Markov Switching Monetary Policy in a two-country DSGE Model

Konstantinos Mavromatis*

October 16, 2011

JOB MARKET PAPER

Abstract

In this paper I show, using both empirical and theoretical analysis, that changes in monetary policy in one country can have important effects on other economies. My new empirical evidence shows that changes in the monetary policy behaviour of the Fed since the start of the Euro have had significant effects on the behaviour of inflation and output in the Eurozone even though ECB's monetary policy is found to be fairly stable. Using a two-country DSGE model, I examine this case theoretically; monetary policy in one of the countries (labelled foreign) switches regimes according to a Markov-switching process and this has non-negligible effects in the other (home) country. Switching by the foreign central bank renders commitment to a time invariant interest rate rule suboptimal for the home central bank. This is because home agents expectations change as foreign monetary policy changes which affects the dynamics of home inflation and output. Optimal policy in the home country instead reacts to the regime of the foreign monetary policy and so implies a time-varying reaction of the home Central Bank. Following this time-varying optimal policy at home eliminates the effects in the home country of foreign regime shifts, and also reduces dramatically the effects in the foreign country. Therefore, changes in foreign monetary regimes should not be neglected in considering monetary policy at home.

Keywords: Markov-switching DSGE, Optimal monetary policy, Dynamic programming, SVAR, real-time data.
JEL Classification: E52, F41, F42.

*I am indebted to Alex Michaelides and Michael McMahon for useful comments at various stages of this paper. I am grateful to Athanasios Orphanides, Dan Waggoner, Anastasios Karantounias, Luca Dedola, Efrem Castelnuovo, Rhys Bidder, Federico Di Pace, Herakles Polemarchakis, Michel Juillard, Paulo Santos Monteiro, Sheheryar Malik and Michail Rousakis. I am also grateful to participants in the 7th Dynare Conference at the Federal Reserve Bank of Atlanta. Finally, I wish to thank my supervisors Mark P. Taylor and Gianna Boero for their excellent guidance and continuous support. Correspondence: Konstantinos D. Mavromatis, Department of Economics, University of Warwick, CV4 7AL, Coventry. E-mail: k.mavromatis@warwick.ac.uk and Warwick Business School, Finance Group, email: Konstantinos.Mavromatis@wbs.ac.uk

1 Introduction

Regime changes in the conduct of monetary policy have been documented largerly over the last ten years. They refer to changes in the way the Central Bank of a country reacts to the key macroeconomic variables, i.e. inflation and output. An example of such a change in monetary policy is that of the US. In particular, Clarida et al. (2001), Lubik and Schorfheide (2004) and Boivin and Giannoni (2006) showed that the reaction of the Fed towards inflation fluctuations until the late '70s was less aggressive compared to that from the early '80s onwards. As a result many authors attribute high inflation volatility in the US during the '70s to the way the Fed was reacting over that period to inflation fluctuations[1]. Moreover, according to these authors, changes in monetary policy are the main reason for the changes in the impulse responses of inflation and output.

A weakness of the above papers is that they do not incorporate changes in monetary policy in a single model. They, rather, calibrate or estimate models conditioning on a specific regime. But, this kind of analysis implies that agents do not expect any regime changes in monetary policy. In other words, this approach implies that agents expect that the Central Bank will be either always hawkish, or always dovish[2]. Therefore, the papers mentioned above do not take into account the effects that potential changes in monetary policy in the future have on agents expectations today.

A popular way of modelling this behavior of the monetary authority and the private sector is by introducing Markov switching in monetary policy. Davig and Leeper (2007), Liu et al. (2008, 2009) and Farmer et al (2011), relying on the empirical estimates about the way monetary policy was conducted in the US from 1970 until recently contruct closed economy DSGE models where the coefficients in the interest rate rule of the Central Bank change over time according to a Markov switching process. All the three papers conclude that the expectation of a future regime shift in monetary policy has significant effects on inflation and output today. Those can be either stabilizing or destabilizing depending on what is the expected future policy.

[1]There is a huge literature over the causes of a change in inflation volatility in the US. Some authors, such as Stock and Watson (2003), attribute that change to different shock sizes, rather than to changes in the way monetary policy was conducted . In other words, according to these authors, heteroskedasticity in shocks variances seems to be the main reason for changes in the volatility of the key macroeconomic variables.

[2]As is standard in the literature, from now on dovish regime will refer to the case where the coefficient on foreign inflation in the foreign interest rate rule is less than one. Hawkish regime will refer to the case where the coefficient on foreign inflation in the foreign interest rate rule is less than one is larger than one.

The existing literature, though, is restricted to a closed economy framework. As a result, so far, in the literuture on Markov-switching DSGE models, the cross country effects of regime shifts in monetary policy have not been analyzed. The monetary policy of one country has effects on other countries as well. Therefore, it is important that we have an open economy framework, so that to analyze the effects that a change in monetary policy of one country has on another country. Changes in the volatilities and the impulse responses of key macroecnomic variables may be the result of changes in domestic conditions. But also, it may be that the volatility of home inflation and output changes as a result of a shift in the monetary policy of a foreign country. Furthermore, it is likely that the impulse responses of those two variables change as well, as foreign monetary policy changes, even if domestic monetary policy has not changed at all. For this reason, I construct a two-country DSGE model where monetary policy of one of the two countries switches regimes. I extend the existing analysis on Markov-switching DSGE models from the closed, to the open economy framework, and, hence, give theoretical evidence, that is missing in the literature, about the international effects of regime switches.

This paper has two objectives. Using a two-country DSGE model, the first, and main objective, is to find the optimal policy of the home Central Bank when the foreign changes its policy, using a welfare criterion derived from the model, in the spirit of Rotemberg and Woodford (1998). The second is to show that when the home Central Bank commits to a time invariant interest rate rule (i.e. when it does not react optimally to changes in foreign monetary policy), the home economy experiences large changes in the volatilities of inflation and output and hence large fluctuations in its welfare.

Before the construction of the theoretical model, I provide some empirical evidence regarding the international dimensions of monetary policy. Specifically, I try to explore whether the behavior of key macroeconomic variables in one country is affected by changes in the monetary policy of another country. I estimate a SVAR model for the Eurozone and the US, using real time monthly data spanning from 1999 through 2010. The empirical model includes seven variables, namely inflation, output gap and the nominal interest rate for both the Eurozone and the US, as well as the real exchange rate. I perform parameter stability tests using the Andrews sup-Wald test, as in

Boivin and Giannoni (2002) and the Andrews-Ploberger test[3]. Both tests find that there have been statistically significant changes in the coefficients in the US interest rate equation. The Andrews-Ploeberger test identifies the break date in June 2004. However, coefficients in the Eurozone interest rate equation seem to be more stable. Therefore, I split the sample into two subsamples, namely before and after that date. The impulse response analysis shows that the responses of Eurozone inflation and output gap are completely different in the two samples. The importance of this result rests on the fact that the responses of of those two variables have changed even though the coefficients in the Eurozone interest rate equation seem to be fairly stable. Additionally, the responses of those two variables are more pronounced, in the second subsample.

But what drives the changes in the impulse responses of inflation and output in the Eurozone? In order to answer that question, I perform a countrefactual analysis in the VAR model. I find that the main reason for the change in the impulse responses of those variables was the change in the US monetary policy. I examine also whether changes in the conditions in the Euro area can account for that. I find that their contribution at causing changes in the responses of inflation and output is tiny. Keeping those findings in mind, I proceed to the construction of an open economy DSGE model.

In the theoretical part, I start with the second objective of the paper. That is, to show that the effets of changes in foreign monetary policy on the home economy are non-negligible, when the latter follows a time invariant interest rate rule. I contruct a two country DSGE model as in Benigno and Benigno (2001) and Benigno (2004). I extend their approach by allowing the coefficients in the interest rate rule to change over time. I assume that it is only the foreign country whose interest rate rule coefficients change over time. Moreover, the home country is assumed to not optimally reacting to foreign monetary policy, initially. It rather adopts the standard Taylor rule with some interest rate smoothing. I show that even though the home Central Bank does not change its interest rate rule, a shift in foreign monetary policy affects the volatilities of home inflation and output, as well as their responses to alternative shocks. Therefore, even though home monetary policy is constantly (and with a constant coefficient) aggresive towards inflation fluctuations, home inflation may exhibit increasing or decreasing volatility over time. Specifically,

[3] I use the Andrews-Ploberger test because of its virtue of identifying the break date.

if there is a positive probability that foreign monetary policy will be dovish in the future, then not only foreign inflation will be more volatile, but also home inflation. This is so because, both home and foreign agents incorporate this probability in their future inflation expectations[4]. The increase in the volatility of home inflation in this case, comes from the increasing volatility of the real exchange rate and relative prices. Commiting, thus, to a specific, regime independent interest rate rule proves not to be enough to stabilize the home economy.

Hence, as a next step, I move to the main objective of this paper. That is, to determine what the optimal policy of the home country should be, when the foreign changes its policy. I solve the dynamic programming problem of the home Central Bank conditional on foreign monetary policy switching regimes over time. I extend Soderlind's (1998) algorithm for solving optimal policy problems in linear rational expectations models to a Markov-switching framework. I show that a time invariant interest rate rule is suboptimal for the home country. Home Central Bank must be always aggressive towards inflation fluctuations. That is, it must always have a coefficient on inflation in its interest rate rule that is greater than one. How much aggresive the home Central Bank should be, depends on the regime which the foreign monetary policy lies in. More specifically, I find that as the probability that the foreign Central Bank becomes dovish rises, the home Central Bank should become even more aggressive towards inflation. The opposite holds as the probability that the foreign Central Bank becomes hawkish increases[5]. The intuition behind that result is consistent with the initial finding of an increasing volatility of home inflation, as the probability of foreign monetary policy becoming dovish in the future rises. When home agents expect that foreign monetary policy will become dovish, they anticipate an increase in the volatility of home inflation. Hence, the home Central Bank must react in such a way so that to offset this effect on home agents expectations. And this, as I show, is achieved by increasing the coefficient on home inflation in the home interest rate rule. Additionally, the coefficient on output gap must increase as well, as the foreign monetary policy becomes dovish. The importance of those findings rests on the fact that, when one country changes its policy, then the other country must adjust (change) its policy appropriately. Markov swithing policy, thus, proves to be Pareto superior for the home

[4]Throughout the paper I assume that the probability of a regime switch is the same for both home and foreign agents.

[5]The way the dynamic programming problem is constructed implies that there is a 'leader' (i.e. the foreign Central Bank) and a 'follower' (i.e. the home Central Bank). One, however, must not confuse this with the timing of the decisions. See also Benigno and Benigno (2001) for a similar characterization.

country. More importantly, I show that when the home Central Bank reacts optimally to changes in foreign monetary policy, the effects of changes in the latter are eliminated completely in the home country, and reduced dramatically in the foreign.

The paper is organized as follows. In section 2 a SVAR model is estimated using real time data for the Eurozone and the US, in order to motivate the theoretical model. In section 3 a two country DSGE model is constructed, allowing for regime switching in monetary policy of the foreign country. In section 4, the model is presented in its loglinear form. In section 5, the welfare criterion for the Home central bank is derived. In section 6 the solution technique of the Markov-Switching DSGE (MSDSGE) is described. In section 7 the model is calibrated and simulated. In section 8 the dynamic programming problem of the Home central bank is solved, in order to find what the optimal reaction of the latter should be, conditional on the fact that foreign monetary policy switches regimes. Section 9 concludes.

2 Stylized facts

2.1 A SVAR model for the Eurozone and the US

In this section I present a structural VAR model for the Eurozone and the US. The objective is to show how key macroeconomic variables respond to various kinds of shocks in both areas. Since the focus of this paper is on changes in monetary policy of either one or, at the same time, both countries I perform parameter stability tests, as in Boivin and Giannoni (2002, 2006), in a reduced form VAR model.

The SVAR model consists of seven variables, namely output gap, inflation rate and nominal interest rates in the Eurozone and the US, and the real exchange rate. Such a model may lead to better policy implications. This is so, because the regions under consideration are close trade partners and, hence, it is likely that changes or shocks in the monetary policy of one country have important effect on the key macroeconomic variables of the other. Additionally, this allows us to draw inference on how each Central Bank should react with respect to the foreign monetary policy. The SVAR model receives the following form.

$$A_0X_t = \Gamma_0 + \Sigma_{i=1}^{p}\Gamma_i X_{t-i} + u_t \quad (1)$$

where A_0 is nonsingular, while the variance-covariance matrix of the fundamental disturbances $\Sigma_u = E(u_t, u_t^{'})$ is assumed to be diagonal. The short-run restictions imposed allow for contemporaneous effects of the CPI rate and the output gap on the policy rate in each region. Therefore, the complete represantation of the SVAR model is summaraized as follows.

$$\begin{pmatrix} 1 & a_{12} & 0 & a_{14} & 0 & a_{16} & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ a_{31} & a_{32} & 1 & 0 & 0 & 0 & 0 \\ a_{41} & 0 & 0 & 1 & a_{45} & 0 & 0 \\ 0 & a_{52} & 0 & a_{54} & 1 & a_{56} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & a_{75} & a_{76} & 1 \end{pmatrix} \begin{pmatrix} CPI_{Euro} \\ Gap_{Euro} \\ i_{Euro} \\ RER \\ CPI_{US} \\ Gap_{US} \\ i_{US} \end{pmatrix}_t =$$

$$= \begin{pmatrix} \gamma_{10} \\ \gamma_{20} \\ \gamma_{30} \\ \gamma_{40} \\ \gamma_{50} \\ \gamma_{60} \\ \gamma_{70} \end{pmatrix} + \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \gamma_{14} & \gamma_{15} & \gamma_{16} & \gamma_{17} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \gamma_{24} & \gamma_{25} & \gamma_{26} & \gamma_{27} \\ \gamma_{31} & \gamma_{32} & \gamma_{33} & \gamma_{34} & \gamma_{35} & \gamma_{36} & \gamma_{37} \\ \gamma_{41} & \gamma_{42} & \gamma_{43} & \gamma_{44} & \gamma_{45} & \gamma_{46} & \gamma_{47} \\ \gamma_{51} & \gamma_{52} & \gamma_{53} & \gamma_{54} & \gamma_{55} & \gamma_{56} & \gamma_{57} \\ \gamma_{61} & \gamma_{62} & \gamma_{63} & \gamma_{64} & \gamma_{65} & \gamma_{66} & \gamma_{67} \\ \gamma_{71} & \gamma_{72} & \gamma_{73} & \gamma_{74} & \gamma_{75} & \gamma_{76} & \gamma_{77} \end{pmatrix} \begin{pmatrix} CPI_{Euro} \\ Gap_{Euro} \\ i_{Euro} \\ RER \\ CPI_{US} \\ Gap_{US} \\ i_{US} \end{pmatrix}_{t-1} + \begin{pmatrix} u_{1,t} \\ u_{2,t} \\ u_{3,t} \\ u_{4,t} \\ u_{5,t} \\ u_{6,t} \\ u_{7,t} \end{pmatrix}$$

The reduced form of the VAR model, thus, is specified as

$$X_t = A_0^{-1}\Gamma_0 + A_0^{-1}\Sigma_{i=1}^{p}\Gamma_i X_{t-i} + \varepsilon_t$$

where $\varepsilon_t = A_0^{-1}u_t$ are the reduced form errors with a variamce-covariance matrix $\Sigma_\varepsilon = E(\varepsilon_t, \varepsilon_t^{'}) = A_0^{-1}E(u_t, u_t^{'})A_0^{-1} = A_0^{-1}\Sigma_u A_0^{-1'}$.

The target of this paper is to figure out whether there have been changes in the way monetary policy was conducted until today by both the ECB and the Fed, and, if so, what does this imply for what the optimal monetary policy of the home country, i.e. the Eurozone, should be. Therefore, in the empirical model stability tests are performed. The strategy followed is similar to that in Boivin and Giannoni (2002). In particular, I am interested in testing for the stability of the parameters of the SVAR model throughout the sample. For each equation of the SVAR model, the stability of all the coefficients is tested[6]. I test for parameter stability using two tests. The first test is the Wald version of the Quandt test, or the Andrews sup-Wald test. The second is the Andrews-Ploberger test[7]. The former has the virtue that it has power against various alternatives, as far as the process of the structural parameters is concerned. The latter is able to locate the timing of the break, if there is one. If there is evidence of parameter instability, then the impulse responses computed using the model estimated for the whole sample are no longer valid. Therefore, if this is the case, I split the sample in smaller sub-samples, depending on the timing of the break, estimated by the Andrews-Ploeberger test.

Given that some authors have argued infavour of changes in the size of shocks hitting the economy, rather than changes in the structural parameters, being the reason for changes in the transmission of monetary policy, heteroskedasticity tests in the estimated residuals are also performed. For each equation specific estimated residual the LM test for $ARCH$ effects is used.

2.2 Data

Monthly real time data were gathered from the ECB statistical warehouse and the Federal Reserve Bank of Philadelphia. The dataset spans from 1999:1 though 2010:6. GDP is proxied by total industrial production. CPI for each region is used as the inflation rate. As far as the policy rates are concerned, the Federal Funds rate for the US and the interbank overnight rate for the Eurozone are used. Finally, the nominal exchange rate is measured by the end of period euro-dollar rate.

[6] Evidence of parameter instability in monetary VAR models is mixed. Boivin and Giannoni (2002), Bernanke, Gertler and Watson (1997) and Boivin (2006) find evidence of parameter instability, while Christiano, Eichenbaum and Evans (1999) find the opposite.

[7] Note that the heteroskedasticity robust version of both tests was used.

2.3 Empirical results

2.3.1 *Stability and heteroskedasticity tests*

Prior to the estimation of the SVAR model[8], I perform stability tests in each equation's coefficients in the reduced form VAR model. At table 1 below the $p - values$ from both tests are reported[9].

Table 1: Stability Tests on Reduced-form VAR coefficients

	Regressors						
Dep. vrb	CPI_{Euro}	Gap_{Euro}	i_{Euro}	RER	CPI_{US}	Gap_{US}	i_{US}
CPI_{Euro}	0.0181	0.9491	0.0189	0.0415	0.0174	0.4007	0.0353
Gap_{Euro}	0.7225	0.2944	0.7338	0.7030	0.7407	0.3018	0.6947
i_{Euro}	0.0508	0.6871	0.1231	0.0432	0.0497	0.5500	0.0825
RER	0.0008	0.5122	0.0002	0.0015	0.0007	0.7031	0.0047
CPI_{US}	0.5558	0.4223	0.2338	0.6056	0.5608	0.4859	0.1903
Gap_{US}	0.0112	0.0561	0.0132	0.0429	0.0112	0.1491	0.0388
i_{US}	0.0025	0.6122	0.0000	0.0030	0.0026	0.2339	0.1093

Notes: $p - values$ reported. Red: Significant at 1% s.l., Blue: Significant at 5% s.l.

Stability tests at Table 1 show that at 1% significance level, monetary policy in the US seems to have changed over the sample considered. Four out of seven coefficients in the equation for the Fed Funds rate have changed over time. On the other hand, monetary policy in the Eurozone has not changed at 1% significance level. At 5% significance level, though, the coefficients on lagged foreign inflation and the real exchange rate appear to have changed. As for the output gap in the Eurozone, it seems to be fairly stable. I derive the same result for CPI in the US. On the other hand the coefficients in the Eurozone CPI and the US output gap equations are subject to breaks at 5 %significance levels. Although, it is easy to interpret breaks in the coefficients in the interest rate equations as changes in the way monetary policy is conducted, breaks in the CPI and the output gap equations are less easy to interpret. As regards Eurozone CPI, it is shown that the cofficients on lagged domestic and foreign CPI rates are subject ot breaks. This could

[8] The lag length of the VAR model was chosen based on the AIC and the BIC criterion. Both criteria showed that 2 lags is optimal.
[9] I report $p - values$ obtained only from the Andrews-Ploberger test in order to save space. The results from the Andrews-Quandt test lead to the same conlcusions.

be attributed to changes in the degree of openness in the Eurozone, or home bias. Taking into account the structure of a New-Keynesian Phillips curve, the break in the coeffiecient on lagged interest rate in the Eurozone CPI equation , could be due to either a change in the frequency of price adjustments, or a change in the degree of backward lookingness in price setting behaviour, or a change in the degree of risk aversion, or change in the degree of habits in consumption, or a combination of all the above. Finally, the changes in the coefficients on lagged Eurozone CPI rate, on lagged Eurozone interest rate, on lagged real exchange rate, on lagged US CPI rate and on lagged US interest rate in the US output gap equation could be attributed to changes in the degree of opennes of the US economy, the degree of risk aversion, the degree of endogenous persistence in output, or to a combination of those three factors. I keep, however, the fact that monetary policy seems to have changed, particularly in the US, which is the main motivation of this paper.

Finally, the Andrews-Ploberger test showed that the break in the US interest rate equation coefficients took place in June 2004. I, thus, use this estimate to split the initial sample into two sub-samples when I will be doing the impulse response analysis in the next section.

The last testing performed was on the variance of the estimated equation specific residuals. As already mentioned, I tested for this using the LM test for $ARCH$ effects. The results are shown at table 2 below.

Table 2: Heteroskedacticity tests

	$p - values$
CPI_{Euro}	0.6088
Gap_{Euro}	0.1550
i_{Euro}	0.0105
RER	0.5734
CPI_{US}	0.2365
Gap_{US}	0.4856
i_{US}	0.4261

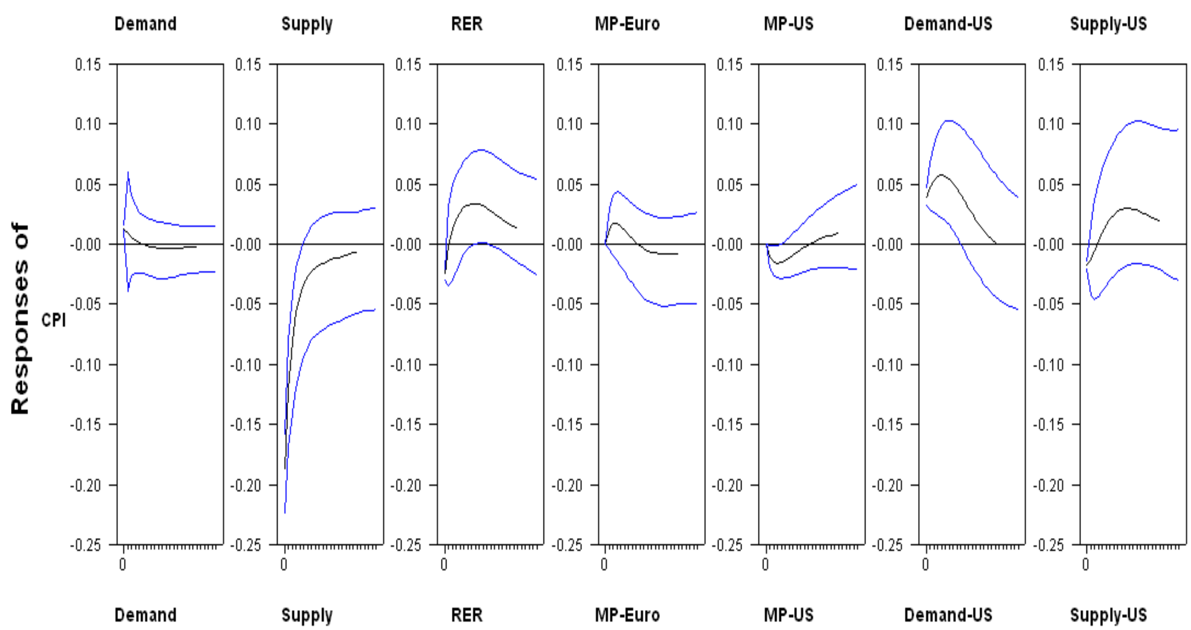
Results at table 2 above show that at 5% significance level only the variance of the residuals from the Eurozone interest rate equation seems to have changed over time.

2.3.2 *Impulse responses*

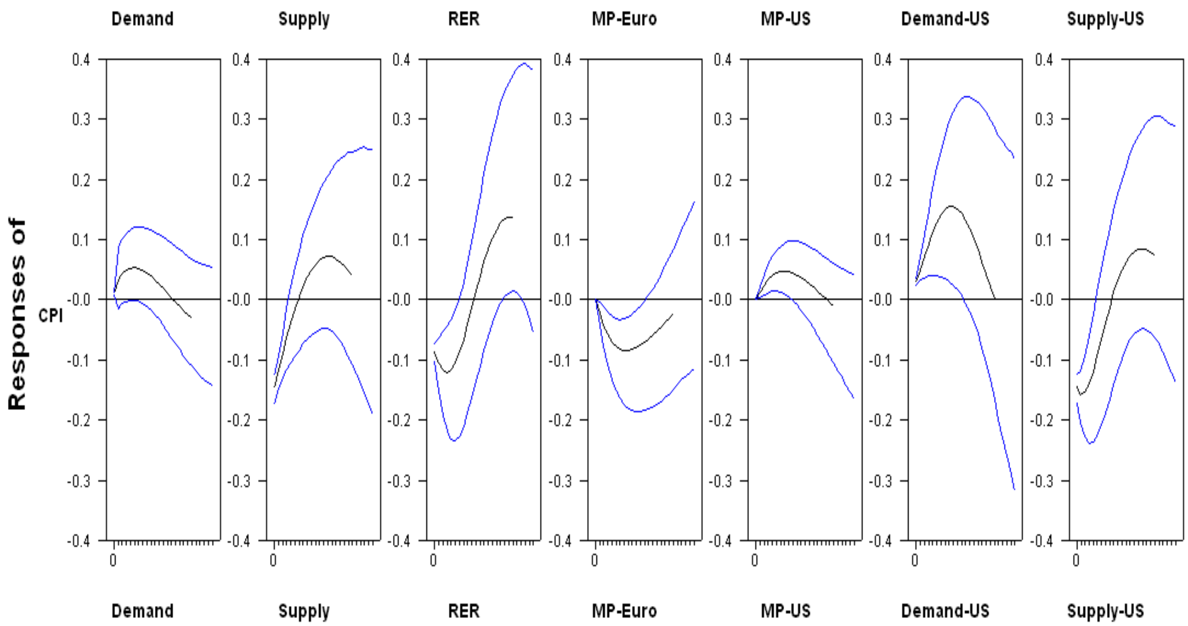
In this section the impulses reponses are computed. Since the Andrews-Ploberger stability test suggested that there has been a change in the US monetary policy at 1% significance level, the sample was split into two sub-samples. Namely, until and after June 2004. The impulse responses of the varialbes of the VAR model are computed for each subsample. Given the presence of breaks in the whole sample, those should be expected to differ depending on the sample used in the estimation in each case. At figure 1 below I present the responses of CPI in the Eurozone following a contractionary monetary policy shock, a postive cost-push shock, a positive demand shock and a postive RER shock in both the Eurozone and the US.

Figure 1: Impulse Responses of Eurozone CPI to alternative shocks.

Sample: 1999:1 - 2004:6



Sample: 2004:7 - 2010:6



Notes: Blue lines: 95% posterior confidence interval.

From the impulse responses I derive the following results. The impulse responses are different in the two samples. In particular, CPI inflation is more volatile and persistent in the second sample for all kinds of shocks considered[10]. Moreover, the sign of the initial impact seems to change as well. For example, CPI initially jumps in the first sample, after a monetary policy shock in the Eurozone. On the contrary, it falls in the second sample. I derive the same result when looking at the impulses following a demand shock in the US.

Counterfactual Analysis with the SVAR

In the previous section, I showed that the Eurzone's and the US response to interest rates fluctuations has changed over time. The stability tests results at table 1 provide evidence in favor of changes in the systematic reaction of the Fed against fluctuations in US and Eurozone inflation, the real exchange rate and the ECB's policy rate. Additionally, it was found that the have been changes in the parameters of the Eurozone CPI and the US output gap equation, respectively. The key point of this paper is to show whether and how domestic variables responses change with

[10]Impulse responses of the output gap lead to the same conclusion. The latter is less volatile and persistent after all kinds of shocks, in the first sample.

respect to changes in foreign monetary policy, and how the domestic Central Bank should react to that. Given that stability tests suggest that coefficients in equations other than that of the US interest rate have changed, it may be that the changes in the impulse responses are due to changes in the coefficients in the nonpolicy part of the VAR rather than the policy one.

For this reason, I now investigate the source of the change in the impulse responses of inflation and output in both countries. I perform a counterfactual exercise on the structural VAR model. The experiment has two stages. At the first, I am trying to figure out whether the observed changes in the impulse responses are explained by the change in the US monetary policy, keeping all other coefficients constant. At the second, I assume that the US monetary policy has not changed at all, contrary to what stability tests suggest. I allow only for the coefficients in the US output gap and the Eurozone CPI equation to change. This allows me to explore the extent to which the changes in the impulse responses can be attributed to changes in the coefficients of part of the nonpolicy block of the SVAR model, rather than the policy one.

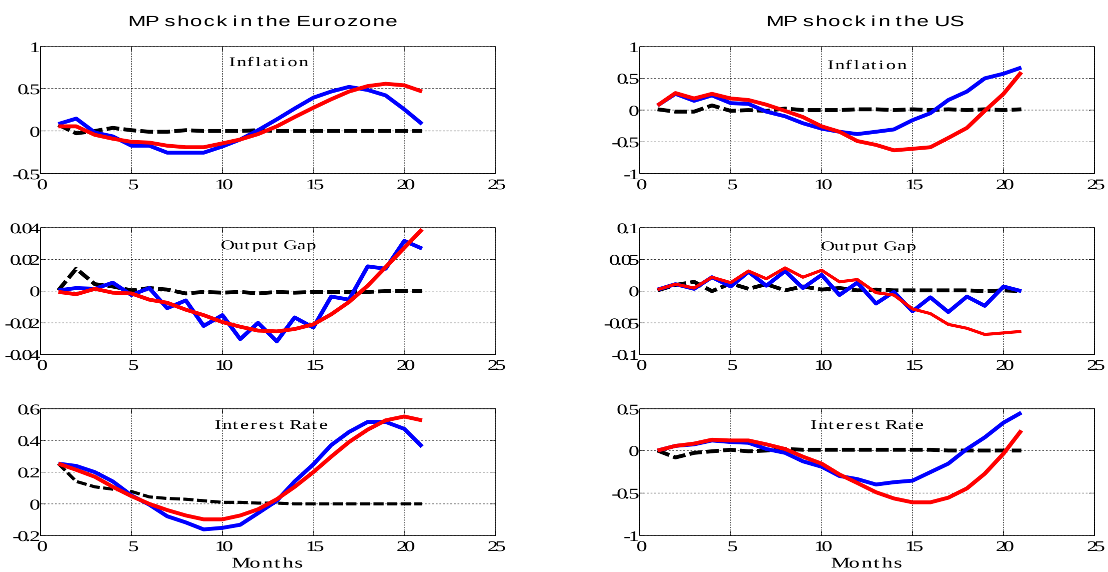
To address the above two questions, let T characterize US monetary policy, K characterize Eurozone CPI and US GDP and N characterize the remaining part of the economy. In particular, T_S is the set of the estimated parameters of the US policy rule, K_S is the set of the estimated parameters in the Eurozone CPI and US GDP equation and N_S is the set of the estimated parameters of the reamaing part of the VAR. Subscript S refers to the period within which those parameters have been estimated. For instance a combination $(T_{pre-2004:6},\ K_{pre-2004:6},\ N_{pre-2004:6})$ denotes the set of all the estimated parameters in the first sub-sample (i.e. form January 1999 to June 2004). This set of parameters characterizes completely the impulse response functions computed for the first sub-sample. On the other hand a combination $(T_{post-2004:6},\ K_{post-2004:6},\ N_{post-2004:6})$ denotes the set of all the estimated parameters in the second sub-sample, and also corresponds to the impulse response funtions obtained for the second sub-sample (i.e. from July 2004 to June 2010).

In order, thus, to answer the first question (i.e. whether the change in the impulse responses is due to a change in the US monetary policy) I will use $(T_{post-2004:6},\ K_{pre-2004:6},\ N_{pre-2004:6})$. That is, keeping all other coefficients fixed and allowing only the coefficients in the US interest rate equation to change, I will compute the new impulse response functions. I will then compare the latter to those obtained using $(T_{pre-2004:6},\ K_{pre-2004:6},\ N_{pre-2004:6})$, in order to see how a change only in the US systematic reaction affects the responses of the variables of the model. Then I will compare the responses from the combination $(T_{post-2004:6},\ K_{pre-2004:6},\ N_{pre-2004:6})$ to those obtained from $(T_{post-2004:6},\ K_{post-2004:6},\ N_{post-2004:6})$, in order to see how close impulse response functions in the former combination move to the ones obtained using the latter combination.

The same strategy will be followed in order to answer the second question. Since, now, the focus is on the effect of changes in the parameters in the Eurozone CPI and the US GDP equations, I will keep all other coefficients fixed. In particular, the impulse response functions are obtained using the combination $(T_{pre-2004:6},\ K_{post-2004:6},\ N_{pre-2004:6})$, and which are compared first to those from $(T_{pre-2004:6},\ K_{pre-2004:6},\ N_{pre-2004:6})$, in order to see how much changes in those coefficients only affect the responses in the first sub-sample. Finally the responses from $(T_{pre-2004:6},\ K_{post-2004:6},\ N_{pre-2004:6})$ will be compared to those from $(T_{post-2004:6},\ K_{post-2004:6},\ N_{post-2004:6})$, in order to explore the extent to which the change in those parameters can capture the behavior of the impulse responses in the second sub-sample. The results are summarized in figure 2 below. In order to save space I show only the responses of the CPI, output gap and interest rate in the Eurozone.

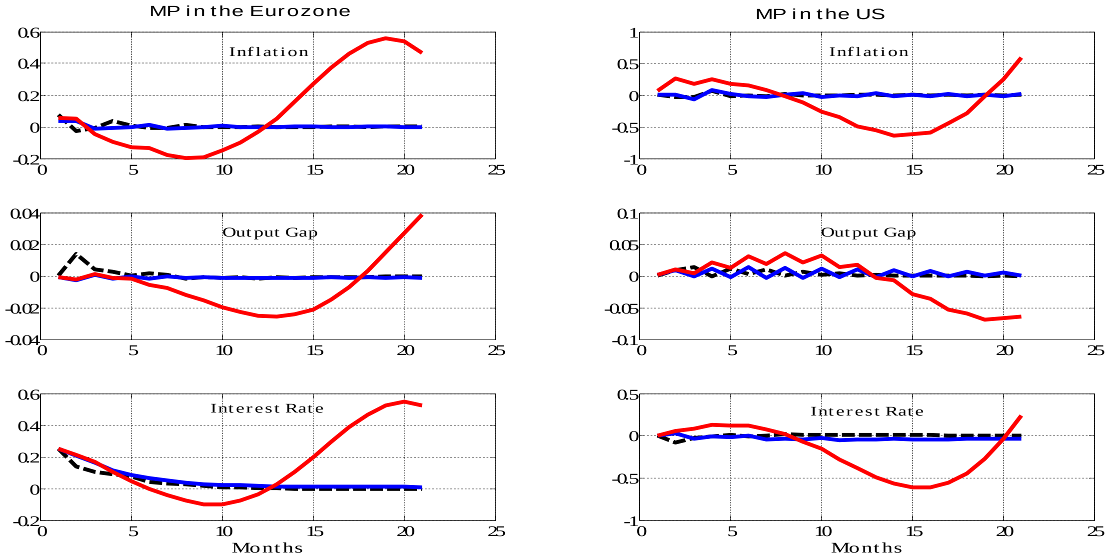
Figure 2: VAR Counterfactual Exercise.

Panel (a): Changes in US policy coefficients only $(T_{post-2004:6},\ K_{pre-2004:6},\ N_{pre-2004:6})$



Notes: Red line: Impulse responses in the second subsample $(T_{post-2004:6},\ K_{post-2004:6},\ N_{post-2004:6})$. Blue line: impulse responses with a change in the US interest rate equation parameters only $(T_{post-2004:6},\ K_{pre-2004:6},\ N_{pre-2004:6})$. Black dashed line: impulse responses in the first sub-sample $(T_{pre-2004:6},\ K_{pre-2004:6},\ N_{pre-2004:6})$.

Panel (b): Changes in US output gap and Eurozone CPI coefficients only ($T_{pre-2004:6}$, $K_{post-2004:6}$, $N_{pre-2004:6}$)



Notes: Red line: Impulse responses in the second subsample ($T_{post-2004:6}$, $K_{post-2004:6}$, $N_{post-2004:6}$). Blue line: impulse responses with a change in the Eurozone CPI and the US output gap equation parameters only ($T_{pre-2004:6}$, $K_{post-2004:6}$, $N_{pre-2004:6}$). Black dashed line: impulse responses in the first sub-sample ($T_{pre-2004:6}$, $K_{pre-2004:6}$, $N_{pre-2004:6}$).

The impulse response functions from figure 2 show that changes in the US interest rate coefficients account more for the change in the impulse responses in the first sub-sample. In fact, in panel (a) the blue line (which is the impulse response when all parameters are fixed but the coefficients in the US ineterest rate equation) moves very close to the red line, which is the impulse response function in the second sub-sample. On the other hand, as shown in panel (b), when only the coefficients in the US output gap and the Eurozone CPI equations change, the impulse response functions in the first sub-sample do not seem to be affected significantly. The blue line, now, moves very close to the dashed black line at all cases. Therefore, I conclude that it is indeed the change in the US systematic reaction that caused the change in the impulse response functions in the Eurozone[11].

2.3.3 Robustness checks

In order to check the sensitivity of the results found so far, various robustness exercises are implemented. The first one considers alternative measures for the output gap. The procedure followed is similar to that in CGG (2000). In particular, instead of using the $hp-filter$, the

[11]Note that the results are the same for US CPI inflation and the output gaps of both countries. I do not present them here, in order to save space.

output gap was measured as the deviation of *log* industrial output from a fitted quadratic function of time. The results do not differ significantly[12]. Both the AIC and the BIC information criteria show that two is the optimal choice of lags in the VAR model. The parameter stability tests do not differ significantly from those reported at table 1 above. The Andrews-Ploberger test locates a break in the parameters in the Federal Funds rate equation in June 2004, as was the case when the $hp - filter$ was used instead. However, what seems to change now is the coefficients only on the lags of the Euro-rate at 1% significance level. The coefficients on the rest the parameters remain unchanged[13]. The LM test for $ARCH$ effects provides the same results as before. That is, only the the variance of the errors in the Euro-rate equation changes at 1% significance level. Finally, the impulse responses lead to the same conclusion as above. Both the CPI and the output gap in the Eurozone responses are different in the two sub-samples.

As a second exercise, a more parsimonious SVAR model was constructed. Given that the dataset is small, it is likely that the impulse responses may not be accurrate, the higher the number of the free parameters to be estimated in matrix A in (1). Therefore, a new SVAR model was estimated allowing for $a_{31}, a_{32}, a_{75}, a_{76}$ to be the only free parameters to be estimated. The key results, found so far, do not change. The impulse responses of the CPI and the the output gap in the Eurozone show that both are more volatile and persistent in sample 2[14].

Moreover, the importance of additional targets in the interest rate rule of both central banks was tested. That is, it was assumed that the each of rest the variables in the system has a contemporaneous effect on the interest rate of each region. At first, the strategy followed was to test the importance of each of the parameters in matrix A individually, so that to avoid the cost of loosing degrees of freedom. Then, the case where both banks reacting to foreign variables or the RER, jointly, was considered. Targeting the RER is beneficial for both central banks only in sample 1. It is enough that only one of the two banks adopts a target for the real exchange

[12] I do not show the results of the robustness exercise here, in order to save space.

[13] Remember that when the $hp - filter$ was used, the Andrews-Ploberger test found that the coefficients on the US and the Euro CPI, the Eurozone output gap and the real exchange rate change, as well, apart from those on the lags of the Euro-rate.

[14] Setting $a_{12} = a_{16} = a_{52} = a_{56} = a_{75} = a_{76} = 0$ does has negligible effects on the impulse responses. Setting, though, $a_{14} = a_{54} = 0$ has nonneglible effects on the impulse responses. That is, allowing for a contemporaneous effect of real exchange rate shocks on the CPI in either country changes the behavior of both the output gap and inflation. In the first subsample, the Eurozone output gap is less volatile after a shock to the RER than when $a_{14}, a_{54} \neq 0$. The same holds for the Eurozone CPI. In the second subsample, the Eurozone CPI is much less volatile after a shock to the RER. Following a demand shock, though, the latter is more volatile. The output gap in the Eurozone is more volatile after a RER shock whenver $a_{14} = a_{54} = 0$. However, as regards the rest of the shocks, the effects of not allowing for contermporaneous effects of RER shocks to the CPI are negligible. Finally, note that still the main conclusion does not change. All variables are more volatile in the secong subsample.

rate. However, the opposite holds in sample 2, where RER targeting does worse than the initial specification in matrix A. Reacting to foreign inflation yields nonneglible gains[15] to both regions. But this holds only for sample 1. Moreover, the sign of the initial responses of some variables, after some shocks, seems to be reversed. When both banks react to the foreign interest rate, there are significant gains regarding inflation fluctuations, in sample 1, especially after a monetary policy shock in the Euro-rate. On the contrary, this no longer holds in sample 2, where reacting to the foreign rate seems not preferable. Finally, foreign output gap targeting allows for lower inflation and output fluctuations in both regions, regardless of the sample.

The possibility, though, of both central banks targeting at the same time foreign variables and/or the real exchange rate was also considered. The results do not change importantly.

2.3.4 A Markov switching interest rate rule for the US

Taking into account the stability test results of section 2.4.1, I now estimate an interest rate rule for the US, which receives the following form

$$i_t = \alpha_0(s_t) + \alpha_\pi(s_t)\pi_t + \alpha_x(s_t)x_t + \varepsilon_t \tag{2}$$

where π_t is inflation and x_t is the output gap. s_t indicates the monetary policy regime and follows a two-state Markov chain. The sample I use is the same as that used for the estimation of the structural VAR model above. Table 4 reports the parameter estimates.

Table 4:Monetary policy rule estimates

States	*Active*	*Passive*
	$s_t = 1$	$s_t = 2$
α_π	1.4562(0.00)	0.3798(0.02)
α_x	0.5934(0.01)	0.4803(0.02)
σ_ε	1.8785e-003	2.2436e-003

Log likelihood value = -235.8437. P-values in parentheses

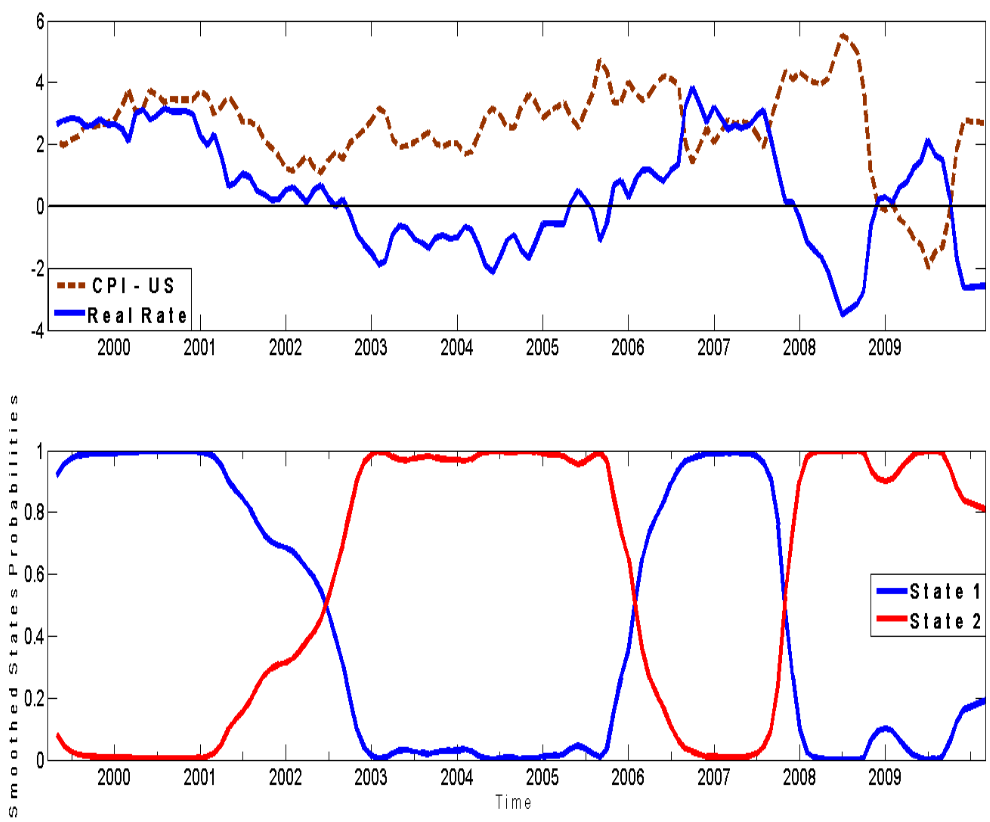
[15] By gains, I meen lower inflation and output gap fluctuations.

the estimated transition matrix is as follows:

$$P = \begin{bmatrix} 0.91 & 0.09 \\ 0.09 & 0.91 \end{bmatrix} \tag{3}$$

Figure 3 below consists of two panels. The upper panel plots the US real interest rate and inflation. The bottom panel plots the estimated transition probabilities for each regime.

Figure 3



The estimated Markov-switching interest rate rule seems to caputre the monetary policy of the fed quite well. The real interest rate starts to fall more than inflation from late 2001 onwards, implying a switch to a more accommodative monetary policy. Indeed, looking at the transition probabilities over the same period, it seems that the probability of staying in state 1 (hawkish) falls (blue line), while the probability of state 2 (dovish) rises (red line). Throughtout the period from 2002 to mid-late 2006, where the real rate is much lower than inflation and negative, the probability of the dovish regime (red line) is close to one. On the other hand the real rate exceeds inflation for the period from late 2006 to late 2007. During that period there is a switch as the Markov switching rule indicates. That is, the probability of switching and then staying in state 1 (hawkish) goes up. Overall, the estimated rule captures fed's policy well. The only exception is the year 2009 where the real rate exceeds inflation.

The results shown above are in line with the existing literature. Bekaert et al. (2011) estimate a closed economy New-Keynesian model where the parameters in the interest rate rule change according to a Markov chain. Using quarterly data for the US, they find that from 1999, which is the start of the sample in this paper, until late 2004, the monetary policy of the fed has been active[16]. From mid 2005, the US monetary policy is accommodative, while the probability of a switch to a high inflation regime rises as well. In particular Bekaert et al. note that "*in 2000 there was a switch to the activist regime, as interest rates rapidly declined"*. Moreover, regarding the accommodative policy during 2002-2006, they note that "*the recent credit crisis starting in 2007 is preceded by a passive montery policy*".

The finding of an accommodative US monetary policy from early 2003 until mid to late 2006 is in line with description of the fed's policy by Ben Bernanke (2010) at the Annual Meeting of the American Economic Association in Atlanta stating the following:

> *"The low policy rates during the 2002-06 period were accompanied at various times by "forward guidance" on policy from the Committee. For example, beginning in August 2003, the FOMC noted in four post-meeting statements that policy was likely to remain accommodative for a considerable period".*

2.3.5 Key Results

From the empirical analysis above, I keep the following key messages. The first is that, there were changes in US monetary policy since the adoption of the common currency in Europe, which seem to have affected the bahavior of key macroeconomic variables not only in the US, but also in

[16] In their paper active is equivalent to the hawkish regime in my paper.

the Eurozone. Moreover, this change in US monetary policy has affected the way macroeconomic aggregates react to various kinds of domestic and foreign shocks. Therefore, changes in the way monetary poliucy is conducted in the foreign country (US) have important implications on the behavior of the home country (Eurozone) macroeconomic variables, even though domestic monetary policy does not change. The degree of openness and, hence, terms of trade effects are likely to be one of the main driving forces for this result. The second is that, there were changes in the behavior of the private sector, as well. This implies that the way expectations were formed, as well as the price setting decisions might have changed over the time considered in this analysis. The counterfactual analysis, though, shows that their effect is small at changing the behavior of inflation and output in either region. Finally, empirical results show that changes in the shock variances do not seem to be the cause for the differences in impulse responses and volatility observed in the two subsamples. Keeping, thus, those facts I proceed to the construction of a two country DSGE model, in order to decompose the effects of those results, theoretically, and to figure out what the optimal policy of the home country should be, given that foreign monetary policy switches regimes. Throughout, the Eurozone is assumed to be the home country, while the US is the foreign.

3 The model

3.1 *Households*

In this section, I specify the structure of the baseline, two country stochastic general equilibrium model. Each country is populated by a continuum of infinitely lived and identical households in the interval $[0, 1]$. Foreign variables are denoted with an asterisk. There are two kinds of households as in Amato and Laubach (2003). Let ψ denote the probability that the household is able to choose its consumption optimally, and which is independent of the household's history. Therefore, by the law of large numbers, in each period a fraction ψ of households will reoptimise, whereas the remaining fraction $1 - \psi$ will not. The latter will choose its consumption in period t according to the following rule of thumb

$$C_t^R = C_{t-1} \tag{4}$$

where C_t denotes aggregate per capita consumption in period t. The remaining $1-\psi$ of households choose C_t^O so as to maximize their ultity. Thus, per capita cunsumption in preiod t is given by

$$C_t = \psi C_t^O + (1-\psi)C_t^R \tag{5}$$

As in Laubach and Amato, this modification to the consumer's problem is based on the assumption that it is costly to reoptimise every period[17]. The households who choose consumtion optimally choose C_t^O to maximize their uitlity function. They derive utility from consumption and disutility from labor supply. The utility function, thus, is specified as

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{(C_s)^{1-\sigma}}{1-\sigma} - \frac{(L_s)^{1+\gamma}}{1+\gamma} \right] \tag{6}$$

where σ is the degree of relative risk aversion. C_t is a composite consumption index described as

$$\begin{aligned} C_t &= \left[\delta^{\frac{1}{\rho}} C_{H,t}^{\frac{\rho-1}{\rho}} + (1-\delta)^{\frac{1}{\rho}} C_{F,t}^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \\ & \qquad\qquad\qquad\qquad\qquad\qquad \rho > 1 \\ C_t^* &= \left[(\delta^*)^{\frac{1}{\rho}} (C_{F,t}^*)^{\frac{\rho-1}{\rho}} + (1-\delta^*)^{\frac{1}{\rho}} (C_{H,t}^*)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}} \end{aligned} \tag{7}$$

where ρ captures the intratemporal elasticity of substitution between home and foreign goods. $\delta > \frac{1}{2}$ is a parameter of home bias in preferences.C_H is the home consumption index. C_F is the foreign consumption index. Consumption indices in the home and the foreign country are defined as

$$\begin{aligned} C_{H,t} &= \left[\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \; C_{F,t} = \left[\int_0^1 c_t(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \\ C_{H,t}^* &= \left[\int_0^1 c_t^*(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}, \; C_{F,t}^* = \left[\int_0^1 c_t^*(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \end{aligned} \tag{8}$$

[17] Amato and Laubach note that Rule (1) has the important feature that rule-of-thumb consumers learn from optimizing households with one period delay. Hence, although Rule (1) is not optimal, it has three important properties. First agents are not required to compute anything. Second, rule-of-thumb households learn from optimizing ones, because last period's decisions by the latter are part of C_{t-1}. Third, the differences between C_t^R and C_t^O are bounded, and will be zero in the steady state.

The aggregate consumption price index for the home and foreign country is specified as

$$P_t = \left[\delta(P_{H,t})^{1-\rho} + (1-\delta)P_{F,t}^{1-\rho}\right]^{\frac{1}{1-\rho}}$$
$$P_t^* = \left[\delta^*(P_{F,t}^*)^{1-\rho} + (1-\delta^*)P_{H,t}^{*\ 1-\rho}\right]^{\frac{1}{1-\rho}} \tag{9}$$

where P_H and P_F are price indices for home and foreign goods, expressed in the domestic currency. The price indices for the Home and Foreign country are defined as

$$P_{H,t} = \left[\int_0^1 p_t(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \quad , \quad P_{F,t} = \left[\int_0^1 p_t(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}}$$
$$P_{H,t}^* = \left[\int_0^1 p_t^*(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \quad , \quad P_{F,t}^* = \left[\int_0^1 p_t^*(z)^{1-\theta}dz\right]^{\frac{1}{1-\theta}} \tag{10}$$

Capital markets are complete. The consumers of both countries purchase state uncontingent bonds denominated in the domestic currency, B_t for domestic agents and B_t^* for foreign agents at price Q_t. That is B_t denotes the home agent's holdings of a one period nominal bond paying one unit of the home currency.

The home agent maximizes her utility subject to the period budget constraint

$$P_tC_t + Q_{t,t+1}B_{t+1} = B_t + W_tL_t + \Pi_t \tag{11}$$

where W_t is the nominal wage and Π_t are nominal profits the individual receives.

3.2 *First order conditions*

Maximizing the utility function (6) subject to the budget constraint (11) yields the following first order conditions

$$Q_{t,t+1} = \frac{\beta P_t}{P_{t+1}}\left(\frac{C_t^O}{C_{t+1}^O}\right)^{\sigma} \tag{12}$$

$$L_t = (C_t^O)^{-\frac{\sigma}{\gamma}} w_t^{\frac{1}{\gamma}} \tag{13}$$

where the first equation is the usual Euler equation while the second determines the labor supply schedule.

Individual demands for each good h produced in the home and in the foreign country respectively are expressed as

$$c_{h,t}(h) = \left(\frac{p_t^h(h)}{P_{H,t}}\right)^{-\theta} \left(\frac{P_{H,t}}{P_t}\right)^{-\rho} \delta C_t \tag{14}$$

$$c_{f,t}(h) = \left(\frac{p_t^*(h)}{P_{F,t}}\right)^{-\theta} \left(\frac{P_{F,t}}{P_t}\right)^{-\rho} (1-\delta) C_t \tag{15}$$

3.3 *Risk sharing*

The fraction of foreign households who choose their consumption optimally (ψ^*) , maximize their utility subject to their budget contraint specified as

$$P_t^* C_t^* + \frac{Q_{t,t+1} B_{t+1}^*}{z_t} = \frac{B_t^*}{z_t} + W_t^* L_t^* + \Pi_t^* \tag{16}$$

where z_t is the nominal exchange rate defined as the domestic currency price of the foreign currency. Therefore, the Euler equation from the foreign agent's maximization problem is

$$Q_{t,t+1} = \frac{\beta P_t^* z_t}{P_{t+1}^* z_{t+1}} \left(\frac{C_t^{O*}}{C_{t+1}^{O*}}\right)^{\sigma} \tag{17}$$

International financial markets are complete. Domestic and foreign households trade in the state contingent one period nominal bonds denominated in the domestic currency. Therefore, combining (12) and (17) , I receive the following optimal risk sharing condition

$$\left(\frac{C_t^{O*}}{C_t^{O}}\right)^{-\sigma} = \varpi q_t \tag{18}$$

where $\varpi \equiv \left(\frac{C_0^f+x}{C_0^h+x}\right)^{-\sigma} \frac{P_0}{z_0 P_0^*}$ depends on initial conditions and $q_t = \frac{z_t P_t^*}{P_t}$ is the real exchange rate.

3.4 ***Price setting***

There are two types of firms, the backward looking and the forward looking. As a result, inflation will depend on both its lagged and forward values. Prices are sticky with a price setting behavior *à ℓa* Calvo (1983). At each date, each firm changes its price with a probability $1-\omega$, regardless of the time since it last adjusted its price. The probability of not changing the price, thus, is ω. The probability of not changing the price in the subsequent s periods is ω^s. Consequently, the price decision at time t determines profits for the next s periods. The price level for home goods at date t will be defined as

$$P_{H,t} = \left[\omega P_{H,t-1}^{1-\theta} + (1-\omega)\widetilde{p}_t(h)^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{19}$$

Firms that are given the opportunity to adjust their prices will either follow a rule of thumb (backward looking firms) or will chose the price that maximizes their expected discounted profits (forward looking firms). The price $\tilde{p}_t(h)$ that will be set at date t is specified as

$$\tilde{p}_t(h) = \zeta p_t^B(h) + (1-\zeta)\, p_t^{For}(h) \tag{20}$$

where $\zeta \in (0,1)$ is the fraction of backward looking firms, $p_t^B(h)$ and $p_t^{For}(h)$ is the price set by the backward and the forward looking firms, respectively. A continuum of firms is assumed for the home economy indexed by $i \in [0,1]$. Each firm produces a differentiated good, with a technology

$$Y_t(z) = A_t L_t(i) \tag{21}$$

where A_t is a country specific productivity shock at date t which is assumed to follow a log stationary process

The strucure of productivity shocks across the two countries receives the following form

$$\begin{bmatrix} \alpha_t \\ \alpha_t^* \end{bmatrix} = \begin{bmatrix} \rho_{\alpha_t} & \rho_{\alpha_t \alpha_t^*} \\ \rho_{\alpha_t^* \alpha_t} & \rho_{\alpha_t^*} \end{bmatrix} \begin{bmatrix} \alpha_{t-1} \\ \alpha_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_{\alpha,t} \\ \varepsilon_{\alpha^*,t}^* \end{bmatrix}$$

where $\begin{bmatrix} \varepsilon_{\alpha,t} \\ \varepsilon_{\alpha^*,t}^* \end{bmatrix} \sim N(0, \Sigma^2)$, with $\Sigma^2 = \begin{bmatrix} \sigma_{\varepsilon_a}^2 & 0 \\ 0 & \sigma_{\varepsilon_{\alpha^*}^*}^2 \end{bmatrix}$.

There is local currency pricing. This implies that each firm chooses one price for the home market and one price for the foreign market.

Backward looking firms.

Backward looking firms set their prices according to the following rule

$$p_t^B(h) = P_{H,t-1} + \pi_{H,t-1} \quad and \quad p_t^{B*}(h) = P_{H,t-1}^* + \pi_{H,t-1}^* \tag{22}$$

Forward looking firms.

Forward looking firms set their prices by maximing their expected discounted profits. Their maximization problem comprises of two decisions. The one concerns the price for the domestic market and the other the price charged in the foreign market, when it exports. Hence their maximization problem is described as

$$max E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} \left\{ \tilde{p}_t(h) y_{t+s}^h(h) + \varepsilon_t \tilde{p}_t^*(h) y_{t+s}^f(h) - W_{t+s}^h L_{t+s}^h \right\} \tag{23}$$

where $y_t^i(h),\ i = h, f$ is the demand for the home good for home and foreign agents specified as

$$y_t^h(p_t(h)) = \left(\frac{\tilde{p}_t(h)}{P_{H,t}} \right)^{-\theta} \left(\frac{P_{H,t}}{P_t} \right)^{-\rho} \delta^* C_t, \tag{24}$$

$$y_t^f(p_t^*(h)) = \left(\frac{\tilde{p}_t^*(h)}{P_{H,t}^*} \right)^{-\theta} \left(\frac{P_{H,t}^*}{P_t^*} \right)^{-\rho} (1 - \delta^*) C_t^* \tag{25}$$

The firm maximizes its objective function (23) subject to (24) in order to find the optimal price for the home good in the home economy. It maximizes subject to (25), in order to find the optimal price for the home good in the foreign economy. The firm chooses a price for the home good in the home economy that satisfies the first order condition

$$E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}(p_t(h)) \left\{ p_t(h) - \frac{\theta}{\theta - 1} MC_{t+s} \right\} = 0$$

where $MC_{t+s} = \frac{W_{t+s}}{A_{t+s}}$ denotes the nominal marginal cost and $\frac{\theta}{\theta-1}$ captures the optimal markup. The optimal price, thus, for the home good in the home country is specified as

$$p_t(h) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} y^h_{t+s}(p_t(h))}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y^h_{t+s}(p_t(h))} \tag{26}$$

Respectively, the optimal price for the home good in the foreign country is specified as

$$p^*_t(h) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} y^f_{t+s}(p^*_t(h))}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y^f_{t+s}(p^*_t(h)) z_{t+s}} \tag{27}$$

Finally, dividing (19) by $P_{H,t-1}$:

$$\Pi_{H,t}^{1-\theta} = \omega + (1-\omega) \left(\frac{\widetilde{p}_t(h)}{P_{H,t-1}} \right)^{1-\theta} \tag{28}$$

where $\Pi_{H,t} \equiv \frac{P_{H,t}}{P_{H,t-1}}$.

Similarly, for the foreign goods consumed in the home economy:

$$\Pi_{F,t}^{1-\theta} = \omega + (1-\omega) \left(\frac{\widetilde{p}_t(f)}{P_{F,t-1}} \right)^{1-\theta} \tag{29}$$

The aggregate price level dynamics are specified, thus, as

$$\Pi_t^{1-\rho} = \delta \left[\left(\frac{P_{H,t-1}}{P_{t-1}} \right) \Pi_{H,t} \right]^{1-\rho} + (1-\delta) \left[\left(\frac{P_{F,t-1}}{P_{t-1}} \right) \Pi_{F,t} \right]^{1-\rho} \tag{30}$$

4 Markov Switching Monetary Policy

Monetary policy in each country is conducted through nominal interest rate rules by each Central Bank. A weakness of the exsiting literature on monetary policy in open economy models is that it fails to take into account potential regime switches in the way monetary policy is conducted in either one of the two or in both countries. As already mentioned, I allow for such changes. I first show that even though domestic monetary policy may not switch, a switch in the foreign monetary policy has effects on the volatility of domestic output and inflation, given the structure of the model. In section 8, it is shown that optimal monetary policy for the home country suggests it changes the coefficients in its interest rate rule, depending on which regime foreign monetary policy lies in and, of course, on the probabilities of a switch.

4.1 ***Policy rules***

In this subsection I describe how Markov switching is introduced into the model. A markov swtiching interest rate rule for the foreign country is specified as

$$i_t^* = i_{t-1}^{*\rho_{s_t}^*} \left(\xi_{s_t}^* \left(\frac{\pi_t^*}{\tilde{\pi}^*} \right)^{\phi_{\pi^*,s_t}^*} \tilde{y}_t^{*\phi_{y^*,s_t}^*} \right)^{1-\rho_{s_t}^*} e^{\varepsilon_t^*} \tag{31}$$

where s_t captures the realized policy regime taking values 1 or 2. Regime follows a Markov process with transition probabilities $p_{ji} = P\left[s_t = i | s_{t-1} = j\right]$, where $i, j = 1, 2$. ξ_t is a scale parameter, $\tilde{\pi}^*$ is the inflation target and $\tilde{y}_t^*$ is the output gap. This specification implies that the policy maker and the private sector does not observe the current regime. Therefore, private sector expectations about future inflation, for example, are specified as $E\left[\pi_{t+1} | \Omega_t^{-s}\right]$, where $\Omega_t^{-s} = \left\{s_{t-1}, \ldots, \varepsilon_t, \varepsilon_{t-1}, \ldots, \varepsilon_t^*, \varepsilon_{t-1}^*, \ldots\right\}$ captures its information set. Having, thus, assumed a two regime markov process for monetary policy, the transition probability matrix P receives the form

$$P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

where p_{11} measures the probability of staying at date t in regime 1 and p_{12} the probability of moving to regime 2 at date t while being in regime 1 at date $t-1$. p_{22} measures the probability of staying in regime 2 at date t and p_{21} the probability of moving to regime 1 at date t while being in regime 2 at date $t-1$.

Monetary policy may switch because of various reasons. One of them could be the switch of the interests of the Central banker. There may be periods, for example, that he is more interested in fighting unemployment than inflation. As a result, the weight on inflation in the interest rate rule could be lower. A monetary policy switch may also be justified by the change of the Central banker. As already mentioned, there is a high number of papers arguing that the US monetary policy has been more tolerant as regards inflation fluctuations in the pre-Volcker period.

The empirical findings in section 2 showed that there was a change in impulse response functions and the volatility of inflation in the Eurozone, even though the monetary policy of the latter remained unchanged. I keep this finding, at first, and assume that the interest rate of home central bank has time invariant coefficients. A standard Taylor rule with interest rate smoothing is adopted which can be summarized as

$$i_t = i_{t-1}^{\rho} \left(\left(\frac{\pi_t}{\tilde{\pi}} \right)^{\phi_\pi} \tilde{y}_t^{\phi_y} \right)^{1-\rho} e^{\varepsilon_t} \tag{32}$$

5 Log linearized model

A log linearized version of the relationships found in the previous section serves in providing a way to deal with the problem of no closed form solution. The model is, thus, loglinearized around a specific steady state. Given the markov switching nature of the model, it is necessary to provide the necessary and sufficient conditions which guarantee that the steady state of the model is unique, and, thus, independent of regime changes. This can be summarized in the following proposition, which is a simple extension to that in Liu, Waggoner and Zha (2008) for the closed economy case

Proposition: *The steady state equilibirum values of aggregate output, consumption and the real wage are independent of monetary policy and are thus invariant to monetary policy regime shifts. Moreover, as long as domestic monetary policy does not change regimes, it is enough that*

$$\xi^*_{s_t} = \frac{1}{\beta}\tilde{\pi}^* \bar{y}^{*-\phi^*_{y^*,s_t}},$$

where $\bar{y}$ is the steady state output gap, so that the steady state nominal variables are given by $\pi = \tilde{\pi}$, $\quad \pi^* = \tilde{\pi}^*$, $R = \frac{\lambda}{\beta}\tilde{\pi}$ and $R^* = \frac{\lambda^*}{\beta}\tilde{\pi}^*$, and which are independent of regime changes as well.

Proof. See appendix A. □

5.1 *Supply side*

I use a first order Taylor approximation around the steady state of zero inflation rate. Log linearized variables are denoted with a hat.

After loglinearizing the first order condition (12), the production function (21) the demand schedules faced by each firm (24) and (25) and optimal price setting rules (26) and (27), I receive the two relations describing the domestically consumed home goods inflation rate and the respective of the home goods consumed in the foreign country

$$\pi_{H,t} = b_{\pi_{H,-1}}\pi_{H,t-1} + b_{\pi^*_{H,-1}}\pi^*_{H,t-1} + \beta E_t \pi_{H,t+1} + b_{\pi^*_H}\pi^*_{H,t} + b_C \hat{C}_t + \ldots$$

$$\ldots + b_T \hat{T}_t + b_{T^*}\hat{T}^*_t + b_q \hat{q}_t + b_a a_t \tag{33}$$

$$\pi^*_{H,t} = b_{\pi_{H,-1}}\pi_{H,t-1} + b_{\pi^*_{H,-1}}\pi^*_{H,t-1} + \beta E_t \pi^*_{H,t+1} + b^*_{\pi_H}\pi_{H,t} + b^*_C \hat{C}_t + \ldots$$

$$\ldots + b^*_T \hat{T}_t + b^*_{T^*}\hat{T}^*_t + b^*_q \hat{q}_t + b^*_a a_t \tag{34}$$

where $T_t = \frac{P_{F,t}}{P_{H,t}}$ and $T^*_t = \frac{P^*_{H,t}}{P^*_{F,t}}$ denote relative prices in the home and foreign country respectively.

The log linearized aggregate price level relation (30) is specified as

$$\pi_t = \pi_{H,t} + (1-\delta)(\pi_{F,t} - \pi_{H,t}) \tag{35}$$

which can be further simplified as[18]

$$\pi_t = \pi_{H,t} + (1-\delta)\Delta\hat{T}_t$$

5.2 ***Demand side***

In this section I proceed to the loglinearization of the Euler equation

$$\hat{C}_t^O = \kappa(i_t - E_t\pi_{t+1}) + E_t\hat{C}_{t+1}^O \tag{36}$$

where $\kappa = -\frac{1}{\sigma}$, and using (5) the Euler equation receives the forward form, which includes both backward and forward looking elements

$$\hat{C}_t = \frac{\kappa\psi}{2-\psi}(i_t - E_t\pi_{t+1}) + \frac{1}{2-\psi}E_t\hat{C}_{t+1} + \frac{1-\psi}{2-\psi}\hat{C}_{t-1} \tag{37}$$

Goods market clearing assumes the following two conditions

$$Y = C_H + C_H^* + G_t \quad \textit{and} \quad Y^* = C_F + C_F^* + G_t^*$$

where G_t and G_t^* capture government expenditures for home and foreign country respectively, assumed to follow an exogenous stationary $AR(1)$ process $g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$ and $g_t^* = \rho_{g^*} g_{t-1}^* + \varepsilon_{g,t}^*$, $\varepsilon_{g,t} \sim N(0, \sigma_{\varepsilon_g}^2)$ and $\varepsilon_{g,t}^* \sim N(0, \sigma_{\varepsilon_g}^{*2})$.

[18] To end up to that expression, I used equation $\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t}$ for the relative price which is reported later in the text.

Combining equation (37) and the market clearing conditions, I derive the aggregate demand equation:

$$\hat{Y}_t = \eta_1 \hat{Y}_{t-1} + \eta_2 E_t \hat{Y}_{t+1} + \eta_3 (i_t - E_t \pi_{t+1}) + \eta_4 \hat{q}_t + \eta_5 \hat{q}_{t+1} + \eta_6 \hat{q}_{t-1} + \dots$$
$$\dots + \eta_7 \Delta \hat{T}_t + \eta_8 E_t \Delta \hat{T}_{t+1} + \eta_9 \Delta \hat{T}^*_t + \eta_{10} E_t \Delta \hat{T}^*_{t+1} \tag{38}$$

where η_i , $i = 1, .., 9$ are defined in detail in appendix B.

5.3 *Real exhange rate and relative prices*

The real exchange rate dynamics are specified by the following relationship

$$\Delta \hat{q}_t = \Delta z_t + \pi^*_t - \pi_t \tag{39}$$

In the home country the price of imported goods relative to that of home goods is specified as $T_t = \frac{P_{F,t}}{P_{H,t}}$, whereas in the foreign country the relative price of home exported goods to foreign goods is specified as $T^*_t = \frac{P^*_{H,t}}{P^*_{F,t}}$. Loglinearizing those two expressions we receive the following

$$\hat{T}_t = \hat{T}_{t-1} + \pi_{F,t} - \pi_{H,t} \quad \hat{T}^*_t = \hat{T}^*_{t-1} + \pi^*_{H,t} - \pi^*_{F,t}$$

5.4 *Flexible price equilibrium*

At the flexible price equilibrium firms adjust their prices at each period. Each firm will set its marginal cost equal to the optimal marginal cost (i.e. $-log\left(\frac{\theta}{\theta-1}\right)$) which is constant over time and equal across firms. Since firms adjust their prices every period, monetary policy will not have any real effects into the economy. The real marginal cost is specified by the following equations

$$mc_t = -log\left(\frac{\theta}{\theta - 1}\right) = -\mu$$

$$mc_t = w_t - \alpha_t - \nu$$

where w_t is the real wage, α_t (log) productivity and ν a subsidy to labor[19]. Solving for the case

[19] This subsidy serves in rendering the flexible price equilibrium efficient. This is achieved by setting the subsidy equal to the mark-up (i.e. $\nu = \mu$), in order to remove the distortion associated with monopolistic competition.

with flexible prices, we receive the following set of equations describing the equilibrium processes for output, consumption, labor, real interest rate[20], given by:

$$y_t^n = \psi_c \bar{c}_{t-1} + \psi_\zeta \zeta + \psi_a \alpha_t + \psi_{a^*} \alpha_t^* + \psi_g g_t + \psi_{g^*} g_t^* \tag{40}$$

$$c_t^n = \tilde{\psi}_c \bar{c}_{t-1} + \psi_\zeta \zeta + \left(\frac{\gamma\delta^* + \sigma}{\delta(\gamma+\sigma) - \gamma(1-\delta^*)} \right) \psi_\alpha \alpha_t - \left(\frac{\gamma}{\sigma} \psi_{\alpha^*} \right) \alpha_t^* - \left(\frac{\gamma}{\sigma} \psi_g \right) g_t - \left(\frac{\gamma}{\sigma} \psi_{g^*} \right) g_t^* \tag{41}$$

$$l_t^n = \tilde{\psi}_c \bar{c}_{t-1} + \psi_\zeta \zeta + \left(\frac{\gamma(\delta^*(1-\sigma) - (1-\delta)) - \sigma(1-\delta)\psi_\alpha}{\delta(\gamma+\sigma) - \gamma(1-\delta^*)} \right) \alpha_t - \psi_{a^*} \alpha_t^* + \psi_g g_t + \psi_{g^*} g_t^* \tag{42}$$

$$r_t^n = \tilde{\tilde{\psi}}_c \bar{c}_{t-1} + \left(\frac{(\gamma\delta^* + \sigma)(1-\rho_a)\psi_a}{\kappa\delta(\gamma+\sigma) - \gamma(1-\delta^*)} \right) \alpha_t - \left(\frac{\gamma(1-\rho_{a^*})\psi_{a^*}}{\kappa\sigma} \right) \alpha_t^* - \left(\frac{\gamma(1-\rho_g)\psi_g}{\kappa\sigma} \right) g_t - \left(\frac{\gamma(1-\rho_{g^*})\psi_{g^*}}{\kappa\sigma} \right) g_t^* \tag{43}$$

5.5 ***Welfare***

The Central Bank sets the interest rate in such a way to minimize a measure of social loss derived by a second order Taylor expansion to the consumer's utility function as in Rotemberg and Woodford (1998), Amato and Laubach (2003), Pappa (2004) and Benigno and Benigno (2006). It is summarized as[21]

$$W_t = -\tfrac{1}{2} u_c C \Xi \{ \lambda_1 (\hat{Y}_t - y_t^n)^2 + \lambda_2 (\hat{Y}_t^* - y_t^{*n})^2 + \lambda_3 (\hat{q}_t - q_t^n)^2 + \lambda_4 \Delta \hat{q}_t^2 + \lambda_5 \Delta \hat{Y}_t^{*2} + \lambda_6 \Delta \hat{Y}_t^2 + \dots$$

$$+ \pi_{H,t}^2 + \lambda_7 (\pi_{H,t} - \pi_{H,t-1})^2 + \lambda_8 (\pi_{H,t}^*)^2 + \lambda_9 (\pi_{H,t}^* - \pi_{H,t-1}^*)^2 + \lambda_{10} (\hat{q}_t + \hat{Y}_t)^2 + \lambda_{11} (\hat{q}_t + \hat{Y}_t^*)^2 + \dots$$

$$\lambda_{12} (\hat{q}_{t-1} + \hat{Y}_t)^2 + \lambda_{13} (\hat{q}_{t-1} + \hat{Y}_t^*)^2 + \lambda_{13} (\hat{q}_{t-1} + \hat{Y}_t^*)^2 + \dots$$

$$\lambda_{14} (\hat{Y}_{t-1}^* - y_{t-1}^{*n})(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{15} (y_{t-1} - y_{t-1}^n)(y_{t-1}^* - y_{t-1}^{*n}) + \lambda_{16} (\hat{C}_t - c_t^n)(\hat{q}_t - q_t^n) + \dots$$

$$\lambda_{17} (\hat{Y}_t + \hat{Y}_{t-1}^*)^2 + \lambda_{18} (\hat{Y}_{t-1} + \hat{Y}_t^*)^2 + \lambda_{19} (\hat{Y}_{t-1} - y_{t-1}^n)(q_{t-1} - q_{t-1}^n) + \dots$$

$$+ \lambda_{20} (\hat{Y}_t^* - \hat{Y}_t^{*n})(\hat{Y}_{t-1}^* - \hat{Y}_{t-1}^{*n}) + \lambda_{21} (\hat{Y}_{t-1}^* + \hat{q}_t)^2 + \lambda_{22} (\hat{Y}_{t-1} + \hat{q}_t)^2 + \lambda_{23} (\hat{Y}_{t-1} - y_{t-1}^n)(\hat{q}_{t-1} - q_{t-1}^n) + \dots$$

$$\lambda_{24} (\hat{C}_{t-1}^* - c_{t-1}^{*n})(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{25} (\hat{q}_t - q_t^n)(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{26} (\hat{Y}_{t-1} - y_{t-1}^n)(\hat{Y}_t - y_t^n) + t.i.p. + O(||\xi||^3) \tag{44}$$

[20] The flexible price expression for the real exchange rate can be easily derived using the risk sharing condition.
[21] The derivation of the loss function is given in detail in the Appendix C.

where the coefficients λ_i, $i = 1, ..., 21$ are functions of the structural parameters and are defined in detail in the appendix.

6 Model Solution

Given the Markov-Switching structure of the model, standard solution techniques cannot be applied in order to find a solution. In the recent literature on Markov-Swithing DSGE models, various alternative techniques for solving such models have been suggested (Farmer, Waggoner and Zha, 2011; Farmer, Waggoner and Zha, 2008; Davig and Leeper, 2007; Svensson and Williams, 2005). The technique I use is that of Farmer, Waggoner and Zha (2011). The virtue of that technique is that it is able to find all possible minimal state variable (MSV) solutions. Moreover, the algorithm is able to find whether the MSV soltuion is stationary (mean square stable) in the sense of Costa, Fragoso and Marques (2004)[22]. The model can be written in the following state space form

$$A(s_t)X_t = B(s_t)X_{t-1} + \Psi(s_t)\varepsilon_t + \Pi(s_t)\eta_t \tag{45}$$

where $X_t = [y_{t+1}, y^*_{t+1}, \pi_{H,t+1}, \pi^*_{H,t+1}, \pi_{F,t+1}, \pi^*_{F,t+1}, q_t, z_{t+1}, T_{t+1}, T_t, y_t, y^*_t, \pi_{H,t}, \ldots$
$\ldots, \pi^*_{H,t}, \pi_{F,t}, \pi^*_{F,t}, q_{t-1}, z_t, T^*_{t+1}, T^*_t, i_t, i^*_t, a_t, a^*_t]$, ε_t is a 6×1 vector of i.i.d. stationary exogenous shocks and η_t is an 8×1 vector of endogenous random variables.
According to that technique the MSV equilibrium of the model takes the form

$$X_t = g_{1,s_t}X_{t-1} + g_{2,s_t}\varepsilon_t \tag{46}$$

In order for the above minimal state variable solution to be stationary it must be that the the eigenvalues of

$$(P \otimes I_{24^2})diag\left[\Gamma_1 \otimes \Gamma_1, \Gamma_2 \otimes \Gamma_2\right] \tag{47}$$

where $\Gamma_j = A(j)V_j$ for $j = 1, 2$. And where V_j is a 24×10 matrix resulting from the Schur decomposition of $A(j)^{-1}B(j)$. In the present model the largest eigenvalue was found to be equal to 0.9174, implying, thus, that our MSV solution is stationary. The impulse responses and the moments of the variables of interest are then derived by that stationary solution.

[22]For an extensive argument regarding the merits of the solution technique used in this paper over the alternative ones see Farmer et al. (2011) and the references therein.

7 Parameterization

In this section, the model is simulated so that to explore what regime switching implies about the dynamic behavior of the key macroeconomic variables. In order to make my argument clearer the impulse responses of inflation and output are compared to those when there is no regime switching, as in Liu et al. (2009). Throughout this section I assume that it is only the foreign central bank switching regimes. The home central bank is assumed to (naively) commit to the Taylor rule, independently of what the foreign central bank does. Therefore, whenever I refer to the hawkish regime, I mean an inflation coefficient in the interest rate rule of the foreign central bank that is greater than one. Whenever I refer to the dovish regime, we mean an inflation coefficient in the interest rate rule of the foreign country that is less than one.

Since it is only the foreign central bank that switches regimes in its monetary policy I have to choose four different parameters for its interest rate rule, depending on the regime. The values assigned are those from the Markov-switching interest rate rule estimated in section 2. That is, $\phi^*_{\pi,1} = 1.4562$, $\phi^*_{\pi,2} = 0.3798$, $\phi^*_{x,1} = 0.5934$, $\phi^*_{x,2} = 0.4803$. I also assume some interest rate smoothing with $\rho^*_1 = \rho^*_2 = 0.6$[23].

As far as the rest of the parameters in the model are concerned, they are regime invariant. Those parameters are the subjective discount factor β, the degree of relative risk aversion σ, the elasticity of substitution between goods produced domestically θ, the elasticisity of substitution between home and foreign goods ρ, the Frisch elasticity of labor supply $1/\gamma$, the degree of price stickiness for the home and the foreign country respectively ω and ω^*, the fractions of rule of thumb firms for each country ζ and ζ^*, the fractions of rule of thumb consumers $1-\psi$ and $1-\psi^*$, the home bias parameters δ and δ^* and the coefficients on the home country interest rate rule ϕ_π, ϕ_x and ρ_i. The values of the parameters are chosen according to the existing empirical and theoretical literature in models similar to mine. They are summarized at table 5.

[23]Note that the results presented in this section hold also for $\rho_1 = \rho_2 = 0$

Table 5: Parameter Values

Structural parameters		
β	0.99	
σ	1.5	
θ	10	(Obstfeld & Rogoff, 2000)
ρ	3	(Obstfeld & Rogoff, 2000)
γ	3	(Pappa, 2004)
$\omega = \omega^*$	0.75	(Adjemian, Paries & Smets, 2008)
$\delta = \delta^*$	0.67	
$\zeta = \zeta^*$	0.5	(Gadzinski & Orlandi, 2004)
$\psi = \psi^*$	0.4	(Adjemian, Paries & Smets, 2008)
Policy Rule Coefficients		
$\phi_\pi = 1.5$	$\phi_y = 0.5$	$\rho = 0.75$
Probabilities		
$p11 = 0.91$	$p22 = 0.91$	

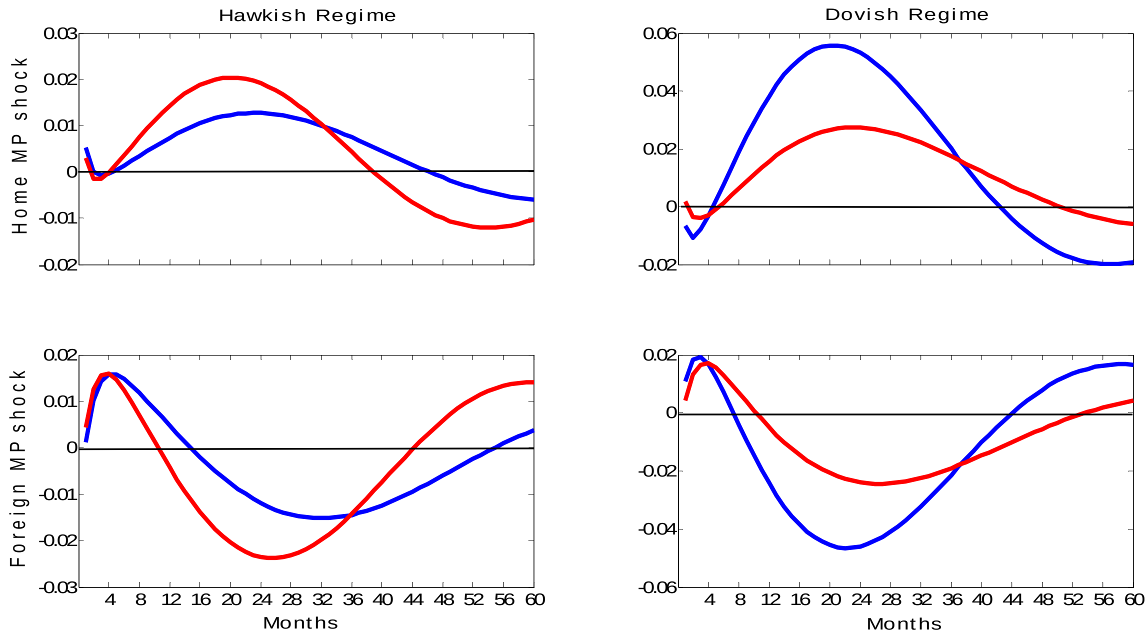
7.1 Impulse responses

To gauge how the possibility of a future switch in foreign monetary affects the dynamics of the domestic macroeconomic variables, I first compute the impulse responses following a one standard deviation shock in monetary policy, demand, productivity in both the domestic and foreign country. In order, thus, to emphasize the importance of expectation effects, the impulse responses of the regime switching model are compared to those of the constant parameter model[24]. That is, the impulse responses when there is a non-zero probability of a change in the regime (red lines) are compared to the absorbing state case (blue line). In the latter case the way agents form their expectations is much simpler, because they do not have to incorporate in their expectations the probability of a future change in regime. On the other hand, when the probability of switching regime in the future is not zero, expectations are affected by the probabilities assigned to each regime. The comparison, thus, of the impulse responses from those two cases allows us to analyze the effects of regime switching on expectations.

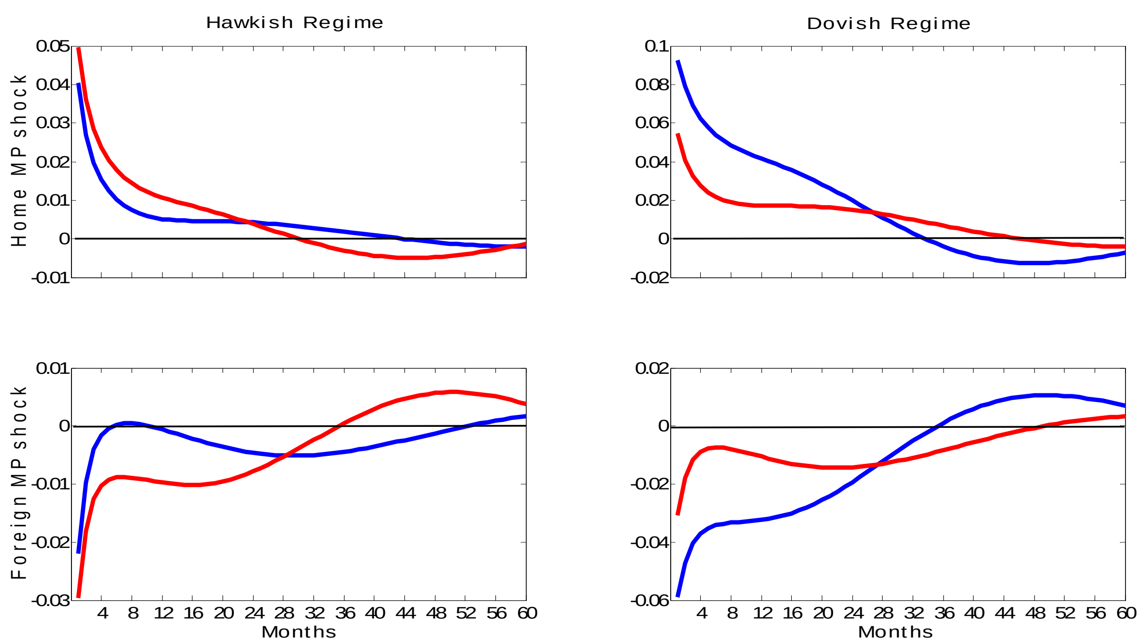
[24] As already mentioned, by constant parameter, I mean the absorbing state, i.e. when there is a zero probability of switching to either the dovish or the hawkish regime.

Figure 5: Home and Foreign inflation responses to a MP shock

(a) Home CPI



(b) Foreign CPI



Notes: The red line impulse responses are from the Markov switching model. The blue line responses are from the constant parameter model. Impulse responses in the hawkish regime are illustrated on the left panel in each graph. Impulse responses in the dovish regime are illustrated on the right panel in each graph.

In figure 5 the impulse responses of the CPI rate are plotted for each of the two regimes. As it is evident, inflation responses, in both countries are dampened[25]in the dovish regime when the probability of a switch to the hawkish regime in the future becomes non zero (redline) after both a home (MP) and a foreign monetary policy shock (FMP). Inflation fluctuates at considerably lower levels than in the absorbing state (blue line). This change in the behavior of inflation, as the probability of switching to the hawkish regime in the future increases, is due to the expectations formation effect. Agents in both countries assign a positive probability on the foreign monetary policy becoming hawkish in the future, affecting, thus, the dynamic behavior of inflation in the home (and the foreign) country. Domestic and foreign inflation are better controled. As far as home inflation is concerned, this result is brought about solely, by agents expectations, without any change in the policy of the home central bank. This is one of the key results in this paper.

> **Result 1:** *In the dovish regime, inflation in the Home country can be better controled without any intervention from the home Central Bank. This result is purely expectations driven. It is enough, that agents in the home country assign a positive probability on the foreign monetary policy becoming hawkish in the future, while it being currently dovish.*

On the other hand, there is an amplifying effect on inflation in the hawkish regime. Following a domestic or foreign monetary policy shock, the effects on either home or foreign inflation are weaker than those in the dovish regime. Inflation responses in both countries seem to be slightly amplified. It is evident that the stabilizing effect, generated in the dovish regime, is stronger than the amplifying effect, generated in the hawkish regime. This can be easily observed by looking at the distance between the red and the blue impulse responses in the hawkish and the dovish regime, respectively. However, as I am showing later, this does not imply that the overall stabilizing effect on either home or foreign inflation is stronger than the amplifying effect. The conlusion drawn until here concerns the two monetary policy shocks only. The dynamics of the model are rich enough and one cannot derive any inference by focusing only on one shock. In order to make this point clearer, I compute the changes in volatilities on inflation and output relative to the absorbing state,

[25]From now on, I will use the term ”stabilizing effect” for the case where the effects of a shock, as measured by the impulse responses, are dampened, and the term ”amplifying effect” when the effects of a shock are amplified.

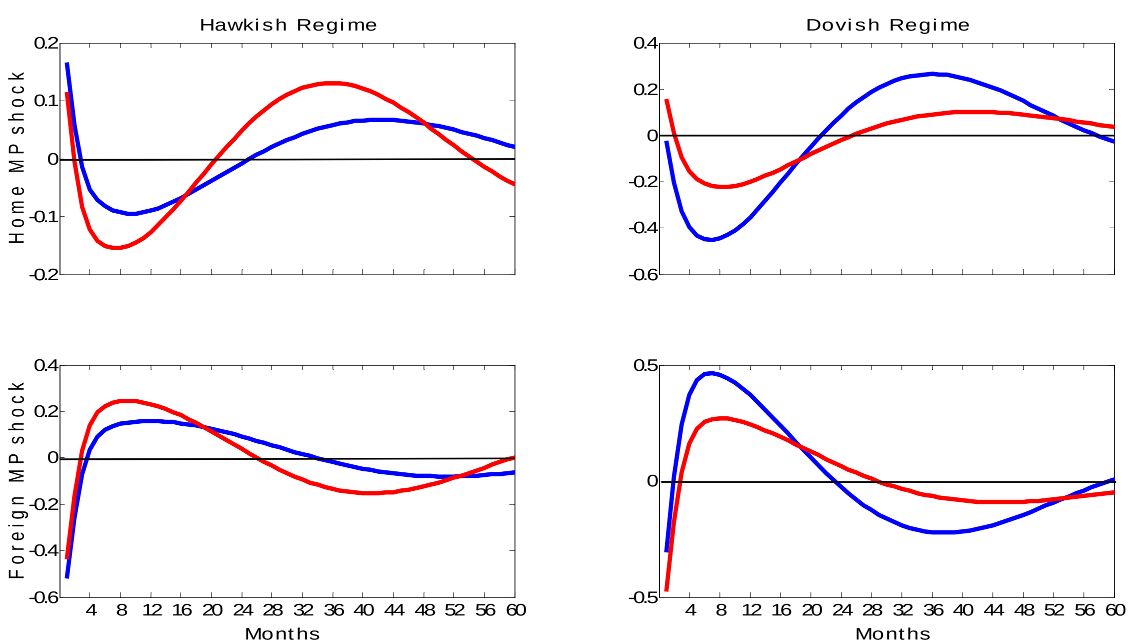
at table 6 below. Note also, the asymmetry in the responses of inflation in each regime, for both countries. The asymmetry in expectation effects causes this asymmetry in inflation responses. The asymmetric expectation effects arise because of the existence of the hawkish regime. The latter is strong enough, so that to make the stabilizing effect stronger than the amplifying. Additionally, the possibility of a future switch to hawkish regime helps anchor agent's expectations (Liu et al., 2009).

Similarly, following a foreign monetary policy shock, inflation respones in both countries are amplified in the hawkish regime and dampened in the the dovish regime.

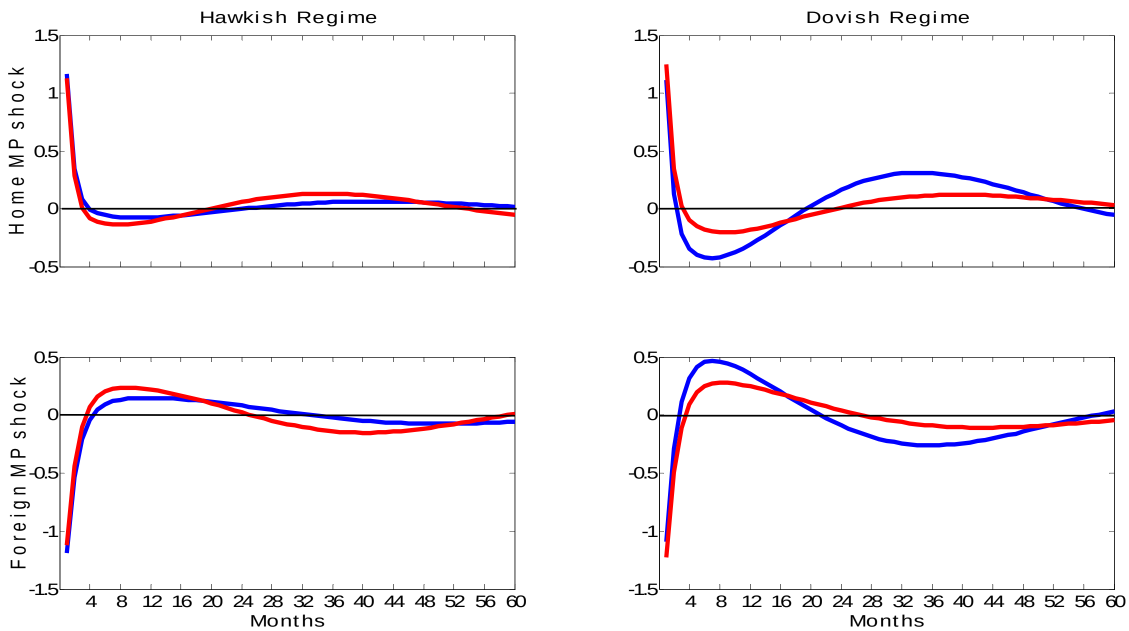
The same reasoning applies to output responses, illustrated in figure 6. The asymmetric effects on output responses, though, are equally pronounced. Following a domestic monetary policy shock, home output response, in the dovish regime, is dampened, as it fluctuates at a lower level and reverts to the teady state faster than if foreign monetary policy stayed dovish forever. On the other hand, in the hawkish regime, its response is slighlty amplified.

Figure 6: Output responses to a MP shock

(a) Home output



(b) Foreign output



Notes: The red line impulse responses are from the Markov switching model. The blue line responses are from the constant parameter model.

Home country's output impulse responses exhibit a pattern similar to those of inflation. Following a foreign monetary policy shock, home and foreign output fluctuate less in the dovish regime for a non zero probability of regime switch, compared to the absorbing state. Output in either country is clearly less volatile in the dovish regime for a positive probability of moving to the hawkish regime (red line). The stabilizing effect is clearly stronger. Home output fluctuations are controled better when home agents attach a positive probability to the foreign monetary policy becoming hawkish in the future, while being currently dovish. In order to make the comparison better, at table 6 I compute the volatility of each variable in each regime relative to the absorbing state. Welfare losses are also shown[26].

[26]Relative losses are computed in the same way as relative volatilities. That is, the loss associated with the Markov-switching model relative to that in the absorbing state.

Table 6: Inflation and Output relative volatilities

	Inflation		*Output*		*Losses*	
	Home	*Foreign*	*Home*	*Foreign*	*Home*	*Foreign*
Hawkish	1.1714	1.7205	1.2709	1.3255	1.6289	1.4633
Dovish	0.7078	0.4495	0.7456	0.7455	0.5610	0.4942

Table 6 shows that there are significant decreases in inflation and output volatility, relative to the absorbing state (i.e. no regime switching case), when foreign monetary policy is dovish. In particular, home country's inflation is 0.7078 times or approximately 30% lower than in the case where the probability of staying in the dovish regime is one. This fall is larger for the foreign country, 0.45 times or 55% lower. On the other hand, a positive probability of a switch to the dovish regime increases home inflation relative to the absorbing state by 17%, while foreign inflation is increased by 72%. The stabilizing effect, thus, on home inflation is much stronger than the amplifying effect. The opposite hods for foreign inflation, where the amplifying effect is much stronger than the stabilizing.

The amplifying effect seems to dominate in output fluctuations, as well. In particular, home output is 27% more volatile in the hawkish regime relative to the absorbing state, while it is 25% less volatile in the dovish regime. Foreign output is 33% more volatile in the hawkish regime and 25% less volatile in the dovish regime.

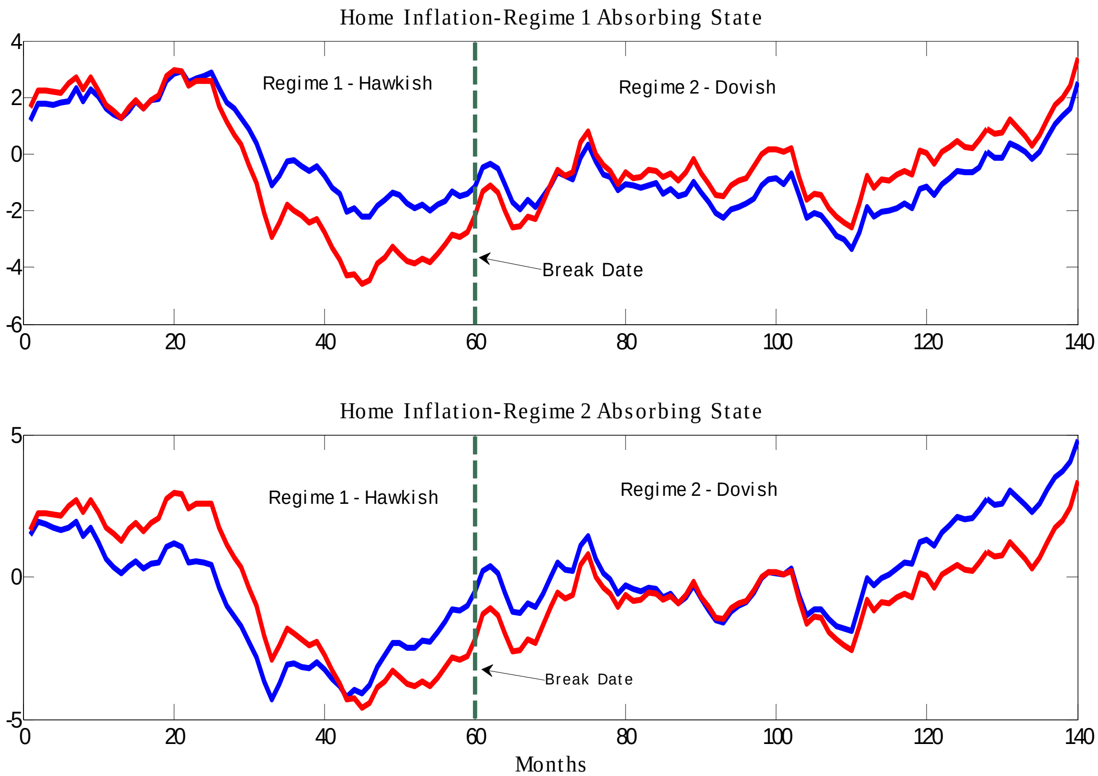
Markov-switching closed economy models examine the effectiveness of regime switching monetary policy by looking at the change in volatilities of inflation and output only. Given the structure of those models, judging such a policy relying on changes in volatility, or on changes in a welfare measure leads to the same conclusions. In an open economy model, as the one in this paper, judging Markov-switching monetary policy by simply looking at the changes in volatilies on inflation and output could lead to the wrong conclusions. As the welfare measure (44) shows the dynamics in the model are far more rich than those in a closed economy model. Therefore, alternative policies would be better compared based on an appropriate welfare measure, rather than by observing changes in volatilities of some variables. I use, thus, the relative changes in the welfare measure (44) as a guide, in order to figure out whether Markov-switching monetary policy generates strong enough stabilzing effects[27] for both economies. As is clear in table 6, the relative fall in home welfare loss in the dovish regime is smaller, in absolute terms, than its relative increase in the hawkish regime. In particular, in the dovish regime, a non-zero probability of a switch to the hawkish regime causes home welfare loss to be 0.5610 times or approximately 44% lower relative to the absorbing state. On the other hand, it is 1.6289 times or 63% higher relative to the absorbing state, in the hawkish regime. Foreign welfare loss rises by 46% in the hawkish regime, and falls by approximately 50% in the dovish regime, relative to the absorbing state. The above results can be summarized as follows.

> **Result 2:** *Markov switching monetary policy in the foreign country generates a stabilizing (dovish regime) and an amplifying (hawkish regime) effect on output and inflation. The stabilizing effect is stronger than the amplifying effect for home inflation. As regards home output and foreign inflation and output, the amplifying effect is stronger*
>
> **Result 3:** *The stabilizing effects are stronger in the foreign country and weaker in the home, in terms of the welfare measure (44).*

So far I have shown that changes in the volatilities and the impulse responses of key macroeconomic variables of the home country may be caused by changes in the way monetary policy is conducted in the foreign country only. In figures 7 and 8 below I show the simulated paths of inflation in each country. The model was simulated for 140 periods allowing for a random date of regime switching in foreign monetary policy. I assumme that the initial regime is the hawkish. The regime changing date is 60 (switch to the dovish regime). For convenienve a green dashed line is drawn on the regime changing date. In the upper panel in both figures, along with inflation in the MSDSGE model (red line) I plot home (foreign) inflation, had foreign monetary policy stayed in the hawkish regime forever (blue line). In the bottom panel inflation in the MSDGE model (red line) is compared to inflation, had foreign monetary policy been always dovish (blue line).

[27] By strong enough stabilizing effects, I mean that the latter is much stronger than the amplifying effects, that is effects caused by the increase in volatility relative to the absorbing state in the hawkish regime.

Figure 7: Home country's inflation



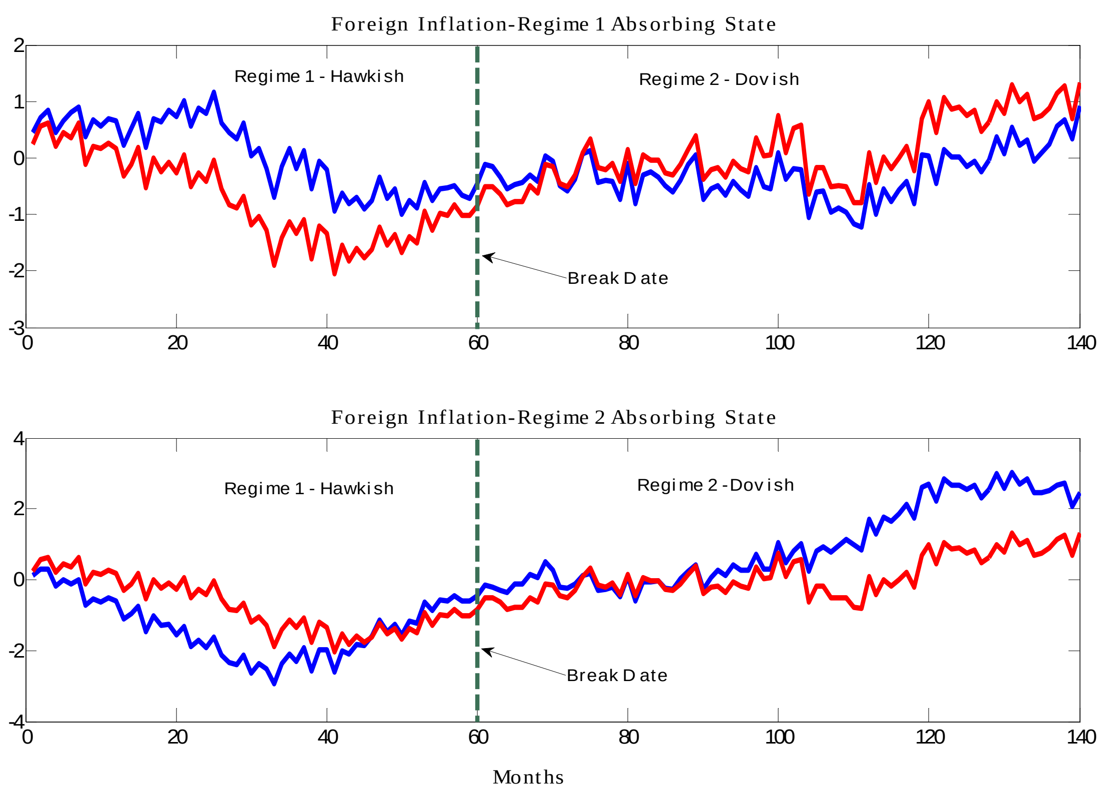
As the upper panel in figure 7 illustrates, inflation in the home country appears to be fluctuating within a certain band while still being in regime 1. For a long period in the regime 1 (until date 60), home inflation behavior resembles that when there is not any regime change (blue line). However, from date 60 onwards, home inflation has higher peaks than it would have, had there not been a change in the foreign monetary policy. The reason for this is the expectations formation effect. As the probability of a future switch in foreign monetary policy rises, inflation in the hawkish regime starts to fluctuate more. After the regime change date home country's inflation fluctuates at constantly higher levels than it would have fluctuated, otherwise. This implies that the home Central Bank should change its policy as well, in order to eliminate as much as possible the additional volatility on domestic inflation.

At the lower panel in figure 7, inflation in the MSDGE model (red line) is illustrated along with inflation when the dovish regime is the absorbing state (blue line). Home inflation fluctuates within a narrower region than it would have otherwise, had foreign monetary policy been always dovish. Comparing the two panels above, it is clear that the stabilizing effect is stronger, since inflation in the MSDGE moves much closer to inflation when the hawkish regime is the absorbing state than when the dovish regime is the absorbing state.

Foreign inflation exhibits the same pattern, as shown in figure 8. A the top panel, foreign inflation fluctuates within a slightly wider region for most of the period in regime 1 (i.e. until date 60). As already mentioned, the reason for this effect is the expectation formation effect becoming stronger as the probability of a regime switch increases and as the regime change date approaches. From date 60 onwards (Regime 2), foreign inflation keeps fluctuating at a constatly wider region than otherwise. Again the blue line shows how inflation fluctuates when the foreign central bank stays in the hawkish regime forever. The red line shows how inflation behaves when the foreign central bank switches from being hawkish to dovish.

On the other hand, foreign inflation is considerably stabilized relative to the absorbing state when the foreign Central Bank is always dovish, as is shown in the bottom panel of figure 8. The red line fluctuates at a much narrower band than the blue line. Comparing the top and bottom panel, it is clear that the stabilizing effect is stronger than the amplifying.

Figure 8: Foreign country's inflation



7.2 Alternative interest rate rules.

Having analyzed the effects of foreign policy regime switching under standard Taylor rules, I turn now the focus to alternative rules. I allow for different or additional targets in the home country's interest rate rule. In particular, I first look at what PPI instead of CPI inflation targeting implies for the home country. Second, I examine the importance of having a real exchange rate target in the home interest rate rule. Third, I introduce foreign variables in the rule. Throughout this section I assume that the interest rate rule of the foreign country is exactly the same as it was in the previous section. That is, the foreign Central Bank keeps targeting foreign CPI and output gap.

Targeting PPI inflation.

When a CPI target is replaced by a target for PPI the interest rate rule of the home Central Bank is specified as

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_{\pi_H}\pi_{H,t} + \phi_y \tilde{y}_t\right) \tag{48}$$

As a first exercise, I compare the performance of rule (48) to the benchmark rule in which the home Central Bank targets CPI inflation and the output gap.

Table 7: Inflation and Output relative volatilities (Rule (48) vs Benchmark)

	Inflation (CPI)		*Output*		*Losses*	
	Home	*Foreign*	*Home*	*Foreign*	*Home*	*Foreign*
Hawkish	0.9532	0.7746	0.9610	0.9282	0.9212	0.9588
Dovish	0.9887	0.9101	0.9431	0.9225	0.8830	1.0067

The result from table 7 show that it is better for the home country to target PPI rather than CPI inflation[28]. Home losses are lower by 8% in the hawkish regime and 12% in the dovish. Foreign loss in the hawkish regime are lower compared to that under the benchmark rule where CPI inflation is targeted by the home Central Bank. On the other hand foreign loss is almost unchanged in the dovish regime. Home output and CPI inflation are marginally less volatile in both regimes. The foreign country has considerable benefits regarding CPI inflation volatility in the hawkish regime. Foreign inflation volatility is 0.7746 times lower in the hawkish when the home central bank targets

[28]The coefficients in rule (48) are exactly the same as in the baseline calibration, that is $\phi_{\pi_H} = 1.5$, $\phi_y = 0.5$ and $\rho = 0.6$.

PPI inflation, and 0.9101 times lower in the dovish regime.

The intuition behind the results above is that, by targeting home PPI inflation, the home central bank isolates the latter from the effects of additional volatility in CPI inflation resulting from higher volatility in imported goods inflation ($\pi_{F,t}$). However, by targeting PPI inflation, the home central bank looses its ability to control imported goods inflation. Imported goods inflation is more volatile in both regimes, by 1.0378 in the hawkish and by 1.0192 in the dovish. Which effect will dominate depends also on the degree of openess of the home country. Not suprisingly, with a degree of home bias in consumption equal to 0.67, the stabilizing effect on home PPI in both regime dominates, leading to lower volatility in CPI inflation.

Lower home output volatility is justified by the lower volatility in the home ineterest rate in both regimes. In particular, it is 0.9203 times less volatile in the hawkish regime and 0.9165 less volatile in the dovish regime.

Targeting the Real Exchange Rate.

I now extend the benchmark interest rate rule of the home Central Bank by adding a real exchange rate target. The rule, thus, receives the following form

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_{H,t} + \phi_y \tilde{y}_t + \phi_q q_t\right) \tag{49}$$

As above, I compare the performance of rule (49) that used in the baseline calibration[29]. Note, though, the substantial differences between rule (49) and the Taylor rule in the baseline calibration. In the former, the home Central Bank targets the home PPI inflation and the real exchange rate[30]. The only common feature is the output gap target.

Table 8: Inflation and Output relative volatilities (Rule (49) vs Benchmark)

	Inflation (CPI)		*Output*		*Losses*	
	Home	*Foreign*	*Home*	*Foreign*	*Home*	*Foreign*
Hawkish	0.9097	0.6770	0.9244	0.8915	0.8518	0.8904
Dovish	0.9978	0.8586	0.9421	0.9060	0.8743	1.1126

[29] The coefficient on the real exchange rate is $\phi_q = 0.1$.

[30] The performance of rule (49) with a CPI inflation target, instead, was also checked. The accrued benefits, however, were negligible.

When the home central bank targets the home PPI inflation along with a target for the real exchange rate the benefits in terms of welfare losses, compared to the benchmark case, are significant. Home loss is almost 15% lower in the hawkish regime and approximately 13% lower in the dovish regime relative to the Taylor rule. The main driving force for the lower volatility in both regimes seems to be the real exchange rate. The latter is almost 7% less volatile in the hawkish regime, and 43% less volatile in the dovish. The most crucial conclusion from rule (49) is that the amplifying effects of a possiblity of a switch to the dovish regime in the future are considerably decreased.

Targeting foreign variables.

One of the important questions in open economy monetary economics has been that of whether Central Banks should target foreign variables or not. Empirically, it seems that such targets can provide the Central Banks some information in order to control better the overall volatility in the domestic economy (Clarida, Gali and Gertler, 1998). One may question the implementability of such rules. Targeting foreign variables implies that the home Central Bank has sufficient information about those, so that to be sure about which direction should it move its instrument. Additionally, in practice, it is not even certain the size and the sign of the effect such variables have on domestic economy. I, however, abstract from this criticism by sticking to the initial assumptions of the model. The class of such rules considered receive the following form[31]

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_{y^*} \tilde{y}_t^*\right) \tag{50}$$

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_{\pi^*} \pi_t^*\right) \tag{51}$$

$$i_t = \rho i_{t-1} + (1-\rho)\left(\phi_\pi \pi_t + \phi_y \tilde{y}_t + \sum_{s=0}^{p} \phi_{i^*,p} i_{t-s}^*\right) \tag{52}$$

The results for the performance of each of the above interest rate rules above are summarized at table 9 below

[31]The coefficients on inflation, the output gap and smoothing are $\phi_\pi = 1.5$, $\phi_y = 0.5$ and $\rho = 0.75$.

Table 9: Inflation and Output relative volatilities (vs Benchmark)

	Inflation (CPI)		*Output*		*Losses*	
	Home	*Foreign*	*Home*	*Foreign*	*Home*	*Foreign*
Rule 50 $\phi_{y^*} = -0.1$						
Hawkish	0.8508	0.3928	0.7775	0.6716	0.6033	0.6407
Dovish	0.9879	0.7395	0.8269	0.7264	0.6578	1.1251
Rule 51 $\phi_{\pi^*} = 0.5$						
Hawkish	0.9371	0.9726	0.9471	0.8988	0.8922	0.9549
Dovish	0.9537	0.8427	0.9737	0.9496	0.9393	1.0918
Rule 52 $\phi_{i^*,p} = -0.1$						
Hawkish	0.7699	0.3683	0.6045	0.4366	0.3735	0.3037
Dovish	0.9335	0.6651	0.7083	0.5441	0.4846	0.8195

The results at table 9 suggest that rule (52) performs much better than any other alternative rule considered in this section. Home country's welfare loss is considerably lower compared to that in the baseline calibration, in both regimes. Welfare loss of the foreign country is dramatically lower than under the benchmark interest rate rule in both regimes.

As for output relative volatilities, they are much lower compared to the benchmark case for both countries in both regimes. As regards home inflation it is 7% less volatile in the dovish regime and 23% less volatile in the hawkish. The effects on foreign inflation are more pronounced. The latter is approximately 63% less volatile in the hawkish regime and 34% less volatile in the dovish.

But the main criterion to judge the overall effects in each country is welfare loss. Since the latter is considerably lower for both countries, it follows that both benefit when the home Central Bank adopts rule (52) instead of the standard Taylor rule.

A direct reaction of the home Central Bank to foreign interest rate fluctuations implies higher weights on both home inflation and output. In fact, by using the UIP condition in rule (52) where the home Central Bank reacts only contemporaneously to the foreign interest rate, I receive the following

$$i_t = (\frac{\rho}{1+\phi_{i^*,0}})i_{t-1} + (1-\rho)\left[\frac{\phi_\pi}{1+\phi_{i^*,0}}\pi_t + \frac{\phi_y}{1+\phi_{i^*,0}}\tilde{y}_t + \frac{\phi_{i^*,0}}{1+\phi_{i^*,0}}\Delta\hat{z}_{t+1}\right]$$

A negative $\phi_{i^*,0}$ implies higher weights on ouput and inflation, hence a more aggresive reaction against their fluctuations. As I am showing in the next section, it is optimal for the home Central Bank to raise the coefficients on inflation and output as the probability of shifting to the dovish regime in the future increases.

8 The Dynamic Programming Problem

So far in the analysis, the parameters in the interest rate rule of the Home country have been assumed to be naively constant over time, independently of what the foreign moentary is and have been set arbitrarily, corresponding to the standard Taylor rule suggested by Taylor (1993). In this section optimized coefficients are computed. The reason for this, is to find how the Home central bank should react for a given policy rule of the foreign country. In other words, I am looking for the optimal policy conditional on the coefficients in the interest rate rule of the foreign country. I am not looking, thus, at the cooperative allocation as in Benigno and Benigno (2006). In this paper I focus on the optimal discretionary policy for the home Central Bank. For this reason, I will make use of dynamic programming techniques. The algorithim I use is that of Soderlind (1998), but extended to a Markov-switching framework.

8.1 Formulation

The procedure followed in this section is similar to that in Zampolli (2006). The policy maker chooses the control i_t (i.e. the interest rate rule) which minimizes the expected value of the intertemporal loss function, stated in the previous section and summarized as

$$\sum_{t=0}^{\infty} \beta^t W(h_t, i_t) \tag{53}$$

subject to h_0, s_0 given, and the model describing the economy

$$h_{t+1} = A(s_{t+1})h_t + B(s_{t+1})i_t + C\varepsilon_{t+1} \qquad t \geq 0 \tag{54}$$

where $L(h_t, i_t)$ is the period loss function, β is the discount factor, h_t is a 21×1 vector of state variables, i_t is the control variable (i.e. the interest rate) and ε_t is a 6×1 vector of white noise shocks with varianve covariance matrix Σ_ε and C is a 21×6 block matrix spcified as

$$C = \begin{pmatrix} C_{11} \\ I \end{pmatrix}$$

where C_{11} is a 23×6 matrix of zeros, I is a 6×6 identity matrix.

The loss function, thus, can be conveniently expressed as follows

$$W(h_t, i_t) = h_t^{'} R h_t + i_t Q i_t \tag{55}$$

where R is a 29×29 positive definite matrix and Q, in our case, is a scalar. The matrices A and B, as already mentioned, are stochastic and take on different values depending on the regime $s_t,\ t = 1, 2$.

8.2 The Bellman equation

The policy maker in a markov switching environment needs to find the interest rate rule that is state-contigent. This rule describes the way that the control variable, the interest rate, should be set as a function of both the state variables and the regime occuring at date t. Therefore, as in Zampolli (2006) a Bellman equation is associated with each regime. In other words, the policy maker solves her minimization problem conditional on the regime. The regime j dependent Bellman equation is specified, thus, as follows

$$V(h_t, j) = max_{i_t} \left\{ W(h_t, i_t) + \beta \Sigma_{i=1}^{2} p_{ji} E_t \left[V(h_{t+1}, i) \right] \right\} \tag{56}$$

where $V(h_t, j)$ is a function of the state variables h_t, the regime prevailing at date t and represents the continuation value of the optimal dynamic programming problem at t.

The value function for this problem is

$$V(h_t, j) = h_t^{'} P_j h_t + d_j, \quad j = 1, 2 \tag{57}$$

where P_j is a 29×29 symmetric positive semidefinite matric, while d_i is a scalar. The optimal policy is given by

$$i(h_t, j) = -F_j h_t, \quad j = 1, 2 \tag{58}$$

where F_j is a 29×1 matrix, depending on P_j. That is, matrix F_j specifies the coefficients in the policy rule of the central bank. Those coefficients are regime specific. Maximizing, thus, the Bellman subject to the constraints, the matrix F_j is specified as

$$F_j = \left(Q + \beta p_{j1} B_1^{'} P_i B_1 + \beta p_{j2} B_2^{'} P_i B_2\right)^{-1} \beta \left(p_{j1} A_1^{'} P_i B_1 + p_{j2} A_2^{'} P_i B_2\right) \tag{59}$$

where matrix P_i has been already determined by a set of interelated Riccati equations, which specify a system with the following form

$$\begin{aligned} P_j = {} & R + \beta p_{j1} A_1^{'} P_i A_1 + \beta p_{j2} A_2^{'} P_i A_2 - \ldots \\ & -\beta^2 \left(p_{j1} A_1^{'} P_i B_1 + p_{j2} A_2^{'} P_i B_2\right) \left(Q + \beta p_{j1} B_1^{'} P_i B_1 + \beta p_{j2} B_2^{'} P_i B_2\right)^{-1} \left(p_{j1} B_1^{'} P_i A_1 + p_{j2} B_2^{'} P_i A_2\right) \end{aligned} \tag{60}$$

8.3 How should home central bank react?

Given foreign monetary policy, I find in this section, the optimized coefficients for the interest rate rule of the home central bank. That is, the coefficients on the output gap and inflation that minimize welfare conditional on both foreign monetary policy and the regime the economy stands. Figures 9 and 10 summarize our key results.

Figure 9: Coefficients in the hawkish regime

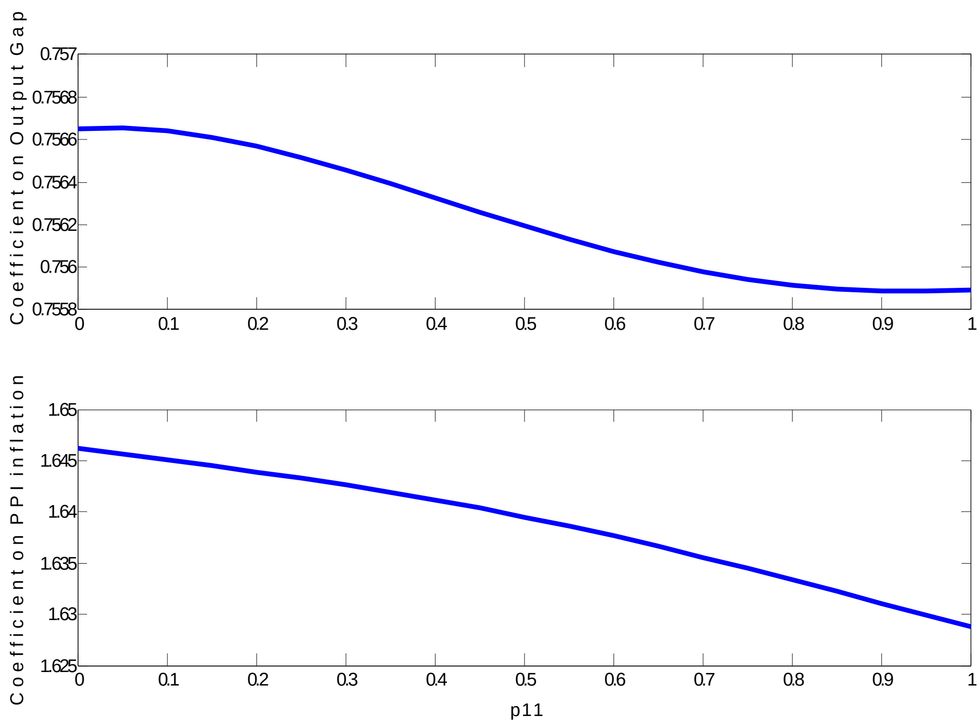
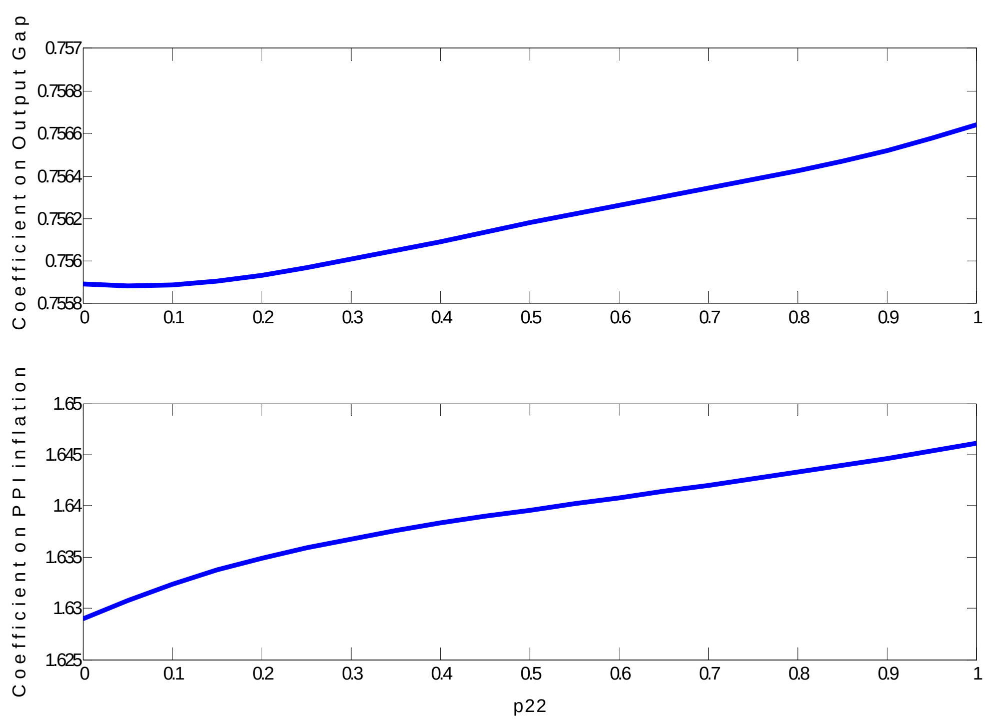


Figure 10: Coefficients in the Dovish regime



The first result from the two figures above is that the home central bank must change the coefficients in its interest rate rule as the probabilities of staying in one regime or the other change. Therefore, it is not optimal to adopt a regime invariant interest rate rule. The second is that, the weight on PPI inflation must increase as the probability of switching to the dovish regime increases[32]. The opposite holds as the probability of switching to the hawikish regime increases. In this case the wieght on PPI inflation falls. The weight on the output gap changes similarly. That is, it rises as the probability of switching to the dovish regime increases, and falls as the probability of moving to the hawkish regime increases. Hence, I end up to the follpwing twp results.

> **Result 4:** *As the probability of the foreign monetary policy switching to the dovish regime increases, the home central bank should become more aggressive to home PPI inflation fluctuations. As the probability of the foreign monetary policy switching to the hawkish regime increases, the home central bank should become less aggressive to home PPI inflation fluctuations.*
>
> **Result 5:** *The home central bank must attach a weight on home PPI inflation that is always greater than one. That is, it must be always hawkish. Moreover, it must even more aggressive to PPI inflation fluctuations, as the foreign central bank becomes dovish.*

Finally, as a last exercise I compare the welfare losses when the home central bank optimally changes the coefficients in its interest rate rule with the case where it naively allows the coefficients in its rule to be time invariant[33]. In this case, the welfare losses and volatilities are computed relative to those in the baseline calibration (and not to those in absorbing state).

8.4 The importance of always reacting optimally.

In this section I am analyzing the importance, in terms of welfare, of an optimal reaction of the home Central Bank to changes in foreign monetary policy. I assume that the home Central Bank

[32] As in Svensson (1998), CPI inflation π_t is not included in the optimal reaction function of the home Central Bank. This is due to the fact that it is not an independent state variable, but, rather, a linear combination of other state variables, i.e. $\pi_{H,t}$ and $\pi_{F,t}$.

[33] The constant parameters used in this exercise are the ones used in the baseline calibration.

always reacts optimally conditional on foreign monetary policy. In particular, I, first, compute the optimal reaction functions of the home Central Bank for each state individually when the latter is an absorbing state, i.e. when foreign policy is in one of the two regimes and is not likely to change. Then, I compare the losses accruing when there is Markov-switching in foreign policy, to those when there is no switching.

Table 10: Relative Losses

	Losses	
	Home	*Foreign*
Hawkish	1.0003	1.0023
Dovish	1.0000	0.9990

The results at table 10 show that when the home Central Bank reacts always optimally conditional on foreign monetary policy, the effects of Markov-switching foreign monetary policy in the home economy are almost zero. More importantly, the foreign country benefits when the home Central Bank reacts optimally to its policy. Foreign welfare loss is only 0.1% higher in the hawkish regime and 0.12% lower in the dovish regime, compared to the absorbing state. Therefore, optimal reaction in the home country is enough to eliminate the huge increase in overall volatility in both countries.

Finally, as a last exercise, I compare rule (52) with the case where the home Central Bank reacts optimally. Given that this rule yields the lowest home welfare losses (relative to the Taylor rule considered in the baseline calibration) than any other of the alternative rules considered in this paper, the comparison of its performance relative to the optimal reaction of the home Central Bank is enough to show how much simple rules are away from the optimal case.

Table 11: *Rule* (52) *vs* *Optimal*

	Losses	
	Home	*Foreign*
Hawkish	3.4663	2.0603
Dovish	3.6832	5.2785

As table 11 shows rule (52) yields losses that are 3.5 times higher in the home country and 2 times higher in the foreign, in the hawkish regime. As regards losses in the dovish regime, they are 3.7

and 5 times higher in the home and the foreign country respectively, relative to the losses accruing under the optimal reaction function.

9 Concluding remarks

Even though the existing literature on MSDSGE modelling focuses on closed economy models, I constructed a two country DSGE model in which foreign monetary switches regimes over time. I gave further insight regarding the effects of regime switching in monetary policy both domestically and abroad. The assumption that it is only the foreign monetary policy that switches regimes was introduced after having estimated a SVAR model for the Eurozone and the US, in which stability tests showed that only the US monetary policy has changed since the adoption of the common currency. I then estimated a Markov-switching interest rate rule for the US. This rule seems to capture the monetary policy of the US for the last 10 years quite well. In the baseline calibration, home monetary policy was assumed to be naively time invariant and follow the Taylor rule with some interest rate smoothing. I made this initial assumption in order to emphasize on the fact that inflation volatility may grow over time, even if domestic monetary policy is stable. Home inflation was shown to be affected both in terms of volatility and in terms of its response to alternative shocks, the only reason being the regime shitfs in foreign monetary policy, and, hence, the change in the way the expectations of the private sector are formed. Foreign monetary policy regime shifts generate a stabilization and an amplifying effect on output and inflation, both in the foreign and home country. Which effect arises depends on which regime the foreign monetary policy lies in. When the latter is dovish there is a stabilization effect, given a non-zero probability of the foreign monetary policy becoming hawkish in the future. When foreign monetary policy is hawkish there is an amplifying effect in both countries, given a non-zero probability of the foreign monetary policy becoming dovish in the future. Moreover, there seems to be an asymmetry on the size of each effect. In particular, I showed that the stabilizing effect is stronger both in the foreign country, based on a welfare measure, derived by a second order approximation of the agents utility function. Finally, through the solution of the dynamic programming problem of the home central banker, conditional on foreign monetary policy switching regimes over time, it was shown that it is optimal to follow a time varying interest rate rule. When the home Central Bank reacts optimally, the effects of regime switches in foreign monetary policy on the home country are completely eliminated. Moreover, the foreign country seems to benefit a lot, in terms of its welfare measure, when the home country reacts optimally to its policy.

Appendix A: The steady State

In this section I compute the steady state of the the real variables, first and then through the proof of proposition 1, the steady state of the nominal variables.

Given that in the steady state each firm will chanrge the same price in both countries, the law of onw price holds and, hence, PPP holds as well. Threfore the real exchangre rate is pegged to one.

$$Q = 1$$

Given an international risk sharing condition, PPP implies that at the steady state consumption levels will be equalized across the two countries. Hence

$$C = C^*$$

From the reresentative household's labor supply decision, I have for each country that

$$L^{\gamma} = C^{-\sigma} \frac{W}{P}$$

$$L^{*\gamma} = C^{*-\sigma} \frac{W^*}{P^*}$$

while from the firms production function in each country, I have that

$$Y = L \quad and \quad Y^* = L^*$$

As already mentioned, firms will set the same price in each country. From their maximization problem it follows that prices at the steady state will be specified as follows

$$p_H = Sp_H^* = P_H = \frac{\theta}{\theta - 1} \frac{W}{A}$$

$$\frac{p_F^*}{S} = p_F = P_F^* = \frac{\theta}{\theta - 1} \frac{W^*}{A^*}$$

and since the law of one price holds, the demand for the home and foreign produced good respectively will be specified as

$$Y_H = \left(\frac{P_H}{P}\right)^{-\rho} C$$

$$Y_F = \left(\frac{P_F^*}{P^*}\right)^{-\rho} C$$

Combining, thus, the above equations, along with the household's optimal labor decision I end up to the following expressions for the consumption levels in the steady state

$$C = \left[\frac{\theta - 1}{\theta}\left(\frac{P_H}{P}\right)^{1+\rho\gamma} A\right]^{\frac{1}{\gamma+\sigma}}$$

$$C^* = \left[\frac{\theta - 1}{\theta}\left(\frac{P_F^*}{P^*}\right)^{1+\rho\gamma} A^*\right]^{\frac{1}{\gamma+\sigma}}$$

As in Benigno (2004), note that both $\frac{P_H}{P}$ and $\frac{P_F}{P}$ are both functions of $T \equiv \frac{P_F}{P_H}$, so that the two equations above uniquely determine C and T. Having specified the steady state values of consumption output and relative prices, I can proceed to the proof of proposition in section 5.

Proof of Proposition in section 5

The foreign households intertemporal decision (12) implies that in the steady state the following will be true for the nominal interest rate

$$i^* = \frac{\pi^*}{\beta}$$

Additionally, the assumed interest rate rule of the foreign country (31) receives the following form in the steady state

$$i = \xi_s \left(\frac{\pi^*}{\tilde{\pi}^*}\right)^{\phi^*_{\pi^*}} y^{*\phi^*_{y^*,s}}$$

Combining the above two equations for the foreign interest rate, solving for ξ_s and recalling that the interest rate in the steady state is such that foreign inflation π^* hits its target $\tilde{\pi}^*$, I receive the following

$$\xi_s = \frac{1}{\beta}\pi^* y^{*\phi^*_{y^*,s}}$$

Therefore the steady state interest rate is

$$i^* = \frac{\tilde{\pi}^*}{\beta}$$

and, as already mentioned, inflation at the steady state is $\pi^* = \tilde{\pi}^*$. Nominal variables, thus, are indepedent of policy regime in the steady state. Moreover, as already shown above, the real variables (i.e. consumption, output, labor) are independent of policy regime, as well, in the steady state.

Appendix B: Aggregate Supply and Aggregate Demand

In this section I derive the PPI inflation rates (33) and (34) and the aggregate demand equation (38) reported in the text.

Aggregate Supply

Forward looking producers in the home country maximize their profits in the home market by choosing the optimal price specified as

$$p_t^{For}(h) = \frac{\theta}{\theta - 1} \frac{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} MC_{t+s} y_{t+s}^h(p_t(h))}{E_t \sum_{s=0}^{\infty} \omega^s Q_{t,t+s} y_{t+s}^h(p_t(h))}$$

where $y_{t+s}^h(p_t(h))$ is specified in (34) in the text. The optimal price above rearranged can be written in the following form

$$E_t \sum_{s=0}^{\infty} (\omega\beta)^s \frac{C_{t+s}^{-\sigma} P_{H,t+s}}{P_{t+s}} \left[\left\{ \frac{p_t^{For}(h)}{P_{H,t+s}} - \left(\frac{\theta}{\theta - 1} \right) \frac{W_{t+s}}{A_{t+s} P_{H,t+s}} \right\} y_{t+s}^h(p_t(h)) \right] = 0$$

and its loglinear approximation is summarized as follows

$$E_t \sum_{s=0}^{\infty} (\omega\beta)^s \left[\hat{p}_{t,t+s}^{For}(h) - \left(\frac{\widehat{W_{t+s}}}{A_{t+s} P_{H,t+s}} \right) \right] = 0 \tag{61}$$

where $\hat{p}_t^{For}(h) = ln \left(\frac{p_t^{For}(h)}{P_{H,t+s}} \right)$. Using the household's optimality condition (13) I can expand the marginal cost term in the above relationship as follows

$$\frac{\widehat{W_{t+s}}}{A_{t+s} P_{H,t+s}} = \gamma \left(\hat{y}_{t+s}(h) - a_t \right) + \frac{\sigma}{\psi} \hat{C}_{t+s} + \frac{(1-\psi)\sigma}{\psi} \hat{C}_{t+s-1} + a_{t+s} + (1-\delta) \hat{T}_{t+s}$$

where I have used the fact that $\hat{C_t^O} = \frac{1}{\psi}\hat{C}_t - \frac{1-\psi}{\psi}\hat{C}_{t-1}$. Furthermore, by suing the demand for the home good $\hat{y}_{t+s}(h)$ can be expanded as follows

$$\hat{y}_{t+s}(h) = -\rho\delta\hat{\tilde{p}}_{t,t+s}(h) + \rho\delta\left(1-\delta\right)\hat{T}_{t+s} + \hat{C}_{t+s} - \rho\left(1-\delta^*\right)\hat{\tilde{p}}^*_{t,t+s}(h)\ldots$$

$$-\rho\delta^*\left(1-\delta^*\right)\hat{T}^*_{t+s} - \frac{\left(1-\delta^*\right)}{\sigma}\hat{q}_{t+s}$$

But $\hat{\tilde{p}}_{t+s}(h)$ and $\hat{\tilde{p}}^*_{t,t+s}(h)$ are specified as

$$\hat{\tilde{p}}_{t+s}(h) = \zeta\hat{p}^{For}_{t+s}(h) + \left(1-\zeta\right)\hat{p}^B_{t+s}(h)$$

$$\hat{\tilde{p}}^*_{t,t+s}(h) = \zeta\hat{p}^{*For}_{t+s}(h) + \left(1-\zeta\right)\hat{p}^{*B}_{t+s}(h)$$

for the home good in the home and the foreign market respectively. From (19) $\hat{\tilde{p}}_t(h)$ and $\hat{\tilde{p}}^*_t(h)$ can be expressed as follows

$$\hat{\tilde{p}}_{t,t+s}(h) = \frac{\omega}{1-\omega}\pi_{H,t} - \sum_{i=1}^{s}\pi_{H,t+i}$$

$$\hat{\tilde{p}}^*_{t,t+s}(h) = \frac{\omega^*}{1-\omega^*}\pi^*_{H,t} - \sum_{i=1}^{s}\pi^*_{H,t+i}$$

Combining the above relationships for the prices set at date t , I can express the price set by the forward looking firms as follows

$$\hat{p}^{For}_t(h) - P_{H,t-1} = \frac{1}{(1-\omega)(1-\zeta)}\pi_{H,t} - \frac{\zeta}{(1-\omega)(1-\zeta)}\pi_{H,t-1}$$

Solving for $\hat{p}^{For}_{t,t+s}(h)$ in (61) and combining all the above relationships I end to the following relationship for PPI inflation

$$\pi_{H,t} = \frac{\zeta}{(\zeta + \omega(1-\zeta) + \theta\gamma\delta\omega(1-\zeta))}\pi_{H,t-1} + \frac{(\omega - \omega^*)(\gamma\theta(1-\delta^*)(1-\zeta)(1-\omega))}{(1-\omega^*)(\zeta + \omega(1-\zeta) + \theta\gamma\delta\omega(1-\zeta))}\pi^*_{H,t} + \dots$$

$$\frac{(1-\omega\beta)(1-\zeta)(1-\omega)}{(\zeta + \omega(1-\zeta) + \theta\gamma\delta\omega(1-\zeta))}\hat{R}_t + \frac{\omega\gamma\theta(1-\delta^*)(1-\zeta)(1-\omega)}{(1-\omega^*)(\zeta + \omega(1-\zeta) + \theta\gamma\delta\omega(1-\zeta))}\left(\beta E_t\pi^*_{H,t+1} - \pi^*_{H,t}\right)$$

where $\hat{R}_t$ is specified as

$$\hat{R}_t = (1+\gamma\rho\delta)(1-\delta)\hat{T}_t + \left(\gamma + \frac{\sigma}{\psi}\right)\hat{C}_t - \gamma\rho\delta^*(1-\delta^*)\hat{T}^*_t - \frac{\gamma(1-\delta^*)}{\sigma}\hat{q}_t - \frac{(1-\psi)\sigma}{\psi}\hat{C}_{t-1} - (\gamma+1)a_{t+s}$$

and from the resource constraint

$$\hat{C}_t = \hat{Y}_t - \rho\delta(1-\delta)\hat{T}_t + \rho(1-\delta^*)\delta^*\hat{T}^*_t + \left(\frac{1-\delta^*}{\sigma}\right)\hat{q}_t$$

The supply of home produced goods in the foreign country is derived by following similar steps. Home producers se t their price in foreign country according to the following maximization rule

$$E_t\sum_{s=0}^{\infty}(\omega^*\beta)^s\frac{C^{-\sigma}_{t+s}P_{H,t+s}}{P_{t+s}}\left[\left\{\frac{p^{*For}_t(h)}{P^*_{H,t+s}}\frac{Z_{t+s}P^*_{H,t+s}}{P_{H,t+s}} - \left(\frac{\theta}{\theta-1}\right)\frac{W_{t+s}}{A_{t+s}P_{H,t+s}}\right\}y^f_{t+s}(p_t(h))\right] = 0$$

and its loglinear approximation is summarized as follows

$$E_t\sum_{s=0}^{\infty}(\omega\beta)^s\left[\hat{\hat{p}}^{*}_{t,t+s}{}^{For}(h) + \widehat{zh_t} - \left(\frac{\widehat{W_{t+s}}}{A_{t+s}P_{H,t+s}}\right)\right] = 0 \qquad (62)$$

where $zh_t = \frac{Z_tP^*_{H,t}}{P_{H,t}}$. And after following similar steps as in the derivation of the supply in the home country I conlcude to the following for the supply of home goods in the foreign country

$$\pi^*_{H,t} = \frac{\zeta}{(\zeta + \omega^*(1-\zeta) + \theta\gamma\delta\omega^*(1-\zeta))}\pi^*_{H,t-1} + \frac{(\omega^* - \omega)(\gamma\theta\delta(1-\zeta)(1-\omega^*))}{(1-\omega)(\zeta + \omega^*(1-\zeta) + \theta\gamma\delta\omega^*(1-\zeta))}\pi_{H,t} + \dots$$

$$\frac{(1-\omega^*\beta)(1-\zeta)(1-\omega^*)}{(\zeta + \omega^*(1-\zeta) + \theta\gamma\delta\omega^*(1-\zeta))}\hat{R}_t + \frac{\omega^*\gamma\theta\delta(1-\zeta)(1-\omega^*)}{(1-\omega)(\zeta + \omega^*(1-\zeta) + \theta\gamma\delta\omega^*(1-\zeta))}(\beta E_t\pi_{H,t+1} - \pi_{H,t})$$

Aggregate Demand

The market clearing condition for home goods market satisfies the following

$$Y_t = C_{H,t} + C^*_{H,t}$$

or

$$Y_t = \left(\frac{P_{H,t}}{P_t}\right)^{-\rho} \delta C_{H,t} + \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-\rho} (1-\delta^*)\, C^*_{H,t}$$

and after loglinearizing and solving for $\hat{C}_t$, I receive the following

$$\hat{C}_t = \hat{Y}_t - \rho\delta\,(1-\delta)\,\hat{T}_t + \rho\,(1-\delta^*)\,\delta^*\hat{T}^*_t + \left(\frac{1-\delta^*}{\sigma}\right)\hat{q}_t$$

Using the Euler equation accruing from the optimizing households loglinearized first order condition (12) and the fact that $\hat{C^O_t} = \frac{1}{\psi}\hat{C}_t - \frac{1-\psi}{\psi}\hat{C}_{t-1}$, I end up to the aggregate demand equation for the home country

$$\hat{Y}_t = -\frac{\psi}{(2-\psi)\,\sigma}\,(i_t - E_t\pi_{t+1}) + \frac{1}{2-\psi}E_t\hat{Y}_{t+1} + \frac{1-\psi}{2-\psi}\hat{Y}_{t-1} - \frac{\rho\delta\,(1-\delta)}{2-\psi}E_t\hat{T}_{t+1} + \frac{\rho\delta^*\,(1-\delta^*)}{2-\psi}E_t\hat{T}^*_{t+1} + \ldots$$
$$\frac{(1-\delta^*)}{(2-\psi)\,\sigma}E_t\hat{q}_{t+1} + \rho\delta\,(1-\delta)\,\hat{T}_t - \rho\delta^*\,(1-\delta^*)\,\hat{T}^*_t - \frac{1-\delta^*}{\sigma}\hat{q}_t - \frac{\rho\delta\,(1-\psi)\,(1-\delta)}{2-\psi}\hat{T}_{t-1} + \ldots$$
$$\frac{\rho\delta^*\,(1-\psi)\,(1-\delta^*)}{2-\psi}\hat{T}^*_{t-1} + \frac{(1-\psi)\,(1-\delta^*)}{(2-\psi)\,\sigma}\hat{q}_{t-1}$$

and similarly for the foreign country

$$\hat{Y}^*_t = -\frac{\psi^*}{(2-\psi^*)\,\sigma}\left(i^*_t - E_t\pi^*_{t+1}\right) + \frac{1}{2-\psi^*}E_t\hat{Y}^*_{t+1} + \frac{1-\psi^*}{2-\psi^*}\hat{Y}^*_{t-1} - \frac{\rho\delta^*\,(1-\delta^*)}{2-\psi^*}E_t\hat{T}^*_{t+1} + \frac{\rho\delta\,(1-\delta)}{2-\psi^*}E_t\hat{T}_{t+1} - \ldots$$
$$-\frac{(1-\delta)}{(2-\psi^*)\,\sigma}E_t\hat{q}_{t+1} + \rho\delta^*\,(1-\delta^*)\,\hat{T}^*_t - \rho\delta\,(1-\delta)\,\hat{T}_t + \frac{1-\delta}{\sigma}\hat{q}_t - \frac{\rho\delta^*\,(1-\psi^*)\,(1-\delta^*)}{2-\psi^*}\hat{T}^*_{t-1} + \ldots$$
$$\frac{\rho\delta\,(1-\psi^*)\,(1-\delta)}{2-\psi^*}\hat{T}_{t-1} - \frac{(1-\psi^*)\,(1-\delta^*)}{(2-\psi^*)\,\sigma}\hat{q}_{t-1}$$

Appendix C: The welfare criterion

In this section I derive the second order approximation (44) to the representative household's utility function (6) in the home country. The steps for the derivation of the welfare measure for the foreign country are exactly the same. I assume that there is a subsidy to labor. This implies that the steady state is efficient, given that the distortions form monopolistic competition are exhausted. Therefore, I derive the welfare criterion for each country using a second-order Taylor series expansion of (6) around the efficient steady state. Moreover, the welfare measure is expressed as deviations from the flexible price equilibrium, which is efficient as well, given the labor subsidy.

The second order approximation of the welfare of the representative optimizing household receives the following form

$$W_t = U + U_C(\hat{C_t^O} + \frac{1}{2}(1 + \frac{U_{CC}C}{U_C})\hat{C_t^O}^2) - U_L(\hat{L}_t + \frac{1}{2}(1 + \frac{U_{LL}L}{U_L})\hat{L}_t^2 \tag{63}$$

where $U_C = C^{-\sigma}$, $U_{CC} = C^{-\sigma-1}$, $U_L = L^{\gamma}$ and $U_{LL} = L^{\gamma-1}$. Using the fact that $\hat{y}_t(h) = a_t + \hat{L}_t$ and approximating it up to a second order I receive the following expression for labor

$$\hat{L}_t = 1 + \frac{y(h)}{L}E_t(\hat{y}_t(h)) + a_t + \frac{y(h)}{2L}var(\hat{y}_t(h)) + a_t^2 - \frac{1}{2}\hat{L}_t^2 \tag{64}$$

Moreover by Woodford (Ch. 6) I have that

$$var(\hat{y}(i)) = \delta\theta^2 var(\tilde{p}_t(h)) + (1-\delta)\theta^2 var(\tilde{p}_t^*(h)) \tag{65}$$

But $\tilde{p}_t(h)$ and $\tilde{p}_t(h)$ are determined according to (20) in the main text. Let $\bar{P}_{H,t} \equiv E_t\left[log(\tilde{p}_t(h))\right]$ and $\Delta_t \equiv var(log(\tilde{p}_t(h))$. Then,

$$\begin{aligned}
\Delta_t &\equiv var(log(\tilde{p}_t(h) - P_{H,t-1}) \\
&= E_t\left[(log(\tilde{p}_t(h) - P_{H,t-1})^2 - (E_t\left[log(\tilde{p}_t(h) - P_{H,t-1}\right])^2\right] \\
&= \omega\Delta_{t-1} + (1-\omega)\zeta(log(p_t^B(h) - \bar{P}_{H,t-1})^2 + (1-\omega)(1-\zeta)(log(p_t^{For}(h) - \bar{P}_{H,t-1})^2 \\
&\quad -(\bar{P}_{H,t} - \bar{P}_{H,t-1})
\end{aligned} \tag{66}$$

where $p_t^B(h)$ and $p_t^F(h)$ are the prices set by the backward and forward looking firms respectively. The same expression holds for $\tilde{p}_t^*(h)$. Before substituting the above expression in (2) and then in (1), note that $\bar{P}_{H,t} = log(\bar{P}_{H,t}) + O(||\xi||^2)$, so that $\bar{P}_{H,t} - \bar{P}_{H,t-1} = \pi_{H,t} + O(||\xi||^2)$. Additionally, the following relationships hold

$$\tilde{p}_t(h) = \zeta p_t^B(h) + (1-\zeta)\, p_t^{For}(h)$$

$$\tilde{p}_t(h) = \frac{\omega}{1-\omega}\pi_{H,t} + P_{H,t}$$

Using the above expressions for $\tilde{p}_t(h)$ I end up to the following expression for the price that is set by the forward looking firms

$$\hat{p}_t^{For}(h) - P_{H,t-1} = \frac{1}{(1-\omega)(1-\zeta)}\pi_{H,t} - \frac{\zeta}{(1-\omega)(1-\zeta)}\pi_{H,t-1}$$

Subsituting the above expression into (61), I receive the following for Δ_t

$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \frac{1}{(1-\omega\beta)}\sum_{t=0}^{\infty}\beta^t\left[\frac{\omega}{1-\omega}\pi_{H,t}^2 + \frac{1-\zeta}{\zeta(1-\omega)}\left(\pi_{H,t} - \pi_{H,t-1}\right)^2\right] + t.i.p. + O(||\xi||^3) \tag{67}$$

Similarly for the price set in the foreign country for the home good I receive the following

$$\sum_{t=0}^{\infty}\beta^t\Delta_t^* = \frac{1}{(1-\omega^*\beta)}\sum_{t=0}^{\infty}\beta^t\left[\frac{\omega^*}{1-\omega^*}\pi_{H,t}^{*2} + \frac{1-\zeta}{\zeta(1-\omega^*)}\left(\pi_{H,t}^* - \pi_{H,t-1}^*\right)^2\right] + t.i.p. + O(||\xi||^3) \tag{68}$$

where $t.i.p.$ respresents terms independent of policy and $O(||\xi||^3)$ stands for terms of order higher than two.

Additionally, note that for the home output the following relationship holds (and similartly for foreign output)

$$\hat{Y}_t = E_t(\hat{y}_t(h)) + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right)var(\hat{y}_t(h)) + O(||\xi||^3)$$

Using the above expression to substitute for $E_t(\hat{y}_t(i))$ in equation (2), I receive the following expression for $\hat{L}_t$

$$\hat{L}_t \approx 1 + \frac{Y}{L}\hat{Y}_t - \frac{1}{2\theta}\frac{Y}{L}var(\hat{y}_t(h)) - \frac{1}{2}\hat{L}_t^2 + t.i.p. \tag{69}$$

Finally, a second order approximation of the resource constraint of the model yields the following

$$\hat{C}_t \approx \frac{1}{2}\hat{Y}_t + \frac{1}{4}\hat{Y}_t^2 + \frac{1}{2}\hat{Y}_t^* + \frac{1}{4}\hat{Y}_t^{*2} + \frac{1}{2\sigma}\hat{q}_t + \frac{1}{4\sigma^2}\hat{q}_t^2 - \frac{1}{2\sigma}\hat{q}_t\hat{C}_t \tag{70}$$

Recalling that

$$C_t = \psi C_t^O + (1-\psi)C_t^R$$

and

$$C_t^R = C_{t-1}$$

so that

$$\hat{C_t^O} = \frac{1}{\psi}\hat{C}_t - \frac{1-\psi}{\psi}\hat{C}_{t-1} \tag{71}$$

Substituting, thus, (8) into (1), I receive the following form for welfare

$$W_t = U + U_C(\frac{1}{\psi}\hat{C}_t - \frac{1-\psi}{\psi}\hat{C}_{t-1} + \frac{1}{2}(1+\frac{U_{CC}C}{U_C})(\frac{1}{\psi}\hat{C}_t^2 + \frac{1-\psi}{\psi}\hat{C}_{t-1}^2 + \frac{1-\psi}{\psi^2}\hat{C}_t\hat{C}_{t-1}))$$
$$-U_L(\hat{L}_t + \frac{1}{2}(1+\frac{U_{LL}L}{U_L})\hat{L}_t^2 + t.i.p. + O(||\xi||^3) \tag{72}$$

Substituting (67), (68), (69) and (70) into (72), I receive the following form for the welfare measure

$W_t = -\frac{1}{2}u_c C\Xi\{\lambda_1(\hat{Y}_t - y_t^n)^2 + \lambda_2(\hat{Y}_t^* - y_t^{*n})^2 + \lambda_3(\hat{q}_t - q_t^n)^2 + \lambda_4\Delta\hat{q}_t^2 + \lambda_5\Delta\hat{Y}_t^{*2} + \lambda_6\Delta\hat{Y}_t^2 + \ldots$
$+\pi_{H,t}^2 + \lambda_7(\pi_{H,t} - \pi_{H,t-1})^2 + \lambda_8(\pi_{H,t}^*)^2 + \lambda_9(\pi_{H,t}^* - \pi_{H,t-1}^*)^2 + \lambda_{10}(\hat{q}_t + \hat{Y}_t)^2 + \lambda_{11}(\hat{q}_t + \hat{Y}_t^*)^2 + \lambda_{12}(\hat{q}_{t-1} + \hat{Y}_t)^2 + \lambda_{13}(\hat{q}_{t-1} + \hat{Y}_t^*)^2 \ldots \lambda_{12}(\hat{q}_{t-1} + \hat{Y}_t)^2 + \lambda_{13}(\hat{q}_{t-1} + \hat{Y}_t^*)^2 + \lambda_{14}(\hat{Y}_{t-1}^* - y_{t-1}^{*n})(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{15}(y_{t-1} - y_{t-1}^n)(y_{t-1}^* - y_{t-1}^{*n}) + \lambda_{16}(\hat{C}_t - c_t^n)(\hat{q}_t - q_t^n) +$
$+\lambda_{17}(\hat{Y}_t + \hat{Y}_{t-1}^*)^2 + \lambda_{18}(\hat{Y}_{t-1} + \hat{Y}_t^*)^2 + \lambda_{19}(\hat{Y}_{t-1} - y_{t-1}^n)(q_{t-1} - q_{t-1}^n) + \lambda_{20}(\hat{Y}_t^* - \hat{Y}_t^{*n})(\hat{Y}_{t-1}^* - \hat{Y}_{t-1}^{*n}) + \lambda_{21}(\hat{Y}_{t-1}^* + \hat{q}_t)^2 + \lambda_{22}(\hat{Y}_{t-1} + \hat{q}_t)^2 + \lambda_{23}(\hat{Y}_{t-1} - y_{t-1}^n)(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{24}(\hat{C}_{t-1}^* - c_{t-1}^{*n})(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{25}(\hat{q}_t - q_t^n)(\hat{q}_{t-1} - q_{t-1}^n) + \lambda_{26}(\hat{Y}_{t-1} - y_{t-1}^n)(\hat{Y}_t - y_t^n) + t.i.p. + O(||\xi||^3)$

where

$\Xi = (\theta\omega)(\sigma/(-1+\sigma))^{-\rho}C^{-\sigma(1-\rho)}L^{\gamma(1+\rho)})/(1-\omega)(1-\omega\beta)$

$\lambda_1 = \Xi(((3(-1+\sigma-2\psi)+16(C-1))(L^\gamma+1))\gamma(\psi^2))/(16(\psi))) + ((3+3\sigma(-1+\psi)-\psi)(-1+\psi)/(16(\psi^2)))-$
$-(\sigma-1)(1-\psi)/(2\sigma(\psi^2)) - (1-\psi)(-1+\sigma)/(4\psi^2) - -(1-\psi)(-1+\sigma)/(4\psi^2))$

$\lambda_2 = -\Xi((3(1-\sigma+2\psi)/(16(\psi^2))) - ((3+3\sigma(-1+\psi)-\psi)(1-\psi)/(16\psi^2)) - (1-\psi)*(-2+2\sigma+\psi)/(8\sigma(\psi^2))-$
$-(1-\psi)(-1+\sigma)/(4\psi^2) - (1-\psi)(-1+\sigma)/(4\sigma\psi^2) - -(1-\psi)(-1+\sigma)/(4\psi^2))$

$\lambda_3 = \Xi(((5-5\sigma+2\psi)/(16((\sigma\psi)^2)))+(-1+\sigma)(1-\psi)/(2\sigma\psi^2)+(1-\psi)(-2+2\sigma+\psi)/(8\sigma\psi^2)+(1-\psi)*(-1+\sigma)/(4\sigma\psi^2))$

$\lambda_4 = -\Xi(-((\sigma-1)(1-\psi)/(2\sigma\psi^2)) + ((-1+\psi)(5+15\psi+\sigma(-5+13\psi))/(16(\sigma\psi)^2)) - (1-\psi)(-1+\sigma)/(4\sigma\psi^2))$

$\lambda_5 = -\Xi((3+3\sigma(-1+\psi)-\psi)*(1-\psi)/(16\psi^2) - (1-\psi)(-2+2\sigma+\psi)/(8\sigma\psi^2) - (1-\psi)(-1+\sigma)/(4\psi^2))$

$\lambda_6 = -\Xi(((3+3\sigma(-1+\psi)-\psi)(-1+\psi)/(16(\psi^2))) - ((-1+\sigma)(1-\psi)/(4\sigma(\psi^2))) - (1-\psi)(-1+\sigma)/(4\psi^2))$

$\lambda_7 = \upsilon\zeta/(\omega(1-\zeta))$,$\lambda_8 = \upsilon\omega^*(1-\omega)(1-\omega\beta)/(\omega(1-\omega^*)(1-\omega^*\beta))$, $\lambda_9 = \upsilon\zeta(1-\omega)(1-\omega\beta)/(\omega(1-\omega^*)(1-$
$\zeta)(1-\omega^*\beta))$ $\lambda_{10} = -\Xi(-1+\sigma+2\psi)/(8\sigma\psi^2)$, $\lambda_{11} = -\Xi(-1+\sigma+2\psi)/(8\sigma\psi^2)$, $\lambda_{12} = -\Xi((\sigma-1)(1-\psi)/(2\sigma\psi^2))$

$\lambda_{13} = -\Xi(1-\psi)(-1+\sigma)/(4\sigma\psi^2)$ $\lambda_{14} = -\Xi(-1+\psi)(1-\psi+\sigma(-1+5\psi))/(8\sigma\psi^2)$, $\lambda_{15} = \Xi(1+\sigma(-1+\psi)-$
$3\psi)(-1+\psi)/(8(\psi^2))$ $\lambda_{16} = -\Xi(-1+\sigma)/(2\sigma\psi^2)$ $\lambda_{17} = -\Xi(-1+\sigma)(1-\psi)/(4\psi^2)$, $\lambda_{18} = -\Xi(-1+\sigma)/(4\psi^2)$,

$\lambda_{19} = -\Xi(-1+\psi)(1-3\psi+\sigma(-1+5\psi))/(8\sigma\psi^2)$
$\lambda_{20} = -\Xi(((-1+\sigma)(1-\psi)/(4\psi^2)) + ((3+3\sigma(-1+\psi)-\psi)(1-\psi)/(8\psi^2)) - (1-\psi)(-2+2\sigma+\psi)/(8\sigma\psi^2) + (1-$
$\psi)(-1+\sigma)/(2\psi^2))$ $\lambda_{21} = -\Xi(1-\psi)(-2+2\sigma+\psi)/(8\sigma\psi^2)$, $\lambda_{22} = \Xi(1-\psi)(-1+\sigma)/(4\psi^2)$

$\lambda_{23} = -\Xi(-1+\psi)(1-3\psi+\sigma(-1+5\psi))/(8\sigma\psi^2)$, $\lambda_{24} = \Xi(-1+\sigma)((-1+\psi)^2)/(2\sigma\psi^2)$ $\lambda_{25} = -\Xi(((-1+\sigma)(1-$
$\psi)/(4(\sigma\psi)^2)) - ((\sigma-1)(1-\psi)/(\sigma\psi^2)) + ((-1+\psi)(5+15\psi+\sigma(-5+13\psi))/(8(\sigma\psi)^2)) - (1-\psi)(-1+\sigma)/(4\sigma\psi^2))$ $\lambda_{26} =$
$-\Xi(((-1+\sigma)(1-\psi)/(2\psi^2)) + ((3+3\sigma(-1+\psi)-\psi)(-1+\psi)/(8\psi^2)) - ((-1+\sigma)(1-\psi)/(4\sigma\psi^2)) + (1-\psi)(-1+\sigma)/(2\psi^2))$

References

[1] Amato, J. D. and T. Laubach (2003). Rule-of-thumb behavior and monetary policy, European Economic Review, 47: 791-831.

[2] Benigno, G. (2004). Real exchange persistence and monetary policy rules, Journal of Monetary Economics, 51(3): 473-502.

[3] Benigno, G. and P. Benigno (2001). Monetary policy rules and the exchange rate, CEPR discussion paper No. 2807.

[4] Benigno, G. and P. Benigno (2006). Designing targeting rules for international monetary policy cooperation, Journal of Monetary Economics, 53(3): 473-506.

[5] Benigno, P and M. Woodford (2006). Linear-quadratic approximation of optimal policy problems, NBER working paper, 12672.

[6] Boivin, J and M. Giannoni (2006). Has monetary policy become more effective?, Review of Economics and Statistics, 88(3): 445-462.

[7] Boivin, J and M. Giannoni (2002). Assessing the changes in monetary transmission mechanism: A VAR approach, Federal Reserve Bank of New York Policy Review 8:1, 97-112.

[8] Calvo, G., (1983). Staggered Prices in a Utility Maximizing Framework, Journal of Monetary Economics, 12: 383-398.

[9] Christiano, L, M. Eichenbaum and C. L. Evans, (1999). Monetary policy shocks: What have we learned and to what end?, in John B. Taylor and Michael Woodford, eds., Handbook of Macroeconomics, 65-148. Amsterdam: Elsevier.

[10] Clarida, R., J. Gali and M. Gertler, (2001). Monetary policy rules and macroeconomic stability: Evidence and some theory, Quarterly Journal of Economics, 147-180.

[11] Clarida, R., J. Gali and M. Gertler, (1998). Monetary policy rules in practice: Some international evidence, European Economic Review, 42: 1033-1097.

[12] Coagley, T. and T. Sargent, (2005). Drifts and volatilities: Monetary policies and outcomes in the Post-WWII US, Review of Economic Dynamics, 8: 262-302.

[13] Davig, T. and E. Leeper (2007). Endogenous monetary policy regime change, American Economic Review.

[14] Farmer, R., D. Waggoner and T. Zha (2011). Minimal state variable solutions to Markov-switching rational expectations models, Journal of Economic Dynamics and Control, forthcoming.

[15] Gali, J., (2002). New perspectives on monetary policy, inflation and the business Cycle. NBER working paper.

[16] Galí, J., M. Gertler, and D. López-Salido (2001). European Inflation Dynamics, European Economic Review, 45(7): 1237-1270.

[17] Galí, J. and M. Gertler (1999). Inflation Dynamics: A Structural Econometric Analysis, Journal of Monetary Economics, 44(2): 195-222.

[18] Levin, A., V. Wieland and J. C. Williams (1999). Robustness of simple monetary policy rules under uncertainty. In: Taylor J. B. (ed), Monetary policy rules, University of Chicago press, Chicago.

[19] Liu, Z., D. Waggoner and T. Zha (2009). Asymmetric expectation effects of regime shifts in monetary policy, Review of Economic Dynamics, 12, 284-303.

[20] Lubik, T. and F. Schorfheide (2004). Testing for indeterminacy: An application to US monetary Policy, American Economic Review, 94: 190-217.

[21] Obstfeld, M. and K. Rogoff, (1998). Risk and exchange rates, NBER working paper 6694.

[22] Orphanides, A. (2001). Monetary policy rules based on real time data, American Economic Review, 964-985.

[23] Rotemberg, J, and M. Woodford, (1998). An optimization based framework for the evaluation of monetary policy, NBER Technical Working Paper, No 233.

[24] Sbordone, A., (1998). Prices and Unit Labor Costs: Testing Models of Pricing Behavior, mimeo

[25] Soderlind, P., (1999). Solution and estimation of RE macromodels with optimal policy, European Economic Review, 43: 813-823.

[26] Svensson, L. (2000). Open-Economy Inflation Targeting, Journal of International Economics, 50(1): 155-183.

[27] Zampolli F. (2006). Opitmal monetary policy in a regime switching economy: The response to abrupt shifts in exchange rate dynamics, Journal of Economic Dynamics and Control, 30: 1527-1567.

[28] Woodford, M., (2003). Interest and Prices: Foundations of a Theory of Monetary Policy, Princeton University Press, New Jersey.