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Non-uniform wage-staggering: european evidence and monetary policy implications

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Abstract

In many countries, wage changes tend to be clustered in the beginning of the year, with wages being set for fixed durations of typically one year. This has been, in particular, documented in recent years for European countries using microeconomic data. Motivated by this evidence we build a model of uneven wage staggering, embedded in a standard DSGE model of the euro area, and investigate the monetary policy consequences of non-synchronised wage setting. The model has the potential to generate responses to monetary policy shocks that differ according to the timing of the shock. Using a realistic calibration of the seasonality in wage-setting, based on a wide survey of European firms, the quantitative difference across quarters turns out however to be moderate. Relatedly, we obtain that the optimal monetary policy rule does not vary much across quarters.

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Summary

Actual patterns of wage setting are a key determinant of how economic shocks affect employment, unemployment and inflation. These patterns include the extent to which wages are indexed to past or future expected wage or price inflation, the extent to which they respond to movements in other costs, the extent to which wages are set differently for newly-employed workers as opposed to existing members of staff, how often wages are renegotiated, whether this renegotiation occurs at regular intervals and whether wages are renegotiated at the same time for the bulk of workers in an economy or wage negotiations are evenly spread out over the year. Recent research by the Eurosystem's Wage Dynamics Network has generated much microeconomic and survey evidence on all of these issues, as well as looked at the macroeconomic implications of this evidence. Some particular findings from the cross-country survey carried out by researchers within this network were that there is substantial heterogeneity in wage-setting institutions across European countries, that wages are typically adjusted once a year, less frequently than prices, and that wage-setting is staggered and synchronised, with a large proportion of wages reset in January.

In this paper, we first document recent evidence on the degree of synchronisation among wage setters in the euro area as whole and in some individual euro-area countries. We then construct a simple model of the euro area to investigate the macroeconomic and monetary policy consequences of these patterns of wage staggering. We construct a model in which, each quarter, a group of workers and employers set their wages for four quarters, but the proportion of workers doing this varies across quarters. With this model, we can study the case of full synchronisation of wage changes in a single quarter, or any particular breakdown of the probability of wage change across quarters that may match the actual bargaining pattern. We embed this set-up in a standard Dynamic Stochastic General Equilibrium model of the euro area. We find that, when wage staggering is uneven, then inflation and output are less persistent, both in general and, more specifically, in the way that they respond to monetary policy changes, than under an even wage staggering scheme. Furthermore, inflation responds by more, and more rapidly, to a given interest rate change if the Central Bank makes this change in the quarter when most workers are renegotiating their wages, ie, in quarter four, than in any other quarter. However, when calibrating the model with the micro data recently produced by the Wage Dynamics Network,

which feature a significant degree of uneven staggering, we find that the quantitative outcome is close to that resulting from an even staggering scheme. And we find that this result is robust to using a US calibration for the degree of wage synchronisation, to alternative ways of modelling when wages are reset, and to reasonable variations in the degree of price stickiness: the quantitative difference between the effects of a monetary policy shock in Q4 and other quarters remained small.

Armed with these results, we then consider the consequence of non-synchronised wage setting for optimal monetary policy. In particular we investigate whether the policy rule should vary from quarter to quarter as a result of seasonality in the wage-setting process. We find that the model has the potential to generate an optimal policy rule that varies considerably across quarters, especially in cases that get close to flexible prices and full synchronisation of wage changes. But, again, we find that under our baseline microeconomic calibration, in spite of some visible unevenness in wage-setting, there is little difference across quarters in the optimal policy response.

1 Introduction

Actual patterns of wage setting are a key determinant of how economic shocks affect employment, unemployment and inflation. These patterns include the extent to which wages are indexed to past or future expected wage or price inflation, the extent to which they respond to movements in other costs, the extent to which wages are set differently for newly-employed workers as opposed to existing members of staff, how often wages are renegotiated, whether this renegotiation occurs at regular intervals and whether wages are renegotiated at the same time for the bulk of workers in an economy or wage negotiations are evenly spread out over the year. Recent research by the Eurosystem's *Wage Dynamics Network* has generated much microeconomic and survey evidence on all of these issues, as well as looked at the macroeconomic implications of this evidence. Some particular findings from the cross-country survey carried out by researchers within this network were that there is substantial heterogeneity in wage-setting institutions across European countries, that wages are typically adjusted once a year, less frequently than prices, and that wage-setting is staggered and synchronised, with a large proportion of wages reset in January. Indeed, according to Druant, Fabiani, Kezdi, Lamo, Martins and Sabbatini (2009), 54% of firms report they change wages in a particular month once a year, with close to 30% of all wage changes take place in January. This observation has been confirmed by quantitative panel wage data (Heckel, Le Bihan and Montornes (2008) and Luennemann and Wintr (2008)).

In this paper, we use a simple model of the euro area to investigate the macroeconomic and monetary policy consequences of these patterns of wage staggering. We construct a model in which wages are set for four quarters as in the Taylor scheme, but the staggering across quarters is non-uniform. With this model, we can study the case of full synchronisation of wage changes in a single quarter, or any particular breakdown of the probability of wage change across quarters that may match the actual bargaining pattern. We embed this set-up in a standard Dynamic Stochastic General Equilibrium (DSGE) model of the euro area. We find that, when wage staggering is uneven, then inflation and output are less persistent, both unconditionally and following a monetary policy shock, than under an even wage staggering scheme. Furthermore, inflation responds by more, and more rapidly, to a given interest rate change if the Central Bank makes this change in the quarter when most workers are renegotiating their wages, ie, in quarter four, than in any other quarter. However when calibrating the model with the micro data recently produced by the WDN, which feature a significant degree of uneven staggering, we find that the quantitative outcome is close to that resulting from an even staggering scheme.

Our paper is closely related to Olivei and Tenreyro (2007), who have investigated similar issues in the case of the United States and argued that non-uniform wage staggering rationalises an asymmetric effect of monetary policy across quarters. Besides focusing on the euro area rather than on the United States, our work differs from theirs in our modelling and calibration approaches. First, to calibrate the model, we are able to rely on a large and representative survey of firms. Second, while wage-setting in the Olivei and Tenreyro (2007), model is based on the Calvo model, with the probability of being able to change wages being quarter-specific, we use the Taylor approach. We argue the latter approach is more in line with the microeconomic and survey evidence. We investigate the implications of these differences in specification choices and argue that to a large extent the contrasts between their results and ours is due to the degree of price rigidity. Finally a specific contribution of the present paper is to investigate the consequences of non-uniform staggerring on optimal monetary policy. Olivei and Tenreyro (2010) have provided time series evidence for several countries that the response of inflation and output to monetary policy shocks is quarter-dependent. They argue that the cross-country differences in the seasonal patterns of the responses can be related to the patterns of wage negotiations. Yet another related paper is Soderberg (2009) who studies a simple tractable model of non-uniform staggering in the case of price setting. Using a DSGE model, we here obtain results that are qualitatively in line with those outlined by Soderberg.

The remainder of the paper is structured as follows. Section 2 discusses some recent microeconomic evidence on wage-setting. Section 3 lays out the model used to analyse these issues. Section 4 derives and compares the time series properties implied by each specification of the model. Section 5 analyzes the responses of variables within our model to monetary policy shocks and compares these responses with what has been found elsewhere in the literature. Section 6 discusses how policy should respond in the face of staggered and synchronised wage-setting and Section 7 concludes.

2 Seasonality in wage setting: recent microeconomic evidence

A substantial amount of microeconomic evidence on the seasonality of the wage-setting process was recently produced, concerning a number of European countries. A first and main contribution to this series of evidence is a harmonised survey of firms initiated by the Eurosystem (Fabiani, Galuscak, Kwapil, Lamo and Room (2010) and Druant *et al* (2009)). This survey has a wide coverage: more than 17000 firms from 15 European countries were surveyed, data being gathered in 2007 and 2009. The studies point out that there is a seasonal pattern in wage-setting. Indeed, 54% firms report that they *change* wages in a specific month (or in specific months) in the year. More specifically there is a 'January effect': 29% of the firms report that they normally implement wage changes during the month of January.[1]

A second source of evidence is the analysis of collective wage agreement data. Recent studies using such data have been carried out for Austria (Knell and Stiglbauer (2009)), France (Avouyi-Dovi, Fougere and Gautier (2010)), and Spain (Izquierdo, Moral and Urtasun (2003)). In each of these three countries, wage agreements are found to typically last one year, and the signing and implementation of agreements are found to be to a large extent clustered in a given period in the year. There is however some heterogeneity across countries as regards the timing of wage agreements. In Spain, *wage changes* tend to occur around the month of June. Izquierdo *et al* (2003) report that 43% of agreements in their observation period (1990 to 2001) were *signed* in May to July. In the case of France, Avouyi-Dovi *et al* (2010) study both industry level and firm level agreements over the period 1999 to 2005. They report that wages tend to be *decided* in the last months of the year, and *wage changes take effect* at the beginning of the following year. They find differences across sectors and according to the type of agreements, since in the metal industry agreements are nearly all signed in December, while agreements are more staggered within the year in the case of firm-level agreements. Knell and Stiglbauer (2009), study a panel of collectively bargained wages in Austria from 1980 to 2006. Austria is characterised by 'pattern bargaining': the metal industry plays a leading role in setting the level of wage increases in the economy. The bargaining period for that sector is September and October, and negotiations in others sectors follow in an unevenly staggered fashion. Overall, new wage agreements are more often signed in the first quarter of the year: on average 46% of agreements are signed in the first quarter.

[1] This figure do not include Germany since this question was not asked in the questionnaire for Germany.

These findings are confirmed by Caju, Gautier, Momferatou and Ward-Warmedinger (2008) who summarise institutional patterns of wage negotiations in 23 European countries, based on answers of national experts to a common questionnaire. du Caju *et al* (2008) find that wage agreements are mainly signed around the first quarter of the year, with some heterogeneity within the euro area.

Finally, two studies for France (Heckel *et al* (2008)) and Luxembourg (Luennemann and Wintr (2008)) have analysed a third type of data: quantitative panel wage data at an infra-annual frequency. Heckel *et al* (2008) analyze a quarterly dataset from a firm-level survey over the period 1998 to 2005, reporting wages at a narrow occupation level. The frequency of wage changes is found to be on average 47.7% in the first quarter, versus 34.8% on average in other quarters (with another spike in the frequency in the third quarter due to the change in the minimum wage occurring in July). Luennemann and Wintr (2008) analyze a monthly social security dataset of individual wages over the period 2001-2006 in Luxembourg. Wage changes are found to be clustered in January. Based on their preferred procedure for addressing measurement errors, the corrected frequency is 20.6% in January versus 3.3% on average in other months.[2] The contrast between January and other months is robust to the procedure used for correcting for measurement errors.

Overall, the European wage-setting process is characterised by a noticeable degree of seasonal synchronisation . Although there is some heterogeneity in the timing of wage changes across countries, and some staggering in each country, there is a tendency for wage changes to be clustered in the first quarter of the year. The above mentioned evidence is summarised in Table 1, which reports the breakdown of the occurrence of wage changes within the year across quarters, or the frequency of wage changes across quarters, from various studies.

It is worthwhile contrasting this evidence with corresponding evidence for the United States and Japan. In Japan it is well known that wage-setting is much synchronised (eg, du Caju *et al* (2008)). Most wages are negotiated in the March to May period, as part of the so-called 'Shunto' process. By contrast, evidence for the United States is scarce: Olivei and Tenreyro (2007) based on 'anecdotal' evidence, and on a survey carried out in New England in 2003, report that there is a tendency for wage agreements to be signed in the fourth quarter of the year,

[2] These figures abstract from the systematic indexation procedure that is triggered whenever inflation exceeds a certain threshold.

Table A: Timing of wage changes across the year: European evidence

Data type	Paper	Country	Q1	Q2	Q3	Q4
	Panel A: Breakdown of wage changes					
Survey	Druant *et al* (2009)	Euro area	38	12	12	6
		EA normalised	55	18	18	9
Collective agreements	Avouyi-Dovi *et al* (2010)	France	37	25	23	11
	Knell and Stiglbauer (2009)	Austria	46	29	9	13
	Izquierdo *et al* (2003)	Spain	20	40	21	19
	Panel B: Frequency of wage change					
Quantitative wage data						
	Heckel *et al* (2008)	France	47.8	36.5	40.5	29.1
	Luennemann and Wintr (2008)	Luxembourg	26.2	10.4	9.5	10.4

Note: In the top panel figure ('Breakdown') in each column is the percentage of *wage changes* that occur in each quarter. In the bottom panel, figures are the frequency of *wage changes* in a given quarter. It is important to note that in almost all these sources, wages tend to be *negotiated* in the quarter before, to the actual changes becoming effective in the current quarter. Sources are : Druant *et al* (2009), figure 2 (numbers were kindly provided by the authors). Avouyi-Dovi *et al* (2010) Table 7 (we use firm level agreements), Knell and Stiglbauer (2009) Table 3, Izquierdo *et al* (2003), Table 5 (reported figures refer to months when wage contracts are signed), Heckel *et al* (2008), figure 4 (numbers were kindly provided by the authors) Luennemann and Wintr (2008) figure 1 (numbers were kindly provided by the authors - we use results after the error correction procedures of the authors and aggregate monthly figures to a quarterly frequency). Due to the nature of each dataset and potential measurement errors, the levels may not be fully comparable across studies. In the case of Druant *et al* (2009) some firms do not report a particularly preferred month. The 'EA normalised' figures assume that the proportions of thse firms changing wages in each quarter are the same as for those firms that report a preferred month. We use these figures for calibration purposes.

and become effective early in the first quarter of the subsequent year. This statement has however recently been challenged by Barattieri, Basu and Gottschalk (2009). Using quantitative data from the Survey of Income and Program Participation (SIPP), these authors find that the probability of wage change does not vary across quarters. Wage data in the SIPP are self-reported data by households, thus viewed as prone to measurement errors. Barattieri *et al* (2009) however also fail to detect seasonality in the frequency of wage change after filtering their data using 'unknown breakpoint' time series procedures.

3 The model

To investigate the effects of synchronisation and staggering in wage setting, we develop a model based on the standard euro-area DSGE model of Smets and Wouters (2003). To capture the microeconomic facts documented in the above section we alter that model along several dimensions. First, we model wage-setting based on the Taylor (1980) model, since the microeconomic evidence suggests that wages tend to change systematically once a year. This evidence does not match with the distribution of wage durations implied by the Calvo model employed by Smets and Wouters (2003). Further, we extend the framework of Taylor (1980) to allow for asymmetric synchronisation in wage setting across the year. A related extension is proposed and studied by Knell (2010), in the context of an analytically tractable model of non-synchronised wage-setting with two cohorts. The standard Taylor (1980) model with wages set evenly over the year is a particular case. Finally, consistent with institutional and microeconomic evidence we allow wages to be determined one period in advance.

3.1 Consumers and demand

The economy is populated by a unit continuum of households indexed by j. Although households will earn different wages, since they bargain wages in different quarters and wages are assumed to be fixed until the same quarter in the following year, we follow Erceg, Henderson and Levin (2000) and assume complete financial markets, so that consumption is the same across all households. The households' problem is to maximise the discounted value of their current and future expected streams of utility, which are positive in consumption and negative in hours worked. Mathematically this can be written as:

$$\text{Maximise } E_0 \sum_{t=0}^{\infty} \beta^t e^{\varepsilon_{b,t}} \left(\frac{\left(c_{j,t} - \psi c_{t-1}\right)^{1-\sigma} - 1}{1-\sigma} - \frac{e^{\varepsilon_{h,t}} h_{j,t}^{1+\phi}}{1+\phi} \right) \quad \textbf{(1)}$$

where c_j denotes consumption of consumer j, c denotes aggregate consumption, h_j denotes total hours worked by j , ε_b is a demand shock and ε_h is a labour supply shock. In this utility function, $1/\sigma$ will be the intertemporal elasticity of substitution and $1/\phi$ will be the elasticity of total hours worked with respect to the real wage in a flexible-price equilibrium. Households own the capital and rent it to firms. Consumers face a standard budget constraint given by:

$$\frac{B_{j,t}}{P_t} = (1+i_{t-1}) \frac{B_{j,t-1}}{P_t} + r_{k,t} z_{j,t} k_{j,t-1} + w_{j,t} h_{j,t} + \Pi_{j,t} - c_{j,t} - I_{j,t} - \frac{a_0}{1+\sigma_z} \left(z_t^{1+\sigma_z} - 1 \right) k_{j,t-1} - T_{j,t} \quad \textbf{(2)}$$

and a capital accumulation constraint given by:

$$k_{j,t} = (1-\delta)\, k_{j,t-1} + \left(1 - \frac{\kappa}{2}\left(\frac{e^{\varepsilon_{I,t}} I_{j,t}}{I_{j,t-1}} - 1\right)^2\right) I_{j,t} \tag{3}$$

where B_j denotes consumer j's holdings of bonds, i is the nominal interest rate, r_k denotes the rental rate for capital, k_j denotes the end-of-period capital stock held by household j, z_j denotes the rate at which household j utilises its capital, w_j denotes the real wage paid to worker j, Π_j denotes the share of the real profits of firms which are transferred lump sum to household j, I_j is investment by household j, T_j is a lump-sum tax and P is the aggregate price level. The function $\frac{a_0}{1+\sigma_z}\left(z_t^{1+\sigma_z} - 1\right) k_{j,t-1}$ represents a cost to the household of overutilising its capital stock and the function $\left(1 - \frac{\kappa}{2}\left(\frac{e^{\varepsilon_{I,t}} I_{j,t}}{I_{j,t-1}} - 1\right)^2\right) I_{j,t}$ represents the costs of adjusting investment. Here, $\varepsilon_{I,t}$ represents an investment-specific technology shock.

We assume that there exist complete markets and so, in equilibrium, consumption, investment, capital holdings and capital utilisation will be identical for all agents. Given this, the first-order conditions imply:

$$\frac{e^{\varepsilon_{b,t}}\left(c_t - \psi c_{t-1}\right)^{-\sigma}}{P_t} = \beta\left(1+i_t\right) E_t \frac{e^{\varepsilon_{b,t+1}}\left(c_{t+1} - \psi c_t\right)^{-\sigma}}{P_{t+1}} \tag{4}$$

$$p_{k,t} = \beta E_t \frac{\left(c_{t+1} - \psi c_t\right)^{-\sigma}}{\left(c_t - \psi c_{t-1}\right)^{-\sigma}} \left(r_{k,t+1} z_{t+1} - \frac{a_0}{1+\sigma_z}\left(z_t^{1+\sigma_z} - 1\right) + p_{k,t+1}\left(1-\delta\right)\right) \tag{5}$$

$$p_{k,t}\left(1 - \frac{\kappa}{2}\left(\frac{e^{\varepsilon_{I,t}} I_t}{I_{t-1}} - 1\right)^2\right) \tag{6}$$

$$= 1 + p_{k,t}\kappa \frac{e^{\varepsilon_{I,t}} I_t}{I_{t-1}}\left(\frac{e^{\varepsilon_{I,t}} I_t}{I_{t-1}} - 1\right) - \beta E_t \frac{\left(c_{t+1} - \psi c_t\right)^{-\sigma}}{\left(c_t - \psi c_{t-1}\right)^{-\sigma}} p_{k,t+1}\kappa\varepsilon_{I,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2\left(\frac{e^{\varepsilon_{I,t+1}} I_{t+1}}{I_t} - 1\right)$$

$$r_{k,t} = a_0 z_t^{\sigma_z} \tag{7}$$

Log-linearising these equations around the non-stochastic, zero-inflation, steady state gives:

$$\widehat{c}_t = \frac{\psi}{1+\psi}\widehat{c}_{t-1} + \frac{1}{1+\psi}E_t\widehat{c}_{t+1} - \frac{1-\psi}{1+\psi}\frac{1}{\sigma}\left(\left(i_t - r\right) - E_t\pi_{t+1} + E_t\varepsilon_{b,t+1} - \varepsilon_{b,t}\right) \tag{8}$$

$$\widehat{p}_{k,t} = E_t\pi_{t+1} - (i_t - r) - E_t\left(\varepsilon_{b,t+1} - \varepsilon_{b,t}\right) + E_t\left((1-\beta(1-\delta))\left(r_{k,t+1} - r - \delta\right) + \beta(1-\delta)\widehat{p}_{k,t+1}\right) + \varepsilon_{pk,t} \quad \textbf{(9)}$$

where we have followed Smets and Wouters (2003) in adding a white-noise equity risk premium shock, ε_{pk}, that was not derived from economic fundamentals.

$$\widehat{I}_t = \frac{1}{1+\beta}\widehat{I}_{t-1} + \frac{\beta}{1+\beta}E_t\widehat{I}_{t+1} + \frac{1}{\kappa(1+\beta)}\widehat{p}_{k,t} + \beta E_t\varepsilon_{I,t+1} - \varepsilon_{I,t} \quad \textbf{(10)}$$

$$\left(r_{k,t} - r - \delta\right) = \sigma_z\widehat{z}_t \quad \textbf{(11)}$$

$$\widehat{k}_t = (1-\delta)\widehat{k}_{t-1} + \delta\widehat{I}_t \quad \textbf{(12)}$$

where π is the inflation rate (assumed to be equal to zero in steady state), r is the steady-state nominal (and real) interest rate and 'hat' variables denote log deviations from steady state.

3.2 Labour supply and wage setting

Following Smets and Wouters (2003) and Erceg *et al* (2000) we assume that households have some market power in wage setting. That is, we assume that labour of type j is only partly substitutable for labour of type k, say. In particular, we assume that the demand for labour of type j is given by:

$$h_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{-\frac{1+\lambda}{\lambda}} h_t \quad \textbf{(13)}$$

where W is the aggregate nominal wage and h is aggregate total hours worked. Households are assumed to set wages with employment being determined by firms. With flexible wages, they would choose the nominal wage consistent with a real wage that was a mark-up, equal to $1+\lambda$, on the ratio of the marginal disutility of work to the marginal utility of consumption, $-\frac{U_{h,t}}{U_{c,t}} = \frac{e^{\varepsilon_{h,t}}h_t^{\phi}}{(c_t - c_{t-1})^{-\sigma}}$.

We follow Taylor (1980) and assume that workers are only able to reset their wages once a year in a known quarter. This departs from the Calvo assumptions used by Erceg *et al* (2000) and Smets and Wouters (2003) of a constant hazard of changing wages. Our assumption is more in line with virtually all the studies that have documented the microeconomic patterns of wage bargaining, in particular those mentioned in Table A. In addition, we assume that wages are 'predetermined': workers negotiate in period t the wage that they will earn in periods $t+1$ through $t+4$, also a pattern consistent with microeconomic evidence on wage bargaining. In this case, we can write the problem for worker able to reset his wage as:

$$\text{Maximise } E_t \sum_{r=1}^{4} \beta^r \left(\frac{U_{c,t+r}}{P_{t+r}} \widetilde{W}_t \left(\frac{\widetilde{W}_t}{W_{t+r}} \right)^{-\frac{1+\lambda}{\lambda}} h_{t+r} - \frac{e^{\varepsilon_{h,t}}}{1+\phi} \left(\left(\frac{\widetilde{W}_t}{W_{t+r}} \right)^{-\frac{1+\lambda}{\lambda}} h_{t+r} \right)^{1+\phi} \right) \quad \textbf{(14)}$$

where we have used $\widetilde{W}_t$ to denote the wage negotiated in period t for those workers who can reset their wages. The first order condition for this problem is:

$$E_t \sum_{r=1}^{4} \beta^r \left(\frac{-U_{c,t+r}}{\lambda P_{t+r}} \left(\frac{\widetilde{W}_t}{W_{t+r}} \right)^{-\frac{1+\lambda}{\lambda}} h_{t+r} + \frac{1+\lambda}{\lambda} \frac{e^{\varepsilon_{h,t}}}{\widetilde{W}_t} \left(\left(\frac{\widetilde{W}_t}{W_{t+r}} \right)^{-\frac{1+\lambda}{\lambda}} h_{t+r} \right)^{1+\phi} \right) = 0 \quad \textbf{(15)}$$

Log linearising this equation and noting that all workers setting wages will set the same wage gives:

$$\left(1 + \phi \frac{1+\lambda}{\lambda} \right) \widehat{\widetilde{W}}_t = \frac{1}{1+\beta+\beta^2+\beta^3} E_t \sum_{r=1}^{4} \beta^r \left(\phi \widehat{h}_{t+r} + \frac{1+\lambda}{\lambda} \phi \widehat{W}_{t+r} - \widehat{U}_{c,t+r} + \widehat{P}_{t+r} \right) + \varepsilon_{h,t} \quad \textbf{(16)}$$

We can write this in terms of real wages as:

$$\left(1 + \phi \frac{1+\lambda}{\lambda} \right) \widehat{\widetilde{\omega}}_t = \frac{1}{1+\beta+\beta^2+\beta^3} E_t \sum_{r=1}^{4} \beta^r \left(\begin{array}{c} \phi \widehat{h}_{t+r} + \frac{1+\lambda}{\lambda} \phi \widehat{\omega}_{t+r} - \widehat{U}_{c,t+r} \\ + \left(1 + \phi \frac{1+\lambda}{\lambda} \right) \sum_{j=1}^{r} \widehat{\pi}_{t+j} \end{array} \right) + \varepsilon_{h,t} \quad \textbf{(17)}$$

where we have used $\widetilde{\omega}_t$ to denote the *real* wage negotiated in period t for those workers who can reset their wages. We can write the aggregate wage index as:

$$w_t = \sum_{k=1}^{4} \frac{\alpha_{t-k} \widetilde{w}_{t-k}}{\prod_{i=0}^{k-1} (1 + \pi_{t-i})} \quad \textbf{(18)}$$

where α_{t-k} is the proportion of workers who negotiated their wages k quarters ago. Log-linearising this equation implies that:

$$\widehat{w}_t = \sum_{k=1}^{4} \alpha_{t-k} \left(\widehat{\widetilde{w}_{t-k}} - \sum_{i=0}^{k-1} \pi_{t-i} \right) \quad \textbf{(19)}$$

Under our seasonal staggering scheme, α_t varies across time in a deterministic fashion: $\alpha_t = \alpha_1^*$ in the first quarter, $\alpha_t = \alpha_2^*$ in the second quarter, $\alpha_t = \alpha_3^*$ in the third quarter, $\alpha_t = \alpha_4^*$ in the fourth quarter, and $\sum_{i=1}^{4} \alpha_i^* = 1$ holds. The standard Taylor staggering structure is a particular case where $\alpha_q^* = 0.25\ \forall q$. We can also consider the case of full synchronisation in one quarter with say $\alpha_4^* = 1$, and $\alpha_q^* = 0\ \forall q \neq 4$, or any weighting structure suggested by available micro-data.

Finally we underline that we have removed the assumption of systematic indexation of wages embodied in Smets and Wouters (2003) and the related literature. Indeed, microeconomic and institutional evidence suggests that systematic indexation of wages is not a pattern of wage setting in most European countries. Formal indexation mechanisms do exist in some European countries, but usually do not operate at a quarterly frequency (du Caju *et al* (2008)).

A consequence of our specification choices (removing indexation and using Taylor rather than Calvo contracts), is that our model will predict less inflation persistence than those of Smets and Wouters (2003) or Olivei and Tenreyro (2007). Though our emphasis is not on fitting a specific set of macro moments, it is likely that our bottom-up approach, while being more consistent with micro facts on the wage bargaining side, may lead us to underpredict the actual degree of inflation persistence. Chari *et al* (2010) illustrate the difficulty of generating macro persistence under Taylor contracts. While we do not elaborate further on this point, we note that increasing persistence while preserving some consistency with micro facts could be done by introducing heterogeneity in price or wage stickiness, as in Carvalho (2006) and Dixon and Kara (2010).

3.3 *Firms and price setting*

As is standard in New Keynesian models, firms are assumed to have some market power and so set price to maximise profits. Following Calvo (1983), we suppose that each period firms are only able to reset their price optimally with probability $1-\xi$. If they do not reset their price optimally, we assume that they partially index their prices to past inflation. Specifically for these firms:

$$P_{k,t} = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\varepsilon} P_{k,t-1} \qquad \textbf{(20)}$$

where ε is the indexation parameter, P_k is the price charged by firm k for its output and P is the aggregate price level.

We suppose that all firms are monopolistically competitive and face a demand curve for their product given by:

$$y_{k,t} = \left(\frac{P_{k,t}}{P_t}\right)^{-\eta} y_t \qquad \textbf{(21)}$$

where y_k is the level of firm k's output and y is aggregate demand.

The production function is assumed to be given by:

$$y_{k,t} = e^{\varepsilon_{a,t}} h_{k,t}^{1-\alpha} \left(z_t k_{k,t-1}\right)^{\alpha} \qquad \textbf{(22)}$$

where h_k is firm k's input of labour, $k_{k,t-1}$ is the amount of capital rented by firm k in period t and ε_a is an economy-wide productivity shock. Log-linearising the production function and aggregating across firms gives us:

$$\widehat{y}_t = \varepsilon_{a,t} + \alpha\left(\widehat{z}_t + \widehat{k}_{t-1}\right) + (1-\alpha)\,\widehat{h}_t \qquad \textbf{(23)}$$

Cost minimisation implies:

$$w_t = \mu_t\,(1-\alpha)\,\frac{y_t}{h_t} \qquad \textbf{(24)}$$

and

$$r_{k,t} = \mu_t \alpha \frac{y_t}{z_t k_{t-1}} \qquad \textbf{(25)}$$

where μ is real marginal cost (the inverse of the mark-up), which, in steady state, will equal $\frac{\eta-1}{\eta}$. Log-linearising these equations gives us:

$$\widehat{w}_t = \widehat{\mu}_t + \widehat{y}_t - \widehat{h}_t \quad \textbf{(26)}$$

and

$$\widehat{r}_{k,t} = \widehat{\mu}_t + \widehat{y}_t - \widehat{z}_t - \widehat{k}_{t-1} \quad \textbf{(27)}$$

Those firms that can change their price set their price optimally; that is, they set their price so as to maximise their expected profit subject to their demand curves and the fact that they may not be able to change their price for a long while. Given that they are assumed to minimise their costs, their expected profit will be given by:

$$E_t \sum_{s=0}^{\infty} (\beta\xi)^s \left(\frac{P_{j,t}}{P_{t+s}} \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\varepsilon} - \mu_{t+s} \right) y_{t+s} \left(\frac{P_{j,t}}{P_{t+s}} \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\varepsilon} \right)^{-\eta} \quad \textbf{(28)}$$

The first-order condition for a price-changing firm (say, firm j) will be given by:

$$E_t \sum_{s=0}^{\infty} (\beta\xi)^s \left(\left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\varepsilon} - \eta \left(\left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\varepsilon} - \mu_{t+s} \frac{P_{t+s}}{P_{j,t}} \right) \right) \frac{y_{t+s}}{P_{t+s}} \left(\frac{P_{j,t}}{P_{t+s}} \left(\frac{P_{t+s-1}}{P_{t-1}} \right)^{\varepsilon} \right)^{-\eta} = 0 \quad \textbf{(29)}$$

The aggregate price level will be given by:

$$P_t = \left(P_{t-1} \left(1 + \pi_{t-1}\right)^{\varepsilon}\right)^{\xi} P_{j,t}^{1-\xi} \quad \textbf{(30)}$$

Log-linearising equations **(29)** and **(30)** around a zero steady-state rate of inflation and combining them gives the New Keynesian Phillips Curve:

$$\pi_t = \frac{\varepsilon}{1+\beta\varepsilon}\pi_{t-1} + \frac{\beta}{1+\beta\varepsilon}E_t\pi_{t+1} + \frac{(1-\xi)(1-\beta\xi)}{\xi(1+\beta\varepsilon)}\left(\widehat{\mu}_t + \varepsilon_{\mu,t}\right) \quad \textbf{(31)}$$

where ε_{μ} is a white-noise price mark-up shock.

3.4 Monetary policy, shock processes and equilibrium

We assume that the central bank operates a Taylor rule of the form:

$$i_t - r = \rho_m \left(i_{t-1} - r\right) + \left(1-\rho_m\right)\left(\phi_\pi \pi_t + \phi_y \left(\widehat{y}_t - \widehat{y}_{fp,t}\right)\right) + \varepsilon_{i,t} \quad \textbf{(32)}$$

where ε_i is a white noise monetary policy shock and y_{fp} represents the level of output that would transpire in a flexible-price version of the model given the current real shocks.

We assume exogenous government spending and, without loss of generality, a zero supply of government bonds. This gives us the aggregate resource constraint:

$$y_t = c_t + I_t + \frac{a_0}{1+\sigma_z}\left(z_t^{1+\sigma_z} - 1\right)k_{t-1} + g_t \tag{33}$$

where g_t is (exogenous) government expenditure. Log-linearising implies:

$$\widehat{y_t} = \frac{c}{y}\widehat{c_t} + \frac{\delta k}{y}\widehat{I_t} + \frac{r_k k}{y}\widehat{z_t} + \frac{g}{y}\varepsilon_{g,t} \tag{34}$$

where c is steady-state consumption, y is steady-state output, k is the steady-state capital stock, g is steady-state government spending and ε_g is a government spending shock. Finally, we assume that the shocks follow the following AR(1) processes:

$$\varepsilon_{a,t} = \rho_a \varepsilon_{a,t-1} + \zeta_{a,t} \tag{35}$$

$$\varepsilon_{b,t} = \rho_b \varepsilon_{b,t-1} + \zeta_{b,t} \tag{36}$$

$$\varepsilon_{g,t} = \rho_g \varepsilon_{g,t-1} + \zeta_{g,t} \tag{37}$$

$$\varepsilon_{I,t} = \rho_I \varepsilon_{I,t-1} + \zeta_{I,t} \tag{38}$$

$$\varepsilon_{h,t} = \rho_h \varepsilon_{h,t-1} + \zeta_{a,t} \tag{39}$$

where the ζs are all white noise innovations.

An equilibrium for the model is where equations **(8)**, **(9)**, **(10)**, **(11)**, **(12)**,**(23)**, **(26)**, **(27)**, **(31)** and **(32)** hold together with **(17)** and **(19)** and **(34)** to **(39)**. These equations solve for the thirteen endogenous variables: y, c, I, π, w, $\widetilde{w}$, r_k, p_k, h, k, z, μ and i together with the five exogenous variables, ε_a, ε_b, ε_g, ε_I and ε_h.

The model is solved numerically using the *Dynare* software. To deal with the issue of agents having different decision rules depending upon what quarter they were in, we used the trick of considering four 'virtual quarters' that are simulated simultaneously within each period of the model: the four virtual quarters have the decision rules associated with quarter 1, quarter 2, quarter 3 and quarter 4, respectively. The procedure is described in an Appendix.[3]

For comparison purposes, we also considered a version of the model in which wage rigidity is described by a seasonal Calvo process as in Olivei and Tenreyro (2007), featuring indexation of wages. The reader is referred to Olivei and Tenreyro (2007) for a complete description of the model of wage rigidity in that case.

3.5 Calibration

We calibrated the model mainly using the mode of the posterior estimates of the parameters reported by Smets and Wouters (2003). In particular, we set the parameters of our model other than those describing the wage process, as shown in Table B.

Following Smets and Wouters (2003) we set the discount factor to 0.99, implying a steady-state real interest rate of 4% per annum, and the wage mark-up to 0.5. The elasticity of output with respect to capital input is set to 0.3 and the depreciation rate on capital to 0.025 (implying 10% depreciation per annum). Following estimates in Smets and Wouters (2003) the intertemporal elasticity of substitution is set to 1.391, the degree of habit formation in consumption to 0.592 and the inverse Frisch elasticity of labour supply to be 2.503. The elasticity of capital utilisation with respect to the rental rate on capital is set to 0.201 and the elasticity of investment adjustment costs with respect to the (shadow) price of capital is set to 0.025. The estimated probability of price change under the Calvo assumption is 0.095 per quarter ($\xi = 0.905$) and the degree of indexation is set to $\varepsilon = 0.477$. All these values are fairly standard in the literature and most of them are in line with those used for the United States by Olivei and Tenreyro (2007). An exception is the degree of price rigidity which is markedly higher than the one used by Olivei and Tenreyro (2007): $\xi = 0.35$ based on Christiano *et al* (2005). The discrepancy could reflect lower price rigidity in the US than in the euro area (see Dhyne *et al* 2006 for micro evidence), but may be a result of the estimation approach chosen by Smets and Wouters (2003) vis-a-vis that used by Christiano *et al* (2005).

[3] We are extremely grateful to Julien Matheron for suggesting this procedure.

Table B: Parameter values

Parameter	Description	Value
β	Discount factor	0.99
σ	Reciprocal of intertemporal elasticity of substitution	1.391
ψ	Degree of habits in consumption	0.592
ϕ	Reciprocal of Frisch elasticity of labour supply	2.503
λ	Wage mark-up	0.5
δ	Capital depreciation rate	0.025
α	Capital share	0.3
σ_z	Elasticity of capital utilisation wrt rental rate	0.201
κ	Elasticity of investment adjustment costs	6.962
ξ	Probability of not changing price	0.905
ε	Degree of price indexation	0.477
η	Elasticity of demand for intermediate goods	11
ϕ_π	Taylor rule coefficient on inflation	1.588
ϕ_y	Taylor rule coefficient on output gap	0.098
ρ_m	Taylor rule coefficient on lagged interest rates	0.956
ρ_a	Persistence of productivity shock	0.811
ρ_b	Persistence of preference shock	0.838
ρ_g	Persistence of government spending shock	0.943
ρ_I	Persistence of investment shock	0.910
ρ_h	Persistence of labour supply shock	0.881
σ_i	Standard-deviation of interest rate shock	0.09
σ_a	Standard-deviation of productivity shock	0.639
σ_b	Standard-deviation of preference shock	0.407
σ_g	Standard-deviation of government spending shock	0.335
σ_I	Standard-deviation of investment shock	0.113
σ_h	Standard-deviation of labour supply shock	3.818
σ_μ	Standard-deviation of price mark-up shock	0.165
σ_{pk}	Standard-deviation of price of capital	0.165

Table C: Alternative calibrations of wage contracts

Calibration	Standard Taylor	Synchronised Q4	WDN	US (OT)
α_1^*	0.25	0	0.18	0.24
α_2^*	0.25	0	0.18	0.02
α_3^*	0.25	0	0.10	0.32
α_4^*	0.25	1	0.54	0.42

For the Taylor rule, the response on inflation is 1.588 and that on the output gap is 0.125 and the coefficient on lagged interest rates is set to 0.92 implying a high degree of persistence in interest rates. The monetary policy shock is white noise with a standard deviation of 0.09 percentage points. Finally, for the AR(1) processes for the other shocks we also use the numbers estimated using euro-area data by Smets and Wouters (2003).

Table C summarises the four variants of the calibration we use for the pattern of wage bargaining. In the first variant, we consider the standard Taylor staggering structure and assume that in each quarter 25% of workers renegotiate their wage (ie, $\alpha_q^* = 0.25\ \forall q$). In the second variant, we assume for illustration purposes that all wages are negotiated in quarter 4 ($\alpha_4^* = 1$). The third variant is a case of asymmetric staggering, calibrated using the results of the European survey reported by Druant *et al* (2009). We assume that 54% of workers renegotiate their wages in Q4 (and their wages change in Q1), 18% in each of Q1 and Q2 (wages change in Q2 and Q3, respectively) and the remainder (10%) in Q3 (wages change in Q4). Finally the last column of Table C reports the calibration used by Olivei and Tenreyro (2007), drawing on a survey in the State of New England and other 'anecdotal evidence' from the United States.

When we consider the version of the model with Calvo wage contracts , we need to calibrate the probabilities of not changing wages a_1, a_2, a_3, a_4. We follow the strategy in Olivei and Tenreyro (2007) for converting the distribution of durations into these Calvo parameters. We calibrate the probabilities a_1, a_2, a_3, a_4, so that the proportion of wage changes that occur in each quarter q is α_q, implying $\alpha_q = (1 - a_q)/(4 - \sum a_j)$. We additionally impose that the average duration, which is the reciprocal of the average frequency of price change, matches that of the model with Taylor contracts (here four quarters by construction). This implies $1/(1 - (a_1 a_2 a_3 a_4)^{\frac{1}{4}}) = 4$.

4 Time series properties under alternative synchronisation patterns

A first question we investigate is the extent to which staggering and synchronisation in wage setting affect the cyclical properties of output, total hours worked, real and nominal wages and inflation. To that end we simulated each of our model variants. We first generated time series for all shocks of the model, each of 30,100 periods (dropping the first 100 observations), used Dynare to generate time series for the endogenous variables in our models conditional on these shocks and, then, calculated the implied second moments.

The results are shown in Tables D through F, below. Each table shows the standard deviations of output, price inflation, the interest rate, total hours worked and the real wage, together with their standard deviation relative to that of output. In addition, it shows the cyclical properties of each of these variables – that is, their correlation with leads and lags of output – and their autocorrelation coefficients of orders one through five.

Table D shows the results for the standard staggered Taylor (1980) model. We can note that output is very volatile, as are total hours and the real wage while quarterly price inflation is much less volatile. Total hours, the real wage and inflation are procyclical with real wages lagging output by three quarters, while inflation and total hours move contemporaneously with output. Finally, the model suggests that output, the real wage and price inflation are highly persistent with first-order autocorrelations of 0.94 and above and significant autocorrelation coefficients for a lag of five quarters. Total hours worked are somewhat less persistent.

Table E shows the extreme case in which all wages are negotiated in Q4 each year (and set in Q1 next year) and subsequently kept fixed throughout the year. In this case, quarterly nominal wage inflation is equal to zero in three quarters out of four. Here, we see more volatility in real wages and, to a lesser degree, price inflation. The structure of wage contracting implies that, at the time of renegotiating wages, the variables more than four quarters ahead are largely irrelevant since all wage-setters will be able to reset wage at that horizon. There is no ‘contract multiplier’ as in the standard staggered wage model. Annual real wage growth, thus marginal cost growth, is much less persistent. As a result inflation, becomes much less persistent: the autocorrelation of order 5 is 0.392 against 0.512 in the above case of even staggering. Recall that prices, unlike wages, are still staggered and indexed in this specification, contributing to maintaining persistence in the

Table D: Simulation results for the Taylor (1980) model with even staggering

Taylor Staggered		Output	Inflation	Interest rate	Real wage	Hours
Standard deviation (% or percentage points)		3.051	0.238	0.161	2.166	2.958
Standard deviation relative to output		1.000	0.078	0.053	0.710	0.970
Correlation of variable at time t with output at time:	$t-4$	0.731	0.578	-0.152	0.768	0.611
	$t-3$	0.823	0.680	-0.241	0.809	0.701
	$t-2$	0.904	0.772	-0.347	0.812	0.786
	$t-1$	0.966	0.844	-0.468	0.776	0.857
	t	1.000	0.882	-0.601	0.705	0.903
	$t+1$	0.966	0.858	-0.670	0.618	0.865
	$t+2$	0.904	0.801	-0.687	0.522	0.803
	$t+3$	0.823	0.722	-0.668	0.426	0.726
	$t+4$	0.731	0.630	-0.622	0.335	0.640
Autocorrelation coefficients	1 lag	0.966	0.958	0.841	0.970	0.919
	2 lags	0.904	0.870	0.685	0.892	0.822
	3 lags	0.823	0.758	0.540	0.781	0.720
	4 lags	0.731	0.634	0.410	0.656	0.619
	5 lags	0.636	0.511	0.298	0.532	0.524

model. This case illustrates a standard result in the sticky price literature: under synchronised price or wage setting, inflation and output are less persistent than under staggering (eg, Knell (2010)).

Finally Table F provides the results in the case of the third calibration based on the WDN survey of European firms. The results, unsurprisingly, lie between cases C and D. However the results are closer to the ones obtained under the standard staggered Taylor model, in particular in terms of inflation persistence: the autocorrelation of order 5 is 0.501 much closer to the figure 0.512 obtained in the case of even staggering than of that in the fully synchronised case (0.392).

5 Effects of monetary policy shocks

In this section, we consider the effects of monetary policy shocks in each of the first three calibrations of wage setting described above. To illustrate the role of the timing of shocks, we consider both the effects of a shock occurring in Q4, where most workers set their wage, and those of shocks occurring in Q1, where most workers will not be able to respond to the shock for a further three quarters.

Chart 1 shows the effects of a one standard deviation monetary policy shock (ie, shock to ε_{it})

Table E: Simulation results for the Taylor (1980) model with all wages reset in Quarter 1

Taylor Q4		Output	Inflation	Interest rate	Real wage	Hours
Standard deviation (% or percentage points)		3.093	0.276	0.153	3.728	2.819
Standard deviation relative to output		1.000	0.089	0.049	1.205	0.911
Correlation of variable at time t	$t-4$	0.666	0.467	-0.098	0.491	0.564
with output at time:	$t-3$	0.773	0.586	-0.180	0.586	0.654
	$t-2$	0.869	0.704	-0.287	0.644	0.743
	$t-1$	0.948	0.806	-0.416	0.670	0.822
	t	1.000	0.869	-0.564	0.672	0.877
	$t+1$	0.948	0.854	-0.657	0.537	0.846
	$t+2$	0.869	0.784	-0.687	0.411	0.785
	$t+3$	0.773	0.682	-0.667	0.297	0.704
	$t+4$	0.666	0.566	-0.608	0.194	0.609
Autocorrelation coefficients	1 lag	0.948	0.944	0.825	0.761	0.912
	2 lags	0.869	0.823	0.653	0.567	0.810
	3 lags	0.773	0.674	0.495	0.404	0.705
	4 lags	0.666	0.522	0.360	0.261	0.603
	5 lags	0.564	0.382	0.252	0.194	0.509

Table F: Simulation results for the Taylor (1980) model with wage-setting synchronised as in Druant et al. (2009).

Taylor WDN calibration		Output	Inflation	Interest rate	Real wage	Hours
Standard deviation (% or percentage points)		3.048	0.242	0.160	2.300	2.940
Standard deviation relative to output		1.000	0.079	0.052	0.755	0.965
Correlation of variable at time t	$t-4$	0.727	0.567	-0.146	0.739	0.606
with output at time:	$t-3$	0.820	0.671	-0.234	0.784	0.696
	$t-2$	0.902	0.766	-0.341	0.792	0.782
	$t-1$	0.965	0.840	-0.463	0.762	0.854
	t	1.000	0.880	-0.598	0.699	0.901
	$t+1$	0.965	0.857	-0.668	0.607	0.863
	$t+2$	0.902	0.799	-0.686	0.510	0.801
	$t+3$	0.820	0.718	-0.668	0.413	0.724
	$t+4$	0.727	0.624	-0.629	0.322	0.638
Autocorrelation coefficients	1 lag	0.965	0.957	0.840	0.941	0.918
	2 lags	0.902	0.866	0.682	0.850	0.820
	3 lags	0.820	0.750	0.535	0.733	0.718
	4 lags	0.727	0.624	0.404	0.615	0.616
	5 lags	0.630	0.498	0.293	0.491	0.522

worth about 36 basis points on the annualised interest rate. Graphs in the upper panel of Chart 1 report the response to a shock occurring in Q1, while graphs in the lower panels of Chart 1 report responses to shocks occurring in Q4. In each simulation, the shock leads interest rates to rise by about 36 basis points initially before falling back to base. The response of the interest rate is not reported in Chart 1: the path of interest rates following a monetary policy shock is basically the same in each of our models. The effect of this movement in interest rates on wages will however depend both on the degree of synchronisation of wages and on when the shock occurs.

Our reference is the standard staggering model (lines with plain circles). Under that specification output and inflation display hump shapes, as in Christiano *et al* (2005) or Smets and Wouters (2003), with both variables displaying a trough about four quarters after the shock. The decrease in output is about 0.8% and in quarterly inflation is about 0.08 percentage points.

To illustrate the mechanism of our model we next consider the effect of monetary policy shocks in the 'fully synchronised' wage-setting version of the model in which all wages are renegotiated in Q4 (IRF is reported black lines). Nominal wages are fixed for one year and cannot react to the shock. As the right-most panel indicates, the real wage increases for the first year in this scenario. This is because, while nominal wages are initially fixed, inflation declines, in part due to expectations of future marginal cost decreases. In this variant, the response of inflation is smooth and quite similar to the standard staggering case. Indeed price stickiness smooths the responses of prices in the same way in the two models. If we consider the other extreme case, namely full synchronisation in Quarter 1 this time, the response of real wages is markedly different. Since all wages can then react with only a one quarter delay, real wages drop from the second quarter of the simulation onwards. This leads to an earlier and more marked trough in inflation than in the two previous cases. Somewhat surprisingly, output also decreases by more in this case than with standard staggering. For both cases of synchronisation (wage bargaining in Q1 or in Q4) the degree of persistence is lower than in the staggered case. Indeed, after eight quarters in these two cases both output and inflation are closer to the steady-state than in the baseline staggered wage model. This echoes the standard outcome that synchronisation leads to less persistence.

We finally consider our arguably more realistic calibration, ie, the model of uneven staggering calibrated to European survey data. Real wages tend to decrease more slowly than in the staggered case reflecting the larger share of workers negotiating in Q4. However the responses of both output and inflation are close to those obtained using the standard staggering scheme.

Graphs in the bottom row of Chart 1 display the effect of a shock occurring in Quarter 4. As expected, in the standard staggered case, the response is identical to that observed for a Q1 shock. As also expected, the IRF of a Q4 shock in the model with synchronised negotiation in Q4 is identical to that of a Q1 shock in the model with wages synchronised in Q1. A more interesting point is the response in the uneven staggering case. As a larger share of workers are now able to reset the 'negotiated wage' at the time of the shock, there is a sharper and earlier decrease in the real wage in this case. As a result, inflation reacts more markedly than with a Q1 shock, as is made more visible in Chart 2 which displays responses in the uneven staggering case, to shocks in Q1 and Q4. One surprising feature is that in the first three quarters, following a Q4 shock, both inflation and output decrease more than following a Q1 shock. This goes against the intuition that following a demand shock there is a trade-off between the size of the price and quantity response. Further investigation led us to conclude that this result is due to the capacity utilisation embodied in the model. Actually, consumption and investment decrease by less after a Q4 shock, but since the rental rate of capital decreases by more, capacity utilisation decreases and the economy economises the amount of output devoted to increasing capacity. However, quantitative differences are quite small, and the inflation and output IRFs remain close to that in the standard staggered case. Thus, while the timing of the shock matters for the response of real and nominal wages, it makes little difference for the dynamics of output and inflation.

To summarise, our model has the potential to produce responses to shocks that vary across quarters, as illustrated by the results for the full synchronisation case. However using a calibration inspired by micro evidence, differences in those responses are quantitatively small, in spite of visible non-uniformity in wage staggering. This result is related to that obtained by Soderberg (2009), who studies a simple, analytically tractable model of price setting under uneven staggering (with 'even' and 'odd' periods). Soderberg (2009) shows that a small deviation from perfect synchronisation , makes the model behave very much like a model with standard staggering. The reason is that the few agents that set prices at a different period from the synchronised majority exert a disproportionate influence and dampen the reaction of the synchronised group. Although we consider a wider scale DSGE model, and focus on non-uniformity in wage-setting, the same mechanism holds here.

In addition to the monetary policy shock, we have also studied the response of the model to a wide range of shocks that our model features. These results are not reported to save space. The bottom line was that, as for the monetary policy shock, the results are quite similar under the empirical WDN calibration and under the standard Taylor calibration.

6 Robustness: on Calvo vs. Taylor and the role of price rigidity

Our results above are to some extent at variance with those reported in Olivei and Tenreyro (2007). Those authors, focusing solely on monetary policy shocks, have emphasised that the empirical output impulse response, derived from VAR models, to such shocks varies substantially depending on the quarter of the shock. They report that a DSGE model à la Christiano *et al*. (2005) modified to incorporate seasonal Calvo contracts can rationalise these contrasts. As is summarised in Chart 2 our model does not produce a large contrast between the response to shocks in Q4 and Q1. We now investigate the reasons for the contrast in our findings. Two main possible alternative reasons stand out: the specification of the model and the values of the parameters. One obvious reason for the calibration to differ is that their model is calibrated to US data while we calibrate our model on euro area results, and in particular the pattern of wage staggering is different.

We first investigated whether the different structure of wage staggering in the US and Europe could explain the difference. More specifically we simulated our model using the calibration of Olivei and Tenreyro (2007) for the share of the wages renegotiated each quarters, ie, the parameters reported in the last columns of Table C. Results are not reported to save space. Impulse Response Functions turned out to be very similar, leading us to rule out this explanation as being relevant.

We then investigated the role of the type of wage contracts used in explaining the discrepancy. For this purpose we simulated our model under the Calvo with uneven staggering specification for wages, leaving the rest of the specification, as well as model parameters, unchanged from our baseline. As mentioned above, for converting the shares of wage changes into Calvo probabilities of changing wages we follow the strategy in Olivei and Tenreyro (2007). Results of our experiment are presented in Chart 3, which reports the IRFs for a Q1 and a Q4 monetary policy shock. As is known in the literature, at least since Kiley (2002), Calvo contracts lead to

Table G: Simulation results for the Calvo (1983) model

		Output	Inflation	Interest rate	Real wage	Hours
Standard deviation (% or percentage points)		3.002	0.235	0.166	1.763	2.858
Standard deviation relative to output		1.000	0.078	0.055	0.587	0.952
Correlation of variable at time t	$t-4$	0.712	0.638	-0.113	0.823	0.619
with output at time:	$t-3$	0.806	0.732	-0.203	0.879	0.712
	$t-2$	0.892	0.819	-0.310	0.919	0.798
	$t-1$	0.961	0.892	-0.431	0.936	0.868
	t	1.000	0.931	-0.565	0.923	0.908
	$t+1$	0.961	0.903	-0.626	0.863	0.853
	$t+2$	0.892	0.843	-0.637	0.784	0.775
	$t+3$	0.806	0.764	-0.615	0.696	0.685
	$t+4$	0.712	0.676	-0.572	0.605	0.592
Autocorrelation coefficients	1 lag	0.961	0.959	0.853	0.960	0.913
	2 lags	0.892	0.879	0.712	0.914	0.809
	3 lags	0.806	0.783	0.581	0.858	0.699
	4 lags	0.712	0.682	0.463	0.802	0.591
	5 lags	0.616	0.583	0.360	0.718	0.490

more inflation persistence (at least if one matches Calvo and Taylor contracts based on average duration, see Dixon and Kara (2006) for an alternative). This result is confirmed here, both in Table G below, which reports times series properties under a Calvo scheme, and in Chart 3. Table G indicates that for the model with the Calvo (1983) specification for wages under the WDN calibration, the fifth order autocorrelation of inflation is 0.578 against 0.512 in the model with Taylor contracts. Comparing Chart 2 and Chart 3 shows that nominal wages respond more smoothly and more persistently – that is take longer to fall to their lowest level and longer to reach their new steady-state level – in the Calvo (1983) model than in the Taylor (1980) model. As is the case with prices and inflation, the response of nominal wages is larger in the Taylor model than in the Calvo model . At the same time however, the contrast in Chart 3 between inflation and output responses to Q1 shocks and to Q4 shocks remain quantitatively small. Thus, it does not seem that the contrast between our findings and Olivei and Tenreyro (2007) can be explained by the contract specification chosen for wages.

Finally, we explored how other mechanisms in the model interact with uneven staggering to create a different response across quarters. Here, we focus on one important parameter found to crucially influence the importance of the timing of shocks: the degree of price rigidity.[4] Our

[4] It is beyond the scope of this paper to report all the exercises. Note that we found the public expenditure rule used in Olivei and

experiment is to perform the simulations of Q1 and Q4 shocks, altering our baseline model by setting the degree of price rigidity to the one used by Olivei and Tenreyro (2007): that is $\xi = 0.35$. Results are presented in Chart 4. Since prices are less rigid, they tend to reflect more the movements in current marginal costs, which are actually differing substantially across quarters, in this simulation. The response of inflation is then quantitatively different: following a Q4 shock quarterly inflation reaches a trough after two quarters at -0.28 percentage points while following a Q1 shock inflation has moved by only -0.11 percentage points after two quarters and reaches a trough after five quarters. Similarly there is a quantitative difference in the response of output. Following a Q4 shock the maximum difference from baseline is -0.5% after two quarters while following a Q1 shock the maximum difference is reached after three quarters with a value of -0.6%. Note that under this parameterisation , the Q4 shock, which generates a larger and faster inflation response, leads to a lower output loss, in line with intuition.

Overall, our conclusion on the contrast in the response to monetary shocks according to the timing of the shocks depends substantially on auxiliary dimensions of the model, in particular the degree of price rigidity. It is debatable however that the estimates from Smets and Wouters (2003) we have used as our baseline in the euro area model are reliable. Indeed the average duration of prices they imply is around ten quarters, contrasting with the micro data estimates that indicate that the average duration of prices in the euro area is around one year (see Dhyne *et al.* 2006). To get of sense of whether incorporating more realistic assumptions on price rigidity changes the conclusion in terms of the effect of the timing of monetary policy shocks, we carried out the experiment again, this time setting the degree of price rigidity to $\xi = 0.75$, consistent with euro area evidence. Results are displayed in Chart 5. The results are unsurprisingly intermediate between the baseline and the case where $\xi = 0.35$. In particular the difference between the response of inflation following a Q4 shock and following a Q1 shock is more visible than in the baseline case. However the quantitative difference remains limited, suggesting the timing of monetary policy shock has only limited implications.

7 Some implications for optimal monetary policy

In this section, we investigate the consequence of non-synchronised wage setting for optimal monetary policy. In particular we investigate whether the policy rule should vary from quarter to

Tenreyro (2007) plays a role in magnifying the contrasts across quarters.

quarter as a result of seasonality in the wage-setting process. There are typically two approaches to optimal stabilisation policy in the literature. One relies on computing the fully optimal 'Ramsey' policy, the other relies on optimal simple rules (OSR). We here use the OSR approach. OSR have been shown to be robust and close to the optimal rules in many models (see Taylor and Williams, 2001, for a survey and discussion) More specifically, this approach allows us to investigate quite easily whether the optimal policy rule varies from quarter to quarter, through comparing parameters. By contrast the parameters of the policy rule resulting from Ramsey optimisation are numerous and difficult to interpret. The results of Ramsey exercises are typically presented as IRF to shocks under optimal policy. In our case it would be difficult to disentangle whether quarter-dependence in IRF would reflect the seasonality in private agent behavior and or that of the policy response.

The simple policy rule we use has the form of a 'super inertial' Taylor rule:

$$
\begin{aligned}
i_t = {} & i_{t-1} + \phi_\pi^1 I_t^1 \pi_t + \phi_\pi^2 I_t^2 \pi_t + \phi_\pi^3 I_t^3 \pi_t + \phi_\pi^4 I_t^4 \pi_t \\
& + \phi_y^1 I_t^1 \left(\widehat{y}_t - \widehat{y}_{fp,t}\right) + \phi_y^2 I_t^2 \left(\widehat{y}_t - \widehat{y}_{fp,t}\right) + \phi_y^3 I_t^3 \left(\widehat{y}_t - \widehat{y}_{fp,t}\right) + \phi_y^4 I_t^4 \left(\widehat{y}_t - \widehat{y}_{fp,t}\right)
\end{aligned}
$$

where I_t^q is a dummy variable equal to 1 in quarter q and 0 otherwise. The 'super inertial' specification allows us to be parsimonious on parameters. In addition, that specification has been shown to be efficient in forward-looking models (see Onatski and Williams (2010) for an example and Taylor and Williams (2011) for a discussion). We optimise over the parameter set $\phi_\pi^1, \phi_\pi^2, \phi_\pi^3, \phi_\pi^4, \phi_y^1, \phi_y^2, \phi_y^3, \phi_y^4$ to minimise the loss function conditional on the log-linearised model:

$$
L = V(\pi_t) + V(y_t) + 0.1V(i_t - i_{t-1})
$$

where $V(\pi_t)$ and $V(y_t)$ are the unconditional variances of inflation and output, across all quarters. Optimisation was carried out numerically using the OSR Dynare module.

As a baseline case we compute optimal policy with the parameter values given in Table B with one alteration: we set the price rigidity parameter to the 'micro founded' value of 0.75 rather than Smets and Wouters' estimates. In addition, to understand better the mechanisms underlying the optimal policy rule parameters, we consider a number of alternative parameter sets. The set of parameterisation s we consider is built by varying:

Table H: Optimal policy parameters

	Specification			ϕ_π^1	ϕ_π^2	ϕ_π^3	ϕ_π^4	ϕ_y^1	ϕ_y^2	ϕ_y^3	ϕ_y^4
1	Pred	IPN	WDN	2.186	2.303	2.088	2.048	2.427	2.398	2.400	2.196
2	Pred	IPN	STAGG	2.074	2.074	2.074	2.074	2.381	2.381	2.381	2.381
3	Pred	IPN	Q4	2.621	4.324	3.874	3.135	2.134	3.047	2.821	2.268
4	Pred	FLEX	WDN	1.244	0.882	0.907	0.708	1.065	0.499	0.647	0.637
5	Pred	FLEX	STAGG	0.893	0.893	0.893	0.893	0.801	0.801	0.801	0.801
6	Pred	FLEX	Q4	1.931	0.547	0.473	0.430	0.956	0.274	0.643	0.003
7	NoPred	IPN	WDN	1.741	1.603	1.663	1.441	2.619	2.621	2.461	2.706
8	NoPred	IPN	STAGG	1.707	1.707	1.707	1.707	2.563	2.563	2.563	2.563
9	NoPred	IPN	Q4	1.874	1.605	1.593	0.827	2.788	2.682	2.457	2.826
10	NoPred	FLEX	WDN	0.168	0.184	0.116	0.407	0.146	0.154	0.176	0.190
11	NoPred	FLEX	STAGG	0.259	0.259	0.259	0.259	0.218	0.218	0.218	0.218
12	NoPred	FLEX	Q4	-0.397	-0.443	-0.343	3.782	0.083	0.408	0.557	-0.113

- predetermination: we consider both predetermined wages and non-predetermined wages
- the degree of price rigidity: we consider (i) 'IPN': the case in which $\xi = 0.75$ (average price duration is 1 year following European micro data) and (ii) 'FLEX': the (nearly) flexible price case in which $\xi = 0.01$
- the degree of wage synchronisation : cases considered are the ones detailed in Table C. (i) 'WDN' refers to our baseline calibration of cohort weights based on the Eurosystem Survey (ii) 'STAGG' refers to standard Taylor staggering and (iii) 'Q4' refers to full synchronisation of wage changes in quarter 4.

Overall we thus have 12 parameters sets, numbered 1 to 12 in Table H, which presents the results. We intentionally focus on discussing the parameters on inflation (the ϕ_π^q's). Results for the responses to output, at least with respect to their pattern across quarters, are broadly similar.

The main result obtained with the baseline parameter set (1) is that the optimal policy parameters do not differ much across quarters. Indeed the optimal parameter set is $(\phi_\pi^1, \phi_\pi^2, \phi_\pi^3, \phi_\pi^4) =$ (2.19,2.30,2.09,2.05). The parameters are quite close to those obtained in the case of standard staggering (row 2) in which case by construction, the optimal rule does not depend on the quarter and we obtain $\phi_\pi^1 = \phi_\pi^2 = \phi_\pi^3 = \phi_\pi^4$ =2.07. The similarity in the two cases echoes the results of simulations in Section 5 of the paper and in Soderberg (2009): a small degree of non-synchronisation of wage setting removes a lot of seasonality in the aggregate response of the economy.

Further insights can be gained looking at our specification (3), namely the polar case of fully synchronised wage change, in which all wages change in Q4. In that case a rather uneven pattern of optimal responses obtains: $(\phi_\pi^1, \phi_\pi^2, \phi_\pi^3, \phi_\pi^4) = (2.62, 4.32, 3.87, 3.14)$. Note, quite counterintuitively, the peak policy response is in Quarter 2 .

It turns out that this pattern is much influenced by the degree of price rigidity. To illustrate this, we consider the cases with flexible prices, that is rows 4-6 of Table H. When wages are evenly staggered (from row 5), the reaction is mechanically the same across quarters. As expected, with flexible prices policy needs to respond less aggressively so the coefficient is lower than with sticky prices: $\phi_\pi^q = 0.89$ for all q compared with $\phi_\pi^q = 2.07$ in the sticky-price case. Now with full synchronisation of wage negotiations in Q4 (row 6) we obtain $(\phi_\pi^1, \phi_\pi^2, \phi_\pi^3, \phi_\pi^4) =$ (1.93,0.55,0.47,0.43). Therefore, under flexible prices and synchronised wage changes it is important for the central bank to react in Q1, that is, when wage are actually changed. This contrasts with the pattern of coefficients in the sticky wages cases discussed above (cases 1 or 3). Our interpretation is that due to sticky wages, the impact on prices of shocks affecting wages is delayed, and it is optimal for the central bank to react when wages are actually on average feeding into inflation, that is later in the year than in Q1.

Finally rows six through twelve report cases where predetermination of wages has been removed. The main finding is that the peak responses are typically observed a quarter earlier than in the case with predetermination. In particular we then find that, with flexible prices and full synchronisation, it is optimal to react to inflation in the quarter in which wages are set, ie, in Q4. This is illustrated by case (12): $\phi_\pi^1, \phi_\pi^2, \phi_\pi^3, \phi_\pi^4 =$ (-0.40,-0.44,-0.34,3.78).

Overall, two main, contrasting results stand out from this exercise. On the one hand, the model has the potential to generate an optimal policy rule that varies considerably across quarters, especially in cases that get close to flexible prices and full synchronisation of wage changes. On the other hand, under our baseline microeconomic calibration, in spite of some unevenness in wage-setting, there is little difference across quarters in the optimal policy response. The first result somehow conforms to the intuition that the optimal policy rule should react aggressively to inflation mainly in quarters when wage changes take place. Two reasons explain why this intuition does not carry out to our preferred specification, which we deem more empirically relevant. First, even a limited amount of non-synchronisation is enough to create strategic complementarity across cohorts of wage setters, and then dampen the quarter-specific idiosyncracies in the response of wages. Second, when prices are stickier, wage changes will take more time (typically two quarters) to affect prices, so it turns out to be optimal, to put more weight on inflation two or three quarters after the period in the year when negotiation occurs.

8 Conclusions

In many countries, wage changes tend to be clustered in the beginning of the year, with wages being set for fixed durations of typically one year. This has been, in particular, documented in recent years for European countries using microeconomic data. In this paper, we have used a simple model of the euro area to investigate the macroeconomic and monetary policy consequences of these patterns of wage staggering. We constructed a model in which wages are set for four quarters as in the Taylor scheme, but the staggering across quarters is non-uniform. Using this model, we found that, when wage staggering is uneven, inflation and output are less persistent, both unconditionally and following a monetary policy shock, than under an even wage staggering scheme. Furthermore, we found that inflation responds by more, and more rapidly, to a given interest rate change if the Central Bank makes this change in the quarter when most workers are renegotiating their wages, ie, in quarter four, than in any other quarter. That said, when calibrating the model with the micro data recently produced by the WDN, which feature a significant degree of uneven staggering, we found that the quantitative outcome is close to that resulting from an even staggering scheme. And we found that this result was robust to using a US calibration for the degree of wage synchronisation, to using Calvo rather than Taylor wage contracts, and to reasonable variations in the degree of price stickiness: the quantitative difference between the effects of a monetary policy shock in Q4 and other quarters remained small.

We then considered the consequence of non-synchronised wage setting for optimal monetary policy. In particular we investigate whether the policy rule should vary from quarter to quarter as a result of seasonality in the wage-setting process. We found that the model has the potential to generate an optimal policy rule that varies considerably across quarters, especially in cases that get close to flexible prices and full synchronisation of wage changes. But, again, we found that under our baseline microeconomic calibration, in spite of some visible unevenness in wage-setting, there is little difference across quarters in the optimal policy response.

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9 Appendix: dealing with seasonal models with Dynare

The time-varying coefficient α_t in our model is deterministic and seasonal, fulfilling $\alpha_t = \alpha_{t-4}$. It cannot easily be treated as a series using standard packages for solving linear rational expectation models due to non-linearity and to the absence of a steady-state. Parameters of the model change every quarter.

Here, we describe a procedure to handle these models using the procedure for simple linear models included in the *Dynare* software (Juillard, 1996).

The procedure relies on considering at each period four parallel sub-models, each governed by the law of motion that is specific to a given quarter q. In simulation exercises, 4 trajectories of each variable are produced, only one of these is selected to report results. When writing leads and lags in the model, one has to acknowledge that the lead of variable x_t at quarter q is provided by the submodel describing the law of motion of quarter $q + 1$.

To illustrate this procedure, we use in this appendix the model of Soderberg (2010) that is analytically tractable.

Prices are preset for 2 periods, with optimal price given by:

$$p_{it} = \frac{1}{2}\{[p_t + E_t p_{t+1}] + \phi[y_t + E_t y_{t+1}]\}$$

In the model there are odd periods (1) and even periods (2). A fraction (1-α) (respectively α) of firms set their price in period 1 (respectively period 2).

The model is closed by introducing a money supply rule and a quantity theory-style determination of output

$$y_t = m_t - p_t$$

$$m_t = m_{t-1} + \varepsilon_t$$

Soderberg (2009) shows that the solution of model is of the form:

$$y_t = \gamma_I y_{t-1} + \omega_I \varepsilon_t$$

where I an indicator variable for the season: $I = 1$ or $I = 2$,

and shows analytically how γ_I depends on the deep parameters (ϕ, α)

The following dynare codes illustrate how to implement and recover results consistent with Soderberg (2009)'s analytical results. Codes for our DSGE model are available upon request.

```
// Dynare code for the example of seasonal model:

// "Non-uniform staggered prices and output persistence" by J.  Soderberg , Upps
//soderberg.mod
var y1 m1 p1 x1 y2 m2 p2 x2;
varexo eps1 eps2 ;
//Parameters
parameters phi alpha sigma;
phi = 0.1;  // degree ofreal rigidity
alpha =0.2;  // share of contract in period 1
sigma =1 ;  // variance of shock
model(linear);
//Equation (5) aggregate demand
y1=m1-p1;
y2=m2-p2;
//Equation (A2) and (A3) price level
p1=alpha*x1+(1-alpha)*x2(-1);
p2=(1-alpha)*x2+alpha*x1(-1);
//Equation (A1) optimal reset price
x1=phi*m1+((1-phi)/2)*(p1+p2(+1));
x2=phi*m2+((1-phi)/2)*(p2+p1(+1));
//Equation money (6) money supply
m1=m2(-1)+eps1;
m2=m1(-1)+eps2;
end;
initval;
y1=0;  m1=0;  p1=0;  x1=0;  y2=0;  m2=0;  p2=0;  x2=0;
end;
steady;
check;
shocks;
var eps1=sigma^2;
var eps2=sigma^2;
```

```
end;
stoch_simul(irf=0) y1 m1 p1 x1 y2 m2 p2 x2;
disp('****************************************************************');
disp(' ');
disp('Analytical results from Soderberg 2009 paper');
A1=alpha*(1-phi)/(2*(phi+alpha*(1-phi)));
A2=(1-alpha)*(1-phi)/(2*(phi+(1-alpha)*(1-phi)));
//**** Use fsolve to recover reduced form parameters
lam0 = [0.5;  0.5];  % Make a starting guess at the solution
options=optimset('Display','iter');  % Option to display output
[lam,fval] = fsolve(@myfun,lam0,options,A1,A2) % Call optimizer
lam1=lam(1);
lam2=lam(2);
omega1=(1-alpha)*lam1+alpha;
omega2=alpha*lam2+(1-alpha);
gam1=lam2*((1-alpha)*lam1+alpha)/(alpha*lam2+(1-alpha));
gam2=lam1*(alpha*lam2+(1-alpha))/((1-alpha)*lam1+alpha);
disp('gam1');
disp(gam1);
disp('omega1');
disp(omega1);
disp('gam2');
disp(gam2);
disp('omega2');
disp(omega2);
disp('Correlation of Y1 and Y1(previous year) ');
disp(gam2*gam1);
```

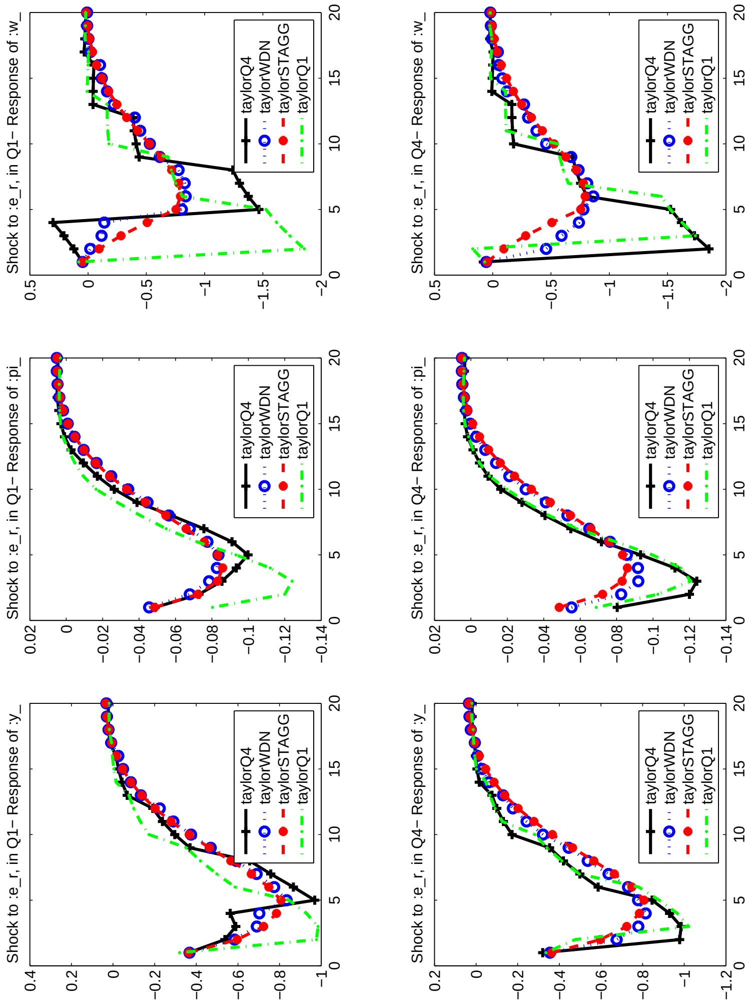

Chart 1: Impulse responses to a monetary shock for alternative staggering specifications

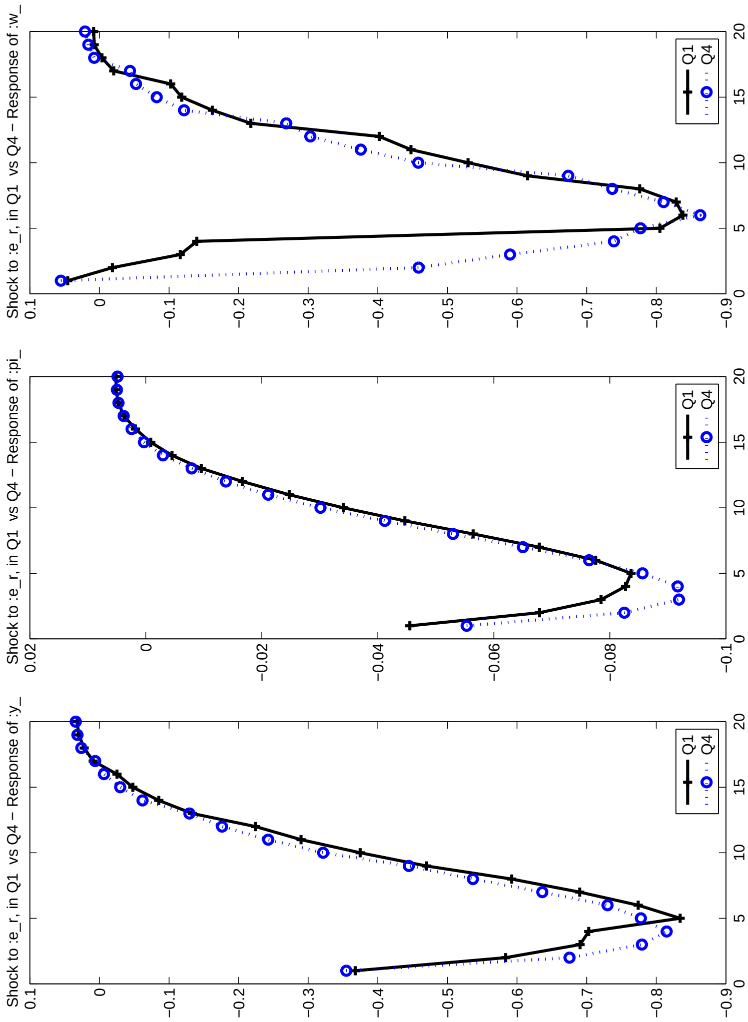

Chart 2: Comparing Shock in Q1 vs Q4. Baseline model

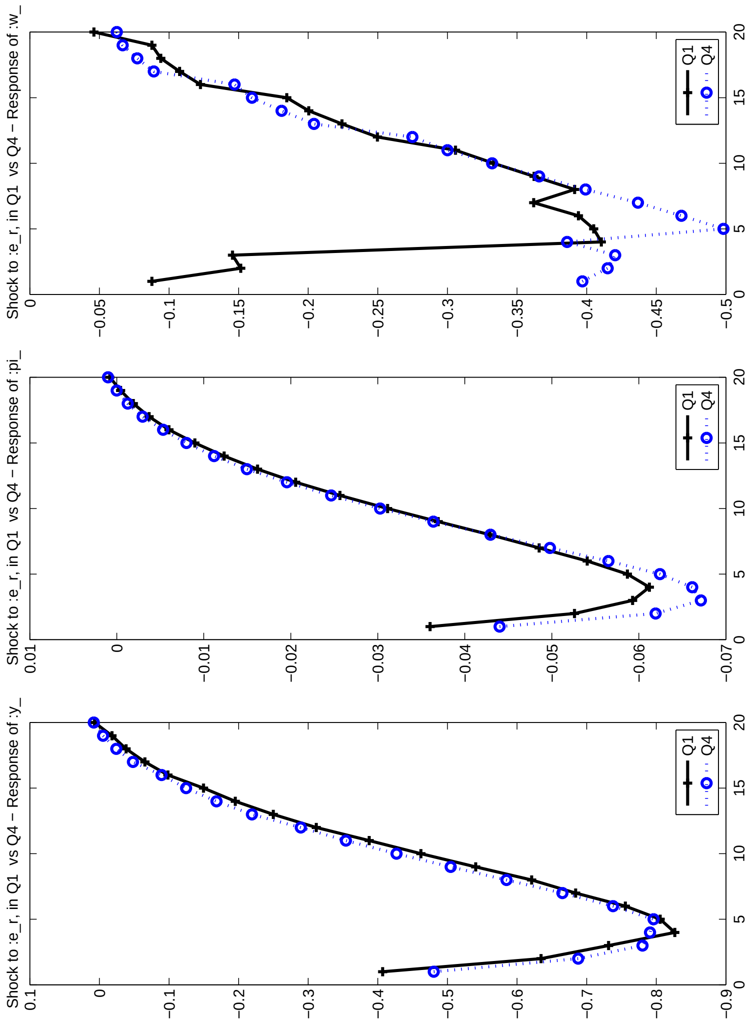

Chart 3: Comparing Shock in Q1 vs Q4. Calvo specification

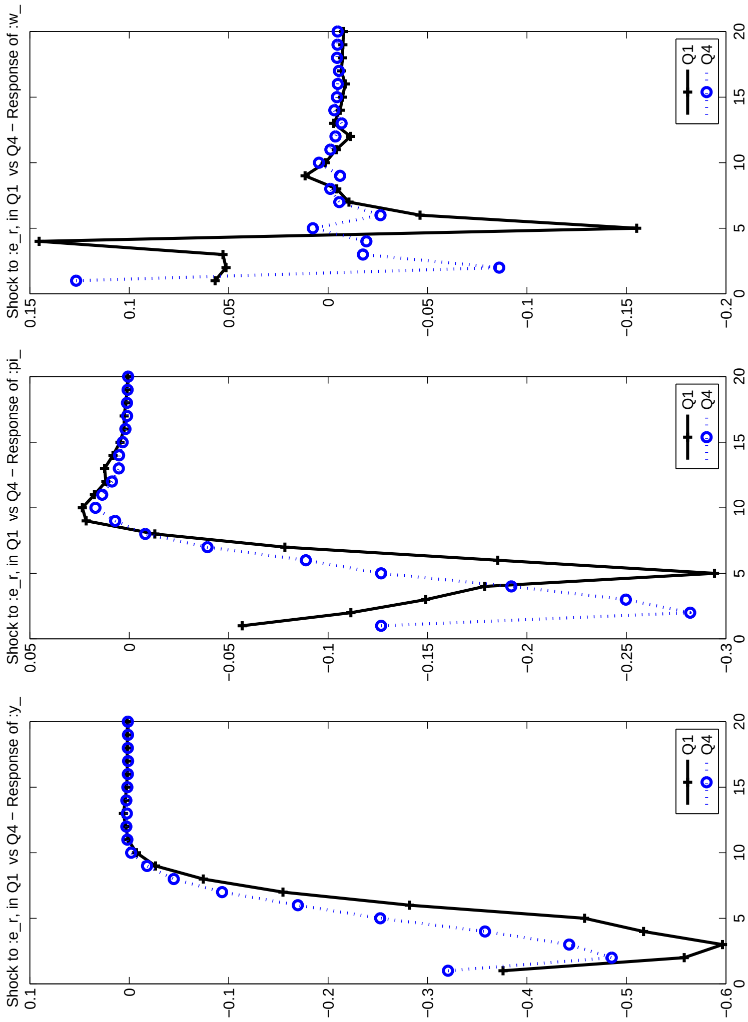

Chart 4: Comparing Shock in Q1 vs Q4. Using a lower degree of rigidity ($\xi = 0.350$)

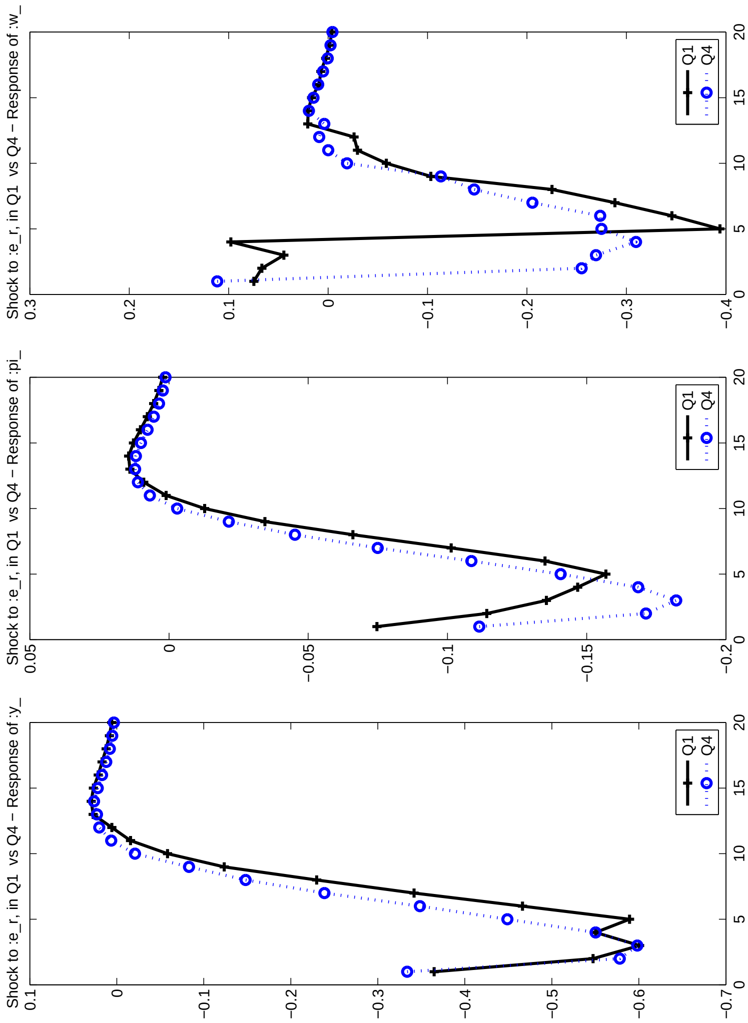

Chart 5: Comparing Shock in Q1 vs Q4. Using an intermediate degree of rigidity ($\xi = 0.750$)