STOCHASTIC DOMINANCE AND NONPARAMETRIC COMPARATIVE REVEALED RISK AVERSION

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ABSTRACT

It is shown how to test revealed preference data on choices under uncertainty for consistency with first and second order stochastic dominance (FSD or SSD). The axiom derived for SSD is a necessary and sufficient condition for risk aversion. If an investor is risk averse, stochastic dominance relations can be combined with revealed preference relations to recover a larger part of an investor's preference. Interpersonal comparison between investors can be based on intersections of revealed preferred and worse sets. Using a variant of Yaari's (1969) definition of "more risk averse than", it is shown that it is sufficient to compare only the revealed preference relations of two investors. This makes the approach operational given a finite set of observations. The central rationalisability theorem provides strong support for this approach to comparative risk aversion. The entire analysis is kept completely nonparametric and can be used as an alternative or complement to parametric approaches and as a robustness check. The approach is illustrated with an application to experimental data of by Choi et al. (2007). Most subjects come close to SSD-rationality, and most subjects are comparable with each other. The distribution of risk attitudes in the population can be described by comparing subjects' choices with any given preference, which is also illustrated.

1 INTRODUCTION

1.1 *Overview*

Suppose an investor who wishes to invest 1 unit of money has the choice between two risky assets representing claims in the harvest of two agricultural goods x_1 and x_2. After the investment has been made, one of two states of the world occur: With probability $\pi_1 \in (0,1)$ it rains very little, in which case only asset x_1 pays off. With probability $\pi_2 = 1 - \pi_1$, it rains heavily and only asset x_2 pays off. The price of one unit of asset x_i is $p_i > 0$. Suppose we observe many different portfolio choices $x^i = (x_1^i, x_2^i)$ of such an investor, each choice for a different price vector $p^i = (p_1^i, p_2^i)$. What testable conditions on the set of observations $\{(x^i, p^i)\}$ are necessary and sufficient to conclude that the investor maximises a utility function which is (i) monotonically increasing, (ii) monotonically increasing and concave, (iii) monotonically increasing and concave and the investor is risk averse in the sense that he always prefers a portfolio x over a portfolio y if y has second order stochastic dominance over (or is a mean preserving spread of) x?

Suppose we have the answer to question (iii). Suppose we observe the portfolio choices of two investors, A and B; then (iv) what is a reasonable way to compare the two investors and conclude that A is more risk averse than B, without relying on particular restrictive forms of risk aversion;

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and, given such a measure, (v) are there practically testable conditions on the set of observations of the two investors to reach such a conclusion?

The sets of alternatives for the investors to choose from correspond to standard competitive budget sets. Thus, the answers to questions (i) and (ii) are given by the well known Afriat's Theorem, and Varian's (1982) Generalised Axiom of Revealed Preference (Garp) is an easily testable and necessary and sufficient condition. This paper gives an answer to question (iii) and adopts a variant of Yaari's (1969) definition of "more risk averse than" to give an answer to question (iv). Based on this answer, it also provide an answer to question (v).

In particular, this papers shows how to combine first or second order stochastic dominance (Fsd or Ssd) relations with revealed preference relations. This allows to test if an investor prefers portfolios which have Fsd or Ssd over other portfolios. The axiom derived for Ssd is a necessary and sufficient condition for risk aversion. If an investor is indeed risk averse, the combined relations allow to recover a larger part of the investor's preference underlying his decisions. In the framework considered here, Garp is not a sufficient condition for the existence of a utility function which obeys Ssd, as Garp does not interpret the assets as such. An investor who has an *intrinsic* taste for one of the assets can still satisfy Garp: Suppose that an investor always invests all his money into asset x_1. Then Garp will be satisfied, but the investor does not necessarily obey Ssd.

Yaari's (1969) definition of "more risk averse than" is useful to analyse the ordering of various classes of utility functions in terms of risk aversion (see, for example, Bommier et al. forthcoming). However, it can also be directly applied to revealed preference relations and used as a nonparametric method to compare the risk aversion of two investors. We show how intersections of revealed preferred and worse sets can be used to make interpersonal comparisons. The variant of Yaari's (1969) definition of "more risk averse than" which is employed here states that investor A is partially more risk averse than investor B if there is are least two portfolios x and y, where x has a higher expected value than y, and A prefers y over x while B prefers x over y. Then if A is partially more risk averse than B, and B is not partially more risk averse than A, we conclude that A is more risk averse than B. While the definition is stated in terms of revealed preferred and revealed worse sets, which can be computed for any portfolio and not just those observed as a choice, it is shown that it is not only necessary but also sufficient to compare only those portfolios which have been observed as a choice by one of the two investors.

The entire analysis is kept completely nonparametric and makes no assumptions on particular functional forms of utility. The approach is illustrated with an application to the experimental data of Choi et al. (2007a). The data is tested for consistency with Ssd and consistency for many subjects is confirmed, based on the Afriat Efficiency Index (or Critical Cost Efficiency Index) supported by Monte-Carlo simulations. The comparative risk aversion approach is then applied to the data.

If neither of two investors is more risk averse than the other, then either (i) they have very similar preferences, or (ii) their extent of risk aversion is different for different income ranges, or (iii) they act according to distinct notions of risk aversion. Case (i) is a helpful result to classify two investors as belonging to the same category of risk preferences, as we cannot reject the hypothesis that the two investors have the same risk preferences. Cases (ii) and (iii) highlight the problem with a "one size fits all approach"; in particular, it shows that comparisons based on parameter estimates rely on the specified form of the utility function. However, most experimental subjects are indeed comparable if choices are corrected by efficiency levels. If neither of two subjects is more risk averse than the other, it is mostly because they have similar preferences.

The analysis provides a strong test of robustness for conclusions based on parameter estimates. Furthermore, while the nonparametric does not give a distribution of parameters of risk aversion in a population, it nonetheless allows to characterise the distribution of risk attitudes: The nonparametric approach tells us what percentage of the population is less or more risk averse than *any* given preference. This is illustrated by comparing the choices of subjects with several parameters of a utility function estimated by Choi et al. (2007a).

While none of the basic elements of the paper are new, it is the combination of several strands of the literature that distinguishes its approach. The theoretical literature on risk preferences, choice under uncertainty, and comparative risk aversion is combined with the nonparametric analysis based on operational revealed preference, and this combination can—and indeed is—applied to data. It is not claimed that the nonparametric approach should replace other approaches. The analysis here complements them and should, at the very least, be applied before further steps are taken, as it allows to draw strong conclusions about preferences without the need of restrictive assumptions on functional form.

1.2 *Related Literature*

This paper is related to the theoretical literature on choice under uncertainty and the discussion of what "risk" is, the comparative risk aversion literature, the revealed preference approach and the nonparametric analysis of choice data within consumer demand theory, and the experimental literature on risk preferences by subjects who are asked to make properly incentivised choices under controlled conditions.

Rothschild and Stiglitz (1970, 1971) provide a definition of "risk" and analyse its economic consequences. In particular, they call a random variable y "more variable" than a random variable x if x is equal to y plus a disturbance term with expected value of 0. Then y is a MPS of x, and x has second order stochastic dominance over x. For two random variables with the same mean, they show that every element u in the set of all concave utility functions yields $u(y) > u(x)$ if and only if x is a mean preserving spread (MPS) of y. Defining risk aversion in terms of second order stochastic dominance is therefore the least restrictive reasonable definition.

Similarly, Hadar and Russell (1969) note that comparing uncertain prospects in terms of moments is problematic if the utility function of an investor is not known. They define dominance of portfolios in terms of first- and second order stochastic dominance and show that the set of all increasing utility functions yield $u(y) > u(x)$ if and only if y has FSD over x, and the all increasing concave utility functions yield $u(y) > u(x)$ if and only if y has SSD over x. See also the early contribution of Hanoch and Levy (1969) in the same year with similar results, and Levy (1992) for a survey.

Yaari (1969) answers the question of when an investor A is more risk averse than B within a framework with one risky asset. Any investment in the risky asset is a gamble, and the acceptance set is the set of all gambles which are preferred to the status quo by an investor. Yaari suggests to call investor A more risk averse than investor B if the acceptance set of A is contained in the acceptance set of B. Similar approaches to uncertainty and ambiguity aversion are developed by Epstein (1999), Ghirardato and Marinacci (2002), and Grant and Quiggin (2005)

A seminal by article by Pratt (1964), and similarly for the work by Kihlstrom and Mirman (1974), analyses a measure of risk aversion based on certainty equivalents. In a recent paper, Bommier et al. (forthcoming) provide a formal framework for analysing comparative risk aversion of different investors, with a focus on intertemporal choice. They use their approach to analyse several classes of utility functions common in the literature.

In the revealed preference approach it is assumed that we only the set of alternatives a decision maker has and the alternative which he actually chooses. Thus, revealed preference relations, like preferences, are binary relations which we observe due to an individual's choices combined with theoretical reasoning about what these choices reveal about the individual's preferences. While with a finite number of observations a revealed preference relation will always be only a partial binary relation, the theoretical reasoning about revelations can allow to recover a lot about underlying preferences. An advantage of the approach is that we do not to need assume any particular functional form of utility (or demand); the revealed preference approach therefore lends itself to a nonparametric analysis of choice data. Afriat's (1967) analysis, for example, makes the revealed preference approach operational when the sets of alternatives are competitive budget sets. Varian (1982, 1983a) refines this approach and provides highly valuable tools for the nonparametric analysis of such data. Clark (2000) considers the problem of recovering expected utility from observed choice behaviour, but does not provide extensive tools for the analysis of revealed preference data.

Varian (1983b) provides a condition which is necessary and sufficient for the existence of an expected utility function which rationalises a set of investment decisions. His condition is linear feasibility system which has to have a solution. He applies his framework to a mean variance model of utility maximisation. The approach described here is more directly rooted in the axiomatic revealed preference approach and shows how to enrich revealed preference relations with Fsd- and Ssd-relations, and the recovered preferred and worse sets are shown to be useful for comparative risk aversion.

Experimental economics allows researchers to collect choice data of subjects under controlled conditions. "Induced budget experiments", where subjects are asked to make choices on competitive budget sets, are increasingly common. Such experiments allow to collect extensive data on individuals' preference. Choi et al. (2007a), in particular, collect fifty decisions of each of ninety three subjects in an induced budget experiment on choice under uncertainty. They test the data for consistency with Garp. Furthermore, they estimate parameters of utility functions to characterise the distribution of risk preferences.

1.3 *Main Results*

In the theoretical part, it is shown how the assumption that investors obey Ssd can be imposed on revealed preference relations. If portfolio x has second order stochastic dominance over portfolio y, then any risk averse consumer will prefer x over y. Second order stochastic dominance leads to the definition of an incomplete binary relation $\succsim_{\text{SSD}}$. Then $\succsim_{\text{SSD}}$ can be combined with the revealed preference relation R to form the new relation R_{SSD}. It shown that adding only finitely many new data points to a set of observations and testing this extended set for consistency with a condition called Ssd-Garp is necessary and sufficient for the existence of a utility function which rationalises the set of observations and obeys Ssd.

The theoretical part then proceeds by translating Yaari's (1969) definition of "more risk averse than" to the framework of this article. The next step is to translate this definition to the revealed preference case: With only finitely many observations, we will never observe a complete preference relation, but we can use the incomplete revealed preference relation to construct "revealed preferred" and "revealed worse" sets for *all* portfolios. This is based on Varian's (1982) framework for nonparametric analysis of demand data. This leads to an important and very useful result: To test whether one investor is more risk averse than another—for *all* portfolios—it is necessary and sufficient to only compare those portfolios which have been observed as a choice by either of the investors. This makes the approach completely operational and allows to compare the risk aversion of two investors without the need of specifying a functional form of utility.

The second part uses experimental data of Choi et al. (2007a) and applies the theoretical concepts to this data. There were two experimental treatments: In the symmetric treatment, the probability that each of the two assets pays off was ½. In the asymmetric treatment, one of the two assets had a ⅓ probability of paying off, and the other paid off with probability ⅔. We find that most subjects in the symmetric treatment show very high efficiency in terms of Ssd-Garp. In the asymmetric treatment, subjects score somewhat lower, but compared to random choices their efficiency is still high.

For those subjects with reasonably high efficiency, the nonparametric comparison of risk aversion shows that most subjects are indeed comparable with most other subjects if choices are corrected by efficiency levels. That is, we find that of two subjects, either one subject is clearly more risk averse than the other, or we cannot reject the hypothesis that the two subjects have the same preferences.

The subjects are also compared with choices generated by a utility function using parameters estimated by Choi et al. (2007a) for different percentiles. For the symmetric treatment these comparisons offer very strong support for the parameter estimates, and somewhat less so for the asymmetric treatment.

1.4 *Outline*

The rest of the paper is organised as follows: Section 2.1 introduces the framework and the notation. Section 2.2 reviews the necessary revealed preference literature and extends the approach using stochastic dominance relations. It derives the Fsd-Garp and Ssd-Garp, both of which are easily testable and which correspond to Varian's (1982) Generalised Axiom of Revealed Preference (Garp). In particular it is shown that Ssd-Garp is necessary and sufficient for the existence of a monotonically increasing and concave utility function which rationalises the observations and which obeys second order stochastic dominance; Ssd-Garp is therefore a necessary and sufficient condition for risk aversion. Section 2.3 introduces the nonparametric approach to compare the extent of risk aversion of two investors. Section 3 applies the methods to the experimental data of

Choi et al. (2007a). Section 4 discusses the results and concludes. All proof can be found in the appendix in Section A.

2 THEORY

2.1 *Preliminaries*

A set of observed investment choices consists of a set of chosen portfolio of assets and the prices and incomes at which these assets were chosen.[1] The asset space is $\mathbb{A} = \mathbb{R}^L_+$ and the price space is $\mathbb{R}^L_{++}$, where $L \geq 2$ denotes the number of different assets.[2] Investors choose portfolios $x^i = (x^i_1, \ldots, x^i_L)' \in X$ when facing a price vector $p^i = (p^i_1, \ldots, p^i_L) \in \mathbb{R}^L_{++}$; these choices are the demand we observe. A budget set is then defined by $B^i = B(p^i) = \{x \in \mathbb{A} : p^i x^i \leq 1\}$; we will sometimes refer to a budget using the characterising price vector. The entire set of N observations on an investor is denoted as $\Omega = \{(x^i, p^i)\}_{i=1}^N$. Unless otherwise noted, we assume that demand is exhaustive (i.e., $p^i x^i = 1$).

There are L different states which can obtain after the portfolio choice has been made. In each state $i \in \{1, \ldots, L\}$, asset i is the only asset that pays off. State i occurs with probability $\pi_i \in \Delta(L)$, where $\Delta(L)$ is the $(L-1)$ probability simplex, i.e., $\pi_i \geq 0$ for all i and $\sum_{i=1}^L \pi_i = 1$. Let $\Pi(x)$ denote the ex post realised payoff of a portfolio x. Let $\mathbb{A}(\pi)$ denote an asset space with the probability vector π; we will often drop the π when the reference is clear or unnecessary.

We assume that an investor can be represented by transitive, complete, and continuous binary relation[3] on $\mathbb{A}$. This binary relation $\succsim \in \mathbb{A} \times \mathbb{A}$ represents his *preference* according to which he decides which portfolio to choose on a budget. The interpretation is as usual, i.e. $(x, y) \in \succsim$, also written $x \succsim y$, means that to the investor x is at least as good as y. For $\succsim$ (and similarly for all other complete relations defined below) $\succ$ denotes the asymmetric part of $\succsim$ and $\sim$ denotes the symmetric part, i.e., $x \succ y$ if $x \succsim y$ and [not $y \succsim x$], and $x \sim y$ if $x \succsim y$ and $y \succsim x$.

Let $\mathbf{E}(x, \pi) = \sum \pi_i x_i$ be the expected value of a portfolio $x \in A(\pi)$. Let $\succsim^\pi_{\mathrm{E}} \in \mathbb{A} \times \mathbb{A}$ be defined as

$$x \succsim^\pi_{\mathrm{E}} y \text{ if } \mathbf{E}(x, \pi) \geq \mathbf{E}(y, \pi).$$

We will drop the π if the reference is clear (i.e., we will write $\mathbf{E}(x)$ and $x \succsim_{\mathrm{E}} y$ if there is no confusion); we will also do this for all other definitions given below.

Let $F : \mathbb{R} \times \mathbb{A} \times \Delta(L) \to [0,1]$ be the cumulative distribution function of a portfolio, i.e., $F(\xi, x, \pi) = \mathrm{Prob}(\Pi(x) \leq \xi)$ gives the probability that the payoff from a portfolio $x \in \mathbb{A}(\pi)$ is less

[1] "Portfolios" correspond to the term "lotteries".

[2] The following notation is used: For all $x, y \in \mathbb{R}^L$, $x \geqq y$ if $x_i \geq y_i$ for all $i = 1, \ldots, L$; $x \geq y$ if $x \geqq y$ and $x \neq y$; $x > y$ if $x_i > y_i$ for all $i = 1, \ldots, L$. We denote $\mathbb{R}^L_+ = \{x \in \mathbb{R}^L : x_i \geqq (0, \ldots, 0)\}$ and $\mathbb{R}^L_{++} = \{x \in \mathbb{R}^L : x > (0, \ldots, 0)\}$.

[3] A binary relation $\succsim$ is *transitive* if $[x \succsim y$ and $y \succsim z]$ implies $x \succsim z$; it is *complete* if for every two bundles x, y, either $x \succsim y$ or $y \succsim x$; it is *continuous* if for all x the sets $\{y : x \succsim y\}$ and $\{y : y \succsim x\}$ are closed.

than or equal to $\xi \in \mathbb{R}$. Let $\succsim_{\text{FSD}}$ and $\succsim_{\text{SSD}}$ be binary relations on $\mathbb{A}$, defined as

$$x \succsim_{\text{FSD}} y \text{ if } F(\xi^i, x, \pi) \leq F(\xi, y, \pi) \text{ for all } \xi^i \in x \text{ or } y$$

and

$$x \succsim_{\text{SSD}} y \text{ if } \sum_{i=1}^{\ell} F(\xi^i, x, \pi)[\xi^{i+1} - \xi^i] \leq \sum_{i=1}^{\ell} F(\xi^i, y, \pi)[\xi^{i+1} - \xi^i] \text{ for all } \ell < n,$$

where the ξ^i, $i = 1, \ldots, n \leq 2L$, are sorted in increasing order and n denotes the number of distinct x_i and y_i. The relations are called the *first and second order stochastic dominance* relations, respectively (see Hadar and Russell 1969): x hast *first order stochastic dominance* (FSD) over y if $x \succsim_{\text{FSD}} y$, and *second order stochastic dominance* (SSD) if $x \succsim_{\text{SSD}} y$. Suppose x has FSD (SSD) order stochastic dominance over y. Then every expected utility maximiser with a monotonically increasing (and concave) utility function will prefer x over y (see, for example, Hanoch and Levy 1969).

Note that $x \succsim_{\text{SSD}} y$ and $y \succsim_{\text{SSD}} x$ if and only if $F(\xi^i, x) = F(\xi^i, y)$. Thus, $x \succ_{\text{SSD}} y$ if and only if $x \succsim_{\text{SSD}} y$ and $F(\xi^i, x) = F(\xi^i, y)$. The same is true for $\succsim_{\text{FSD}}$.

Axiom *A preference $\succsim$ satisfies the* Axiom of First Order Stochastic Dominance *(AFSD) if* $\succsim_{\text{FSD}} \subset \succsim$. *A preference $\succsim$ satisfies the* Axiom of Second Order Stochastic Dominance *(ASSD) if* $\succsim_{\text{SSD}} \subset \succsim$.

Note that ASSD $\Rightarrow$ AFSD but not vice versa. We will also say that investors whose preferences satisfy AFSD or ASSD are FSD-rational or SSD-rational.

Let $2^{\mathbb{A}}$ be the set of all subsets of $\mathbb{A}$. We then define the correspondence $\mathcal{P} : \mathbb{A} \times (\mathbb{A} \times \mathbb{A}) \times \Delta(L) \to 2^{\mathbb{A}}$ as

$$\mathcal{P}(x, \mathrm{Q}, \pi) = \{y \in \mathbb{A} : y \, \mathrm{Q} \, x \text{ given the probability distribution } \pi\},$$

for some arbitrary binary relation Q on $\mathbb{A}$. We record a first lemma to be used later but already worth mentioning.

Lemma 1 *The relation $\succsim_{SSD}$ is quasi-concave, i.e., $\mathcal{P}(x, \succsim_{SSD})$ is convex.*

All proofs can be found in the appendix.

The *convex hull* CH of a set of points $Y = \{y^i\}$ and its *convex monotonic hull* CMH are defined as

$$\mathrm{CH}(Y) = \Big\{x \in \mathbb{R}^L_+ : x = \sum_i \lambda_i y^i, \lambda \geq 0, \sum_i \lambda_i = 1\Big\}$$
$$\mathrm{CMH}(Y) = \text{interior of } \mathrm{CH}\Big(\big\{x \in \mathbb{R}^L_+ : x \geqq y^i \text{ for some } i\big\}\Big),$$

and $\overline{\text{CMH}}$ is the closure of CMH. For some binary relation Q on $\mathbb{A}$ we also write $\text{CMH}(x, \text{Q}) = \text{CMH}(\{y \in \mathbb{A} : y\,\text{Q}\,x\})$.

Define recursively for some sequence of indices $\{i_j\}_{j=1}^{n}$, $n \le L-1$, $1 \le i_j \le L$,

$$M(x, \{i_1\}) = \left\{y \in \mathbb{A} : y = \arg\max_{\{\tilde{y} \in \mathcal{P}(x, \succsim_{\text{SSD}} \cap \sim_{\text{E}})\}} \tilde{y}_i\right\},$$
$$M(x, \{i_j\}_{j=1}^{n}) = \left\{y \in \mathbb{A} : y = \arg\max_{\{\tilde{y} \in \mathcal{P}(x, \succsim_{\text{SSD}} \cap \sim_{\text{E}}) \cap M(x, \{i_j\}_{j=1}^{n-1})\}} \tilde{y}_{i_j}\right\}.$$

Then by construction $y \in M(x, \{i_j\}_{j=1}^{L-1})$ has second order stochastic dominance over x and the same expected value as x. Note that x is a *mean preserving spread* (Mps) of all elements in $\mathcal{P}(x, \succsim_{\text{SSD}} \cap \sim_{\text{E}})$, which plays an important role in the analysis of Rothschild and Stiglitz (1970). Let $\hat{M}(x)$ denote the union of all $M(x, \{i_j\}_{j=1}^{L-1})$ for every permutation of indices from 1 to L.

Lemma 2 *For all $x \in \mathbb{A}$,*
(i) $\mathcal{P}(x, \succsim_{SSD} \cap \sim_E) = CH(\hat{M}(x))$
(ii) $\mathcal{P}(x, \succsim_{SSD}) = \overline{CMH}(x, \succsim_{SSD} \cap \sim_E)$, *and thus* $\mathcal{P}(x, \succsim_{SSD}) = \overline{CMH}(\hat{M}(x))$

See also Figure 1.(a) below for an example of $\mathcal{P}(x, \succsim_{\text{SSD}})$.

2.2 *Revealed Preference*

Revealed preference relations, like preferences, are binary relations on $\mathbb{A}$ which we observe due to an investor's choices combined with theoretical reasoning about what these choices reveal. While with a finite number of observations a revealed preference relation will always be only a partial binary relation, we would like to recover the greatest possible part of an investor's preference given a set of observations $\Omega = \{(x^i, p^i)\}_{i=1}^{N}$.

Let $\text{Q} \subseteq \mathbb{A} \times \mathbb{A}$ be any binary relation. Then the *transitive closure* $(\text{Q})^+$ of Q is defined as the smallest transitive relation that contains Q, that is,

$$(\text{Q})^+ = \{(x, y) \in \mathbb{A} \times \mathbb{A} : x\,\text{Q},\ x', x'\,\text{Q}\,x^*, \ldots, x^\circ\,\text{Q}\,x^\bullet, x^\bullet\,\text{Q}\,y \quad \text{for some sequence of portfolios } x', x^*, \ldots, x^\circ, x^\bullet\}.$$

We the use the following definitions to recover an investor's preference that is implicit in a set of portfolio choices:

- The portfolio x^i is *directly revealed preferred* to a portfolio x, written $x^i\,\text{R}^0\,x$, if $p^i x^i \ge p^i x$.
- The portfolio x^i is *strictly directly revealed preferred* to a portfolio x, written $x^i\,\text{P}^0\,x$, if $p^i x^i > p^i x$.
- Let $\text{R} = (\text{R}^0)^+$. Then the portfolio x^i is *revealed preferred* to a portfolio x if $x^i\,\text{R}\,x$.
- The portfolio x^i is *strictly revealed preferred* to a portfolio x, written $x^i\,\text{P}\,x$, if for some sequence of observations $x^i\,\text{R}\,x^j$, $x^j\,\text{P}^0\,x^k$, $x^k\,\text{R}\,x$.

Axiom (Varian 1982) *A set of observations Ω satisfies the* Generalised Axiom of Revealed Preference *(Garp) if* $[$*not* $x^i\,P^0\,x^j]$ *whenever* $x^j\,R\,x^i$.

The strength of GARP is based on the fact that it is an easily testable condition and is a necessary and sufficient condition for utility maximisation, as Afriat's Theorem demonstrates. We say that a utility function $u : \mathbb{A} \Rightarrow \mathbb{R}$ *rationalises* a set of observations Ω if $u(x) \geq u(y)$ whenever $x \mathrm{R} y$. Let $\mathcal{U}$ denote the set of all continuous, non-satiated, monotonic, and concave utility functions.

Afriat's Theorem (Afriat 1967, Diewert 1973, Varian 1982) *The following conditions are equivalent:*

1. *there exists a $u \in \mathcal{U}$ which rationalises the set of observations Ω;*
2. *the set of observations Ω satisfies GARP.*

The revealed preference relations can be extended by imposing axioms AFSD or ASSD. If the hypotheses are correct, then R is the subset of some preference $\succsim$. If the investor's preference satisfies first order stochastic dominance, then $\succsim_{\mathrm{FSD}}$ is a subset of the same preference $\succsim$. Thus $(\mathrm{R} \cup \succsim_{\mathrm{FSD}}) \subset \succsim$, and similarly for $\succsim_{\mathrm{SSD}}$. Define

$$\begin{aligned} &\mathrm{R}_{\mathrm{FSD}} = (\mathrm{R} \cup \succsim_{\mathrm{FSD}})^+, \quad \mathrm{R}_{\mathrm{SSD}} = (\mathrm{R} \cup \succsim_{\mathrm{SSD}})^+, \quad \mathrm{P}^0_{\mathrm{FSD}} = \mathrm{P}^0 \cup \succ_{\mathrm{FSD}}, \quad \mathrm{P}^0_{\mathrm{SSD}} = \mathrm{P}^0 \cup \succ_{\mathrm{SSD}}, \\ &\mathrm{P}_{\mathrm{FSD}} = \{(x, y) \in \mathbb{A} \times \mathbb{A} : x \,\mathrm{R}_{\mathrm{FSD}}\, z \,\mathrm{P}^0_{\mathrm{FSD}}\, z' \,\mathrm{R}_{\mathrm{FSD}}\, y \text{ for some } z, z' \in \mathbb{A}\}, \\ &\mathrm{P}_{\mathrm{SSD}} = \{(x, y) \in \mathbb{A} \times \mathbb{A} : x \,\mathrm{R}_{\mathrm{SSD}}\, z \,\mathrm{P}^0_{\mathrm{SSD}}\, z' \,\mathrm{R}_{\mathrm{SSD}}\, y \text{ for some } z, z' \in \mathbb{A}\}. \end{aligned} \tag{1}$$

Let $\sigma_\ell(x)$ denote the ℓth permutation of x, with $\sigma_1(x) = x$. Let $L!$ denote the factorial of L. Define

$$\sigma(\Omega) = \left\{ y \in \mathbb{A} : y = \sigma_\ell(x^i) \text{ for some } i = 1, \ldots, N \text{ and some } \ell = 1, \ldots, L! \right\}.$$

We will refer to the elements in $\sigma(\Omega)$ as s^i. Note that all $x^i \in \sigma(\Omega)$; let the set be sorted such that $\sigma(\Omega)^i = x^i$ for $i = 1, \ldots, N$. Define

$$\tau(\Omega) = \left\{ y \in \mathbb{A} : y \in \hat{M}(x^i) \text{ for some } i = 1, \ldots, N \right\}.$$

We will refer to the elements in $\tau(\Omega)$ as t^i. Again we have $x^i \in \tau(\Omega)$; let $\tau(\Omega)$ be sorted in the same way as $\sigma(\Omega)$.

Axiom *A set of observations Ω satisfies the FSD-GARP if for all $s^i \in \sigma(\Omega)$,*

$$[\textit{not } s^i \, P_{FSD} \, s^j] \textit{ whenever } s^j \, R_{FSD} \, s^i.$$

It satisfies the SSD-GARP if for all $r^i \in \tau(\Omega)$,

$$[\textit{not } t^i \, P_{SSD} \, t^j] \textit{ whenever } t^j \, R_{SSD} \, t^i.$$

We say that a utility function u *FSD-rationalises* a set of observations Ω if $u(x) \geq u(y)$ whenever $x \,\mathrm{R}_{\mathrm{FSD}}\, y$; it *SSD-rationalises* Ω if $u(x) \geq u(y)$ whenever $x \,\mathrm{R}_{\mathrm{SSD}}\, y$.

Theorem 1 *The following conditions are equivalent:*

1. *there exists a $u \in \mathcal{U}$ which* FSD*-rationalises (*SSD*-rationalises) the set of observations Ω;*
2. *the set of observations Ω satisfies* FSD-GARP *(*SSD-GARP*).*

Note that SSD-GARP is a necessary and sufficient condition for risk aversion in the SSD-sense.

Following Varian (1982), we now turn to the question of recoverability of preferences. Let $\mathrm{Ax}(\mathrm{Q})$ denote the axiom associated with the relation Q, that is, $\mathrm{Ax}(\mathrm{R})$ is GARP, $\mathrm{Ax}(\mathrm{R}_{\mathrm{FSD}})$ is FSD-GARP, and $\mathrm{Ax}(\mathrm{R}_{\mathrm{SSD}})$ is SSD-GARP. Let $\phi(\mathrm{Q})$ be the strict relation associated with Q, that is, $\phi(\mathrm{R}) = \mathrm{P}$ etc. Given some portfolio $x^0 \in \mathbb{A}$ which was not necessarily observed as a choice, the set of prices which *support* x^0 is defined as

$$S(x^0, \mathrm{Q}) = \left\{p^0 \in \mathbb{R}^L_{++} : \ \{(x^i, p^i)\}_{i=0}^N \text{ satisfies } \mathrm{Ax}(\mathrm{Q}) \text{ and } p^0 x^0 = 1\right\}.$$

Varian (1982) uses $S(x^0, \mathrm{R})$ to describe the set of all bundles (here: portfolios) which are revealed worse and revealed preferred to a portfolio x^0: If for any price vector at which x^0 can be demanded without violating GARP x^0 must be revealed preferred to x, then x is in the set of all portfolios revealed worse to x^0, and similarly for revealed preferred sets. Thus, the set of all portfolios which are *revealed worse* than x^0 is given by

$$\mathcal{RW}(x^0, \mathrm{Q}) = \left\{x \in \mathbb{A} : \text{ for all } p^0 \in S(x^0, \mathrm{Q}),\ x^0\, \phi(\mathrm{Q})\, x\right\}$$

and the set of all portfolios which are *revealed preferred* to x^0 is given by

$$\mathcal{RP}(x^0, \mathrm{Q}) = \left\{x \in \mathbb{A} : \text{ for all } p \in S(y, \mathrm{Q}),\ x\, \phi(\mathrm{Q})\, x^0\right\}.$$

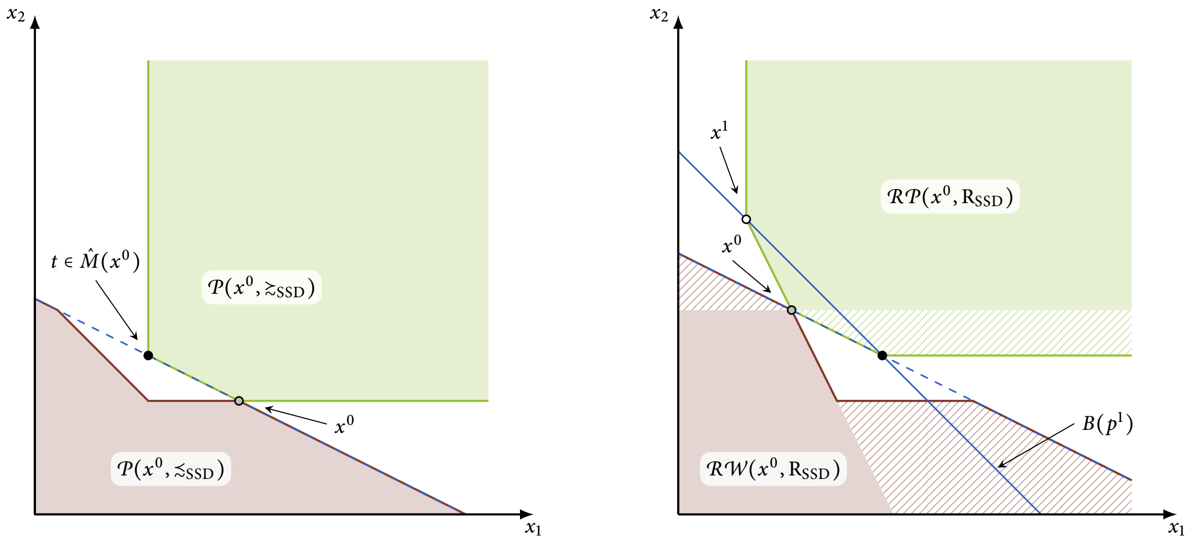
These definitions are well motivated by the equivalence of GARP with the existence of a concave utility function which rationalises the data: *Any* utility function which rationalises a set of observations must have $u(x) > u(x^0)$ if $x \in \mathcal{RP}(x^0, \mathrm{R})$, etc. Note that if a utility function which rationalises a set of observations represents a preference $\succsim$, then GARP implies that $\mathrm{R} \subseteq \succsim$, $\mathrm{P} \subseteq \succ$, $\mathcal{RW}(x^0, \mathrm{R}) \subseteq \mathcal{P}(x^0, \precsim)$, and $\mathcal{RP}(x^0, \mathrm{R}) \subseteq \mathcal{P}(x^0, \succsim)$. See Figure 1 for an example.

Proposition 1 *For all $x \in \mathbb{A}$ and all $Q \in \{R, R_{FSD}, R_{SSD}\}$,*

$$CMH(x^0, Q) \subseteq \mathcal{RP}(x^0, Q) \subseteq \overline{CMH}(x^0, Q)$$

Varian (1982) and Knoblauch (1992) prove the proposition for Q = R. We omit the proof, which is along the lines of Knoblauch's (1992) proof; Lemma 2 makes the extension quite simple.

As opposed to the usual proofs of Afriat's Theorem, no utility functions are constructed in the proof of Theorem 1. However, if a set of observations satisfies SSD-GARP, there exist price vectors for every x^0 such that $\{(x^i, p^i)\}_{i=0}^N$ satisfies SSD-GARP. Any $p \in S(x^0, \mathrm{R}_{\mathrm{SSD}})$ can be chosen, and thus, we can augment a set of observations by arbitrarily many new observations, and construct utility functions, using for example the algorithms in Varian (1982). In particular, we can find price vectors for all $t^i \notin \{x^i\}_{i=1}^N$ and construct utility functions accordingly.



(a) (b)

Figure 1: Example with probabilities $(\pi_1, \pi_2) = (⅓, ⅔)$. The dashed line shows all portfolios with the same expected value as the portfolio x^0. (a): The set of portfolios which have second order stochastic dominance over x^0, and the set of portfolios over which x^0 has second order stochastic dominance. (b): Revealed preferred and revealed worse set of x^0 with one observation (x^1, p^1), based on the extended relation $\mathrm{R}_{\mathrm{SSD}}$. The dashed parts show what is added by combining R and $\succsim_{\mathrm{SSD}}$.

2.3 *Interpersonal Comparison*

Let $\trianglerighteq \in \times_{i=1}^{4} \mathbb{A}$ be the *more risk averse than* relation. For two preferences $\check{\succsim}$ and $\hat{\succsim}$ which satisfy Assd (and therefore Afsd), define

$$\check{\succsim} \trianglerighteq \hat{\succsim} \text{ if } [\hat{\succsim} \cap \prec_{\mathrm{E}}] \subseteq [\check{\succsim} \cap \prec_{\mathrm{E}}].$$

That is, an investor $\check{\succsim}$ is more risk averse than an investor $\hat{\succsim}$ if the set of portfolios with a lower expected value than x which are preferred to x by $\hat{\succsim}$ is a subset of the corresponding set of $\check{\succsim}$. Let $\triangleright$ be the asymmetric part of $\trianglerighteq$, that is, $\check{\succsim}$ is *strictly more risk averse than* $\hat{\succsim}$, written $\check{\succsim} \triangleright \hat{\succsim}$, if $\check{\succsim} \trianglerighteq \hat{\succsim}$ and [not $\hat{\succsim} \trianglerighteq \check{\succsim}$].

The definition of more risk averse is closely modelled on Yaari's (1969) concept, who considers acceptance sets of gambles. If investor A prefers all gambles over the status quo which investor B also prefers over the status quo, and there are additional gambles which A prefers but B does not, then B is more risk averse than A. The definition of $\trianglerighteq$ is translatess this concept to the framework considered here.

Let $\mathcal{RPL}(x^0, \mathrm{R}) = \mathcal{RP}(x^0, \mathrm{R}) \cap \mathcal{P}(x^0, \prec_{\mathrm{E}})$ and $\mathcal{RWL}(x^0, \mathrm{R}) = \mathcal{RW}(x^0, \mathrm{R}) \cap \mathcal{P}(x^0, \prec_{\mathrm{E}})$. We will now consider two investors, on which we have sets of observations $\check{\Omega}$ and $\hat{\Omega}$, and we will refer to these two investors by their revealed preference relations $\check{\mathrm{R}}$ and $\hat{\mathrm{R}}$.

How can $\trianglerighteq$ be made operational given a finite set of observations on an investor and the revealed preference relation based on these observations? One problem is that any revealed relation Q is only a partial relation, and therefore $x \notin \mathcal{RP}(x^0, \mathrm{Q})$ does *not* imply $x \in \mathcal{RW}(x^0, \mathrm{Q})$. Thus, it would be presumptuous to base the statement that investor $\check{\mathrm{R}}$ is more risk averse than $\hat{\mathrm{R}}$ on the fact that $\mathcal{RPL}(x^0, \hat{\mathrm{R}}) \subset \mathcal{RPL}(x^0, \check{\mathrm{R}})$. In fact, a single observation on a slightly risk averse investor would make this investor "more risk averse" than any investor on which we do not have any observations. We therefore introduce a more careful concept: If, for some x, there is a portfolio y with a lower expected value than which is preferred to x by investor $\check{\mathrm{R}}$, and at the same time investor $\hat{\mathrm{R}}$ prefers x to y, then investor A is at least partially more risk averse than $\hat{\mathrm{R}}$. If $\check{\mathrm{R}}$ is partially more risk averse than $\hat{\mathrm{R}}$, but $\hat{\mathrm{R}}$ is not partially more risk averse than $\check{\mathrm{R}}$, then we conclude that $\check{\mathrm{R}}$ is more risk averse than $\hat{\mathrm{R}}$.

Define $\trianglerighteq_{\mathrm{RA}} \in \mathbb{A} \times \mathbb{A}$ as

$$\check{\mathrm{Q}} \trianglerighteq_{\mathrm{RA}} \hat{\mathrm{Q}} \text{ if there exists } x \in \mathbb{A} \text{ such that } \mathcal{RPL}(x, \check{\mathrm{Q}}) \cap \mathcal{RWL}(x, \hat{\mathrm{Q}}) \neq \varnothing; \tag{2}$$

if $\check{\mathrm{Q}} \trianglerighteq_{\mathrm{RA}} \hat{\mathrm{Q}}$, we say that $\check{\mathrm{Q}}$ is *partially revealed more risk averse than* $\hat{\mathrm{Q}}$. Then $\check{\mathrm{Q}}$ is *revealed more risk averse than* $\hat{\mathrm{Q}}$, written $\check{\mathrm{Q}} \vartriangleright_{\mathrm{RA}} \hat{\mathrm{Q}}$, if $\check{\mathrm{Q}} \trianglerighteq_{\mathrm{RA}} \hat{\mathrm{Q}}$ and [not $\hat{\mathrm{Q}} \trianglerighteq_{\mathrm{RA}} \check{\mathrm{Q}}$].

Define

$$\delta(\check{\Omega}, \hat{\Omega}) = \begin{cases} 1 & \text{if there are } \check{x}^i \prec_{\mathrm{E}} \hat{x}^j \text{ and} \\ & \left([\check{x}^i \, \check{\mathrm{R}} \, \hat{x}^j \text{ and } \hat{x}^j \, \hat{\mathrm{P}} \, \check{x}^i] \text{ or } [\check{x}^i \, \check{\mathrm{P}} \, \hat{x}^j \text{ and } \hat{x}^j \, \hat{\mathrm{R}} \, \check{x}^i]\right), \\ 0 & \text{otherwise.} \end{cases}$$

The following theorem only considers data which satisfy the Ssd-Garp. To see why, consider two portfolios x and y and let $L = 2$, $\pi = (1/3, 2/3)$, $x = (12, 0)$ and $y = (6, 6)$, such that $y \succ_{\mathrm{E}} x$ and $y \succ_{\mathrm{SSD}} x$. An investor may prefer x over y even though y has a higher expected value, but this cannot be the result of risk aversion. Such an investor can satisfy Garp, but not Ssd-Garp, and his behaviour cannot (should not) be considered a sign of risk aversion.

Theorem 2 *Suppose $\check{\Omega}$ and $\hat{\Omega}$ satisfy Ssd-Garp.*

1. *The following conditions are equivalent:*
   - *(i)* $\delta(\check{\Omega}, \hat{\Omega}) = 1$ *and* $\delta(\hat{\Omega}, \check{\Omega}) = 0$*;*
   - *(ii)* $\check{R}_{SSD} \vartriangleright_{RA} \hat{R}_{SSD}$*;*
   - *(iii) there exist* $\check{u}, \hat{u} \in \mathcal{U}$ *which Ssd-rationalise* $\check{\Omega}$ *and* $\hat{\Omega}$*, respectively, and there do not exist* $\check{v}, \hat{v} \in \mathcal{U}$ *which Ssd-rationalise* $\check{\Omega}$ *and* $\hat{\Omega}$*, respectively, such that for all* $x, y \in \mathbb{A}$ *with* $\boldsymbol{E}(x) < \boldsymbol{E}(y)$, $\hat{u}(x) > \hat{u}(y) \Rightarrow \check{u}(x) > \check{u}(y)$ *and* $\check{v}(x) > \check{v}(y) \Rightarrow \hat{v}(x) > \hat{v}(y)$.
2. *The following conditions are equivalent:*
   - *(i)* $\delta(\check{\Omega}, \hat{\Omega}) = \delta(\hat{\Omega}, \check{\Omega}) = 1$*;*
   - *(ii)* $\check{R}_{SSD} \trianglerighteq_{RA} \hat{R}_{SSD}$ *and* $\hat{R}_{SSD} \trianglerighteq_{RA} \check{R}_{SSD}$*;*
   - *(iii) there do not exist* $\check{u}, \hat{u} \in \mathcal{U}$ *which Ssd-rationalise* $\check{\Omega}$ *and* $\hat{\Omega}$*, respectively, such that for all* $x, y \in \mathbb{A}$ *with* $\boldsymbol{E}(x) < \boldsymbol{E}(y)$, $\hat{u}(x) > \hat{u}(y) \Rightarrow \check{u}(x) > \check{u}(y)$ *or* $\check{u}(x) > \check{u}(y) \Rightarrow \hat{u}(x) > \hat{u}(y)$.
3. *The following conditions are equivalent:*
   - *(i)* $\delta(\check{\Omega}, \hat{\Omega}) = \delta(\hat{\Omega}, \check{\Omega}) = 0$*;*

(ii) $[\text{not } \check{R}_{SSD} \trianglerighteq_{RA} \hat{R}_{SSD}]$ *and* $[\text{not } \hat{R}_{SSD} \trianglerighteq_{RA} \check{R}_{SSD}]$;

(iii) *there exist* $\check{u}, \hat{u} \in \mathcal{U}$ *and* $\check{v}, \hat{v} \in \mathcal{U}$ *which* Ssd*-rationalise* $\check{\Omega}$ *and* $\hat{\Omega}$*, respectively, such that for all* $x, y \in \mathbb{A}$ *with* $\boldsymbol{E}(x) < \boldsymbol{E}(y)$, $\hat{u}(x) > \hat{u}(y) \Rightarrow \check{u}(x) > \check{u}(y)$ *and* $\check{v}(x) > \check{v}(y) \Rightarrow \hat{v}(x) > \hat{v}(y)$.

Theorem 2 is quite powerful: It shows that it is necessary and sufficient to compare only choices observed by one of the two investors, even though the definition of $\trianglerighteq_{\mathrm{RA}}$ uses *all* $x \in \mathbb{A}$. The theorem therefore provides a nonparametric way to compare the risk aversion of two investors with only a finite number of comparisons. The third statement in the three parts of Theorem 2 provides strong support for the suggested definition of "revealed more risk averse than".

We say that two investors are (a) *similar* if $[\text{not } \check{\mathrm{R}}_{\mathrm{SSD}} \trianglerighteq_{\mathrm{RA}} \hat{\mathrm{R}}_{\mathrm{SSD}}]$ and $[\text{not } \hat{\mathrm{R}}_{\mathrm{SSD}} \trianglerighteq_{\mathrm{RA}} \check{\mathrm{R}}_{\mathrm{SSD}}]$ and (b) *not comparable* if $\check{\mathrm{R}}_{\mathrm{SSD}} \trianglerighteq_{\mathrm{RA}} \hat{\mathrm{R}}_{\mathrm{SSD}} \trianglerighteq_{\mathrm{RA}} \check{\mathrm{R}}_{\mathrm{SSD}}$. Cases (a) and (b) are the two possible cases if $[\text{not } \check{\mathrm{R}}_{\mathrm{SSD}} \triangleright_{\mathrm{RA}} \hat{\mathrm{R}}_{\mathrm{SSD}}]$ and $[\text{not } \hat{\mathrm{R}}_{\mathrm{SSD}} \triangleright_{\mathrm{RA}} \check{\mathrm{R}}_{\mathrm{SSD}}]$.

Case (a) implies that the two investors have very similar preferences which do not, in the strict sense, disagree which each other. The two investors are, in a different sense, still comparable: The comparison leads to the conclusion that the preferences of the two investors are not sufficiently different. Indeed, we cannot reject the hypothesis that the two investors have the same preferences underlying their choices, and we can find rationalising utility functions which either imply that the first investor is more risk averse than the second or vice versa (see Theorem 2.3.iii). Case (b) implies that either (1) the extent of risk aversion of at least one of the investors is not constant over the entire income range, or (2) that the two investors have different notions of risk.

3 APPLICATION

3.1 *Preliminaries*

Theorem 1 provides a testable condition for Ssd-rationalisation. If an investor does not satisfy Ssd-Garp (or not even Garp), we would like to have a test for "almost optimising" behaviour, or a measure for the severity of the violation of the axiom. One such measure is the Afriat Efficiency Index (Aei, Afriat 1972) or Critical Cost Efficiency Index, which is arguably the most popular of such measures. Reporting the Aei is a standard in experimental economics.[4]

To obtain the Aei for Garp, budgets are shifted towards the origin until a set of observations satisfies Garp. We will use the same idea to measure efficiency of choices in terms of Ssd-Garp: For $e \in [0,1]$, define the relations $\mathrm{R}^0(e)$ and $\mathrm{P}^0(e)$ as $x^i\, \mathrm{R}^0(e)\, x^j$ if $e\, p^i x^i \geq p^i x$ and $x^i\, \mathrm{P}^0(e)\, x^j$ if $e\, p^i x^i > p^i x$, and let $\mathrm{R}(e) = (\mathrm{R}^0(e))^+$ be the transitive closure. Then define $\mathrm{R}_{\mathrm{SSD}}(e)$ and $\mathrm{P}_{\mathrm{SSD}}(e)$ accordingly as is Eq. (1). The relation $\gtrsim_{\mathrm{SSD}} (e)$ is defined as $x \gtrsim_{\mathrm{SSD}} (e) y$ if $e\, x \gtrsim_{\mathrm{SSD}} (e)\, y$. We then say that Ω satisfies Ssd-Garp(e) if $[\text{not } x^i\, \mathrm{P}_{\mathrm{SSD}}(e)\, x^j]$ whenever $x^j\, \mathrm{R}_{\mathrm{SSD}}(e)\, x^i$. Then the Ssd-Aei is the largest number e such that Ssd-Garp(e) is satisfied. The Aei, of course, is defined in the same way, applied to the R relation. Note that the Aei can be interpreted as a measure of

[4] See, for example, Sippel (1997), Mattei (2000), Harbaugh et al. (2001), Andreoni and Miller (2002), Février and Visser (2004), Choi et al. (2007b), Fisman et al. (2007).

wasted income; that is, an investor with an Ssd-Aei of, say, 9/10 could have obtained the same level of utility by spending only 90% of what he actually spent to obtain this level. This is, however, based on the assumption that the investor is Ssd-rational; an investor who satisfies Garp but has a low Ssd-Aei should not be considered risk averse.

Bronars (1987) suggests a Monte Carlo approach to determine the power the test has against random behaviour. The approximate power of the test is the percentage of random choices which violate Garp; this can also be applies to Ssd-Garp. A high power does not, however, imply that the power remains high once we "allow" investors to deviate from 100% efficiency. This is also related to the problem that there is no natural definition for what constitutes a "high" or "low" Aei. But it is important to know what efficiency levels can be considered as high enough when screening the data for efficiency before further analytical steps are taken. Heufer (forthcoming) provides a detailed discussion of this point together with a procedure based on Monte-Carlo simulations and the reduction of the power the test has against random behaviour to determine which set of observations can be considered close enough to Garp. This can easily be adopted for Ssd-Garp. For the application to data in Section 3.2 we use the "measure of success" adaptation in Heufer (forthcoming) to determine which subjects to use. It is based on Selten's (1991) measure of predictive success for area theories and maximises the difference between the fraction of subjects and the fraction of random choice sets accepted as close enough to an axiom based on the efficiency index.

3.2 *Data Analysis*

We are using data by Choi et al. (2007a); for a detailed description the reader is referred to their article. Choi et al. asked ninety three subjects to choose one portfolio on each of fifty budget sets. In the symmetric treatment, the two assets paid off with probabilities $(\pi_1, \pi_2) = (1/2, 1/2)$. In the asymmetric treatment, the two assets paid off with probabilities $(\pi_1, \pi_2) = (1/3, 2/3)$. In one of the sessions the probabilities were $(\pi_1, \pi_2) = (2/3, 1/3)$ which is taken into account.

Only one of the subjects satisfies Garp, but even this subject does not satisfy Ssd-Garp.[5] Like Choi et al. (2007a), we therefore compute efficiency indices for the subjects and for generated sets of random choices. Figures 2 and 3 show the distribution of the Ssd-Aei for subjects and random choices, for the two different treatments, based on 1860 random choice sets. While most subjects in the asymmetric treatment show substantially higher Ssd-efficiency than random choices, a notable fraction of 41.3% (17.39%) has an efficiency level of less than .9 (.8), while this is the case for only 21.28% (12.77%) of subjects in the symmetric treatment. Subjects in the symmetric treatment have generally somewhat higher efficiency levels, but stochastic dominance is a rather simple concept with equal probabilities. It might indicate that a few subjects have some minor difficulties applying the concept of stochastic dominance in the asymmetric case.

Tables 1 and 2 summarise some results. For the symmetric treatment, based on the procedures described in Heufer (forthcoming), we should consider an Aei and and Ssd-Aei of $\bar{e} = .8401$ as

[5] In fact, this subject has an Ssd-Aei of .7341, which is the 5th lowest of all subjects in the asymmetric treatment. The choices indicate that this subject treated x_1 and x_2 as homogeneous goods despite the asymmetric probabilities. This highlights the importance of testing Ssd-Garp.

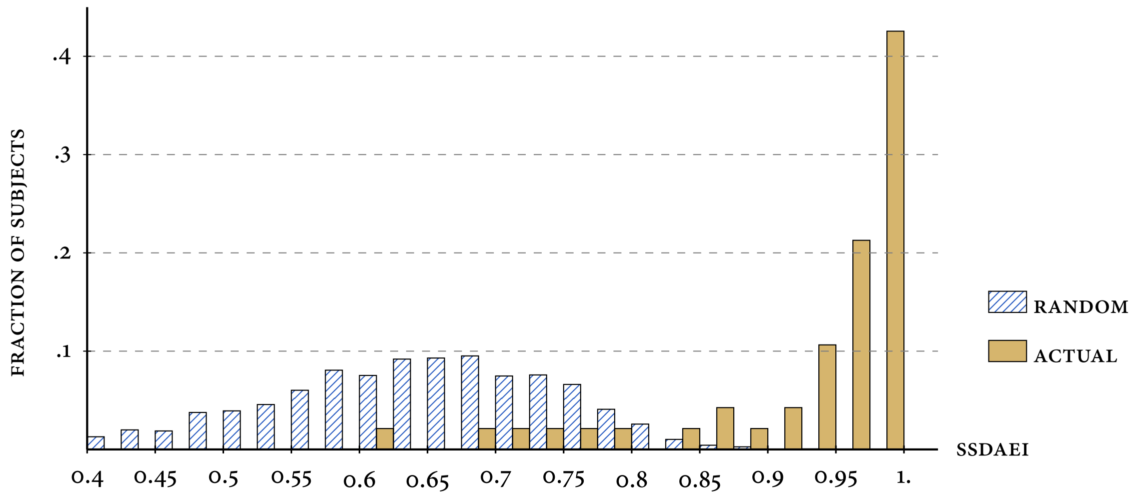


Figure 2: Ssd-Aei for symmetric treatment: ▨ shows the distribution for random choices, ■ for actual subjects. Data from Choi et al. (2007a).

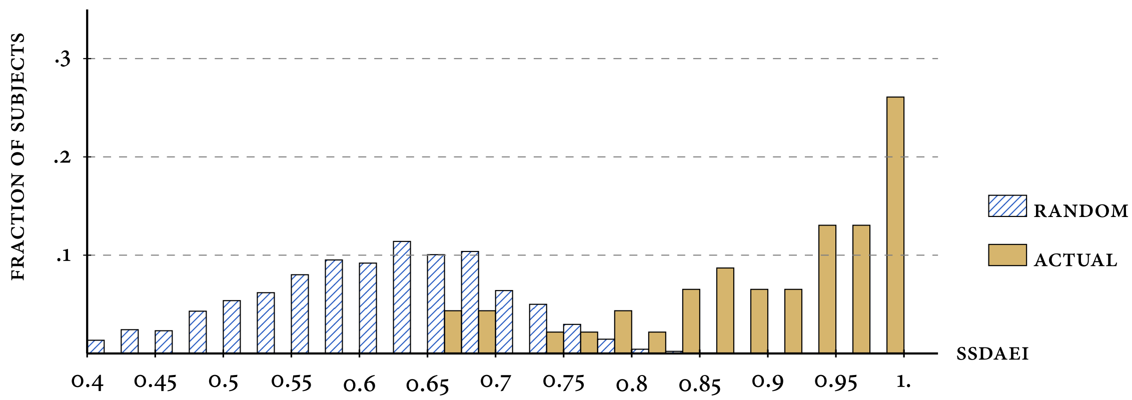


Figure 3: Ssd-Aei for asymmetric treatment: ▨ shows the distribution for random choices, ■ for actual subjects. Data from Choi et al. (2007a).

sufficient. For the asymmetric treatment, these values are $\bar{e} = .8396$ for the Aei and $\bar{e} = .7791$ for the Ssd-Aei. We require that subjects satisfy both requirements.

We compare the choices of subjects *corrected by their individual Ssd-Aei-level*, that is, we base the comparison on the $R_{SSD}(e)$ relation, where e is the subject's Ssd-Aei.[6] With 41 accepted subjects for the symmetric treatment (39 for the asymmetric treatment) we have 1640 (1482) comparisons. In 63.54% of all cases we find that one of the subject is revealed less or more risk averse than the other (54.25% for the asymmetric treatment). In 8.29% (12.96%) of the cases, neither subject is partially more risk averse than the other, that is, these subjects have similar preferences. In 28.17% (32.79%) of all cases, both subjects are partially revealed preferred to each other, rendering them incomparable.

We also compare subjects at the minimum Ssd-Aei-level of each pair of subjects, that is, we apply the same (low) efficiency standard to both of them, which somewhat increases the fraction of subjects who are comparable. Tables 1 and 2 summarise these main results.

SYMMETRIC TREATMENT			
	AEI	SSD-AEI	BOTH
EFFICIENCY REQUIREMENT $\bar{e}$	.8401	.8401	
NO. OF SUBJECTS WITH $e \geq \bar{e}$	41	41	41
CORRELATION BETWEEN	PEARSON		SPEARMAN RANK
SUBJECTS' AEI AND SSD-AEI	.9954		.9936
RANDOM AEI AND SSD-AEI	.9811		.9786
OF THOSE SUBJECTS WHICH SATISFY $\bar{e}$ REQUIREMENTS:			
CORRELATION BETWEEN	PEARSON		SPEARMAN RANK
AEI AND SSD-AEI	.9721		.9904
COMPARABILITY OF RISK AVERSION	MORE/LESS	NEITHER	BOTH
FRACTION AT INDIVIDUAL SSD-AEI	63.54%	8.29%	28.17%
FRACTION AT MINIMUM SSD-AEI	63.66%	20.73%	15.61%

Table 1: Summary statistics for the symmetric treatment with $(\pi_1, \pi_2) = (½, ½)$. See text for a description. Data from Choi et al. (2007a).

Choi et al. (2007a) estimate parameters α and ρ of a utility function $U : \mathbb{A} \to \mathbb{R}$, where $U(x) = \min\{(\pi_2/\pi_1)\,\alpha\, u(x_1) + u(x_1), u(x_1) + (\pi_2/\pi_1)\,\alpha\, u(x_2)\}$ and $u : \mathbb{R}_+ \to \mathbb{R}$ takes the form of a power utility function $u(x_i) = x_i^{1-\rho}/(1-\rho)$. If $\alpha > 1$, this utility function exhibits disappointment aversion (Gul 1991). Thus, α is a measure of disappointment aversion, and ρ is the Arrow-Pratt measure of relative risk aversion.

We compare all subjects to choices generated by maximising the utility function U for different parameters. As parameters, we choose the α and ρ for different percentiles, that is, we use α and ρ such that 5%, 25%, 50%, 75%, and 95% of all subjects have the same or lower individual estimates.

[6] We subtract an additional .001 from the efficiency level, as the computation of the efficiency levels is only an approximation.

ASYMMETRIC TREATMENT			
	AEI	SSD-AEI	BOTH
EFFICIENCY REQUIREMENT $\bar{e}$	.8396	.7791	
NO. OF SUBJECTS WITH $e \geq \bar{e}$	42	40	39
CORRELATION BETWEEN	PEARSON		SPEARMAN RANK
SUBJECTS' AEI AND SSD-AEI	.7192		.6671
RANDOM AEI AND SSD-AEI	.8866		.8585
OF THOSE SUBJECTS WHICH SATISFY $\bar{e}$ REQUIREMENTS:			
CORRELATION BETWEEN	PEARSON		SPEARMAN RANK
AEI AND SSD-AEI	.6653		.6967
COMPARABILITY OF RISK AVERSION	MORE/LESS	NEITHER	BOTH
FRACTION AT INDIVIDUAL SSD-AEI	54.25%	12.96%	32.79%
FRACTION AT MINIMUM SSD-AEI	44.26%	35.63%	20.11%

Table 2: The same summary statistics as in Table 1, here for the asymmetric treatment with $(\pi_1, \pi_2) = (⅓, ⅔)$. Data from Choi et al. (2007a).

Table 3 shows the result for the symmetric treatment for which we find that the nonparametric comparison corresponds very well to the parameter estimates. For example, using the median α and ρ we find that at individual Ssd-Aei-levels 31.71% of subjects are less risk averse, 9.76% of subjects have similar preferences, and 34.15% of subjects are more risk averse. Table 4 shows the same result for the asymmetric treatment, where only 2.56% of subjects are less risk averse while 58.97% of subjects are more risk averse than the preferences described by a utility function with median parameters.

SYMMETRIC TREATMENT						
	CRRA		SUBJECT RISK AVERSION			
PERCENTILE	α	ρ	LESS	NEITHER	MORE	BOTH
5TH:	1.000	0.048	0.00%	0.00%	100.00%	0.00%
25TH:	1.000	0.165	0.00%	7.32%	87.80%	4.88%
50TH:	1.179	0.438	31.71%	9.76%	34.15%	24.39%
75TH:	1.477	0.794	68.29%	12.2%	4.88%	14.63%
95TH:	2.876	3.871	80.49%	9.76%	0.00%	9.76%

Table 3: Nonparametric comparison of subjects' risk aversion with a choices generated by a utility function with different parameters, here for the symmetric treatment. See text for a description. Data from Choi et al. (2007a).

As Choi et al. (2007a) estimate a two-parameter utility function, they cannot represent risk aversion as a single parameter. They therefore compute a risk premium r for every subject, which is the fraction of initial wealth that gives the same utility as a lottery with 50-50 odds of winning or losing the initial amount. We can compare the ranking of subjects' risk aversion obtained by r with the nonparametric interpersonal comparison. If of two subjects, the first has a higher r than

ASYMMETRIC TREATMENT						
	CRRA		SUBJECT RISK AVERSION			
PERCENTILE	α	ρ	LESS	NEITHER	MORE	BOTH
5TH:	1.000	0.048	0.%	2.56%	92.31%	5.13%
25TH:	1.000	0.165	0.%	2.56%	82.05%	15.38%
50TH:	1.179	0.438	2.56%	17.95%	58.97%	20.51%
75TH:	1.477	0.794	41.03%	20.51%	17.95%	20.51%
95TH:	2.876	3.871	56.41%	0.00%	2.56%	41.03%

Table 4: The same statistics as in Table 3, here for the asymmetric treatment. Data from Choi et al. (2007a).

the second, then ideally the first subject is revealed more risk averse than the second. If this is not the case, and the second subject is revealed more risk averse than the first or both have similar preferences, then the difference ranking of the two subjects by r should be small. Table 5 shows how often the ranking of two subjects, of which one is revealed more risk averse than the other, differ by more than 1, 2, 4, 8, and 12 ranks.

For the symmetric treatment, a measure of risk aversion can also be obtained by computing the share of tokens allocated to the cheaper asset. The higher the share, the less risk averse a subject should be. Table 5 also shows how often the ranking of two subjects differs by this measure of risk aversion, where we use the average share of tokens and call this measure $\tilde{r}$.

SYMMETRIC TREATMENT						
	BY MORE THAN . . . RANKS					
FRACTION OF COMPARISONS WHICH	0	1	2	4	8	12
DISAGREE WITH RANKING BY r	23.55%	20.04%	17.56%	14.26%	8.88%	6.20%
DISAGREE WITH RANKING BY $\tilde{r}$	23.84%	21.70%	19.18%	15.00%	9.47%	6.71%
ASYMMETRIC TREATMENT						
	BY MORE THAN . . . RANKS					
FRACTION OF COMPARISONS WHICH	0	1	2	4	8	12
DISAGREE WITH RANKING BY r	27.36%	25.05%	22.02%	17.33%	11.12%	7.94%

Table 5: Difference in ranking of subjects by measures of risk aversion and their nonparametric comparisons. See text for a description.

Tables 6 and 7 in the appendix give the complete list of interpersonal comparisons between all subjects in the symmetric and asymmetric treatment, respectively, at individual SSD-AEI-levels. Figure 4 shows examples of revealed preferred and revealed worse sets of four different subjects based on the extended relation R_{SSD}. The first one is revealed more risk averse than most other subject, the second on is revealed less risk averse than most other subjects. The third one is an intermediate case which is similar to several other subjects, and revealed more and revealed less risk averse to some others. The last one is a subject that is incomparable with several others. The last one is particularly interesting as it nicely illustrates why some subjects are not comparable:

This subject exhibits almost risk neutrality around the 45° line, with a sudden sharp increase in risk aversion as the amount of any assets drops below 15.

4 DISCUSSION AND CONCLUSION

We have provided a method to account for first and second order stochastic dominance when analysing choice under uncertainty. This allows to test if there exists a well behaved utility function which rationalises such data and obeys stochastic dominance, and to extend the revealed preference relations recovered from such data. The application to the experimental data of Choi et al. (2007a) shows that while most subjects are reasonably close to such Ssd-rationality, although some clearly are not. On the one hand, the result therefore confirms previously drawn conclusions to a large extent. On the other hand, it shows that there are, albeit few, subjects who come close to Garp but exhibit strong violations of Ssd-rationality. This highlights that it is important to apply the tests for Ssd.

We have also provided a way to make Yaari's (1969) idea for comparative risk aversion operational based on revealed preferred and revealed worse sets. The central rationalisability theorem shows that if and only if the conditions for "revealed more risk averse" are satisfied, there exist utility functions which rationalise the two observations on two investors, such that the utility function of the more risk averse investor exhibits greater risk aversion for every portfolio. Furthermore there do not exist rationalising utility functions which exhibit greater risk aversion for the less risk averse investor.

The theorem also shows that it is sufficient to only compare a finite number of portfolios, namely those observed as choices, even though the revealed more risk averse relation is defined in terms of the revealed preferred and worse sets of all portfolios. It therefore leads to a nonparametric way to compare the risk aversion of two investors without relying on particular forms of utility.

Testing the experimental data of Choi et al. (2007a) for consistency with Ssd-rationality shows that, compared to random choices, strong consistency of most subjects is confirmed. The nonparametric approach to comparative risk aversion is useful as an alternative or complement to parametric estimation of risk aversion. It can serve as a robustness check for the parametric approach; the analysis in Choi et al. (2007a) is found to be quite robust for both treatments, but more so for the symmetric treatment. Obviously a nonparametric approach does not offer a distribution of parameters to describe risk attitudes in a given sample. However, it can be used to compute the fraction of investors which is less or more risk averse than *any* given preference and can therefore also offer a characterisation of risk preferences in a population.

If the nonparametric comparison is used as more than a robustness check, refinements based on efficiency levels are easy to apply. For example, one can compute the highest e at which two investors are comparable. This can also be used to measure how different preferences are.

Interpersonal comparisons based on revealed preferred and worse sets can also be usefully applied to other aspects of preferences, such as sense of fairness (Karni and Safra 2002a,b) or impartiality (Nguema 2003). For example, Karni and Safra (2002b) apply Yaari's (1969) notion of "is more risk averse than" to the concept of "has a stronger sense of fairness than". The results here

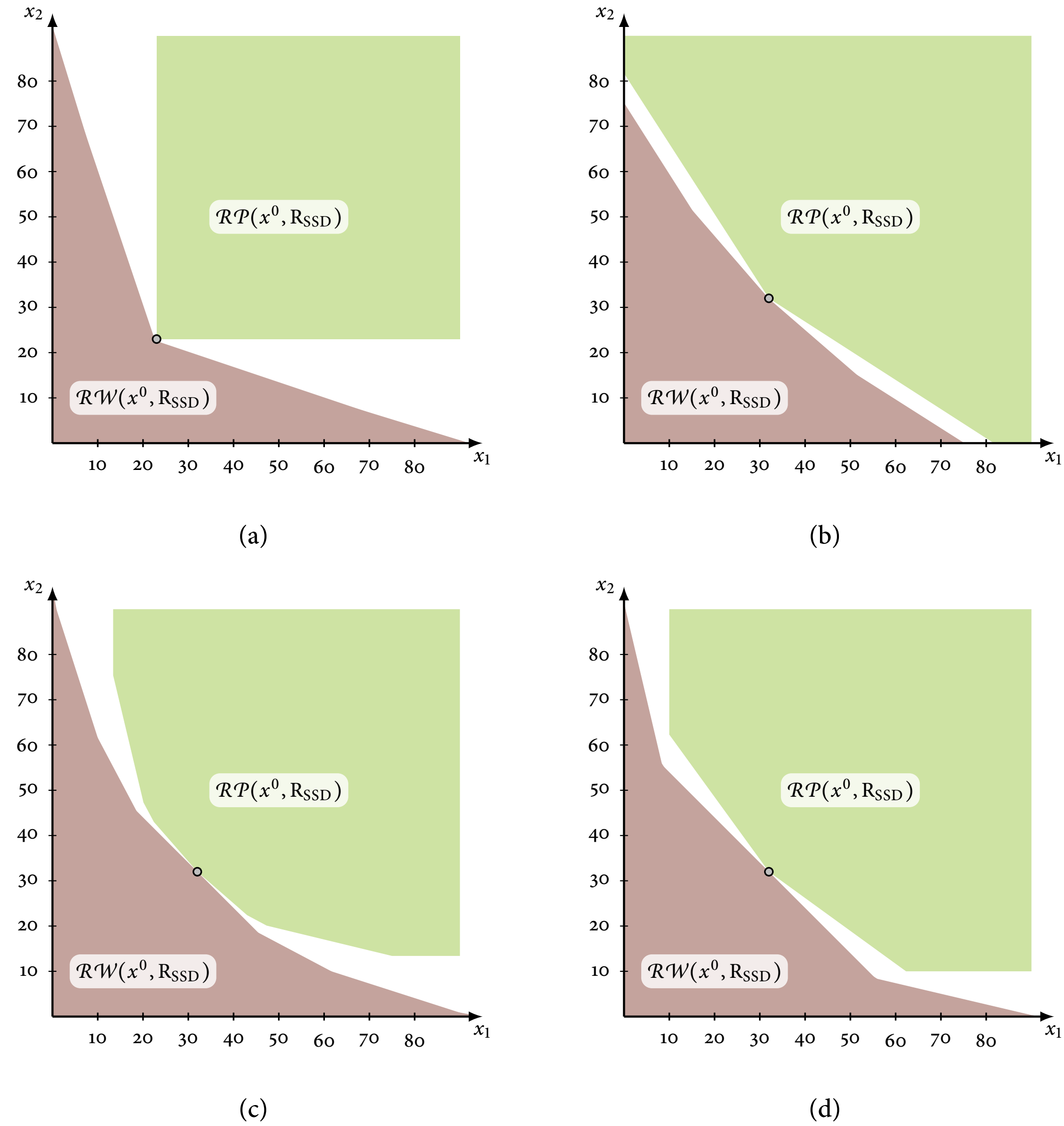


Figure 4: Examples of subjects' revealed preferred and revealed worse sets, from the symmetric treatment. (a) Subject number 23 (ID 304): A subject who is revealed more risk averse than most other subjects. (b) Subject number 26 (ID 307): A subject who is revealed less risk averse than most other subjects. (c) Subject number 8 (ID 208): A subject who is revealed more risk averse and revealed less risk averse than some other subject and has similar preferences as many other subjects. (d) Subject number 5 (ID 205): A subject who is incomparable with some other subjects. Data from Choi et al. (2007a).

can be translated to suit this interpersonal comparison of the sense of fairness. In particular, it is possible to compare two decision makers with only a finite number of comparisons between observed choices.

A APPENDIX

We only consider the SSD case here; proofs for the FSD case are simpler.

A.1 *Proof of the Lemmata*

Proof of Lemma 1 This follows directly from the fact that every risk averse expected utility maximiser will prefer x over y whenever $x \succsim_{\text{SSD}} y$ (see above): Let $\mathbf{EU}_u(x)$ denote the expected utility of $x \in \mathbb{A}$ with $u : \mathbb{A} \to \mathbb{R}$ being a continuous, increasing, and concave utility function. Then $x \succsim_{\text{SSD}} y$ if and only if $\mathbf{EU}_u(x) \geq \mathbf{EU}_u(y)$ for all such u. Suppose $z = \mu x + (1-\mu)y$ for $\mu \in (0,1)$; then $\mathbf{EU}_u(x) \geq \mathbf{EU}_u(y)$ implies $\mathbf{EU}_u(z) \geq \mathbf{EU}_u(y)$, and thus $z \succsim_{\text{SSD}} y$. ■

Proof of Lemma 2

(i) Let $ma(x,i)$ denote the maximal value of y_i such that $y \succsim_{\text{SSD}} x$. Then the set

$$HC(x) = \{y \in \mathbb{A} : \min(y) \geq \min(x) \text{ and } y_i \leq ma(x,i) \text{ for all } i = 1,\dots,L\}$$

is a hypercube in $\mathbb{R}^L_+$ which intersects the hyperplane $\mathcal{P}(x, \sim_{\text{E}})$ (except when $x_i = x_j$ for all $i,j = 1,\dots,L$, in which case the two sets only share the point x). Then $\mathcal{P}(x, \succsim_{\text{SSD}} \cap \sim_{\text{E}}) \subseteq HC(x) \cap \mathcal{P}(x, \sim_{\text{E}})$. By construction of $\hat{M}(x)$, $HC(x) \cap \mathcal{P}(x, \sim_{\text{E}}) = \text{CH}(\hat{M}(x))$ and $y \succsim_{\text{SSD}} x$ for all $y \in \hat{M}(x)$. Then by Lemma 1, $\text{CH}(\hat{M}(x)) \subseteq \mathcal{P}(x, \succsim_{\text{SSD}} \cap \sim_{\text{E}})$, and the first part of Lemma 2 follows.

(ii) It is obvious that $\mathcal{P}(x, \succsim_{\text{SSD}}) \subseteq \overline{\text{CMH}}(x, \succsim_{\text{SSD}} \cap \sim_{\text{E}})$. As $y \succsim_{\text{E}} x$ is a necessary condition for $y \succsim_{\text{SSD}} x$, consider any $y \succ_{\text{E}} x$, $y \notin \overline{\text{CMH}}(\hat{M}(x))$, and suppose $y \succsim_{\text{SSD}} x$. Let $y_j = \max(y)$ and let $z \sim_{\text{E}} x$ be such that $z_i = y_i$ for all $i \neq j$ and $z_j < y_j$. Then $F(\xi^i, y) = F(\xi^i, z)$ for all $\ell < n$. Thus if $y \succsim_{\text{SSD}} x$ then $z \succsim_{\text{SSD}} x$. But that contradicts the first part of Lemma 2.

■

A.2 *Proof of Theorem 1*

Note that $y \,\text{R}\, z$ implies $y \in \{x^i\}_{i=1}^N$ and that $\succsim_{\text{SSD}}$ is transitive.

Lemma 3 *If $x^i \, R^0 \, y \succsim_{SSD} x^j$, then $x^i \, R_{SSD} \, x^j$. If $x^i \, P^0 \, y \succsim_{SSD} x^j$, then $x^i \, P_{SSD} \, x^j$. This holds for all $y \in \mathbb{A}$, not just for $y \in \tau(\Omega)$.*

Proof We have $y \succsim_{\text{SSD}} x^j \Leftrightarrow y \in \mathcal{P}(x^j, \succsim_{\text{SSD}}) = \overline{\text{CMH}}(\hat{M}(x^j))$ by Lemma 2. Then $x^i \, \text{R}^0 \, y \Leftrightarrow y \in B(p^i)$ implies $B(p^i) \cap \overline{\text{CMH}}(\hat{M}(x^j)) \neq \varnothing$. Then for some $t^k \in \hat{M}(x^j)$ we must have $t^k \in B(p^i)$, thus $x^i \, \text{R}^0 \, t^k$. Because $t^k \in \tau(\Omega)$ and $t^k \succsim_{\text{SSD}} x^j$ it follows that $x^i \, \text{R}_{\text{SSD}} \, x^j$. Similarly for P^0. ■

Proof of Theorem 1
(1) ⇒ (2): This follows from non-satiation of the utility function which SSD-rationalises the set of observations. The proof is very similar to the proofs that can be found in Varian (1982) or Forges and Minelli (2009), and we omit it.

(2) ⇒ (1): The existence of a utility function $u \in \mathcal{U}$ which rationalises Ω follows from Afriat's Theorem. It is obvious that there are $u \in \mathcal{U}$ which rationalise $\succsim_{\mathrm{SSD}}$ (i.e., $u(x) \geq u(y)$ whenever $x \succsim_{\mathrm{SSD}} y$; see also the proof of Lemma 1). The existence of a continuous and non-satiated (but not necessarily concave) utility function which rationalises the set $\tau(\Omega)$ follows from Forges and Minelli's (2009, Proposition 3) generalisation of Afriat's Theorem.

We need to show that a utility function which rationalises $\tau(\Omega)$ is also concave and SSD-rationalises the data. Suppose $x \,\mathrm{R}_{\mathrm{SSD}}\, y$; we will show that [not $y \,\mathrm{P}_{\mathrm{SSD}}\, \mu(x,y)$] for all $\mu \in (0,1)$ with $\mu(x,y) = \mu x + (1-\mu) y$: $x^i \,\mathrm{R}\, x^j \,\mathrm{P}\, \mu(x^i, x^j)$ is excluded by GARP, $x \succsim_{\mathrm{SSD}} y \succ_{\mathrm{SSD}} \mu(x,y)$ is impossible by Lemma 1. So suppose $x^i \,\mathrm{R}\, z \succsim_{\mathrm{SSD}} x^j \,\mathrm{R}\, x^k \,\mathrm{P}^0\, x^\ell \,\mathrm{R}^0\, \mu(x^i, x^j)$. Then by Lemma 3 and transitivity, $x^i \,\mathrm{P}\, x^\ell$ and $x^j \,\mathrm{P}\, x^\ell$. But $x^\ell \,\mathrm{R}^0\, \mu(x^i,x^j)$ is equivalent to $\mu(x^i,x^j) \in B(p^\ell)$, and all budgets boundaries are hyperplanes which separate $\mathbb{A}$ into two half-spaces; therefore either $x^\ell \,\mathrm{R}^0\, x^i$ or $x^\ell \,\mathrm{R}^0\, x^j$ or both. But this is excluded by SSD-GARP, and the existence of a $u \in \mathcal{U}$ which rationalises $\tau(\Omega)$ follows. That this u also SSD-rationalises the data follows with Lemma 3. ■

A.3 *Proof of Theorem 2*

Lemma 4 *Suppose $\check{\Omega}$ and $\hat{\Omega}$ satisfy Ax(Q). Then there are $\check{x}^j$ and $\hat{x}^i$ such that [$\hat{x}^i \,\hat{\mathrm{Q}}\, \check{x}^j$ and $\check{x}^j \,\phi(\check{\mathrm{Q}})\, \hat{x}^i$] or [$\check{x}^i \,\check{\mathrm{Q}}\, \hat{x}^j$ and $\hat{x}^j \,\phi(\hat{\mathrm{Q}})\, \check{x}^i$] if and only if $\mathcal{RP}(x^0, \check{\mathrm{Q}}) \cap \mathcal{RW}(x^0, \hat{\mathrm{Q}}) \neq \varnothing$; this holds for all $(\check{\mathrm{Q}}, \hat{\mathrm{Q}}) \in \{(\check{R}, \hat{R}), (\check{R}_{FSD}, \hat{R}_{FSD}), (\check{R}_{SSD}, \hat{R}_{SSD})\}$.*

Proof By GARP there is no $x \in \mathcal{RP}(x^0, \hat{\mathrm{R}})$ such that $x^0 \geq x$. Then by the definition of $\mathcal{RW}(\cdot, \check{\mathrm{R}})$, for all $x \in \mathcal{RW}(x^0, \check{\mathrm{R}})$, $\check{p}^i \check{x}^i \geq \check{p}^i x \Leftrightarrow x \in B(\check{p}^i)$ for at least one $i = 1, \ldots, \check{N}$. As $B(\check{p}^i)$ is a hyperplane and, by Proposition 1, $\mathcal{RP}(x^0, \hat{\mathrm{R}})$ is a convex polytope whose vertices are x^0 and all $\hat{x}^j \,\hat{\mathrm{R}}\, x^0$, there is at least one $\hat{x}^j \in \mathcal{RP}(x^0, \hat{\mathrm{R}}) \cap \mathcal{RW}(x^0, \check{\mathrm{R}})$. By definition, $\hat{x}^j \in \mathcal{RW}(x^0, \check{\mathrm{R}})$ implies that $\hat{x}^j$, if chosen by consumer $\check{\mathrm{R}}$, cannot be revealed preferred to x^0 without violating GARP: If $\hat{x}^j \,\check{\mathrm{R}}\, x^0$, then $\hat{x}^j \,\check{\mathrm{R}}\, \check{x}^k$ and $\check{x}^k \,\check{\mathrm{P}}\, \hat{x}^j$. But $\hat{x}^j \,\hat{\mathrm{R}}\, x^0$, thus $\hat{x}^j \,\hat{\mathrm{R}}\, \check{x}^k$. Then $\hat{x}^j \,\hat{\mathrm{R}}\, \check{x}^k$ and $\check{x}^k \,\check{\mathrm{P}}\, \hat{x}^j$; and similarly for [$\check{x}^i \,\check{\mathrm{R}}\, \hat{x}^j$ and $\hat{x}^j \,\hat{\mathrm{P}}\, \check{x}^i$]. Thus the Lemma holds for $\check{\mathrm{R}}$ and $\hat{\mathrm{R}}$. The rest follows from the fact that $\succsim_{\mathrm{FSD}}$ and $\succsim_{\mathrm{SSD}}$ are the same for both investors. ■

Lemma 5 *Suppose $\check{\Omega}$ and $\hat{\Omega}$ satisfy SSD-GARP. Then $\check{R}_{SSD} \trianglerighteq_{RA} \hat{R}_{SSD}$ if and only if $\delta(\check{\Omega}, \hat{\Omega}) = 1$.*

Proof The theorem states that $\check{\mathrm{R}}_{\mathrm{SSD}} \trianglerighteq_{\mathrm{RA}} \hat{\mathrm{R}}_{\mathrm{SSD}} \Leftrightarrow \delta(\check{\Omega}, \hat{\Omega}) = 1$. It is obvious that $\delta(\check{\Omega}, \hat{\Omega}) = 1 \Rightarrow \check{\mathrm{R}}_{\mathrm{SSD}} \trianglerighteq_{\mathrm{RA}} \hat{\mathrm{R}}_{\mathrm{SSD}}$. We will show that $\delta(\check{\Omega}, \hat{\Omega}) = 0$ implies [not $\check{\mathrm{R}}_{\mathrm{SSD}} \trianglerighteq_{\mathrm{RA}} \hat{\mathrm{R}}_{\mathrm{SSD}}$].

Suppose $\delta(\check{\Omega}, \hat{\Omega}) = 0$ and $\check{\mathrm{R}}_{\mathrm{SSD}} \trianglerighteq_{\mathrm{RA}} \hat{\mathrm{R}}_{\mathrm{SSD}}$. Then there does not exist a $\check{x}^i \precsim_{\mathrm{E}} \hat{x}^j$ such that $\check{x}^i \,\check{\mathrm{R}}_{\mathrm{SSD}}\, \hat{x}^j \,\hat{\mathrm{P}}_{\mathrm{SSD}}\, \check{x}^i$, but still $\mathcal{RPL}(z^0, \check{\mathrm{R}}_{\mathrm{SSD}}) \cap \mathcal{RWL}(z^0, \hat{\mathrm{R}}_{\mathrm{SSD}}) \neq \varnothing$. Then by Proposition 1 and Lemma 4, there is an $\check{t}^i \in \tau(\check{\Omega})$ such that $\check{t}^i \in \mathcal{RPL}(z^0, \check{\mathrm{R}}_{\mathrm{SSD}}) \cap \mathcal{RWL}(z^0, \hat{\mathrm{R}}_{\mathrm{SSD}})$. By SSD-GARP and Theorem 1, we cannot have $z^0 \succ_{\mathrm{SSD}} \check{t}^i$, and because $z^0 \succ_{\mathrm{E}} \check{t}^i$, we cannot have $\check{t}^i \succsim_{\mathrm{SSD}} z^0$. Then either

$\check{t}^i = \check{x}^i \,\check{\mathrm{R}}\, z^0$ or there is an $\check{x}^i$ such that $\check{t}^i \succsim_{\mathrm{SSD}} \check{x}^i \,\mathrm{R}\, z^0$; in either case, $\check{x}^i \in \mathcal{RPL}(z^0, \check{\mathrm{R}}_{\mathrm{SSD}}) \cap \mathcal{RWL}(z^0, \hat{\mathrm{R}}_{\mathrm{SSD}})$.

As $\check{x}^i \in \mathcal{RWL}(z^0, \hat{\mathrm{R}}_{\mathrm{SSD}})$ and [not $z^0 \succsim_{\mathrm{SSD}} \check{t}^i$], there must be some $\hat{t}^j \,\hat{\mathrm{R}}_{\mathrm{SSD}}\, \check{x}^i$, such that either (i) $z^0 \,\hat{\mathrm{R}}_{\mathrm{SSD}}\, \hat{t}^j$ or (ii) $z^0 \succsim_{\mathrm{SSD}} \mu\,\hat{t}^j + (1-\mu)\check{x}^i$ for some $\mu \in (0,1)$. In case (ii), $\hat{t}^j = \hat{x}^j$, $\hat{x}^j \succ_{\mathrm{E}} z^0$, and $\hat{x}^j \,\hat{\mathrm{R}}\, \check{x}^i$; but then $\delta(\check{\Omega}, \hat{\Omega}) = 1$, a contradiction. Thus, $z^0 \,\hat{\mathrm{R}}_{\mathrm{SSD}}\, \hat{t}^j$. Because $\hat{t}^j = \hat{x}^j = z^0$ implies $\delta(\check{\Omega}, \hat{\Omega}) = 1$, $z^0 \,\hat{\mathrm{R}}_{\mathrm{SSD}}\, \hat{t}^j$ implies $z^0 \succsim_{\mathrm{SSD}} \hat{t}^j$ as z^0 cannot be preferred to $\hat{t}^j$ in any other way.

Then $\check{x}^i \,\check{\mathrm{R}}_{\mathrm{SSD}}\, z^0$ and $z^0 \succsim_{\mathrm{SSD}} \hat{t}^j$ imply $\check{x}^i \,\check{\mathrm{R}}_{\mathrm{SSD}}\, \hat{t}^j \succsim_{\mathrm{SSD}} \hat{x}^j$, where $\hat{t}^j \in \hat{M}(\hat{x}^j)$. But then $\check{x}^i \,\check{\mathrm{R}}_{\mathrm{SSD}}\, \hat{x}^j$, thus $\delta(\check{\Omega}, \hat{\Omega}) = 1$ implies that $\check{x}^i \succ_{\mathrm{E}} \hat{x}^j$. Then $\hat{t}^j \sim_{\mathrm{E}} \hat{x}^j$ implies [not $\hat{t}^j \succsim_{\mathrm{SSD}} \check{x}^i$], thus $\hat{x}^j \,\hat{\mathrm{R}}_{\mathrm{SSD}}\, \check{x}^j$.

To summarise, we have $z^0 \succsim_{\mathrm{SSD}} \check{x}^i$, $\check{x}^i \succ_{\mathrm{E}} \hat{x}^j$, $\check{x}^i \,\check{\mathrm{R}}_{\mathrm{SSD}}\, \hat{x}^j$, and $\hat{x}^j \,\hat{\mathrm{R}}_{\mathrm{SSD}}\, \check{x}^i$. Then with

$$\mathcal{P}(z^0, \precsim_{\mathrm{SSD}}) \cap \mathcal{P}(\check{x}^i, \prec_{\mathrm{E}}) \subseteq \mathcal{P}(\check{x}^i, \prec_{\mathrm{SSD}}),$$

we obtain that $z^0 \succsim_{\mathrm{SSD}} \hat{x}^j$ and $\check{x}^i \succ_{\mathrm{E}} \hat{x}^j$ implies $\check{x}^i \succ_{\mathrm{SSD}} \hat{x}^j$. But $\hat{x}^j \,\hat{\mathrm{R}}_{\mathrm{SSD}}\, \check{x}^i$, which contradicts Ssd-Garp. ■

Proof of Theorem 2 The equivalence of (i) and (ii) for all three parts of the theorem follows immediately from Lemma 5. The equivalence of (i) and (iii) then follows from the definition of $\mathcal{RP}$ and $\mathcal{RW}$, the equivalence of (i) and (ii), and Theorem 1. ■

A.4 *Tables: Interpersonal Comparisons of Subjects*

Tables 6 and 7 show the complete list of interpersonal comparisons between all subjects in the symmetric and asymmetric treatment, respectively.

Symmetric Treatment: Part I																					
	2	3	4	5	6	7	8	9	10	12	13	14	15	16	17	18	19	20	21	22	23
2	•	¤	–	–	▶	–	◁	◁	▶	¤	◁	–	–	–	–	–	◁	▶	◁	▶	▶
3	¤	•	–	–	–	◁	▶	–	–	–	◁	▶	–	◁	◁	–	◁	¤	◁	¤	–
4	–	–	•	–	–	◁	–	–	▶	◁	◁	–	–	◁	–	–	◁	–	◁	¤	▶
5	–	–	–	•	▶	–	–	▶	▶	▶	◁	▶	–	–	–	¤	–	▶	–	–	▶
6	◁	–	–	◁	•	◁	◁	◁	–	▶	◁	–	–	◁	◁	◁	–	–	◁	–	–
7	–	▶	▶	–	▶	•	▶	◁	▶	▶	▶	–	–	▶	–	–	▶	▶	◁	▶	▶
8	▶	◁	–	–	▶	◁	•	◁	▶	¤	◁	▶	–	¤	▶	–	¤	▶	◁	¤	▶
9	▶	–	–	◁	▶	▶	▶	•	▶	▶	¤	–	–	▶	▶	–	▶	▶	◁	▶	–
10	◁	–	◁	◁	–	◁	◁	◁	•	◁	◁	◁	◁	◁	◁	◁	◁	◁	◁	◁	¤
12	¤	–	▶	◁	◁	◁	¤	◁	▶	•	◁	–	–	◁	▶	–	◁	▶	◁	◁	▶
13	▶	▶	▶	▶	▶	◁	▶	¤	▶	▶	•	▶	¤	¤	▶	▶	▶	▶	◁	▶	▶
14	–	◁	–	◁	–	–	◁	–	▶	–	◁	•	–	◁	¤	◁	◁	–	◁	▶	▶
15	–	–	–	–	–	–	–	–	▶	–	¤	–	•	–	–	–	▶	–	–	–	▶
16	–	▶	▶	–	▶	◁	¤	◁	▶	▶	¤	▶	–	•	▶	▶	▶	▶	◁	▶	▶
17	–	▶	–	–	▶	–	◁	◁	▶	◁	◁	¤	–	◁	•	–	◁	▶	◁	¤	▶
18	–	–	–	¤	▶	–	–	–	▶	–	◁	▶	–	◁	–	•	–	–	–	–	▶
19	▶	▶	▶	–	–	◁	¤	◁	▶	▶	◁	▶	◁	◁	▶	–	•	–	◁	▶	▶
20	◁	¤	–	◁	–	◁	◁	◁	▶	◁	◁	–	–	◁	◁	–	–	•	◁	◁	▶
21	▶	▶	▶	–	▶	▶	▶	▶	▶	▶	▶	▶	–	▶	▶	–	▶	▶	•	▶	▶
22	◁	¤	¤	–	–	◁	¤	◁	▶	▶	◁	◁	–	◁	¤	–	◁	▶	◁	•	▶
23	◁	–	◁	◁	–	◁	◁	–	¤	◁	◁	◁	◁	◁	◁	◁	◁	◁	◁	◁	•
24	–	–	–	–	–	◁	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
25	◁	◁	–	◁	◁	◁	◁	◁	▶	◁	◁	–	–	◁	–	◁	◁	◁	◁	◁	▶
26	▶	▶	▶	–	▶	¤	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	¤	▶	▶
27	◁	¤	¤	–	¤	◁	¤	◁	▶	◁	◁	¤	◁	◁	◁	◁	¤	¤	◁	◁	▶
28	¤	▶	–	◁	¤	◁	¤	◁	–	–	¤	▶	◁	¤	–	◁	▶	▶	◁	◁	▶
30	–	▶	▶	–	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	–	–	▶	–	▶	▶
31	▶	▶	▶	–	▶	◁	¤	–	▶	▶	–	¤	–	◁	▶	–	▶	▶	◁	¤	▶
32	–	▶	▶	–	▶	◁	¤	◁	▶	◁	◁	▶	–	◁	▶	–	◁	–	◁	¤	▶
33	–	▶	–	–	–	▶	–	▶	▶	▶	▶	▶	–	▶	–	–	▶	▶	▶	▶	▶
34	–	–	–	◁	–	◁	◁	¤	–	–	¤	–	–	–	◁	¤	◁	▶	◁	◁	–
35	◁	¤	◁	◁	¤	◁	◁	◁	▶	–	◁	◁	◁	◁	◁	–	◁	◁	◁	◁	¤
36	◁	¤	–	◁	–	◁	–	◁	▶	◁	◁	◁	◁	◁	◁	◁	¤	◁	◁	¤	▶
37	–	▶	▶	–	▶	¤	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	◁	▶	▶
38	–	▶	▶	–	▶	–	▶	◁	▶	▶	◁	▶	–	◁	▶	–	¤	▶	◁	▶	▶
39	▶	–	–	◁	▶	–	▶	▶	▶	▶	◁	▶	–	–	–	¤	–	▶	–	–	▶
41	▶	▶	–	–	▶	◁	¤	◁	▶	▶	◁	▶	–	◁	–	▶	–	▶	◁	–	▶
42	◁	◁	–	◁	–	◁	◁	◁	▶	¤	◁	▶	–	◁	–	◁	◁	–	◁	◁	▶
43	▶	▶	▶	▶	▶	¤	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	▶	¤	▶	▶
45	–	¤	¤	◁	–	◁	◁	–	▶	–	▶	▶	◁	◁	–	–	◁	▶	◁	¤	▶
46	▶	–	–	◁	▶	◁	◁	◁	–	▶	–	–	–	◁	–	◁	–	–	–	–	–

Table 6: Part I of the "more risk averse than" table for the symmetric treatment with $\pi = (½, ½)$ at individual Ssd-Aei-level. A ◁ indicates that the row subject is revealed more risk averse than the column subject, ▶ indicates that the column subject is revealed more risk averse than the row subject, and ¤ indicates that neither of the subjects is partially revealed more risk averse to the other. A "–" indicates that both subjects are partially more risk averse than the other. Subject numbers correspond to subject IDs 201-219 and 301-328, i.e. number 20 has ID 301 etc. Data from Choi et al. (2007a).

Symmetric Treatment: Part II																				
	24	25	26	27	28	30	31	32	33	34	35	36	37	38	39	41	42	43	45	46
2	–	▶	◁	▶	¤	–	◁	–	–	–	▶	▶	–	–	◁	◁	▶	◁	–	◁
3	–	▶	◁	¤	◁	◁	◁	◁	◁	–	¤	¤	◁	◁	–	◁	▶	◁	¤	–
4	–	–	◁	¤	–	◁	◁	◁	–	–	▶	–	◁	◁	–	–	–	◁	¤	–
5	–	▶	–	–	▶	–	–	–	–	▶	▶	▶	–	–	▶	–	▶	◁	▶	▶
6	–	▶	◁	¤	¤	◁	◁	◁	–	–	¤	–	◁	◁	◁	◁	–	◁	–	◁
7	▶	▶	¤	▶	▶	◁	▶	▶	◁	▶	▶	▶	¤	–	–	▶	▶	¤	▶	▶
8	–	▶	◁	¤	¤	◁	¤	¤	–	▶	▶	–	◁	◁	◁	¤	▶	◁	▶	▶
9	–	▶	◁	▶	▶	◁	–	▶	◁	¤	▶	▶	◁	▶	◁	▶	▶	◁	–	▶
10	–	◁	◁	◁	–	◁	◁	◁	◁	–	◁	◁	◁	◁	◁	◁	◁	◁	◁	–
12	–	▶	◁	▶	–	◁	◁	▶	◁	–	–	▶	◁	◁	◁	◁	¤	◁	–	◁
13	–	▶	◁	▶	¤	◁	–	▶	◁	¤	▶	▶	◁	▶	▶	▶	▶	◁	◁	–
14	–	–	◁	¤	◁	◁	¤	◁	◁	–	▶	▶	◁	◁	◁	◁	◁	◁	◁	–
15	–	–	◁	▶	▶	◁	–	–	–	–	▶	▶	◁	–	–	–	–	◁	▶	–
16	–	▶	◁	▶	¤	◁	▶	▶	◁	–	▶	▶	◁	▶	–	▶	▶	◁	▶	▶
17	–	–	◁	▶	–	◁	◁	◁	–	▶	▶	▶	◁	◁	–	–	–	◁	–	–
18	–	▶	◁	▶	▶	–	–	–	–	¤	–	▶	◁	–	¤	◁	▶	◁	–	▶
19	–	▶	◁	¤	◁	–	◁	▶	◁	▶	▶	¤	◁	¤	–	–	▶	◁	▶	–
20	–	▶	◁	¤	◁	◁	◁	–	◁	◁	▶	▶	◁	◁	◁	◁	–	◁	◁	–
21	–	▶	¤	▶	▶	–	▶	▶	◁	▶	▶	▶	▶	▶	–	▶	▶	¤	▶	–
22	–	▶	◁	▶	▶	◁	¤	¤	◁	▶	▶	¤	◁	◁	–	–	▶	◁	¤	–
23	–	◁	◁	◁	◁	◁	◁	◁	◁	–	¤	◁	◁	◁	◁	◁	◁	◁	◁	–
24	•	–	◁	–	–	–	▶	–	–	◁	–	–	◁	–	–	–	–	–	–	–
25	–	•	◁	◁	–	◁	◁	◁	◁	◁	▶	◁	◁	◁	◁	◁	◁	◁	◁	–
26	▶	▶	•	▶	▶	◁	▶	▶	▶	▶	▶	▶	▶	▶	–	▶	▶	◁	▶	▶
27	–	▶	◁	•	▶	◁	¤	¤	◁	–	▶	¤	◁	◁	◁	◁	–	◁	¤	–
28	–	–	◁	◁	•	◁	▶	¤	◁	¤	▶	▶	◁	–	◁	◁	–	◁	–	◁
30	–	▶	▶	▶	▶	•	▶	▶	▶	–	▶	▶	▶	▶	–	▶	▶	▶	▶	–
31	◁	▶	◁	¤	◁	◁	•	▶	◁	▶	▶	¤	◁	◁	–	–	▶	◁	¤	–
32	–	▶	◁	¤	¤	◁	◁	•	◁	–	▶	▶	◁	◁	–	◁	▶	◁	¤	–
33	–	▶	◁	▶	▶	◁	▶	▶	•	–	▶	▶	▶	▶	–	▶	▶	▶	▶	–
34	▶	▶	◁	–	¤	–	◁	–	–	•	◁	¤	◁	–	¤	◁	–	◁	–	◁
35	–	◁	◁	◁	◁	◁	◁	◁	◁	▶	•	◁	◁	◁	◁	◁	◁	◁	◁	◁
36	–	▶	◁	¤	◁	◁	¤	◁	◁	¤	▶	•	◁	◁	◁	◁	▶	◁	¤	◁
37	▶	▶	◁	▶	▶	◁	▶	▶	◁	▶	▶	▶	•	▶	–	▶	▶	¤	▶	–
38	–	▶	◁	▶	–	◁	▶	▶	◁	–	▶	▶	◁	•	–	–	▶	◁	¤	–
39	–	▶	–	▶	▶	–	–	–	–	¤	▶	▶	–	–	•	–	▶	◁	–	▶
41	–	▶	◁	▶	▶	◁	–	▶	◁	▶	▶	▶	◁	–	–	•	▶	◁	–	–
42	–	▶	◁	–	–	◁	◁	◁	◁	–	▶	◁	◁	◁	◁	◁	•	◁	–	◁
43	–	▶	▶	▶	▶	◁	▶	▶	◁	▶	▶	▶	¤	▶	▶	▶	▶	•	▶	▶
45	–	▶	◁	¤	–	◁	¤	¤	◁	–	▶	¤	◁	¤	–	–	–	◁	•	–
46	–	–	◁	–	▶	–	–	–	–	▶	▶	▶	–	–	◁	–	▶	◁	–	•

Asymmetric Treatment: Part I																				
	1	2	3	4	5	7	8	9	10	11	13	14	15	16	17	18	19	20	22	23
1	●	◁	¤	▶	▶	–	¤	▶	▶	◁	–	¤	◁	▶	¤	¤	▶	¤	◁	◁
2	▶	●	◁	▶	–	–	–	▶	▶	◁	–	▶	–	–	▶	▶	▶	◁	–	–
3	¤	▶	●	–	–	–	–	–	–	¤	–	¤	¤	–	¤	▶	–	¤	▶	¤
4	◁	◁	–	●	▶	–	–	◁	◁	◁	–	◁	◁	◁	◁	◁	◁	◁	◁	◁
5	◁	–	–	◁	●	–	–	◁	–	◁	–	◁	◁	◁	◁	¤	–	◁	◁	¤
7	–	–	–	–	–	●	◁	◁	◁	◁	◁	–	–	◁	–	◁	–	–	◁	–
8	¤	–	–	–	–	▶	●	◁	¤	◁	–	▶	–	¤	◁	–	▶	–	–	–
9	◁	◁	–	▶	▶	▶	▶	●	▶	◁	▶	¤	–	¤	¤	¤	▶	◁	◁	–
10	◁	◁	–	▶	–	▶	¤	◁	●	◁	–	◁	–	¤	–	–	▶	◁	–	–
11	▶	▶	¤	▶	▶	▶	▶	▶	▶	●	▶	▶	–	▶	▶	▶	▶	▶	▶	▶
13	–	–	–	–	–	▶	–	◁	–	◁	●	–	–	–	–	▶	–	◁	–	–
14	¤	◁	¤	▶	▶	–	◁	¤	▶	◁	–	●	–	◁	¤	¤	▶	◁	◁	◁
15	▶	–	¤	▶	▶	–	–	–	–	–	–	–	●	–	▶	▶	–	–	–	▶
16	◁	–	–	▶	▶	▶	¤	¤	¤	◁	–	▶	–	●	◁	¤	▶	◁	–	–
17	¤	◁	¤	▶	▶	–	▶	¤	–	◁	–	¤	◁	▶	●	¤	▶	¤	¤	¤
18	¤	◁	◁	▶	¤	▶	–	¤	–	◁	◁	¤	◁	¤	¤	●	▶	¤	◁	◁
19	◁	◁	–	▶	–	–	◁	◁	◁	◁	–	◁	–	◁	◁	◁	●	◁	–	◁
20	¤	▶	¤	▶	▶	–	–	▶	▶	◁	▶	▶	–	▶	¤	¤	▶	●	▶	–
22	▶	–	◁	▶	▶	▶	–	▶	–	◁	–	▶	–	–	¤	▶	–	◁	●	▶
23	▶	–	¤	▶	¤	–	–	–	–	◁	–	▶	◁	–	¤	▶	▶	–	◁	●
24	¤	◁	◁	▶	¤	–	–	◁	–	◁	–	◁	◁	◁	¤	¤	▶	¤	–	◁
26	◁	◁	–	▶	–	–	–	◁	◁	◁	–	◁	–	◁	◁	¤	¤	◁	–	–
27	◁	◁	◁	▶	¤	▶	–	◁	▶	◁	–	¤	–	–	◁	¤	▶	◁	◁	◁
28	◁	–	–	▶	◁	¤	▶	◁	▶	◁	–	–	–	¤	◁	¤	–	◁	◁	–
29	◁	◁	◁	▶	▶	▶	–	◁	–	◁	–	¤	–	–	◁	▶	▶	◁	◁	–
30	◁	◁	–	¤	¤	▶	–	◁	–	◁	–	¤	◁	◁	¤	¤	▶	◁	◁	◁
31	¤	–	◁	▶	▶	▶	▶	▶	▶	◁	▶	▶	–	▶	▶	▶	▶	–	–	–
32	◁	–	–	–	–	¤	◁	◁	◁	◁	◁	◁	–	◁	–	◁	–	◁	◁	–
33	–	–	–	–	–	¤	◁	◁	–	◁	◁	◁	–	◁	–	¤	–	◁	◁	–
34	–	¤	¤	▶	▶	–	–	–	–	¤	–	◁	◁	–	¤	–	–	¤	–	¤
36	–	–	–	–	–	▶	▶	◁	▶	◁	▶	–	–	–	–	–	▶	◁	–	–
37	▶	▶	¤	▶	▶	▶	▶	▶	▶	¤	–	▶	◁	▶	▶	▶	▶	¤	▶	▶
38	▶	–	¤	▶	▶	▶	–	▶	▶	¤	–	–	¤	▶	▶	▶	▶	▶	▶	▶
39	¤	◁	◁	▶	–	–	–	▶	¤	◁	–	¤	–	–	◁	¤	–	◁	–	◁
41	◁	–	◁	▶	▶	▶	▶	¤	▶	◁	▶	¤	–	◁	◁	◁	–	▶	¤	–
42	¤	◁	◁	–	◁	–	–	–	–	◁	–	◁	◁	–	◁	◁	–	◁	◁	◁
43	¤	◁	–	–	¤	–	–	¤	▶	◁	–	¤	◁	◁	◁	◁	▶	◁	¤	◁
44	▶	◁	–	–	▶	–	◁	◁	◁	◁	◁	◁	◁	◁	◁	¤	–	◁	◁	◁
46	–	–	¤	–	¤	–	–	▶	▶	◁	–	¤	◁	–	¤	▶	–	¤	▶	▶

Table 7: Part I of the "more risk averse than" table for the asymmetric treatment with $\pi = (1/3, 2/3)$ at individual SSD-AEI-level. Subject numbers correspond to subject IDs 401-417, 501-520, and 601-609, i.e. number 18 has ID 501, number 38 has ID 601, etc.

Asymmetric Treatment: Part II																			
	24	26	27	28	29	30	31	32	33	34	36	37	38	39	41	42	43	44	46
1	¤	➤	➤	➤	➤	➤	¤	➤	–	–	–	◁	◁	¤	➤	¤	¤	◁	–
2	➤	➤	➤	–	➤	➤	–	–	–	¤	–	◁	–	➤	–	➤	➤	➤	–
3	➤	–	➤	–	➤	–	➤	–	–	¤	–	¤	¤	➤	➤	➤	–	–	¤
4	◁	◁	◁	◁	◁	¤	◁	–	–	◁	–	◁	◁	◁	◁	–	–	–	–
5	¤	–	¤	➤	◁	¤	◁	–	–	◁	–	◁	◁	–	◁	➤	¤	◁	¤
7	–	–	◁	¤	◁	◁	◁	¤	¤	–	◁	◁	◁	–	◁	–	–	–	–
8	–	–	–	◁	–	–	◁	➤	➤	–	◁	◁	–	–	◁	–	–	➤	–
9	➤	➤	➤	➤	➤	➤	◁	➤	➤	–	➤	◁	◁	◁	¤	–	¤	➤	◁
10	–	➤	◁	◁	–	–	◁	➤	–	–	◁	◁	◁	¤	◁	–	◁	➤	◁
11	➤	➤	➤	➤	➤	➤	➤	➤	➤	¤	➤	¤	¤	➤	➤	➤	➤	➤	➤
13	–	–	–	–	–	–	◁	➤	➤	–	◁	–	–	–	◁	–	–	➤	–
14	➤	➤	¤	–	¤	¤	◁	➤	➤	➤	–	◁	–	¤	¤	➤	¤	➤	¤
15	➤	–	–	–	–	➤	–	–	–	➤	–	➤	¤	–	–	➤	➤	➤	➤
16	➤	➤	–	¤	–	➤	◁	➤	➤	–	–	◁	◁	–	➤	–	➤	➤	–
17	¤	➤	➤	➤	➤	¤	◁	–	–	¤	–	◁	◁	➤	➤	➤	➤	➤	¤
18	¤	¤	¤	¤	◁	¤	◁	➤	¤	–	–	◁	◁	¤	➤	➤	➤	¤	◁
19	◁	¤	◁	–	◁	◁	◁	–	–	–	◁	◁	◁	–	–	–	◁	–	–
20	¤	➤	➤	➤	➤	➤	–	➤	➤	¤	➤	¤	◁	➤	◁	➤	➤	➤	¤
22	–	–	➤	➤	➤	➤	–	➤	➤	–	–	◁	◁	–	¤	➤	¤	➤	◁
23	➤	–	➤	–	–	➤	–	–	–	¤	–	◁	◁	➤	–	➤	➤	➤	◁
24	•	–	–	➤	◁	➤	◁	–	–	◁	–	◁	◁	➤	–	◁	➤	➤	◁
26	–	•	◁	◁	◁	¤	◁	–	◁	–	◁	◁	◁	◁	–	–	◁	◁	–
27	–	➤	•	➤	➤	¤	◁	➤	➤	◁	–	◁	◁	◁	➤	–	◁	➤	–
28	◁	➤	◁	•	–	¤	◁	¤	➤	–	◁	◁	–	–	◁	–	◁	◁	–
29	➤	➤	◁	–	•	➤	◁	–	–	¤	–	◁	◁	➤	◁	–	➤	➤	¤
30	◁	¤	¤	¤	◁	•	◁	◁	◁	–	–	◁	◁	◁	–	–	◁	¤	–
31	➤	➤	➤	➤	➤	➤	•	➤	➤	–	➤	◁	◁	➤	➤	➤	➤	➤	➤
32	–	–	◁	¤	–	➤	◁	•	¤	–	◁	◁	◁	◁	◁	–	–	–	–
33	–	➤	◁	◁	–	➤	◁	¤	•	–	◁	◁	–	◁	◁	–	–	➤	–
34	➤	–	➤	–	¤	–	–	–	–	•	–	◁	¤	–	–	➤	–	–	¤
36	–	➤	–	➤	–	–	◁	➤	➤	–	•	◁	◁	➤	◁	–	–	–	–
37	➤	➤	➤	➤	➤	➤	➤	➤	➤	➤	➤	•	◁	➤	➤	➤	➤	➤	➤
38	➤	➤	➤	–	➤	➤	➤	➤	–	¤	➤	➤	•	➤	–	➤	➤	➤	➤
39	◁	➤	➤	–	◁	➤	◁	➤	➤	–	◁	◁	◁	•	–	¤	➤	¤	◁
41	–	–	◁	➤	➤	–	◁	➤	➤	–	➤	◁	–	–	•	–	¤	➤	–
42	➤	–	–	–	–	–	◁	–	–	◁	–	◁	◁	¤	–	•	–	–	◁
43	◁	➤	➤	➤	◁	➤	◁	–	–	–	–	◁	◁	◁	¤	–	•	–	–
44	◁	➤	◁	➤	◁	¤	◁	–	◁	–	–	◁	◁	¤	◁	–	–	•	–
46	➤	–	–	–	¤	–	◁	–	–	¤	–	◁	◁	➤	–	➤	–	–	•

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