History-based price discrimination in markets with switching costs: who gains and who loses?

Andrew Rhodes*

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Abstract

In some markets firms try to attract their competitors' customers by offering them a discount if they switch. This paper seeks to understand the impact of this history-based price discrimination, in a model where consumers incur costs when switching between suppliers. We show that price discrimination leads to fiercer competition, and therefore often benefits consumers but harms firms. In contrast with other papers, price discrimination can be welfare-enhancing. This is because price discrimination effectively causes firms to lend money to young consumers, and this lending is socially valuable if consumers discount the future sufficiently highly relative to firms.

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*Contact: *andrew.rhodes@economics.ox.ac.uk* Department of Economics, Manor Road Building, Oxford, OX1 3UQ. I would like to thank Natalia Fabra, Paul Klemperer, Alexei Parakhonyak, John Thanassoulis and Michael Waterson for their helpful comments. Financial assistance from the ESRC (UK) and British Academy is also gratefully acknowledged. Any remaining errors are my own.

1 Introduction

Price discrimination - the practice where a firm charges different consumers a different price for the same product - is ubiquitous. For example students receive money, railcards or other free products (effectively a negative price) when they open a current account. Amongst older consumers, those who switch their current account are rewarded with more favourable terms, such as interest-free overdrafts or preferential rates of interest on savings accounts. Energy companies also often advertise discounts to new customers. These examples have two things in common. Firstly, what a consumer pays is to some extent shaped by where she bought from in the past - so-called history-based price discrimination. Secondly, switching supplier involves some cost, such as the time and hassle required to cancel an old contract and start a new one. An interesting question is therefore what effect price discrimination has on prices and welfare in such markets.

It is well-known that in a static setting without switching costs, the ability to price discriminate *may* cause oligopolists to compete more fiercely, and therefore cause industry profit to be lower (see for example the classic papers by Thisse and Vives 1988 and Holmes 1989). Gehrig *et al* (2011) similarly show that price discrimination can harm firms but benefit consumers, in a one-shot model with switching costs. However these static results may not apply in many markets, because consumers often buy products repeatedly and therefore competition is dynamic. One paper which does analyse history-based price discrimination in a dynamic context with switching costs is Chen (1997), in which consumers live for two periods and have heterogeneous switching costs. Firms sell homogeneous products, and offer old consumers discounts in order to entice them to switch. As in the static models, the ability to price discriminate leads old consumers to pay lower prices. However this changes competition for (and hence the prices paid by) young consumers. As a result price discrimination reduces industry profit, and can also make consumers worse off. It always reduces welfare because it encourages more consumers to switch (a socially wasteful activity) in search of a lower price. Taylor (2003) extends Chen's model to multiple periods and firms, and also shows that discrimination is bad for firms and welfare.

In reality it is very rare to find firms offering completely homogeneous products - either because they use advertising to artificially create differentiation, or because they tailor their products to meet the needs of certain people. For this reason, and in contrast with Chen, we build a model in which firms offer differentiated products. Consumers live for two periods and, in order to isolate the interaction of switching costs with price discrimination, we assume that consumer tastes change over time.[1] We show that when firms are allowed to discriminate, they do offer discounts to other firms' customers. However unlike in Chen's model, history-based discrimination allow firms to earn more profits on old consumers, because it enables them to better exploit their own past consumers' switching costs. At the same time this leads to fiercer competition for young consumers, compared with the situation where price discrimination is banned. In fact we show competition is so fierce that, in general, firms are made worse off by the ability to discriminate but consumers are better off.

Our most interesting and novel result concerns the overall welfare effect of price discrimination. As far as we are aware, all papers on this subject find that history-based price discrimination leads to lower welfare. The reason (which is also true in our model) is that firms offer discounts to people who switch, and this leads to an inefficiently large amount of switching. However in our model there is a countervailing force, which arises because we allow firms and consumers to have different rates of time preference. If consumers are sufficiently impatient relative to firms, it would be socially optimal for firms to lend money to young consumers. But in our model price discrimination does exactly this - it leads to lower prices for young consumers but then forces old consumers to pay more. We show that the social gain from transferring money across periods in this way, can easily outweigh the additional deadweight loss

[1]In particular we assume that a consumer's purchase decision in one period reveals nothing about her likely tastes in the next period. This contrasts with for example Fudenberg and Tirole (2000) in which preferences are perfectly correlated over time, but consumers are able to switch suppliers costlessly. Combining correlated tastes and positive switching costs is difficult, because no pure strategy equilibrium exists unless switching costs are sufficiently high (see Klemperer 1987 for example).

caused by excessive switching. Hence under a wide range of parameters history-based price discrimination is socially beneficial.

2 Model

We build a simple model with overlapping generations of consumers, and use it to evaluate the effects of price discrimination on consumers, firms, and overall welfare. To be precise, time is infinite and discrete, and consumers live for two periods. As is standard within the literature, we use the terms 'young' and 'old' to denote consumers in the first and second years of their lives respectively. Each period a unit mass of consumers is born whilst a unit mass dies and exits the model. In each period consumers are interested in buying one unit of a differentiated product, which is sold by two infinitely-lived firms called A and B. Consumer preferences over these two products are captured using a Hotelling line, with firm A located at one end and firm B located at the other. In each period every consumer is randomly assigned an allocation on the Hotelling line $x \in [0, 1]$, and a consumer located at point x derives gross surplus $V - x$ from product A and $V - (1 - x)$ from product B.[2] If an old consumer previously bought from firm i and wishes to switch to firm $j \neq i$ she must pay a switching cost s which is socially wasteful, and is used to capture the hassle, effort and time required to change supplier. In order to guarantee existence of equilibrium, we restrict attention to switching costs which satisfy $s \in [0, 1/2]$.[3] We assume that both firms and consumers are rational and forward-looking, with discount factors $\delta_f, \delta_c \in (0, 1)$ respectively.

The move order of the game is as follows. Each period the two firms set prices non-cooperatively (the next paragraph explains in more detail how firms make their decision). Young consumers learn their location on the Hotelling line in that period,

[2] As is usual, we assume V to be sufficiently high that in equilibrium every consumer always buys one of the products. The assumption of linear transport costs is necessary to solve for the equilibrium in the no-discrimination case. We have also normalised the transport cost parameter to unity, although this is purely for convenience and has no effect on the results.

[3] When the switching cost goes above this threshold, firm profit functions in the no-discrimination case may not be quasiconcave.

and then buy whichever product is best for them, taking into account how this purchase decision will affect their utility in the following period. When consumers become old they receive a brand new location on the Hotelling line, and decide whether to stay with their existing supplier or pay the switching cost and buy the other product.

In Section 3.1 we solve the model under the assumption that firms are able to use history-based price discrimination. Firms are able to charge three different prices - one price to young consumers, and then two prices to old consumers depending upon whether they previously bought product A or product B. Section 3.2 then analyses a ban on price discrimination, which forces firms to charge all consumers in any given time period the same price. Since the firms are infinitely-lived, in principle the game could have many collusive equilibria. Our focus however will be on symmetric competitive Markov perfect equilibria - meaning that firm pricing strategies depend (at most) on whatever prices were played in the previous period.[4] We first derive expressions for prices, consumer surplus and industry profit, before comparing them in Section 4.

3 Equilibrium prices

3.1 With discrimination

As explained when outlining the model, we look for a symmetric equilibrium where firms charge the same prices in each period. The model in this Section is related to Klemperer 1987, except that we allow firms to charge old consumers different prices depending upon their purchase history. We use the following notation: a firm charges young consumers a price p^y; charges p^l to old consumers who are 'locked' to it (that is, consumers who bought from it in the past) and a price p^u to people who are 'unlocked' (that is, consumers who previously bought from its rival).

[4] If the game were played a large but finite number of times, it is simple to show that the only equilibria would have such a Markov perfect form, so this is a natural class of equilibria to investigate.

The prices paid by old consumers can be characterised as follows. Consider an old consumer who previously bought product A. She can either remain with firm A and earn a payoff $V - x - p^l$, or she can switch to firm B and get $V - (1 - x) - p^u - s$. The old consumer therefore buys product A again if $x \leq \left(1 + p^u - p^l + s\right)/2$, and otherwise switches to firm B. Firm A's profits on its 'locked' consumers can then be written as $p^l \left(1 + p^u - p^l + s\right)/2$; taking a first order condition, the latter is maximised when $p^l = (1 + p^u + s)/2$. Similarly B earns profit $p^u \left(1 - p^u + p^l - s\right)/2$ on its 'unlocked' consumers, and this is maximised when $p^u = \left(1 + p^l - s\right)/2$. Combining these two best responses, old consumers pay prices[5]

$$p^u = 1 - s/3 \quad \text{and} \quad p^l = 1 + s/3 \tag{1}$$

The prices in (1) have several interesting features. Firstly $p^l > p^u$ which reflects the fact that each firm has some market power over its own locked consumers, which it exploits by charging them a high price. Secondly however $p^l < p^u + s$, so old consumers are more likely to stay with their original supplier rather than switch. This makes it valuable for firms to lock-in consumers when they are young.[6] Thirdly p^u and p^l are independent of how many young consumers each firm has previously sold to. Intuitively there are two different aftermarkets - one for people who previously bought product A, and another for people who previously bought product B. The markets are independent because firms can perfectly discriminate between consumers in the two markets; consequently the sizes of the two markets merely scale up demands, but do not affect marginal pricing incentives. (See also Chen 1997 and Taylor 2003.) Now let p_i^y denote the price charged by firm i to young consumers. The following corollary is immediate

Corollary 1 *Young consumers buy from A if $x \leq (1 + p_B^y - p_A^y)/2$ and buy from B otherwise*

[5] Clearly since everything is symmetric, if we looked at old consumers who previously bought from firm B, B would charge $p^l = 1 + s/3$ and A would charge $p^u = 1 - s/3$.

[6] It also implies that consumers never switch to get a better price, only to get a better match. This contrasts with models in which products are homogeneous, such as Chen 1997.

It is easy to see that if young consumers do not care at all about the future (that is, if $\delta_c = 0$) then they should buy product A if and only if $x \leq \left(1 + p_B^y - p_A^y\right)/2$. According to Corollary 1 the same is true even when $\delta_c > 0$ and consumers do care about the future. This is because 1). tastes are independent across periods and 2). irrespective of which product you buy when young, when old your previous supplier will charge you $p^l = 1 + 3/s$ and its competitor will charge you $p^u = 1 - s/3$. This then implies that the future expected consumer surplus from being locked to firm A or firm B is the same. Consequently a young consumer should always just act 'myopically' and choose the product which offers her the greatest surplus in the present.

We can now characterise the prices paid by young consumers. The total profit which firm A expects to earn on any given consumer is equal to

$$\frac{1 + p_B^y - p_A^y}{2}\left[p_A^y + \frac{\delta_f}{2}\left(1 + \frac{s}{3}\right)^2\right] + \frac{1 + p_A^y - p_B^y}{2}\left[\frac{\delta_f}{2}\left(1 - \frac{s}{3}\right)^2\right] \tag{2}$$

This comprises two parts. Firstly a fraction $\left(1 + p_B^y - p_A^y\right)/2$ of consumers buy product A when they are young, and each pays a price p_A^y. When these consumers become old, they stay at firm A with probability $\left(1 + p^u - p^l + s\right)/2 = 1/2 + s/6$ and pay it a price $p^l = 1 + s/3$. Secondly a fraction $\left(1 + p_A^y - p_B^y\right)/2$ of consumers buy product B when they are young. When they become old, they switch to firm A with probability $1/2 - s/6$ and pay it a price $p^u = 1 - s/3$. Maximising (2) with respect to p_A^y yields a first order condition; if we impose $p_A^y = p_B^y$ we find[7] that young consumers pay a price p^y which is equal to

$$p^y = 1 - \frac{2}{3}\delta_f s \tag{3}$$

This has the following interpretation. On average a firm makes an extra profit of $2s/3$ on every old consumer who is locked to it rather than to its rival. Therefore firms decrease their prices by an amount equal to the (discounted) future value of

[7] Alternatively we can explicitly derive firm B's profit function using the same steps, and then prove there is only one equilibrium, and that it is characterised by $p_A^y = p_B^y$.

locking somebody in. Finally direct computation reveals that

Lemma 2 *When firms price discriminate*

$$\begin{aligned} \textit{Consumer surplus} &= CS|_{s=0} + \frac{2\delta_f s}{3} + \frac{\delta_c s^2}{36} - \frac{\delta_c s}{2} \\ \textit{Total industry profit} &= \Pi|_{s=0} + \frac{2\delta_f s}{3}\left(\frac{s}{6} - 1\right) \end{aligned}$$

where $CS|_{s=0}$ and $\Pi|_{s=0}$ denote respectively consumer surplus and industry profit when there are no switching costs. Consumer surplus refers to the total expected lifetime surplus earned by any single cohort of consumers. Similarly industry profit refers to the total expected profit earned by firms on any single cohort of consumers.

3.2 Without discrimination

We now assume that in any time period t, a firm must offer its product to all consumers at the same price irrespective of whether they are new or where they bought from in the past. The process of how to derive equilibrium prices is lengthy and has been studied elsewhere[8], so we simply review the key features of the equilibrium and leave derivations to Appendix A. The main contribution comes in the following Section where we evaluate the welfare effects of price discrimination.

As explained in Section 2 we look for a symmetric linear Markov perfect equilibrium. In particular suppose that there exists some threshold $\tilde{x}^{t-1} \in (0,1)$ such that in period $t-1$ all young consumers with location $x^{t-1} \leq \tilde{x}^{t-1}$ bought product A and all other young consumers bought product B. We look for a subgame perfect equilibrium where in period t the two firms charge prices which have the following form:

$$q_{A,t} = J + K\left(\tilde{x}^{t-1} - 1/2\right) \tag{4}$$

$$q_{B,t} = J - K\left(\tilde{x}^{t-1} - 1/2\right) \tag{5}$$

[8]In the same framework Rhodes (2011) analytically derives conditions under which switching costs are pro-competitive. Somaini and Einav (2011) also solve a related model and use it to study the effect of mergers.

Appendix A shows how to derive analytic expressions for the parameters J and K. Before we describe the equilibrium in more detail, it is helpful to look at consumer behaviour, since this will play an important role in understanding the welfare comparisons in Section 4. The decision of *old consumers* is not much different when price discrimination is banned - as earlier, an old consumer who previously bought product A will buy it again provided that $x \leq (1 + q_{B,t} - q_{A,t} + s)/2$ and otherwise switches to product B. However the purchase decision of *young consumers* is more complicated:

Lemma 3 *Young consumers in period t buy product A if and only if*

$$x \leq \frac{1}{2} + \frac{q_{B,t} - q_{A,t}}{2(1 + K\delta_c s)} \tag{6}$$

Comparing Lemma 3 with Corollary 1, young consumers no longer make the same as choice as would a myopic individual. More specifically provided that $K > 0$ young consumers' demand is less elastic when price discrimination is forbidden. The intuition (which is standard in the literature - see for example Klemperer 1995) is as follows. If firm A reduces $q_{A,t}$ then it attracts more young consumers, which means that come the next period (according to equation 4) A increases its price. Young consumers understand this, and fearing the possibility they may become stuck with firm A in the future, view its price cut in period t slightly less favourably (and consequently are less likely to be persuaded to buy B over A if A cuts its price slightly).

Lemma 4 *There is a subgame perfect equilibrium where in every period, the firms split the market equally and charge the same price*

$$J = \frac{2 + 2K\delta_c s + \delta_f K}{2 + K\delta_c s + \delta_f s} \tag{7}$$

where $K \in [s/3, 2s/5]$ solves the following quartic equation

$$\delta_f K^3 (2 + K\delta_c s) - 3K(2 + K\delta_c s)(1 + K\delta_c s)^2 + 2s(1 + K\delta_c s)^3 = 0 \tag{8}$$

The price expression in Lemma 4 is substantially more complicated than the prices charged under discrimination. This is natural and reflects the fact that when discrimination is banned, a firm's price must balance the differing pricing incentives on each of the three consumer groups, *and* (through equations 4 and 5) account for how a change in price now affects the optimal strategy of the rival many periods ahead. Nevertheless despite the complexity of the pricing expression, it turns out that we are still able to make sharp welfare comparisons. Before doing so, we note that direct calculation can be used to prove the following:

Lemma 5 *When firms do not price discriminate*

$$\begin{aligned}\textit{Consumer surplus} &= CS|_{s=0} + \frac{\delta_c s^2}{4} - \frac{\delta_c s}{2} - (J-1)(1+\delta_c) \\ \textit{Total industry profit} &= \Pi|_{s=0} + (J-1)(1+\delta_f)\end{aligned}$$

where $CS|_{s=0}$ and $\Pi|_{s=0}$ again denote consumer surplus and industry profit when there are no switching costs.

4 Welfare comparison

It is useful to begin by comparing prices under the two scenarios. Recall that with discrimination, young consumers pay p^y whilst old consumers pay p^l or p^u depending upon whether they stay with their original supplier or switch. Let $p^o = (1/2 + s/6)\,p^l + (1/2 - s/6)\,p^u = 1 + s^2/9$ be the average price paid by old consumers when firms are able to price discriminate. Also let $q = J$ be the price charged to everybody when price discrimination is infeasible.

Lemma 6 *There exists a set $A \in \mathbb{R}^2$ such that*

1). if $(\delta_c, \delta_f) \in A$ then $p^y < q < p^o$

2). If $(\delta_c, \delta_f) \notin A$ then $p^y < p^o \leq q$

The set A itself includes all (δ_c, δ_f) which satisfiy $\delta_c \leq \delta_f$

Before explaining the Lemma, Figure 1 plots the set A for the case where $s = 1/2$ (qualitatively things do not change when s takes other values, just that A becomes

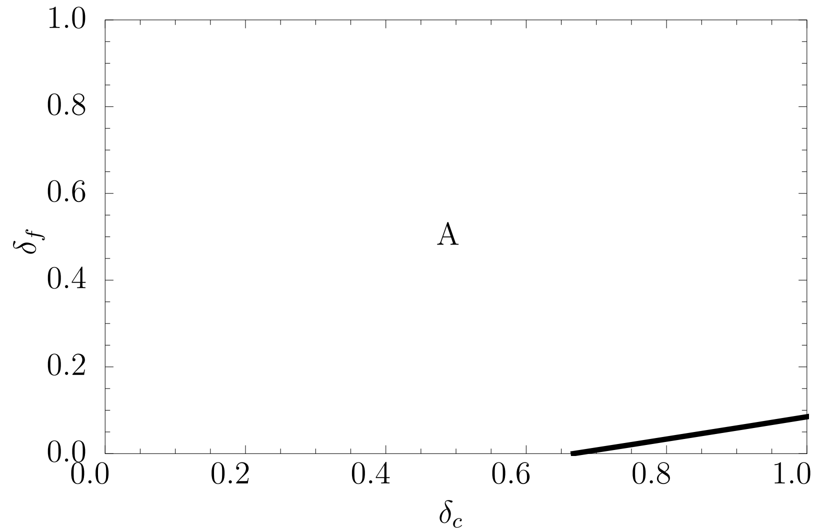

Figure 1: The region A when $s = 1/2$

smaller). It is clear that A covers much of $[0, 1]^2$, including all cases in which consumers are less forward-looking than firms. According to the first part of Lemma 6, whenever we are in the set A, price discrimination causes old consumers to pay more but young consumers to pay less. Intuitively the ability to price discriminate helps firms to better target old consumers and exploit their reluctance to switch by charging them more. However firms then have greater incentives to lock-in young consumers, who consequently pay lower prices compared to when discrimination is infeasible. The second part of the Lemma then considers the more unusual situation in which consumers are very patient relative to firms. In these scenarios *all* consumers pay a lower price when price discrimination is feasible. Recalling the discussion in Section 3.2, this happens because young consumers become very concerned about future adverse price changes, and are less tempted to change behaviour in response to price cuts by one of the retailers. This makes demand less elastic, pushing up the price for everybody.

Lemma 7 *Industry profit is always higher when discrimination is infeasible*

Clearly when $(\delta_c, \delta_f) \notin A$ industry profit must be lower when discrimination is feasible, since both young and old consumers end up paying less. What Lemma 7 shows is that the same conclusion holds for all parameters. To understand why, initially assume that $\delta_c = 0$ and that firms cannot price discriminate. Each firm understands that by cutting its price, it can attract more young consumers in the present, and that these consumers are then more likely to buy from it again in the future. However each firm also understands that (according to equations 4 and 5) such an action will cause its rival to charge a lower price in the following period. This aggressive future behaviour on the part of the rival, discourages firms somewhat from cutting price in the present. On the other hand no such issue arises when discrimination is permitted. As shown earlier, the prices charged to old consumers p^l and p^u are independent of past market share. This creates a stronger link between current market share amongst young consumers and future profitability, which in turn encourages firms to be more aggressive when pricing to young consumers. This extra aggressiveness results in prices and profits being lower overall. When $\delta_c > 0$ and consumers care about the future, the above difference is only strengthened because when discrimination is not possible, consumers' demands are less elastic and this pushes prices even higher.

Lemma 8 *Consumers are almost always better off when price discrimination is permitted*

Price discrimination leads to fiercer competition which, according to Lemma 8, almost always benefits consumers. One might expect that if consumers were very patient, and if $(\delta_c, \delta_f) \in A$, then price discrimination could harm consumers by forcing them to pay very high prices in the future. This turns out not to be the case, because as consumers become more patient, their demands in the without-discrimination case become progressively more inelastic, which pushes up prices in that case as well.[9]

[9] The Lemma says 'almost always' because there are some situations in which consumers are marginally better off if price discrimination is banned. However we show in the Appendix that, amongst other things, this can only happen when $\delta_f \leq 0.009$ - or equivalently an extremely high discount rate.

It is clear from Lemmas 7 and 8 that consumers and firms almost always have opposite views on the desirability of price discrimination. We therefore now consider total surplus, which is defined as the discounted sum of consumer surplus and industry profits (from any single cohort of consumers). Clearly switching costs themselves introduce a deadweight loss to society - both direct (since they have no social benefit) and indirect (since they distort consumer choices, preventing some consumers from purchasing the product that is best for them). However *in any single period*, price discrimination introduces an *additional* deadweight loss. This is because from society's point of view, old consumers should switch from A to B if and only if $V - x \leq V - (1 - x) - s$ - or simply $x \geq (1 + s)/2$. Similarly old consumers should switch from B to A if and only if $x \leq (1 - s)/2$. When price discrimination is banned and firms charge the same price, old consumers do switch under precisely these conditions. However when discrimination is permitted, we showed earlier that old consumers would, for example, switch from A to B whenever $x \geq (1 + s/3)/2$ - meaning too much switching is observed in equilibrium. The reason is that firms offer a discount when trying to poach consumers, meaning that some consumers end up switching (due to the favourable price differential) even when the new product does not give enough extra surplus to justify the cost (s) to society. In light of this additional deadweight loss, the next Lemma is somewhat surprising:

Lemma 9 *Fix s and δ_f. Then there exists a threshold $\bar{\delta}_c \in (0, \delta_f)$ such that total surplus is maximised under discrimination if $\delta_c \leq \bar{\delta}_c$ and otherwise is maximised by banning price discrimination*

In general there may exist a large range of parameters for which price discrimination can improve total surplus, as is shown by Figure 2, which plots the threshold $\bar{\delta}_c$ for various levels of δ_f (for the case of $s = 1/2$). This is an unusual result which contrasts with most other papers, although the intuition is actually straightforward. Although price discrimination creates a deadweight loss in any period, it also transfers money intertemporally from the future to the present. If consumers are relatively impatient compared to firms, this can be a good trade, since consumers get a discount early on (when they value money a lot) and firms are partly compensated by

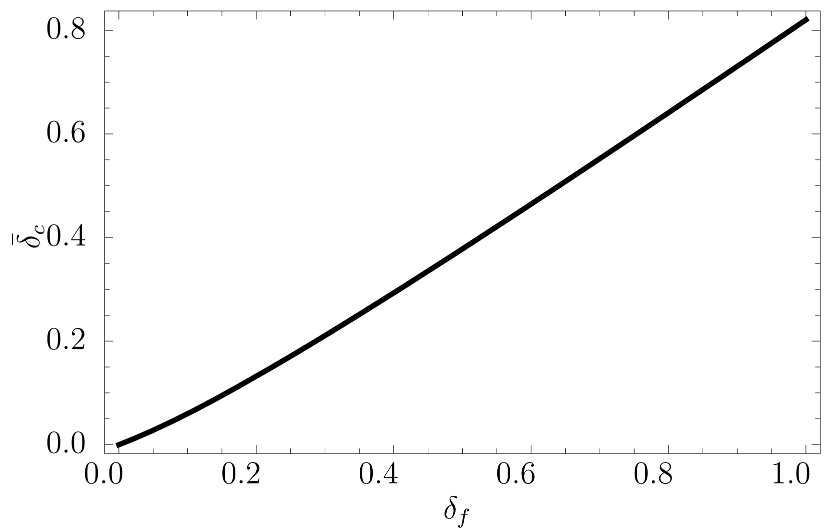

Figure 2: The critical δ_c such that welfare is equal with and without discrimination (for the case $s = 1/2$)

getting more money in the future. If δ_c is low relative to δ_f, this trade can improve the surplus generated by the economy. Hence it can be socially optimal to introduce a policy which creates additional distortions.

A Appendix

A.1 Lemma 4

Proof of Lemma 3. Suppose young consumers (rationally) anticipate facing prices $q_{A,t+1}$ and $q_{B,t+1}$ when they become old. Then the expected lifetime payoff from buying product A in period t can be written as

$$V - x^t - q_{A,t} + \delta_c \left[V - \frac{1}{2} - q_{A,t+1} + \int_{\dot{x}}^{1} \left[-(1-2y) + q_{A,t+1} - q_{B,t+1} - s \right] dy \right] \quad (9)$$

where $\dot{x} = (1 + q_{B,t+1} - q_{A,t+1} + s)/2$ is the location of the old consumer who is just indifferent about switching from A to B in period $t+1$. (9) is composed of a certain payoff $V - x^t - q_{A,t}$ when young, plus an expected payoff when old which is then discounted. To interpret the latter, when the consumer becomes old she could remain with firm A and get an average payoff $\int_0^1 (V - y - q_{A,t+1})\, dy = V - \frac{1}{2} - q_{A,t+1}$. However when her preference for product B turns out to be strong enough, she switches and earns some additional surplus given by the integral term in (9). Similarly the expected lifetime payoff from buying product B in period t can be written as

$$V - (1-x) - q_{B,t} + \delta_c \left[V - \frac{1}{2} - q_{B,t+1} + \int_0^{\ddot{x}} \left[(1-2y) + q_{B,t+1} - q_{A,t+1} - s \right] dy \right] \quad (10)$$

where $\ddot{x} = (1 + q_{B,t+1} - q_{A,t+1} - s)/2$ is the location of the old consumer who is just indifferent between switching from B to A.

Provided $|q_{B,t} - q_{A,t}|$ is not too large, there exists a young consumer located at $x = \tilde{x}$ who is just indifferent between buying the two products. To find this consumer, substitute $x = \tilde{x}$ into equations (9) and (10) and set them equal to each other. After some algebraic manipulations, $\tilde{x}$ satisfies

$$\tilde{x}^t = \frac{1}{2} + \frac{q_{B,t} - q_{A,t} + \delta_c s \left(q_{B,t+1} - q_{A,t+1} \right)}{2} \quad (11)$$

Using equations (4) and (5) we know that for example $q_{A,t+1} = J + K\left(\tilde{x}^t - 1/2\right)$ so substituting this into (11) gives

$$\tilde{x}^t = \frac{1}{2} + \frac{q_{B,t} - q_{A,t}}{2\left(1 + K\delta_c s\right)} \quad (12)$$

■

Now for the remainder of the proof. Since pricing functions in (4) and (5) are linear we expect value functions (discounted sum of present and future profits) to be quadratic. Therefore write period-t value functions for the two firms as:

$$V_{A,t} = M + N\left(\tilde{x}^{t-1} - 1/2\right) + R\left(\tilde{x}^{t-1} - 1/2\right)^2 \quad (13)$$
$$V_{B,t} = M - N\left(\tilde{x}^{t-1} - 1/2\right) + R\left(\tilde{x}^{t-1} - 1/2\right)^2 \quad (14)$$

Next using (12) and the switching decisions of old consumers outlined in the text, we can write firm A's demand in period t as

$$D_{A,t}\left(q_{A,t}, q_{B,t}, \tilde{x}^{t-1}\right) = \frac{1}{2} + \frac{q_{B,t} - q_{A,t}}{2\left(1 + K\delta_c s\right)} + \frac{1 + q_{B,t} - q_{A,t}}{2} + s\left(\tilde{x}^{t-1} - 1/2\right) \quad (15)$$

and its flow profit in period t as $\pi_{A,t}\left(q_{A,t}, q_{B,t}, \tilde{x}^{t-1}\right) = q_{A,t} D_{A,t}\left(q_{A,t}, q_{B,t}, \tilde{x}^{t-1}\right)$. Take $\pi_{A,t}\left(q_{A,t}, q_{B,t}, \tilde{x}^{t-1}\right) + \delta_f V_{A,t+1}\left(\tilde{x}^{t}\right)$ and use equations (13) and (12) to substitute out for $V_{A,t+1}\left(\tilde{x}^{t}\right)$. Then maximise with respect to $q_{A,t}$ to get a first order condition

$$D_{A,t}\left(q_{A,t}, q_{B,t}, \tilde{x}^{t-1}\right) - \frac{q_{A,t}}{2}\frac{2 + K\delta_c s}{1 + K\delta_c s} - \frac{\delta_f N}{2\left(1 + K\delta_c s\right)} - \frac{\delta_f R\left(q_{B,t} - q_{A,t}\right)}{2\left(1 + K\delta_c s\right)^2} = 0 \quad (16)$$

Substitute out $q_{A,t}$ and $q_{B,t}$ using equations (4) and (5), collect terms, and then rewrite (16) in the form $\alpha_1 + \alpha_2\left(\tilde{x}^{t-1} - \frac{1}{2}\right) = 0$. Setting $\alpha_1 = \alpha_2 = 0$ gives the following conditions

$$1 - \frac{J}{2}\frac{2 + K\delta_c s}{1 + K\delta_c s} - \frac{\delta_f N}{2\left(1 + K\delta_c s\right)} = 0 \quad (17)$$

$$s - \frac{3K}{2}\frac{2 + K\delta_c s}{1 + K\delta_c s} + \frac{\delta_f R K}{\left(1 + K\delta_c s\right)^2} = 0 \quad (18)$$

To find an expression for A's period-t valuation, take $\pi_{A,t}\left(q_{A,t}, q_{B,t}, \tilde{x}^{t-1}\right) + \delta_f V_{A,t+1}\left(\tilde{x}^{t}\right)$ and again use equations (13) and (12) to substitute out for $V_{A,t+1}\left(\tilde{x}^{t}\right)$. Then use equations (4) and (5) to eliminate $q_{A,t}$ and $q_{B,t}$. After collecting terms, A's period-t valuation can we expressed in the form $\alpha_3 + \alpha_4\left(\tilde{x}^{t-1} - \frac{1}{2}\right) + \alpha_5\left(\tilde{x}^{t-1} - \frac{1}{2}\right)^2$. Since we assumed in equation (13) that this value equals $M + N\left(\tilde{x}^{t-1} - \frac{1}{2}\right) + R\left(\tilde{x}^{t-1} - \frac{1}{2}\right)^2$, we can equate coefficients and get three equations

$$\alpha_3 = J + \delta_f M = M \quad (19)$$

$$\alpha_4 = Js - JK\frac{2 + K\delta_c s}{1 + K\delta_c s} + K - \frac{\delta_f K N}{1 + K\delta_c s} = N \quad (20)$$

$$\alpha_5 = Ks - K^2\frac{2 + K\delta_c s}{1 + K\delta_c s} + \frac{\delta_f R K^2}{\left(1 + K\delta_c s\right)^2} = R \quad (21)$$

Now obtain an equation containing only K. Note that if $s > 0$, then from equation (18) $K \neq 0$. Therefore rewrite equation (18) as

$$R = \frac{3\left(2 + K\delta_c s\right)\left(1 + K\delta_c s\right)}{2\delta_f} - \frac{s\left(1 + K\delta_c s\right)^2}{\delta_f K} \tag{22}$$

and then substitute this into equation (21) and rearrange to find

$$\delta_f K^3\left(2 + K\delta_c s\right) - 3K\left(2 + K\delta_c s\right)\left(1 + K\delta_c s\right)^2 + 2s\left(1 + K\delta_c s\right)^3 = 0 \tag{23}$$

After rearranging the other equations, we find the expression for J provided in the text.

The next task is to bound K. When setting up demand in (15) we assumed that after any history, each firm sells to some young consumers, and each firm has old consumers both switching to and away from it. First, to ensure each firm has old consumers both switching to and away from it, we require $1 - s \geq q_{A,t} - q_{B,t}$, or alternatively $1 - s \geq 2K\left(\tilde{x}^{t-1} - 1/2\right)$. Note that $2K\left(\tilde{x}^{t-1} - 1/2\right) \leq |K|$, therefore it is necessary to have $|K| < 1 - s$. (Similarly to have $\dot{x} \leq 1$, it is again necessary to have $|K| < 1 - s$.) Second, the expression for $\tilde{x}^t$ in equation (12) is only well-defined if $1 + K\delta_c s \neq 0$; it is simple to show that this holds provided that $|K| < 1 - s$. Thirdly, to ensure that each firm sells to some young consumers, $\tilde{x}^t$ defined in equation (12) must satisfy $\tilde{x}^t \in (0, 1)$ or alternatively $|K| < 1 + K\delta_c s$; again this condition holds provided that $|K| < 1 - s$.

Aim: in light of the above, we first show that equation (23) has exactly one solution on the interval $\left[-\left(1 - s\right), 1 - s\right]$, and that it lies in $[s/3, 2s/5]$.

Step 1: Show that equation (23) has exactly one solution on the interval $[0, (1 - s)]$.

Step 1a. Show that $\frac{\partial\phi(K)}{\partial K} < 0$. To prove this, differentiate (23) with respect to K:

$$\frac{1}{2}\frac{\partial\phi\left(K\right)}{\partial K} = K^2\delta_f\left(3 + 2K\delta_c s\right) + 3\left(1 + K\delta_c s\right)\left[-1 - 4K\delta_c s - 2\left(K\delta_c s\right)^2 + \delta_c s^2 + K\delta_c^2 s^3\right]$$

Since we are considering $K \geq 0$, $3 + 2K\delta_c s \leq 3(1 + K\delta_c s)$ and therefore

$$\frac{1}{2}\frac{\partial \phi(K)}{\partial K} \leq 3(1 + K\delta_c s)\left[K^2\delta_f - 1 - 4K\delta_c s - 2(K\delta_c s)^2 + \delta_c s^2 + K\delta_c^2 s^3\right]$$

Notice that $-4K\delta_c s + K\delta_c^2 s^3 < 0$ since $\delta_c s^2 < 4$, and that $K^2\delta_f - 1 + \delta_c s^2 \leq (1-s)^2 - 1 + s^2 = 2s(s-1) < 1$. Therefore $\frac{\partial \phi(K)}{\partial K} < 0$.
Step 1b. Note that $\phi(0) = 2s > 0$. Now prove that $\phi(1-s) < 0$. Substituting $K = 1 - s$ into $\phi(K)$ and then simplifying, we find that:

$$\phi(1-s) = \delta_f (1-s)^3 [2 + \delta_c s(1-s)] - [1 + \delta_c s(1-s)]^2 \left(6 - 8s + 3\delta_c s - 8\delta_c s^2 + 5\delta_c s^3\right)$$

The first term $\delta_f (1-s)^3 [2 + \delta_c s(1-s)]$ is decreasing in s, so setting $s = 0$, it is at most $2\delta_f$. The final bracketed expression $6 - 8s + 3\delta_c s - 8\delta_c s^2 + 5\delta_c s^3$ is positive and decreasing in s, and therefore (substituting in $s = 1/2$) is at least $2 + \delta_c/8 \geq 2$.[10] The other term $[1 + \delta_c s(1-s)]^2$ is increasing in s for any $s \in [0, 1/2]$ and is therefore at least 1. Combining all this information, $\phi(1-s) \leq 2\delta_f - 2 < 0$.
Step 1c. Combining steps 1a and 1b, it is clear that on $[0, 1-s]$ there is a unique K which solves $\phi(K) = 0$.
Step 2: Show that the solution to $\phi(K) = 0$ lies on $[s/3, 2s/5]$.
Step 2a. Show that $\phi(s/3) > 0$. The terms $-3K(2 + K\delta_c s)(1 + K\delta_c s)^2 + 2s(1 + K\delta_c s)^3$ become $(1 + K\delta_c s)^2 K\delta_c s^2 > 0$. Also $\delta_f K^3 (2 + K\delta_c s) > 0$ since $K > 0$.
Step 2b. Show that $\phi(2s/5) < 0$. First write

$$\begin{aligned}
\phi\left(\frac{2s}{5}\right) &= \delta_f \left(\frac{2s}{5}\right)^3 \left(2 + \frac{2\delta_c s^2}{5}\right) - \frac{6s}{5}\left(2 + \frac{2\delta_c s^2}{5}\right)\left(1 + \frac{2\delta_c s^2}{5}\right)^2 + 2s\left(1 + \frac{2\delta_c s^2}{5}\right)^3 \\
\phi\left(\frac{2s}{5}\right) &= \delta_f \left(\frac{2s}{5}\right)^3 \left(2 + \frac{2\delta_c s^2}{5}\right) - \frac{8s}{125}\left(2 + \frac{2\delta_c s^2}{5}\right)\left(1 + \frac{2\delta_c s^2}{5}\right)^2 \\
&\quad - \frac{142s}{125}\left(2 + \frac{2\delta_c s^2}{5}\right)\left(1 + \frac{2\delta_c s^2}{5}\right)^2 + 2s\left(1 + \frac{2\delta_c s^2}{5}\right)^3
\end{aligned}$$

[10] It is positive because $s \leq 1/2$ therefore $8s + 8\delta_c s^2 < 6$. The first derivative with respect to s is $-8 + 3\delta_c - 16\delta_c s + 15\delta_c s^2$, which is negative because $s \leq 1/2$ and therefore $3\delta_c + 15\delta_c s^2 < 8$.

The first two terms are negative, so it is sufficient to show that the last two terms are also negative. They are provided that $2\left(1+2\delta_c s^2/5\right) < (142/125)\left(2+2\delta_c s^2/5\right)$, or alternatively if $\delta_c s^2 < 85/108$ - which always holds since $s < 1/2$.

Step 2c. Since $\phi\left(K\right)$ strictly decreases on $[0, 1-s]$, and $\phi\left(s/3\right) > 0$ but $\phi\left(2s/5\right) < 0$, then the solution to $\phi\left(K\right) = 0$ must lie on $[s/3, 2s/5]$.

Step 3. Now show there is no solution to $\phi\left(K\right) = 0$ for $K \in [-\left(1-s\right), 0]$.

Step 3a. Using Step 1a

$$\frac{1}{6}\frac{\partial^2\phi\left(K\right)}{\partial K^2} = 2K\delta_f\left(1+K\delta_c s\right) + \delta_c s\left(-5 - 12K\delta_c s - 6\left(K\delta_c s\right)^2 + 2\delta_c s^2 + 2K\delta_c^2 s^3\right)$$

Since $K \in [-\left(1-s\right), 0]$, the first term $2K\delta_f\left(1+K\delta_c s\right)$ is negative. To show the remainder is also negative, it is sufficient to show that $-5 - 12K\delta_c s + 2\delta_c s^2 < 0$. The latter is toughest to satisfy when K is very negative and δ_c is large, so substitute in $K = -\left(1-s\right)$ and $\delta_c = 1$; it is then sufficient to prove that $-5 + 12s\left(1-s\right) + 2s^2 < 0$ - which is then easily seen to hold for all $s \in [0, 1/2]$. Therefore $\partial^2\phi\left(K\right)/\partial K^2 < 0$ ($\phi\left(K\right)$ is concave) for all $K \in [-\left(1-s\right), 0]$.

Step 3b. Show that $\phi\left(-\left(1-s\right)\right) > 0$. Rewrite $\phi\left(K\right)$ as

$$\begin{aligned} & -K\left(2+K\delta_c s\right)\left[3\left(1+K\delta_c s\right)^2 - \delta_f K^2\right] + 2s\left(1+K\delta_c s\right)^3 \\ = \; & \left(1-s\right)\left(2-\delta_c s\left(1-s\right)\right)\left[3\left(1-\delta_c s\left(1-s\right)\right)^2 - \delta_f^2\left(1-s\right)^2\right] + 2s\left(1+\delta_c s\left(1-s\right)\right) \end{aligned}$$

which by inspection in positive. We also showed in Step 1b that $\phi\left(0\right) > 0$. Therefore since $\phi\left(K\right)$ is concave on $[-\left(1-s\right), 0]$ and positive at the boundaries of that set, it must be true that $\phi\left(K\right) > 0 \forall K \in [-\left(1-s\right), 0]$, hence there is no root to $\phi\left(K\right)$ on that interval.

A.2 Welfare expressions

Proof of Lemma 2. *Consumer surplus.* Young consumers all pay p^y and either buy from A if $x \leq 1/2$ or from B if $x \geq 1/2$. Therefore young consumer surplus is

$$V - \int_0^{1/2} x dx - \int_{1/2}^{1} (1-x)\, dx - p^y = V - \frac{5}{4} + \frac{2}{3}\delta_f s \tag{24}$$

As stated earlier, old consumers have the same expected surplus irrespective of whether they previously bought from A or B, so consider an old consumer who previously bought product A. If $x \leq 1/2 + s/6$ she stays with firm A and pays p^l otherwise she switches to firm B and pays $p^u + s$. Therefore old consumers enjoy a surplus

$$\begin{aligned} & V - \int_0^{\frac{1}{2}+\frac{s}{6}} x dx - \left(\frac{1}{2} + \frac{s}{6}\right) p^l - \int_{\frac{1}{2}+\frac{s}{6}}^{1} (1-x)\, dx - \left(\frac{1}{2} - \frac{s}{6}\right)(p^u + s) \\ = \; & V - \frac{5}{4} + \frac{s^2}{36} - \frac{s}{2} \end{aligned} \tag{25}$$

The discounted sum of (24) and (25) is

$$\left(V - \frac{5}{4}\right)(1 + \delta_c) + \frac{2}{3}\delta_f s + \frac{\delta_c s^2}{36} - \frac{\delta_c s}{2}$$

To get the expression in Lemma 2 set $CS|_{s=0} = \left(V - \frac{5}{4}\right)(1 + \delta_c)$.

Industry profit. Evaluating expression (2) at the p_y defined in equation (3), each firm earns profit

$$\frac{1 + \delta_f}{2} + \frac{\delta_f s}{3}\left(\frac{s}{6} - 1\right)$$

Then double this and let $\Pi|_{s=0} = 1 + \delta_f$ to get the expression in Lemma 2. ■

Proof of Lemma 5. *Consumer surplus.* Young consumers all pay J and buy A if $x \leq 1/2$ and buy B otherwise, therefore young consumer surplus is

$$V - \int_0^{1/2} x dx - \int_{1/2}^{1} (1-x)\, dx - J = V - \frac{5}{4} - (J - 1) \tag{26}$$

By symmetry to calculate old consumer surplus, it is sufficient to look at surplus of consumers locked into firm A. These consumers buy product A if $x \leq (1+s)/2$ and otherwise pay s and switch to firm B. Therefore old consumer surplus is equal to

$$V - \int_0^{\frac{1+s}{2}} x dx - \int_{\frac{1+s}{2}}^1 (1-x)\, dx - J - s\left(1 - \frac{1+s}{2}\right) = V - \frac{5}{4} - (J-1) + \frac{s^2}{2} - \frac{s}{2} \quad (27)$$

The discounted sum of (26) and (27) is

$$\left(V - \frac{5}{4}\right)(1+\delta_c) - (J-1)(1+\delta_c) + \frac{\delta_c s^2}{2} - \frac{\delta_c s}{2}$$

To get the expression in Lemma 5 set $CS|_{s=0} = \left(V - \frac{5}{4}\right)(1+\delta_c)$. ■

A.3 Welfare comparisons

Proof of Lemma 6. To prove that $p^y < J$ note that J is increasing in K and that $K \geq s/3$, therefore

$$J \geq J|_{K=s/3} = \frac{2 + 2\delta_c s^2/3 + \delta_f s/3}{2 + \delta_c s^2/3 + \delta_f s} = 1 + \frac{\delta_c s^2/3 - 2\delta_f s/3}{2 + \delta_c s^2/3 + \delta_f s} > 1 - \frac{2\delta_f s}{3} = p^y$$

Now suppose that $\delta_c \leq \delta_f$. We again know that J is increasing in K therefore

$$J \leq J|_{K=2s/5} = \frac{2 + 4\delta_c s^2/5 + 2\delta_f s/5}{2 + 2\delta_c s^2/5 + \delta_f s}$$

Since $\delta_c \leq \delta_f$ this implies that

$$J \leq 1 + \frac{2\delta_c s^2/5 - 3\delta_f s/5}{2 + 2\delta_c s^2/5 + \delta_f s} \leq 1 < p^o$$

Therefore A is non-empty and includes all (δ_c, δ_f) which satisfy $\delta_c \leq \delta_f$. Finally to show that A^c is non-empty, suppose $\delta_c = 1$ and $\delta_f = 0$. Then

$$J = \frac{2 + 2s\ K|_{\delta_c=1,\delta_f=0}}{2 + s\ K|_{\delta_c=1,\delta_f=0}} \geq \frac{2 + 2s^2/3}{2 + s^2/3} = 1 + \frac{s^2/3}{2 + s^2/3} > 1 + \frac{s^2}{9} = p^o$$

where the final inequality follows because $s < 1/2$. ■

Proof of Lemma 7. We wish to show that $(J-1)(1+\delta_f) \geq \frac{2\delta_f s}{3}\left(\frac{s}{6} - 1\right)$. Note that $K \geq s/3$ therefore

$$(J-1)(1+\delta_f) \geq \frac{2 + \delta_c s^2/3 + \delta_f s/3}{2 + \delta_c s^2/3 + \delta_f s}(1+\delta_f) \tag{28}$$

Also note that the righthand side of (28) is increasing in δ_c. Therefore if we can show that

$$\left(\frac{2 + \delta_f s/3}{2 + \delta_f s} - 1\right)(1+\delta_f) \geq \frac{2\delta_f s}{3}\left(\frac{s}{6} - 1\right)$$

then we are done. However the latter can be simplified to

$$\left(1 - \frac{s}{6}\right)(2 + \delta_f s) \geq 1 + \delta_f$$

which is least likely to hold when $\delta_f = 1$. However even substituting in $\delta_f = 1$, the condition simplifies to $s^2 \leq 4s$ which clearly holds. ■

Proof of Lemma 8. Let Δ be the difference in consumer surplus when discrimination is not and is feasible, that is

$$\Delta = \frac{2\delta_c s^2}{9} - \frac{2\delta_f s}{3} - \left(\frac{K\delta_c s + \delta_f K - \delta_f s}{2 + K\delta_c s + \delta_f s}\right)(1+\delta_c) \tag{29}$$

We want to find conditions under which this is positive, and show they are very stringent. Notice that (29) is increasing in K, so we can place an upper bound on parameters for which the condition holds by substituting in $K = s/3$. Doing this and tidying up, we find that (29) can only ever be positive if

$$-\frac{s}{3}\delta_f^2+\delta_f\left[-\frac{1-\delta_c}{3}\right]+\delta_c s\left[\frac{1}{18}+\frac{\delta_c s^2}{27}-\frac{\delta_c}{6}\right]\geq 0$$

By inspection if $\delta_f=0$ there exists a range of δ_c such that this inequality. More generally the inequality is satisfied provided that

$$\delta_f\leq\frac{3\left(\delta_c-1\right)+\sqrt{9-18\delta_c+9\delta_c^2+6\delta_c s^2+4\delta_c^2 s^4-18\delta_c^2 s^2}}{6s}$$

We now show that this can only ever be satisfied if $\delta_f\leq 0.009$. This is true provided that

$$\begin{aligned}\sqrt{9-18\delta_c+9\delta_c^2+6\delta_c s^2+4\delta_c^2 s^4-18\delta_c^2 s^2} &\leq 0.054s+3\left(1-\delta_c\right)\\ 6\delta_c s+4\delta_c^2 s^3-18\delta_c^2 s &\leq 0.002916s+\frac{81}{250}\left(1-\delta_c\right)\end{aligned}$$

which is clearly satisfied if $6\delta_c\leq 0.002916$. If instead $6\delta_c>0.002916$ then the inequality becomes harder to satisfy. Hence if it holds when $s=1/2$ it must hold for all $s\in[0,1/2]$. Evaluating it at $s=1/2$, the inequality becomes

$$3\delta_c+\delta_c^2/2-9\delta_c^2-0.001458-\frac{81}{250}\left(1-\delta_c\right)\leq 0 \tag{30}$$

The lefthand side of (30) is maximised by setting $\delta_c=831/4250$. However it is easy to verify that (30) is negative when evaluated at $\delta_c=831/4250$. Therefore consumer surplus can be higher without price discrimination, but only if $\delta_f\leq 0.009$. ■

Proof of Lemma 9. Firstly if $\delta_c=0$ then total surplus is highest under discrimination. This is readily seen by direct computation, after noting that since $(\delta_c,\delta_f)\in A$, then (using the earlier proof of Lemma 7) $J\leq 1$. At the same time direct computation also shows that when $\delta_c=\delta_f>0$ then total surplus is highest without discrimination. Rhodes (2011) also shows that K is increasing in δ_c, and hence J must be increasing in δ_c. Hence fixing s and δ_f there must be a unique $\delta_c\in(0,\delta_f)$

such that the following holds

$$\frac{\delta_c s^2}{4} - \frac{\delta_c s}{2} - (J-1)(1+\delta_c) + (J-1)(1+\delta_f) = \frac{2\delta_f s}{3} + \frac{\delta_c s^2}{36} - \frac{\delta_c s}{2} + \frac{2\delta_f s}{3}\left(\frac{s}{6}-1\right)$$

But the latter is just the condition under which total surplus is equal under the two pricing schemes. Therefore for δ_c above $\bar{\delta}_c$ no discrimination is preferred, and for δ_c below $\bar{\delta}_c$ permitting discrimination is preferred. ■

References

[1] Chen, Y. (1997): 'Paying customers to switch', Journal of Economics and Management Strategy 6(4), 877–897

[2] Farrell, J. and Klemperer, P. (2007): 'Coordination and Lock-In: Competition with Switching Costs and Network Effects', *Handbook of Industrial Organization*, Vol 3. M. Armstrong and R. Porter (eds.), North-Holland

[3] Fudenberg, D. and Tirole, J. (2000): 'Customer poaching and brand switching', RAND Journal of Economics 31(4), 634-657

[4] Gehrig, T., Shy, O. and Stenbacka (2011): 'A welfare evaluation of history-based price discrimination', Journal of Industry Competition and Trade, forthcoming

[5] Holmes, T. (1989): 'The Effects of Third-Degree Price Discrimination in Oligopoly', The American Economic Review 79(1), 244-250

[6] Klemperer, P. (1987): 'The competitiveness of markets with switching costs', RAND Journal of Economics 18(1), 138–150

[7] Klemperer, P. (1995): 'Competition when consumers have switching costs', Review of Economic Studies 62(4), 515–539

[8] Rhodes, A. (2011): 'When are small switching costs pro-competitive?', University of Oxford

[9] Somaini, P. and Einav, L. (2011): 'A model of market power in customer markets', Stanford University

[10] Taylor, C. (2003): 'Supplier surfing: Competition and consumer behavior in subscription markets', RAND Journal of Economics 34(2), 223–246

[11] Thisse, J. and Vives, X. (1988): 'On The Strategic Choice of Spatial Price Policy', The American Economic Review 78(1), 122-137