Unmediated and Mediated Communication Equilibria of Battle of the Sexes with Incomplete Information*

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Abstract

We consider the Battle of the Sexes game with incomplete information and allow two-sided cheap talk before the game is played. We characterise the set of fully revealing symmetric cheap talk equilibria. The best fully revealing symmetric cheap talk equilibrium, when it exists, has a desirable characteristic. When the players' types are different, it fully coordinates on the ex-post efficient pure Nash equilibrium. We also identify conditions under which less information disclosure at the cheap talk stage (either because of partial revelation by one type of each player or one-sided talk) can be welfare improving. We also analyse the mediated communication equilibria of the game.

Keywords: Battle of the Sexes, Cheap Talk, Mediation, Coordination, Efficiency.

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1 Introduction

We study the Battle of the Sexes game with private information about each player's "intensity of preference" for the other player's favorite outcome. Recall that the complete information Battle of the Sexes game is a coordination game with two pure (and one mixed) strategy Nash equilibria. The players need to coordinate their actions in order to achieve one of these equilibria. If the players use strategies corresponding to different (pure) equilibria, then they both end up in a miscoordinated outcome that is worse than either of the (pure) Nash equilibria. With incomplete information, while coordination is clearly desirable, it is not obvious who of the two players should concede and go along to the other's favorite outcome. Ex post efficiency (based on the unweighted sum of the two players' ex post utilities) demands that the concession is made by the player who suffers a smaller loss in utility.

We ask if the two players, when faced with a coordination problem of the above kind, can communicate with each other about their intensity of preference for the other's favorite outcome to achieve coordination and (ex-post) efficiency. We have in mind a cheap talk phase in which the two players make announcements about their respective intensity of preference or a mediated communication in which they report to and get recommendations from a mechanism. We are interested in addressing whether fully revealing cheap talk (i.e., when players simultaneously and truthfully reveal their intensity of preference) and symmetric actions can achieve coordinated, and possibly even ex-post efficient outcomes in some circumstances.

To address these questions, we take as our starting point an incomplete information version of the Battle of the Sexes game. This kind of a coordination game is appropriate for modelling issues like market entry (Dixit and Shapiro, 1985), product compatibility (Farrell and Saloner, 1988), networking (Katz and Shapiro, 1985) among other problems (such as public goods, credence goods, R&D problems). We however consider the Battle of the Sexes game with incomplete information in order to assess to which extent fully revealing cheap talk helps in achieving coordinated outcomes.

Following the seminal paper by Crawford and Sobel (1982), much of the cheap talk literature has focused on the sender-receiver framework whereby one player has private information but takes no action and the other player is uninformed but is responsible for taking a payoff relevant decision. This framework can be restrictive when we want to model social situations involving multiple players who all have private information and can take actions. For instance, in most standard complete information two-player games that are commonly studied, both players choose strategies from their respective strategy sets. If we want to analyse incomplete information variants of these games, we would naturally keep the structure of the action phase similar to the complete information game. This could potentially give rise to new issues that cannot be dealt with by extrapolating from the sender-receiver framework. Among the many new complications that one can bring to these incomplete information games with two-sided actions, the important ones are one-sided or two-sided private information, one-sided or two-sided cheap talk and simultaneous or sequential cheap talk.

Farrell (1987) has considered coordination in an entry-game that is similar to the Battle of the Sexes game with complete information. He has shown that cheap talk communication among the players can reduce the probability of miscoordination. Banks and Calvert (1992) characterised incentive compatible, (ex-ante) efficient mechanisms for a similar game and proved that in a cheap-talk set-up, ex-ante efficiency can be achieved under certain conditions. In related literature, Park (2002) considered a similar entry game and identified conditions for achieving efficiency and coordination.

We model the communication between two players as direct cheap talk and then as mediated communication using a mechanism. In the cheap talk protocol, the Battle of the Sexes game with incomplete information is augmented by an initial stage of cheap talk before the action phase. This cheap talk is two-sided, i.e., both players can make announcements simultaneously. The messages are directly related to the incomplete information of each player.

We analyse two-sided cheap talk equilibria and characterise the set of fully revealing symmetric cheap talk equilibria. We note that the best (in terms of ex-ante expected payoffs) fully revealing symmetric cheap talk equilibrium, when exists, has a desirable characteristic. When the players' types are different, it fully coordinates on the ex-post efficient pure Nash equilibrium. In this outcome, when players are of different types, a certain type makes some sort of a compromise or sacrifice by agreeing to coordinate on a less preferred Nash equilibrium of the underlying complete information game. Casual observation of anecdotal evidence suggests that people do exhibit such behavior that apparently contradicts traditional concepts of self-interested rationality. Why do some people make altruistic sacrifices and why are they concerned about fairness? Instead of just assuming that people have concerns about fairness or other peoples' utilities and looking at the implications of such behavior, we derive this kind of behavior as part of an equilibrium in a game with communication.

We also analyse mediated communication equilibria of the game, following Banks and Calvert (1992) who fully characterised the ex-ante efficient incentive compatible mechanism for a similar framework. It is well-known that any unmediated equilibrium can be obtained as a mediated equilibrium. We focus here on our best fully revealing symmetric cheap talk equilibrium and achieve this outcome as a mediated equilibrium. We show that the range of the prior for which this outcome exists as a mediated equilibrium is strictly larger than the range for the cheap talk equilibrium.

This paper is organised as follows. In Section 2 we introduce our Battle of the Sexes game with incomplete information, which can be preceded by a communication stage with a single round of simultaneous cheap talk. In Sections 3 and 4 respectively, we report our main results as to what outcomes can be achieved under fully revealing symmetric cheap talk and using symmetric mediated mechanisms. Section 5 offers some remarks on asymmetric fully revealing cheap talk equilibria and compares the payoffs from cheap talk and mediated equilibria, using an example. Section 6 concludes.

2 The Model

2.1 The Game

We first consider the standard Battle of the Sexes game with complete information as given below. Each of the two players has two strategies, namely, F (Football) and C (Concert). The payoffs corresponding to the outcomes are as in the following table. We will call this a Battle of the Sexes game with values t_1 and t_2, where $0 < t_1,\ t_2 < 1$.

		Wife (Player 2)	
		Football	Concert
Husband (Player 1)	Football	$1, t_2$	$0, 0$
	Concert	$0, 0$	$t_1, 1$

This game has two pure Nash equilibria: (F, F) and (C, C) and a mixed Nash equilibrium in which player 1 plays F with probability $\frac{1}{1+t_2}$ and player 2 plays C with probability $\frac{1}{1+t_1}$.

Now consider the Battle of the Sexes game with private information, in which the value of t_i is the private information for player i. We assume that t_i is a random variable whose realisation is only observed by player i. For $i = 1, 2$, we henceforth refer to t_i as player i's type. For simplicity, we assume that each t_i is a discrete[1] random variable that takes only two values L and H (where, $0 < L < H < 1$), that is, each player's type is independently drawn from the set $\{L, H\}$, according to a probability distribution with $\Pr(t_i = H) \equiv p \in [0, 1]$.

2.2 Unmediated Cheap Talk

We now consider a situation in which the players first have a round of cheap talk before they play the game. We thus study an extended game, in which an

[1] One may also consider a continuum of types. Ray (2009) indeed analyses an implementation problem in the spirit of Kar, Ray, Serrano (2009) for this game with continuum of types.

unmediated communication stage precedes the actual play of the above Battle of the Sexes game.

In the first (cheap talk) stage of this extended game, each player i simultaneously chooses a costless and nonbinding announcement τ_i from the set $\{L, H\}$. Then, given a pair of announcements (τ_1, τ_2), in the second (action) stage of this extended game, each player i simultaneously chooses an action s_i from the set $\{F, C\}$. The strategies of this extended games are formally described as follows.

An announcement strategy in the first stage for player i is a function $a_i : \{L, H\} \to \Delta(\{L, H\})$, $t_i \mapsto a_i(t_i)$, where $\Delta(\{L, H\})$ is the set of probability distributions over $\{L, H\}$. We write $a_i(H \,|t_i\,)$ for the probability that strategy $a_i(t_i)$ of player i with type t_i assigns to the announcement H. Thus, player i with type t_i's announcement τ_i is a random variable drawn from $\{L, H\}$ according to a probability distribution with $\Pr(\tau_i = H) \equiv a_i(H \,|t_i\,)$.

In the second stage, a strategy for player i is a function $\sigma_i : \{L, H\} \times \{L, H\} \times \{L, H\} \to \Delta(\{F, C\})$, $(t_i, \tau_1, \tau_2) \mapsto \sigma_i(t_i, \tau_1, \tau_2)$, where $\Delta(\{F, C\})$ is the set of probability distributions over $\{F, C\}$. We write $\sigma_i(F \,|t_i; \tau_1, \tau_2\,)$ for the probability that strategy $\sigma_i(t_i; \tau_1, \tau_2)$ of player i with type t_i assigns to the action F when the first stage announcements are (τ_1, τ_2). Thus, player i with type t_i's action choice s_i is a random variable drawn from $\{F, C\}$ according to a probability distribution with $\Pr(s_i = F) \equiv \sigma_i(F \,|t_i; \tau_1, \tau_2\,)$. Given a pair of action choices $(s_1, s_2) \in \{F, C\} \times \{F, C\}$, the players' actual payoffs are given by the relevant entry in the above type-specific payoff matrix of the Battle of the Sexes game.

Definition 1 *A strategy profile in the action phase is symmetric if* $\forall t, \tau_1, \tau_2 \in \{H, L\}$,$\sigma_i(F \,|t; \tau_1, \tau_2\,) = \sigma_j(C \,|t; \tau_2, \tau_1\,)$.

3 Cheap Talk Equilibrium

3.1 Two-sided cheap talk, fully revealing equilibrium

We consider a specific class of strategies in this section where we impose the property that the cheap talk announcement should be *fully revealing*.

Definition 2 *In the extended game, cheap talk is said to be fully revealing if the announcement strategy a_i for each player $i = 1, 2$ has the following property: if $t_i = H$, we have $a_i(H \,|H\,) = 1$, and if $t_i = L$, we have $a_i(H \,|L\,) = 0$.*

The above definition simply asserts that, in the communication stage, each player makes an announcement that coincides with that player's type: $\tau_i = t_i$.

Having fixed each player's announcement strategy a_i so that it is fully revealing, we next ask what form the second-stage strategies σ_1 and σ_2 should take. First note that, under fully revealing announcements, for player i with type t_i, a strategy in the second stage can be written as $\sigma_i(t_i, t_j)$.

We now restrict our attention to symmetric strategies in this stage of the extended game. We formally define symmetry assuming full revelation in the cheap talk stage.

Definition 3 *Under fully revealing cheap talk, a strategy profile in the action phase is symmetric if $\forall t_1, t_2 \in \{H, L\}$, $\sigma_i \left[F \,|t_1, t_2\,\right] = \sigma_j \left[C \,|t_2, t_1\,\right] \forall i, j \in \{1, 2\}$.*

Note that the above definition preserves symmetry for both players and the types for each player. We are interested in (Nash) equilibria[2] of the extended game in symmetric fully revealing strategies.

[2] We could also consider a perfect Bayesian equilibrium of this two-stage game. One would require beliefs μ_1 and μ_2, so as to render a fully revealing symmetric strategy-profile $((a_1, \sigma_1), (a_2, \sigma_2))$ a perfect Bayesian equilibrium. A belief μ_i for player i is a probability distribution over $\{L, H\}$, which represents player i's belief about player j's type, conditional on player j's announcement ($j = 1, 2$, $j \neq i$). It is obvious that the natural set of beliefs that would support a fully revealing symmetric equilibrium is the belief that corresponds with the announced type.

Definition 4 *A strategy-profile* $((a_1, \sigma_1), (a_2, \sigma_2))$ *is called a fully revealing symmetric cheap talk equilibrium if the announcement strategy* a_i *is fully revealing, the action strategy* σ_i *is symmetric for each player* i *and the profile is a Nash equilibrium of the extended game.*

We characterise the set of fully revealing symmetric cheap talk equilibria in this section. This two-stage game has possibly many (Nash) equilibria. As a first step towards the characterisation of this set, we observe the following[3].

Claim 1 *In a fully revealing symmetric cheap talk equilibrium* $((a_1, \sigma_1), (a_2, \sigma_2))$*, the players' strategies in the action phase must constitute a (pure or mixed) Nash equilibrium of the corresponding Battle of the Sexes game with complete information; that is,* $(\sigma_1(t_1, t_2), \sigma_2(t_1, t_2))$ *is a (pure or mixed) Nash equilibrium of the Battle of the Sexes game with values* t_1 *and* t_2*,* $\forall t_1, t_2 \in \{H, L\}$*.*

Claim 2 *In a fully revealing symmetric cheap talk equilibrium* $((a_1, \sigma_1), (a_2, \sigma_2))$*, conditional on the announcement profile* (H, H) *or* (L, L)*, the strategy profile in the action phase must be the mixed strategy Nash equilibrium of the corresponding complete information Battle of the Sexes game; that is, whenever* $t_1 = t_2$*,* $(\sigma_1(t_1, t_2), \sigma_2(t_1, t_2))$ *is the mixed Nash equilibrium of the Battle of the Sexes game with values* $t_1 = t_2$*.*

Based on the above claims, we can now identify all candidate equilibrium strategy profiles of the extended game that are fully revealing and symmetric.

Claim 2 implies that these candidate strategy profiles $(\sigma_1(t_1, t_2), \sigma_2(t_1, t_2))$ in the action stage has the property that $\sigma_i [F |t, t] = \sigma^i(tt)$ with $t = H,\ L$, where $\sigma^i(tt)$ is the mixed Nash equilibrium of the complete information Battle of the Sexes game with values t and t, $t = H,\ L$. Therefore these profiles are differentiated only by the actions played when $t_1 \neq t_2$, that is, when the players' types are (H, L) and (L, H).

As the strategies are symmetric, it is sufficient to characterise these candidate profiles only by $\sigma_1 [F |H, L]$. By symmetry, one could identify the full profile of

[3] The proofs of these claims are obvious and hence omitted.

actions, based on $\sigma_1 [F | H, L]$.

From Claim 1, there are only three possible candidates for $\sigma_1 [F | H, L]$ as the complete information Battle of the Sexes game with values H and L has three (two pure and one mixed) Nash equilibria. They are (i) $\sigma_1 [F | H, L] = \sigma^1(HL)$ where $\sigma^1(HL)$ is the probability of playing F in the mixed Nash equilibrium strategy of player 1 of the complete information Battle of the Sexes game with values H and L (ii) $\sigma_1 [F | H, L] = 1$ and (iii) $\sigma_1 [F | H, L] = 0$.

Therefore there are only three fully revealing symmetric strategy profiles that are candidate equilibria of the extended game. In these three candidate equilibria, in the cheap talk phase, players announce their types truthfully, i.e., player i(H-type) announces H and player i(L-type) announces L and then in the action phase, the players' strategies are one of the following:

In the first candidate strategy profile, the players play the mixed Nash equilibrium strategies of the complete information Battle of the Sexes game for all type profiles. We call this profile S_m.

In the second candidate strategy profile, the players play the mixed Nash equilibrium strategies of the complete information Battle of the Sexes game when both players' types are identical (by Claim 2), and they play (F, F) $((C, C))$, when only player 1's type is H (L). Note that in this profile players fully coordinate on one of the pure Nash equilibrium outcomes when the types are different; however, the outcome they coordinate to generates payoffs $(1, L)$ and thus is not (ex-post) efficient in the corresponding Battle of Sexes game with different types. We call this profile S_{ineff}.

In the third candidate strategy profile, the players play the mixed Nash equilibrium strategies of the complete information Battle of the Sexes game when both players types' are identical (by Claim 2), and they play (C, C) $((F, F))$, when only player 1's type is H (L). Note that in this profile players fully coordinate on a pure Nash equilibrium when the players' types are different and that the outcome they coordinate to generates the ex-post efficient payoff of $(1, H)$ in the corresponding Battle of the Sexes game with different types. We call this profile S_{eff}. Clearly among these three candidates, the third, whenever exists, is the best in terms of payoffs.

We now look at cases when these candidates are indeed equilibrium profiles.

Lemma 1 *S_m is not an equilibrium of the extended Battle of the Sexes game with fully revealing cheap talk.*

Proof. Under S_m, H-type player will reveal his type truthfully only if $p(\frac{H}{1+H}) + (1-p)(\frac{H}{1+H}) \geq p(\frac{H}{1+L}) + (1-p)(\frac{H}{1+L})$ where the LHS is the expected payoff from truthfully announcing H and the RHS is the expected payoff from announcing L and choosing the optimal action in the action phase given the deviation in the cheap talk phase. This inequality implies $\frac{1}{1+H} \geq \frac{1}{1+L}$ which can never be satisfied as $H > L$. Therefore, S_m is not an equilibrium of the extended game. ■

Lemma 2 *S_{ineff} is an equilibrium of the extended incomplete information Battle of the Sexes game with cheap talk only when $\frac{1+H}{1+L+L^2+L^2H} \leq p \leq \frac{1+L+HL-H^2}{1+L+HL+H^2L}$.*

Proof. Under S_{ineff}, H-type player will reveal his type truthfully only if

$p(\frac{H}{1+H}) + (1-p)(1) \geq p(H) + (1-p)(\frac{H}{1+L})$ where the LHS is the expected payoff from truthfully announcing H and the RHS is the expected payoff from announcing L and choosing the optimal action in the action phase given the deviation in the cheap talk phase. This inequality implies $p \leq \frac{1+L+HL-H^2}{1+L+HL+H^2L}$. Similarly, L-type player will reveal his type truthfully only if $p(L) + (1-p)(\frac{L}{1+L}) \geq p(\frac{H}{1+H}) + (1-p)(1)$ which implies $\frac{1+H}{1+L+L^2+L^2H} \leq p$. Hence the proof. ■

Lemma 3 *S_{eff} is an equilibrium of the extended incomplete information Battle of the Sexes game with cheap talk only when $\frac{L^2+L^2H}{1+L+L^2+L^2H} \leq p \leq \frac{HL+H^2L}{1+L+HL+H^2L}$.*

Proof. Under S_{eff}, H-type player will reveal his type truthfully only if $p(\frac{H}{1+H}) + (1-p)(H) \geq p(1) + (1-p)(\frac{H}{1+L})$ where the LHS is the expected payoff from truthfully announcing H and the RHS is the expected payoff from announcing L and choosing the optimal action in the action phase given the deviation in the cheap talk phase. This inequality implies $p \leq \frac{HL+H^2L}{1+L+HL+H^2L}$. Similarly, L-type player will reveal his type truthfully only if $p(1) + (1-p)(\frac{L}{1+L}) \geq$

$p(\frac{H}{1+H}) + (1-p)(L)$ which implies $\frac{L^2+L^2H}{1+L+L^2+L^2H} \leq p$. Hence the proof. ■

Based on the above lemmas, the following theorem now fully characterises the set of fully revealing symmetric equilibria of the extended Battle of the Sexes game.

Theorem 1 *(i) There does not exist any fully revealing symmetric equilibrium of the extended Battle of the Sexes game when* $p < \frac{L^2+L^2H}{1+L+L^2+L^2H}$ *and when* $p > \frac{1+L+HL-H^2}{1+L+HL+H^2L}$.

(ii) S_{eff} *is the only fully revealing symmetric equilibrium of the extended Battle of the Sexes game for* $\frac{L^2+L^2H}{1+L+L^2+L^2H} \leq p \leq \frac{1+H}{1+L+L^2+L^2H}$.

(iii) S_{ineff} *and* S_{eff} *are the only fully revealing symmetric equilibria of the extended Battle of the Sexes game for* $\frac{1+H}{1+L+L^2+L^2H} \leq p \leq \frac{HL+H^2L}{1+L+HL+H^2L}$.

(iv) S_{ineff} *is the only fully revealing symmetric equilibrium of the extended Battle of the Sexes game for* $\frac{HL+H^2L}{1+L+HL+H^2L} \leq p \leq \frac{1+L+HL-H^2}{1+L+HL+H^2L}$.

Proof. We observe that $\frac{L^2+L^2H}{1+L+L^2+L^2H} < \frac{1+H}{1+L+L^2+L^2H}$ and $\frac{HL+H^2L}{1+L+HL+H^2L} < \frac{1+L+HL-H^2}{1+L+HL+H^2L}$ as both L and H are less than 1. The theorem now follows immediately from the lemmas above. ■

As noted earlier, the equilibrium profile S_{eff} has a desirable characteristic. When the players' types are different, it fully coordinates on an outcome which is the ex-post efficient pure Nash equilibrium outcome. However, this equilibrium exists only for a specific range of, p, $\frac{L^2+L^2H}{1+L+L^2+L^2H} \leq p \leq \frac{HL+H^2L}{1+L+HL+H^2L}$. Note that $\frac{HL+H^2L}{1+L+HL+H^2L} < \frac{1}{2}$.

For $\frac{1+H}{1+L+L^2+L^2H} \leq p \leq \frac{HL+H^2L}{1+L+HL+H^2L}$, S_{eff} clearly is the best (in terms of payoffs) fully revealing symmetric equilibrium of the extended Battle of the Sexes game.

3.2 Two-sided cheap talk, partially revealing H - type, fully revealing L - type

Consider the following class of strategy profiles (call it S_3) of the extended incomplete information Battle of the Sexes game with cheap talk.

In the cheap talk phase, Player i(H-type) announces H with probability r and L with probability $1-r$ and Player i(L-type) announces L with probability 1. The following theorem characterises all possible equilibria within this class.

Theorem 2 *There exist a continuum of equilibria belonging to the class S_3 of the extended incomplete information Battle of the Sexes game with cheap talk for $p > \frac{H}{1+H}$. All these equilibria are characterised by the following:*

(i) $\sigma_1(F\,|H;H,H) = \sigma_1(F\,|H;H,L) = \frac{1}{1+H}$

(ii) $\sigma_1(F\,|H;L,H) = \sigma_1(F\,|H;L,L) = \frac{rp-p+H-Hp}{rp+rpH-p-pH}$

(iii) $\sigma_1(F\,|L;L,H) = \sigma_1(F\,|L;L,L) = 1$

The ex ante expected utility to Player 1 from an S_3 equilibrium is $\frac{H}{1+H}$.

Proof. See appendix. ■

3.3 Two-sided cheap talk, partially revealing H - type and L - type

Now, we allow both types of each player to randomise and cheat in the cheap talk announcement phase.

Consider the following class of strategy profiles (call it S_4) of the extended incomplete information Battle of the Sexes game with cheap talk.

In the cheap talk phase, Player i(H-type) announces H with probability r_1 and L with probability $1-r_1$ and Player i(L-type) announces H with probability r_2 and L with probability $1-r_2$.

The following theorem shows that there are equilibria in S_4 that improve on the equilibria in S_3 under certain conditions.

Theorem 3 *There is a continuum of equilibria belonging to the class S_4 of the extended incomplete information Battle of the Sexes game with cheap talk that improve on the equilibria in S_3 when $p > \frac{1}{1+H}$.*

Proof. See appendix. ■

3.4 One-sided cheap talk, fully revealing equilibrium

In this section, we allow only Player 1 to make cheap talk announcements and we assume he reports his type truthfully. Player 2 does not announce anything. Call this class of strategy profiles S_5.

Theorem 4 *There does not exist any equilibrium in the class S_5.*

3.5 One-sided cheap talk, partially revealing H - type, fully revealing L - type

Consider the following class of strategy profiles (call it S_6) of the extended incomplete information Battle of the Sexes game with cheap talk.

In the cheap talk phase, Player 1(H-type) announces H with probability r and L with probability $1-r$ and Player 1(L-type) announces L with probability 1. Player 2 does not announce anything. The following theorem characterises all possible equilibria within this class.

The following theorem shows that there are equilibria in S_6 that improve on the equilibria in S_3 under certain conditions.

Theorem 5 *There is a continuum of equilibria belonging to the class S_6 of the extended incomplete information Battle of the Sexes game with cheap talk that improve on the equilibria in S_3 when $p > \frac{1}{1+H}$. These equilibria in S_6 provide the same ex ante expected utility for Player 1 but strictly greater expected utility for Player 2 compared to S_3 as long as $p > \frac{1}{1+H}$.*

Proof. See appendix. ■

4 Mediated Equilibrium

Having characterised the fully revealing symmetric equilibria of the game with cheap talk, one may also analyse this game with mediated communication. Consider a situation in which players have access to a mediator who, based on the players' announcements in the communication stage, makes a non-binding recommendation to each player as to which action that player should adopt in the Battle of the Sexes game. Considering such mediated mechanisms is useful because they inform us about the limits to communication possibilities via cheap talk.

A mediated mechanism is a probability distribution over the product set of actions $\{(F,F),(F,C),(C,F),(C,C)\}$ for every profile of types. In a (direct) mediated communication process the players first report their types (H or L) to the mediated mechanism (mediator) and then the mediator picks an action profile according to the given probability distribution and informs the respective action to each player privately. The players then play the game.

As in the earlier section, we consider symmetric communication.

Definition 5 *A symmetric mediated mechanism is a probability distribution over the product set of actions $\{(F,F),(F,C),(C,F),(C,C)\}$ of the Battle of the Sexes game for every profile of reported types, as below:*

	F	C
F	$\frac{1-v_6-v_7}{2}$	v_7
C	v_6	$\frac{1-v_6-v_7}{2}$

HH

	F	C
F	$1-v_3-v_4-v_5$	v_5
C	v_4	v_3

HL

	F	C
F	v_3	v_5
C	v_4	$1-v_3-v_4-v_5$

LH

	F	C
F	$\frac{1-v_1-v_2}{2}$	v_2
C	v_1	$\frac{1-v_1-v_2}{2}$

LL

where all v_i's lie in the closed interval $[0,1]$.

Mediated mechanisms have been studied by Banks and Calvert (1992) in essentially the same setup as the one we consider here. It is easy to see that our version of the Battle of the Sexes with incomplete information can readily be obtained from Banks and Calvert's (1992) setup through a linear transformation of the players' payoffs. Our setting however differs in the sense that our mediator makes non-binding recommendations, and therefore needs to provide incentives for each player to follow that recommendation.

Definition 6 *A (symmetric) mediated mechanism is called a (symmetric) mediated equilibrium if it provides the players with incentives to truthfully reveal their types to the mediator, and provides the players with incentives to follow the mediator's recommendations following their type announcements.*

A (symmetric) mediated equilibrium thus can be characterised by a set of incentive-compatibility constraints. To be in equilibrium, a symmetric mediated mechanism as above must satisfy the following incentive compatibility constraints.[4]

IC1: Incentive compatibility for H-type to report $H \Longrightarrow$

$$(1-p)(1+H)\frac{v_1+v_2}{2}-p(1+H)\frac{v_6+v_7}{2}-(1-H)(v_3-\frac{1}{2})-[1-(1+H)p](v_4+v_5) \geq 0. \tag{IC1}$$

IC2: Incentive compatibility for L-type to report $L \Longrightarrow$

$$(1+L)p\frac{v_6+v_7}{2}-(1+L)(1-p)\frac{v_1+v_2}{2}+(1-L)(v_3-\frac{1}{2})+[1-(1+L)p](v_4+v_5) \geq 0. \tag{IC2}$$

IC3: Incentive compatibility for H-type to choose F when F has been recommended $\Longrightarrow$

$$(1-p)(1-v_3-v_4-v_5)+p\frac{1}{2}(1-v_6-v_7)-H[(1-p)v_5+pv_7] \geq 0. \tag{IC3}$$

IC4: Incentive compatibility for H-type to choose C when C has been recommended $\Longrightarrow$

[4] These contraints are for the player 1 and by symmetry, the set of constraints for player 2 is mathematically identical.

$$H[(1-p)v_3 + p\frac{1}{2}(1-v_6-v_7)] - (1-p)v_4 - pv_6 \geq 0. \quad \text{(IC4)}$$

IC5: Incentive compatibility for L-type to choose F when F has been recommended $\Longrightarrow$

$$(1-p)\frac{1}{2}(1-v_1-v_2) + pv_3 - L[(1-p)v_2 + pv_5] \geq 0. \quad \text{(IC5)}$$

IC6: Incentive compatibility for L-type to choose C when C has been recommended $\Longrightarrow$

$$L[(1-p)\frac{1}{2}(1-v_1-v_2) + p(1-v_3-v_4-v_5)] - (1-p)v_1 - pv_4 \geq 0. \quad \text{(IC6)}$$

Among the class of symmetric mediated equilibria, one could characterise the ex-ante efficient (in terms of ex-ante expected payoffs) symmetric mediated equilibrium in our setup following the results in Banks and Calvert (1992) who indeed have characterised a similar ex ante efficient incentive-compatible mechanism.

We however focus on the issue of obtaining any unmediated equilibrium from the previous section as a mediated equilibrium. It is well-known that any unmediated equilibrium can indeed be obtained using a mediated mechanism. Typically, however the mediator can improve upon the set of unmediated equilibria.

Here we are interested in achieving S_{eff}, the efficient fully revealing symmetric unmediated equilibrium as a symmetric mediated equilibrium. Consider the following symmetric mediated mechanism defined by the probabilities over action profiles for each type-profile induced by the strategy profile S_{eff}.

	F	C
F	$\frac{H}{(1+H)^2}$	$\frac{1}{(1+H)^2}$
C	$\frac{H^2}{(1+H)^2}$	$\frac{H}{(1+H)^2}$

HH

	F	C
F	0	0
C	0	1

HL

	F	C
F	1	0
C	0	0

LH

	F	C
F	$\frac{L}{(1+L)^2}$	$\frac{1}{(1+L)^2}$
C	$\frac{L^2}{(1+L)^2}$	$\frac{L}{(1+L)^2}$

LL

Define the symmetric mediated mechanism equivalent to the above distribution as M_{eff}. So, M_{eff} is the symmetric mediated mechanism where $v_1 = \frac{L^2}{(1+L)^2}, v_2 = \frac{1}{(1+L)^2}, v_3 = 1, v_4 = 0, v_5 = 0, v_6 = \frac{H^2}{(1+H)^2}$ and $v_7 = \frac{1}{(1+H)^2}$. We observe the following.

Proposition 1 *M_{eff} is a (symmetric) mediated equilibrium when*

$$\frac{2L^2H+L^2H^2+L^2}{L+H+LH^2+L^2H+L^2H^2+L^2+H^2+1} \leq p \leq \frac{-L+H+LH^2+L^2H+L^2H^2+H^2}{L+H+LH^2+L^2H+L^2H^2+L^2+H^2+1}.$$

Proof. Substituting the above values of $v_1, v_2, v_3, v_4, v_5, v_6$ and v_7 into the six incentive-compatibility constraints, one can easily check that IC3 and IC6 will be satisfied for all p. Also, IC4 will hold if $p \leq 1$ and IC5 will hold if $0 \leq p$. Finally, note that IC1 will be satisfied if $p \leq \frac{-L+H+LH^2+L^2H+L^2H^2+H^2}{L+H+LH^2+L^2H+L^2H^2+L^2+H^2+1}$ and IC2 will require that $\frac{2L^2H+L^2H^2+L^2}{L+H+LH^2+L^2H+L^2H^2+L^2+H^2+1} \leq p$. Hence the proof. ■

One might be interested in comparing the above range for p for M_{eff} to be in equilibrium with the range for p that we found for S_{eff} to be in equilibrium. It can be shown that the first range strictly contains the latter with respect to both the lower and upper bounds, i.e., $\frac{2L^2H+L^2H^2+L^2}{L+H+LH^2+L^2H+L^2H^2+L^2+H^2+1} < \frac{L^2+L^2H}{1+L+L^2+L^2H}$ and $\frac{HL+H^2L}{1+L+HL+H^2L} < \frac{-L+H+LH^2+L^2H+L^2H^2+H^2}{L+H+LH^2+L^2H+L^2H^2+L^2+H^2+1}$.

The proposition thus implies that the outcome generated by the profile S_{eff}, with the desirable characteristic, can be obtained as a mediated equilibrium M_{eff} for a larger range of p.

5 Remarks

5.1 Asymmetric equilibria

We have characterised the set of fully revealing symmetric equilibria of the cheap talk game. There are of course many fully revealing but asymmetric equilibria of this extended game. Clearly, babbling equilibria exist in which the players ignore the communication and just play one of the Nash equilibria of the complete information Battle of the Sexes game for all type-profiles.

There are other asymmetric equilibria. Consider for example the following strategy profile $(\sigma_1(t_1,t_2),\sigma_2(t_1,t_2))$ that the players play in the action stage. $(\sigma_1(H,H),\sigma_2(H,H)) = (\sigma_1(H,L),\sigma_2(H,L)) = (C,C)$, $(\sigma_1(L,H),\sigma_2(L,H)) = (F,F)$, and $\sigma_i(L,L) = \sigma^i(LL)$, where $\sigma^i(LL)$ is the mixed Nash equilibrium of the complete information Battle of the Sexes game with values L, L. The outcome can be generated by the following distribution (mediated mechanism).

	F	C
F	0	0
C	0	1

HH

	F	C
F	0	0
C	0	1

HL

	F	C
F	1	0
C	0	0

LH

	F	C
F	$\frac{L}{(1+L)^2}$	$\frac{1}{(1+L)^2}$
C	$\frac{L^2}{(1+L)^2}$	$\frac{L}{(1+L)^2}$

LL

Call this strategy profile S_{asymm}.

Proposition 2 *S_{asymm} is a fully revealing equilibrium of the extended incomplete information Battle of the Sexes game with cheap talk only when $L^2 \leq p \leq \frac{LH}{1-H+L}$.*

Proof. Under S_{asymm}, H-type player will reveal his type truthfully only if $p(H) + (1-p)(H) \geq p(1) + (1-p)(\frac{H}{1+L})$ where the LHS is the expected payoff from truthfully announcing H and the RHS is the expected payoff from announcing L and choosing the optimal action in the action phase given the deviation in the cheap talk phase. This inequality implies $p \leq \frac{LH}{1-H+L}$. Also, L-type player will reveal his type truthfully only if $p(1)+(1-p)(\frac{L}{1+L}) \geq p(L)+(1-p)(L)$ which implies $L^2 \leq p$. Hence the proof. ■

5.2 Payoffs

One may be interested in the payoff generated by the best fully revealing symmetric equilibrium. Note that the ex-ante expected payoff for any player from S_{eff} and M_{eff} is identical and is given by $EU = p^2 \frac{H}{1+H} + p(1-p)(1+H) + (1-p)^2 \frac{L}{1+L}$.

This EU is concave in p and has an unique interior maximum in $[0,1]$. However, $\frac{\partial EU}{\partial p} > 0$ at $p = \frac{HL+H^2L}{1+L+HL+H^2L}$ (the upper bound for the range of p for which S_{eff} is an equilibrium). So, EU is increasing over this range of p. Hence, the best achievable payoff from S_{eff} is

$$EU = \left[p^2 \frac{H}{1+H} + p(1-p)(1+H) + (1-p)^2 \frac{L}{1+L}\right]_{p=\frac{HL+H^2L}{1+L+HL+H^2L}}$$
$$= \frac{L}{(LH^2+LH+L+1)^2}\left(L+H+2LH^2+2LH^3+LH^4+LH+2H^2+H^3+1\right)$$

Similarly, the best achievable payoff from M_{eff} is

$$EU = \left[p^2 \frac{H}{1+H} + p(1-p)(1+H) + (1-p)^2 \frac{L}{1+L}\right]$$

achieved at $p = \frac{-L+H+LH^2+L^2H+L^2H^2+H^2}{L+H+LH^2+L^2H+L^2H^2+L^2+H^2+1}$

5.3 An example

We may illustrate all our results using an example. Take for example, $L = \frac{1}{3}$, $H = \frac{2}{3}$.

For these values, the range of the prior p for which the efficient fully revealing symmetric unmediated equilibrium S_{eff} exists is $0.12 \leq p \leq 0.22$.

On the other hand, the range of the prior p for which M_{eff} is a mediated equilibrium is given by $0.11 \leq p \leq 0.37$.

The best payoff from S_{eff} (at $p = 0.22$) is 0.46 while the corresponding best payoff from M_{eff} (at $p = 0.37$) is 0.54.

6 Conclusion

In this paper, we consider an incomplete information version of the Battle of the Sexes game. The game has two-sided private information, two-sided cheap talk and of course, two-sided actions. Cheap talk is modeled by adding a stage of announcements by players about their own types before going into the action stage.

Strategic information transmission and communication in games has been recognised as an important determinant of outcomes in these games. The seminal work by Crawford and Sobel (1982) first illustrated this point, following which a burgeoning literature has been trying to investigate different aspects of this issue. The sender-receiver framework has been extended in different directions. Extensions include introducing multiple senders (Gilligan and Krehbiel 1989; Austen-Smith 1993; Krishna and Morgan 2001a, 2001b; Battaglini 2002) and multiple receivers (Farrell and Gibbons 1989); however, these extensions are not helpful in analyzing two-player games with two-sided actions and two-sided private information because only receivers take actions and only senders have private information. What happens in two-player games where both might have private information and both can indulge in cheap talk and both can take decisions or choose actions? Some authors have pursued these problems in a complete information environment (Rabin 1994; Santos 2000) as well as in an incomplete information setting (Matthews and Postlewaite 1989; Austen-Smith 1990; Banks and Calvert 1992; Baliga and Morris 2002; Baliga and Sjostrom 2004). The issue of multiple rounds of cheap talk has also been discussed in the literature (Aumann and Hart 2003; Krishna and Morgan 2004; R. Vijay Krishna 2007) but only with one-sided private information.

A different avenue of research considers what happens when a static cheap talk game is repeated. Repetition gives rise endogenously to reputational concerns and this might impose additional constraints on what can be communicated via cheap talk (Sobel 1985; Benabou and Laroque 1992; Morris 2001; Avery and Meyer 2003; Ottaviani and Sorensen 2006a, 2006b; Olszewski 2004). Analysing cheap talk in this repeated framework would require us to make assumptions about the nature of these reputational concerns. Does the sender care about appearing to be well informed or does he want to be perceived as not having a large conflicting bias? He might have a bigger incentive to mask the truth and create a false perception now because this will affect his future credibility and hence future payoffs.

We achieve a desirable (ex-post efficient) outcome as a cheap-talk equilibrium outcome. The desirability criterion is mainly related to altruistic concerns for fairness whereby different players sacrifice or compromise under different states of nature. We should mention here that Borgers and Postl (2009) consider a set-up in which a compromise outcome is chosen.

In a follow-up paper, Ganguly and Ray (2009) consider cheating during the announcement phase. When cheap talk fails to achieve the exact desirable outcome, their results indicate how one can use partially revealing cheap talk to approximate the desired outcome and derive how close an achievable outcome can be to the desired outcome. They consider more general strategy profiles which allow for some degree of randomization at the cheap talk stage itself. Clearly, this would lead to outcomes that are somewhat different from the desirable outcomes we want to achieve. Nevertheless, characterising these equilibria will help us analyse how close to the desirable outcome we can get using some form of cheap talk. Under certain conditions, they also show that cheating or randomisation by both types of players during the announcement phase can be welfare improving compared to cheating by just one type.

Finally, in this context, one may think of a planner who may be able to help the players coordinate using a social choice function that may be fully implemented. Ray (2009) has illustrated this point by using implementation and correlated equilibrium distributions, in the spirit of Kar, Ray and Serrano (2009). Ray considers any social choice function that chooses one of the two pure

Nash equilibria in two different states from the class of all correlated equilibrium distributions and asks whether it can be implemented in Nash equilibrium or not.

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