# Time-Separability, Wealth Effects and International Business Cycles

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**Abstract**

This paper addresses the international comovement puzzle. International real business cycle models tend to predict negative cross-country correlations of investment and employment. In the data these correlations are positive. To reconcile the theory with the data the literature has resorted to financial frictions and new sources of disturbances. We show that a model driven by productivity shocks alone can account for the comovement puzzle in a complete markets environment. Our two-country model with imperfectly substitutable traded intermediate goods features time nonseparable preferences that allow arbitrarily small wealth effects on labor supply. It reconciles with the data by predicting (i) positive cross-country correlations of investment and hours worked; (ii) a plausible cross-country correlation of output; and (iii) countercyclical net exports. Although our model cannot reproduce the observed volatility of relative prices, it improves slightly on standard approaches. Unlike models with incomplete markets, ours show little sensitivity to the degree of persistence or international spillovers of the shocks.

## 1   Introduction

The predictions of two-country equilibrium business cycle models with complete markets conflict with the data in several respects. These discrepancies arise from the models' tendency to predict (i) negative cross-country correlations of investment and employment (the "international comovement puzzle", Baxter, 1995); and (ii) international consumption correlations in excess of output correlations (the "quantity anomaly", Backus, Kehoe, and Kydland, 1992, 1995) when the opposite is seen in the data. A number of well-known international business cycle models also fail to generate realistic within-country behavior of the trade balance (e.g. Backus, Kehoe, and Kydland, 1992; Kehoe and Perri, 2002; Chari, Kehoe, and McGrattan, 2002). Such models predict an acyclical or procyclical net export-to-output ratio, when in the data the ratio is countercyclical.

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Existing efforts to tackle these discrepancies follow the route of restricting international asset markets (Baxter and Crucini, 1995; Kollmann, 1996; Kehoe and Perri, 2002; Corsetti, Dedola, and Leduc, 2008), and/or introducing new types of disturbances, such as preference shocks, monetary shocks or investment-specific technology shocks (Stockman and Tesar, 1995; Ravn, 1997; Wen, 2007; Johri and Lahiri, 2008; Raffo, 2010; Benigno and Thoenissen, 2008). Rather than present a new variation on either of these themes, this paper instead considers whether financial market frictions are necessary for resolving international comovement puzzles. We find that the answer is "no". When we relax the assumption of time-separable Cobb-Douglas preferences, a model with complete markets driven only by productivity shocks can resolve the "international comovement puzzle",[1] and can account for countercyclical net exports. Furthermore, our model outperforms standard models in accounting for the "quantity anomaly".

The environment we consider features complete markets for contingent claims, product markets for both intermediate and final goods, and capital adjustment costs. Preferences are time-nonseparable and characterized by arbitrarily small wealth effects on labor supply. Our model matches three key features of the data: (i) positive cross-country correlations of investment and hours worked; (ii) plausible cross-country correlations of output; and (iii) countercyclical net exports. While it reduces the gap between international correlations of output and consumption, it fails to change their order. There are similarities between the empirical performance of our model and that of of Kehoe and Perri's (2002) model with endogenously incomplete markets. While their model predicts positive correlations of factors but cannot generate countercyclical net exports, our benchmark reproduces both features of the data. Unlike models with restricted financial markets, ours shows little sensitivity to the parameterization of the forcing process.

Our model deviates from Backus, Kehoe, and Kydland (1994) (hereafter BKK) along two important dimensions. First, it incorporates the preference structure introduced by Greenwood, Hercowitz, and Huffman (1988) (henceforth GHH) that eliminates the wealth effect on labor supply. Second, it includes internal habit formation in consumption, which makes preferences nonseparable over time. Habit formation re-introduces the wealth effect on labor supply and intertemporal substitution in leisure. The magnitude of both effects depends on the intensity of habit formation.[2]

The introduction of GHH preferences has its strongest effect in the labor market. When there is no wealth effect on labor supply, foreign individuals increase their hours in response to a positive productivity shock in the home country. To paraphrase Baxter and Crucini (1995, p. 841), residents of the less-productive country no longer take advantage of risk-sharing to "take a paid vacation". The lack of a wealth effect, combined with

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[1]The importance of this result is emphasized by Baxter (1995, p. 1859) who claims that "...a major challenge to the theory is to develop a model which can explain international comovement in labor input and investment". In line with the above, Canova and Ubide (1998, p. 558) argue that "...the magnitude and the sign of the cross-country investment correlations constitute an important regularity previously under-emphasized by the literature. For the largest 9 OECD countries the size of pairwise investment correlation ranges between [-0.01, 0.77] with the median value around 0.45. A successful model of the international business cycle must therefore be able to reproduce this important feature of the data..."

[2]There appears to be little consensus in the empirical literature regarding the strength of the wealth effect on labor supply. Kimball and Shapiro (2008) report empirical evidence that the wealth effect is large, and offsets a similarly large substitution effect. In contrast, Schmitt-Grohé and Uribe (2008) estimate the parameters of the utility function introduced by Jaimovich and Rebelo (2009) and find that the wealth effect is close to zero.

a surge in the terms of trade that favors the foreign country, induces positive comovement of hours worked. Positive labor comovement, in turn, results in a more realistic cross-country output correlation.

Relaxing the assumption of time separability reconfigures the dynamics of consumption and investment. Habit-forming individuals suffer disutility from rapid changes to the time-profile of their consumption. They seek to smooth not just consumption but also its rate of change. This has implications for investment. Following a positive productivity shock at home, domestic output peaks on impact while consumption exhibits a hump-shaped response. Consumers need time to adjust their habits, so they save most of the extra output. A large part of this saving is initially channeled into investment at home. Domestic absorption rises by more than output on impact, causing net exports to decline. Subsequently, accumulated saving is transmitted to the foreign country via the export channel. The wealth effect induces foreign agents to increase their consumption, but habit formation compels them to do so gradually. Thus, investment abroad also rises.

The mechanisms that account for the positive comovement of hours and investment in our model are mutually reinforcing. As foreign hours increase following a rise in home productivity, the response of global output is greater. This creates savings that translate into foreign as well as home investment, bolstering the investment comovement. Meanwhile, habit formation augments the response of the terms of trade to a productivity shock. This boosts income in the foreign country, producing a stronger response of foreign hours than would be seen in the absence of habits.

The GHH preference structure has been widely used in two-country models. Our work is related to Devereux, Gregory, and Smith (1992) who first introduced GHH preferences in a two-country business cycle model. Their model predicts a realistic international consumption correlation provided that productivity shocks are uncorrelated. More recently Raffo (2008) has demonstrated that an international RBC model with GHH preferences and tradable intermediate inputs reproduces countercyclical net exports, although it fails to account for the "international comovement puzzle". Raffo (2010) introduces investment-specific technology (IST) shocks into a two-country, two-good model with GHH preferences and variable capacity utilization. His model reconciles with the observed behavior of quantity aggregates and relative prices. However, Mandelman, Rabanal, Rubio-Ramírez, and Vilán (2010) show that Raffo's results are sensitive to the calibration of IST shocks, and break down when an estimated VECM process is used. Engel and Wang (2011) combine GHH preferences with a composite of durable and non-durable consumption. They show that a two-country, two-sector model accounts for the observed pro-cyclicality and volatility of imports and exports.

Habit formation and GHH preferences have both been used successfully in small open-economy models (Letendre, 2004; Correia, Neves, and Rebelo, 1995; Mendoza, 1991). However, to date the literature has given little attention to the implications of time-nonseparability for the predictions of two-country international business cycle models. Dmitriev and Krznar (2011) emphasize the role of time-nonseparable preferences in the international transmission of productivity shocks. Their one-good model succeeds in producing a plausible

cross-country investment correlation, but only at the cost of almost perfectly negatively correlated hours worked.

The rest of the paper proceeds as follows. The next section describes the model economy. Section 3 discusses our calibration of the model. Section 4 presents our quantitative results. It first explains how each feature of the model contributes to reproducing observed features of the data. Then, it provides an overview of the sensitivity analysis. Section 5 concludes.

# 2  The Model Economy

The economic environment is similar to BKK or Heathcote and Perri (2002). The world consists of two countries. The same parameters describe technology and preferences in both countries. Each country $j = 1, 2$ is populated by a continuum of identical infinitely lived individuals.

Labor is immobile across countries. In each period $t$, the world economy experiences an event $s_t$ drawn from the countable set of events, $S$. Let $s^t = (s_0, s_1, ..., s_t) \in S^t$ be the history of events from time 0 to time $t$. The probability at time 0 of any given history $s^t$ is denoted by $\pi(s^t)$.

## 2.1  Consumers

Habit-forming consumers have their preferences defined over stochastic sequences of consumption, habits, and hours worked

$$U = \sum_{t=0}^{\infty} \beta^t \sum_{s^t \in S^t} \pi\left(s^t\right) u(c_j\left(s^t\right), h_j\left(s^{t-1}\right), n_j\left(s^t\right)), \tag{1}$$

where $\beta \in (0, 1)$ is the discount factor. Let $c_j\left(s^t\right)$ denote household consumption in country $j$ after realization of history $s^t$, and let $n_j\left(s^t\right) \in [0, 1]$ denote individual labor supply. The time endowment in each period is normalized to one.

The way we model habits has three distinct features. First, habits are internal. The agent cares about her consumption relative to her own past consumption. Second, habits are persistent. The stock of habits depends on the entire past consumption history. Third, habits are additive. The agent smoothes the quasi-difference between her consumption, $c_j\left(s^t\right)$, and her current stock of habits, $h_j\left(s^{t-1}\right)$.[3]

Following Ferson and Constantinides (1991), we define the stock of habits, $h_j\left(s^t\right)$, with which the agent begins the next period as a convex combination of her current consumption and her current stock of habits:

$$h_j\left(s^t\right) = \lambda c_j\left(s^t\right) + (1 - \lambda) h_j\left(s^{t-1}\right). \tag{2}$$

Under this specification, the habit stock depreciates at a constant rate as in Campbell and Cochrane (1999). The parameter $\lambda \in (0, 1]$ determines the degree of habit persistence. The higher the $\lambda$, the more weight

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[3]Rozen (2010) provides axiomatic foundations for this class of preferences.

agents place on recent consumption history relative to the past. That is, as $\lambda$ increases, the persistence of habits declines. When $\lambda = 1$, the next period's habit stock is just the level of current consumption.

The instantaneous utility function takes the GHH form defined over habit-adjusted consumption, $c_j\left(s^t\right) - Bh_j\left(s^{t-1}\right)$, and hours worked:

$$u(c_j\left(s^t\right), h_j\left(s^{t-1}\right), n_j\left(s^t\right)) = \frac{1}{1-\sigma}\left(c_j\left(s^t\right) - Bh_j\left(s^{t-1}\right) - \chi\frac{n\left(s^t\right)^{1+\nu}}{1+\nu}\right)^{1-\sigma},$$

where $\sigma$ is the curvature parameter, and $\chi$ governs disutility from labor effort. The parameter $B \in [0,1)$ denotes the intensity of habit formation and introduces time non-separability of preferences. The Frisch elasticity of labor supply is given by $1/\nu$.

This specification of preferences nests several well-known special cases. When $B = 0$ it reduces to GHH preferences. When $\lambda = 1$, the model features a time non-separable version of GHH preferences. This has been used by Monacelli and Perotti (2008) to explore the transmission of government spending shocks, and by Boileau and Normandin (2005) to study current account fluctuations in a small open economy model. As $\nu$ tends to infinity, the labor supply becomes inelastic and the preference structure reduces to internal linear habit formation preferences. The non-persistent version ($\lambda = 1$) was popularized by Constantinides (1990), and was generalized by Ferson and Constantinides (1991) to include the persistence of memory, $\lambda \in (0,1]$.

Households in country $j$ own the domestic capital stock, $k_j\left(s^t\right)$, and rent it to domestic intermediate good firms. Installed capital is immobile across countries. It evolves according to the following law of motion

$$k_j\left(s^t\right) = (1-\delta)k_j\left(s^{t-1}\right) + \phi\left(\frac{i_j\left(s^t\right)}{k_j\left(s^{t-1}\right)}\right)k_j\left(s^{t-1}\right), \text{ for } j \in \{1,2\}, \tag{3}$$

where $\delta$ is the depreciation rate, and $i_j\left(s^t\right)$ is gross investment. Capital accumulation is subject to convex adjustment costs described in Hayashi (1982). The adjustment cost function $\phi$ satisfies $\phi\left(\cdot\right) > 0$, $\phi'\left(\cdot\right) > 0$, and $\phi''\left(\cdot\right) < 0$. The restrictions $\phi\left(\delta\right) = \delta$ and $\phi'\left(\delta\right) = 1$ ensure that incorporation of the adjustment cost does not affect the deterministic steady state of the model.

## 2.2 Producers

The are two types of firms in this environment: intermediate good firms and final good firms.

**Intermediate good firms** in each country specialize in the production of a single country-specific good. Country 1 produces good $a$, whereas Country 2 produces good $b$. Households supply labor to the intermediate firms at the wage $w_j\left(s^t\right)$, and capital at the rental price $r_j\left(s^t\right)$. The firms have access to a constant returns-to-scale technology. Production is subject to country-specific exogenous random shock, $z_j\left(s^t\right)$, to total factor productivity (TFP). Output in country $j$ after history $s^t$ is given by

$$y_j\left(s^t\right) = z_j\left(s^t\right)f(k_j\left(s^{t-1}\right), n_j\left(s^t\right)), \tag{4}$$

where $f$ is Cobb-Douglas production function given by $f(k,n) = k^\alpha n^{1-\alpha}$. The TFP shocks follows a stationary vector autoregressive process in logs:

$$
\begin{bmatrix} \log(z_1(s^t)) \\ \log(z_2(s^t)) \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{12} & \rho_{1a1} \end{bmatrix} \begin{bmatrix} \log(z_1(s^{t-1})) \\ \log(z_2(s^{t-1})) \end{bmatrix} + \begin{bmatrix} \varepsilon_1(s^t) \\ \varepsilon_2(s^t) \end{bmatrix}.
$$

Diagonal elements of the transition matrix, $\rho_{11}$, determine the degree of persistence in productivity within each country. Off-diagonal elements, $\rho_{12}$, determine the speed with which productivity innovations spill over national borders. The innovations to the productivity process are serially uncorrelated bivariate normal random variables with zero mean and the contemporaneous covariance matrix

$$
E\left[\varepsilon_t \varepsilon_t'\right] = \sigma_\varepsilon^2 \cdot \begin{bmatrix} 1 & \rho_\varepsilon \\ \rho_\varepsilon & 1 \end{bmatrix}.
$$

**Final good firms** in country 1 use as inputs the quantities $a_1(s^t)$ and $b_1(s^t)$ of the two intermediate goods. They produce an amount $G(a_1(s^t), b_1(s^t))$ of the final good, using a constant returns-to-scale technology

$$
G(a,b) = \left(\omega a^{1-\eta} + (1-\omega) b^{1-\eta}\right)^{\frac{1}{1-\eta}},
$$

where $1/\eta$ is the elasticity of substitution between intermediate inputs. The constant $\omega \in (1/2, 1)$ reflects the extent of home bias. In the same way, the firms in country 2 produce an amount $G(b_2(s^t), a_2(s^t))$ of the final good, using $a_2(s^t)$ and $b_2(s^t)$ of the intermediate inputs.

## 2.3 Markets

Agents have access to a complete set of state-contingent claims traded internationally. The claims are denominated in terms of the intermediate good $a$. Let $Q(s^t, s_{t+1})$ be the price of a claim sold after realization of $s^t$, that delivers a unit of good $a$ at time $t+1$ provided that the state $s_{t+1}$ is realized. Let $B_2(s^t, s_{t+1})$ be the quantity of such claims held by the residents of country 2 after history $s^t$. Then, the consumers in country 2 face the following budget constraint:

$$
c_2(s^t) + i_2(s^t) + q_2^a(s^t) \sum_{s_{t+1} \in S} Q(s^t, s_{t+1}) B_2(s^t, s_{t+1}) \tag{5}
$$

$$
= q_2^b(s^t) \left[ r_2(s^t) k_2(s^{t-1}) + w_2(s^t) n_2(s^t) \right] + q_2^a(s^t) B_2(s^{t-1}, s_t),
$$

where $q_2^a(s^t)$ and $q_2^b(s^t)$ denote the prices of good $a$ and $b$ respectively, in terms of the final good of country 2. The agents in country 1 face a similar constraint.

## 2.4 Equilibrium

The equilibrium consists of the state-contingent sequences of factor prices $\left\{\{r_j(s^t), w_j(s^t)\}_{j\in\{1,2\}}\right\}_{t=0, s^t\in S^t}^{\infty}$, intermediate good prices $\left\{\{q_j^a(s^t), q_j^b(s^t)\}_{j\in\{1,2\}}\right\}_{t=0, s^t\in S^t}^{\infty}$, bond prices $\left\{\{Q(s_{t+1}, s^t)\}_{s_{t+1}\in S}\right\}_{t=0, s^t\in S^t}^{\infty}$ and allocations $\left\{\{c_j(s^t), i_j(s^t), n_j(s^t), k_j(s^t), a_j(s^t), b_j(s^t), \{B_j(s_{t+1}, s^t)\}_{s_{t+1}\in S}\}_{j\in\{1,2\}}\right\}_{t=0, s^t\in S^t}^{\infty}$ that satisfy the following conditions. Given the prices:

i) Consumers in country $j$ choose state contingent sequences $\{c_j(s^t)\}_{t=0}^{\infty}$, $\{n_j(s^t)\}_{t=0}^{\infty}$, $\{i_j(s^t)\}_{t=0}^{\infty}$ and bond holdings $\{B_j(s_{t+1}, s^t)\}_{s_{t+1}\in S}$ for all $s^t \in S^t$, to maximize expected utility (1) subject to the budget constraint (5), the laws of motion (3) and (2) and the initial conditions.

ii) Intermediate firms in country $j$ choose $n_j(s^t)$ and $k_j(s^{t-1})$ to maximize profits

$$y_j(s^t) - r_j(s^t)k_j(s^{t-1}) - w_j(s^t)n_j(s^t),$$

subject to the technological constraint (4), and the non-negativity constraints $n_j(s^t) \geq 0$ and $k_j(s^{t-1}) \geq 0$.

iii) The final good producers in country $j$ choose $a_j(s^t)$ and $b_j(s^t)$ to maximize profits $\Pi_j(s^t)$ given by

$$\Pi_j(s^t) = \begin{cases} G(a_j(s^t), b_j(s^t)) - q_j^a(s^t)a_j(s^t) - q_j^b(s^t)b_j(s^t), & \text{for } j = 1; \\ G(b_j(s^t), a_j(s^t)) - q_j^a(s^t)a_j(s^t) - q_j^b(s^t)b_j(s^t), & \text{for } j = 2. \end{cases}$$

The prices ensure that for all $t \geq 0$ and for all $s^t \in S^t$ :

iv) Intermediate good markets clear

$$\begin{aligned} a_1(s^t) + a_2(s^t) &= y_1(s^t), \\ b_1(s^t) + b_2(s^t) &= y_2(s^t). \end{aligned}$$

v) Final good markets clear

$$\begin{aligned} c_1(s^t) + i_1(s^t) &= G(a_1(s^t), b_1(s^t)), \\ c_2(s^t) + i_2(s^t) &= G(b_2(s^t), a_2(s^t)). \end{aligned}$$

vi) Asset markets clear

$$B_1(s_{t+1}, s^t) + B_2(s_{t+1}, s^t) = 0, \text{ for all } s_{t+1} \in S.$$

## 2.5 Other Variables of Interest

We measure the terms of trade in country 1 as

$$TOT(s^t) = q_1^b(s^t)/q_1^a(s^t).$$

The real exchange rate for country 1 is defined as

$$RER\left(s^t\right) = q_1^a(s^t)/q_2^a(s^t).$$

We define the trade balance as the net exports for country 1 as a fraction of GDP for country 1, both measured in current prices

$$NX\left(s^t\right) = \frac{q_1^a(s^t)a_2(s^t) - q_1^b(s^t)b_1(s^t)}{q_1^a(s^t)y_1(s^t)}.$$

# 3   Calibration and Solution

## 3.1   Parameter Values

We solve the benchmark model numerically using the parameter values reported in Table 1. Several of these values are common to the international business cycle literature. The capital-income share $\alpha$ and utility curvature $\sigma$ take the values found in BKK and Kehoe and Perri (2002). The Frisch elasticity of labor supply $1/\nu$ is set to 1.43, as in Correia, Neves, and Rebelo (1995) who incorporated GHH preferences in a small open economy setting. We rely on BKK's estimates of parameters governing the stochastic process for productivity.

Table 1: Parameter Values

| | |
|---|---|
| *Benchmark Model* | |
| Preferences | $\beta = 0.989, \sigma = 2, B = 0.69, \nu = 1/1.43, \eta = 1/1.5$ |
| Technology | $\alpha = 0.36, \delta = 0.025$ |
| Productivity shocks | $\rho_{11} = 0.906, \rho_{12} = 0.088, \sigma_\varepsilon = 0.00852, \rho_\varepsilon = 0.258$ |
| | |
| *Variations* | |
| KP (2002) productivity process | $\rho_{11} = 0.95, \rho_{12} = 0, \sigma_\varepsilon = 0.007, \rho_\varepsilon = 0.25$ |
| Frisch elasticity | $1/\nu \in [0.9, 1.8]$ |
| Trade elasticity | $1/\eta = \in [0.6, 2.5]$ |
| GHH preferences | $B = 0$ |
| GHH preferences / persistent habits | $B = 0.69, \lambda = 2/3$ |
| Standard preferences | $B = 0, \sigma = 1.8248$ |

Note: One period of time corresponds to one quarter. The adjustment cost parameter, $\xi$, is set to fit the standard deviation of investment relative to the standard deviation of output in the data. The preference parameter $\chi$, or $1-\gamma$ under standard preferences, that controls disutility from labor is set to ensure that hours worked in the steady state equal $1/3$. Other parameters in the variations are the same as in the benchmark model.

Similar to BKK we set the elasticity of substitution between traded intermediate goods $1/\eta$ to 1.5. We consider a symmetric steady-state in which $y_1 = y_2$ and $b_1 = a_2$. The degree of home bias $\omega$ is set to match the observed long-run import share of GDP. The average value of the import share for the US, Japan and the EU-16 for the period 1995-2010 is 0.15. Equating $a_1/y_1$ to 0.15 and noting that $TOT = 1$ in the steady state, we obtain the home bias parameter from

$$\omega = \frac{(a_1/b_1)^\eta}{1 - (a_1/b_1)^\eta}.$$

Braun, Constantinides, and Ferson (1993) report econometric estimates for the intensity of habits in the G7 countries ranging from 0.57 (in Japan) to 0.93 (in Canada). In the benchmark model we set the habit intensity parameter $B$ to the median of their estimates (0.69), although in variations we consider a wider range of values. [4]

We calibrate the other parameters to match long-run averages in the US data as described in Cooley (1997). One period of time corresponds to one quarter. The quarterly depreciation rate $\delta$ is set to ensure that the steady-state investment-GDP ratio is 0.25 and the capital-GDP ratio is 10. Once $\delta$ is set, the discount factor $\beta$ follows from the consumer's intertemporal condition in the steady state. Given the values of $\alpha, \delta$ and the steady-state capital-output ratio, we compute the discount factor as $\beta = (\alpha(qy/k) + 1 - \delta)^{-1}$. The coefficient that controls disutility from labor, $\chi$, is set so that the agents spend $1/3$ of their unit time endowment on market activities in the non-stochastic steady state. This requires that, from the steady-state labor supply equation

$$\chi = \left(1 - \frac{\lambda \beta B}{1 - \beta(1 - \lambda)}\right) \frac{w}{n^{\nu}},$$

where $w = q(1 - \alpha)(k/n)^{\alpha}$ is the steady-state real wage. The parameter $\chi$ depends on the strength and persistence of habits. In our benchmark, the habit persistence parameter is set to $\lambda = 1$, but we consider a variation in which $B = 0.69$, $\lambda = 2/3$.

As a further variation, we also consider time-separable Cobb-Douglas preferences

$$u(c, n) = \frac{[c^{\gamma}(1 - n)^{1-\gamma}]^{1-\sigma}}{1 - \sigma},$$

where the weight of leisure in the instantaneous utility function, $1 - \gamma$, ensures that the steady-state level of hours worked remains at $1/3$. As in Raffo (2008) we keep the Frisch elasticity of labor supply $\varepsilon_f = 1/\nu$ constant across models. To achieve this, we must adjust the curvature parameter $\sigma$:

$$\sigma = \frac{(1 - \gamma)(1 - n)}{\varepsilon_f n - \gamma(1 - n)}.$$

The curvature parameter $\sigma$ takes the value 1.8248 for time-separable Cobb-Douglas preferences.

Since we do not rely on log-linearization methods to solve the model, we must specify the functional form for capital adjustment costs explicitly. We adopt the following formulation from Boldrin, Christiano, and Fisher (2001)

$$\phi(x) = \frac{\kappa_1}{1 - 1/\xi}(x)^{1-1/\xi} + \kappa_2,$$

where $\kappa_1 = \delta^{1/\xi}$, $\kappa_2 = \delta/(1-\xi)$, and $\xi$ is the elasticity of investment with respect to Tobin's $q$. The restrictions $\phi(\delta) = \delta$ and $\phi'(\delta) = 1$ imposed on the constants $\kappa_1$ and $\kappa_2$ ensure that incorporation of the adjustment

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[4]In the asset pricing literature values for habit intensity within the range of 0.6 to 0.9 have been reported to help explain the equity premium puzzle (Boldrin, Christiano, and Fisher (2001); Constantinides (1990); Jermann (1998)). In the sensitivity analysis, we report results from simulations of the model for this range.

cost does not affect the deterministic steady state of the model. The capital adjustment cost parameter $\xi$ is set to match the observation that the standard deviation of investment is 2.93 times higher than that of output. Costly capital adjustment enhances the fit of our model to the data, but in variations we show that the main results are robust to its absence.

## 3.2 Solution Method

We solve the model numerically using a variant of the ergodic set methods described by Maliar, Maliar, and Judd (2011). The algorithm we use is classified by Judd, Maliar, and Maliar (2009) as belonging to the stochastic simulation class of methods. An advantage of this approach is that it allows us to avoid computing solutions in areas of the state space that are never visited in equilibrium. It also enables us to avoid the approximation errors associated with linearization of the optimality conditions. The algorithm parameterizes decision rules derived from the first order conditions using a polynomial approximation function of the current state variables. It then uses a fixed point iteration method to compute fixed-point values of the polynomial coefficients, evaluating conditional expectations using a combination of Monte-Carlo integration and regression. The details of implementation are described in the Computational Appendix.[5]

# 4  The Results

## 4.1  Positive Cross-Country Correlations Investment, Employment and Output

We start by reviewing the puzzles. The first column of Table 2 displays cross-country and within-country moments observed in the data. The fourth column reports predictions of the model with time-separable Cobb-Douglas preferences. Consider Panel A (international comovements). The standard model fails along two dimensions. First, it predicts negative cross-country correlations in investment and hours worked when these are positive in the data. Second, in the data international correlations of output and consumption are positive and fairly high, with the former exceeding the latter. In the model, the reverse holds. The output correlation is much lower than in the data, and the consumption correlation exceeds that of output.

Replacing Cobb-Douglas preferences with GHH preferences (column 5) eliminates the negative correlation of hours worked, but cannot reverse the negative comovement of investment. The contribution of our benchmark model (column 2) is that it resolves what Baxter (1995) calls the "international comovement puzzle". It matches the data by predicting positive cross-country correlations of investment and hours worked. The model moves in the right direction in accounting for the "quantity anomaly" of BKK. It predicts a realistic international correlation of output (0.53 vs. 0.59 in the data), but it fails to reverse the order of the consumption and output correlations.

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[5]Available at http:\\research.economics.unsw.edu.au\admitriev

## 4.2 Countercyclical Net Exports

All four models deliver a countercyclical net export to output ratio, matching an important feature of the data. In many respects, the performance of our benchmark model is similar to Kehoe and Perri's (2002) model with limited commitment. However, while their model generates positive factor correlations but fails to deliver countercyclical net exports, our benchmark reproduces both regularities.

Table 2: Business Cycle Statistics: Implications for Quantities

| | Data (1973:1 2010.2) | Model Economy with Time- | | | |
|---|---|---|---|---|---|
| | | Nonseparable Preferences | | Separable Preferences | |
| | | Simple Habits | Persistent Habits | Cobb-Douglas | GHH |
| **Panel A - *Cross-country correlations* - (US - EU15)** | | | | | |
| Output | 0.59 | 0.53 | 0.53 | 0.26 | 0.37 |
| Consumption | 0.46 | 0.84 | 0.84 | 0.78 | 0.71 |
| Investment | 0.51 | 0.23 | 0.18 | -0.35 | -0.55 |
| Employment | 0.49 | 0.59 | 0.59 | -0.38 | 0.37 |
| **Panel B - *Volatilities* - Standard deviation (in %)** | | | | | |
| Output | 1.55 | 1.70 | 1.72 | 1.32 | 1.70 |
| Net Export/Output | 0.48 | 0.10 | 0.13 | 0.17 | 0.33 |
| **Standard deviations relative to output** | | | | | |
| Consumption | 0.81 | 0.61 | 0.55 | 0.54 | 0.78 |
| Investment | 2.93 | 2.93 | 2.93 | 2.93 | 2.43 |
| Employment | 0.75 | 0.60 | 0.61 | 0.35 | 0.59 |
| **Panel C = *Correlations with output*** | | | | | |
| Consumption | 0.87 | 0.80 | 0.85 | 0.94 | 0.98 |
| Investment | 0.95 | 0.91 | 0.94 | 0.95 | 0.88 |
| Employment | 0.89 | 0.99 | 0.99 | 0.94 | 0.99 |
| Net Exports/Output | -0.52 | -0.46 | -0.46 | -0.55 | -0.54 |
| **Panel D - *Autocorrelations*** | | | | | |
| Output | 0.88 | 0.73 | 0.72 | 0.70 | 0.70 |
| Consumption | 0.89 | 0.92 | 0.89 | 0.73 | 0.72 |
| Investment | 0.90 | 0.61 | 0.65 | 0.66 | 0.66 |
| Employment | 0.92 | 0.74 | 0.73 | 0.66 | 0.70 |
| Net Exports/Output | 0.77 | 0.75 | 0.68 | 0.65 | 0.64 |

Note: International statistics in the Data column are calculated from U.S. data and aggregated data for 15 European countries. The sample consists of quarterly time series covering 1973:1-2010:2. The model's statistics are computed from a single simulation of 100,000 periods. All the statistics are based on logged (except for the net exports) and HP-filtered data with a smoothing parameter of 1600.

The within-country business cycle properties of our model are comparable to those of the model with Cobb-Douglas preferences. In particular, the volatilities of consumption, employment and output fit the data more closely. As expected, internal habits induce higher persistence in consumption, which results in a better fit to the data. These improvements are balanced by some deteriorations, notably in the within-country

correlations with output. Allowing some habit persistence ($\lambda < 1$) improves the within-country correlations of consumption and investment. This is at the expense of slightly reducing the comovement of investment between countries.

## 4.3   Some Intuition

Figure 1 provides some further insights. Consider the aftermath of a positive productivity shock in home country. Internal habit formation in our benchmark model means that home consumption exhibits a hump-shaped response to the shock. The behavior of consumption has important implications for investment. Habit-forming consumers want to save much of the increase in output arising from higher productivity. In the first instance, this saving is largely channeled into domestic investment. The rise in absorption exceeds that of output, initially causing net exports to decline. In subsequent periods, savings are channeled abroad, making additional resources available in the foreign country. Habit-forming foreign agents only increase their consumption gradually in response to the resulting boost to wealth. As a result, foreign investment also rises in response to a positive shock at home. This prediction of our model contrasts to those of models with time-separable preferences.

The time-nonseparable version of GHH preferences reintroduces the wealth effect on labor supply. However, an improvement in the terms of trade favoring the foreign country raises income abroad, dominating the wealth effect. This means that foreign employment rises immediately in response to an increase in domestic productivity.

The mechanisms that deliver comovement in investment and employment are mutually reinforcing. Since foreign hours rise after a surge in domestic productivity, there is a much larger response of global output. This creates savings that translate into foreign as well as home investment, enhancing the investment comovement. Habit formation amplifies the response of the terms of trade to a productivity shock. For this reason, the response of foreign employment in our benchmark model is even greater than it is under the GHH specification.

## 4.4   Elements of the model

In this section we discuss the role played by the three key ingredients of the model, and how each contributes to our result.

### 4.4.1   GHH Preferences

GHH preferences eliminate the wealth effect on labor supply as well as intertemporal substitution of leisure. The labor supply equation (6) shows that marginal rate of substitution between consumption and leisure is

Figure 1: Impulse Response Functions: Quantity Aggregates
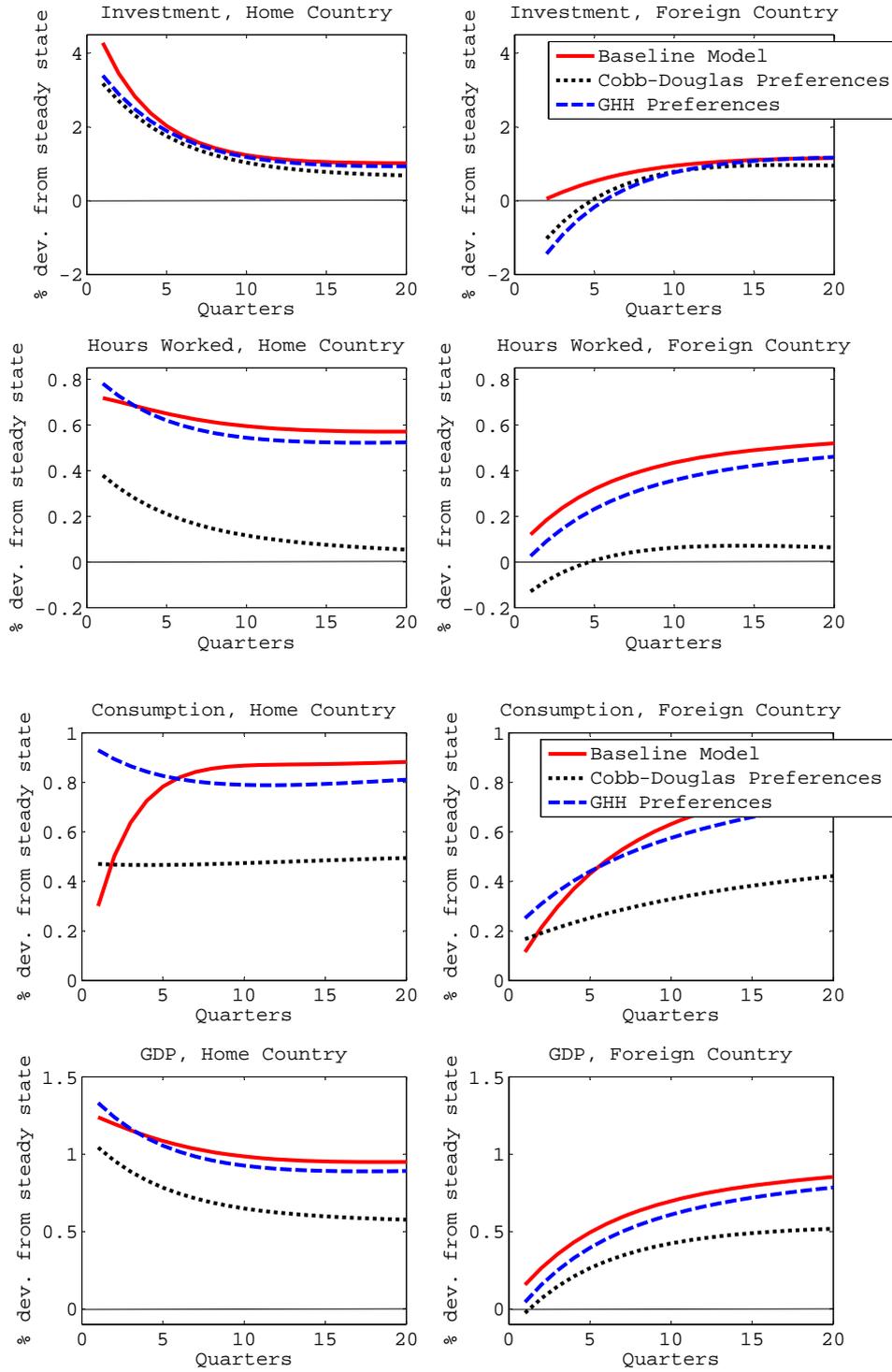


Note: The figure plots the percentage changes in investment, hours worked, consumption and GDP in response to a one standard deviation positive productivity shock in country 1.

Figure 2: Impulse Responses: Prices and Quantities
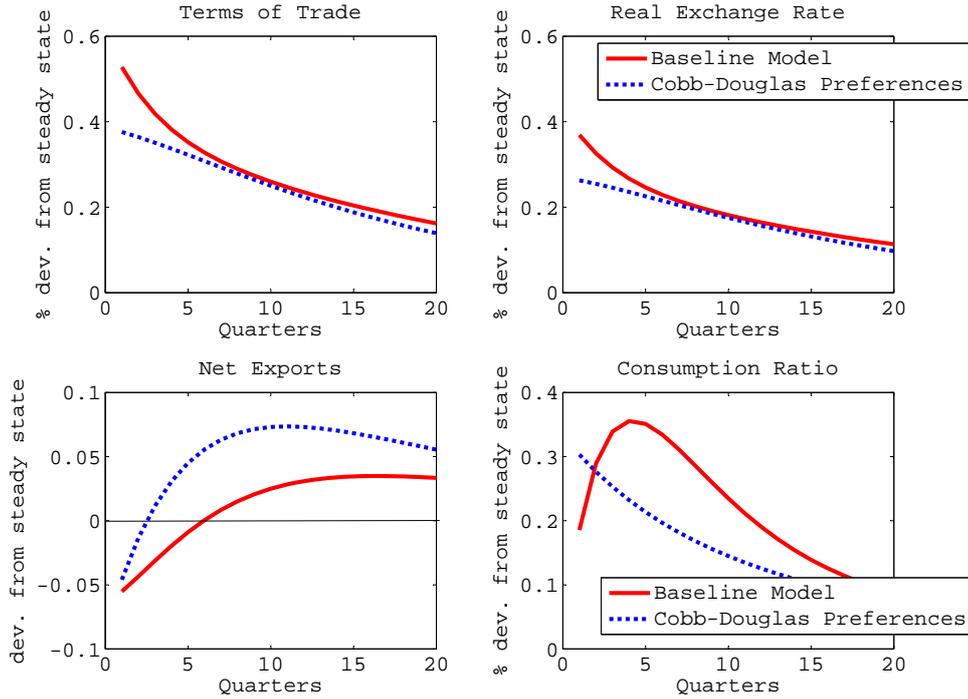
Note: The figure plots the percentage changes in the terms of trade, the real exchange rate, net exports and the consumption ratio in response to a one standard deviation positive productivity shock in country 1.

independent of consumption:

$$\chi n_1(s^t)^\eta = q_1^a(s^t)w_1(s^t). \tag{6}$$

Two mechanisms work to ensure a positive international correlation in employment. To see this, consider the log-linearized version of equation (6)

$$(\eta + \alpha)\widehat{n}_1 = \widehat{q}_1^a + \widehat{z}_1 + \alpha\widehat{k}_1, \tag{7}$$

where $\widehat{q}, \widehat{n}, \widehat{z}$ and $\widehat{k}$ denote percentage deviation of the corresponding variable from its steady state. Since $k(s^{t-1})$ is predetermined at $t$, positively correlated innovations to productivity tend to create a positive comovement of hours worked. This mechanism is augmented by another one: variation in the terms of trade.

A positive shock to domestic productivity causes relative scarcity of the foreign-made good $b$. This leads to an increase in income abroad and boosts labor supply. As a result, variation in the terms of trade enhances the comovement of hours worked. Labor comovement reinforces the productivity comovement and translates into a sizeable positive correlation in GDP across countries.

Figure 1 and Table 2 show that a model featuring GHH preferences fails to account for the comovement puzzle. It cannot get the investment comovement right. The tendency that Backus, Kehoe, and Kydland (1995, p. 340) call to "make hay where sun shines" is still present. Investment rapidly relocates to the most

14

productive location, causing a negative cross-country correlation.

### 4.4.2 Internal Habit Formation

When linear intrinsic consumption habits are introduced, the marginal utility of consumption consists of two terms. The first term captures an increase in utility from consuming an extra unit. The second term measures expected discounted future disutility from today's increase in consumption.

$$\Lambda_j\left(s^t\right) = u_1\left(s^t\right) + \lambda\beta \sum_{\tau=t+1}^{\infty} [\beta(1-\lambda)]^{\tau-t-1} \sum_{s^\tau} \pi\left(s^\tau \mid s^t\right) u_2\left(c_j\left(s^\tau\right), h_j\left(s^{\tau-1}\right), n_j\left(s^\tau\right)\right).$$

Habit-forming households seek to smooth not only the level of consumption but also its rate of change. In the aftermath of a positive shock, they increase consumption gradually and allow their habits to adjust. At the same time, foreign households want to raise their consumption due to the wealth effect. However, habit formation punishes rapid changes in consumption. The result is that world output rises with only modest changes to world consumption. If habits are sufficiently strong, an increase in world saving raises investment abroad.

When we incorporate consumption habits in the GHH class of preferences, the marginal utility of consumption becomes forward looking

$$
\begin{aligned}
\Lambda_1\left(s^t\right) \;=\; & \left(c_1(s^t) - Bh_1(s^{t-1}) - \chi\frac{n_1(s^t)^{1+\nu}}{1+\nu}\right)^{-\sigma} \\
& -\lambda\beta \sum_{\tau=t+1}^{\infty} [\beta(1-\lambda)]^{\tau-t-1} \sum_{s^\tau} \pi\left(s^\tau \mid s^t\right) B\left(c_1(s^\tau) - Bh_1(s^{\tau-1}) - \chi\frac{n_1(s^\tau)^{1+\nu}}{1+\nu}\right)^{-\sigma},
\end{aligned}
$$

while the marginal disutility from labor effort does not

$$\frac{\Lambda_1\left(s^t\right)}{-u_3\left(c_1\left(s^t\right), h_1\left(s^{t-1}\right), n_1\left(s^t\right)\right)} = \frac{1}{q_1^a\left(s^t\right) z_1\left(s^t\right) f_2(k_1\left(s^{t-1}\right), n_1\left(s^t\right))}.$$

The static optimality condition that controls labor supply now becomes

$$\chi n_1(s^t)^\nu \frac{1}{\Delta_1(s^t)} = q_1^a\left(s^t\right) z_1\left(s^t\right) f_2(k_1\left(s^{t-1}\right), n_1\left(s^t\right)), \tag{8}$$

where

$$\Delta_1(s^t) = 1 - \lambda\beta \sum_{\tau=t+1}^{\infty} [\beta(1-\lambda)]^{\tau-t-1} \sum_{s^\tau} \pi\left(s^\tau \mid s^t\right) \frac{u_2\left(c_1\left(s^\tau\right), h_1\left(s^{\tau-1}\right), n_1\left(s^\tau\right)\right)}{u_1\left(c_1\left(s^t\right), h_1\left(s^{t-1}\right), n_1\left(s^t\right)\right)}. \tag{9}$$

The term $\Delta_1(s^t)$ measures the difference between the overall and immediate increase in utility from an extra unit of consumption. Since $\Delta_1(s^t)$ depends on current consumption, its presence in (7) re-introduces the wealth effect on labor supply. The magnitude of the welath effect depends on the intensity of habits, $B$. Thus, the wealth effect can be made arbitrarily small by choosing $B$ close to zero. This feature can also be seen in the model of Jaimovich and Rebelo (2009), which incorporates time-nonseparable preferences consistent with arbitrarily small wealth effects.

The log-linearized versions of equation (8) for both coutnries are given by

$$(\eta + \alpha)\widehat{n}_1 = \widehat{q}_1^a + \widehat{z}_1 + \alpha\widehat{k}_1 + \widehat{\Delta}_1,$$

$$(\eta + \alpha)\widehat{n}_2 = \widehat{q}_2^b + \widehat{z}_2 + \alpha\widehat{k}_2 + \widehat{\Delta}_2.$$

As we have already observed, $k_j(s^{t-1})$ is predetermined at $t$. Risk-sharing ensures that the $\Delta_j(s^t)$ will comove, although this comovement is partly offset by fluctuations in the real exchange rate. International comovement of hours will depend on movements in the terms of trade $\widehat{q}_1^b - \widehat{q}_1^a$ and the covariance of $\widehat{z}$ and $\widehat{\Delta}$. The terms $\widehat{z}$ and $\widehat{\Delta}$ comove positively. Yet Figure 1 shows that foreign employment increases immediately when home country productivity rises. This is because the wealth effect on labor supply in the foreign country is dominated by the improvement in the terms of trade. As Table 2 shows, the cross-country correlation of employment is noticeably higher in our benchmark than under GHH (0.59 vs. 0.37). The explanation is that movements in the terms of trade are stronger in the presence of habits. The flatter response of consumption under habit formation ensures that home investment rises more quickly, leading to higher imports and further increasing the relative scarcity of good $b$.

The positive comovement of hours reinforces the mechanism behind the positive comovement of investment. Since foreign hours rise in response to an increase in home productivity, there is a much greater response of global output. This creates additional savings that are channeled into foreign as well as home investment, enhancing the investment comovement.

### 4.4.3 Adjustment Costs to Capital Formation

The role of adjustment costs is more subtle. Capital adjustment costs curb the volatility of investment relative to output.[6] They also interact with habit-formation preferences to generate a hump-shaped response of domestic absorption to productivity disturbances. This enhances the positive cross-country correlation of investment. Pakko (2003) shows that even in the case of standard preferences, strong capital adjustment costs can generate positive investment comovement at the cost of counterfactually low investment volatility.

However it is important to note that in our model, adjustment costs are not essential for delivering positive cross-country correlations of investment and hours. As observed by BKK, in models with imperfect substitutability investment is naturally less volatile than when foreign and domestic goods are perfect substitutes. After a shock to home productivity, the complementarity between domestic and imported goods dampens the tendency for investment to relocate immediately to the most productive location. Investment therefore responds less aggressively in both countries, moderating the negative comovement of investment and hours. Figure 3 shows that, in even the absence of adjustment costs, our model delivers positive factor comovements provided that habit intensity is high enough. However, as habit intensity declines towards zero,

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[6]Most two-country business cycle models rely on adjustment costs to curb investment volatility. This applies to complete market models (Raffo, 2008), as well as models with restricted markets (Baxter and Crucini, 1995). A notable exception is Kehoe and Perri (2002) where a limited commitment friction endogenously delivers plausible investment volatility.

Figure 3: Habit Intensity and International Comovements: No Adjustment Costs.
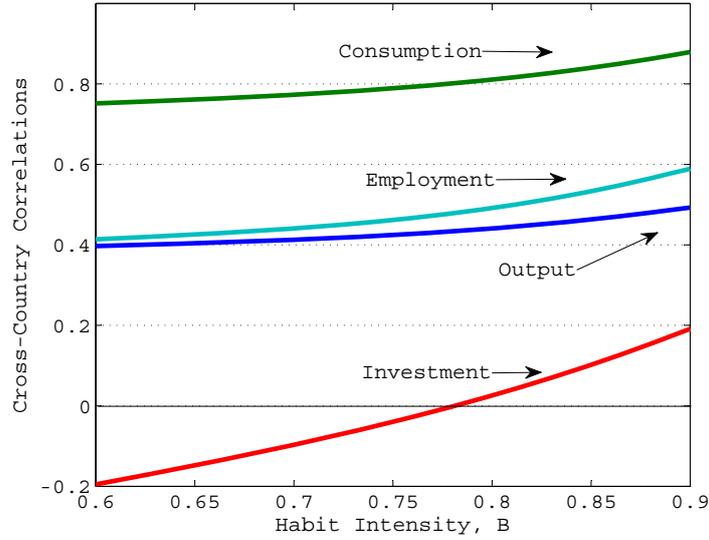


Note: To examine the sensitivity of the model's predictions to the parameterization of habits we vary the intensity of habits. The figure shows that even in the absence of capital adjustment costs the model delivers positive factor comovements for sufficiently high habit intensity.

households become less inclined to smooth the change rather than the level of their consumption, and the model resembles a setup with time-separable preferences.

There are two established methods for directly inducing sluggish adjustment in the law of motion for capital: capital adjustment costs (CAC) and investment adjustment costs (IAC). Both types have previously been used in the context of international real business cycle models.

Section 2.1 describes the capital adjustment costs of Hayashi (1982), which are used in our benchmark model. When the model features the investment adjustment costs of Christiano, Eichenbaum, and Evans (2005), the stock of capital instead evolves according to the following equation of motion:

$$k_j \left( s^t \right) = (1 - \delta) k_j \left( s^{t-1} \right) + \psi \left( i_j \left( s^t \right), i_j \left( s^{t-1} \right) \right), \text{ for } j \in \{1, 2\}, \tag{10}$$

where $\delta$ is the depreciation rate, $i_j \left( s^t \right)$ denotes gross investment, and

$$\psi \left( i_j \left( s^t \right), i_j \left( s^{t-1} \right) \right) = \left[ 1 - \Delta \left( \frac{i_j \left( s^t \right)}{i_j \left( s^{t-1} \right)} \right) \right] i_j \left( s^t \right).$$

Imposing the restrictions $\Delta (1) = \Delta' (1) = 0$ ensures that the steady state of the model is independent of the adjustment cost. We adopt the following functional form from Schmitt-Grohé and Uribe (2004):

$$\Delta (x) = \frac{\chi}{2} (x - 1)^2,$$

where $\chi = \Delta'' (\cdot) > 0$ controls the penalty for changing the level of investment. This formulation implies

that it is costly to change the level of investment. In addition, the cost is increasing in the magnitude of the change.

Investment adjustment costs have been shown to have advantages over capital adjustment costs in a closed economy setting. In a closed economy New Keynesian model, Christiano, Eichenbaum, and Evans (2005) show that IAC outperform CAC in accounting for hump-shaped investment responses to monetary shocks. Beaubrun-Diant and Tripier (2005) demonstrate that in a closed economy real business cycle setup, IAC better account for business cycle and asset pricing phenomena than CAC.

As far as quantities are concerned the model's predictions show little sensitivity to the type of adjustment cost introduced (see Table 4). However, the next section shows that capital adjustment costs do a better job at reproducing the observed behavior of prices.

## 4.5 Relative Prices

In this section we discuss the implications of our model for prices. Introducing non-separability of preferences helps bring the variability of the terms of trade and the real exchange rate closer to the data. Similar to other models in the literature, however, ours is unable to reproduce the magnitude of fluctuations that are observed empirically. Like the model of BKK, our model replicates the observed "S-curve" relationship between net exports and the terms of trade.

### 4.5.1 Volatilities

A well-known puzzle described by BKK concerns the volatility of relative prices. Two-country business cycle models with Cobb-Douglas preferences tend to predict very little variance in the terms of trade and the real exchange rate at business cycle frequencies, whereas in the data both are very volatile. Backus, Kehoe, and Kydland (1995) refer to this puzzle as the "price anomaly".

Replacing Cobb-Douglas preferences with GHH preferences reduces the volatility of prices even further. This reflects the fact that following a home productivity shock, the larger response of domestic absorption offsets the increased relative scarcity of the foreign good. However, introducing habits to the GHH preference structure almost doubles the volatilities of the terms of trade and the real exchange rate. The explanation is that the hump-shaped response of consumption to productivity shocks creates savings that are channeled into exports as well as investment. The increased supply of home goods on the international market complements the scarcity effect, placing further upward pressure on the relative price of the foreign good. Even so, relative prices are still not nearly as volatile as in the data.

Table 3: Business Cycle Statistics: Implications for Relative Prices

| | Data | Model Economy with Time- | | | |
| --- | --- | --- | --- | --- | --- |
| | | Nonseparable Preferences | | Separable Preferences | |
| | | Habits | Persistent Habits | Standard | GHH |
| Panel A - Volatilities - St. deviation (in %) | | | | | |
| Terms of trade | 2.73 | 0.78 | 0.74 | 0.61 | 0.45 |
| Real exchange rate | 4.85 | 0.55 | 0.51 | 0.43 | 0.32 |
| Panel B - Correlation with relative consumption | | | | | |
| Real exchange rate | 0.12 | 0.83 | 0.94 | 0.99 | 0.70 |

Note: International statistics in the Data column are calculated from U.S. data and aggregated data for 15 European countries. The sample consists of quarterly time series covering 1973:1-2010:2. The model's statistics are computed from a single simulation of 100,000 periods. All the statistics are based on logged (except for the net exports) and HP-filtered data with a smoothing parameter of 1600.

### 4.5.2 The Backus-Smith Puzzle

Two-country business cycle models typically also display the puzzle highlighted by Backus and Smith (1993). In the data, the correlation of the real exchange rate with the consumption of one country relative to the other is close to zero. [7] In complete markets models featuring perfect risk sharing, however, the correlation is usually close to one.

Habits do not seem to be key to resolving the Backus-Smith puzzle. Under persistent habits the correlation of the real exchange rate with relative consumption is not substantially different to that under standard preferences. This casts doubt on the conjecture of Huang and Liu (2007) that "non-time-separable preferences... may help break the tight link between the real exchange rate and relative consumption". In our benchmark model, and under time-separable GHH preferences, the response of relative consumption ($c_1/c_2$) to a home productivity shock is much larger than under standard preferences. Yet, as we have already seen, habit-formation in consumption increases rather than reduces the volatility of relative prices. These two features unfortunately move the correlation between the real exchange rate and relative consumption in the wrong direction.
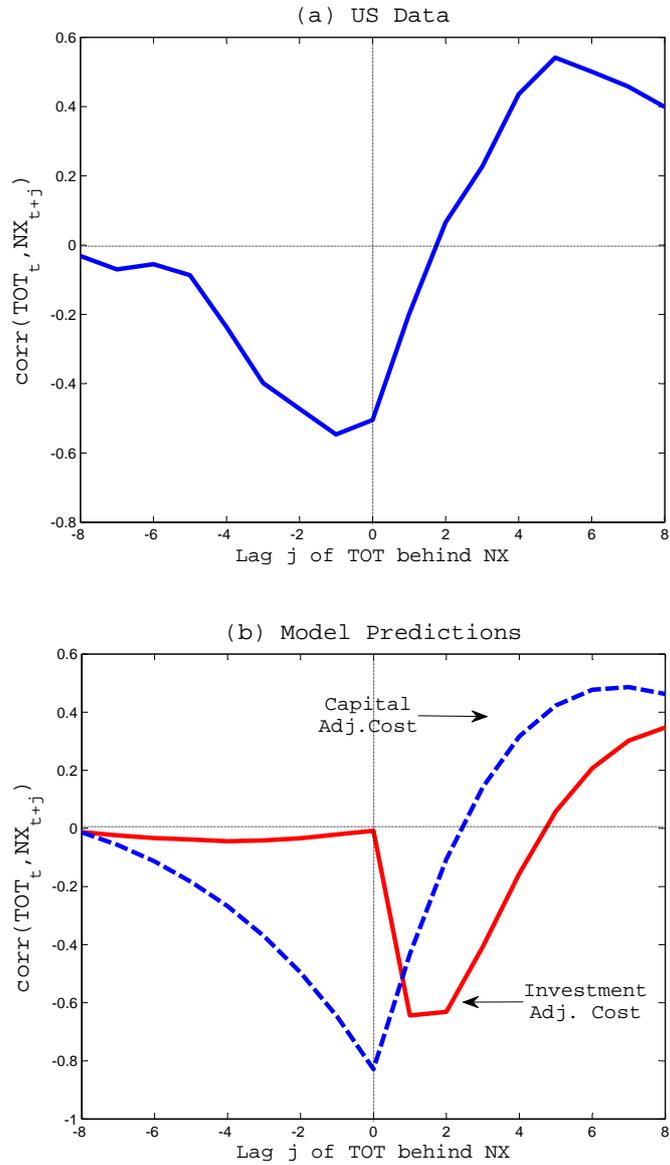
### 4.5.3 The S-Curve? Cross-correlation between Net Exports and the Terms of Trade

Finally, we consider the model's ability to reproduce the empirical "S-curve" relationship between net exports and the terms of trade. An S-shaped cross-correlation function between the trade balance and the terms of trade has been widely documented for both developed economies (BKK, Bahmani-Oskooee and Ratha, 2007) and emerging economies (Senhadji, 1998). Figure 4(a) shows this regularity for recent data. The S-curve is characterized by two properties. First, the correlation between $NX(t+k)$ and $TOT(t)$ is negative for $k = 0$, and for several first positive lags $k$. Second, this correlation is increasing for the few first positive lags $k$.

Figure 4(b) shows model-based results under (i) capital adjustment costs; and (ii) investment adjustment

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[7]The correlation between the real exchange rate and the relative consumption is found to be negative in some studies (Chari, Kehoe, and McGrattan, 2002; Corsetti, Dedola, and Leduc, 2008).

Figure 4: Cross-Correlation Function between Net Exports and the Terms of Trade.



(a) US Data

(b) Model Predictions

Note: The top panel of the figure displays the S-shaped cross-correlation function between the trade balance and the terms of trade for recent US data. The lower panel shows that the "S-curve" relationship is predicted by the model with capital adjustment costs, but not by the model with investment adjustment costs.

costs. The model featuring capital adjustment costs is consistent with the S-curve pattern of cross-correlations between net exports and the terms of trade observed in the data, while the model with investment adjustment costs is not.

The explanation is related to the hump-shaped response of investment under IAC. Under capital adjustment costs, a home productivity shock causes domestic investment to jump on impact. As a result, domestic absorption increases by more than domestic output, prompting an immediate fall in the trade balance. Net exports then increase as the effect of the productivity shock on investment subsides. The terms of trade rise on impact, then gradually decline. These patterns result in a negative but increasing correlation between the terms of trade and leads of the trade balance, consistent with the S-curve pattern observed in the data.

Under investment adjustment costs, the response of domestic investment (and absorption) to a positive productivity shock is hump-shaped. The response of output imitates that of productivity: a jump followed by a gradual decline. The response of net exports, measured as the gap between output and absorption, is thus initially close to zero. It reaches a maximum after a few quarters and then declines gradually towards its steady-state level. The trade balance therefore displays an inverted hump-shaped profile in response to a positive shock. However, the response of the terms of trade is a jump followed by a steady decline. This pattern sees the cross-correlation function decline after $k = 0$ and reach its minimum after only a few quarters. These dynamics are at odds with the S-curve shown in Figure 4(a).

## 4.6   Robustness

This section discusses how robust our model's predictions are to variations in the elasticity of substitution between domestic and foreign goods, the Frisch elasticity of labor supply, the parameterization of the forcing process and the persistence of habits. The results are reported in Table 4, Figure 5 and Figure 6.

### The Trade Elasticity

The susbstitution elasticity between domestic and imported goods is the most important parameter. Higher complementarity between domestic goods and imports removes firms' incentive to switch locations in response to a productivity shock, leading to greater synchronization in output, investment and employment. As shown by Pakko (2003), this mechanism is enhanced by the presence of capital adjustment costs. Furthermore, as Heathcote and Perri (2002, p. 621) argue, "greater complementarity is associated with a larger return to relative scarcity". Thus, a lower elasticity produces higher volatility in the terms of trade.

There is no consensus in the literature on the "correct" value of this parameter. Our benchmark uses the value of 1.5 as in BKK. However, microeconomic studies such as Bernard, Eaton, Jensen, and Kortum (2003) and Feenstra, Obstfeld, and Russ (2010) provide much higher estimates than macroeconomic studies (e.g. Hooper, Johnson, and Marquez, 2000). We therefore consider the range between 0.6 and 2.5. As the elasticity increases the model's predictions become closer to those of a one-good economy. Variations in relative prices diminish. Since restricting the substitutability between domestic and foreign goods increases comovement

Figure 5: Sensitivity to the Elasticity of Substitution.
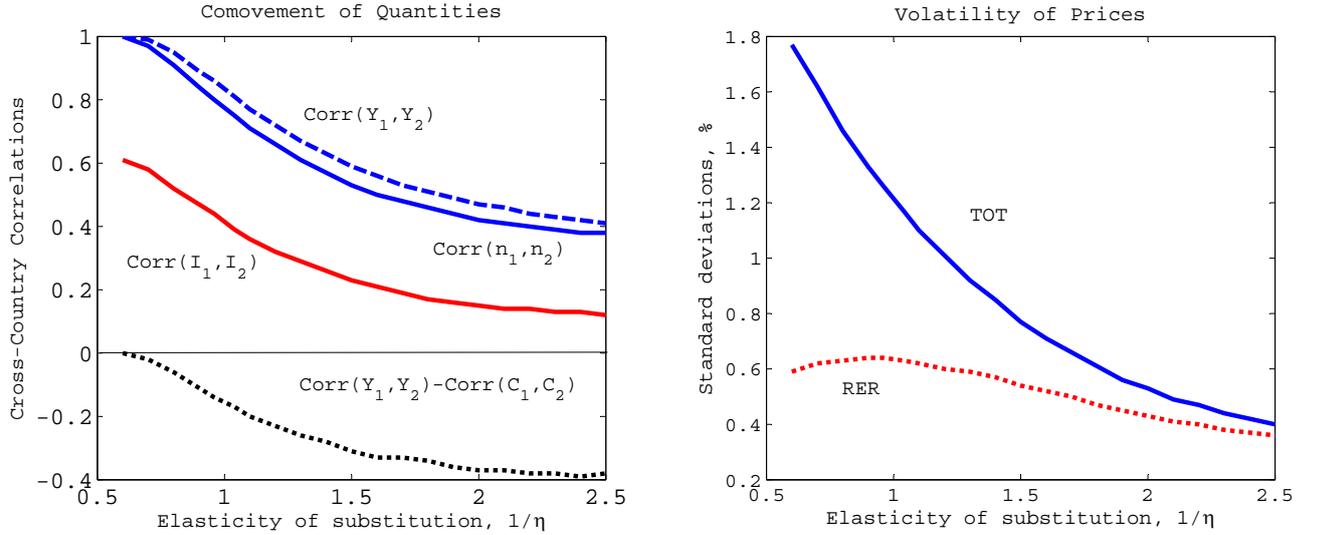


Note: The figure depicts the sensitivity of quantity comovements, and the volatility of the terms of trade and real exchange rate, to variation in the elasticity of substitution between intermediate and final goods. The figure shows that as the trade elasticity rises, comovements moderate but remain positive, while the volatility of relative prices falls.

in investment and labor supply, increasing the elasticity tends to reduce the cross-country correlations of quantity aggregates. However, they remain positive for the whole range.

### The Frisch Elasticity of Labor Supply

The literature disagrees on the appropriate size of the labor supply elasticity, $1/\nu$. In our experiments we considered a range between 0.9 and 1.8, compared with the benchmark value of 1.43. This range corresponds to the estimates by Chang and Kim (2006) and Fiorito and Zanella (2008). Table 4 refers to these cases as "low" and "high" Frisch elasticities. It is clear from the table that the models's predictions are not particularly sensitive to this parameter. Reducing the labor supply elasticity lowers the variability of output and hours worked, but the remaining business cycle statistics are largely unaffected.

### The Stochastic Process for TFP

There is also little consensus in the literature regarding the degree of persistence or spillovers characterizing the forcing process for productivity shocks. To assess the robustness of our results to deviations from the BKK process, we consider three alternative cases: (i) low persistence without spillovers ($\rho_{11} = 0.91$, $\rho_{12} = 0$) , (ii) high persistence without spillovers ($\rho_{11} = 0.97$, $\rho_{12} = 0$) and (iii) the process of Kehoe and Perri (2002) with moderately high persistence and no spillovers ($\rho_{11} = 0.95$, $\rho_{12} = 0$).

Table 4 shows how these variations affect our model's ability to account for international comovements. The positive cross-correlations of investment and hours worked are not sensitive to variations in the forcing process. Nor are the model's predictions for the quantity anomaly. In contrast, the ability of incomplete

market models to reproduce international comovements is quite sensitive to changes in the productivity process (Kehoe and Perri, 2002; Baxter and Crucini, 1995).

In our model, the implications for net exports are most sensitive. Eliminating spillovers results in a procyclical trade balance unless the persistence of shocks is high. The reason is as follows. First, reducing spillovers increases the need for foreign agents to borrow on international markets to smooth consumption following a home productivity shock. This raises the correlation between net exports and output in the home country. Second, when the persistence of shocks is low, households are less inclined to change their consumption profiles, making more output available for both home investment and exports. Together, these two mechanisms generate a procyclical trade balance. If shocks are highly persistent, however, the response of home consumption is strong enough to lower net exports both on impact and in the immediate aftermath of the shock. In this case, the response of domestic absorption is large enough to generate a countercyclical trade balance.
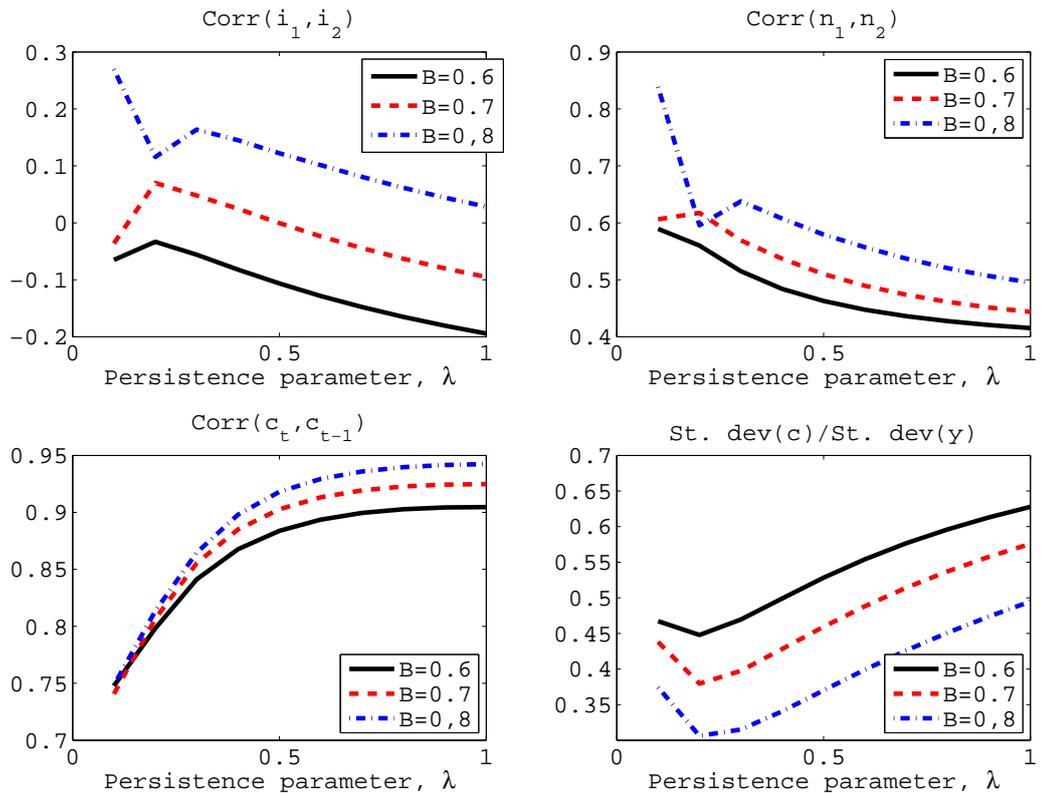
**Habit Persistence**

International comovements depend on the strength and intensity of habit formation. As we have already seen in Figure 3, factor correlations are positive in our model for sufficiently high $B$, even in the absence of sluggish capital adjustment. Figure 6 considers the robustness of the model's predictions to variations in habit persistence, $\lambda$, for different degrees of habit intensity. Results are shown for the model without costly capital adjustment.

The more persistent their habits, the less willing households are to change the growth of consumption in response to shocks. Greater habit persistence is therefore associated with less volatile and less autocorrelated consumption flows. Figure 6 demonstrates that if $B$ is high enough, the cross-correlation of investment is positive for the entire range of $\lambda$. As habit persistence rises, the cross-correlation of investment tends to increase. The explanation is as follows. Higher habit persistence flattens the hump-shaped response of home consumption following a positive shock. This means that consumption increases more rapidly following the shock, but peaks later. Home investment increases less rapidly on impact, but expands more strongly in subsequent periods. The flatter response of consumption implies that in the aftermath of the shock there is a larger surplus of output that can be exported abroad and utilized in foreign capital formation. This pattern increases the correlation between home and foreign investment.

# 5   Concluding Remarks

Our results provide evidence that financial frictions are not necessary to reconcile the quantity predictions of two-country theoretical models with the data. A complete markets model driven by productivity shocks alone, which deviates from time-separable Cobb-Douglas preferences, succeeds in matching two well-established

Figure 6: Sensitivity to Habit Persistence: No Adjustment Costs.



Note: To further examine the sensitivity of the model's predictions to the parameterization of habits we vary the persistence parameter for different levels of habit intensity. Even in the absence of capital adjustment costs the model can deliver positive investment and employment comovements, regardless of habit persistence, for high enough habit intensity. Factor comovements tend to increase as habit persistence rises.

business cycle facts: positive cross-relations of factors (the "international comovement puzzle") and the countercyclical trade balance. Our model also improves on standard approaches with regard to the "quantity anomaly".

The model we study incorporates a time non-separable version of GHH preferences featuring arbitrarily small wealth effects on labor supply. Compared with approaches that restrict financial markets, the results of our model are relatively robust to variations in the persistence of shocks and the presence of cross-country spillovers.

While our model's implications for quantities compare favorably to those of standard models featuring financial frictions, more work is needed to reconcile the model with empirical evidence on international relative prices. It is here that financial market incompleteness may play a more important role. Heathcote and Perri (2002) have shown that a drastic strategy of eliminating financial markets entirely ("financial autarky") pays dividends in terms of reconciling theory with the empirical behavior of relative prices. A promising line of enquiry might be to extend two-country models with credit frictions, such as Kehoe and Perri's (2002) limited enforcement environment, to incorporate time non-separability and/or limited wealth effects. The extension of our model to allow for a more complex asset market structure may help it to account better for price anomalies, such as counterfactually low exchange rate volatility and the "Backus-Smith puzzle". This is an avenue we leave for future research.

Table 4: Variations

| | Data | Benchmark | Adjustment costs | | | Forcing process | | | Frisch elasticity of labor supply | | Elasticity of substitution | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | No adj. cost | | IAC | Persistence | | | | | | |
| | | $B=0.69$ | $B=0.69$ | $B=0.9$ | $B=0.69$ | High | Low | KP | High | Low | High | Low |
| **Panel A - *Cross-country correlations* - (US - EU15)** | | | | | | | | | | | | |
| Output | 0.59 | 0.53 | 0.41 | 0.49 | 0.54 | 0.48 | 0.52 | 0.50 | 0.54 | 0.53 | 0.38 | 0.84 |
| Consumption | 0.46 | 0.84 | 0.77 | 0.88 | 0.84 | 0.64 | 0.62 | 0.62 | 0.83 | 0.86 | 0.76 | 0.95 |
| Investment | 0.51 | 0.23 | -0.11 | 0.19 | 0.23 | 0.39 | 0.61 | 0.48 | 0.23 | 0.24 | 0.12 | 0.47 |
| Employment | 0.49 | 0.59 | 0.44 | 0.59 | 0.62 | 0.49 | 0.51 | 0.50 | 0.60 | 0.59 | 0.41 | 0.89 |
| **Panel B - *Volatilities* - Standard deviation (in %)** | | | | | | | | | | | | |
| Output | 1.55 | 1.70 | 1.81 | 1.89 | 1.66 | 1.40 | 1.27 | 1.36 | 1.81 | 1.54 | 1.82 | 1.53 |
| Net Export/Output | 0.48 | 0.10 | 0.33 | 0.29 | 0.10 | 0.04 | 0.07 | 0.03 | 0.12 | 0.07 | 0.07 | 0.23 |
| *Standard deviations relative to output* | | | | | | | | | | | | |
| Consumption | 0.81 | 0.61 | 0.58 | 0.35 | 0.62 | 0.53 | 0.46 | 0.50 | 0.62 | 0.59 | 0.58 | 0.65 |
| Investment | 2.93 | 2.93 | 3.33 | 4.01 | 2.93 | 2.93 | 2.93 | 2.93 | 2.93 | 2.93 | 2.93 | 2.93 |
| Employment | 0.75 | 0.60 | 0.62 | 0.67 | 0.60 | 0.60 | 0.54 | 0.58 | 0.66 | 0.51 | 0.61 | 0.59 |
| **Panel C = *Correlations with output*** | | | | | | | | | | | | |
| Consumption | 0.87 | 0.80 | 0.79 | 0.50 | 0.79 | 0.81 | 0.79 | 0.80 | 0.80 | 0.79 | 0.79 | 0.82 |
| Investment | 0.95 | 0.91 | 0.90 | 0.95 | 0.91 | 0.94 | 0.96 | 0.95 | 0.91 | 0.92 | 0.92 | 0.88 |
| Employment | 0.89 | 0.99 | 0.99 | 0.99 | 0.98 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 |
| Net Exports/Output | -0.52 | -0.46 | -0.52 | -0.48 | -0.28 | -0.30 | 0.46 | 0.24 | -0.47 | -0.40 | -0.40 | -0.30 |
| **Panel D - *Autocorrelations*** | | | | | | | | | | | | |
| Output | 0.88 | 0.73 | 0.72 | 0.72 | 0.80 | 0.74 | 0.72 | 0.74 | 0.73 | 0.73 | 0.72 | 0.74 |
| Consumption | 0.89 | 0.92 | 0.92 | 0.96 | 0.94 | 0.93 | 0.92 | 0.93 | 0.93 | 0.92 | 0.92 | 0.93 |
| Investment | 0.90 | 0.61 | 0.60 | 0.67 | 0.74 | 0.64 | 0.64 | 0.64 | 0.61 | 0.61 | 0.60 | 0.61 |
| Employment | 0.92 | 0.74 | 0.72 | 0.72 | 0.86 | 0.75 | 0.74 | 0.75 | 0.74 | 0.74 | 0.72 | 0.78 |
| Net Exports/Output | 0.77 | 0.75 | 0.65 | 0.64 | 0.68 | 0.96 | 0.51 | 0.54 | 0.74 | 0.78 | 0.78 | 0.68 |

Note: International statistics in the Data column are calculated from U.S. data and aggregated data for 15 European countries. The sample consists of quarterly time series covering 1973:1-2010:2. The model's statistics are computed from a single simulation of 100,000 periods. All the statistics are based on logged (except for the net exports) and HP-filtered data with a smoothing parameter of 1600.

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