-0.044**	**-0.045**	**-0.045**	**-0.046**	**-0.046**	**-0.047**	**-0.048**	**-0.049**	**-0.050**
Rt-10	**-0.037**	**-0.038**	**-0.039**	**-0.039**	**-0.039**	**-0.041**	**-0.042**	**-0.043**	**-0.043**	**-0.044**
Rt-11	**-0.032**	**-0.033**	**-0.033**	**-0.033**	**-0.035**	**-0.036**	**-0.037**	**-0.037**	**-0.038**	**-0.039**
Rt-12	**-0.028**	**-0.028**	**-0.028**	**-0.029**	**-0.030**	**-0.031**	**-0.032**	**-0.032**	**-0.034**	**-0.035**
Rt-13	**-0.023**	**-0.023**	**-0.024**	**-0.025**	**-0.027**	**-0.027**	**-0.027**	**-0.029**	**-0.030**	**-0.029**
Rt-14	**-0.020**	**-0.021**	**-0.023**	**-0.024**	**-0.024**	**-0.024**	**-0.026**	**-0.027**	**-0.026**	**-0.026**
Rt-15	**-0.018**	**-0.019**	**-0.020**	**-0.021**	**-0.021**	**-0.023**	**-0.023**	**-0.023**	**-0.023**	**-0.023**
Rt-16	**-0.016**	**-0.017**	**-0.018**	**-0.018**	**-0.019**	**-0.020**	**-0.020**	**-0.020**	**-0.020**	**-0.020**
Rt-17	**-0.013**	**-0.014**	**-0.015**	**-0.016**	**-0.017**	**-0.016**	**-0.016**	**-0.016**	**-0.017**	**-0.017**
Rt-18	**-0.011**	**-0.011**	**-0.013**	**-0.014**	**-0.013**	**-0.013**	**-0.013**	**-0.014**	**-0.014**	**-0.015**
Rt-19	**-0.008**	**-0.009**	**-0.011**	**-0.010**	**-0.010**	**-0.010**	**-0.010**	**-0.010**	**-0.012**	**-0.011**
Mean R^2	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.08	0.08
Median R^2	0.07	0.07	0.07	0.06	0.06	0.06	0.06	0.06	0.06	0.06

Table 2: Cross-sectional variations in liquidity and market performance

In this table, stocks are divided annually into quartiles based on 1) last year's stock-specific estimates of Amihud's (2002) *ILLIQ*-measure (calculated as in Amihud, 2002, *ILLIQ* is defined as the annual average of the ratios where stocks' absolute returns are divided by their dollar volumes) and 2) their market capitalization at the end of the previous year. These rankings include all stocks listed on NYSE or Amex that satisfy all the criteria stated in the text, that were active at the end of the previous year, and have at least 200 daily observations in CRSP during the previous year. This table shows for each liquidity quartile, defined as described above, 1) the average first-order autocorrelation, $\rho(1)$ 2) our estimates of 5-day and 20-day return reversal, 3) the parameter of the persistence of price impact, *B*, whose estimation is described in the text, 4) the average of the firm-specific ratios of transitory to total volatility, $\sigma_P/(\sigma_P+\sigma_\varepsilon)$, where transitory volatility is estimated as the difference between the daily volatility and the monthly volatility adjusted using the square root rule, and total volatility equals daily volatility. The volatilities are calculated from excess returns with equal-weighted industry indexes as reference indexes, 5) the average daily volatility, and 6) the average daily turnover 7) our proxies for the returns from providing immediacy, R_{IMM} *(EqualW)* and R_{IMM} *(Exp.Ret.W),* whose estimations are described in the text and 8) daily return, which equals the equal-weighted average return of all the stocks in the corresponding liquidity quartile.

Rankings by Amihud ILLIQ-measure

Liquidity quartiles	ρ*(1)*	*5-day return reversal*	*20-day return reversal*	*Persistence of price impact, B*	*Transitory volatility %* $\sigma_P^2/(\sigma_P^2+\sigma_\varepsilon^2)$	*Total volatility* $\sigma_P^2+\sigma_\varepsilon^2$	*Daily Turnover*	R_{IMM} *(EqualW)*	R_{IMM} *(Exp.Ret.W)*	*Daily Return*
Q1 (= illiquid)	-0.129	30.1 %	36.8 %	0.41	27 %	4.25 %	0.16 %	6.47 %	8.05 %	0.16 %
Q2	-0.082	19.8 %	26.0 %	0.61	20 %	2.64 %	0.21 %	4.91 %	5.98 %	0.07 %
Q3	-0.070	16.5 %	21.6 %	0.65	17 %	2.11 %	0.28 %	3.43 %	3.97 %	0.05 %
Q4 (= liquid)	-0.052	13.7 %	17.6 %	0.66	13 %	1.69 %	0.29 %	2.06 %	2.55 %	0.04 %

Rankings by Market Capitalization

Liquidity quartiles	ρ*(1)*	*5-day return reversal*	*20-day return reversal*	*Persistence of price impact, B*	*Transitory volatility %* $\sigma_P^2/(\sigma_P^2+\sigma_\varepsilon^2)$	*Total volatility* $\sigma_P^2+\sigma_\varepsilon^2$	*Daily Turnover*	R_{IMM} *(EqualW)*	R_{IMM} *(Exp.Ret.W)*	*Daily Return*
Q1 (= small)	-0.124	29.5 %	36.3 %	0.43	27 %	4.27 %	0.20 %	5.66 %	6.48 %	0.15 %
Q2	-0.083	20.0 %	25.6 %	0.61	20 %	2.65 %	0.24 %	4.82 %	5.64 %	0.07 %
Q3	-0.069	16.2 %	21.2 %	0.65	17 %	2.08 %	0.27 %	3.23 %	3.94 %	0.06 %
Q4 (= large)	-0.055	14.6 %	19.8 %	0.65	13 %	1.66 %	0.23 %	2.20 %	2.89 %	0.05 %

Table 3: Time-series correlations between mean reversion and various measures of liquidity

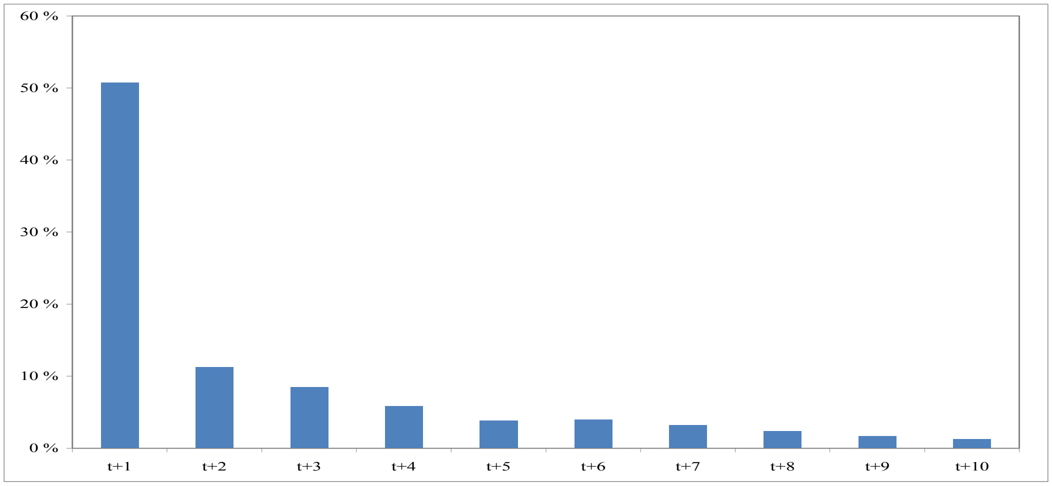
In this table, we present the correlations in market level average estimates of 1) stocks' first-order autocorrelation *ρ(1)*, 2) the parameter of the persistence of price impact, *B*, whose estimation is described in the text, 3) the firm-specific ratios of transitory to total volatility, $\sigma_P/(\sigma_P+\sigma_\varepsilon)$. Here, transitory volatility is estimated as the difference between the daily volatility and the monthly volatility adjusted using the square root rule; and total volatility equals daily volatility. All volatilities are calculated using excess returns with equal-weighted industry indexes as reference indexes, 4) Amihud's (2002) *ILLIQ*-measure (as in Amihud, 2002, *ILLIQ* is defined as the annual average of the ratios where stocks' absolute returns are divided by their dollar volumes. We adjust the dollar volumes using the consumer price index), 5) our estimates for 5-day and 20-day return reversal, 6) the average daily volatility, 7) the average daily turnover and 8) our proxies for the returns from providing immediacy, R_{IMM} *(EqualW)* and R_{IMM} *(Exp.Ret.W)*, whose estimations are described in the text. The correlations between our three measures of liquidity, defined in Proposition 9, and the Amihud's ILLIQ-measure are highlighted.

	Persistence of price impact, B	Transitory volatility %	Amihud's ILLIQ	5-day return reversal	20-day return reversal	Total volatility	Daily turnover	Liquidity Premium	R_{IMM} (EqualW)	R_{IMM} (Pred. Ret.W)
ρ(1)	93.7 % ***	-81.5 % ***	-45.4 % ***	-88.3 % ***	-79.4 % ***	-39.5 % ***	25.2 % **	3.5 %	-70.9 % ***	-45.6 % ***
Persistence of price impact, B		-67.2 % ***	-41.0 % ***	-76.8 % ***	-66.7 % ***	-30.2 % ***	15.4 % *	-4.7 %	-52.3 % ***	-36.6 % ***
Transitory volatility %			21.5 % **	73.2 % ***	68.1 % ***	18.9 % **	-31.8 % ***	-34.7 % ***	67.6 % ***	16.2 % *
Amihud's ILLIQ				52.1 % ***	46.7 % ***	80.9 % ***	-2.5 %	7.0 %	52.4 % ***	50.4 % ***
5-day return reversal					96.3 % ***	45.3 % ***	-47.1 % ***	3.0 %	78.8 % ***	55.5 % ***
20-day return reversal						37.0 % ***	-62.7 % ***	9.4 %	79.6 % ***	60.5 % ***
Total volatility							19.7 % **	11.1 %	59.0 % ***	47.9 % ***
Daily turnover								-5.4 %	-40.2 % ***	-34.5 % ***
Liquidity Premium									8.0 %	50.9 % ***
R_{IMM} (EqualW)										73.4 % ***

Figure 1: Exponential pattern of return reversal and autocorrelations

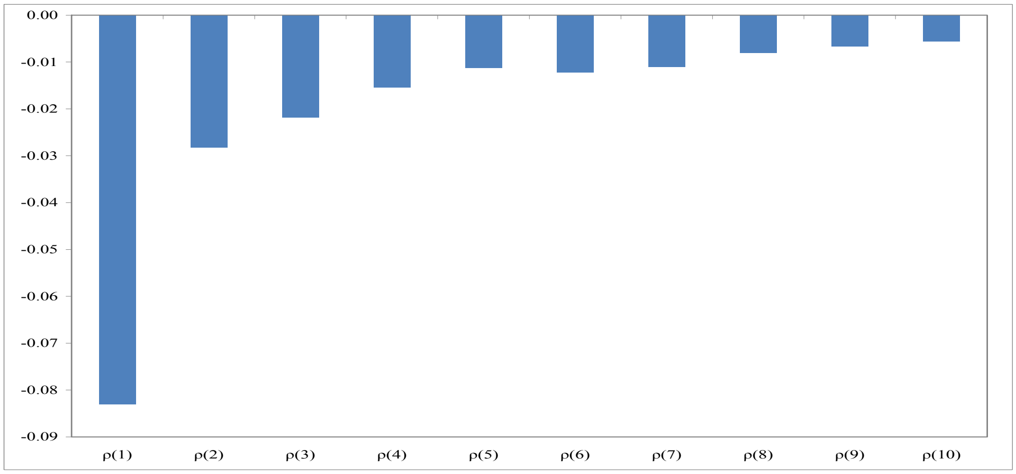
Panel A: Proportion of the predictable mean reversion for R_t, that occurs on day *t+Z* (calculated from data in Table 1).[31]

t+1	t+2	t+3	t+4	t+5	t+6	t+7	t+8	t+9	t+10
50.8%	11.3%	8.5%	5.9%	3.9%	4.0%	3.2%	2.4%	1.7%	1.3%



Panel B: Estimated autocorrelation coefficients

ρ(1)	ρ(2)	ρ(3)	ρ(4)	ρ(5)	ρ(6)	ρ(7)	ρ(8)	ρ(9)	ρ(10)
-0.083	-0.028	-0.022	-0.015	-0.011	-0.012	-0.011	-0.008	-0.007	-0.006
***	***	***	***	***	***	***	***	***	***



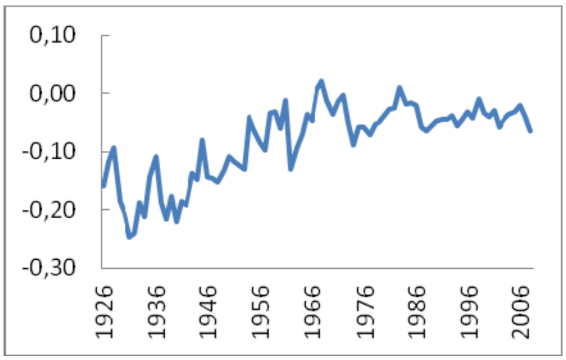
*** Denotes statistical significance at the 1% level

* For *Z=1*, the percentage equals $\hat{\beta}_{1,0}/\hat{\beta}_{20,0}$. When $Z > 1$, it equals $\left(\hat{\beta}_{Z,0}-\hat{\beta}_{Z-1,0}\right)/\hat{\beta}_{20,0}$

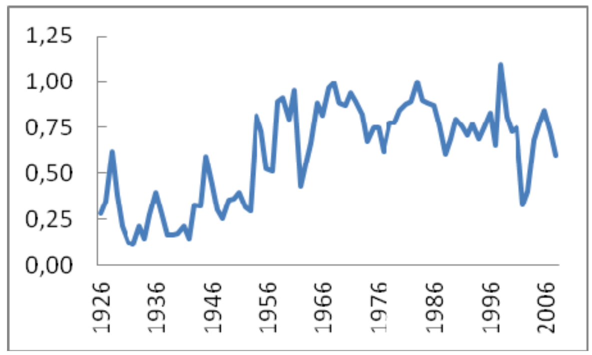
Figure 2: Time series of our measures of liquidity and other key variables of interest

We present graphically the time series evolution of the market level average estimates of A) stocks' first-order autocorrelation *ρ(1)*, B) the parameter of the persistence of price impact, B, whose estimation is described in the text, C) the firm-specific percentage of transitory volatility, $\sigma_P/(\sigma_P+\sigma_\varepsilon)$. Here, transitory volatility is estimated as the difference between the daily and the monthly volatility estimates, adjusted using the square root rule. The total volatility equals daily volatility. All volatilities are calculated using excess returns with equal-weighted industry indexes as reference indexes, D) illiquidity premium (defined as the difference in risk premium between stocks with high expected illiquidity and low expected illiquidity as defined in Acharya and Pedersen (2005), E) our estimates for 5-day and 20-day return reversal, and F) our proxies for the returns from providing immediacy, R_{IMM} (EqualW) and R_{IMM} (Exp.Ret.W), whose estimation is described in the text. Time period is 1926 to 2008 (1927-2008 for panel F).

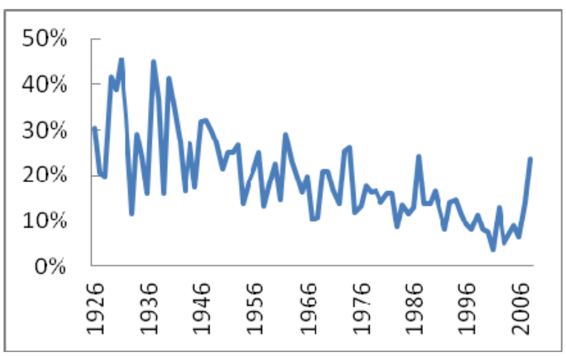
A. First-order autocorrelation *ρ(1)*



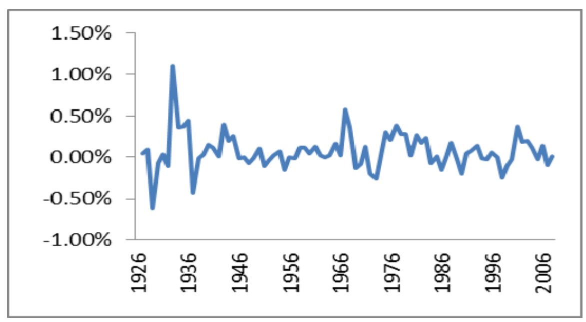
B. Persistence of price impact, *B*



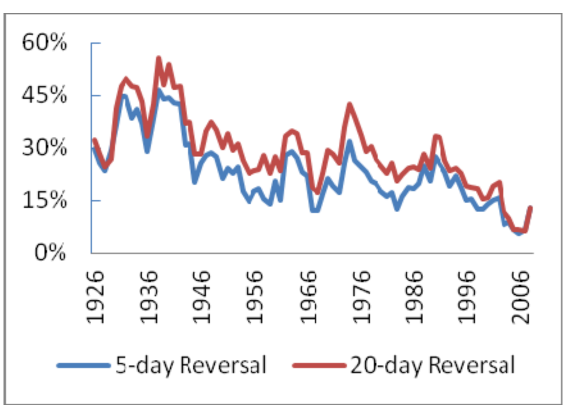
C. Transitory volatility %



D.Illiquidity premium



E.Return reversal (5-and 20 day)



F. R_{IMM} (Equal W and Pred.Ret.W)