Communication and Commitment in Contests*

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Abstract

Players often engage in high-profile public communications to demonstrate their confidence of winning before they carry out actual competitive activities. This paper investigates players' incentives to conduct such pre-contest communication. We assume that a player suffers a cost when he sends a "message of confidence", but misses his stated goal thereafter. However, this cost increases the player's incentive to win the competition and allows him to commit to a tougher stance in the subsequent contest. In a formal model, we show that communicating confidence may be beneficial for several reasons. First, it may discourage the opponent, and deter his entry into the contest and second, it may function as an information transmission device that signals private information about the competitor's own strength. We further show that such communications also lead to rich strategic trade-offs in the players' incentives to supply their effort and may help improve the allocative efficiency of the contest.

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1 Introduction

Many competitive events resemble contests in that economic agents forfeit scarce resources to compete for a limited number of prizes. A typical example of such a competitive event is an R&D race where firms attempt to develop cutting-edge technology before others. Other examples of contests include the rivalry between firms who increase their marketing expenditure to become market leaders, between politicians who go to great lengths to win votes during political campaigns, between rent-seekers who make political contributions so as to be able to influence policy or secure the patronage of powerful politicians, and between parties involved in legal disputes who incur great costs gathering evidence so that they can prevail in court. Such contests are prevalent in society. Other similar phenomena include war and international conflict, sports competitions and the market for internal labor.[1]

Contest contenders, whatever the context, often conduct high-profile communications in public before the actual contests take place. During a 1961 mission statement before the U.S. Congress, President John F. Kennedy expressed his goal of beating the Soviet Union in the race to reach the Moon by the end of the 1960s : "I believe that this nation should commit itself to achieving the goal, before this decade is out, of landing a man on the Moon and returning him safely to the Earth". The goal he boldly announced was ultimately achieved by the crew of the Apollo 11 in 1969. Similarly, Intel's president and chief executive Paul Otellini took a high-profile stance when announcing Intel's entry into the unfolding tablet market: "We are going to utilize all the assets at our disposal to win this segment" and "we will win in the tablet market". Such high-profile statements are also made in the world of sports. Former boxing world champion Mike Tyson very publicly demonstrated his confidence in winning his next fight by making media statements such as the following: "it's no doubt I am going to win this fight and I feel confident about winning this fight." Similarly, after being eliminated from the title race in 2011, New York Jets head coach Rex Ryan said: "Our goal for next year? I got news for you, it won't change. And it'll never change. We're gonna chase that Super Bowl, we're gonna chase it til we get it. And then we'll chase it again after that."

The many incidents of high-profile pre-competition communication have led to extensive discussions on pre-competition communication in research on business strategy.[2] As Porter (1980) argues, a competitor sends a verbal message as "a direct or indirect indication of its intentions, motives, goals, internal situations". Such a "message" is intended to modify the structure of subsequent competition and to trigger a favorable reaction from rivals. Porter broadly defines such actions as "market signals", and regards them as a critical element of competitive strategies. Despite this, the economics literature has made relatively little con-

[1] See Konrad (2009) for a survey of the literature.

[2] See Heil and Robertson (1991) for a review of the literature.

tribution in formal modelling to understanding the strategic value of such communications, and the incentives of players to engage in them. The current paper attempts to fill in this gap. We develop a formal model that allows for the understanding of the strategic effects of public communication that precedes a winner-take-all contest.

To fully understand the strategic trade-offs involved in pre-contest communications, it is worth noting that such activities can be costly. High-profile communications focus public attention on the ultimate performance of the participant sending the message. If the message sender does not achieve his stated goal (winning the contest), he risks public embarrassment and reputational damage. In the event of such a failure, the "message of confidence" or publicly stated resolve would be read by the public or press as a "bluff" (Kreps and Wilson 1982, Milgrom and Roberts 1982) that amplifies the player's failure, jeopardizes his credibility, and handicaps him in future strategic maneuvers. For instance, before Germany's federal elections in 2002, Guido Westerwelle of the Free Democratic Party (FDP) initiated the (ambitious) so-called "project 18" and committed publicly to the objective to win 18 per cent of the votes. When the FDP won only 7.4 percent of the votes, Westerwelle was subsequently taunted by the media and public. This tarnished his reputation as an effective political leader. In contrast, Microsoft and Lotus have long been said to compromise their own market reputation by "repeatedly not fulfilling pre-announcement signals about software upgrades." According to a survey by London-based Aon Ltd., most companies surveyed in the United Kingdom view such a loss of reputation as the greatest risk they face in the conduct of their business.

The message sender bears additional costs when he misses the stated goal. However, these costs enlarge the stake he places in the contest. This allows him to commit credibly to a tougher stance in the competition. This commitment could eventually pay off because it may discourage competitors and thus attenuate their incentive to exert effort. We label this strategic effect as the "discouragement effect".

To provide an account of communicative activities prior to contests, we consider a model where two players compete for a prize and the players are allowed to send a public "message of confidence" prior to the contest. Our analysis demonstrates that the discouragement effect on its own is insufficient to overcome the cost of sending the message of confidence, i.e. the cost when the stated goal is missed. Consequently, in a standard complete-information contest, where players' "strengths" (valuations for the prize) are commonly known, there exists no equilibrium where either player sends a message of confidence with a positive probability.

Our subsequent analysis, however, depicts two main contexts where the value of pre-contest communication can be identified. First, we provide an "entry deterrence" argument, and demonstrate that the discouragement effect plays a more significant role when the contest involves endogenous entry. Continuing in a complete-information setting, we allow contenders to decide autonomously whether to participate in a contest, with participation involving a fixed entry cost. Hence, the game proceeds in three stages. In the first stage, players decide whether to conduct communication. In the second stage, they decide whether to participate. In the third stage, they choose their actual efforts and these determine the winner of the contest. We show first that the equilibrium depends critically on the size of the fixed cost for entering the contest. Further, communicative activities arise in the equilibrium only if the entry cost is sufficiently high. When entry costs are high enough, one player's commitment of a tough stance (through communication) could effectively prevent another's entry. The value of the discouragement effect is thus amplified under this circumstance as it may potentially deter rivals from entering.

Second, we demonstrate that high profile pre-contest communications would arise when the contest involves incomplete and asymmetric information. This would occur even if entry was free, in which case pre-contest communication will not serve as an entry deterrence device. To illustrate this point, we allow one player's strength (i.e. his valuation of the prize) to take either a high or a low value, which is privately known. The presence of asymmetric information breeds rich strategic trade-offs. The uninformed player, whose strength is commonly known, never conducts the communication. However, the informed player may send a message of confidence, which functions not only as a commitment device but also as a signaling device that conveys private information. Communicative activities are revived in this context through two possible avenues.

1. There could exist a separating equilibrium which demonstrates a "confirmation effect". In the equilibrium, the player with private information sends a message of confidence if and only if he has a high valuation of the prize. The separating equilibrium is made possible because the stronger participant incurs a lower cost in sending the message of confidence as he stands a lesser chance of losing. In this case, the uninformed player discerns the true strength of the other party by observing his communicative activity. The informative signal allows the stronger player to credibly convince his rival of his superior strength. The confirmed advantage of the stronger party, together with the discouragement effect of his commitment to the tough stance, successfully weakens the incentive of his rival to put forth effort.

2. There could also exist a pooling equilibrium, which embodies the "bluffing effect". In this equilibrium, the informed party sends the message of confidence regardless of his relative strength as a contender. Such an equilibrium tends to emerge when the prior is less favorable (from the perspective of the uninformed player), i.e. when (1) the informed player is more likely to be of the high (stronger) type; and/or (2) the high-

type informed player possesses substantial advantages in the contest. In this case, the stronger type benefits from sending the message to maintain the pessimistic belief of his rival and thus discourages high effort from the rival. Interestingly, the weaker type may also want to send such a message, as by doing so, he hides his weakness and takes advantage of the pessimistic opponent.

The analysis of each of these yields several efficiency implications. We investigate two main issues: (1) How does pre-contest communication affect rent dissipation?; and (2) How does it affect the allocative efficiency of the contest, i.e. would a player who values the prize more win more frequently when such communication is in place?

In the complete information setting, communication may allow either player to successfully deter the entry of his competitor and to subsequently win the contest. Such a mechanism leads to lesser rent dissipation and may improve social efficiency in certain scenarios, such as rent seeking and political campaigning, where the outlays are viewed as pure wastes of social resources. The implications, however, would be reversed in alternative contexts where the contenders' outlays are productive and valuable (e.g., in R&D races and internal labor market tournaments). Our analysis, however, yields ambiguous implications on the second issue of allocative efficiency. Either the stronger player or the weaker play may successfully deter the entry of his rival and end up winning the prize in the equilibria. However, our analysis demonstrates that the inefficient equilibrium (where the weaker player wins) is less likely to emerge.

In the incomplete information setting, we show that communication exerts countervailing effects on players' incentives to supply effort and the expected overall effort may either increase or decrease. Despite its ambiguous effects on effort supply, communication always reduces the likelihood of "misallocation", by helping the stronger player win more often.

The remainder of the paper is organized as follows. First, we briefly discuss the relation of the current work to the existing literature. In Section 2, we describe the complete-information model and present its solution. Section 3 deals with the incomplete-information case and Section 4 concludes the paper. All formal proofs are presented in the Appendix.

1.1 Relation to Literature

As outlined in the previous section, the message of confidence plays the role of a commitment device in the context of the current paper. The literature on strategic commitment in contests includes prominent works by Dixit (1987), Baik and Shogren (1992), Morgan (2003), Yildirim (2005), Fu (2006) and Morgan and Várdy (2007). However, this strand of literature typically focuses on players' commitment to particular timing patterns of their moves. The players in our paper exert effort simultaneously and are allowed to commit to tougher stances through pre-contest communication.

Our paper can be linked to the small but growing literature on communication and feedback in contests. However, again, our current study stands in contrast to these papers, in that most of them analyze vertical communications between a contest organizer and the contestants (e.g., Gershkov and Perry 2009, Aoyagi 2010, Ederer 2010, Gürtler and Harbring 2010, Goltsman and Mukherjee 2011). These authors also typically focus on dynamic contest settings, and the question of whether the contest designer should reveal intermediate results to the contestants. A notable exception is the paper by Sutter and Strassmair (2009) who conduct experiments on contests between teams and allow for horizontal communication between contestants. They find that within-team communication leads to higher efforts, while between-team communication leads to lower efforts. The latter finding provides indirect evidence for the discouragement effect.

The message of confidence that we concern ourselves with in this study can function as a signaling device as well when the game involves private information. The small literature on contests with private and incomplete information features Hurley and Shogren (1998a, b) and Malueg and Yates (2004) who assume that contestants have independent valuations of the prize. Fu (2006, 2008) and Wärneryd (2003, 2009) consider common-value contests, but allows a subset of contestants to privately know the true prize purse. Fey (2008) and Münster (2009) further allow contestants to possess private information concerning their own abilities or effort costs. However, only a few of these studies introduce an information transmission device into the contest setting. Fu (2006) analyzes games where an informed contestant moves ahead of his uninformed opponent, with his rent-seeking outlay conveying his private information. Wärneryd (2007) and Fu (2008) further allow potential contestants to negotiate for settlement before they enter conflicts, and their actions in the negotiations reveal their private information. Wärneryd (2007) specifically requires the uninformed party to demand settlement, with the settlement demand serving as a screening device in the model, while Fu (2008) let the informed party propose a transfer payment, potentially as a way to signal his information. The current paper introduces a novel signaling device (public communication) into the contest setting and reveals the rich strategic trade-offs triggered by the signaling activities. In this regard, our paper contributes to this ongoing research agenda.

More generally, the paper is linked also to the broad and diverse literature on signaling and information transmission (see Spence, 1973 and 1974 and Crawford and Sobel, 1982). Two remarks are in order. First, it should be noted that the pre-contest communication in our context is defined more broadly than in the information economics literature. In the current paper, the pre-contest communication is primarily a commitment device that ex post compels the sender to "act tough", without necessarily being an information transmission device. The message of confidence is labeled as "public communication". It directly affects the environmental factors which determine the payoff structure of the game; while it does not merely communicate private information between the two active players in the game. Second, in the incomplete-information setting, the current modeling approach differs from conventional frameworks in that the cost of the signal (message of confidence) is endogenous in our model, as the sender bears the cost if and only if he loses the contest.

2 Complete Information

Two players, indexed by $i = 1, 2$, are involved in a contest and they compete for an indivisible object. Each player i values the object at $v_i \geq 0$, which is commonly known. We assume that player 1 values the object more than the other, i.e. $v_1 > v_2$.

The game proceeds in three stages. In the first stage, the two players simultaneously choose $s_i \in \{s, n\}$, where $s_i = s$ means that player i sends a public message of confidence, which indicates his confidence of winning the contest; while $s_i = n$ means that he does not. In the second stage, the players decide whether to enter the contest. Entry entails a fixed cost $c \geq 0$. When the entry cost drops to zero, the game degenerates into a two-stage contest as players always obtain non-negative expected payoffs from the contest and their entry decision is trivial. In the third stage, the active contestants choose their efforts $x_i \in \mathbb{R}_+$.

If both players have entered the contest, the winner is determined through a standard lottery contest, i.e., contestant i wins with probability $p_i = \frac{x_i}{x_i + x_j}$ if $x_i + x_j > 0$.[3] The prize is randomly assigned if both enter the contest but neither exerts positive effort. If only one of the players has entered the contest, he receives the prize automatically.

We assume that one's effort incurs a unity marginal cost and that a player bears the cost of his own effort regardless of whether he wins or loses. If a player sends the message $s_i = s$, but loses in the subsequent contest, he suffers an additional cost $k > 0$.[4]

The solution concept is subgame perfect Nash equilibrium (henceforth equilibrium) in the complete-information setting. For expositional clarity, we further assume that each player would enter the contest when he is indifferent between entry and exit. However, this assumption is by no means crucial for our results.

[3] Many of the results that will be derived in this paper also hold for a more general ratio-form contest success function (which has been axiomatized by Skaperdas 1996 and Clark and Riis 1998), and the completely discriminating contest (Baye et al. 1996). Formal proofs are available from the authors upon request.

[4] In Section 4, we discuss alternative assumptions surrounding the pre-contest communication and their effects on contest outcomes.

2.1 Effort Stage

The game is solved by backward induction. Accordingly, we begin with the analysis of behavior in the third stage. In the third stage, positive efforts are provided only if both players have entered the contest. For notational convenience, let I_i be an indicator variable that equals k if player i has chosen $s_i = s$ and zero otherwise. Player i maximizes

$$E\left[u_i\right] = \frac{x_i}{x_i + x_j}\left(v_i + I_i\right) - x_i - I_i - c.$$

Standard technique allows us to obtain the equilibrium efforts

$$x_i = \frac{\left(v_i + I_i\right)^2\left(v_j + I_j\right)}{\left(v_i + v_j + I_i + I_j\right)^2}, i = 1, 2.$$

A player i's equilibrium payoff is given by

$$E\left[u_i\right] = \frac{\left(v_i + I_i\right)^3}{\left(v_i + v_j + I_i + I_j\right)^2} - I_i - c.$$

The strategic value of the message of confidence (i.e. choosing $I_i = k$) is demonstrated by closer inspection of the equilibrium effort and payoff functions. The commitment fostered by the communication ($s_i = s$) compels the sender to step up his effort, as a loss would incur additional costs. The commitment of player i, however, triggers ambiguous reactions from the rival. It lowers the equilibrium effort of his rival j if and only if the incentives of player j to win are sufficiently weak, i.e. $v_j + I_j < \sqrt{\left(v_i + k\right)v_i}$. In such a case, player j would appear to be the underdog, and a better committed player i would disincentivize player j further. We label the strategic interaction a *discouragement effect* and its logic is similar to the conventional wisdom prevalent in the literature on strategic pre-commitment in contests (see, for example, Dixit 1987).

2.2 Entry Stage

We now consider the subgame during which players decide whether or not to enter the contest. If player i does not enter, he receives a payoff of $-I_i$. If he enters, his payoff depends on various factors, including both players' communicative activities in the first stage, and the entry decision of his rival.

If an excessive entry cost is involved, the outcome becomes relatively straightforward and trivial: neither player will enter if $c > v_1 + I_1$ while only player 1 enters if $v_1 + I_1 \geq c > v_2 + I_2$. With moderate entry costs (i.e. $c \leq v_2 + I_2$), three alternative cases are possible depending on the size of c. For the sake of brevity, we provide only a short qualitative overview of the results in the following paragraphs. More detailed analytical results are presented in Appendix A.

1. If c is sufficiently small, both players enter the contest.

2. If the entry cost remains in the medium range, only the ex post stronger player (i.e. the player i with $v_i + I_i \geq v_j + I_j$) enters. The entry cost is sufficiently small such that player i must enter regardless of player j's entry decision, while at the same time, the cost is high enough to ensure that his rival stays out given player i's entry.[5]

3. If c is sufficiently large (but still below $v_2 + I_2$), there are two pure strategy equilibria where exactly one player enters. In this case, each player prefers to stay out when the other enters, and to enter if the other stays out. Furthermore, there also exists a mixed equilibrium where players randomize their entry.

2.3 Communication Stage

The discussion on equilibrium play in the communication stage begins with a few interesting preliminary results, which will be used frequently in subsequent analysis.

Lemma 1 *There exists no pure strategy equilibrium where a player i sends the message $s_i = s$ but does not enter the contest in the subsequent subgame.*

Lemma 2 *Suppose that player j would enter the contest if i has sent the message $s_i = s$. Anticipating this, player i strictly prefers not to send the message.*

Lemma 2 yields important implications. A player never finds it optimal to send the message of confidence if he anticipates that his rival would eventually enter the contest (which is more likely when the entry cost c is sufficiently small). The implication of this result is that the benefit accrued from the discouragement effect cannot offset the potential cost of the pre-contest communication.

Lemmas 1 and 2 directly lead to the following.

Proposition 1 *There is no equilibrium in pure strategies where both players send the message.*

Despite the limited role of the discouragement effect in the competition, the message of confidence may play a nontrivial role in moderating players' ex ante incentive to enter the contest. The message sender commits himself to stepping up his effort, which limits the rent available to the other player and in turn may lead him to concede. The following lemma thus demonstrates the possible benefit of entry deterrence which may compel a player to proceed with the pre-contest communication.

[5] Since $v_1 > v_2$, player 1 typically has the higher incentive to win. The only exception is the case $v_2 + k > v_1$, where player 2 has the higher incentives in the subgames in which only player 2 has sent the message.

Lemma 3 *Suppose $v_i > c$. Player i strictly prefers to send $s_i = s$ if the message deters player j from entering the contest (i.e., if player j enters if and only if i has not send the message).*

By Proposition 1, in any pure strategy equilibrium, either (1) neither player sends the message, or (2) exactly one player does. We discuss these possibilities in subsections 2.3.1 and 2.3.2. In 2.3.3, we consider mixed equilibria where both players engage in pre-contest communication with positive probabilities.

2.3.1 Equilibrium with No Communication

We first analyze the case where neither player communicates. Such an equilibrium emerges under two possible circumstances, either when the entry cost is sufficiently low, or when entry is excessively costly. The latter possibility is relatively trivial. Consider, for instance, the case of $v_1 > c > v_2$. Player 2 will never enter in such a scenario, while player 1 is indifferent between sending and not sending the message as the cost will never be realized. Therefore, there exists an equilibrium where neither communicates and only player 1 enters.[6]

We focus on the former possibility. The following can be obtained.

Proposition 2 *An equilibrium where neither player communicates and both of them enter the contest exists if and only if $c \leq \frac{v_2^3}{(v_1+v_2+k)^2}$. Moreover, under this parameter restriction, the equilibrium is unique.*

Proposition 2 yields important implications. It states that neither player sends the message, while both enter the contest in equilibrium if the cost of entry is sufficiently low. The result formalizes and generalizes the insight we obtained from Lemma 2. The benefit from the discouragement effect alone can never overcome the concomitant cost. One has no incentive to engage in such commitment if the tough stance cannot successfully deter the entry of the rival, i.e. when the entry cost is sufficiently low.

It should be noted that the result is not an artifact of the particular lottery contest model. Proposition 2 continues to hold for more generally defined ratio-form contest success functions and also for a perfectly discriminating contest (all-pay auction). We omit the details for brevity, but the analysis is available from the authors upon request. Nevertheless, it is possible to construct examples of non-canonical contest models where a message of confidence pays off sufficiently because of the discouragement effect alone.

[6] Under certain parameter constellations, there also exists an equilibrium where neither communicates and only player 2 enters. To illustrate this, suppose that $\frac{(v_1+k)^3}{(v_1+v_2+k)^2} < c \leq v_2$. Suppose further that player 2 does not communicate but enters the contest regardless of the message sent by player 1. In such a situation, it would be optimal for player 1 to stay out in these subgames, since $\frac{v_1^3}{(v_1+v_2)^2} < \frac{(v_1+k)^3}{(v_1+v_2+k)^2} < c$. It does not pay for player 1 to communicate either since he will not enter. Thus there is an equilibrium where no player communicates and only 2 enters.

2.3.2 Equilibria Where One Player Communicates

We now consider equilibria where exactly one player communicates. By Lemmas 1 and 2, the player who did communicate enters, whereas the other player stays out. We first consider the equilibrium where only player 1 enters.

Proposition 3 *An equilibrium where only player* 1 *communicates and only player* 1 *enters exists if and only if*

$$c \in \left(\frac{v_2^3}{(v_1 + v_2 + k)^2}, v_1 \right].$$

Proposition 3 demonstrates the possible benefit of the message of confidence in that it serves as a deterrence to rival's entry. As a player commits to a tougher stance in the subsequent contest, the rival player may find it no longer worthwhile to participate in the competition. This "entry-deterrence effect" compels a player to engage in high-profile communication. By the same logic as that explaining Proposition 2, player 2 can be prevented from entering the contest only if the contest requires substantially costly entry.

Unlike the parameter range involved in Proposition 2, the equilibrium is in general not unique when c falls in the interval $\left(\frac{v_2^3}{(v_1+v_2+k)^2}, v_1 \right]$. There can be an equilibrium where only player 2 sends the message and enters. The next result demonstrates this possibility and establishes the conditions under which such an equilibrium exists.

Proposition 4 *An equilibrium where only player* 2 *communicates and only player* 2 *enters exists if and only if either (i)*

$$\frac{v_1^3}{(v_1 + v_2 + k)^2} < c \leq \min \left\{ v_2, \frac{(v_2 + k)^3}{(v_1 + v_2 + 2k)^2} \right\},$$

or (ii)

$$\frac{(v_1 + k)^3}{(v_1 + v_2 + 2k)^2} < c \leq v_2.$$

Two facts deserve to be noted. First, the two sets defined by conditions (i) and (ii), respectively, are disjoint.[7] Second, the union of the two sets is included in $\left(\frac{v_2^3}{(v_1+v_2+k)^2}, v_1 \right]$.[8] This implies that the ex ante favorite (player 1) is more likely to engage in pre-contest communication. Whenever there exists an equilibrium where only player 2 sends the message

[7] To appreciate this, note that $\frac{(v_1+k)^3}{(v_1+v_2+2k)^2}$ is greater than both $\frac{(v_2+k)^3}{(v_1+v_2+2k)^2}$ and $\frac{v_1^3}{(v_1+v_2+k)^2}$. The latter is true because $\frac{\partial \frac{(v_1+u)^3}{(v_1+v_2+k+u)^2}}{\partial u} = \frac{3(v_1+u)^2(v_1+v_2+k+u)-2(v_1+u)^3}{(v_1+v_2+k+u)^3} = \frac{(v_1+u)^2[3(v_1+v_2+k+u)-2(v_1+u)]}{(v_1+v_2+k+u)^3} > 0$.

[8] To see that, note $\frac{(v_1+k)^3}{(v_1+v_2+2k)^2} > \frac{v_1^3}{(v_1+v_2+k)^2} > \frac{v_2^3}{(v_1+v_2+k)^2}$ and $v_2 < v_1$.

and enters, there must exist an equilibrium where only player 1 sends the message and enters. The converse, however, is not true.

These results yield interesting efficiency implications. First, pre-contest communication may allow one player to successfully prevent the entry of his rival, thereby decreasing rent dissipation in the contest. In this way, such communication improves social welfare in contexts where efforts are unproductive, for example in political campaigns, as it helps avoid the waste of resources. However, the communication may backfire in circumstances where the competition leads to productive efforts as is the case in R&D races, architectural design competitions and internal labor competitions. In these situations, pre-contest communication instead facilitates coordination and dilutes the competition.

Second, pre-contest communication does not exert a definitive impact on allocative efficiency. Allocative efficiency, in the current context, requires that the player who values the prize more wins it. However, as demonstrated by Propositions 2 and 3, multiple equilibria exist when the entry cost is sufficiently high. Even player 2 may successfully deter the entry of the other and win the prize, thereby jeopardizing the allocative efficiency of the contest. However, as observed from Propositions 2 and 3, the more efficient equilibrium (where player 1 prevails) would emerge in a wider setting.

2.3.3 (Mixed) Equilibria Where Both Communicate With Positive Probabilities

As implied by Proposition 1, equilibria where both players communicate must involve mixed strategies in the communication stage. We briefly discuss the possibility of such mixed-strategy equilibria. They may emerge when sufficiently high entry costs lead to multiple pure-strategy equilibria, as shown in Propositions 2 and 3. Under these circumstances, the strategic interaction in the current context (in terms of communication and entry) resembles that of a standard coordination game.

Suppose, for instance, that entry cost c falls in the interval $(\frac{v_1^3}{(v_1+v_2+k)^2}, \frac{v_2^3}{(v_1+v_2)^2})$. In this case, there exists an equilibrium where both players send $s_i = s$ with positive probabilities. The players' strategy plays in the entry subgames are as follows.

1. If only player i has sent the message, i enters and j stays out.
2. If neither player has sent the message, both enter the contest subsequently.
3. If both have sent the message, then both enter.[9]

[9] In this subgame, it is optimal for player 2 to enter. Note that he receives an expected payoff $\frac{(v_2+k)^3}{(v_1+v_2+2k)^2} > \frac{v_2^3}{(v_1+v_2)^2} > c$. The first inequality follows from the fact that the left hand side is strictly increasing in k: $\frac{\partial}{\partial u}\left(\frac{(v_2+u)^3}{(v_1+v_2+2u)^2}\right) = \frac{(v_2+u)^2(3v_1+2u-v_2)}{(v_1+v_2+2u)^3} > 0$.

Anticipating such strategic play in the entry stage, $s_i = s$ is strictly optimal for player i if $s_j = n$ by Lemma 3, whereas $s_i = n$ is strictly optimal for i if $s_j = s$ by Lemma 2. Hence, there must exists a $p_j \in (0, 1)$ such that, if j sends $s_j = s$ with probability p_j, then i is indifferent between sending and not sending the message.

Therefore, a mixed-strategy equilibrium exists where both players send the signal with positive probabilities and both may enter the contest.

Such equilibria may explain why in many real world cases, competing parties both send messages of confidence and they then engage in an actual competition for the prize - a phenomenon that is not literally captured by the pure strategy equilibrium of our model. The discussion on mixed strategy equilibrium thus complements our main analysis. For the sake of expositional efficiency, we do not present an exhaustive account of all possible mixed equilibria. Two remarks are in order. First, all the equilibria exhibit qualitatively similar properties while the main purpose of the study is to explore players' incentives to either engage or not engage in pre-contest communication. Second, the strategic trade-offs that underpin the mixed-strategy equilibria are analogous to those that lead to the pure-strategy equilibria depicted by Propositions 2 and 3.

3 Incomplete Information

In this section, we provide an alternative rationale for pre-contest communication. We demonstrate in the complete-information setting (1) that sending $s_i = s$ pays off only if it helps deter an opponent's entry into the contest and (2) that the direct benefit from its commitment value (the discouragement effect) does not outweigh the possible cost of communication. However, communication could emerge when the game involves incomplete and asymmetric information, even if entry costs are trivial and entry deterrence is impossible.

We consider a direct variation of the basic model. Tullock type contest models have limited tractability under incomplete information. Hence, we focus on a simple case to highlight the main implications of information asymmetry on contenders' incentives to engage in pre-contest communication. We allow the valuation of player 1 to be uncertain. There can be two types of player 1, with his type (t) being either high (h) or low (l). For simplicity, we normalize the valuation of a type-l player 1 to one, which occurs with probability $\lambda \in (0, 1)$. At the same time, the valuation of the type-h player takes a value of $v > 1$, which occurs as the complementary event. The realization of v_1 is privately known to player 1, while its distribution is common knowledge. The valuation of player 2 for the prize is low, with $v_2 = 1$, and it is commonly known.

Such one-sided asymmetric information is widespread. Consider for instance the case of two oil exploration firms competing for entitlement to an oil field with ambiguous property rights, with one oil company being the incumbent and the other a new entrant. A firm's stake in the dispute ultimately depends on how much it stands to profit from the operation. The profitability of the entrant is arguably less known than that of the incumbent.

To highlight the role played by information asymmetry, we focus throughout the subsequent analysis on the limiting case where $c = 0$. This simplification allows us to avoid the confounding influence of other factors. Entry deterrence is impossible under this condition as one always expects positive payoffs from the contest. The game essentially reduces to two stages because entry decisions are trivial. Players simultaneously choose their message s_i in the first stage. They then expend their efforts vying for the prize in the second stage of the game. We establish in the subsequent analysis that player 1 may send $s_1 = s$ in equilibrium even when entry-deterrence is impossible.

Before we proceed with equilibrium analysis, let us briefly analyze a benchmark case, where communication prior to the contest is absent. Player 2 chooses his effort to maximize his expected payoff based on the prior while player 1 chooses type-dependent effort. A unique Bayesian Nash equilibrium exists in the game, which is detailed in the following lemma.

Lemma 4 *If no pre-contest communication is possible, there exists a unique Bayesian Nash equilibrium. In the equilibrium, player* 2 *exerts effort* $x_2 = \left(\frac{\lambda v+(1-\lambda)\sqrt{v}}{(1-\lambda)+\lambda v+v}\right)^2$. *The high-type player* 1 *exerts effort* $x_{1h} = -\left(\frac{\lambda v+(1-\lambda)\sqrt{v}}{(1-\lambda)+\lambda v+v}\right)^2 + \left(\frac{\lambda v+(1-\lambda)\sqrt{v}}{(1-\lambda)+\lambda v+v}\right)\sqrt{v}$, *whereas the low-type player* 1 *exerts effort* $x_{1l} = -\left(\frac{\lambda v+(1-\lambda)\sqrt{v}}{(1-\lambda)+\lambda v+v}\right)^2 + \left(\frac{\lambda v+(1-\lambda)\sqrt{v}}{(1-\lambda)+\lambda v+v}\right)$.

We now consider the possibility of pre-contest communication. We adopt the solution concept of Perfect Bayesian equilibrium to analyze the game. The trade-off faced by the uninformed player 2 is analogous to that in the complete-information setting. As in the complete information case, the commitment value of the communication is insufficient to overcome its cost and there is no equilibrium where player 2 sends the message. As player 1 possesses private information, his communicative activities become a signaling device, which allows him to strategically manipulate the belief of his rival. We demonstrate in subsequent analysis that the informed player may reap additional benefit from pre-contest communications when the message of confidence also conveys private information.

Before we proceed with this analysis, the following can be established.

Proposition 5 *There exists no equilibrium where only the low-type player 1 sends the message.*

Proposition 5 results from the fact that the high-type player 1 always benefits more from the reduction of the opponent's effort due to the communication than his low-type counterpart. Hence, a low-type player 1 never conducts pre-contest communications alone.

We demonstrate that communication can appear in equilibrium through two mechanisms: (1) the "confirmation effect"; and (2) the "bluffing effect". The former effect is exercised through a separating equilibrium where only the high-type player 1 sends a message of confidence; while the latter emerges in a pooling equilibrium where both types of player 1 participate in the communication.

3.1 Confirmation Effect: Separating Equilibrium

We first consider the possibility of separating equilibrium where only the high-type player 1 chooses $s_1 = s$, while the low type chooses $s_1 = n$. In this case, player 2 perfectly infers the true type of player 1, which gives $\Pr(t = l | n) = 1$ and $\Pr(t = l | s) = 0$. The following result establishes the conditions under which the separating equilibrium exists.

Proposition 6 *(a) Suppose that $v \geq 81/16$. There is a critical value $\tilde{k} \in (0, \infty)$ such that a separating equilibrium with the high-type player 1 choosing $s_1 = s$ and the low-type player 1 choosing $s_1 = n$ exists if and only if $k \geq \tilde{k}$.*

(b) Suppose that $v < 81/16$. There are two cutoff values $\hat{k}$ and $\tilde{k}$, with $0 < \tilde{k} < \hat{k} < \infty$, such that the separating equilibrium exists if and only if $k \in \left[\tilde{k}, \hat{k}\right]$.

Proposition 6 demonstrates that sending the message of confidence can be optimal in a setting with asymmetric information, even though the entry-deterrence effect is absent. The commitment allows the high type to credibly reveal his type, which yields two types of strategic benefits. First, the discouragement effect, which is exercised through the commitment of k, continues to exist. Second, the transmission of information provides a credible verification of a high-type player's competitive advantage, thereby even further discouraging player 2 from exerting competitive effort. The latter effect is referred to as the *confirmation effect*. The combination of the two mechanisms compels the high type to send the message.

The high type is able to separate from his low-type counterpart for two reasons. First, recall that the cost of sending $s_1 = s$ is endogenous, as a player bears the cost k only when he loses. In equilibrium, the low type is more likely to lose and thus his expected costs from the commitment are higher. Second, both the discouragement effect and the confirmation effect lead player 2 to reduce his effort. The high type, however, benefits more from a less aggressive opponent: because of his higher valuation of the prize, any given effort reduction from player 2 will matter more to the high-type player 1. In summary, the high type bears a lesser cost when sending the message of confidence; while he also reaps a larger gain from it.

We then interpret the conditions that ensure the existence of the separating equilibrium. First, the high type has an incentive to send $s_1 = s$ if and only if either (a) $v \geq 81/16$, or (b)

$v < 81/16$ but k is below a critical value $\hat{k} \in (0, \infty)$. A larger v amplifies the gains to the high type from the confirmation effect because: (1) it implies a more significant advantage in the competition, which allows him to disincentivize player 2 further; and (2) he benefits more from the concession of player 2 when his valuation for the prize increases. He prefers to send the message $s_1 = s$ when v is sufficiently large. When v is small, i.e. $v < 81/16$, his incentive to send the message depends on k as well, and he would not send $s_1 = s$ if such a message is too costly, i.e., $k > \hat{k}$.

Second, the low type has no incentive to mimic the high type (by choosing $s_1 = s$) if and only if k is sufficiently high, i.e., $k \geq \tilde{k}$. The low type is always tempted to "bluff" by misrepresenting his type. By doing so, he fools his opponent into believing that he is a high type, and this allows him to disincentivize his rival and to reduce the competition. A separating equilibrium requires that such an incentive be checked by a nontrivial k: the low type would refrain from misrepresentation only if he would be punished severely if he loses.

The size of v exerts competing effects on the existence of separating equilibrium. Two interesting observations are highlighted as follows.

Corollary 1 *Both the lower bound $\tilde{k}$ and the upper bound $\hat{k}$ increase with v.*

As implied by Corollary 1, a larger v is a double-edged sword that may either facilitate or preclude the equilibrium. First, it increases the lower bound $\tilde{k}$. The incentive compatibility condition that prevents the low type from mimicry is less likely to be met. A larger v amplifies the gain from the confirmation effect, as player 2 would be disincentivized further when the disadvantage widens. Hence, the low type is tempted more to misrepresent himself, and a larger k (higher cost) is required to deter the mimicry. Second, the same effect also increases the upper bound $\hat{k}$. Recall that the high-type player 1 is willing to send $s_1 = s$ when v falls below $\frac{81}{16}$ only if the cost of the message is moderate, i.e. $k \leq \hat{k}$. A larger v encourages the high type to engage in the communication. The magnified benefits from the discouragement and confirmation effects would allow him to overcome even higher cost (i.e. tolerating a larger k).

To complement the discussion and to obtain more intuition, consider the limiting cases of $v \uparrow \infty$ and $v \downarrow 1$, respectively. The separating equilibrium breaks down under either circumstance. The former case infinitely amplifies the benefit of confirmation effect. Player 2 would give up if he is convinced that he will encounter the high-type player 1. At the same time, there exists no force to prevent the low-type player 1 from mimicry: he would win with certainty if he successfully deceives his rival and the cost k will not be realized in that case.[10] In the latter case, the high-type player 1 does not possess significant advantage and the benefit of confirmation effect is diminished. The discouragement effect alone, as

[10] Formally, as v approaches infinity, $\tilde{k}$ approaches infinity as well.

discussed above, is insufficient to overcome the cost k. The separating equilibrium would not emerge.

In what follows, we discuss the efficiency implications of pre-contest communication in the incomplete-information setting. We explore two main issues: (1) Does the communication help increase or decrease equilibrium efforts? (2) Does the communication improve allocative efficiency, i.e., does it result in the stronger player receiving the prize more often?

3.1.1 Effort Comparison

We address the first issue by comparing players' equilibrium efforts with those in the benchmark case where pre-contest communication is absent.

We begin with player 2, whose efforts in different situations can be ranked unambiguously as follows:

Remark 1 *The effort of player* 2 *is lowest in the separating equilibrium when the type of player* 1 *is high, and highest in the separating equilibrium when the type of player* 1 *is low. In the benchmark case without pre-competition communication, the effort of player* 2 *is between these two values.*

The separating equilibrium leads to full information revelation. When the low-type player 1 is present, players know that they are evenly matched, and this leads to the fiercest competition and the highest effort. In contrast, a high-type player is able to verify himself as the favorite in the equilibrium, which maximally disincentivizes player 2. In the benchmark case, player 2 chooses his effort based on the prior. He takes into account both possibilities when choosing his effort. The uncertainty limits his incentives. His effort thus remains in the middle of the above cases.

We next consider the equilibrium effort of the low-type player 1.

Remark 2 *The effort of the low-type player* 1 *is higher in the separating equilibrium than in the benchmark case.*

The logic we discussed above also explains Remark 2. Player 2 is handicapped in the benchmark case. He may encounter a strong rival and have a lesser chance of winning. The uncertainty disincentives player 2, thereby allowing the low-type player 1 to slack off. However, the separating equilibrium, dismisses the uncertainty and intensifies the competition.

Finally, the separating equilibrium triggers mixed responses from the high-type player 1.

Remark 3 *The effort of the high-type player* 1 *can be either higher or lower in the separating equilibrium than in the benchmark case. If* v *and* k *are sufficiently high, the high type's effort is lower in the separating equilibrium.*

There exists no unambiguous ranking of equilibrium efforts for the high-type player 1. The communication in the equilibrium exerts countervailing effects on the incentive of the high-type player 1 to supply his effort. On the one hand, he is allowed to slack off because he successfully convinces his rival of superior strength. On the other hand, he ends up with a larger stake in the contest because of his commitment k, which compels him to step up his effort in order to avoid a loss.

As a result of the countervailing effects, the ranking of expected overall efforts between the cases also remains unclear and it depends on various environmental factors, i.e. on v, k and λ.

We provide two numerical examples to illustrate the ambiguity. Denote by X_b the equilibrium expected overall effort in the benchmark case, and by X_s that in the separating equilibrium. Let $\Delta \triangleq X_b - X_s$ be the difference between the two cases. First consider a case of $v = 2$, $k = 0.075$, and $\lambda = 1/10$. A separating equilibrium exists in this setting: $\tilde{k}$ is about 0.071, and $\hat{k}$ is about 0.217. We observe in this case $\Delta < 0$: expected overall effort is higher in the separating equilibrium. In contrast, suppose that $v = 10$, $k = 2$, and $\lambda = 1/10$. A separating equilibrium also exists in this case ($\tilde{k}$ is about 1.27) but $\Delta > 0$. Despite the ambiguity, the following regularity can still be observed.

Proposition 7 *Δ strictly increases in v and strictly decreases in k. For a given k, there exists a unique $\hat{v}(k) \in (1, \infty)$ such that $\Delta < 0$ for all $v \in (1, \hat{v}(k))$ and $\Delta > 0$ for all $v > \hat{v}(k)$. The critical value $\hat{v}(k)$ is increasing in k.*

Proposition 7 shows that communication is more likely to reduce effort supply in the separating equilibrium in the presence of a larger v and a lesser k. To understand why X_b must eventually leapfrog X_s when v increases, it should be noted that v affects both X_b and X_s indefinitely. On the one hand, a larger v tends to incentivize the high-type player 1 as he has a larger stake. On the other hand, it disincentivizes player 2, which further allows player 1 to slack off. However, the latter (negative) effect tends to be weaker in the benchmark case: player 2 is uncertain about the type of his rival in the benchmark and would therefore rationally discount the "unconfirmed" advantage. Hence, he responds to the increase of v less sensitively in the benchmark case, and this contributes to our observations.

It is less difficult to see why Δ decreases with k. The commitment k affects only X_s but does not affect X_b. It also exerts competing effects. An increase in k tends to further incentivize the high-type player 1 in the separating equilibrium; while it also disincentivizes player 2. However, the latter effect is an indirect one and is dominated by the former direct effect. Hence, a larger k always increases X_s (and decrease Δ) whenever the separating equilibrium exists.

The same logic applies to the observation that the critical value $\hat{v}(k)$ increases in k. An increasing k offsets the positive effect on Δ exerted by v, and therefore prevents X_b from leapfrogging X_s. Hence, a larger v is required for rebalance.

3.1.2 Allocative Efficiency

While no definitive conclusion can be reached for effort comparison, our results can demonstrate that allocative efficiency would improve if pre-contest communication is possible and a separating equilibrium exists.

Note that the allocation of the prize does not make a difference if player 1 turns out to be of the low type, because both players have the same valuation. We only need to consider the probability of player 1 winning when he has a high valuation for the prize. Simple analysis leads to the following.

Proposition 8 *"Misallocation" (i.e., a high-type player 1 loses) is less likely to occur in the separating equilibrium than in the benchmark case.*

Proposition 8 formally establishes that pre-contest communication improves allocative efficiency. With pre-contest communication, a high-type player 1 can successfully convince the rival of his advantage. The communication discourages player 2 as he learns that he has to compete against a stronger and better committed opponent.

3.2 Bluffing Effect: Pooling Equilibrium

In this subsection, we demonstrate the possibility of a pooling equilibrium where even the low-type player 1 would send the message $s_1 = s$. In a pooling equilibrium, no additional information is transmitted through the communicative activity, hence $\Pr(t = l|\, s) = \lambda$. The following lemma will be used repeatedly below.

Lemma 5 *In any pooling equilibrium where both types of player 1 send the message, the effort of player 2 is given by*

$$x_2 = \left(\frac{\lambda (v+k) \sqrt{1+k} + (1-\lambda)(1+k)\sqrt{v+k}}{(1+k)(v+k) + \lambda (v+k) + (1-\lambda)(1+k)} \right)^2 .$$

It decreases in v and k and it increases in λ.

The following proposition provides conditions under which such a pooling equilibrium exists.

Proposition 9 *For every given k, there exists a unique cutoff $\overline{v} > 1$, and an associated cutoff probability $\overline{\lambda}(v) \in (0,1)$, such that a pooling equilibrium, where player 1 sends the message regardless of his type, exists if and only if $v > \overline{v}$ and $\lambda \leq \overline{\lambda}(v)$.*

The logic that underlies the result is as follows: The pooling equilibrium is underpinned by the incentive of the low-type player 1 to hide his type by mimicking his high-type counterpart. Uncertainty remains in such a pooling equilibrium and allows the low type to avoid revealing his weakness. This further discourages player 2 from putting forth effort, and benefits the low type. Such a "bluff" pays off significantly if and only if (1) the prior is sufficiently unfavorable to player 2 ($\lambda \leq \overline{\lambda}(v)$) and (2) the high-type player 1 possesses substantial advantage ($v > \overline{v}$). The former let player 2 believe that he is more likely to meet the high-type rival; while the latter let him believe that he is handicapped more severely in the competition when he meets the high-type rival. As Lemma 5 shows, both of the possibilities lead him to behave less competitively. The benefit of bluffing thus outweighs the possible cost of the message when these conditions are met.

3.2.1 Effort Comparison

As in the preceding sections, we analyze how pre-competition communication affects total effort. We compare players' efforts here to those in the benchmark case. The results are qualitatively similar to those of the separating equilibrium and therefore our discussion here will be brief. Player 2 exerts less effort in the pooling equilibrium than he does in the benchmark case (this follows from Lemmas 4 and 5). However, pre-contest communication has countervailing effects on player 1's incentives. On the one hand, player 1's stake in the contest has risen. On the other hand, player 2 is discouraged by the prospect of facing a better committed rival, and this allows player 1 to slack off. It turns out that the equilibrium effort of the high-type player 1 can both be higher and lower than in the benchmark case. Moreover, expected overall effort can also be higher or lower than in the benchmark case.

We provide two contrasting numerical examples to illustrate the ambiguity. First, consider a case of $v = 10$, $\lambda = 0.01$ and $k = 0.5$. A pooling equilibrium exists in this setting. The high-type player 1 exerts an equilibrium effort $x_{1h}^{*} = 0.842$, and the expected overall effort amounts to 0.9175. In the corresponding benchmark case, the high-type player 1 exerts an equilibrium effort $x_{1h} = 0.83$, and the expected overall effort is 0.9149. In this case, equilibrium communication increases effort supply. The high-type player 1 exerts a higher effort in the pooling equilibrium. Expected overall effort also rises.

Second, consider a case of $v = 10$, $\lambda = 0.1$ and $k = 0.5$. A pooling equilibrium also exists in this setting. The high-type player 1 exerts an equilibrium effort $x_{1h}^{*} = 0.9133$, and the expected overall effort amounts to 0.9477. In the corresponding benchmark case, the high-type player 1 exerts an equilibrium effort $x_{1h} = 0.9175$, and the expected overall effort is 0.9522. In this case, equilibrium communication reduces effort supply. The high-type player 1 exerts a lower effort in the pooling equilibrium. Expected overall effort also declines.

3.2.2 Allocative Efficiency

We further consider the allocative efficiency of the contest when the pooling equilibrium is being played.

Proposition 10 *"Misallocation" is less likely in the pooling equilibrium than in the benchmark case.*

Despite the competing effects pointed out above, the high-type player 1 always wins with a greater likelihood in the pooling equilibrium. Equilibrium communication thus improves allocative efficiency.

4 Concluding Remarks

The current paper offers an economic model which helps us to understand why contest participants often make statements of confidence even before they carry out any actual competitive activities. While it is generally shown that pre-contest communication increases one's incentives to win and helps disincentivize the opponent, we demonstrate that the commitment value of such communication never offsets its cost in a standard contest setting. The statement of confidence nevertheless may be valuable for several reasons. First, it may deter rival's entry into the contest. Second, it can function as a signaling device to strategically manipulate the belief of the opponent when the contest involves incomplete and asymmetric information. Pre-contest communications also yield interesting efficiency implications. We show, for instance, that such communications may help improve the allocative efficiency and reduce wasteful rent dissipation under certain circumstances.

Although we have assumed in our model that a player who has sent a message of confidence prior to the contest suffers a fixed cost if he does not succeed eventually, our setup can be extended in a number of ways. First, players can be allowed to choose different levels of commitment when they state their confidence prior to the contest. To put it more formally, I_i may be chosen from a set of more than two values or even allowed to be continuous. Such extension would enrich our strategic analysis, but the main results of this paper would not change qualitatively from such a move. For instance, Proposition 2 states that neither player engages in the communication if the entry cost is sufficiently small. This result holds for all k although the cutoff for entry cost depends quantitatively on the size of k.

Second, players could also make pessimistic statements like "I am going to lose the contest!". A player may receive additional gains if his performance exceeds expectations and he ultimately wins. Thus, a pessimistic statement may also increase one's incentive to win the contest. A strategic analysis of players' incentive to make pessimistic statements would complement the current work.

Analytical complexity and expositional efficiency have limited our analysis into a stylized setting. The setup, however, can also be generalized in various other ways, and the key insights would extend to wider contexts. For example, we have indicated that some findings are robust with respect to the form of the contest-success function. In addition, varying other assumptions (e.g. the number of contestants) might affect the magnitude of the strategic effects highlighted in this paper, but it would not qualitatively alter the main predictions. These extensions will be attempted by the authors in future. We believe, however, that the current model captures the most important forces at work in the context of pre-competition communication.

A Appendix: The Entry Stage

Here we give details of players' behavior at the entry stage. We focus on play in pure strategies. If player i does not enter, he receives a payoff of $-I_i$. If a player enters, his payoff depends on the message combinations chosen in the first stage. The equilibrium payoffs for the different message combinations conditional on entry are displayed in Table 1.

	$s_2 = s$	$s_2 = n$
$s_1 = s$	$\frac{(v_1+k)^3}{(v_1+v_2+2k)^2} - k - c, \frac{(v_2+k)^3}{(v_1+v_2+2k)^2} - k - c$	$\frac{(v_1+k)^3}{(v_1+v_2+k)^2} - k - c, \frac{v_2^3}{(v_1+v_2+k)^2} - c$
$s_1 = n$	$\frac{v_1^3}{(v_1+v_2+k)^2} - c, \frac{(v_2+k)^3}{(v_1+v_2+k)^2} - k - c$	$\frac{v_1^3}{(v_1+v_2)^2} - c, \frac{v_2^3}{(v_1+v_2)^2} - c$

Table 1: Equilibrium payoffs for different message combinations conditional on entry

As noted in the main text, the case where $c > v_2 + I_2$ is straightforward: only player 1 enters when $c \leq v_1 + I_1$, and no player enters when $c > v_1 + I_1$. The following lemmas formally characterize behavior at the entry stage in case $c \leq v_2 + I_2$. We provide a formal proof of Lemma A.1. The proofs of the other lemmas are analogous and therefore omitted.

Lemma A.1 *Suppose $s_1 = s_2 = s$ and $c \leq v_2 + k$.*
(i) If $c \leq \frac{(v_2+k)^3}{(v_1+v_2+2k)^2}$, then both players enter the contest. Payoffs are $E[u_1] = \frac{(v_1+k)^3}{(v_1+v_2+2k)^2} - k - c$ and $E[u_2] = \frac{(v_2+k)^3}{(v_1+v_2+2k)^2} - k - c$.
(ii) If $c \in \left(\frac{(v_2+k)^3}{(v_1+v_2+2k)^2}, \frac{(v_1+k)^3}{(v_1+v_2+2k)^2}\right]$, only player 1 enters the contest and payoffs are $E[u_1] = v_1 - c$ and $E[u_2] = -k$.
(iii) If $c > \frac{(v_1+k)^3}{(v_1+v_2+2k)^2}$, only one of the player enters and receives $E[u_i] = v_i - c$, while the other player receives $E[u_j] = -k$ ($j = 1, 2$, $i \neq j$).

Proof of Lemma A.1. It is straightforward to see that entry is the dominating choice for player 1 if $c \leq \frac{(v_1+k)^3}{(v_1+v_2+2k)^2}$. Parts (i) and (ii) then follow from a comparison of the entry payoff to the non-entry payoff of player 2. If $c > \frac{(v_1+k)^3}{(v_1+v_2+2k)^2}$, no player would enter if the opponent player enters the contest, while each player would enter if the other player stays out. ■

Lemma A.2 *Suppose $s_1 = s_2 = n$ and $c \leq v_2$.*
(i) If $c \leq \frac{v_2^3}{(v_1+v_2)^2}$, then both players enter the contest. Payoffs are $E[u_1] = \frac{v_1^3}{(v_1+v_2)^2} - c$ and $E[u_2] = \frac{v_2^3}{(v_1+v_2)^2} - c$.
(ii) If $c \in \left(\frac{v_2^3}{(v_1+v_2)^2}, \frac{v_1^3}{(v_1+v_2)^2}\right]$, only player 1 enters the contest and payoffs are $E[u_1] = v_1 - c$ and $E[u_2] = 0$.
(iii) If $c > \frac{v_1^3}{(v_1+v_2)^2}$, only one of the player enters and receives $E[u_i] = v_i - c$, while the other player receives $E[u_j] = 0$.

Lemma A.3 *Suppose $s_1 = s$, $s_2 = n$, and $c \leq v_2$.*
(i) If $c \leq \frac{v_2^3}{(v_1+v_2+k)^2}$, then both players enter the contest. Payoffs are $E[u_1] = \frac{(v_1+k)^3}{(v_1+v_2+k)^2} - k - c$ and $E[u_2] = \frac{v_2^3}{(v_1+v_2+k)^2} - c$.
(ii) If $c \in \left(\frac{v_2^3}{(v_1+v_2+k)^2}, \frac{(v_1+k)^3}{(v_1+v_2+k)^2}\right]$, only player 1 enters the contest and payoffs are $E[u_1] = v_1 - c$ and $E[u_2] = 0$.
(iii) If $c > \frac{(v_1+k)^3}{(v_1+v_2+k)^2}$, only one of the player enters and receives $E[u_i] = v_i - c$, while the other player receives $E[u_j] = -I_j$.

Lemma A.4 *Suppose $s_1 = n$, $s_2 = s$, and $c \leq v_2 + k$.*
(i) If $c \leq \min\left\{\frac{v_1^3}{(v_1+v_2+k)^2}, \frac{(v_2+k)^3}{(v_1+v_2+k)^2}\right\}$, then both players enter the contest. Payoffs are $E[u_1] = \frac{v_1^3}{(v_1+v_2+k)^2} - c$ and $E[u_2] = \frac{(v_2+k)^3}{(v_1+v_2+k)^2} - k - c$.
(ii) If $c \in \left(\min\left\{\frac{v_1^3}{(v_1+v_2+k)^2}, \frac{(v_2+k)^3}{(v_1+v_2+k)^2}\right\}, \max\left\{\frac{v_1^3}{(v_1+v_2+k)^2}, \frac{(v_2+k)^3}{(v_1+v_2+k)^2}\right\}\right]$, only the player with the higher incentive enters the contest and receives $E[u_i] = v_i - c$, while the other player receives $E[u_j] = -I_j$.
(iii) If $c > \max\left\{\frac{v_1^3}{(v_1+v_2+k)^2}, \frac{(v_2+k)^3}{(v_1+v_2+k)^2}\right\}$, only one of the player enters and receives $E[u_i] = v_i - c$, while the other player receives $E[u_j] = -I_j$.

B Appendix: Proofs of Main Results

B.1 Complete Information

Proof of Lemma 1. If i chooses $s_i = s$ and stays out, he gets $-k$. But i can guarantee himself a payoff of zero by choosing $s_i = n$ and staying out. ■

Proof of Lemma 2. Suppose that j enters if $s_i = s$. There are two cases to consider (they differ in whether or not j enters if $s_i = n$).

First, suppose that j enters regardless of the choice of s_i. If i chooses $s_i = s$, he gets

$$\max\left\{\frac{(v_i + k)^3}{(v_i + k + v_j + I_j)^2} - k - c, -k\right\}.$$

If i chooses $s_i = n$, he gets

$$\max\left\{\frac{v_i^3}{(v_i + v_j + I_j)^2} - c, 0\right\}.$$

Let

$$h(u) := \frac{(v_i + u)^3}{(v_i + u + v_j + I_j)^2} - u - c.$$

Since for all $u \geq 0$,

$$h'(u) = -(I_j + v_j)^2 \frac{3u + I_j + 3v_i + v_j}{(v_i + u + v_j + I_j)^3} < 0,$$

we have $h(0) > h(k)$. It follows that i 's payoff is strictly higher if he chooses $s_i = n$.

Second, suppose that j enters if and only if i communicates. If this were part of j 's strategy, the payoff of i upon not communicating is $\max\{v_i - c, 0\}$, and upon communicating it is $\max\{h(k), -k\}$. Since $v_i - c > h(0) > h(k)$, not communicating is strictly better. ■

Proof of Proposition 1. Suppose $s_1 = s_2 = s$. Then i enters the contest in the resulting subgame by Lemma 1. But then j should choose $s_j = n$ by Lemma 2. ■

Proof of Lemma 3. If i chooses $s_i = s$ and j does not enter, i gets $v_i - c$. If i chooses $s_i = n$ and j enters, i either stays out and gets zero; or i enters, spends a positive effort in equilibrium, and wins with probability less than one. Therefore his payoff is strictly smaller than $v_i - c$. ■

Proof of Proposition 2. "If" part. Let $c \leq \frac{v_2^3}{(v_1+v_2+k)^2}$. Then both players enter the contest regardless of messages sent in the first stage. To see this, note that the relevant conditions for entry of both players in Lemma A.1 to A.4 are fulfilled. This is straightforward to see for the conditions in Lemma A.2 to A.4. For Lemma A.1, we have to show that

$$\frac{(v_2 + k)^3}{(v_1 + v_2 + 2k)^2} \geq \frac{v_2^3}{(v_1 + v_2 + k)^2}.$$

To show that this condition is fulfilled, consider the derivative

$$\frac{\partial}{\partial u}\frac{(v_2 + u)^3}{(v_1 + v_2 + k + u)^2} = \frac{3(v_2 + u)^2(v_1 + v_2 + k + u) - 2(v_2 + u)^3}{(v_1 + v_2 + k + u)^3},$$

which is clearly positive. The result then follows from Lemma 2.

"Only if" part. Suppose that $c \in \left(\frac{v_2^3}{(v_1+v_2+k)^2}, \frac{v_2^3}{(v_1+v_2)^2} \right]$. Consider an equilibrium candidate where $s_1 = s_2 = n$ and both players enter in the subgame after $s_1 = s_2 = n$. Note that

$$\frac{v_2^3}{(v_1+v_2)^2} < \frac{v_1^3}{(v_1+v_2)^2} < \frac{(v_1+k)^3}{(v_1+v_2+k)^2},$$

(the last inequality follows from the fact that the right hand side is strictly increasing in k). Therefore, if player 1 deviates and chooses $s_1 = s$, Lemma A.3 part ii applies: only player 1 enters. That is, by choosing $s_1 = s$ player 1 deters player 2 from entering. From Lemma 3, it follows that the payoff of player 1 is strictly higher if he sends the message. Therefore there is no equilibrium where $s_1 = s_2 = n$ and both players enter.

Finally, suppose that $c > \frac{v_2^3}{(v_1+v_2)^2}$. Then after $s_1 = s_2 = n$, player 2 will not enter. Hence there cannot be an equilibrium where no player sends the message and both enter. ■

Proof of Proposition 3. "If" part. Suppose that $c \in \left(\frac{(v_2+k)^3}{(v_1+v_2+2k)^2}, v_1 \right]$.

Consider strategies with the following features. Player 1 communicates, player 2 does not. Player 1 enters in subgame $(s_1, s_2) = (s, n)$ and in subgame $(s_1, s_2) = (s, s)$, while player 2 does not enter in these subgames.

Suppose 2 behaves as described above and consider 1. At the entry stage, it is optimal for 1 to enter whenever he has chosen $s_1 = s$ since in theses subgames 2 stays out. Moreover, choosing $s_1 = s$ is optimal, again for the reason that in the resulting subgame 2 stays out.

Now suppose 1 behaves as described above and consider player 2. First consider the entry stage. Staying out if 1 has chosen $s_1 = s$ is optimal for 2 since

$$\frac{(v_2+I_2)^3}{(v_1+k+v_2+I_2)^2} \leq \frac{(v_2+k)^3}{(v_1+k+v_2+k)^2} < c.$$

Second, consider the communication stage. Since 1 will enter even if 2 chooses $s_2 = s$, choosing $s_2 = n$ is optimal for 2 by Lemma 2.

It remains to consider the case where

$$c \in \left(\frac{v_2^3}{(v_1+k+v_2)^2}, \min\left\{ \frac{(v_2+k)^3}{(v_1+v_2+2k)^2}, v_1 \right\} \right].$$

Here the construction above does not work since in subgame $(s_1, s_2) = (s, s)$, both players will enter. However, there still is an equilibrium with the desired features. It differs from the construction above only off the equilibrium path, in the subgame starting after $(s_1, s_2) = (s, s)$.

Consider strategies with the following features. Player 1 communicates, player 2 does not. Player 1 enters in subgame $(s_1, s_2) = (s, n)$ and in subgame $(s_1, s_2) = (s, s)$. Player 2 does not enter in the subgame $(s_1, s_2) = (s, n)$, but enters in subgame $(s_1, s_2) = (s, s)$.

On the entry stage, entry is optimal for player 1 in subgame $(s_1, s_2) = (s, n)$ since 2 stays out and $c \leq v_1$. In the same subgame, staying out is optimal for player 2 since $c > \frac{v_2^3}{(v_1+v_2+k)^2}$. In the subgame $(s_1, s_2) = (s, s)$, entry is optimal for both since

$$c \leq \frac{(v_2+k)^3}{(v_1+v_2+2k)^2} < \frac{(v_1+k)^3}{(v_1+v_2+2k)^2}.$$

On the communication stage, player 1 has no incentive to deviate since player 2 does not enter on the equilibrium path. Moreover, by Lemma 2, player 2 has no incentive to deviate on the communication stage, since player 1 enters after $(s_1, s_2) = (s, s)$.

"Only if" part. If $c > v_1$, clearly there is no equilibrium where 1 chooses $s_1 = s$. Moreover, by Proposition 2 there is no equilibrium where only 1 communicates if $c \leq \frac{v_2^3}{(v_1+k+v_2)^2}$. ■

Proof of Proposition 4. "If" part. Under condition (i), there is an equilibrium where only 2 communicates, only 2 enters after $(s_1, s_2) = (n, s)$, and only 2 enters after $(s_1, s_2) = (s, s)$.

Now suppose that $c \leq \frac{(v_1+k)^3}{(v_1+v_2+2k)^2}$. Then this construction does not work since after $(s_1, s_2) = (s, s)$, player 1 will enter. However, if (ii) holds, there is an equilibrium where after $(s_1, s_2) = (s, s)$ both players enter. (On the equilibrium path, after $(s_1, s_2) = (n, s)$, only player 2 enters.) Entry after $(s_1, s_2) = (s, s)$ is optimal for both since $c \leq v_2$ and

$$c \leq \frac{(v_2+k)^3}{(v_1+v_2+2k)^2} < \frac{(v_1+k)^3}{(v_1+v_2+2k)^2}.$$

It remains to show that 1 has no incentive to choose $s_1 = s$. This follows from the fact that 2 enters even after $(s_1, s_2) = (s, s)$ together with Lemma 2.

"Only if" part. If $c > v_2$, there is clearly no equilibrium where 2 chooses $s_2 = s$ and enters. If $c \leq \frac{v_1^3}{(v_1+k+v_2)^2}$, player 1 will enter after $(s_1, s_2) = (n, s)$; hence there is no equilibrium where only 2 communicates by Lemma 2. Finally, if $\frac{(v_2+k)^3}{(v_1+v_2+2k)^2} < c \leq \frac{(v_1+k)^3}{(v_1+v_2+2k)^2}$, after $(s_1, s_2) = (s, s)$ only player 1 enters. Therefore, player 1 should deviate to choosing $s_1 = s$ by Lemma 3. ■

B.2 Incomplete Information

Proof of Lemma 4. Suppose that no pre-contest communication is possible. By standard arguments, there is a unique Bayesian Nash equilibrium. Player 2, whose type is known to be $v_2 = 1$ solves

$$\max_{x_2} \lambda \frac{x_2}{x_{1l} + x_2} + (1-\lambda) \frac{x_2}{x_{1h} + x_2} - x_2,$$

where x_{1l} is the effort of the low-type player 1, and x_{1h} of the high type. The first order condition is

$$\lambda \frac{x_{1l}}{(x_{1l} + x_2)^2} + (1-\lambda) \frac{x_{1h}}{(x_{1h} + x_2)^2} = 1.$$

Note that the payoff function is globally concave in x_2 and hence the first order condition is sufficient for a global maximum.

Player 1 with private information solves

$$\max_{x_{1t}} \frac{x_{1t}}{x_{1t}+x_2}v_t - x_{1t} \quad t = h, l$$

where $v_h = v$ and $v_l = 1$. The first order condition is

$$\frac{x_2}{(x_{1t}+x_2)^2} = \frac{1}{v_t}.$$

Solving for x_{1t} gives the reaction functions of the high and low type:

$$x_{1t} = -x_2 + \sqrt{x_2 v_t}$$

Solving the three first order conditions simultaneously gives the equilibrium efforts. Note that, since $v > 1$, all these efforts are strictly positive. ■

Proof of Proposition 5. Suppose there is such an equilibrium. If player 2 chooses weakly higher effort after observing $s_1 = s$ than after $s_1 = n$, the low-type player 1 should deviate to $s_1 = n$. So suppose that player 2 chooses lower effort after observing $s_1 = s$. Then the high-type player 1 has an incentive to deviate to sending the message, because for him the reduction in the opponent's effort is even more valuable than for the low type. To see this, consider the following decision problem where the opponent's effort $x_2 \in (0, w)$ is a parameter:

$$\max_{x} \frac{x}{x+x_2}w - x.$$

The indirect utility function (i.e. the maximized value of this objective function) is

$$\left(1 - \sqrt{\frac{x_2}{w}}\right) w.$$

Obviously, the indirect utility function decreases in x_2:

$$\frac{\partial}{\partial x_2}\left(1 - \sqrt{\frac{x_2}{w}}\right) w = -\sqrt{w}\frac{\partial \sqrt{x_2}}{\partial x_2} < 0.$$

Moreover, the indirect utility function decreases more rapidly if w is higher:

$$\frac{\partial^2}{\partial x_2 \partial w}\left(1 - \sqrt{\frac{x_2}{w}}\right) w = -\left(\frac{\partial \sqrt{w}}{\partial w}\right)\left(\frac{\partial \sqrt{x_2}}{\partial x_2}\right) < 0$$

Thus, a reduction in the opponent's effort x_2 is more valuable if the value of winning is higher. ■

Proof of Proposition 6. Suppose that a separating equilibrium where only the high type communicates is played. We begin the analysis by calculating equilibrium efforts and payoffs. Player 2 perfectly infers player 1's type from the observation of his message, and he forms the posterior $\Pr(v_1 = v|\, s) = 1$ and $\Pr(v_1 = v|\, n) = 0$. Upon the high type sending $s_i = s$, the two players in the subsequent contest would exert their efforts, respectively,

$$x_{1h} = \frac{(v+k)^2}{(v+k+1)^2}, \text{ and } x_{2h} = \frac{(v+k)}{(v+k+1)^2}.$$

The high type receives an expected payoff

$$\pi_{1h}(k) = \frac{(v+k)^3}{(v+k+1)^2} - k.$$

Upon the low type sending $s_1 = n$, the two players' subsequent efforts are $x_{1l} = x_{2l} = \frac{1}{4}$. The low-type player 1 receives an expected payoff $\pi_{1l} = \frac{1}{4}$.

We then establish the incentive compatibility conditions which ensure that neither type has the incentive to misrepresent. If the high type deviates to $s_1 = n$, his opponent believes that he competes against a player with low valuation and he would choose $x_{2l} = 1/4$ in the subsequent contest. Player 1's optimal effort x_{hd} maximizes $\frac{x_{hd}}{x_{hd}+\frac{1}{4}}v - x_{hd}$. Thus he exerts the effort $x_{hd} = \frac{\sqrt{v}}{2} - \frac{1}{4}$. The payoff from this deviation is given by

$$\pi_{hd} = \frac{\frac{\sqrt{v}}{2} - \frac{1}{4}}{\frac{\sqrt{v}}{2}}v - \left(\frac{\sqrt{v}}{2} - \frac{1}{4}\right) = \left(\sqrt{v} - \frac{1}{2}\right)^2.$$

If the low type deviates by sending $s_1 = s$, his opponent chooses $x_{2h} = \frac{(v+k)}{(v+k+1)^2}$. The low type then chooses his effort x_{ld} to maximize his expected payoff $\frac{x_{ld}}{x_{ld}+\frac{(v+k)}{(v+k+1)^2}}(1+k) - x_{ld}$. The optimal effort is given by $x_{ld} = \frac{\sqrt{(v+k)(1+k)}}{v+k+1} - \frac{v+k}{(v+k+1)^2}$. The payoff from deviation is thus given by

$$\begin{aligned}\pi_{ld}(k) &= \left(1 - \frac{\frac{(v+k)}{(v+k+1)^2}}{\frac{\sqrt{(v+k)(1+k)}}{v_h+k+1}}\right)(1+k) - k - \left(\frac{\sqrt{(v+k)(1+k)}}{v+k+1} - \frac{v+k}{(v+k+1)^2}\right) \\ &= 1 - \frac{2\sqrt{(v+k)(1+k)}\,(v+k+1) - (v+k)}{(v+k+1)^2}.\end{aligned}$$

The equilibrium requires

$$\begin{aligned}\pi_{1h}(k) &\geq \pi_{hd}, \\ \pi_{1l} &\geq \pi_{ld}(k).\end{aligned}$$

Consider the high type. Define

$$G(k) := \pi_{1h}(k) - \pi_{hd} = \frac{(v+k)^3}{(v+k+1)^2} - k - \left(\sqrt{v} - 0.5\right)^2$$

Note that

$$G(0) = \frac{v^3}{(v+1)^2} - \left(\sqrt{v} - 0.5\right)^2 > 0$$

and

$$G'(k) = \frac{3(v+k)^2(v+k+1) - 2(v+k)^3}{(v+k+1)^3} - 1 = -\frac{3k+3v+1}{(k+v+1)^3} < 0.$$

Moreover,

$$\begin{aligned} \lim_{k\to\infty} G(k) &= \lim_{k\to\infty}\left(\frac{k^2v - 2k^2 + 2kv^2 - 2kv - k + v^3}{k^2 + 2kv + 2k + v^2 + 2v + 1}\right) - \left(\sqrt{v} - 0.5\right)^2 \\ &= \sqrt{v} - \frac{9}{4} \end{aligned}$$

thus $\lim_{k\to\infty} G(k) = (<,>)\, 0$ iff $v = (<,>)\, 81/16$. Together with $G(0) > 0$ and $G'(k) < 0$, this implies that, if $v \geq \frac{81}{16}$, then $G(k) > 0$ for all $k \geq 0$. On the other hand, if $v < \frac{81}{16}$, there exists a unique $\hat{k} \in (0, \infty)$ such $G\left(\hat{k}\right) = 0$, $G(k) > 0$ for all $k < \hat{k}$, and $G(k) < 0$ for all $k > \hat{k}$. In fact, $\hat{k}$ can be calculated explicitly. The equation $G(k) = 0$ has exactly one possibly positive solution, namely

$$\hat{k} = \frac{2 - (v+1)(3 - 2\sqrt[4]{v})}{3 - 2\sqrt[4]{v}}.$$

Even though it follows from the considerations above that $\hat{k} > 0$ if and only if $v \in (1, \frac{81}{16})$, we directly verify this here. This direct approach also yields the comparative static properties of $\hat{k}$. So suppose $v \in (1, \frac{81}{16})$. Then $3 > 2\sqrt[4]{v}$. It remains to show that $f(v) := 2 - (v + 1)(3 - 2\sqrt[4]{v}) > 0$. We have $\lim_{v\downarrow 1} f(v) = 0$ and $\lim_{v\uparrow \frac{81}{16}} f(v) = 2$. Differentiating $f(v)$ yields $f'(v) = (2v^{\frac{1}{4}} - 3) + \frac{1}{2}v^{-\frac{3}{4}}(v+1) = \frac{5}{2}v^{\frac{1}{4}} + \frac{1}{2}v^{-\frac{3}{4}} - 3$. Further differentiating $f'(v)$ yields $f''(v) = \frac{5}{8}v^{-\frac{3}{4}} - \frac{3}{8}v^{-\frac{7}{4}} = \frac{1}{8}v^{-\frac{3}{4}}(5 - \frac{3}{v})$, which must be strictly positive because $v > 1$. Further, $\lim_{v\downarrow 1} f'(v) = 0$. It implies that $f(v)$ must be increasing and therefore $f(v) > 0$ over the interval $(1, \frac{81}{16})$. Note also that these observations imply that $\hat{k}$ is increasing in v, and that $\lim_{v\downarrow 1} \hat{k} = 0$, as we have claimed in the main text. If $v > 81/16$, then $3 < 2\sqrt[4]{v}$ and the $\hat{k}$ given in the formula above is clearly negative. To sum up this discussion:

Claim 1 *If $v > 81/16$, the high valuation type never has an incentive to deviate. On the other hand, if $v < 81/16$, then the high valuation type has no incentive to deviate if and only if $k \leq \hat{k}$. The critical value $\hat{k}$ is increasing in v, and $\lim_{v\downarrow 1} \hat{k} = 0$*

Next, we consider the incentives of the low valuation type. Define

$$H(k) := \pi_{1l} - \pi_{ld}(k) = -0.75 + \frac{2\sqrt{(v+k)(1+k)}(1+v+k) - (v+k)}{(1+v+k)^2}.$$

Note that

$$H(0) = -0.75 + \frac{2\sqrt{v}(1+v) - v}{(1+v)^2} < 0.$$

In addition, $H(k)$ can be written as

$$H(k) = -0.75 + \frac{2\sqrt{(v+k)(1+k)}}{(1+v+k)} - \frac{(v+k)}{(1+v+k)^2}$$

and hence

$$H'(k) = \frac{\sqrt{(k+1)(k+v)}(v^2 + kv + k + 1)}{(k+1)(k+v)(k+v+1)^2} + \frac{k+v-1}{(k+v+1)^3} > 0$$

(recall that $v > 1$). Moreover, note that

$$\lim_{k\to\infty} H(k) = \frac{5}{4}.$$

Together with $H(0) < 0$ and $H'(k) > 0$, this implies that there is a unique $\tilde{k} \in (0, \infty)$ such that $H\left(\tilde{k}\right) = 0$, and $H(k) < (>)\, 0$ if $k < (>)\, \tilde{k}$. To see how the critical value $\tilde{k}$ depends on v, note that $H(k)$ is decreasing in v, as can be shown by partial differentiation. From the implicit function rule,

$$\frac{d\tilde{k}}{dv} = -\frac{\frac{\partial}{\partial v} H(k)}{H'(k)} > 0.$$

Claim 2 *There is a unique $\tilde{k} \in (0, \infty)$ such that the low type has no incentive to mimic the high type if and only if $k \geq \tilde{k}$. The critical value $\tilde{k}$ is increasing in v.*

Now we are in position to consider existence of a separating equilibrium. The case where v is so big that the high type never wants to deviate is straightforward.

Claim 3 *If $v \geq \frac{81}{16}$, a separating equilibrium exists whenever $k \geq \tilde{k}$.*

For the case where $v < 81/16$, we now show that $\hat{k} > \tilde{k}$. To see this, note that $H\left(\hat{k}\right)$ is strictly positive iff

$$2\sqrt{\left(v+\hat{k}\right)\left(1+\hat{k}\right)} > 0.75\left(1+v+\hat{k}\right) + \frac{v+\hat{k}}{\left(1+v+\hat{k}\right)}$$

$$\Leftrightarrow 4\left(v+\hat{k}\right)\left(1+\hat{k}\right) - \left(0.75\left(1+v+\hat{k}\right) + \frac{v+\hat{k}}{\left(1+v+\hat{k}\right)}\right)^2 > 0.$$

Insert $\hat{k}$ into the left hand side of this expression. The condition becomes

$$\frac{1}{4\sqrt{v}-12\sqrt[4]{v}+9}\left(8v-16\sqrt{v}+16\sqrt[4]{v}+16v^{\frac{3}{2}}+16v^{\frac{3}{4}}-32v^{\frac{5}{4}}-8\right)>0$$
$$\Leftrightarrow v-2\sqrt{v}+2\sqrt[4]{v}+2v^{\frac{3}{2}}+2v^{\frac{3}{4}}-4v^{\frac{5}{4}}-1>0,$$

since $4\sqrt{v}-12\sqrt[4]{v}+9>0$, for $v<\frac{81}{16}$. The latter condition can be shown to be fulfilled for all $v>1$.

Since $H\left(\hat{k}\right)>0=H\left(\tilde{k}\right)$ and $H'\left(k\right)>0$, it follows that $\hat{k}>\tilde{k}$. Therefore

Claim 4 *If $v<81/16$, a separating equilibrium exists if and only if $k\in\left[\tilde{k},\hat{k}\right]$.*

■

Proof of Corollary 1. Contained in proof of Proposition 6. ■

Proof of Remark 1. We have to show that

$$\frac{v+k}{\left(v+k+1\right)^{2}}<\left(\frac{\lambda v+\left(1-\lambda\right)\sqrt{v}}{\left(1-\lambda\right)+\lambda v+v}\right)^{2}<\frac{1}{4}.$$

Let

$$g\left(v,\lambda\right):=\left(\frac{\lambda v+\left(1-\lambda\right)\sqrt{v}}{\left(1-\lambda\right)+\lambda v+v}\right)^{2}$$

Note that g is strictly increasing in λ. Moreover, $g\left(v,1\right)=1/4$, which proves the second inequality in the lemma. Similarly, $g\left(v,0\right)=\frac{v}{\left(v+1\right)^{2}}$. Thus

$$\frac{v}{\left(v+1\right)^{2}}<\left(\frac{\lambda v+\left(1-\lambda\right)\sqrt{v}}{\left(1-\lambda\right)+\lambda v+v}\right)^{2}<\frac{1}{4}$$

Moreover,

$$\frac{v+k}{\left(v+k+1\right)^{2}}<\frac{v}{\left(v+1\right)^{2}}$$

where the inequality follows from the fact that the left hand side is strictly decreasing in k. This proves the first inequality in the lemma. ■

Proof of Remark 2. We have to show that

$$1/4>-\left(\frac{\lambda v+\left(1-\lambda\right)\sqrt{v}}{\left(1-\lambda\right)+\lambda v+v}\right)^{2}+\left(\frac{\lambda v+\left(1-\lambda\right)\sqrt{v}}{\left(1-\lambda\right)+\lambda v+v}\right).$$

This follows from

$$\begin{aligned}&\frac{\partial}{\partial\lambda}\left(-\left(\frac{\lambda v+\left(1-\lambda\right)\sqrt{v}}{\left(1-\lambda\right)+\lambda v+v}\right)^{2}+\left(\frac{\lambda v+\left(1-\lambda\right)\sqrt{v}}{\left(1-\lambda\right)+\lambda v+v}\right)\right)\\ =\;& v\left(1-\lambda\right)\frac{\left(\sqrt{v}-1\right)^{4}}{\left(v-\lambda+v\lambda+1\right)^{3}}>0\end{aligned}$$

and

$$\lim_{\lambda\to 1}\left(-\left(\frac{\lambda v+(1-\lambda)\sqrt{v}}{(1-\lambda)+\lambda v+v}\right)^2+\left(\frac{\lambda v+(1-\lambda)\sqrt{v}}{(1-\lambda)+\lambda v+v}\right)\right)=\frac{1}{4}.$$

■

Proof of Remark 3. Let

$$\delta=-\left(\frac{\lambda v+(1-\lambda)\sqrt{v}}{(1-\lambda)+\lambda v+v}\right)^2+\left(\frac{\lambda v+(1-\lambda)\sqrt{v}}{(1-\lambda)+\lambda v+v}\right)\sqrt{v}-\frac{(v+k)^2}{(v+k+1)^2}$$

denote the difference in effort of the high-type player 1 in the benchmark and the separating equilibrium. If v and k are sufficiently high, the high type's effort is lower in the separating equilibrium. To see this formally, note that δ approaches infinity as $v\to\infty$, and correspondingly $k\to\infty$ in order that a separating equilibrium exists, independently of the order in which the limits are taken.

On the other hand, if v is sufficiently small, the effort can be higher in the separating equilibrium. An example where $\delta<0$ is $v=5$, $\lambda=9/10$, and $k\to\infty$. ■

Proof of Proposition 7. We know that $\Delta=X_b-X_s$ with

$$\begin{aligned} X_b &= \left(\frac{\lambda v+(1-\lambda)\sqrt{v}}{(1-\lambda)+\lambda v+v}\right)\left(\lambda+(1-\lambda)\sqrt{v}\right),\\ X_s &= \lambda\left(\frac{1}{4}+\frac{1}{4}\right)+(1-\lambda)\left(\frac{(v+k)^2}{(v+k+1)^2}+\frac{(v+k)}{(v+k+1)^2}\right). \end{aligned}$$

We first show that Δ is increasing in v. Note that

$$\begin{aligned} \frac{\partial}{\partial k}\frac{\partial\Delta}{\partial v} &= 0-\frac{\partial}{\partial k}\frac{\partial X_s}{\partial v}\\ &= -\frac{\partial}{\partial k}\frac{1-\lambda}{(k+v+1)^2}>0. \end{aligned}$$

Hence it is sufficient to show that $\frac{\partial\Delta}{\partial v}>0$ at $k=0$. So suppose that $k=0$. Then we have

$$\begin{aligned} &\frac{\partial\Delta}{\partial v}|_{k=0}\\ =\ &\frac{\lambda(1-\lambda)\left(\sqrt{v}-1\right)^2\left(1-\lambda+v(7-5\lambda)+v^2(7+\lambda)+v^3(1+\lambda)+2\sqrt{v}+v^{\frac{3}{2}}(4+2\lambda)+2(1+\lambda)v^{\frac{5}{2}}\right)}{2\sqrt{v}(v+1)^2(v-\lambda+v\lambda+1)^2}\\ >\ &0. \end{aligned}$$

Second, it is straightforward to show that

$$\frac{\partial\Delta}{\partial k}=-\frac{1-\lambda}{(k+v+1)^2}<0.$$

Third, fix some k and consider what happens if $v \to \infty$ or $v \to 1$. We have $\lim_{v\to\infty} X_b = \infty$, and $\lim_{v\to\infty} X_s = \lambda\frac{1}{2} + 1 - \lambda$. Therefore, $\lim_{v\to\infty} \Delta > 0$. Moreover, $\lim_{v\to 1} X_b = 1/2$ and $\lim_{v\to 1} X_s = \lambda\frac{1}{2} + (1-\lambda)\frac{k+1}{k+2} > \frac{1}{2}$ for all $k > 0$. Therefore, $\lim_{v\to 1} \Delta < 0$. Since Δ is continuous, it follows from the intermediate value theorem that there exists a $\hat{v}(k) \in (1, \infty)$ such that $\Delta < (=, >)\, 0$ whenever $v < (=, >)\, \hat{v}(k)$. Finally,

$$\frac{d\hat{v}}{dk} = -\frac{\frac{\partial \Delta}{\partial k}}{\frac{\partial \Delta}{\partial v}} > 0.$$

■

Proof of Proposition 8. Using players' best response functions in the benchmark setting, we obtain $\frac{x_2}{x_{1h}+x_2} = \frac{x_2}{\sqrt{x_2 v}}$ as player 2's winning probability against a high-type player 1. We now claim $\frac{x_2}{\sqrt{x_2 v}} > \frac{1}{1+v}$, or equivalently $\sqrt{x_2} > \sqrt{v}\frac{1}{1+v}$.

Plug in x_2 from Lemma 4 to obtain

$$\frac{\lambda v + (1-\lambda)\sqrt{v}}{(1-\lambda) + \lambda v + v} > \sqrt{v}\frac{1}{1+v}.$$

Dividing by $\sqrt{v}$ and rearranging it, the inequality is rewritten as

$$\left(\lambda\sqrt{v} + (1-\lambda)\right)(1+v) > (1-\lambda) + \lambda v + v.$$

Since

$$\left(\lambda\sqrt{v} + (1-\lambda)\right)(1+v) - ((1-\lambda) + \lambda v + v) = \lambda\sqrt{v}\left(\sqrt{v} - 1\right)^2 > 0$$

the condition $\frac{x_2}{\sqrt{x_2 v}} > \frac{1}{1+v}$ is fulfilled. Finally, because $\frac{1}{1+v} > \frac{1}{1+v+k}$ (which is the winning probability of player 2 against a high-type player 1 in the separating equilibrium) the claim is established. ■

Proof of Lemma 5. Suppose that both types of player 1 send the message. The analysis of the effort stage is completely analogous to that in Lemma 4, with the exception that communication has changed the valuation of losing for player 1 (but player 2's belief regarding player 1's type is still the same). This leads to the formula for x_2 given in the lemma.

To establish the comparative static of x_2, let $z = \frac{1}{\sqrt{(1+k)}}$ and $y = \frac{1}{\sqrt{(v+k)}}$. Then we can write

$$\sqrt{x_2} = \frac{\lambda z + (1-\lambda)\, y}{1 + \lambda z^2 + (1-\lambda)\, y^2}$$

We show that this expression increases in y, keeping all else constant.

$$\begin{aligned} & \frac{\partial}{\partial y}\left(\frac{\lambda z + (1-\lambda)\, y}{1 + \lambda z^2 + (1-\lambda)\, y^2}\right) \\ = \;& (1-\lambda)\frac{\left(1 + \lambda z^2 + (1-\lambda)\, y^2\right) - \left(\lambda z + (1-\lambda)\, y\right) 2y}{\left(1 + \lambda z^2 + (1-\lambda)\, y^2\right)^2} \end{aligned}$$

which is strictly positive iff

$$1 > (1-\lambda)\, y^2 + \lambda z\,(2y - z)$$

which is true since

$$z\,(2y-z) < \max_{\zeta} \zeta\,(2y-\zeta)) = y^2 < 1.$$

It follows that x_2 is decreasing in v. A similar argument shows that

$$\frac{\partial}{\partial z}\left(\frac{\lambda z + (1-\lambda)\, y}{1+\lambda z^2 + (1-\lambda)\, y^2}\right) > 0.$$

Together with the observation above, this implies that x_2 decreases in k. Finally,

$$\frac{\partial}{\partial \lambda}\left(\frac{\lambda z + (1-\lambda)\, y}{1+\lambda z^2 + (1-\lambda)\, y^2}\right) = \frac{(z-y)\,(1-yz)}{\left(1+\lambda z^2 + (1-\lambda)\, y^2\right)^2} > 0$$

where the inequality holds since $y < z < 1$. It follows that x_2 increases in λ. ■

Proof of Proposition 9. Suppose both types of player 1 send a message of confidence. The first order condition of type $t \in \{l, h\}$ of player 1 in such a pooling equilibrium gives

$$x_{1t} = \sqrt{x_2 v_t} - x_2$$

where $v_h = v + k$ and $v_l = 1 + k$. The payoff of type t of player 1 can thus be written as

$$\begin{aligned}
\pi_t^* &= \frac{x_{1t}}{x_{1t}+x_2} v_t - x_{1t} - k \\
&= \frac{(-x_2 + \sqrt{x_2 v_t}) v_t - k\sqrt{x_2 v_t}}{\sqrt{x_2 v_t}} + x_2 - \sqrt{x_2 v_t} \\
&= (v_t - k) + x_2 - 2\sqrt{x_2 v_t}.
\end{aligned}$$

where x_2 is given in Lemma 5.

To demonstrate that the pooling equilibrium exists, we have to show that neither type of player 1 wants to deviate by not sending a message of confidence. We thus have to consider the out-of-equilibrium belief of player 2 if she unexpectedly observes no message. Denote this belief by $\mu' = \Pr(t = l|\, n)$. The derivation of payoffs when player 1 deviates from the pooling equilibrium proceeds again in the same way as outlined in the proof of Lemma 5. Just replace λ by μ' and let k approach 0. Player 2 exerts effort

$$x_2(\mu') = \left(\frac{\mu' v + (1-\mu')\sqrt{v}}{v + \mu' v + (1-\mu')}\right)^2$$

while the two types of player 1 receive, respectively,

$$\begin{aligned}
\pi_l' &= 1 + x_2(\mu') - 2\sqrt{x_2(\mu')}, \\
\pi_h' &= v + x_2(\mu') - 2\sqrt{x_2(\mu') v}.
\end{aligned}$$

The pooling equilibrium with both types of player 1 sending a message of confidence exists if and only if there exists $\mu' \in [0,1]$ such that $\pi_t^* \geq \pi_t^{/}$ for both types. The conditions depend on the out-of-equilibrium belief μ' and the corresponding effort $x_2(\mu')$ of player 2. They can be written as

$$\begin{aligned} 1 + x_2 - 2\sqrt{x_2(1+k)} &\geq 1 + x_2(\mu') - 2\sqrt{x_2(\mu')}, \\ v + x_2 - 2\sqrt{x_2(v+k)} &\geq v + x_2(\mu') - 2\sqrt{x_2(\mu')v}, \end{aligned}$$

or

$$\begin{aligned} x_2 - x_2(\mu') &\geq 2\left(\sqrt{x_2(1+k)} - \sqrt{x_2(\mu')}\right), \\ x_2 - x_2(\mu') &\geq 2\left(\sqrt{x_2(v+k)} - \sqrt{x_2(\mu')v}\right). \end{aligned}$$

Two observations are important. First, the equilibrium conditions cannot hold if $\mu' = \lambda$ and subsequently $x_2(\mu') = x_2$. Player 1 has an incentive to send the message in the pooling equilibrium only if he would be punished by a more unfavorable belief ($\mu' > \lambda$) otherwise, which leads player 2 to exert higher effort (note that $x_2(\mu')$ is increasing in μ'). The harshest punishment for deviation is therefore a belief with $\mu' = 1$ and $x_2(\mu') = \frac{1}{4}$. Second, the equilibrium condition for the high type is redundant, because $\sqrt{x_2(1+k)} - \sqrt{x_2(\mu')} > \sqrt{x_2(v+k)} - \sqrt{x_2(\mu')v}$ whenever $v > 1$. To see this, consider the partial derivative of $\left(\sqrt{x_2(v+k)} - \sqrt{x_2(\mu')v}\right)$ with respect to v. This derivative is given by $\frac{1}{2}\left(\sqrt{\frac{x_2}{v+k}} - \sqrt{\frac{x_2(\mu')}{v}}\right)$. Because $x_2 < x_2(\mu')$ and $v + k > v$, it is always negative.

Together, the two observations imply that the pooling equilibrium exists if and only if the equilibrium payoff of the low-type player 1 is bigger than 1/4, his deviation payoff when player 2 holds belief $\mu' = 1$ and chooses effort $x_2(\mu') = \frac{1}{4}$. The corresponding condition

$$\pi_l^* = 1 + x_2 - 2\sqrt{x_2(1+k)} \geq \frac{1}{4}$$

is equivalent to

$$(\sqrt{x_2})^2 - 2\sqrt{1+k}(\sqrt{x_2}) + \frac{3}{4} \geq 0,$$

or

$$\begin{aligned} &\frac{[\lambda(v+k)\sqrt{1+k} + (1-\lambda)(1+k)\sqrt{v+k}]^2}{[(1+k)(v+k) + \lambda(v+k) + (1-\lambda)(1+k)]^2} \\ &-2\sqrt{1+k}\frac{[\lambda(v+k)\sqrt{1+k} + (1-\lambda)(1+k)\sqrt{v+k}]}{(1+k)(v+k) + \lambda(v+k) + (1-\lambda)(1+k)} + \frac{3}{4} \geq 0. \end{aligned} \tag{1}$$

We learn that the size of x_2 (under prior beliefs) matters critically. When λ approaches 0 and v tends to infinity, x_2 approaches 0 and π_l^* approaches 1. The low-type player 1 clearly has an incentive to play the equilibrium strategy. By continuity, a pooling equilibrium exists if v is big enough and λ is sufficiently small.

Now consider the limit $v \to 1$. Here, condition (1) simplifies to

$$\frac{[\lambda\sqrt{1+k}+(1-\lambda)\sqrt{1+k}]^2}{[(1+k)+\lambda+(1-\lambda)]^2}-2\sqrt{1+k}(\frac{[\lambda\sqrt{1+k}+(1-\lambda)\sqrt{1+k}]}{(1+k)+\lambda+(1-\lambda)})+\frac{3}{4}\geq 0$$

$$\Leftrightarrow \frac{1+k}{[2+k]^2}-\frac{2+2k}{2+k}+\frac{3}{4}\geq 0 \Leftrightarrow \frac{4+4k-8(1+k)(2+k)+3[2+k]^2}{4[2+k]^2}\geq 0$$

$$\Leftrightarrow \frac{-8k-5k^2}{4[2+k]^2}\geq 0.$$

Obviously, the condition can never be fulfilled. No pooling equilibrium exists since equilibrium payoffs are approaching those from the complete information case. Similarly, if $\lambda \to 1$, a pooling equilibrium cannot exist since the belief of player 2 cannot get any worse than the prior.

The payoff of the low-type player 1 in the candidate equilibrium is continuous in v and λ. Moreover, it is monotonically increasing in v and monotonically decreasing in λ (because the payoff is decreasing in x_2 and player 2 becomes discouraged more strongly if the expected strength of his rival increases by Lemma 5). This completes the proof. ■

Proof of Proposition 10. Allocative efficiency is an issue only if player 1 turns out to be of the high type. Consider the pooling equilibrium and suppose that player 1 has a high type. Using the first order condition of the high type of player 1, the probability that he wins is

$$\frac{x_{1h}}{x_{1h}+x_2}=\frac{\sqrt{x_2(v+k)}-x_2}{\sqrt{x_2(v+k)}}=1-\sqrt{\frac{x_2}{v+k}}.$$

From Lemma 5, we know that x_2 decreases in k. Thus the probability that 1 wins, conditional on his type being high, increases in k. ■

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C Additional Pages For The Referees (Not For Publication)

In the following, we show that important results from the complete-information model (in particular, Proposition 2) continue to hold (qualitatively) for more general classes of contest-success functions. Yet, we also provide a counterexample, in which Proposition 2 does not hold.

C.1 Asymmetric Ratio-Form Contest Success Function

Here we consider a more general contest success function:

$$p_i = \frac{a_i x_i^r}{a_i x_i^r + a_j x_j^r} \tag{2}$$

where a_i, a_j and r are positive parameters. We assume that, if $x_i = x_j = 0$, player i wins with some probability α_i, where $\alpha_1 + \alpha_2 = 1$. The precise value of α_i does not matter for our results. It is well known that a pure strategy Nash equilibrium of the contest stage exists if and only if r is sufficiently small. We assume that r is small enough such that there exists a pure strategy Nash equilibrium in any subgame starting at stage 3. In part 3, we also consider the case where r is so big that no pure strategy Nash equilibrium exists.

To simplify notation, let $a = a_1/a_2$ and $w_2 = v_2 + I_2$. Payoffs are

$$E[u_1] = \frac{a x_1^r}{a x_1^r + x_2^r}(v_1 + I_1) - x_1 - I_1 - c \text{ and}$$
$$E[u_2] = \frac{x_2^r}{a x_1^r + x_2^r} w_2 - x_2 - I_2 - c.$$

The first-order conditions are

$$\begin{aligned} \frac{r a x_1^{r-1} x_2^r}{(a x_1^r + x_2^r)^2}(v_1 + I_1) &= 1, \\ \frac{r a x_2^{r-1} x_1^r}{(a x_1^r + x_2^r)^2} w_2 &= 1. \end{aligned}$$

Therefore, equilibrium efforts are

$$\begin{aligned} x_1 &= \frac{r a (v_1 + I_1)^r w_2^r}{(a (v_1 + I_1)^r + w_2^r)^2}(v_1 + I_1), \\ x_2 &= \frac{r a (v_1 + I_1)^r w_2^r}{(a (v_1 + I_1)^r + w_2^r)^2} w_2. \end{aligned}$$

The equilibrium payoff of player 1 (neglecting entry cost c) is equal to

$$\frac{a (v_1 + I_1)^r}{a (v_1 + I_1)^r + w_2^r}(v_1 + I_1) - r(v_1 + I_1)\frac{a (v_1 + I_1)^r w_2^r}{(a (v_1 + I_1)^r + w_2^r)^2} - I_1.$$

We now show that, qualitatively, Proposition 2 holds in this more general setting. Suppose that $I_1 = k$. Then the payoff of 1, considered as a function of k, is

$$f(k) := \frac{a(v_1+k)^r}{a(v_1+k)^r + w_2^r}(v_1+k) - r(v_1+k)\frac{a(v_1+k)^r w_2^r}{\left(a(v_1+k)^r + w_2^r\right)^2} - k.$$

Taking the derivative,

$$f'(k) = -\frac{w_2^r\left(a^2(k+v_1)^{2r} + w_2^{2r} - a^2r^2(k+v_1)^{2r} + 2aw_2^r(k+v_1)^r + ar^2w_2^r(k+v_1)^r\right)}{\left(a(k+v_1)^r + w_2^r\right)^3}.$$

We show that $f'(k) < 0$, or equivalently,

$$a^2(k+v_1)^{2r}\left(1-r^2\right) + w_2^{2r} + 2aw_2^r(k+v_1)^r + ar^2w_2^r(k+v_1)^r > 0.$$

It is sufficient to show that

$$a(k+v_1)^r(1+r)(1-r) + 2w_2^r + r^2w_2^r > 0.$$

In order that a pure strategy equilibrium exists, the payoff of player 2 must be nonnegative. A necessary condition for this to be the case is

$$\frac{w_2^r}{a(v_1+k)^r + w_2^r}w_2 - rw_2\frac{a(v_1+k)^r w_2^r}{\left(a(v_1+k)^r + w_2^r\right)^2} \geq 0$$

or, equivalently,

$$a(v_1+k)^r(1-r) + w_2^r \geq 0.$$

Thus

$$\begin{aligned}
& a(k+v_1)^r(1+r)(1-r) + 2w_2^r + r^2w_2^r \\
\geq\ & (1+r)\left(-w_2^r\right) + 2w_2^r + r^2w_2^r \\
=\ & w_2^r\left(r^2 - r + 1\right) > 0.
\end{aligned}$$

The next proposition summarizes the result.

Proposition 11 *Consider an asymmetric ratio type contest success function (2) and suppose that entry costs are sufficiently low that the opponent enters even if i sends a message of confidence. Then in equilibrium no player will send a message of confidence.*

C.2 The Completely Discriminating Contest (All-Pay Auction)

Next we consider the case of a perfectly discriminating contest or all-pay auction. Suppose first that entry costs are zero. The payoff of $i = 1, 2$ is

$$u_i = p_i (v_i + I_i) - x_i - I_i,$$

where $p_i = 1$ if $x_i > x_j$, $p_i = 0$ if $x_i < x_j$, and $p_i = 1/2$ if $x_i = x_j$.

It is well known that the all-pay auction has a unique equilibrium, which is in mixed strategies (Baye et al. 1996). The expected equilibrium payoff of i is

$$\max\{(v_i + I_i - v_j - I_j), 0\} - I_i.$$

Let

$$D := \max\{(v_i + k - v_j - I_j), 0\} - k - \max\{(v_i - v_j - I_j), 0\}$$

denote the benefit from sending a message of confidence.

Proposition 12 *In the all-pay auction with complete information and zero entry cost, the benefit from sending a message of confidence is zero or negative:* $D \leq 0$.

Proof. If $v_i \geq v_j + I_j$, then $D = 0$. If $v_i + k \geq v_j + I_j > v_i$, then $D = v_i - v_j - I_j < 0$. If $v_j + I_j > v_i + k$, then $D = -k$. ■

There is, however, a difference between the all-pay auction and the imperfectly discriminating contests considered above. If $c > 0$, in the all-pay auction, in any pure strategy equilibrium only one player will enter. Moreover, as long as $c \leq v_1$, there exists an equilibrium where only 1 communicates and enters. Similarly, if $c \leq v_2$ and $v_2 + k > v_1$, there exists an equilibrium where only 2 communicates and enters.

C.3 Tullock Contest With $r > 2$

Finally, we consider the case of a ratio-form contest where no pure strategy equilibrium exists. Suppose that

$$p_i = \frac{x_i^r}{x_i^r + x_j^r}$$

and assume that $r > 2$. It is well known that the contest has no pure strategy Nash equilibrium under these assumptions. There is, however, a mixed strategy equilibrium where players have the same payoffs as in the all-pay auction (see Alcalde and Dahm 2010). Therefore, the analysis of the all-pay auction in the last subsection applies.

C.4 An Example Where Proposition 2 Does Not Hold

It seems that the discouragement effect should, in some conceivable circumstances, be strong enough to give incentives to send a message of confidence. For player 1, this might be the case if the reaction function of player 2 is decreasing very rapidly. As an illustration, we consider a contest where draws are possible and assume that a draw is as bad as losing for the players. This allows us to disentangle p_1 from p_2 and to illustrate the idea more clearly. In particular, by introducing the possibility of a draw we can keep the reaction function of player 1 as in the standard contest, but are able to manipulate the reaction function of player 2 to make it sufficiently steep.

Consider an example where $v_1 = v_2 = k = 1$, $c = 0$ and suppose that

$$p_1 = \frac{x_1}{x_1 + x_2}$$
$$p_2 = f(x_1) \frac{x_2}{x_1 + x_2}$$

and that there is a draw with probability

$$(1 - f(x_1)) \frac{x_2}{x_1 + x_2}.$$

[11]

Let the function f be nonnegative and equal to 1 for $x_1 \leq 1/4$. For $x_1 > 1/4$, assume

$$f(x_1) = \max \left\{ \left(1 - 100 \left(x_1 - \frac{1}{4}\right)\right) x_1, 0 \right\}.$$

That is, if x_1 gets bigger than 1/4, increasing x_1 increases the probability that player 1 wins the contest, but also the probability of a draw. Note that p_i is increasing and concave in x_i, so the game is well behaved. As mentioned before, a draw is assumed to be as bad as losing the contest. Hence, winning gives player i v_i, while losing or a draw gives $-I_i$.

Suppose $I_1 = k = 1$. Then the reaction functions are (in the relevant range)

$$x_2 = \sqrt{\left(1 - 100\left(x_1 - \tfrac{1}{4}\right)\right) x_1} - x_1$$
$$x_1 = \sqrt{2x_2} - x_2$$

Equilibrium is approximately at $x_1 = 0.25644$ and $x_2 = 4.5621 \times 10^{-2}$. The payoff of player 1 is approximately

$$0.84897 * 2 - 0.25644 - 1 = 0.4415.$$

This payoff is bigger than 1/4 (the payoff when choosing $I_1 = 0$).

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[11] The assumption $v_1 = v_2$ is only made to keep the example simple. It is not crucial for the argument.

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