New Goods and Long-Run Trends in Labor Supply

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Abstract

Conventional economic theory, coupled with standard preference parameters, predicts that labor supply should fall steadily over time. Yet this contradicts the fact that labor hours fall as an economy develops, but subsequently tend to stabilize. I present a model which explains long-run trends in labor supply by the interaction of two opposing forces: a rising real wage, which tends to lower labor supply, and increasing product variety, which tends to raise it. Both are manifestations of the same underlying force—innovation—and, on a balanced growth path, their interaction can sustain stable labor hours. I consider applications of the model to cross-sectional differentials in labor hours and the rise in labor hours prior to the Industrial Revolution.

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Introduction

A striking feature of time series data, indeed a stylized fact, is the relative stability of hours worked over time. Although labor hours per capita tend to fall as an economy develops, they subsequently tend to stabilize. This ensuing stability is often listed along with the so-called "great ratios" as one of the necessary features of a balanced growth path—a prediction of any convincing model of growth. As shown in Figure 1, for example, despite a tripling of GDP per capita in the United States in the past fifty years, hours per capita have remained roughly stable.

Yet from the perspective of economic theory, the stability of hours worked is not obvious. According to standard preference parameters, rising consumption causes marginal utility to fall quickly, leading to a steady fall in labor supply as real wages rise. But this prediction has not materialized. In this paper, I present a theory of labor supply consistent with standard preference parameters and observed patterns in labor hours over time.

Underlying the prediction of falling labor hours in the standard model is the assumption of a single good. Because the model implicity treats all goods as perfect substitutes, consumers quickly become satiated in the face of diminishing marginal utility. Introspection leads one to the same conclusion: would people continue to supply labor, as more and more of the *same* good became plentiful? That is, if the products available today were identical to that of a hundred years ago, but today's real wages still prevailed, would the average person today still work the same hours? Although this seems unlikely, standard models answer in the affirmative: labor hours are independent of the product space.

According to the model presented here, trends in product variety are central to any explanation of labor supply patterns. Rather than consuming more of the same, consumers purchase an increasing variety of goods as real wages grow. New products and better qualities of existing products continually come on stream, and an entire industry—advertising—exists to inform us of these goods. Most importantly, increasing product variety counters the diminishing marginal utility of consumption: a tenth desktop computer, for instance, hardly provides as much extra utility as a new laptop.

In the paper, I endogenize labor supply as a function of product variety. By product variety, I

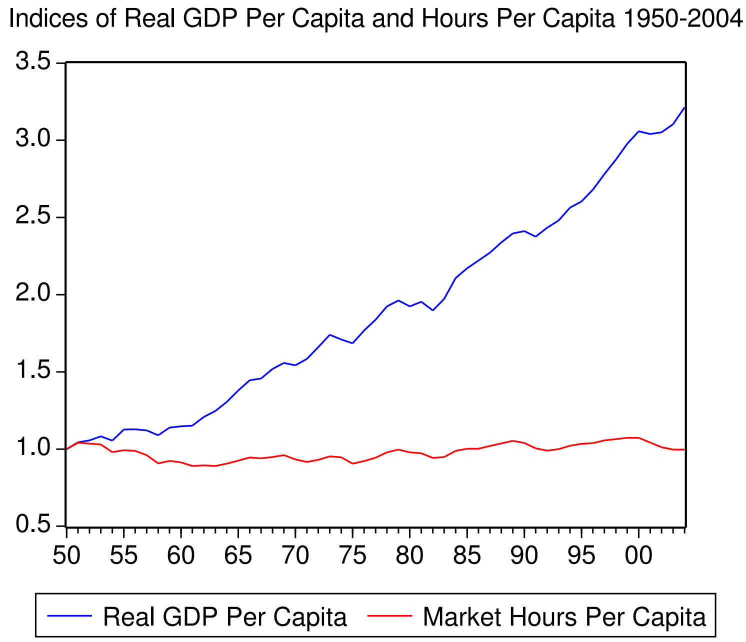

Figure 1: SOURCE: PENN WORLD TABLES AND FRANCIS AND RAMEY (2006)

mean distinct product groups—such as cars—and relatively similar brands within each group—such as Volvos and Fords. As we shall see, growth in the number of product groups increases labor supply, while brand growth reduces it. But with a sufficiently high taste for new product groups over brands, the net effect of increasing variety is to raise labor hours. By raising the marginal utility of income, increases in product variety attenuate the income effect of a rising wage. This dynamic is not just incidental. A rising real wage and increasing product variety are manifestations of the same forces: innovation and specialization.

To see the basic idea, consider a simple static model, where the real wage is W, consumption is C, labor hours are l, and utility is $u(C) = u(Wl) - v(l)$, $u' > 0, u'' < 0$. As W rises over time, the marginal utility of consumption falls; and assuming a dominant income effect, this depresses labor supply over time. But suppose now there are n relatively distinct goods, with all goods additively separable in utility; utility reduces to: $u(C) = nu(\frac{Wl}{n})$. Hence, if n is rising too, this tempers the diminishing marginal utility arising from increases in W. As a result, increasing product variety can sustain stable labor hours in the face of a rising real wage. The rising earnings accruing to this stable labor supply can act as a market for more innovation, leading to further wage and variety growth, and so on—thus keeping the entire dynamic in train.

I proceed as follows. After reviewing current approaches to modeling labor supply in Section 1, I present a partial equilibrium model in Section 2. This model presents the main insight of the paper: how increasing product variety affects labor hours. I then discuss implications of the model for time series and cross-sectional variation in labor hours. Because real wages differ across nations, while distinct product groups are widely available, the framework here also has implications for cross-sectional differentials in labor hours. Following this, I turn to long-run growth. Although labor supply is the fundamental source of growth, so far the labor/leisure margin has received little attention in the growth literature. To address this gap, in Section 3 I develop a dynamic general equilibrium model, which incorporates labor supply into a model of long-run growth. In the model, a higher labor supply effectively raises the size of the market, which increases the expected profits from innovation. By increasing the incentive to innovate, this causes more wage and variety growth, and so on. On the balanced growth path, there is sustained growth and constant labor hours. In Section 4, I show how the model confers insights on the rise in labor hours in the fifty years prior to the Industrial Revolution—the "Industrious Revolution." Finally, Section 5 concludes.

1 Review of Labor Supply Modeling

Before turning to the model, I briefly review standard theory and common approaches to modeling trends in hours worked.

Central to the analysis of labor supply trends is the elasticity of marginal utility with respect to consumption—here denoted $\sigma > 0$—which mediates how fast the marginal utility of consumption declines as consumption rises. To see why, consider the standard first order labor/leisure optimality condition, where consumption is denoted by C, labor by l, and the real wage by W. Assuming an interior solution and consumption/labor separability, we have:

$$Wu'(C) = V'(l), \tag{1}$$

where $u' > 0, u'' < 0, V' > 0$, and $V'' > 0$. Solving for l (noting V' is increasing) gives: $l = V'^{-1}(Wu'(C)) \Rightarrow l'(\cdot) = \frac{1}{V''(\cdot)} > 0$. So if $Wu'(C)$ falls, l will also fall. As a result, the level of labor supply depends on the interaction of the real wage W—the source of the substitution effect—and the marginal utility of consumption $u'(C)$—the source of the income effect. Specifically, a permanent one percent rise in the real wage a rise in W of 1 percent and a fall in $u'(C)$ by σ percent. As a result, labor supply changes by $1-\sigma$ percent; formally, we have $\frac{dl}{dW} < 0 \Leftrightarrow \sigma > 1$. Determining the magnitude of σ is central to determining the path of l.

For concreteness, consider now the case of standard isoelastic, time-separable utility, which is separable in consumption and labor: $u(C) = \frac{C^{1-\sigma}}{1-\sigma}$, and $V(l) = \frac{l^{1+\theta}}{1+\theta}$, $\theta > 0$. Setting $C = Wl$ for simplicity, making substitutions, and taking growth rates of (1) yields

$$(\sigma+\theta)\frac{\dot{l}}{l} = \frac{\dot{W}}{W}(1-\sigma). \tag{2}$$

Given $\frac{\dot{W}}{W} > 0$, only the knife-edge case of logarithmic utility (i.e., $\sigma = 1$) ensures stable labor hours. By contrast, if $\sigma > 1$, the model predicts labor hours fall steadily over time. Irrespective of the magnitude of σ, the model predicts labor hours will either fall or rise steadily, or simply remain static over time.

According to the model, the size of σ has a large bearing on labor supply trends over time. A large body of empirical evidence indicates that σ exceeds one. For example, Campbell (2001) notes that "direct evidence on the elasticity of intertemporal substitution suggests that it is fairly low, certainly well below one [i.e., $\sigma > 1$]." Barro (2005) maintains that time series estimates on savings rates restrict σ to lie between 0.2 and 0.5. Hall (1988) and Campbell and Mankiw (1989) estimate the elasticity to be close to zero.[1]

[1] Hall's (1988) value is representative, and he claims the IES "may well be zero". Cochrane (2008) remarks that "more recent macro literature has tended to side with Hall". Carroll and Summers (1989) observe that a value below 0.25 is standard. Studies based on micro data find estimates a little higher, but not by much. For example, using micro data, Attanasio and Weber (1995) report estimates in the range $[0.48, 0.67]$. Finally, measures of risk aversion almost uniformly exceed unity and generally lie in the range $(1, 5)$. In this standard power utility setting, the coefficient of relative risk aversion is σ. According to standard expected utility theory, risk aversion is also mediated by the degree of diminishing marginal utility. Thus, measures of risk aversion also provide evidence of the degree to which marginal utility changes as consumption fluctuates.

1.1 Common Approaches to Labor Supply Modeling

Although empirical evidence suggests $\sigma > 1$, this is hard to reconcile with standard isoelastic preferences and a long-run stable labor supply.[2] Here, I briefly review standard approaches to modeling labor supply, emphasizing how they explain stable long-run hours.

1.1.1 Cobb-Douglas Utility

Because the Cobb Douglas function—i.e., $\sigma = 1$—is the only standard function with consumption/labor separability that predicts stable hours, most growth and real business cycle models impose $\sigma = 1$. Yet primarily this utility formulation is chosen *so as* to yield a stable labor supply. While this function is consistent with stable labor hours, imposing of $\sigma = 1$ contradicts the empirical evidence on σ.

1.1.2 King - Plosser - Rebelo Utility

The King-Plosser-Rebelo utility function can maintain stable hours even when $\sigma > 1$. This function is:

$$u(C, l) = \begin{cases} \frac{c^{1-\sigma} v(l) - 1}{1-\sigma} & \text{if } \sigma \neq 1 \\ \log C + v(l) & \text{if } \sigma = 1, \end{cases}$$

where $v' > 0$, $v'' > 0$, when $\sigma > 1$, and $v' < 0$, $v'' > 0$ otherwise. To ensure stable labor hours when $\sigma > 1$, this function imposes complementarity between consumption and labor; i.e., $U_{Cl} > 0$. By raising the incentive to supply labor as consumption rises, this counters the income effect of a rising wage. Given certain restrictions, labor supply is independent of permanent wage changes. Yet whether this complementarity exists is unclear. Conceivably the marginal utility of consumption is *decreasing* in work hours, $U_{Cl} < 0$: long hours at work means less time to enjoy consumption goods—the case of a vacation being an obvious example. According to Barro and Xala I Martin (2004), this case is "introspectively more plausible". Empirically, there is little evidence of complementarity. Chetty (2006) discusses

[2]Isoelastic preferences are necessary for steady state analysis, in that they ensure constant interest rates and risk premia in the face of a secular trend in income.

evidence on consumption patterns when a person becomes unemployed or experiences poor health. Because labor supply is now lower, the King-Plosser-Rebelo function predicts optimal consumption falls. Yet empirical evidence indicates that consumption only falls marginally for those with liquid wealth. This suggests optimal consumption levels are not much lower when labor supply is low. In addition, those with higher unemployment benefits experience less pronounced falls in consumption. Such observations suggest that any consumption drop is largely due to imperfect insurance markets or liquidity constraints—and not complementarity. Finally, this function cannot explain patterns in labor supply over time. In this framework, labor supply is independent of permanent changes in the real wage.

1.1.3 Home Production

Another explanation of stable hours focusses on the relative trends in market and home production (see e.g., Benhabib, Rogerson and Wright (1991)). In home production models, consumers allocate time between home and market production to produce consumption goods. Depending on the relative productivity of the two activities, consumers substitute between them. But if the productivity of home and market production grow at the same rate, there is no reason for substitution, and market hours remain stable. However, the degree of substitutability between home and market activity, together with whether the trends in home and market productivity are equal, remains unclear.[3]

1.1.4 Overview

To summarize, existing models have problems reconciling theory with long-run trends in labor supply. Ideally we seek a model that is consistent with an intertemporal elasticity of substitution below unity, trends in labor supply as an economy develops, and one that permits the possibility of stable long run labor hours—a stylized fact and a convenient modeling assumption in steady state growth analysis. In the next section, I present such a model.

[3] Coulibaly (2006) presents a model where increases in job quality lowers the disutility of labor supply, thereby preventing a fall in hours. For most of the twentieth century, however, higher paid workers—who typically have higher quality jobs—worked shorter hours. Thus it is unclear how strong this effect is.

2 Partial Equilibrium Model of Labor Hours and Variety

Here I present a model showing how expanding product variety affects labor supply, and how this feature can sustain stable labor hours in the long run. For now, I take variety growth as exogenous; in SECTION, the general equilibrium model will illustrate how labor hours affect the level of product variety.

2.1 The Economic Environment

There is a single representative consumer who lives for T periods. Consumers have preferences defined over a continuum of existing and potential goods, including leisure. The time endowment each period is unity. There is latent demand for all potential goods, and no good is essential. The consumer receives income from supplying labor and savings. My main concern is labor supply per capita over the long run, so what I mean by "labor supply" is broad: it captures labor effort and intensity and generally encompasses any activity directed to supplying labor in the marketplace. As such, because it is a purposeful means of raising one's *effective* or quality-adjusted labor supply, it also incorporates human capital accumulation. On the other hand, leisure incorporates any nonmarket oriented activity; for example, home production, child rearing, idleness, and so on.

Consumption consists of different components, and there are two margins of differentiation: product groups and brands. Product groups represent broad categories of goods without close substitutes, for which demand is relatively inelastic; for example, new medicines/treatments, cars, microwave ovens, VCRs, printers, laptops, lasik surgery, tvs, air-conditioning, cell phones, and so on. There is a continuum of differentiated product groups indexed along the infinite interval $(0, \infty)$, but at any time t, only a measure $n_t < \infty$ of groups is available for purchase.[4] There is a common elasticity of substitution between all groups, and since groups are imperfect substitutes, I assume this is less than one. New goods on this margin represent "breakthrough" innovations.

Associated with each group is a continuum of brands, also indexed on $(0, \infty)$. Brands constitute different varieties or characteristics of goods within a given group—in terms of attributes

[4] Technically, this is the Lebesgue measure of the Borel-measurable set, $(0, m_t]$.

like function, style, flavor, size, and color. Brand growth also incorporates quality improvements within each group. For example, the release of a new yoghurt containing extra nutrients would be an example of a quality increase embodied in a new brand. At time t, there is a measure $m_t < \infty$ of brands available in each group.[5] Because brands are relatively good substitutes, demand on this margin is elastic, and the common elasticity of substitution between brands exceeds one.

The distributions of m_t, n_t, and the wage w_t are given exogenously. Brand, group, and wage growth have means g_m, g_n, and g_w respectively. As for notation, groups are indexed by $j \in [0, n_t]$ and brands by $i \in [0, m_t]$. Thus $c_{jit} \geq 0$ denotes the consumption service flow from brand i in group j at time t; c_{jt} denotes the service flow from group j at time t. Finally, all existing goods have a price of one, while non-existent goods have infinite prices.

The consumer has a love of variety for groups and brands. For groups, one can explain this by the welfare improvement associated with large innovations: new groups satisfy previously unmet needs. Broadly, one can explain a love of variety for brands in two ways. First, with diminishing marginal utility to each good, there is a welfare gain to smoothing consumption over more goods. Faced with a variety of yoghurts, for example, the consumer might prefer to consume a little of each flavor; this is the Dixit-Stiglitz (1977) setup. Second, more variety enables consumers to attain their ideal brand or bundle of characteristics; this is the Lancaster (1979) formulation.[6] Either way, welfare of the representative consumer rises as the number of brands increases. Mostly for simplicity, I use the more tractable Dixit-Stiglitz formulation.

2.2 Consumer Preferences

Period utility is a function of consumption services from product groups, c_{jt}, and labor, $l_t \in [0, 1]$. Both consumption services from individual groups and labor are separable in utility. Consumption services from a group c_{jt} are given by the constant elasticity of substitution index:

[5] For convenience, I ignore indivisibilities and the non-integral nature of the variables n and m, and from now on refer to them loosely as numbers. More generally, it is convenient to view m and n as continuous indices of brand and group variety.

[6] If consumers derive utility from the novelty of a new good, this would constitute another rationale for love of variety.

$$c_{jt} \equiv \mathfrak{m}_t^{v+1-\frac{1}{\alpha}} \left(\int_0^{m_t} c_{jit}^{\alpha}\, di \right)^{\frac{1}{\alpha}}, \tag{3}$$

where $c_{jit} \geq 0$, $\alpha \in (0,1)$, and $\mathfrak{m}_t > 0$ denotes the measure of brands actually consumed. Distinguishing between the number of brands available, m_t, and the number actually consumed, $\mathfrak{m}_t$, ensures that only goods consumed affect welfare. For clarity, I set the upper integral limit to m_t and not to infinity, but technically utility is defined over a continuum of goods.[7] Since there is a continuum of brands, the elasticity of substitution between brands within a group is $\frac{1}{1-\alpha} \in (1, \infty)$.

Following Benassy (1996), $v \in (0, \infty)$ mediates the taste for brand variety, and governs the elasticity of the marginal utility of consumption with respect to the number of brands consumed. The parameter v disentangles the distinct concepts of love of variety and substitutability between goods (which is also the elasticity of demand for each brand).[8] Hence, this formulation can handle situations where the consumer might be highly responsive to price changes, but still have a large taste for variety; or cases where the consumer has little taste for variety, but perceives goods as imperfect substitutes.[9]

Now define:

$$u(c_{jt}) = \frac{(c_{jt} + \epsilon)^{1-\theta}}{1-\theta},$$

where $\theta > 1$, $\epsilon > 0$, and $c_{jt} \gg \epsilon \approx 0$. For now, assume there is only a single existing group, c_{jt}. Then the utility from the consumption services of the group is given by

$$u(c_{jt}) - u(0) = \frac{(c_{jt} + \epsilon)^{1-\theta}}{1-\theta} - \frac{\epsilon^{1-\theta}}{1-\theta} \geq 0.$$

The constant ϵ is arbitrarily small and governs the utility gain from consuming any positive quantity of the group. Since $\theta > 1$, ϵ ensures that, first, the utility flow from consuming a group is positive and, second, that period utility is well-defined when $c_{jt} = 0$. Otherwise, it

[7] That is, $c_{jt} \equiv \mathfrak{m}_t^{v+1-\frac{1}{\alpha}} \left(\int_0^{\infty} c_{jit}^{\alpha}\, di \right)^{\frac{1}{\alpha}}$, where $\mathfrak{m}_t^{v+1-\frac{1}{\alpha}} \left(\int_{m_t}^{\infty} c_{jit}^{\alpha}\, di \right)^{\frac{1}{\alpha}} = 0$.

[8] To see why, suppose consumption expenditure on a group is C_t. Given symmetry and strict concavity, all available goods are consumed in equal quantities, so $\mathfrak{m}_t = m_t$, and $c_{jt} = m_t^{v+1-\frac{1}{\alpha}} m_t^{(\frac{1}{\alpha}-1)} C_t = m_t^v C_t$. It follows that $v > 0$ now mediates the marginal utility gain to consuming more brands.

[9] By comparison, the standard Dixit-Stiglitz function conflates the degree of love of variety with the elasticity of substitution (and elasticity of demand), and implies $v = \frac{1}{\alpha} - 1 > 0$.

plays no role in the results. Since $\epsilon \approx 0$, even a small amount of consumption on a new group raises utility significantly. Thus there is a sizable welfare gain to distinct new innovations, irrespective of the quantity consumed. Subsequently, utility increases at a diminishing rate, as consumption of the group rises.

More generally, period utility from consumption services when n_t groups are available for purchase is

$$\mathfrak{n}_t^{\phi} \int_0^{n_t} u(c_{jt}) - u(0)\ dj = \mathfrak{n}_t^{\phi} \int_0^{n_t} \frac{(c_{jt}+\epsilon)^{1-\theta}}{1-\theta} - \frac{\epsilon^{1-\theta}}{1-\theta}\, dj, \tag{4}$$

where $\phi > -1$, and $\mathfrak{n}_t$ denotes the number of groups actually consumed.[10] The constant, $\frac{1}{\theta} < 1$, is the elasticity of intertemporal substitution of consumption services from each group across time.[11] Because groups are separable in utility, $\frac{1}{\theta}$ is also the elasticity of substitution between groups. As a result, consumption services in different periods and consumption services of different groups are equally substitutable. By assumption, $\theta > 1$, while the standard view is that the intertemporal elasticity of substitution is also below one, so this is a reasonable simplification.[12]

Here, ϕ plays a role similar to that of v in the discussion of brands: it disentangles the degree of love of variety for groups from the elasticity of substitution between groups. To see why, let $\overline{c}_{jt}$ denote the equilibrium level of consumption services in each group at time t. If $\mathfrak{n}_t$ groups are consumed in equilibrium, then by symmetry the equilibrium level of utility is given by: $\mathfrak{n}_t^{\phi+1}\big(u(\overline{c}_{jt}) - u(0)\big)$. Holding the level of consumption services in each group constant, $\phi > -1$ mediates the marginal utility gain to group consumption. Holding c_{jt} constant, this restriction ensures utility is rising in the number of groups consumed.

Keeping c_{jt} on other groups fixed, if $-1 < \phi < 0$, there is decreasing marginal utility to the number of groups consumed. When $\phi = 0$, groups are independent and additively separable

[10] More precisely, utility from consumption services is

$$\mathfrak{n}_t^{\phi} \int_0^{n_t} u(c_{jt}) - u(0)\ dj = \mathfrak{n}_t^{\phi} \left(\int_0^{n_t} u(c_{jt}) - u(0)\ dj + \int_{n_t}^{\infty} u(0) - u(0)\ dj \right).$$

[11] Strictly speaking, this is an approximation that is only true as $c_{jt} \to \infty$. The elasticity of intertemporal substitution is $\frac{c_{jt}+\epsilon}{\theta c_{jt}}$. But since $\epsilon \approx 0$, $\frac{c_{jt}+\epsilon}{\theta c_{jt}} \approx \frac{1}{\theta}$.

[12] In equilibrium, the parameter $\frac{1}{\theta}$ is the elasticity of intertemporal substitution of real consumption expenditure.

in utility. In contrast, if $\phi > 0$, there is increasing marginal utility to the number of groups consumed: new goods now supplement the usefulness of existing ones. As an example, suppose there are only two groups: food and recreation. Then a third group—say, cars—is introduced. Keeping expenditure on all groups fixed, the introduction of cars now has two effects. Because the consumer can now travel to nice restaurants, the marginal utility of food rises. Since the consumer can now easily travel to airports for vacations, the marginal utility of recreation also rises. Thus the consumption of a new group can have a positive effect on the marginal utility of other groups.

2.3 The Complete Problem

Each consumer has a Frisch elasticity of labor supply of $\frac{1}{\theta} > 0$ and a rate of time preference of $\rho > 0$. Assets are denoted by b. Labor and consumption are separable in utility. Consumers take the interest rate r_t, the real wage W_t, n_t, m_t and the initial level of assets, b_0, as given and solve:

$$\max_{(0<l_t<1,\{c_{jit}\geq 0\})_{t=0}^{t=\infty}} \int_0^\infty \left(\mathfrak{n}_t^\phi \int_0^{n_t} u(c_{jt}) - u(0)\, dj - \beta \frac{l_t^{1+\theta}}{1+\theta} \right) \exp(-\rho t)\, dt,$$

subject to:

$$\dot{b}_t \leq r_t b_t + W_t l_t - \int_0^{n_t} \int_0^{m_t} c_{jit}\, di\, dj, \tag{5}$$

$$\lim_{t\to\infty} \frac{b_t}{\Pi_{j=0}^t (1+r_j)} \geq 0. \tag{6}$$

2.4 The Solution to the Intratemporal Problem

I solve the dynamic problem by two-stage budgeting. First I solve the intratemporal problem under certainty in period t. Given that utility is both time-separable and separable in consumption and leisure, I find the optimal allocation of goods each period, subject to some given level of consumption expenditure, C_t. First, given the concavity of the consumption index, all brands are consumed in equal quantities in equilibrium. As for groups, if they are all are consumed in equilibrium the period utility function reduces to approximately $\frac{n_t^{\phi+\sigma}(m_t^v C_t)^{1-\sigma}}{1-\sigma} - \frac{n_t^{\phi+1}\epsilon^{1-\sigma}}{1-\sigma}$.

Considering that ϵ is arbitrarily small and $\theta > 1$, I assume utility is always increasing in n, and hence all existing groups are consumed in equilibrium.[13] That is, $\mathfrak{n}_t = n_t$ and so, by symmetry, the consumer spends $\frac{C_t}{n_t}$ on each existing group. Combining these results and noting prices of unity for each existing good, the quantity demanded of each existing good is $c_{jit} = \frac{C_t}{m_t n_t} > 0, \quad \forall j \in [0, n_t], \quad \forall i \in [0, m_t]$. Plugging these demands into Eq. (3) implies that the optimal level of consumption services from group j at time t is

$$\max_{\{c_{jit} \geq 0\}} \left\{ c_{jt} : \int_0^{m_t} c_{jit} \, di = \frac{C_t}{n_t} \right\} = m_t^v \frac{C_t}{n_t}.$$

Substituting the optimal c_{jt} into Eq. (4) gives the period indirect utility function for consumption:

$$V(C_t, m_t, n_t) = \frac{n_t^{\phi+1}(m_t^v \frac{C_t}{n_t} + \epsilon)^{1-\sigma}}{1-\sigma} + \frac{n_t^{\phi+1}}{(\sigma-1)\epsilon^{\sigma-1}}.$$

For convenience, define $\zeta = \phi + \theta > 0$. Then noting that $\frac{m_t^v C_t}{n_t} \gg \epsilon \approx 0$, I approximate to get

$$V(C_t, n_t, m_t) \approx \frac{n_t^\zeta m_t^{v(1-\sigma)} {C_t}^{1-\sigma}}{1-\sigma} + \frac{n_t^{\phi+1}}{(\sigma-1)\epsilon^{\sigma-1}}. \tag{7}$$

Finally, the reduced form expected life-time utility is

$$\mathbb{U} = \int_0^\infty \left(\frac{n_t^\zeta m_t^{v(1-\sigma)} {C_t}^{1-\sigma}}{1-\sigma} + \frac{n_t}{(\sigma-1)\epsilon^{\sigma-1}} - \beta \frac{l_t^{1+\theta}}{1+\theta} \right) \exp(-\rho t) \, dt.$$

2.5 Discussion of Intertemporal Problem

Two important points follow from Eq. (7). First, in contrast to the standard model—where $V(C_t) = \frac{{C_t}^{1-\sigma}}{1-\sigma}$ —the indirect utility function for consumption now depends on the variety of goods consumed. In particular, when m_t and n_t vary over time and states of nature, $V(C_t)$ and hence $V'(C_t)$ become time and state-dependent. For this reason, the traditional interpretation of the permanent income hypothesis—that is, smoothing real consumption *expenditure* over time and states—is not necessarily optimal in this setting. Significantly, by setting $n_t = m_t = 1$,

[13]Technically, this is true if $\epsilon < \left(\frac{\phi+\sigma}{\phi+1}\right)^{\frac{1}{1-\sigma}} \frac{C_t m_t^v}{n_t}$. Because ϵ is arbitrary small by assumption, and that, for all reasonable parameter values and trend growth rates, trend growth in n_t is almost surely smaller than that of $C_t m_t^v$, I assume this condition is satisfied.

notice that the resulting utility function represents the same preferences as in the standard one-good model. Thus, the standard model is a special case of a multi-good setup, where the level of variety is constant. Another interpretation is that the standard case assumes no love of variety, and all goods—such as cars and new medical treatments—are perfect substitutes in the eyes of consumers.

Second, the intertemporal elasticity of substitution for real consumption *expenditure* is $\frac{1}{\sigma}$. All else constant, a high σ indicates sharp diminishing marginal utility to consumption expenditure at any given point in time. But because of product group growth, marginal utility does not necessarily fall this fast *over* time. Later on, this feature plays a key role in reconciling low intertemporal elasticity of substitution at any given point in time with apparently high intertemporal substitution over the long run.

2.6 The Effect of Variety on Marginal Utility

From Eq. (7), marginal utility is

$$V'(C_t) = \frac{n_t^{\zeta}}{m_t^{v(\theta-1)} C_t^{\theta}}. \tag{8}$$

Therefore, noting $\theta > 1$, $\frac{\partial^2 V}{\partial m_t \partial C_t} < 0$ and $\frac{\partial^2 V}{\partial n_t \partial C_t} > 0$. Why is marginal utility decreasing in m_t and but increasing in n_t? To see this, recall that the optimal value of c_{jt} is

$$c_{jt} = m_t^v \frac{C_t}{n_t}.$$

Expressed this way, one can view $\frac{C_t}{n_t}$ and m_t as distinct substitutable inputs, all combining to produce consumption services, c_{jt}.[14] Because these inputs enter in Cobb-Douglas form, the intratemporal elasticity of substitution between them is unity. Now setting $\epsilon = 0$ for simplicity, utility from $u(c_{jt})$ becomes

$$u(c_{jt}) = \frac{n_t^{\phi} (m_t^v \frac{C_t}{n_t})^{1-\theta}}{1-\theta}. \tag{9}$$

[14] By contrast, in the standard one-good model, $c_{jt} = C_t$; that is, there is one group and utility derives solely from real consumption expenditure.

For expenditure allocation, the consumer has preferences defined over two margins. The first margin relates to the composition of each group; i.e., allocating inputs—$\frac{C_t}{n_t}$ and m_t^v—to produce consumption services from each group. The second relates to the allocation of the level of consumption services, c_{jt}, across time and groups.[15] Given that $\theta > 1$, the elasticity of substitution between m_t^v and $\frac{C_t}{n_t}$—i.e., 1—exceeds the elasticity of substitution of consumption services across time and groups, $\frac{1}{\theta}$. Therefore, compared to consumption services in each period and group, m_t and $\frac{C_t}{n_t}$ are relatively good substitutes. As a result, consumers are more concerned about attaining similar levels of c_{jt} per group across time than with equating the level of each input within groups.[16]

A rise in the number of groups, n_t, in a period has two effects on marginal utility. First, since consumers smooth expenditure, C_t, over groups, notice from Eq. (9) that a rise in n_t reduces the consumption of each group, $\frac{C_t}{n_t}$. Because consumers strongly desire to smooth the level of consumption services across groups and time, this consumption "widening" raises the marginal utility of consumption for each group in period t. Second, there is a direct effect due to ϕ, capturing the degree of complementarity between groups. Holding expenditure on all groups fixed, increasing the number of groups consumed affects the marginal utility of consuming existing groups.

Because they raise the level of consumption services from a given amount of expenditure on each existing group in a period, increases in m_t and A_t lead to consumption "deepening." And since consumers prioritize the smoothing of c_{jt} over j and time, they quickly become satiated as consumption services rise in a given period. Intuitively, since m_t and $\frac{C_t}{n_t}$ are relatively good substitutes, increases in m_t act as a substitute for consumption expenditure, C_t. In turn, this tends to satiate the consumer.[17] Rather than wanting to consume more, consumers now desire

[15]Implicitly the standard one-good model assumes the consumer only cares about levels of consumption expenditure over time, and is risk-neutral over composition in a given period. But there is no a priori reason to believe this is the case.

[16]Because consumers always spread expenditure equally across all goods in each period, the relative magnitudes of both elasticity of substitution between brands, groups and time has no bearing on the analysis (apart from the issue of whether the elasticity is above or below unity.) Instead, the chief tension is the difference between the elasticity of substitution between inputs of the Cobb Douglas index and the elasticity of substitution of product groups, c_{jt}, over time.

[17]Equivalently, when m rises in a period, this causes a fall in the price of consumption services that period; i.e.,

to shift real consumption resources, C_t, to other periods. As a result, a rise in m_t in a period reduces marginal utility in that period. To summarize, we have the following propositions:

Proposition 1 *: A rise in the number of brands, m_t, in a period reduces the marginal utility of consumption in that period; i.e., $\frac{\partial^2 V}{\partial m_t \partial C_t} < 0$.*

Proposition 2 *: A rise in the number of groups, n_t, in a period raises the marginal utility of consumption in that period; i.e., $\frac{\partial^2 V}{\partial n_t \partial C_t} > 0$.*

2.7 Solution—Long-Run Trends in Labor Supply

Taking logarithms and differentiating the labor optimality condition, $W_t V'(C_t) = \beta l_t^\theta$, with respect to time gives:

$$\theta \frac{\dot{l}_t}{l_t} = \zeta \frac{\dot{n}_t}{n_t} + v(1-\sigma)\frac{\dot{m}_t}{m_t} - \sigma \frac{\dot{C}_t}{C_t} + \frac{\dot{W}_t}{W_t}.$$

As a result of the transversality condition, on a balanced growth path we have $\frac{\dot{C}_t}{C_t} = \frac{\dot{W}_t}{W_t}$, and so

$$\theta \frac{\dot{l}_t}{l_t} = \zeta \frac{\dot{n}_t}{n_t} + v(1-\sigma)\frac{\dot{m}_t}{m_t} - \frac{\dot{W}_t}{W_t}(\sigma - 1). \tag{10}$$

For our purposes, this is an important condition. Whether labor supply rises or falls depends on the interaction of three forces. First, real wage growth lowers labor supply; given $\sigma > 1$, this is due to the dominance of the income effect. Second, growth in the number of brands, m, reduces marginal utility and depresses labor supply. Third, because growth in product groups raises marginal utility, group growth raises labor supply. The effects of variety growth are strengthened if the tastes for variety, ζ and v, are higher.

Total resources devoted to consumption are $C_t = \int_0^{n_t} \int_0^{m_t} c_{jit}\, di\, dj = n_t m_t \overline{c_t}$, where $\overline{c_t}$ is the consumption of each good in period t. The standard one-good model treats these $n_t m_t \overline{c_t}$ units of consumption as perfect substitutes that are effectively transformed into a single good C_t - i.e., $n_t = m_t = 1$ - implying $V(C_t) = \frac{(n_t m_t \overline{c_t} + \epsilon)^{1-\sigma}}{1-\sigma} + \frac{1}{(\sigma-1)\epsilon^{\sigma-1}}$. With a rising wage rate, labor supply trends downwards over time in this one-good world when $\sigma > 1$. Another way to see this is by setting $v = \zeta = 0$ above, in which case labor trends downwards over time.

a fall in the "welfare-based" price index. Because $\theta > 1$, the income effect of this price fall dominates, inducing consumers to smooth the welfare gain and shift consumption services—specifically, real consumption—to other periods. That is, demand for consumption in other periods rises. As a result, when m rises in a given period, the marginal utility of consumption falls in that period.

2.7.1 Stable Labor Hours

To maintain stable labor hours, growth in product groups must grow in tandem with real wage and brand growth. Setting $\dot{l}_t = 0$ in equation (10) yields

$$\frac{\dot{W}_t}{W_t} = \frac{\zeta}{\sigma - 1}\frac{\dot{n}_t}{n_t} - v\frac{\dot{m}_t}{m_t}. \tag{11}$$

If this condition holds, labor hours are stable. This suggests an explanation for the stability of labor hours over time. Associated with rising income is the growth of groups, brands and wages. As such, the variables in equation (11) are manifestations of the same fundamental forces: innovation and specialization. As already noted, group growth counters diminishing marginal utility and attenuates the income effect of wage and brand growth. Provided that the taste for groups, ζ, is sufficiently high, equation (11) ensures stable labor hours. Because consumers likely place more valuation on distinct groups—such as cell phones—than on new brands—such as different colored phones—a relatively high ζ seems reasonable.

Yet there is no *a priori* reason to believe this condition will always hold. When $\frac{\dot{W}_t}{W_t} \geq \frac{\zeta}{\sigma-1}\frac{\dot{n}_t}{n_t} - v\frac{\dot{m}_t}{m_t}$ labor hours fall, and vice versa. For example, relatively high wage growth at the start of the century might explain the associated downward trend in hours displayed in Figure 2. The theory permits a lot of medium run movement, and is reconcilable with what Pissarides and Ngai (2006) describe as a "U-shape for market hours" in the United States over the 20th century, and the "nonstationarity features" in hours noted by Gali (2005).

2.8 Discussion

The model can reasonably account for differences in labor supply across countries. While trade ensures considerable convergence in the number of tradable product groups, n, across countries, productivity and wage rates differ significantly. Consistent with Figure 3, the model predicts higher labor hours in poorer countries. In particular, relatively poor to middle-income countries that are open to trade would exhibit high labor hours; Asia seems a relevant example here. Figure 3 shows little evidence of a systematic relationship between income and labor hours in richer countries. Within the framework of the model, this could be explained in two ways. First, the presence of the nontradable sector affects the product space and increases in importance as a country develops; for instance, there are large differences in the nature of European and American nontradable sectors (see e.g., Freeman and Schettkat (2002)). Second, disparate policies and institutions across countries affect wage rates. For example, unionization and social welfare systems reduce within country disparities in wage rates, thus affecting average aggregate hours worked. I shall return to these issues in Section 4.

Regarding within-country variation in hours, the model predicts that higher wage earners work less. Although this has been the situation for most of economic history—the "working class" being those less well-off—this has not been a feature of the United States economy in the past twenty years or so. Likewise, Blundell and MacCurdy (1999) observe that "it is the higher educated group in the United Kingdom that has tended to work fewer weekly hours on average", but they find an upward trend in labor hours of well-paid workers since the late seventies. Kuhn and Lozano (2005) attribute this recent phenomenon to some relatively new incentive mechanisms increasing labor hours of higher paid workers. Tournaments, signalling, and human capital accumulation all act to increase labor hours of higher income earners, and all are associated with the ascendance of professional services. Such observations could simply reflect the intertemporal substitution of labor over the life-cycle.

Turning to time series evidence, as shown in Figure 2, labor hours in the United States fell markedly at the start of the century, but subsequently stabilized. Vandenbroucke (2005) attributes the fall in hours in the first half of the century to a dominant income effect of wage growth. Discussing post-war trends, Hall (1997) maintains that movements in preferences or tastes—what he calls MRS (marginal rate of substitution) or preference shifts—have been an important feature of the United States economy in recent decades. Analyzing labor hours at medium frequencies, Hall concludes that "the only possible explanation of the large movement in hours per member of the population at medium frequencies is the MRS shift".[18] By affecting one's taste for consumption over leisure, variety growth is a natural candidate for such a taste shift.

Is there any direct evidence for the underlying mechanism of the model? To my knowledge,

[18]Hall downplays the role of other factors - such as taxation - that affect the marginal rate of substitution between consumption and leisure. Similarly, in a detailed analysis of the labor optimality condition in the United States over the twentieth century, Mulligan (2002) claims (referring to the post 1980s period) that these "behavioral changes remain unexplained".

there is only a single study attempting to ascertain the effects of new products on labor supply. Rempel (2006) studies how the work patterns of teenagers change upon the release of new video games. Because teenagers' labor supply is relatively flexible, and video games are an important good for teenagers, this is a useful way to identify the importance of the mechanism stressed here. Consistent with the model, Rempel finds that 16-17 year old males increase their labor supply by ten percent in response to the introduction of a new video game. Also, a number of studies have noted a positive relationship between advertising and labor hours (see e.g., Brack and Cowling (1983), Fraser and Paton (2003)). Another piece of evidence is the low levels of labor input at the very early stages of development together with abundant evidence of a backward bending supply curve. Milton Friedman (1962) maintains that "in a primitive society, the initial low wage rate at which the income effect becomes dominant reflects a lack of familiarity with market goods.... As tastes develop and knowledge spreads, the point at which the income effect dominates begins to rise." Essentially, this is the dynamic modeled in this paper. Finally, Clark (2007) observes how males in tribal communities work fewer hours than males in modern America. To the extent that consumption variety is markedly lower in tribal communities, this observation is consistent with the analysis here.

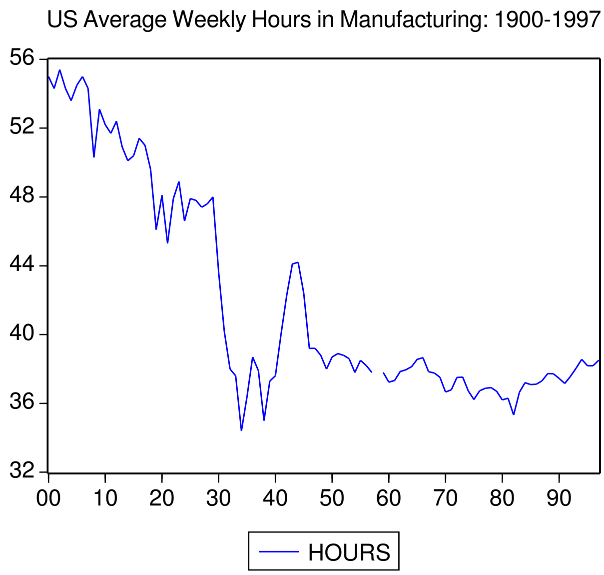

Figure 2: Source: Historical Statistics of the United States.

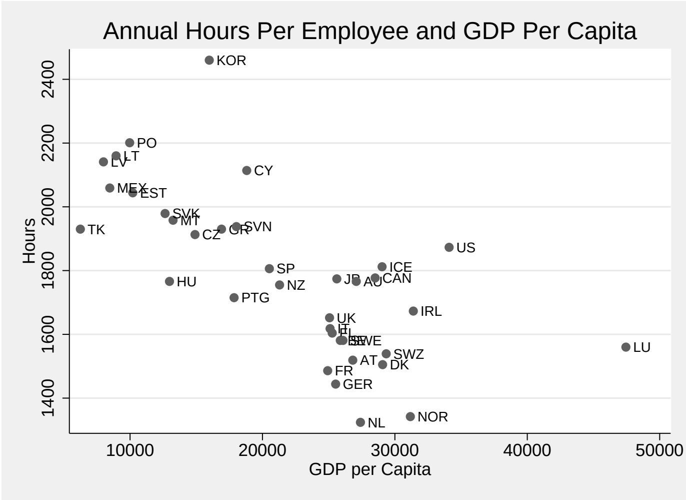

Figure 3: Source: Groningen Total Economy Database. (Data is from 2002).

3 Labor Supply and Long-Run Growth

In this section, I incorporate the previous model into a general equilibrium setting. While the partial equilibrium model showed how new products affect labor supply, the general equilibrium analysis confers insights on the other side of this dynamic: how labor supply impacts the innovation process. In this setting, a greater labor supply effectively increases the size of the market, which raises the incentive to innovate. In the model, new innovations raise not only productivity and the real wage but also the level of product variety. In turn, these forces can sustain a stable labor supply in equilibrium. On the balanced growth path, the level of equilibrium labor supply will, among other things, be a function of the consumption technology: how good is the economy at creating new consumption opportunities?

Turning to the details of the model, there is an expanding range of monopolistically competitive input firms. Consumers purchase inputs and use them to produce consumption goods. There is also a competitive R & D sector, which uses inputs to produce blueprints for new firms. To finance the upfront cost of R & D, entrepreneurs issue debt to households. Input firms take demand curves as given and hire labor in a competitive labor market. They set price as a markup over unit cost; this wedge then permits profits, which firms secure by infinitely lived patents. Because households finance the up-front cost of R & D, they have a claim on these profits. Ultimately firms use all profits to pay dividends, so there are no supernormal profits in equilibrium.

More generally, new firms represent increases in the stock of accumulated knowledge. In the spirit of New Growth models, some knowledge is nonrivalrous, so innovators do not capture all the gains from their investment. As a result of knowledge spillovers, labor productivity—and the real wage—increase along with with the number of input firms. To simplify the analysis, there is no physical capital; wealth is knowledge, embodied in the firms, and the "capital" stock is the sum of firm values.

The household decides between consumption and leisure each period and supplies labor to all the input firms. The household's income comprises wage income, W_t, and dividends (i.e., interest income) from input firms. Each period, the consumer purchases new inputs from all existing firms, and, via a home production technology, transforms the inputs into consumer goods. Because of knowledge spillovers, the home production technology permits the growth of new commodities all the time, coincident with the number of input firms.

On the balanced growth path, the number of input firms, and hence the real wage, increases along with the number of consumer products. For certain parameter restrictions, the interaction of increasing variety and the real wage sustains stable hours. The deep parameters of the model, in particular the sophistication of the home production technology, determine the actual steady state level of labor supply. A more efficient home technology, for example, raises the marginal utility of any given real wage, motivating the consumer to supply more labor.

Before proceeding, some points are worth noting. The price of input i at time t is given by p_{it}. The productivity of each worker at time t is denoted by A_t, as is common in the New Growth literature. Input firms are subscripted by i and consumer products by j. In contrast to the previous model, inputs now replace what were previously called brands. In addition, I assume the number of inputs has no effect on utility and that there is no externality from new groups affecting the utility derived from existing ones; i.e., $\zeta = 0$. These assumptions are made for simplicity, and the insights of the model are robust to these changes. Finally, the model is derived for a balanced growth path and, as such, applies in the long run. The full solution is in the Appendix.

3.1 The Economic Environment

An imperfectly competitive equilibrium in this economy is an allocation:

$\{b_t, Y_t, I_t, \{l^d_{it}\}, l^s_t, A_t, n_t, \{\pi_{it}\}_{i=0}^{A_t}, \{x_{jit}\}, \{x^R_{it}\}_{i=0}^{A_t}\}_{t=0}^{t=\infty}$

and a price system: $\{W_t, r_t, p_{At}, \{p_{it}\}_{i=0}^{A_t}\}_{t=0}^{t=\infty}$ where:

The Consumer: There is an infinitely lived representative consumer who takes the path of $\{W_t, r_t, \{p_{it}\}, A_t\}_{t=0}^{\infty}$, and the production technology for new consumption products as given. The variable $A_t \in [0, \infty)$ denotes the measure of input firms that exist at time t. The consumer then solves:

$$\max_{(0 \le l_t \le 1, \{x_{jit} \ge 0\})} \int_0^\infty \left(\int_0^{n_t} (u(c_{jt}) - u(0))\, dj - \beta \frac{l_t^{1+\theta}}{1+\theta} \right) \exp(-\rho t)\, dt,$$

where:

$$u(c_{jt}) = \frac{(c_{jt} + \epsilon)^{1-\sigma}}{1-\sigma}, \quad \epsilon > 0, \quad c_{jt} \gg \epsilon; \qquad c_{jt} = A_t^{1-\frac{1}{\alpha}} \left(\int_0^{A_t} x_{jit}^\alpha \, di \right)^{\frac{1}{\alpha}},$$

where $0 < \alpha < 1$, and $\sigma > 1$. Here, c_{jt} is a composite commodity, and the consumer buys inputs, x_{jit}, from firms $i \in [0, A_t]$ to produce it.[19]

The budget constraint is

$$\dot{b}_t \le r_t b_t + W_t l^s_t - \int_0^{A_t} p_{it} x^C_{it}\, di, \tag{12}$$

where $x^C_{it} = \int_0^{n_t} x_{jit}\, dj$ denotes the total quantity of inputs purchased by the representative consumer from firm $i \in [0, A_t]$. The boundary conditions—the no-Ponzi game and initial condition—are:

$$\lim_{t \to \infty} \frac{b_t}{\Pi_{j=0}^t (1 + r_j)} \ge 0, \qquad b_0 \quad \text{given.} \tag{13}$$

The number of consumption goods the household can produce at time t, n_t, is given by the household consumption technology:

[19]To ensure a bounded integral, I impose $\rho > g_n$, where g_n denotes the steady state growth rate of n.

$$n_t = \phi A_t^{\gamma}, \qquad \phi, \gamma > 0.$$

Therefore, A_t is the level of nonrival knowledge available to consumers to produce consumer products. The parameters, ϕ and γ, govern how easy it is to exploit available knowledge, A_t, to produce new consumption goods. They mediate the efficiency with which the household can create new consumption goods.

The solution to the household problem gives the time path: $\{\{x_{jit}\}, n_t, l_t^s, b_t\}_{t=0}^{t=\infty}$.

Research and Development: The R&D sector is perfectly competitive. There is free entry into blueprint production, and each new entrepreneur can borrow funds on the capital market so as to manufacture blueprints. The entrepreneur takes $\{r_t, A_t, p_{At}, \{p_{it}\}\}_{t=0}^{t=\infty}$ as given and chooses $\{x_{it}^R\}_{t=0}^{t=\infty}$ to create blueprints. The constant returns to scale technology for blueprint manufacture is:

$$\dot{A}_t = \frac{A_t^{1-\frac{1}{\alpha}}}{\alpha\eta} \left(\int_0^{A_t} (x_{it}^R)^{\alpha} \, di \right)^{\frac{1}{\alpha}}.$$

The technology conveys the notion that, in equilibrium, the use of $\frac{\alpha\eta}{A_t}$ units of each input produces a single blueprint.[20] Thus, the manufacture of each blueprint uses $\alpha\eta$ resources, and the cost of producing a blueprint is $\frac{\alpha\eta}{A_t} \int_0^{A_t} p_{it} \, di$; hence, one can interpret a high η as a high startup cost. For blueprint production, an increase in A makes it both easier and harder to come up with new innovations; these are the respective "standing on shoulders" (i.e., greater stock of public knowledge to avail of) and "stepping on toes" (i.e., many discoveries have already been made) effects. Here, both effects just offset.

Entrepreneurs borrow I_t on the capital market at net interest rate, r_t, and solve:

$$\max_{\{x_{it}^R\}} \; p_{At}\dot{A}_t \qquad \text{subject to} \qquad \int_0^{A_t} p_{it} x_{it}^R = I_t.$$

This gives $\{x_{it}^R\}_{t=0}^{t=\infty}$.

[20] This will prove to be a convenient and innocuous normalization.

Monopolistically Competitive Input Firms: Input firms are monopolistically competitive and take $\{W_t, A_t, p_{At}, \{p_{it}\}\}_{t=0}^{\infty}$ as given. Firms sell inputs to consumers and the R & D sector and purchase patents from entrepreneurs. The only factor of production is labor, and the labor market is competitive. The constant returns to scale production function for the representative firm $i \in [0, A_t]$ is $y_{it} = A_t l_{it}$. As is standard in New Growth theory models, worker efficiency is given by A_t. Hence, it takes $\frac{1}{A_t}$ units of labor to produce a unit of output, and so the marginal cost of production is $\frac{W_t}{A_t}$, where W_t is the nominal wage.

Each firm $i \in [0, A_t]$ ignores its impact on the average price level and chooses $\{p_{it}\}_{t=0}^{\infty}$ to solve:

$$\max_{(p_{it})} \pi_{it} = x_{it}(p_{it})\Big(p_{it} - \frac{W_t}{A_t}\Big),$$

where the total demand x_{it} is given from the consumer and R & D problem, that is, $x_{it}(p_{it}) = x_{it}^C + x_{it}^R$. Derived demand for labor is then: $l_{it}^d = \frac{x_{it}}{A_t}$. This solution gives $\{p_{it}, l_{it}^d, \pi_{it}\}$ for all t and $i \in [0, A_t]$.

Market Clearing:

- The labor market clears: $\int_0^{A_t} l_{it}^d \, di = l_t^s$, where $l_{it}^d = \frac{x_{it}}{A_t}$; this gives W_t.
- The asset market clears: the interest rate on debt, r_t, maintains capital market equilibrium, $b_t = p_{At} A_t$, where p_{At} denotes the price of a patent at time t.
- By arbitrage, assets have equal returns:

$$r_t = \frac{\pi_t}{p_{At}} + \frac{\dot{p}_{At}}{p_{At}}.$$

  The interest rate on the capital market equals the rate of return from investing in a firm, which comprises the per-period rate of return, $\frac{\pi_t}{p_{At}}$, and capital appreciation, $\frac{\dot{p}_{At}}{p_{At}}$. This asset pricing equation gives p_{At}.
- The resource constraint in nominal terms is: $Y_t = C_t + I_t$, where $Y_t = \int_0^{A_t} p_{it} y_{it} \, di$, and $C_t = \int_0^{A_t} p_{it} x_{it}^C \, di$, where $x_{it}^C = \int_0^{n_t} x_{jit} \, dj$. This gives I_t.

Technology: $y_{it} = A_t l_{it}^d$.

3.2 Solving for the Equilibrium Allocation

3.2.1 Resource Constraint

The resource constraint is:

$$Y_t = W_t l_t + r_t b_t = C_t + \dot{A}_t p_{At},$$

that is, income derives from labor and capital and is spent on consumption and investment. In equilibrium, the patent price, labor supply, interest rate, and profits are all constant (this will be confirmed later). Thus, dividing across by A_t indicates that Y_t, C_t, W_t, and A_t all grow at the same rate on a balanced growth path.

3.2.2 Consumer Problem

Now, combining the standard Euler equation with the static labor/leisure condition and imposing constant labor supply gives:

$$\frac{\dot{C}_t}{C_t} = \frac{\dot{W}_t}{W_t} = \frac{\dot{A}_t}{A_t} = r_t - \rho.$$

Differentiate the static labor optimality condition, $W_t V'(C_t) = V'(l_t) \Rightarrow \frac{W_t p_t^{1-\sigma} n_t^{\sigma}}{C_t^{\sigma}} = \beta l_t^{\theta}$, with respect to time.[21] Setting $\frac{\dot{C}_t}{C_t} = \frac{\dot{W}_t}{W_t}$ and labor hours constant gives:

$$\frac{\dot{n}_t}{n_t} = \frac{\sigma - 1}{\sigma} \frac{\dot{A}_t}{A_t}. \tag{14}$$

3.2.3 Firms

From the consumer and R & D problems, total demand for input $i \in [0, A_t]$ is $x_{it}^C + x_{it}^R = \frac{p_{it}^{\frac{-1}{1-\alpha}}}{\left(\int_0^{A_t} p_{it}^{\frac{-\alpha}{1-\alpha}} di\right)}(C_t + I_t)$. Each firm takes $\int_0^{A_t} p_{it}^{\frac{-\alpha}{1-\alpha}} di$ and aggregate demand, $C_t + I_t$, as given, so faces demand of elasticity $\frac{1}{1-\alpha}$. Then, given constant elasticity of demand of $\frac{1}{1-\alpha}$ and setting $\frac{W_t}{A_t}$ as numeraire, the firm's price is a fixed markup over marginal cost:

[21] Prices set by all firms are equal at time t, and p_t denotes this common price.

$$p_{it} = \frac{1}{\alpha}\frac{W_t}{A_t} = \frac{1}{\alpha} \equiv p,$$

which is constant and the same for all firms. Thus, by symmetry, each firm $i \in [0, A_t]$ will produce the same quantity, x, and will demand the same amount of labor. This also gives the firm's labor/hours demand and *real wage*, $\frac{W_t}{p} = \alpha A_t$. Substituting the constant price into the representative demand curve, gives the equilibrium quantity produced by each firm:

$$x_i = x = \frac{C_t + I_t}{pA_t} = \frac{\alpha(C_t + I_t)}{A_t}, \qquad \forall i \in [0, A_t].$$

From the resource constraint, $Y_t = C_t + I_t$, Y_t and A_t grow at the same rate on the balanced growth path. Hence, quantities produced are also constant, and expansion of input firms means an increase in the number of firms, not more existing firms.

3.2.4 Profits of Input Firms

Because $\frac{1}{\alpha} - 1$ is the profit per unit of output, on the balanced growth path profits are:

$$\pi_t = (\frac{1}{\alpha} - 1)x = (1 - \alpha)\frac{C_t + I_t}{A_t}.$$

3.3 Capital Market Equilibrium

Substituting $p = \frac{1}{\alpha}$ into the equation for the price of a patent implies that the production cost of a patent is $\frac{\alpha\eta}{A_t}\int_0^{A_t} p_{it}\, di = \eta$. Because of free entry, the present discounted value of all future profits is equal to the cost of a patent:

$$\eta = p_{At} = \int_t^\infty (1 - \alpha)\frac{C_t + I_t}{A_t} \exp(-\overline{r}(t, s)(s - t))\, ds,$$

where $\overline{r}(t, s) = \frac{1}{s-t}\int_t^s r(\kappa)\, d\kappa$. Note that the patent price is constant, so $\dot{p}_{At} = 0 \Rightarrow \eta = \frac{\pi}{\overline{r}_t} \Rightarrow \overline{r}_t p_{At} = \pi$; i.e., the interest payments are just equal to profits. This also indicates that the interest rate, r_t, is constant, so we can put $r_t \equiv r$. In equilibrium, therefore:

$$\eta = \frac{\pi}{r} = (1 - \alpha)\frac{C_t + I_t}{rA_t} \qquad \Rightarrow r = (1 - \alpha)\frac{C_t + I_t}{A_t \eta}. \tag{15}$$

Now, in capital market equilibrium, the total value of assets is: $b_t = p_{At}A_t = \eta A_t \Rightarrow \dot{b}_t \equiv I_t = \eta \dot{A}_t$. From the household budget constraint, $\dot{b}_t = W_t l_t + r b_t - C_t$:

$$C_t + \dot{b}_t = W_t l_t + r b_t = W_t l_t + r\eta A_t.$$

The demand facing each firm on the balanced growth path is then:

$$\frac{C_t + I_t}{A_t} = \frac{C_t + \dot{b}_t}{A_t} = \frac{W_t l_t + r\eta A_t}{A_t} = l + r\eta. \tag{16}$$

There is a scale effect here. The greater are equilibrium hours, the greater the demand and the higher the level of production per firm. Combining equations (15) and (16) yields:

$$r = \frac{(1-\alpha)}{\alpha}\frac{l}{\eta}.$$

Observe that we also have a multiplier effect: namely, a high rate of return, via the budget constraint, raises demand and production, which raises profits and hence the rate of return itself.

The Keynes-Ramsey rule now reduces to:

$$\frac{\dot{C}_t}{C_t} = r - \rho = \frac{(1-\alpha)}{\alpha}\frac{l}{\eta} - \rho. \tag{17}$$

3.4 Labor Market Equilibrium

In solving for equilibrium labor supply, l^*, to simplify things, I restrict the analysis to linear disutility, setting $\theta = 0$. Substituting the equation for the real wage into the consumer's labor/leisure condition gives:

$$l^* = \frac{n}{\beta^{\frac{1}{\sigma}}\alpha^{1-\frac{1}{\sigma}}A^{\frac{\sigma-1}{\sigma}}} - \eta\rho.$$

As already noted, to ensure stable labor hours we must have: $\frac{\dot{n}}{n} = \frac{\sigma-1}{\sigma}\frac{\dot{A}}{A}$. Then, recalling the form of the household technology, $n_t = \phi A_t^{\gamma}$, on the balanced growth path, the parameter restriction, $\gamma = \frac{\sigma-1}{\sigma}$, must be satisfied. Then steady state labor supply reduces to:

$$l^* = \frac{\phi}{\beta^{\frac{1}{\sigma}}\alpha^{1-\frac{1}{\sigma}}} - \eta\rho \qquad \in [0,1].$$

Altogether, we have:

$$\frac{\dot{C}_t}{C_t} = \frac{\dot{A}_t}{A_t} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{W}_t}{W_t} = \frac{\sigma}{\sigma - 1}\frac{\dot{n}_t}{n_t} = \frac{\phi(1-\alpha)}{\beta^{\frac{1}{\sigma}}\eta\alpha^{2-\frac{1}{\sigma}}} - \frac{\rho}{\alpha} \equiv g.$$

At any point, t, we have GDP per capita, $Y_t = A_t l^*$, where $A_t = A_0 \exp(gt)$, and $A_0 = \eta b_0$.

3.4.1 Discussion of Model Implications

The model confers a number of insights. First, economic growth is an increasing function of labor hours per person. The fruits of labor supply create a market for new innovations, which ultimately drives growth. The fact labor supply has a growth effect is a strong result, but given knowledge diffuses across borders, this model applies properly to the world. Over long spans of history, this is a reasonable prediction: *market* oriented labor activity is in fact a relatively recent phenomenon and is associated with sustained growth. I shall return later to the Industrial Revolution, where this model seems especially relevant.

Labor supply is constant in the long run. The stability of labor supply is due to two countervailing effects of new innovations. First, new innovations raise the real wage by inducing labor augmenting technical change; this lowers labor supply. Second, innovations raise new consumption opportunities; this raises labor supply. In the long run, these effects counter each other and sustain a constant labor supply. The model also incorporates structural change since, as productivity rises, people devote labor hours to new sectors. This roughly corresponds to the kind of structural change that occurs in economies as they develop. Although labor hours are fixed, efficiency units of labor are always rising, along with labor augmenting technical change.

The level of labor supply is a function of the deep parameters of the model. In particular, labor supply is increasing in ϕ: a higher ϕ translates into greater consumption variety for any given level of technology. In turn, this raises the marginal utility of income and therefore labor supply. Higher labor hours yield a growth effect, leading to a virtuous cycle with more wage and variety growth, and so on. While the exogeneity of ϕ seems arbitrary, it is easy to conceive of ways to increase ϕ; for example, opening up to trade; urbanization; a shift away from agriculture; and a shift to free markets, among others. Moreover, the fact that ϕ can increase quickly gives the framework the semblance of a “big push” model, common in models of development. Since there has been no shortage of such "growth miracles" in economic history, this is a desirable feature.

In addition, ϕ itself is surely a function of average labor supply too (i.e., $\phi \equiv \phi(l)$), and this feature could easily be introduced. That is, at any point, the more people engaged in labor, the greater the variety of goods available, which raises the incentive for others to work. Such a feature would lend the model a strong multiplier effect, effectively making workers strategic complements.

What sustains growth in the model is *market* oriented labor supply. Home production, common in Europe and developing countries, does not permit a larger *market* for goods. In particular, shifting labor from home to market activity yields two effects. First, it leads to a greater *demand* for goods, since people are no longer self-sufficient. Second, it induces greater productivity, due to specialization and the division of labor. In this setting, labor market activity does not necessarily translate into less leisure—it could just signify a substitution of market for home production. For centuries of historical experience, most people were self-sufficient, so the mechanics of the model were inoperative. The introduction of formal market-oriented labor markets was associated with sustained growth. Therefore the model can reasonably explain long-run historical trends in market-oriented labor supply, which rose significantly upon the introduction of a market economy. I return to this issue in Section 4.

There are positive externalities to supplying labor in the model. By supplying labor hours, workers raise prospective profits for innovators, thereby creating research incentives for new innovations—which then raises productivity, wages, and consumption variety. This improves welfare for others, who are not party to the labor contract. Because the worker does not internalize this externality, the social planner would seek to increase labor hours. Additionally, given knowledge has public good properties, the social return to investment is higher than the private return.

Since all goods have a relative price of one, there is no allocative inefficiency in goods production. Nonetheless, there is a distortion in the labor market, since workers are paid less than their marginal product; more precisely, $\alpha A < A$ (i.e., there is a wedge between the marginal rate of substitution and the real wage.) Yet this distortion actually enhances growth. Since $\sigma > 1$ and the income effect dominates, a lower real wage rate increases steady state labor supply.[22]

In contrast with standard models of growth, the model emphasizes the role of consumer demand. Fundamentally, it is consumption demand that drives growth in the model, which then induces technological improvements on the supply side. Equilibrium labor supply, and hence the rate of growth, is a function of the extent to which the economy produces new consumption products, ϕ.

A higher start up cost for firms, η, reduces labor supply. This is a standard pure income effect. Higher start up costs raise the equilibrium value of firms, thereby increasing non labor income and depressing labor supply.

Although there is no physical capital in the model, to capture conditional convergence - certainly, an empirical regularity - capital accumulation could easily be introduced.[23] Indeed, the dynamics presented here would induce faster capital accumulation, since higher labor supply (broadly construed) raises the marginal product of capital. Currently, what comprises "capital" are new firms and innovations. For developed nations, which rely on entrepreneurship and innovation for sustained growth, this de-emphasis on physical capital seems reasonable.

Finally, because of the love of variety, welfare increases in n. This captures the notion that, despite the smaller real income, a poor person today most likely has a higher level of welfare than a richer person a hundred years ago. By contrast, the standard Ramsey and New Growth models do not capture this effect. In these settings, there is a single consumption good, meaning real wealth is the sole determinant of welfare.

4 The Industrious Revolution

For most of historical experience, real GDP per capita and real wages stagnated. There was little formal labor market activity at all, and feudalism, guilds, and other very informal labor contracts were mostly what constituted pre-industrial labor markets. Production in these

[22]This result, however, would not hold on the extensive margin since, here, there is only a pure negative substitution effect, which would reduce labor supply.

[23]Currently, as in an AK model, there are no transitional dynamics. As in Jones and Manuelli (1990), we could capture conditional convergence with a function like $Y = f(k) + Al$ where k denotes capital stock and $f'(k) > 0$, $f''(k) < 0$.

agricultural economies was largely directed at home production and not at market activity. Moreover, any commercial activity was highly regulated, and merchants were continually exposed to the threat of expropriation. Because this frustrated the development of a commercial society, there was little variety in the goods available to purchase in the pre-industrial economy. Clark (2007), for example, reports how "for most of human history.....the bulk of material consumption has been food, shelter, and clothing."

In this pre-industrial economy, the labor market was well-described by the backward bending supply curve. Max Weber (1930), for instance, describes the preindustrial labor market: "For centuries it was an article of faith that low wages were productive i.e., they increased the material results of labor....the people only work because and so long as they are poor". That is, workers were "target incomers", who reduced labor supply upon a wage rise. In addition, rest days and religious holidays abounded in the pre-industrial economy. And, as for early forager societies, Clark (2007) refers to what one anthropologist, Marshall Sahlins, calls the "primitive affluence" - abundance of leisure - of such societies. This coincidence of low labor input and little, if any, product variety growth is consistent with the model outlined earlier.

Clearly the preindustrial world was not fertile ground for an Industrial Revolution. Work by De Vries (1994) confirms dynamics in eighteenth century England consistent with the key mechanisms of the model. In the century prior to the Industrial Revolution, there was what De Vries calls an "Industrious Revolution." After the Glorious Revolution in 1688, merchants were granted more freedom to engage in commerce. This was a result of greater property rights and economic deregulation. Shops opened, imports increased, and because urbanization was on the rise, consumers and producers interacted more. As a result, the variety of goods available to consume increased greatly.

De Vries points out how many literary works dating from this period convey impressions of a resurgence in commercial activity and a new fixation on material goods. Other historians offer similar evidence. McCracken (1991), for example, documents "an explosion of consumer choices" in this period and describes how "the world of goods expanded dramatically to include new opportunities for the purchase of furniture, pottery, silver, mirrors, cutlery, gardens, pets, and fabric." Similarly, McKendrick (1982) describes a "consumer revolution," "rampant consumer behavior," "an orgy of spending," and emphasizes a new concern with fashion, and social emulation. McKendrick claims that "necessities underwent a dramatic metamorphosis in style, variety and availability".

According to De Vries, the desire to purchase new products induced people to move out of household production and towards the production of market-oriented goods to sell. DeVries argues that the new desire to consume promoted work effort and "unleash[ed] a beneficial industriousness" that preceded and laid the ground for the Industrial Revolution. Consistent with the model in Section 3, this increased the supply and demand for goods. In this vein, he quotes Steuart (1767): "Men are forced to labor now because they are slaves to their own wants." By the end of the eighteenth century, labor hours in Britain had risen markedly. Clark (2007) observes:

> *"Despite popular images of the Industrial Revolution herding formally happy peasants into a life of unrelenting labor in gloomy factories, this transition seems to have occurred before the Industrial Revolution, rather than as a result of it...On the eve of the Industrial Revolution the typical male worked 10 or more hours a day for 300 or more days a year for a total annual labor input in excess of 3000 hours."*

This environment, then, was fertile ground for the Industrial Revolution, which could never have taken place in the pre-industrial world of target incomers. Moreover, for decades before the Industrial Revolution—when work effort increased—real wages did not rise. According to estimates by Lindert and Williamson (1985), wages barely changed for workers from 1755-1819 and indeed fell for full time blue collar workers. Even so, according to estimates by Voth (2003), work hours per year rose by almost 500 in the period 1750 to 1800—the very eve of the Industrial Revolution. Such observations are consistent with a shift outwards in the labor supply curve. But once wages experienced sustained growth around 1830, labor hours started to subside, and women and children began to withdraw from the workforce. In terms of the model, product variety increased markedly initially, thereby stimulating labor supply, which subsequently fell as wages rose.

Finally, Lucas (2001) maintains that a prevailing challenge for the theory of economic development is to reconcile the Malthusian dynamics with those of the neoclassical growth model. The model presented here points in this direction. In centuries prior to the Industrial Revolution, increases in income induced higher fertility. One likely reason is that there was simply little else to consume, and therefore children comprised one of the few consumption goods. But once there is sustained growth in the level of product variety, fertility falls. An interesting extension of the model, then, would be to apply this analysis to the Malthusian trap. Within the framework of the model, a demographic transition seems likely.

5 Conclusion

This paper has presented a model that endogenizes labor supply as a function of product variety. It explains long-run trends in labor supply trends by the interaction of two main forces: variety growth, which raises labor supply, and wage growth, which reduces it. In contrast with the standard one-good setup, the model here is consistent with stable long-run hours, together with trends in hours as an economy develops. Incorporating the idea into a general equilibrium model of long-run growth yields further insights. By raising labor supply, variety growth increases the size of the market and stimulates innovation. On the balanced growth path, labor hours are stable, and the economy exhibits sustained growth.

The framework has a number of applications, both methodological and explanatory. The paper presents a time-separable utility function consistent with stable labor hours in equilibrium. More generally, the framework introduces variety growth into a model of intertemporal choice, and so has broad implications for dynamic macroeconomic models. Especially in the context of long-run growth, this confers useful insights on how living standards rise over time. An interesting question is whether the model is consistent with trends in the capital market. According to the model trend group growth would raise expected future marginal utility, thereby causing a rise in savings. This might explain the fall in interest rates upon the start of sustained growth in economic levels. In contrast, the stardard one-good model predicts a rise in interest rates.

Lack of empirical work testing the underlying mechanisms of the model is a shortcoming of the analysis. Yet casual observation suggests people care a lot about what they buy. Moreover, the idea of "love of variety" has successfully explained economic phenomena in such fields as trade theory and industrial organization. Hence, although the underlying mechanism seems reasonable, more rigorous empirical testing of the driving mechanism is needed.

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Appendix

Partial Equilibrium

The current valued Hamiltonian is:

$$H = \frac{n^{\zeta} m_t^{v(1-\sigma)} C_t^{1-\sigma}}{1-\sigma} + \frac{n_t^{\phi+1}}{(\sigma-1)\epsilon^{\sigma-1}} - \beta \frac{l_t^{1+\theta}}{1+\theta} + \lambda_t (r_t b_t + W_t l_t - C_t)$$

where λ_t is the costate variable. Then, invoking the Pontryagin Maximum Principle gives the necessary first order conditions:

$$\frac{\partial H}{\partial C_t} = 0 \Rightarrow \frac{n_t^{\zeta} m_t^{v(1-\sigma)}}{C_t^{\sigma}} = \lambda_t$$

$$\frac{\partial H}{\partial l_t} = 0 \Rightarrow \beta l_t^{\theta} = \lambda W_t$$

and the costate equation:

$$\frac{\dot{\lambda}_t}{\lambda_t} = \rho - r_t$$

To ensure a bounded integral, I assume $(\phi + 1)\frac{\dot{n}}{n} < \rho$. The transversality condition (TVC) reduces to $lim_{t\to\infty} \lambda_t b_t \exp(-\rho t) = 0 = \lambda_0 b_0 \exp(\rho - r + g - \rho)$ or $r > g$. And given $g = r - \rho$, the TVC reduces to $\rho > 0$, which is trivially satisfied. Given the concavity of the utility and the linearity of constraints, Mangasarian's theorem ensures the foregoing conditions are sufficient for optimality.

Taylor Expansion

Taking a Taylor expansion of the utility function gives:

$$\frac{(\frac{m_t^v C}{n_t} + \epsilon)^{1-\sigma}}{1-\sigma} = \frac{(\frac{m_t^v C_t}{n_t})^{1-\sigma}}{1-\sigma} + \frac{\epsilon}{(\frac{m_t^v C}{n_t})^{\sigma}} - \frac{\sigma \epsilon^2}{(\frac{m_t^v C}{n})^{\sigma+1}} + O(3)$$

Hence given $\epsilon \approx 0$, $\frac{(\frac{m_t^v C}{n_t} + \epsilon)^{1-\sigma}}{1-\sigma} = \frac{(\frac{m_t^v C_t}{n_t})^{1-\sigma}}{1-\sigma}$.

General Equilibrium Derivations

5.0.2 Resource Constraint

The resource constraint is:

$$Y_t = W_t l_t + r_t b_t = C_t + p_{At}\dot{A}_t,$$

that is, income derives from labor and capital and is spent on consumption and investment. In equilibrium, the patent price, labor supply, interest rate, and profits are all constant (this will be confirmed later). Thus, dividing across by A_t indicates that Y_t, C_t, W_t, and A_t all grow at the same rate on a balanced growth path.

5.1 The Consumer Problem

Let C_t denote the consumer's nominal consumption expenditure in period t. By symmetry and concavity, consumers spend an equal amount, $\frac{C_t}{n_t}$, of each existing good $j \in [0, n_t]$. To maximize the production of each composite commodity, c_{jt}, the consumer then solves:

$$\max_{\{x_{jit}\}} \quad A_t^{1-\frac{1}{\alpha}} \left(\int_0^{A_t} x_{jit}^{\alpha}\, di \right)^{\frac{1}{\alpha}} + \lambda_t \Big(\frac{C_t}{n_t} - \int_0^{A_t} p_{it} x_{jit}\, di \Big),$$

where x_{jit} is the input i demand for product j in period t, and λ_t is the Lagrangian multiplier on the budget constraint. Total demand for the input from firm i is $x_{it}^C = \int_0^{n_t} x_{jit}\, dj$.

Given the Dixit-Stiglitz form of the production function, x_{jit} is given by:

$$x_{jit} = \frac{p_{it}^{\frac{-1}{1-\alpha}}}{\left(\int_0^{A_t} p_{it}^{\frac{-\alpha}{1-\alpha}}\, di \right)} \frac{C_t}{n_t},$$

that is, consumer demand for each input is dependent on the inputs's relative price. By symmetry, total consumer demand for input i, x_{it}^C, is:

$$x_{it}^C = \int_0^{n_t} x_{jit}\, dj = \frac{p_{it}^{\frac{-1}{1-\alpha}}}{\left(\int_0^{A_t} p_{it}^{\frac{-\alpha}{1-\alpha}}\, di \right)} C_t.$$

As we shall see, in a symmetric equilibrium, prices will be constant and equal. By concavity, the consumer will consume all n_t products in equal amounts, so $c_{jt} = \frac{C_t}{pn_t}$. As in partial equilibrium, consumers then solve (noting the approximation given $c_{jt} \gg \epsilon$):

$$\max_{(C_t>0,\, 0\leq l_t\leq 1)} \int_0^\infty \left(\frac{p^{\sigma-1} n_t^\sigma C_t^{1-\sigma}}{1-\sigma} + \frac{n_t}{(\sigma-1)\epsilon^{\sigma-1}} - \beta \frac{l_t^{1+\theta}}{1+\theta} \right) \exp(-\rho t)\, dt,$$

subject to the constraints.

5.1.1 Intertemporal Problem

Instantaneous utility is:

$$V(C_t, l_t, n_t) = \frac{p^{\sigma-1} n_t^\sigma C_t^{1-\sigma}}{1-\sigma} + \frac{n_t}{(\sigma-1)\epsilon^{\sigma-1}} - \beta \frac{l_t^{1+\theta}}{1+\theta}$$

Consumers then solve:

$$\max_{(C_t, l_t)} \int_0^\infty \left(\frac{p^{\sigma-1} n_t^\sigma C_t^{1-\sigma}}{1-\sigma} + \frac{n_t}{(\sigma-1)\epsilon^{\sigma-1}} - \beta \frac{l_t^{1+\theta}}{1+\theta} \right) \exp(-\rho t)\, dt$$

subject to:

$$\dot{b}_t = rb_t + W_t l_t - C_t$$

$$0 \leq l_t \leq 1$$

$$b_0 > 0 \qquad \textit{given.}$$

The transversality condition is:

$$lim_{t\to\infty} \lambda_t b_t \exp(-\rho t) = 0$$

The current valued Hamiltonian and solution is:

$$H = \frac{p^{\sigma-1} n_t^\sigma C_t^{1-\sigma}}{1-\sigma} + \frac{n_t}{(\sigma-1)\epsilon^{\sigma-1}} - \beta \frac{l_t^{1+\theta}}{1+\theta} + \lambda_t (r_t b_t + W_t l_t - C_t)$$

with necessary first order conditions:

$$\frac{\partial H}{\partial C} = 0 \Rightarrow \frac{p_t^{1-\sigma} n_t^{\sigma}}{C_t^{\sigma}} = \lambda_t$$

$$\frac{\partial H}{\partial l} = 0 \Rightarrow \beta l_t^{\theta} = \lambda_t W_t$$

and costate equation:

$$\frac{\dot{\lambda}_t}{\lambda_t} = \rho - r_t$$

Combining and imposing constant labor supply implies: $\frac{\dot{C}_t}{C_t} = \frac{\dot{A}_t}{A_t} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{W}_t}{W_t}$.

Now imposing constant labor supply on the balanced growth path and invoking the resource constraint - as we will see, profits will be constant in equilibrium -, the Keynes-Ramsey rule reduces to:

$$\frac{\dot{C}_t}{C_t} = \frac{\dot{W}_t}{W_t} = \frac{\dot{A}_t}{A_t} = r_t - \rho.$$

Differentiating the static labor optimality condition, $WV'(C_t) = V'(l_t)$, with respect to time and setting $\frac{\dot{C}_t}{C_t} = \frac{\dot{W}_t}{W_t}$ gives:

$$\frac{\dot{n}_t}{n_t} = \frac{\sigma - 1}{\sigma} \frac{\dot{A}_t}{A_t}.$$

5.2 The R & D Problem

The representative entrepreneur borrows funds, I_t, on the capital market and chooses x_{it}^R each period to maximize the number of blueprints produced:

$$\max_{(x_{it}^R)_{i=0}^{A_t}} \frac{A_t^{1-\frac{1}{\alpha}}}{\alpha\eta} \Big(\int_0^{A_t} x_{it}^{R\alpha} \, di \Big)^{\frac{1}{\alpha}} + \lambda_t \Big(I_t - \int_0^{A_t} p_{it} x_{it} \, di \Big).$$

From the production technology, $\frac{\alpha\eta}{A_t}$ units of each input combine to produce one blueprint. Denoting the cost of a blueprint by p_{At}, we have $\dot{A}_t = \frac{I_t}{p_{At}}$.

Now, from the consumer and R & D problems, total demand for input $i \in [0, A_t]$ is $x_{it}^C + x_{it}^R = \frac{p_{it}^{\frac{-1}{1-\alpha}}}{\left(\int_0^{A_t} p_{it}^{\frac{-\alpha}{1-\alpha}} \, di \right)} (C_t + I_t)$. Each firm takes $\int_0^{A_t} p_{it}^{\frac{-\alpha}{1-\alpha}} \, di$ and aggregate demand, $C_t + I_t$, as given, so faces demand of elasticity $\frac{1}{1-\alpha}$.

5.3 Input Firms

Given the production function, $y_{it} = A_t l^d_{it}$, for all firms $i \in [0, A_t]$ and noting that $l^d_{it} = \frac{x^d_{it}}{A_t}$, profit is:

$$\pi_{it} = p_{it}x^d_{it} - \frac{W_t}{A_t}x^d_{it} = x^d_{it}\Big(p_{it} - \frac{W_t}{A_t}\Big),$$

where $x_{it} \equiv x_{it}(p_{it}) = x^C_{it} + x^R_{it})$. From the resource constraint, the marginal cost, $\frac{W_t}{A_t}$, is constant, so I set marginal cost as the numeraire; that is, $\frac{W_t}{A_t} = 1$. Given constant elasticity of demand of $\frac{1}{1-\alpha}$, the firm's price is a fixed markup over marginal cost:

$$p_{it} = \frac{1}{\alpha}\frac{W_t}{A_t} = \frac{1}{\alpha} \equiv p,$$

which is constant and the same for all firms. Thus, by symmetry, each firm $i \in [0, A_t]$ will produce the same quantity, x, and will demand the same amount of labor. This also gives the firm's labor/hours demand and *real wage*, $\frac{W_t}{p} = \alpha A_t$.[24] The worker is thus paid less than his marginal value product, A_t; this wedge will create a distortion in the labor/leisure choice. Equal prices across firms implies equal quantities produced. Substituting the constant price into the representative demand curve, gives the equilibrium quantity produced by each firm:

$$x_i = x = \frac{C_t + I_t}{pA_t} = \frac{\alpha(C_t + I_t)}{A_t}, \qquad \forall i \in [0, A_t].$$

From the resource constraint, $Y_t = C_t + I_t$, Y_t and A_t grow at the same rate on the balanced growth path; and so quantities produced are also constant, $\frac{\dot{x}}{x} = 0$, so expansion of input firms means more new firms, not more existing firms.

5.3.1 Profits of Input Firms

On the balanced growth path profits are:

$$\pi_t = (\frac{1}{\alpha} - 1)x = (\frac{1}{\alpha} - 1)\frac{\alpha(C_t + I_t)}{A_t} = (1 - \alpha)\frac{C_t + I_t}{A_t}.$$

The wedge, $\frac{1}{\alpha} - 1$, is the profit per unit produced. This enables the entrepreneur to recoup the sunk cost of production and pay dividends to households. Two important points follow

[24] Combining this with the household's labor/leisure first order condition gives equilibrium labor supply.

from the equation for profits. First, note that an increase in demand, $Y_t = C_t + I_t$, will increase profits. Other things equal, an increase in the number of firms, A, will decrease profits, since demand is now spread over a greater product space. But on the balanced growth path, both Y_t and A_t will grow at the same rate so production per firm and profits will remain constant. Second, note that each firm will require fewer labor hours as productivity, A, increases. Those labor hours will then be supplied to new firms.

5.4 Equilibrium

Because there is free entry into patent production, the present discounted value of all future profits is equal to the cost of a patent:

$$p_{At} = \int_t^\infty (1-\alpha)\frac{C_t + I_t}{A_t} \exp(-\overline{r}(t,s)(s-t))\, ds,$$

where $\overline{r}(t,s) = \frac{1}{s-t}\int_t^s r(\kappa)\, d\kappa$. Substituting $p = \frac{1}{\alpha}$, this implies the cost of a blueprint is:

$$\frac{\alpha\eta}{A_t}\int_0^{A_t} p_{it}\, di = \eta.$$

Now, free entry into blueprint production implies:[25]

$$\eta = p_{At} = \int_t^\infty (1-\alpha)\frac{C_t + I_t}{A_t} \exp(-\overline{r}(t,s)(s-t))\, dt.$$

Note that the patent price is constant, so $\dot{p}_{At} = 0 \Rightarrow \eta = \frac{\pi}{\overline{r}_t} \Rightarrow \overline{r}_t p_{At} = \pi$; i.e., the interest payments are just equal to profits. This also indicates that the interest rate, r_t, is constant, so we can put $r_t \equiv r$. In equilibrium:

$$\eta = \frac{\pi}{r} = (1-\alpha)\frac{C_t + I_t}{rA_t} \qquad \Rightarrow r = (1-\alpha)\frac{C_t + I_t}{A_t\eta}.$$

Now, in capital market equilibrium, the total value of assets is: $b_t = p_{At}A_t = \eta A_t \Rightarrow \dot{b}_t \equiv I_t = \eta\dot{A}_t$. From the household budget constraint, $\dot{b}_t = W_t l_t + rb_t - C_t$:

$$C_t + \dot{b}_t = W_t l_t + rb_t = W_t l_t + r\eta A_t.$$

[25] Note that if $p_{At} > \frac{\alpha\eta}{A_t}\int_0^{A_t} p_{it}\, di = \eta$, an infinite amount of resources will be assigned to R & D. Conversely, no resources are assigned if the inequality sign is reversed.

The demand facing each firm on the balanced growth path is then:

$$\frac{C_t + I_t}{A_t} = \frac{C_t + \dot{b}_t}{A_t} = \frac{W_t l_t + r\eta A_t}{A_t} = l + r\eta.$$

There is a scale effect here. The greater are equilibrium hours, the greater the demand and the higher the level of production per firm. It follows that:

$$r = (1-\alpha)\frac{C_t + I_t}{A_t \eta} = (1-\alpha)\frac{l + r\eta}{\eta} \Rightarrow r = \frac{(1-\alpha)}{\alpha}\frac{l}{\eta}.$$

Observe that we also have a multiplier effect: namely, a high rate of return, via the budget constraint, raises demand and production, which raises profits and hence the rate of return itself.

The Keynes-Ramsey rule now reduces to:

$$\frac{\dot{C}_t}{C_t} = r - \rho = \frac{(1-\alpha)}{\alpha}\frac{l}{\eta} - \rho.$$

5.5 Labor Condition

In solving for equilibrium labor supply, l^*, I restrict the analysis to linear disutility, setting $\theta = 0$.

Labor Supply

Using the first order condition for consumption:

$$\frac{p^{\sigma-1} n_t^{\sigma}}{C_t^{\sigma}} = \lambda_t$$

The labor/leisure condition (which implicitly defines optimal labor supply) is:

$$\lambda_t W_t = \beta l^{\theta}$$

$$W_t \frac{p^{\sigma-1} n_t^{\sigma}}{C_t^{\sigma}} = \beta l^{\theta}$$

$$W_t p^{\sigma-1} n_t^{\sigma} = \beta(r_t b_t + W_t l_t - \dot{b}_t)^{\sigma}$$

Then dropping time scripts and substituting in equilibrium values:

$$W^{\frac{1}{\sigma}} p^{1-\frac{1}{\sigma}} n = \beta^{\frac{1}{\sigma}} (r\eta A + Wl - g\eta A)$$

Recall now that $g = r - \rho \Rightarrow \rho = r - g$ and that the real wage, $\frac{W}{p}$, is αA. Substituting the equation for the real wage into the labor supply equation and assuming linear labor disutility gives:

$$l^* = \frac{n}{\beta^{\frac{1}{\sigma}} \alpha^{1-\frac{1}{\sigma}} A^{\frac{\sigma-1}{\sigma}}} - \eta\rho.$$

Substituting for $n_t = \phi A_t^{\gamma}$ and noting our parameter restriction $\gamma = \frac{\sigma-1}{\sigma}$ gives:

$$l^* = \frac{\phi}{\beta^{\frac{1}{\sigma}} \alpha^{1-\frac{1}{\sigma}}} - \eta\rho \qquad \in [0,1].$$

$$\frac{\dot{n}}{n} = \frac{\sigma - 1}{\sigma} \frac{\dot{A}}{A}.$$

Then, recalling the form of the household technology, $n_t = \phi A_t^{\gamma}$, on the balanced growth path we then have the parameter restriction: $\gamma = \frac{\sigma-1}{\sigma}$.

Then steady state labor supply reduces to:

$$l^* = \frac{\phi}{\beta^{\frac{1}{\sigma}} \alpha^{1-\frac{1}{\sigma}}} - \eta\rho \qquad \in [0,1].$$

Then, from equation (17), the per capita growth rate is given by:

$$\frac{\dot{C}_t}{C_t} = \frac{(1-\alpha)}{\alpha} \frac{l^*}{\eta} - \rho > 0.$$

Altogether, we have:

$$\frac{\dot{C}_t}{C_t} = \frac{\dot{A}_t}{A_t} = \frac{\dot{Y}_t}{Y_t} = \frac{\dot{W}_t}{W_t} = \frac{\sigma}{\sigma - 1} \frac{\dot{n}_t}{n_t} = \frac{\phi(1-\alpha)}{\beta^{\frac{1}{\sigma}} \eta \alpha^{2-\frac{1}{\sigma}}} - \frac{\rho}{\alpha} \equiv g.$$

At any point, t, we have GDP per capita, $Y_t = A_t l^*$, where $A_t = A_0 \exp(gt)$, and $A_0 = \eta b_0$.