

Universal banking, competition and risk in a macro model ^{*}

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Abstract

A stylized macroeconomic model is developed with an indebted, heterogeneous ‘investment banking sector’ funded by borrowing from a retail banking sector. The government guarantees retail deposits. Investment banks choose how risky their activities should be. We find that the financial sector can move very sharply from safe to risky investment strategies and that the degree of competitiveness is important for risk premia. The incidence of banking default is considered under different constellations of shocks and degrees of competitiveness. The benefits of universal banking rise in the volatility of idiosyncratic shocks to trading strategies and are positive even for very bad common shocks, even though government bailouts, which are costly, are larger compared to the case of separated banking entities. The benefits of universal banking are positive but decreasing in the value and volatility of shocks to the ‘quality of financial capital’. Banks with some market power have a cushion of profits against adverse shocks which is beneficial since there is an excess burden associated with government bailouts. Hence the optimal degree of competitiveness of separate banking firms is lower than for universal firms. So, the welfare assessment of the structure of banks may depend crucially on the kinds of shock hitting the economy as well as on the efficiency of government intervention.

JEL Classification: E13, E44; G11; G24; G28.

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1. Introduction

Recent events in world financial markets have convinced, perhaps reminded, many economists and policymakers that maintaining financial and macroeconomic stability are not separate policy challenges, at least some of the time. As a result, the issue of macro-prudential regulation has moved centre-stage and seems set to become an evolving feature of macroeconomic policies. The responsibilities of central banks look likely to increase.¹ Perhaps more controversially, policymakers are also debating whether, and in what form, constraints on so-called universal banking, as in the US Banking Act of 1933, should be reintroduced.²

That debate over the merits of universal banks (that is, of banks which combine both retail/utility banking and investment banking operations) is being fiercely contested. On one side, some are concerned that universal banks benefit from a more or less explicit taxpayer guarantee and hence a de facto funding subsidy. The consequent moral hazard may then push banks to be larger and more risky than otherwise. Related to these concerns are worries in some countries, such as the UK, that parts of the banking sector are not sufficiently competitive. On the other side, it is argued that economies of scale and scope exist that can be exploited with universal banking such that their products and services are cheaper and better tailored to clients' needs, thus benefiting the wider economy. The debate is further complicated by a lack of agreement as to what constitutes the dividing line between different kinds of banks and products.

In this paper a simple macroeconomic model is developed to study the competitiveness of the banking sector and the desirability of universal banking. The relationship between financial intermediaries and the real economy is modelled: A final-goods producing sector is reliant for its production on the output of a stylized investment banking sector. In turn, the investment banking sector is reliant on loans from a retail banking sector which is funded by retail deposits. Initially we set out the problems facing the commercial banks and the investment banks separately; we then 'merge' these institutions to model the implications of universal banking.

Three key features are incorporated in the model in order to assess whether universal banking is likely to be welfare improving. First, investment banks are assumed to make a

¹Many economists and commentators argue that central banks need to be involved in both systemic regulation as well as monetary policy. See for example the analysis and recommendations contained in The Squam Lake Report (2010). Recent reforms in the UK appear to be in step with the broad thrust of those recommendations.

²The 1933 Act is popularly known as the Glass-Steagall Act. Amongst other things it separated retail banking from investment banking. That provision was effectively repealed in 1999 by the Gramm-Leach-Bliley Act.

decision concerning the riskiness of their balance sheets, having access to both risky and riskless trading strategies. As this risk is leveraged on commercial bank loans, the possibility of insolvency is present if the strategy does not pay off. And since the final goods producer is also reliant on the investment banking sector, it too is vulnerable to events in the financial sector.

The second feature concerns the stochastic structure of the economy. Investment banking may be vulnerable to a combination of common and idiosyncratic shocks. As a result, the economy can be well-insured against shocks or extremely vulnerable to shocks, depending on whether common or idiosyncratic shocks are dominant. In addition, a common shock to the output of the investment banks is introduced, similar to the quality of capital shock in Gertler and Kiyotaki (2010).

The third feature is that in the event of default by an investment bank, commercial banks are bailed out in order to protect retail depositors. Thus depositors have no need to monitor commercial banks and commercial banks have no need to monitor investment banks. However, the bail out is costly to agents as a whole since remedial government action is distortive.³

This simple framework provides initial insights into a number of issues of current interest. We find that more competition in the investment banking sector increases the probability of default. That is because more competition lowers profit margins making banks less able than otherwise to absorb losses. The increased probability of default increases the spread on commercial bank loans as a risk premium is factored in. Ultimately, though, increased competition is no bad thing; it lowers markups, reduces the price of the investment banks' output and ultimately increases final goods production and agents' welfare.

The relative roles of idiosyncratic and systematic shocks to the investment banks are analyzed. The cost to the taxpayer following a banking bail-out, under various assumptions about the relative dominance of these shocks, is undertaken. When idiosyncratic shocks dominate, this cost is relatively low. That is because price expectations errors are relatively modest, on average. On the other hand, when shocks are common, investment banks' forecast errors of future prices, and hence of their revenue stream, can sometimes be very large. In that case, the government bail-out can be rather costly.

Finally, the issue of universal banking versus separated banking is considered, again under a number of scenarios about the constellation of shocks hitting the investment banks. Absent

³Among academic papers comparing the separated and universal banking structures are Boyd, Chang and Smith (1998) who argue that a universal banking structure requires a larger FDIC involvement. See also Boot and Thakor (1997). However, to our knowledge, the present paper is the first one providing a relevant welfare analysis.

quality of capital shocks, universal banking seems to welfare dominate separated banks. The key trade-off that emerges is one between double marginalization and relatively low default when banks are separated, against competitive pricing and increased government bailouts with universal banks. The welfare dominance of universal banking is rising in the volatility of trading strategies as the risk premium that would otherwise impact the lending between commercial and investment banks is eradicated. It appears that double marginalization combined with the required credit spread is, in welfare terms, a more costly distortion than distortive government bail-outs. On the other hand, when shocks to the overall efficiency of the financial system predominate, universal banking remains attractive, but perhaps less so. Despite boosting the output of the investment banks, and hence final goods production (and hence consumption), the universal banking results in larger taxpayer bail-outs and a higher level of equilibrium labour supply. Finally, we briefly examine the optimal degree of competition. We find that the optimal degree of competitiveness of separate banking firms is lower than for universal firms. That follows because banks with some market power have a cushion of profits against adverse shocks which is beneficial as there is an excess burden associated with government bailouts.

1.1. Outline of paper

Sections 2 through 5 set out the components of the model covering the behavior of private agents, commercial banks, final goods producers and investment banks. Section 6 analyses credit spreads, the effects of idiosyncratic shocks and the costs of the commercial bank bail out. In Section 7 the aggregate equations of the baseline model are set out. In Section 8 the impact of common shocks (to the investment banks) on the cost of bank bail outs is examined. In Section 9 the model's implications for the costs and benefits of universal versus separate retail-investment banking are examined. Sections 10 and 11 examine the aggregate model equations under various specifications for the stochastic structure of the investment banking sector and under various policy environments (i.e., universal versus separate banking). These sections also analyze the implications of the quality of capital shock for universal versus separate banking. We conclude in Section 12.

2. The Model: Households

There is a large number of identical agents in the economy who evaluate their utility using the following criterion:

$$E_0 \sum_{t=0}^{\infty} \beta^t (\log(C_t) - \lambda N_t). \quad (2.1)$$

E_t denotes the expectations operator at time t , β is the discount factor, C_t is consumption and $N_t = \int_i N_t(i) di$ is labour, where $N_t(i)$ is the quantity of labour supplied to firm i . $\nu \geq 0$ measures the labour supply elasticity while λ is a preference parameter.

Consumption is defined over a basket of goods and indexed by i , $C_t \equiv \left[\int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}$ where $\theta > 1$ is the elasticity of substitution. The price-level, P_t , is $P_t = \left[\int_0^1 p_t(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}$. The demand for each good is given by

$$Y_t(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t^d, \quad (2.2)$$

where $p_t(i)$ is the nominal price of the final good produced by firm i and Y_t^d denotes aggregate demand. All firms pay the same real wage for the same labour. As a result, $w_t(i) = w_t, \forall i$. And since all households provide the same share of labour to all firms the agent's nominal flow budget constraint is

$$P_t C_t + D_{t+1}^h = R_t^h D_t^h + W_t N_t + \Pi_t. \quad (2.3)$$

D_t^h is the nominal value of deposits in the commercial bank at the start of date t . Between date $t-1$ and the start of t these balances earn a nominal gross interest return of R_t^h . In period t agents have to decide how much of their current wealth to place in the retail bank, D_{t+1}^h . W_t is the nominal wage in period t , and Π_t is profits remitted to the individual net of the cost of deposit insurance and bailing out of banks. So, it is assumed that any retail bank that makes a loss has those losses made good by the taxpayer and is allowed to go on trading.

Necessary conditions for an optimum include:

$$C_t = \frac{w_t}{\lambda}; \quad (2.4)$$

and

$$E_t \left\{ \frac{\beta C_t}{C_{t+1}} \frac{P_t}{P_{t+1}} \right\} = \frac{1}{R_t^h}. \quad (2.5)$$

The environment is such that there is a unique stochastic discount factor,

$$\Theta_{t,t+k} = \beta \frac{C_t P_t}{C_{t+k} P_{t+k}}, \text{ and } E_t \{ \Theta_{t,t+k} \} = E_t \prod_{j=0}^k \frac{1}{R_{t+j}^h}. \quad (2.6)$$

3. The retail bank sector

There is a continuum of retail (or commercial) banks indexed by i . Banks pay an interest rate on their deposits of R_t^h . That deposit rate will be common across banks and need not be indexed by i . In the loans market banks are monopolistic competitors and set loan rates, $R_t^c(i)$. So, following Aksoy et. al. (2009), banks face the following demand for loans

$$B_t^c(i) = \left(\frac{R_t^c(i)}{R_t^c} \right)^{-\delta} B_t^c. \quad (3.1)$$

Here $B_t^c(i)$ is bank i 's lending, R_t^c is a measure of the average interest rate on loans, $R_t^c = \left[\int_0^1 R_t^c(i)^{1+\delta} di \right]^{\frac{1}{1+\delta}}$, and B_t^c is aggregate demand for loans, $B_t^c = \left[\int_0^1 B_t^c(i)^{\frac{\delta-1}{\delta}} di \right]^{\frac{\delta}{\delta-1}}$, where $\delta > 1$ is the elasticity of substitution between loans. The objective of each bank, therefore, is to maximize expected profits by choosing the rate charged on lending. If all borrowers remains solvent, the commercial bank will earn nominal return R_t^c per unit loaned. In the case of default, the assets of the borrower are repossessed by the commercial bank. The commercial bank's problem is studied in full in section 6.

4. The final goods sector

The production of final goods depends on A_t , common to all producers, and a financial product, $X_t(i)$:

$$Y_t(i) = A_t X_t(i). \quad (4.1)$$

We wish to think of $X_t(i)$ as a bundle of services from an investment banking sector, comprising financial and consultancy services. The assumption is clearly that these financial services are necessary for production of the final output and that when the output of such services is low so too is the output of the final goods sector, *ceteris paribus*. The behavior of the investment bank is described in more detail presently. A_t may be thought of as an aggregate shock to the quality of financial capital, or as a utilization shock, reflecting factors such as the costs of using the financial system.

Firms set prices to maximize their profit

$$\max_{P_t(i)X_t(i)} \left(\left(\frac{P_t(i)}{P_t} \right) Y_t(i) - q_t X_t(i) \right);$$

where $Y_t(i) = \left(\frac{P_{t+k}(i)}{P_{t+k}} \right)^{-\theta} Y_t$ and $q_t = \frac{Q_t}{P_t}$ are the demand for final good i and the real price of the output of the financial sector, respectively.

In a symmetric equilibrium $\frac{P_t(i)}{P_t} = 1$, and using the first order conditions for firms, it is straightforward to derive an aggregate real price, (4.2), and the aggregate demand for the financial product (4.3):

$$q_t = \frac{(\theta - 1)}{\theta} A_t; \quad (4.2)$$

$$X_t^d = Y_t / A_t. \quad (4.3)$$

5. The investment bank sector

Agents deposit savings in commercial banks. The commercial banks bundle and pass these funds to an investment banking sector. The investment banks in the model need loans from commercial banks to pay the wage bill ahead of selling their output to the final goods sector. However the amount of output is affected by investment banks' production or trading

strategies. The more risky the investment banks' trading strategy, the more uncertain is the amount of input they can provide to the final goods sector. Investment banks have to decide on the riskiness of their activities, given their expectations of how profitable they will be; so the degree of risk is an endogenous variable⁴. The idea is to reflect in a simple way both the importance and potential riskiness of financial intermediation in the economy. If the investment bank turns out to be less profitable than expected it could have negative net assets. In that event, the bank's losses are in effect made good by the taxpayer and it is allowed, next period, to continue trading. If the bank makes a profit, this is remitted to private agents. These assumptions limit the horizon of the investment banks' optimization problem and ensure a demand for funds from the commercial banking sector.

The investment bank produces output at $t + 1$, $X_{t+1}(j)$, by employing labour at time t . Labour is employed in combination with a choice two strategies. One is a 'safe strategy' with output per unit labour of $\alpha_t > 0$, which is known ex ante. Denote the output using the safe strategy by $X_{t+1,1}^s(j)$ (where the superscript s indicates 'supply' and the subscript 1 indicates the safe technology) so that

$$X_{t+1,1}^s(j) = \alpha_t N_t(j). \quad (5.1)$$

$N_t(j)$ is the labour input employed by investment bank j . Labour is homogeneous and can be used in conjunction with either the safe strategy just described or with a 'risky strategy', to which all banks have access. The risky strategy is described by

$$X_{t+1,2}^s(j) = \Omega_t \varepsilon_{t+1}(j) N_t(j). \quad (5.2)$$

Here, $\Omega_t > 0$ is the time t expected return from employing the risky strategy; this expectation is common to all investment banking firms. $\varepsilon_{t+1}(j)$ reflects a bank-specific shock in period t . It is constructed so that $\varepsilon_t \geq 0$, and $E_t \varepsilon_{t+1} = 1$, all t, j . The cumulative distribution of ε_{t+1} is denoted by $F(\varepsilon)$, is time-invariant and common to all banks.

Ex post, the average return to labour will in general differ across strategies. It is important to note that nothing precludes the investment banking firm employing a mixed strategy. Thus, it may employ some proportion of labour $s_t \in [0, 1]$ in the safe strategy and $(1 - s_t)$ in the risky strategy so that

$$X_{t+1}^s(j) = [(1 - s_t) \Omega_t \varepsilon_{t+1}(j) + s_t \alpha_t] N_t(j). \quad (5.3)$$

At the start of period t the investment bank borrows $B_t(j) = W_t N_t(j)$ from retail banks. At the start of the next period, the investment bank receives $Q_{t+1}(j) X_{t+1}(j)$, and pays $B_t(j) R_t^C$

⁴For example, consider an investment bank that has made losses while trading on world financial markets on its own account. Other things constant, that bank will then be less able to provide financial or other services (e.g., underwriting) relating to raising of equity for the final goods firms. Of course, these activities are not modelled explicitly, but these are the kind of issues that we have in mind.

to the retail bank, where $Q_{t+1}(j)$ denotes the price per unit $X_{t+1}(j)$, and R_t^C is the interest due on the loan.

The market for the output of the investment banking sector is assumed to be imperfectly competitive and the demand for output of bank j is

$$X_t^d(j) = \left(\frac{Q_t(j)}{Q_t} \right)^{-\eta} X_t^d. \quad (5.4)$$

The superscript d indicates ‘demand’, and $\eta > 1$ is the demand elasticity. The aggregate price next period, Q_{t+1} , and aggregate demand, X_{t+1} , are exogenous to the bank’s decision, and not fully predictable. It will turn out that the expectation of the next period price and demand does not affect the investment bank’s optimal choice of s_t .

Clearly, the optimal strategy includes selling everything demanded ex-post:

$$X_{t+1}^d(j) = X_{t+1}^s(j) = X_{t+1}(j). \quad (5.5)$$

Combining (5.3), (5.4) and (5.5) and assuming a symmetric equilibrium, $N_t(j) = N$, shows that the ex-post price depends on the realization of the relative rates of return across the two strategies,

$$Q_{t+1}(j) = Q_{t+1} \left([(1-s_t)\Omega_t\varepsilon_{t+1}(j) + s_t\alpha_t] \frac{N_t}{X_{t+1}^d} \right)^{-1/\eta}. \quad (5.6)$$

Ex-ante, the investment bank needs to decide on s_t and the level of borrowing/labour input. So, expected profit, $E_t\Pi_{t+1}$, of a bank with shock ε_{t+1} is

$$\begin{aligned} E_t(\Pi_{t+1}|\varepsilon_{t+1}) &= \max [E_t Q_{t+1}(j) X_{t+1}(j) - W_t N_t R_t^C, 0], \\ &= \max \left[[(1-s_t)\Omega_t\varepsilon_{t+1} + s_t\alpha_t] N_t \right]^{1-1/\eta} E_t X_{t+1}^{1/\eta} Q_{t+1} - W_t N_t R_t^C, 0 \end{aligned} \quad (5.7)$$

showing that default occurs when ε_{t+1} is less than some optimally determined critical threshold, ε_t^D (where D indicates default). A straightforward manipulation shows

$$\varepsilon_t^D = \frac{1}{(1-s_t)\Omega_t} \left[\left(\frac{W_t N_t^{1/\eta} R_t^C}{E_t X_{t+1}^{1/\eta} Q_{t+1}} \right)^{\frac{\eta}{\eta-1}} - s_t \alpha_t \right]. \quad (5.8)$$

It follows that the expected default probability is $F(\varepsilon_t^D)$. It is convenient to define $D_t = \varepsilon_t^D (1-s_t)\Omega_t + s_t\alpha_t$ as the output of the threshold investment bank, for a choice of s_t and N_t , given (functions of) expected demand and price, such that profits are zero in expectation and the probability of default is also zero in expectation. Then, (5.8) implies that the demand for labour is

$$N_t = (D_t)^{\eta-1} \left(\frac{E_t X_{t+1}^{1/\eta} Q_{t+1}}{W_t R_t^C} \right)^\eta. \quad (5.9)$$

Equation (5.8) shows that the investment bank can reduce the probability of default by demanding less labour, since $(\partial\varepsilon_t^D/\partial N_t) > 0$. That means that an expanding investment banking sector increases risk, other things constant. It also says that $(\partial\varepsilon_t^D/\partial Q_{t+1}) < 0$, so

that a higher rate of expected default is associated with a lower expected price for financial intermediation services.

Given this threshold value of ε^D , expected profit of an investment bank is

$$E_t \Pi = E_t \int_{\varepsilon^D}^{+\infty} \left[Q_{t+1} (X_{t+1})^{1/\eta} \left([(1-s_t) \Omega_t \varepsilon + s_t \alpha_t] N_t \right)^{(\eta-1)/\eta} \right] f(\varepsilon) d\varepsilon - [1 - F(\varepsilon_t^D)] W_t R_t^C N_t. \quad (5.10)$$

Analytically, this is a complex problem as it is defined over the joint density of macroeconomic and microeconomic factors⁵. To retain tractability we therefore assume that investment banks do not take the spread of possible outturns for Q into account; loosely speaking they are ‘ignoring’ macro risk. Simplifying (5.10) and using (5.8) to substitute for N_t , one can write the bank’s maximand as $\frac{1}{W_t R_t^C} \left(\frac{W_t R_t^C}{E_t X_{t+1}^{1/\eta} Q_{t+1}} \right)^\eta E_t \Pi$. Hence, one may state the investment bank’s problem:

$$\max_{D_t, \varepsilon_t^D, s_t} D_t^{\eta-1} \left(\int_{\varepsilon^D}^{+\infty} \left(\frac{(1-s_t) \Omega_t \varepsilon + s_t \alpha_t}{D_t} \right)^{(\eta-1)/\eta} f(\varepsilon) d\varepsilon - [1 - F(\varepsilon_t^D)] \right) \quad (5.11)$$

$$\text{s.t. } D_t = [(1-s_t) \Omega_t \varepsilon_t^D + s_t \alpha_t]; \quad (5.12)$$

$$\varepsilon_t^D \geq 0; \quad (5.13)$$

$$s_t \geq 0; \quad (5.14)$$

$$-s_t \geq -1. \quad (5.15)$$

The problem facing the j -th investment bank is to maximize expected profits by: supplying an amount $X_{t+1}^s(j)$ to the final goods producer; choice of trading strategy, s_t ; hiring an amount of labour, $N_t(j)$, and hence incurring debt, $W_t N_t(j)$ plus interest. These choices are made given: the interest charge on borrowing, R_t^C ; the expected return from employing the safe and risky strategies; and expected demand $X_t^d(j)$. A key consideration is that the probability of default is rising in the amount borrowed for a given share of risky investment. Recognition of that factor led to the definition of ε_t^D above. Thus, a solution to (5.11)-(5.15) includes a choice of (ε^{D*}, s^*) , for each t , where a corner solution is defined as $\varepsilon^{D*} = 0$ and/or $s^* = 0$. A Lagrangian for this problem is analyzed fully in the appendix. Optimality requires

$$\mu (1 - s_t) \Omega_t + \lambda_d = 0, \quad (5.16)$$

where λ_d and μ are multipliers associated with (5.12) and (5.13), respectively. It is necessary either that $\lambda_d > 0$ and $\varepsilon_t^{D*} = 0$, or $\lambda_d = \mu = 0$. Whenever $\lambda_d > 0$, optimality for ε_t^D requires that $s_t^* \neq 1$ and $\mu < 0$. In this case, $\varepsilon_t^{D*} = 0$; *any* realization of ε ensures nonnegative

⁵That is, writing the problem in full,

$$E_t \Pi(N_t(j), \varepsilon^D(j), s(j)) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \Pi(N_t(j), \varepsilon^D(j), s(j), Q_{t+1}, X_{t+1}) f(Q_{t+1}, X_{t+1}) dQ_{t+1} dX_{t+1}.$$

expected profits and no expected default. On the other hand, when $\varepsilon_t^{D^*} > 0$, expected profits are still positive but there is potential for default. Hence, $\mu = 0$ as the marginal investment bank no longer produces D_t . Thus:

Proposition 5.1. *There is no internal solution to (5.11)-(5.15).*

Proposition 5.2. *There are only two types of local maxima: (i) The first is where the probability of default is zero; $F(\varepsilon_t^{D^*}) = 0$. (ii) The second is where investment banks choose $s_t^* = 0$.*

Proposition 5.1 is established in the appendix by demonstrating that an internal equilibrium violates the second order necessary conditions for a local maximum. Since the solution to (5.11)-(5.15) is characterized by the pair $(\varepsilon_t^{D^*}, s_t^*)$, the two candidate optima are those of Proposition 5.2. Furthermore, the first of these candidates will also entail $0 < s_t^* < 1$, whilst the second necessarily implies $F(\varepsilon_t^{D^*}) > 0$.

For a given distribution of risky returns, the optimal choice of s_t depends on ϕ_t , the expected relative rates of return across the risky and safe strategies, $\phi_t \equiv \Omega_t/\alpha_t$. To see this, note that the optimality condition with respect to D_t is

$$(\eta - 1) D_t^{\eta-2} \left[\frac{\eta - 1}{\eta} \int_{\varepsilon^D}^{+\infty} \left(\frac{(1 - s_t) \Omega_t \varepsilon + s_t \alpha_t}{D_t} \right)^{(\eta-1)/\eta} f(\varepsilon) d\varepsilon - [1 - F(\varepsilon^D)] \right] - \mu = 0.$$

With positive default probability, it has been established that $\mu = 0$ and $s^* = 0$. Thus:

Corollary 5.3. *There is a level of ϕ_t , ϕ^l , such that the local maximum with positive probability of default exists if and only if $\phi_t \geq \phi^l$. If the probability of default is positive then $s^* = 0$ and the default threshold satisfies equality*

$$\int_{\varepsilon^D}^{+\infty} \left(\frac{\varepsilon_t}{\varepsilon^D} \right)^{(\eta-1)/\eta} f(\varepsilon) d\varepsilon = \frac{\eta}{\eta - 1} [1 - F(\varepsilon^D)]. \quad (5.17)$$

Combining (5.17) with (5.10) one sees that in this case investment banks arrange things so that expected profit is a constant proportion, $1/(\eta - 1)$, of expected costs. This is a generalization of the standard result of a fixed mark up in a deterministic production environment under monopolistic competition.

Corollary 5.4. *There exists ϕ^h , defined in the appendix, such that the local maximum with $\varepsilon^{D^*} = 0$ exists if and only if $\phi_t \leq \phi^h$. The optimal s is deduced in the next Corollary 5.5.*

Following Proposition 5.1 one can compare expected profits associated with the risky strategy described in Corollary 5.3 with the maximum expected profit which can be obtained when the expected probability of default is zero.

To find the optimal safe strategy, investment banks solve the following problem

$$\max_{s, N} E_t \Pi = E_t \int_0^{+\infty} \left[Q_{t+1} (X_{t+1})^{1/\eta} \left([(1-s)\Omega_t \varepsilon + s\alpha_t] N_t \right)^{(\eta-1)/\eta} \right] f(\varepsilon) d\varepsilon - W_t R_t^C N_t. \quad (5.18)$$

subject to $0 \leq s \leq 1$ and that expected profits are positive, even when $\varepsilon = 0$,

$$E_t Q_{t+1} (X_{t+1})^{1/\eta} (s\alpha_t N_t)^{(\eta-1)/\eta} - W_t R_t^C N_t > 0. \quad (5.19)$$

This optimization problem is set out in full in the appendix. We state the results as follows:

Corollary 5.5. *The solution of problem (5.18-5.19) depends on ϕ_t as follows:*

- i) *If $\phi_t \leq 1$, it is optimal to employ only the safe strategy ($s^* = 1$).*
- ii) *If $1 \leq \phi_t \leq \phi^h$, (same as Corollary 5.4) there is an internal solution; neither of the constraints are binding ($0 < s^* < 1$) and the solution satisfies*

$$\int_0^{+\infty} (\phi\varepsilon - 1) \left[\left(\frac{1-s}{s} \right) \phi\varepsilon + 1 \right]^{-1/\eta} f(\varepsilon) d\varepsilon = 0. \quad (5.20)$$

Moreover, applying the implicit function theorem to (5.20) which defines optimal s^* , one can establish (see the appendix Section 13.5) that s^* declines in ϕ .

- iii) *if $\phi_t^h \leq \phi_t$ there is only a corner solution with binding constraint (5.19) and optimal s^* defined as*

$$\int_0^{+\infty} (\phi_t \varepsilon - 1) \left[\left(\frac{1-s}{s} \right) \phi_t \varepsilon + 1 \right]^{-1/\eta} f(\varepsilon) d\varepsilon - (\eta - 1) \int_0^{+\infty} \left[\frac{(1-s)}{s} \phi_t \varepsilon + 1 \right]^{1-1/\eta} f(\varepsilon) d\varepsilon + \eta = 0.$$

Having characterized expected profits under alternate strategies, it is now possible to clarify the nature of the global maximum.

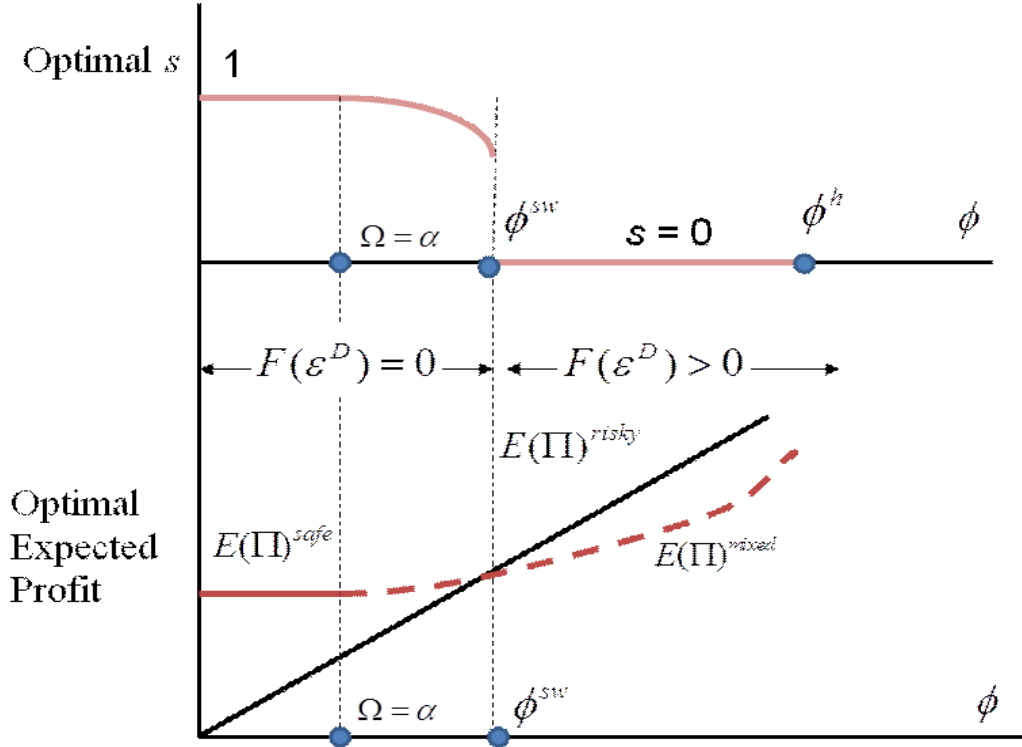
Proposition 5.6. *There exists a unique switching level of relative rates of return, ϕ^{sw} , where $\phi^l \leq \phi^{sw} \leq \phi^h$, such that a local maximum defined in Corollary 5.5 ($\varepsilon^{D*} = 0$) is a global one if and only if $\phi_t \leq \phi^{sw}$, while the solution defined in Corollary 5.3 is a global maximum if and only if $\phi_t \geq \phi^{sw}$.*

Consider the case where $\phi^{sw} > 1$:

Corollary 5.7. *If $\phi_t^{sw} > 1$, then for $\phi_t \in [1, \phi_t^{sw}]$ The optimal choice of s is $0 < \frac{\phi^h}{\phi^h + k^*} < s^* < 1$.*

The proofs and expressions for ϕ^l , ϕ^{sw} , ϕ^h , k^* are in the appendix. Proposition 5.6 shows that as ϕ_t declines from a value greater than ϕ^{sw} to a value less than ϕ^{sw} , the investment bank's optimal choice of s^* switches from a completely 'risky' one, $s^* = 0$, to one where there is a significant share of investment in the safe technology $s^* > 0$. So a smooth change in the economic environment can result in a 'jump' in investment decisions. Figure 5.1 summarizes the choice of optimal s as ϕ varies.

Figure 1: Profit and optimal strategy.



To conclude, given ϕ , investment banks decide on the optimal level of default probability. In what follows the case where that default probability is positive will be of particular interest; that is where $\phi_t > \phi_t^{sw}$ and $F(\varepsilon_t^{D*}) > 0$.

5.1. Competition and risk taking

The above simple model can be used to look at the interaction between risk, default and the intensity of competition. To that end equation (5.17) is studied to investigate the relation between competition in the investment banking sector (reflected in η) and risk (measured as the probability of default, $F(\varepsilon^D)$). The next proposition demonstrates that the default rate increases with competition.

Proposition 5.8. For $F(\varepsilon^D) > 0$, the default threshold, ε^D , and the probability of default $F(\varepsilon^D)$, increase with the intensity of competition, η .

Proof. Define $G(\varepsilon_t^D, \eta) = \int_{\varepsilon^D}^{+\infty} \left(\frac{\varepsilon_t}{\varepsilon_t^D}\right)^{(\eta-1)/\eta} f(\varepsilon) d\varepsilon - \frac{\eta}{\eta-1} [1 - F(\varepsilon_t^D)]$. Then, by equation (5.17), $G(\varepsilon_t^D, \eta) = 0$. It follows from the implicit function theorem that

$$\frac{d\varepsilon^D}{d\eta} = -\frac{\partial G}{\partial \varepsilon^D} / \frac{\partial G}{\partial \eta} > 0. \quad (5.21)$$

Also,

$$\frac{\partial G}{\partial \eta} = \frac{1}{\eta^2} \int_{\varepsilon^D}^{\varepsilon^m} \left(\frac{\varepsilon_t}{\varepsilon_t^D}\right)^{(\eta-1)/\eta} \log\left(\frac{\varepsilon_t}{\varepsilon_t^D}\right) f(\varepsilon) d\varepsilon + \frac{1}{(\eta-1)^2} [1 - F(\varepsilon_t^D)] > 0$$

and $G(\varepsilon_t^D, \eta) = 0$ were derived from the first order condition with respect to ε_t^D , for a profit maximization problem under the assumption $s = 0$. Therefore, the second order condition is equivalent to $\frac{\partial G}{\partial \varepsilon^D} < 0$ when ε^D is a local maximum. That proves

$$\frac{d\varepsilon^D}{d\eta} > 0; \text{ and } \frac{dF(\varepsilon^D)}{d\eta} = f(\varepsilon^D) \frac{d\varepsilon^D}{d\eta} > 0. \quad (5.22)$$

■

The intuition for these results seem straightforward enough; the lower mark-up shrinks the cushion of excess profits that absorbs the impact of low ε draws. This basic result seems, in spirit, consistent with arguments already in the literature that competition in financial markets may promote risk taking (see for example, Hellmann et.al. (2000), Bolt and Tieman, (2004), Repullo (2004) and Allen and Gale(2004)). The real issue, of course, is whether increased competition is welfare enhancing. For example, some microeconomic models suggest that competition not only increases risk but may also improves entrepreneurs' access to credit (Bolt and Tieman, 2004) and reduces the loan rate (Boyd and De Nicoló 2005). Thus higher risk may be positively correlated with higher investment, and therefore may promote production and welfare. That positive relation is also documented in some recent empirical work (see Claessence and Laeven 2005). This is exactly what is observed in our model, since employment, and therefore the output of the financial sector, is positively related to the expected default threshold (See equation (5.9)). Later it is shown that in the present framework, increased competition will result in higher production and lower prices for financial services, which in turn will result in higher consumption and welfare.

5.2. Aggregate behavior of the investment banking sector

It is straightforward now to characterize the aggregate behavior of the investment banking sector. Using the formula for aggregate supply, $X_t = \left[\int_0^1 X_t(i)^{\frac{\eta-1}{\eta}} di \right]^{\frac{\eta}{\eta-1}}$, one can derive a relation between labour and aggregate output of the investment banking sector:

$$E_t X_{t+1} = N_t \left[\int_0^{\infty} [(1-s_t)\Omega\varepsilon + s_t\alpha_t]^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon \right]^{\frac{\eta}{\eta-1}}. \quad (5.23)$$

Then, equation (5.9) in combination with (5.23) provide an expression for the expected price,

$$E_t \frac{Q_{t+1}}{W_t R_t^C} = E_t D_t^{-\frac{\eta-1}{\eta}} \left[\int_0^\infty [(1-s)\Omega_t \varepsilon + s\alpha_t]^\frac{\eta-1}{\eta} f(\varepsilon) d\varepsilon \right]^{-\frac{1}{\eta-1}}. \quad (5.24)$$

Formula (5.24) shows that the average markup in the investment banking sector declines with risk, ε^D .

Consider the case where the investment bank optimally sets $s = 0$. Then

$$E_t X_{t+1} = \Omega_t N_t \Delta_t; \quad (5.25)$$

$$E_t Q_{t+1} = \frac{W_t R_t^C}{\Omega_t} (\varepsilon^D)^{-\frac{\eta-1}{\eta}} \Delta^{-\frac{1}{\eta}}, \quad (5.26)$$

where $\Delta := \left[\int_0^\infty \varepsilon^\frac{\eta-1}{\eta} f(\varepsilon) d\varepsilon \right]^\frac{\eta}{\eta-1}$ is less than 1, increases in η , and converges to 1 as η approaches infinity. Formula (5.25) shows that the expected output of the investment banking sector is increasing in the degree of competition, while formula (5.26) shows that the expected mark up over effective labour costs declines in the intensity of competition.

6. Commercial Banking and the Credit Spread

As described earlier, commercial banks attract deposits and offer loans to investment banks. Commercial banks benefit from credit diversification so that loans of commercial bank i , $B(i)$ are fully diversified across all investment banks. They act as monopolistic competitors and maximize expected profit, $E_t \Psi_{t+1}$, given the demand for loans, (3.1), where

$$E_t \Psi_{t+1}(R_t^c(i)) = E_t R_t^c(i) B_t^c(i) - R_t^h B_t^c(i). \quad (6.1)$$

If a borrower remains solvent, the commercial bank will earn nominal return R_t^c per unit loaned. In the case of default, the assets of the borrower are repossessed by the commercial bank. So, for the borrowers for whom $\varepsilon_{t+1}(j) < \varepsilon^D$, the expected repayment is $E_t Q_{t+1}(j) \Omega_t \varepsilon_{t+1}(j) N_t$ only. Note that at the expected default threshold for the investment bank defined as $E_t Q_{t+1}(\varepsilon^D) \Omega_t \varepsilon^D N_t = B^c(i) R_t^c(i)$, where $E_t Q_{t+1}(j) = E_t Q_{t+1}(\varepsilon_j)^{-1/\eta}$ (from (5.6) in assumption that $s = 0$) and in general the assets to liabilities ratio of borrower j can be expressed as

$$E_t \frac{Q_{t+1}(j) \Omega_t \varepsilon_{t+1}(j)}{R_t^c(i) W_t} = E_t \left(\frac{\varepsilon_{t+1}(j)}{\varepsilon_t^D} \right)^\frac{\eta-1}{\eta}.$$

The expected return of the commercial bank may then be written as

$$E_t B_t^c(i) R_t^c(i) = B_t^c(i) R_t^c(i) \left[\int_0^{\varepsilon^D} \left(\frac{\varepsilon}{\varepsilon^D} \right)^\frac{\eta-1}{\eta} f(\varepsilon) d\varepsilon + [1 - F(\varepsilon^D)] \right]. \quad (6.2)$$

Consequently, one can re-write the commercial bank's objective in a slightly more convenient form by combining (3.1), (6.1) and (6.2),

$$E_t \Psi_{t+1}(R_t^c(i)) = \left(\Gamma \frac{R_t^c(i)}{R^c} - \frac{R_t^h}{R_t^c} \right) \left(\frac{R_t^c(i)}{R_t^c} \right)^{-\delta} B_t^C R_t^c,$$

where

$$\Gamma := \int_0^{\varepsilon^D} \left(\frac{\varepsilon}{\varepsilon^D}\right)^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon + [1 - F(\varepsilon^D)] < 1. \quad (6.3)$$

$1/\Gamma$ is interpreted as an insurance premium against default. By definition Γ is a ratio of expected to contract return. In a risk-free market Γ_t should be equal 1. The value $1/\Gamma$ corresponds to a minimum relative spread required for a zero expected profit. It is easy to see that for a given distribution f , the ratio $1/\Gamma$ increases with the default threshold, ε^D .

The first order condition with respect to the relative interest rate requires

$$\frac{R_t^C(i)}{R_t^c} \Gamma = \frac{\delta}{\delta - 1} \frac{R_t^h}{R_t^c}$$

and since in equilibrium all banks charge the same interest rate

$$R_t^C = \frac{\delta}{\delta - 1} \frac{1}{\Gamma} R_t^h. \quad (6.4)$$

Once again we can see that if the credit market were perfectly competitive ($\delta \rightarrow \infty$), the relative spread would be $1/\Gamma$. Expression (6.4) provides a measure of the spread,

$$spread = \ln R_t^C - \ln R_t^h = \ln \frac{\delta}{\delta - 1} - \ln \left[\int_0^{\varepsilon^D} \left(\frac{\varepsilon}{\varepsilon^D}\right)^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon + [1 - F(\varepsilon^D)] \right]. \quad (6.5)$$

The following proposition is readily established:

Proposition 6.1. *The credit spread increases with competition in the investment banking sector,*

$$\frac{d(spread)}{d\eta} > 0.$$

Proof. It follows immediately from (5.22) and (6.3) that

$$\frac{d(spread)}{d\eta} = -\frac{1}{\Gamma_t} \left(\frac{\partial \Gamma_t}{\partial \varepsilon^D} \frac{d\varepsilon^D}{d\eta} + \frac{d\Gamma_t}{d\eta} \right) > 0$$

since $\frac{\partial \Gamma}{\partial \varepsilon^D} = -\frac{1}{(\varepsilon_t^D)^2} \int_0^{\varepsilon^D} \varepsilon f(\varepsilon) d\varepsilon < 0$ and $\frac{d\Gamma_t}{d\eta} = \frac{1}{\eta^2} \int_0^{\varepsilon^D} \left(\frac{\varepsilon}{\varepsilon^D}\right)^{\frac{\eta-1}{\eta}} \ln\left(\frac{\varepsilon}{\varepsilon^D}\right) f(\varepsilon) d\varepsilon < 0$.

Moreover, from (6.5) it is easy to formulate Proposition 6.2.

Proposition 6.2. *The credit spread declines with competition in the commercial banking sector, $\frac{d(spread)}{d\delta} < 0$.*

■

This result is consistent with Proposition 5.8 above. Since competition increases the expected default threshold, commercial bank loans are more risky and the spread is consequently higher.

6.1. Actual and expected default: pure idiosyncratic shock

First consider the case when shocks to investment banks are purely idiosyncratic. Then aggregate output and expected price will be as in (5.23) and (5.24). If banks are investing solely in the risky strategy, $s_t = 0$, then

$$X_{t+1} = E_t X_{t+1} = \Omega_t N_t \Delta_t; \quad (6.6)$$

$$E_t Q_{t+1} = \frac{W_t R_t^C}{\Omega_t} (\varepsilon^D)^{-\frac{\eta-1}{\eta}} \Delta_t^{-\frac{1}{\eta}}. \quad (6.7)$$

For investment bank i total revenue is recovered from (5.3) and (5.6)

$$Q_{t+1}(i) X_{t+1}(i) = Q_{t+1} X_{t+1} \left(\frac{\Omega_t N_t \varepsilon_{t+1}(i)}{X_{t+1}} \right)^{1-1/\eta} = Q_{t+1} X_{t+1} \left(\frac{\varepsilon_{t+1}(i)}{\Delta_t} \right)^{1-1/\eta}. \quad (6.8)$$

That bank will default when $\varepsilon_{t+1}(i) < \varepsilon^A$, where the actual default threshold (ε^A) is defined as

$$Q_{t+1} X_{t+1} \left(\frac{\varepsilon^A}{\Delta_t} \right)^{1-1/\eta} = W_t R_t^C N_t. \quad (6.9)$$

On the other hand, the expected default threshold is

$$E_t Q_{t+1} X_{t+1} \left(\frac{\varepsilon^D}{\Delta_t} \right)^{1-1/\eta} = W_t R_t^C N_t. \quad (6.10)$$

So, combining these equations it follows that

$$\ln \varepsilon^A = \ln \varepsilon^D - \frac{\eta}{\eta-1} (\ln Q_{t+1} - \ln E_t Q_{t+1}). \quad (6.11)$$

One concludes that competition reduces the impact of price misperceptions on equilibrium outturns. In other words, actual and expected prices are closer on average as competition increases and so too are expected and actual default levels.

The realized value of Q_{t+1} can be derived from equation (4.2)

$$Q_{t+1} = P_{t+1} A_{t+1} \frac{(\theta-1)}{\theta}$$

So the investment banking sector will be in default if either the financial shock ε_{t+1} or macroeconomic shock A_{t+1} is small

$$\frac{(\theta-1)}{\theta} A_{t+1} \Omega_t \varepsilon_{t+1} < \frac{w_t R_t^C}{\pi_{t+1}}. \quad (6.12)$$

Formula (6.12) also shows that inflation reduces the probability of default as it reduces the value of liabilities. So, in this model, inflation acts in part like a subsidy to investment banking.

6.2. Cost of bail out of commercial bank

It is assumed throughout that the government guarantees the safety of deposits. It has also been assumed that all commercial banks fully diversify their loan portfolios across all

investment banks. So in the event of default, all banks in default will have identical negative net worth. In that case the aggregate nominal cost of the bailout, $P_t G_t$, is straightforward to calculate; it will be equal to the difference between total deposit liabilities of the commercial banking sector and the revenue of the financial sector. That is,

$$P_t G_t = \max(-\Psi_t(R_{t-1}^c), 0).$$

Here $\Psi_t(R_{t-1}^c)$ denotes the aggregate assets of the commercial banking sector less the aggregate liabilities. These assets at the end of the period are the loans that are repaid. One may calculate an explicit formula for Ψ_t as follows.

A commercial bank will receive $R_t^c B_t^c$ from investment bank j if the the bank's ε draw is larger than ε^A , defined above. The funds repaid by solvent investment banks are then equal to $R_t^c B_t^c (1 - F(\varepsilon^A))$.

On the other hand, the amount recovered from an insolvent investment bank is $Q_{t+1}(j)X_{t+1}(j)$, which is the same as (6.8). So the aggregate loan recovery across all insolvent investment banks is

$$Q_{t+1}X_{t+1} \left[\int_0^{\varepsilon^A} \left[\frac{\varepsilon}{\Delta} \right]^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon \right].$$

Thus, it follows that

$$\Psi_{t+1}(\varepsilon^A) = Q_{t+1}X_{t+1} \left[\int_0^{\varepsilon^A} \left[\frac{\varepsilon}{\Delta} \right]^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon \right] + R_t^c W_t N_t (1 - F(\varepsilon^A)) - R_t^h W_t N_t. \quad (6.13)$$

Recalling (6.6) and (6.7), and using (6.9), yields

$$\Psi_{t+1}(\varepsilon^A) = Q_{t+1}X_{t+1} \left[\int_0^{\varepsilon^A} \left[\frac{\varepsilon}{\Delta} \right]^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon + \left(\frac{\varepsilon^A}{\Delta} \right)^{\frac{\eta-1}{\eta}} \left(1 - F(\varepsilon^A) - \frac{R_t^h}{R_t^c} \right) \right]. \quad (6.14)$$

It seems intuitively plausible⁶ that Ψ_{t+1} declines in ε^A and that commercial banks will have negative net assets only when the actual default rate is greater than some threshold value, $\varepsilon^A > \varepsilon^C$, where ε^C solves the zero profit condition, $\Psi_{t+1}(\varepsilon^C) = 0$.

Furthermore, recalling the definition of the credit spread, (6.3) and (6.4), one can show that the critical value of ε for default is larger than the expected default rate, $\varepsilon^C > \varepsilon^D$ and that $\Psi_{t+1}(\varepsilon^D) > 0$. This, together with equation (6.11), implies that government intervention is not required when the mistake in price prediction is relatively low: $Q_{t+1} \approx E_t Q_{t+1}$.

It is straightforward to show that the probability of commercial bank insolvency and the size of the government bail out increase with competition in the commercial banking

⁶Although intuitively plausible, there are some technical issues. These are explained in the appendix, Section 13.7.

sector⁷, represented by δ . However, cheaper financial intermediation may offset the costs of bankruptcy. This is considered further below.

We assume that government intervention is costly. Such costs, $g(G_t)$, can be associated with monitoring costs and distortive taxation. It is also worth mentioning that rising government expenditure will increase the gap between production and consumption since

$$Y_t = C_t + g(G_t).$$

So, when the prices for financial intermediation are reasonably predictable, it has been shown in (6.11) that $\varepsilon^A \approx \varepsilon^D$, and therefore $\Psi_{t+1}(\varepsilon^A)$ is positive and government intervention is not required, $G_t = 0$. Therefore, the implications for social welfare from purely idiosyncratic shocks are limited. However, when the investment banking sector faces common, or systematic, shocks then the impact on welfare may be substantial.

7. Aggregate equations of the model economy

Clearly the equilibrium relations of the model change depending on the nature of the shocks and whether or not universal banking is considered. Here the set of equilibrium equations is set out for the baseline model with separated banking and idiosyncratic shocks. As we consider variations on the baseline, we indicate which key equations change and refer in some cases to the appendix for further details.

7.1. Idiosyncratic shock with separate banking

The equations of the model in the case of purely idiosyncratic shocks with separate banking, are set out below.

Definition 7.1. *A monopolistically competitive equilibrium is a set of plans, $\{C_{t+k}, Y_{t+k}, N_{t+k}, w_{t+k}, X_{t+k}, q_{t+k}, \pi_{t+k}, R_{t+k}^c, R_{t+k}^h, q_{t+1}^e\}_{k=0}^\infty$, given initial conditions, $\{A_{t-1}, N_{t-1}, R_{t-1}^h, R_{t-1}^c, w_{t-1}\}$, and dynamics of policy variables, $\{\pi_{t+k}\}_{k=0}^\infty$, and exogenous shocks, $\{A_{t+k}, \Omega_{t+k}\}_{k=0}^\infty$, and satisfying conditions ((7.1)-(7.11)).*

⁷Given everything else constant, it reduces the spread as proved in Proposition 6.2,

$$E_t \left\{ \frac{\beta C_t}{C_{t+1}} \frac{1}{\pi_{t+1}} \right\} = \frac{1}{R_t^h} \quad (7.1)$$

$$C_t = \frac{w_t}{\lambda}; \quad (7.2)$$

$$q_t = A_t \frac{(\theta - 1)}{\theta} \quad (7.3)$$

$$Y_t = A_t X_t \quad (7.4)$$

$$X_{t+1} = \Omega_t N_t \Delta \quad (7.5)$$

$$\left(\frac{w_t R_t^C}{\Omega_t E_t \pi_{t+1} q_{t+1}} \right)^\eta = \Delta [\varepsilon_t^D]^{\eta-1} \quad (7.6)$$

$$R_t^C = \frac{\delta}{\delta - 1} \frac{1}{\Gamma} R_t^h \quad (7.7)$$

$$C_t = Y_t - g(G_t) \quad (7.8)$$

$$G_t = \max(-\Psi_t(\varepsilon^A), 0) \quad (7.9)$$

$$\Psi_{t+1}(\varepsilon^A) = \frac{q_{t+1} X_{t+1}}{\Delta^{\frac{\eta-1}{\eta}}} \left[\int_0^{\varepsilon^A} \varepsilon^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon + (\varepsilon^A)^{\frac{\eta-1}{\eta}} \left[1 - F(\varepsilon^A) - \frac{R_t^h}{R_t^c} \right] \right] \quad (7.10)$$

$$\left(\frac{\varepsilon^A}{\varepsilon^D} \right)^{\frac{\eta-1}{\eta}} = \frac{E_t q_{t+1}}{q_{t+1}} \quad (7.11)$$

Where ε^D , Γ and Δ are defined in ((7.12)-(7.14))

$$1 - F(\varepsilon^D) = \frac{\eta - 1}{\eta} \int_{\varepsilon^D}^{+\infty} \left(\frac{\varepsilon}{\varepsilon^D} \right)^{(\eta-1)/\eta} f(\varepsilon) d\varepsilon \quad (7.12)$$

$$\Gamma = \int_0^{\varepsilon^D} \left(\frac{\varepsilon}{\varepsilon^D} \right)^{(\eta-1)/\eta} f(\varepsilon) d\varepsilon + [1 - F(\varepsilon^D)] \quad (7.13)$$

$$\Delta = \left[\int_0^{\infty} \varepsilon^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon \right]^{\frac{\eta}{\eta-1}} \quad (7.14)$$

8. Systematic shock

Following the recent financial crisis many researchers (for example, see Haldane, 2010) have highlighted the importance of the common or systematic component of shocks hitting the financial system. In this section we derive the key formulas characterizing financial intermediation when the shock in the investment banking sector is common across all banks, and is perceived as such. Later on, a different type of common shock is analyzed which affects the usefulness of the output of investment banks.

When the shock to investment banks is not purely idiosyncratic, then expected prices and demand can be significantly different from actual. If the shock is purely systematic, all investment banking firms will have the same output and will charge the same price

$$X_{t+1} = \Omega_t \varepsilon_{t+1} N_t. \quad (8.1)$$

Since the shock is common to all banks, all expect to charge the same price and produce the same quantity

$$E_t X_{t+1}(j) = E_t X_{t+1} = \Omega_t N_t; \quad (8.2)$$

$$E_t Q_{t+1}(j) = E_t Q_{t+1}. \quad (8.3)$$

As a consequence, the discrepancy between actual and expected output is

$$X_{t+1} = \varepsilon_{t+1} E_t X_{t+1}. \quad (8.4)$$

8.1. Spread

To set R_t^C , the commercial banks need to compute the expected return on credit. If $\varepsilon_{t+1} < \varepsilon^D$, only a fraction, $\varepsilon_{t+1}/\varepsilon^D$ of loans will be repaid. Therefore, the spread component will be slightly different to the situation when the shock is idiosyncratic,

$$\Gamma_t^s := \int_0^{\varepsilon_t^D} \left(\frac{\varepsilon}{\varepsilon^D} \right) f(\varepsilon) d\varepsilon + [1 - F(\varepsilon_t^D)] < 1. \quad (8.5)$$

8.2. Actual default and the size of bailout: systematic shock

Default occurs when

$$Q_{t+1} X_{t+1} < W_t R_t^C N_t, \quad (8.6)$$

or equivalently if $\varepsilon_{t+1} < \varepsilon^A$, where ε^A is defined as

$$\varepsilon_t^A = \frac{W_t R_t^C}{\Omega_t Q_{t+1}} = \frac{E_t Q_{t+1}}{Q_{t+1}} \frac{W_t R_t^C}{\Omega_t E_t Q_{t+1}} = \frac{E_t Q_{t+1}}{Q_{t+1}} \varepsilon_t^D.$$

In other words,

$$\ln \varepsilon_t^A - \ln \varepsilon_t^D = -(\ln Q_{t+1} - \ln E_t Q_{t+1}).$$

When the shock is purely systematic, every commercial bank recovers either $R_t^c B_t^c$ or $Q_{t+1} X_{t+1}$, whichever is the smaller. Consider the threshold value ε^C when the profit of the banking industry is zero:

$$\begin{aligned} \Psi_{t+1} &= Q_{t+1} X_{t+1} - R_t^h B_t^c; \\ &= N_t R_t^C W_t \left(\frac{Q_{t+1} \Omega_t}{R_t^C W_t} \varepsilon^C - \frac{R_t^h}{R_t^C} \right) = 0. \end{aligned}$$

Rearranging this last expression, one finds that

$$\varepsilon^C = \frac{R_t^h}{R_t^C} \frac{R_t^C W_t}{E_t Q_{t+1} \Omega} \frac{E_t Q_{t+1}}{Q_{t+1}} = \frac{R_t^h}{R_t^C} \frac{E_t Q_{t+1}}{Q_{t+1}} (\varepsilon^D)^{\frac{\eta-1}{\eta}}.$$

One can conclude from this that commercial banks fail less often than investment banks since,

$$\varepsilon^C = \frac{R_t^h}{R_t^C} \varepsilon^A < \varepsilon^A. \quad (8.7)$$

When shock ε_{t+1} hits the banking system, the net assets of the commercial banking sector are

$$\Psi_{t+1} = Q_{t+1} X_{t+1} - R_t^h B_t^c = N_t R_t^h W_t \left(\frac{\varepsilon_{t+1}}{\varepsilon^C} - 1 \right). \quad (8.8)$$

A bail out occurs only when Ψ_{t+1} is negative. Thus, the bail out will be

$$G_t = R_t^h W_t N_t \max \left[\left(1 - \left(\frac{\varepsilon_{t+1}}{\varepsilon^C} \right) \right), 0 \right]. \quad (8.9)$$

8.3. Aggregate equations: systematic shocks and separate banking

In considering separated banking, the system (7.1-7.11) is slightly transformed. First, the dispersion term, Δ_t , is replaced by 1. Second, there is a slight change in the risk premium. The new constant Γ_s is defined as in (8.5). The formulas for the expected and actual output of investment banks become (14.13) and (14.14). There will be equations defining the default threshold level of the trading strategies, which determines the solvency of commercial banks as in (8.7). Finally, the formula for government support will be (8.9). The full set of equations is set out in the appendix 14.2.

9. Cost and benefit of vertical integration

The desirability of universal banks versus separate investment and commercial banking entities is the subject of much current debate. The above framework allows one to begin to examine some key underlying themes in that debate. To that end, consider the model economy where one commercial bank is integrated with one investment bank. There is an obvious cost associated with this course of action since risk will be less diversified⁸. On the other hand, a benefit ought to come from addressing double marginalization and the insurance premium associated with two separate entities.

Assume that the integrated bank can directly invest in a risky or safe strategy. As before, intermediation requires labour input. However, now the integrated bank cost of borrowing is equal to the deposit rate, R_t^h . From equation (7.7) one sees the benefits of integrated banking. The reduction in cost consists of the elimination of a monopoly distortion among commercial banks, $\frac{\delta}{\delta-1}$, which in the industrial organization literature is known as double marginalization. There will also be savings associated with the premium $\frac{1}{\Gamma_t}$. So, in the economy with integrated banking one replaces (7.7) with the simple equality

$$R_t^C = R_t^h.$$

However, on the downside, the cost of bail out, G_t , may increase significantly.

Consider the purely idiosyncratic shock case in the absence of macroeconomic instability ($Q_{t+1} = E_t Q_{t+1}$). When commercial banks are separated from investment banks, the commercial banking sector generates positive profit and the cost of government intervention is zero, on average.

In an economy with global banking, $\varepsilon^A = \varepsilon^C$ and the government will need to bail out the sector if the idiosyncratic shock is smaller than ε^A . In this case, the total cost of the

⁸The benefits from risk diversification was clearly shown in Greenwood and Jovanovic (1990). In our model the absence of risk diversification in the banking sector is partly compensated by the government deposit insurance policy.

bailout will be

$$G_t = R_t^h W_t N_t \int_0^{\varepsilon^A} \left(1 - \left(\frac{\varepsilon}{\varepsilon^A}\right)^{\frac{\eta-1}{\eta}}\right) f(\varepsilon) d\varepsilon.$$

There always will be a proportion of global banks in trouble even when the shock is purely idiosyncratic. In this case the small proportion of insolvent banks will need to be bailed out, which will require the fiscal authorities to raise tax revenue. However, the whole economy may benefit from cheaper financial intermediation resulting in lower costs of production.

On the other hand, when the shock is purely systematic, $\varepsilon^C = \varepsilon^A$, the cost of the bail out will be,

$$G_t = R_t^h W_t N_t \max \left[\left(1 - \left(\frac{\varepsilon}{\varepsilon^A}\right)\right), 0 \right].$$

Purely systematic negative shocks can be very costly under universal banking, more so than with separated banks. However, as noted previously, the cost of government intervention is offset by the lower cost of financial intermediation that universal banking permits.

In what follows we parameterize the model in order to form a preliminary view as to how the model evaluates the potential trade-offs between universal and separate banking in the face of different kinds of shocks.

10. Numerical analysis

The analysis of the impact of universal banking proceeds by comparison of welfare in the stochastic steady states, given the assumed shock process. We begin by analyzing the impact of idiosyncratic versus systematic shocks. Then we turn to shocks to the quality of financial capital. Further details of the parameterization and the full sets of equations for permutations of the model are given in the appendix, Section 14.

10.1. Idiosyncratic shocks

In the stochastic steady state with separated banking, $\varepsilon^A = \varepsilon^D$. When commercial banks are separated from investment banks, it is possible to diversify the risk completely. All commercial banks have the same positive profit $\Psi_{t+1} > 0$, there is no default in the commercial banking industry and $\varepsilon^C = 0$. Hence, there is a wedge between deposit and loan rates. Furthermore, no government intervention is required, $G_t = 0$. On the other hand, under universal banking, it is the case that: $\varepsilon^C = \varepsilon^A = \varepsilon^D$. The equilibrium equations of the model change so that

$$R_t^C = R_t^h; \tag{10.1}$$

$$C_t = Y_t - g(G_t); \tag{10.2}$$

$$G_t = -Q_{t+1} X_{t+1} \Delta^{-\frac{\eta-1}{\eta}} \left[\int_0^{\varepsilon^D} \left(\varepsilon^{\frac{\eta-1}{\eta}} - (\varepsilon^D)^{\frac{\eta-1}{\eta}} \right) f(\varepsilon) d\varepsilon \right]. \tag{10.3}$$

In other words, there is no wedge between the interest rates and government expenditure is non-zero. We assume that the government's intervention is distortive in the sense that every £1 of bailout costs the taxpayer £1.20; so the net distortion is 20%, $g(G_t) = 0.2G_t$.⁹ This costs is levied lump-sum across agents.

Table 1 contains the results of the numerical comparison between universal and separated banking with idiosyncratic shocks. Social welfare is computed assuming a lognormal distribution for ε .¹⁰

Table 1: Idiosyncratic shocks	R^C	C	G	G/C	N	Welfare Gain ²
<i>EDR</i> ¹ = 3.8% ($\sigma = 0.20$)						
Separate	16.0%	0.59	0	0%	0.58	
Universal	4.2%	0.66	0	0.2%	0.65	4.2%
<i>EDR</i> ¹ = 10% ($\sigma = 0.25$)						
Separate	16.7%	0.60	0	0%	0.59	
Universal	4.2%	0.67	0	0.6%	0.66	4.3%
<i>EDR</i> = 19% ($\sigma = 0.30$)						
Separate	18.3%	0.61	0.00	0.0%	0.60	
Universal	4.2%	0.69	0.01	1.5%	0.69	4.5%
<i>EDR</i> = 40% ($\sigma = 0.40$)						
Separate	25.0%	0.63	0.00	0.0%	0.63	
Universal	4.2%	0.76	0.04	5.9%	0.77	4.9%
1 <i>EDR</i> is the expected default rate, $F(\varepsilon^D)$. 2. Welfare is measured in consumption equivalent.						

It is worth recalling that $F(\varepsilon^D)$ is endogenous. From Table 1 it appears that universal banking is not only welfare superior to separated banking, but the gain is rising in the variance of ε ; the increase in consumption required to compensate for separated banking is monotonically rising in the variance of ε . Higher volatility increases the insurance premium under separated banking firms. And even in the extreme case of very high volatility, the model suggests that universal banking is preferable to separated banking. In short higher government expenditure (by way of a consumption-reducing, distortive bail-out) is offset by a reduction in the insurance premium.

Another interesting observation is that volatility stimulates the economy. Since financial

⁹Allgood and Snow (1998) estimate the marginal cost of raising tax revenue to be between 5% and 26% depending on the assumption about the elasticity of labour supply. That is why we chose 20% for our simulation.

¹⁰We also experimented with the gamma distribution and for different parameters. The conclusions in the text were unaltered.

intermediaries have an option to be bailed out, increasing volatility can be viewed as providing increased upside risk. That stimulates the production of the investment banking sector. One observes this in the higher level of employment, N . However, these result are obtained under the restrictive assumption that the shock is purely idiosyncratic and therefore diversifiable from a macroeconomic perspective. In the following section we turn to the opposite extreme and assume shocks are purely systematic, that is common across investment banks.

10.2. Systematic shocks

Assuming that shocks are systematic, we now enquire how large would a negative shock have to be for universal banking to be welfare reducing. Since we assume that $\pi_t = 1$ in steady state, there is no uncertainty about the price of financial intermediation, so that

$$\begin{aligned} Q &= q = A \frac{(\theta - 1)}{\theta}; \\ \varepsilon^A &= \varepsilon^D. \end{aligned}$$

For the case of separated banking we use system (14.9)-(14.20) from appendix 14.2 and make the following substitutions: $R_t^C = \frac{\delta}{\delta-1} \frac{1}{\Gamma_s} R_t^h$, $\varepsilon^C = \frac{R_t^h}{R_t^C} \varepsilon^D$; $w_t = \lambda C_t$; $Y_t = A_t X_t$; $X_{t+1} = \Omega N_t \varepsilon_{t+1}$, and $\pi_t = 1$. So,

$$E_t \left\{ \frac{1}{C_{t+1}} \right\} = \frac{1}{\beta C_t R_t^h}; \quad (10.4)$$

$$\left(C_t R_t^h \right)^{-1} = [\varepsilon^D]^{-\frac{\eta-1}{\eta}} \frac{\lambda}{q \Omega} \frac{\delta}{\delta-1} \frac{1}{\Gamma_s}; \quad (10.5)$$

$$C_t = A \Omega N_{t-1} \varepsilon_t - g(G_t); \quad (10.6)$$

$$G_{t+1} = R_t^h \lambda C_t N_t \max \left[\left(1 - \left(\frac{\varepsilon_{t+1}}{\varepsilon^D} \frac{\delta}{\delta-1} \frac{1}{\Gamma_s} \right) \right), 0 \right]. \quad (10.7)$$

However, to proceed we now have to construct equations for the stochastic steady state, in particular for consumption and labour.

From equation (10.5) one concludes that $C_t R_t^h$ is constant over time. Now define new constants

$$u = \left(\frac{1}{\varepsilon^D} \frac{\delta}{\delta-1} \frac{1}{\Gamma_s} \right)^{-1}$$

and

$$u_1 = \left([\varepsilon^D]^{-\frac{\eta-1}{\eta}} \frac{\lambda}{q \Omega} \frac{\delta}{\delta-1} \frac{1}{\Gamma_s} \right)^{-1}.$$

Then equations (10.4) and (10.5) can be rewritten as

$$C_t R_t^h = u_1; \quad (10.8)$$

$$E_t \left\{ \frac{1}{C_{t+1}} \right\} = \frac{1}{\beta u_1}. \quad (10.9)$$

One can now easily compute $E_t \left\{ \frac{1}{C_{t+1}} \right\}$. Consider equation (10.6) with a one period lead which in combination with (10.7) implies

$$C_{t+1} = N_t \left(A\Omega\varepsilon_{t+1} - \lambda \frac{g}{u} u_1 \max[(u - \varepsilon_{t+1}), 0] \right).$$

$$\frac{N_t}{C_{t+1}} = \begin{cases} \frac{1}{A\Omega\varepsilon_{t+1}}, & \text{if } \varepsilon_{t+1} \geq u \\ \frac{1}{A\Omega\varepsilon_{t+1} - \lambda \frac{g}{u} u_1 (u - \varepsilon_{t+1})} & \text{if } \varepsilon_{t+1} < u. \end{cases}$$

Therefore, the expectation of $\left\{ \frac{N_t}{C_{t+1}} \right\}$ will depend on current consumption and the deposit interest rate

$$E_t \left\{ \frac{N_t}{C_{t+1}} \right\} = \int_0^u \frac{1}{A\Omega\varepsilon - \lambda \frac{g}{u} u_1 (u - \varepsilon)} f(\varepsilon) d\varepsilon + \int_u^{+\infty} \frac{1}{A\Omega\varepsilon} f(\varepsilon) d\varepsilon.$$

Denote the right hand side of this equation by χ_s . Now use (10.9) in order to get N_t

$$N_t = u_1 \beta \chi_s,$$

which is also constant over time. From this one sees that labour demand does not change with the realization of the shock, a key aspect of the way the investment bank's problem is set up.

Finally, one can combine (10.6) and (10.7) to yield

$$C_t(\varepsilon_t) = u_1 \beta \chi_s \left(A\Omega\varepsilon_t - \lambda \frac{g}{u} u_1 \max[(u - \varepsilon_t), 0] \right).$$

Hence, the equilibrium distribution of consumption and labour, and thus expected utility, may be constructed, given an assumed distribution function for ε .

It also emerges that, in steady state, consumption and the interest rate are inversely related, given price stability (as in formula 10.8). Higher consumption increases wage demands and therefore the interest rate needs to be lower to stimulate output.

For universal banking one obtains the distribution of consumption and labour in a similar way to find that

$$C_t(\varepsilon_t) = u_{u1} \beta \chi_u \left(A\Omega\varepsilon_t - \lambda \frac{g}{u_u} u_{u1} \max[(u_u - \varepsilon_t), 0] \right)$$

and

$$N_g = u_{u1} \beta \chi_u.$$

Note, that the constants differ from the previous case.¹¹

¹¹Specifically, $u_u = \varepsilon^D$; $u_{u1} = \left([\varepsilon^D]^{\frac{\eta-1}{\eta}} \frac{\lambda}{q\Omega} \right)^{-1}$; and $\chi_u = \int_0^{u_u} \frac{1}{A\Omega\varepsilon - \lambda \frac{g}{u_u} u_{u1} (u_u - \varepsilon)} f(\varepsilon) d\varepsilon + \int_{u_u}^{+\infty} \frac{1}{A\Omega\varepsilon} f(\varepsilon) d\varepsilon.$

Table 2: systematic shocks	R^h	R^C	C	G	G/C	N	Welfare Gain ²
$EDR^1 = 3.8\%$ ($\sigma = 0.20$)							
$F(\varepsilon) = 0.5$							
Separate	2.1%	13.8%	0.60	0	0%	0.60	
Universal	2.0%	2.0%	0.67	0	0%	0.67	4.0%
$F(\varepsilon) = 0.1$							
Separate	31.9%	47.0%	0.39	0	0%	0.60	
Universal	31.8%	31.8%	0.43	0	0.6%	0.67	3.1%
$F(\varepsilon) = 0.01$							
Separate	62.7%	81.3%	0.38	0.00	0.3%	0.60	
Universal	66.3%	66.3%	0.41	0.05	11.8%	0.67	1.7%
$F(\varepsilon) = 0.001$							
Separate	100.4%	123.3%	0.31	0.05	17.1%	0.60	
Universal	107.5%	107.5%	0.34	0.11	31.4%	0.67	1.2%
1 and 2, see Table 1.							

Given the above one can compare average welfare with separated banking with expected welfare conditional on a bad systematic shock occurring and banks being universal in nature. Table 2 contains the results. We find that to benefit from the separation of investment and commercial banking, an economy has to be hit by a negative systematic shock which is drawn from the bottom 0.1% of the distribution. Although this result clearly depends on the distribution, $f(\varepsilon)$, it appears to be robust to reasonable change of parameters.

The Table also suggests that, although a negative systematic shock may increase government spending when banks have a universal structure, it also increases the interest rate premium, and therefore the credit spread, when banks are separated.

The key message from Tables 1 and 2 appears to be that the combination of double marginalization and the interest premium is, in welfare terms, a more costly distortion compared with distortive government bail-outs.

11. Shock to quality of financial intermediation

In the previous section, the analysis seemed to suggest that universal banking may have merit regardless of whether shocks to the investment banking sector were common or idiosyncratic. Now consider a situation where there is uncertainty as regards the quality of financial intermediation, A_t . Assume that $\log A_t$ follows a first order autoregressive process

$$\log A_t = \rho \log(A_{t-1}) + \log(u_t)$$

where $\log(u_t)$ is normally distributed with zero mean and variance σ_A . When the aggregate price for final goods is stable, the price setting equation implies that the price for financial intermediation is,

$$Q_t = \frac{(\theta - 1)}{\theta} A_t.$$

From this one can easily compute the expected price for financial intermediation

$$E_t Q_{t+1} = \frac{(\theta - 1)}{\theta} A_t^\rho \exp\left(\frac{1}{2}\sigma_A^2\right).$$

Therefore the actual default rate may be higher than expected when returns to the trading strategy are smaller than expected and from (6.11) can be computed as in (11.1)

$$\varepsilon^A = \left(\frac{\exp(\frac{1}{2}\sigma_A^2)}{u_{t+1}} \right)^{\frac{\eta}{\eta-1}} \varepsilon^D. \quad (11.1)$$

11.1. Separate banking

It is apparent that government intervention occurs when actual default is higher than a certain threshold, ε^c , below which commercial banks make zero profit. This critical value is recovered from equation (14.20) which equivalent to the following equation (

$$\int_0^{\varepsilon^c} \varepsilon^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon + (\varepsilon^c)^{\frac{\eta-1}{\eta}} \left(1 - F(\varepsilon^c) - \Gamma \frac{\delta - 1}{\delta} \right) = 0.$$

In other words, if $\varepsilon^A < \varepsilon^c$, there is no government intervention, and $G_t = 0$. From (11.1) we can conclude that $\varepsilon^A < \varepsilon^c$ when A or u is sufficiently large,

$$u_t > u^* = \left(\frac{\varepsilon^D}{\varepsilon^c} \right)^{\frac{\eta-1}{\eta}} \exp\left(\frac{1}{2}\sigma_A^2\right).$$

To simplify the system, (14.21-14.31), first introduce a new function

$$\begin{aligned} G_N(u_t) &= \frac{G_t}{N_{t-1} A_{t-1}^\rho} \\ &= \max \left\{ -\frac{(\theta - 1) u_t \Omega \Delta^{\frac{1}{\eta}}}{\theta} \left[\int_0^{\varepsilon^A(u)} \varepsilon^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon + (\varepsilon^A(u))^{\frac{\eta-1}{\eta}} \left(1 - F(\varepsilon^A(u)) - \frac{R_t^h}{R_t^c} \right) \right], 0 \right\} \end{aligned} \quad (11.2)$$

Consumption in period $t + 1$ will depend on labour in the current period, N_t and the realization of u_t . Thus,

$$C_{t+1} = N_t A_t^\rho (u_{t+1} \Omega \Delta - g G_N(u_{t+1})). \quad (11.3)$$

The expected value of the inverse of consumption may now be computed

$$E_t \frac{1}{C_{t+1}} = \frac{1}{N_t A_t^\rho} \left[\int_0^{u^*} \frac{1}{u \Omega \Delta - g G_N(u)} dF_A(u) + \int_{u^*}^{+\infty} \frac{1}{u \Omega \Delta} dF_A(u) \right] = \frac{1}{\beta R_t^h C_t},$$

where $F_A(u)$ is the CDF of u .

Again, define a new constant

$$\Lambda_u = \int_0^{u^*} \frac{1}{u\Omega\Delta - gG_N(u)} dF_A(u) + \int_{u^*}^{+\infty} \frac{1}{u\Omega\Delta} dF_A(u). \quad (11.4)$$

Then expected inverse consumption can be computed as

$$E_t \frac{1}{C_{t+1}} = \frac{\Lambda_u}{N_t A_t^\rho} = \frac{1}{\beta R_t^h C_t}.$$

Combining this with the Euler equation recovers a value for labour

$$N_t = \Lambda_u A_t^{-\rho} R_t^h C_t. \quad (11.5)$$

And finally combining this with (14.26) and the risk premium yields

$$N_t = \Lambda_u \frac{\Omega}{\lambda} \Delta^{1/\eta} [\varepsilon^D]^{\frac{\eta-1}{\eta}} \frac{(\theta-1)}{\theta} \exp\left(\frac{1}{2}\sigma_A^2\right) \Gamma \frac{\delta-1}{\delta}.$$

Therefore, labour supply is constant and consumption can be computed from (11.5). That allows us to perform a welfare analyses reported in the next section.

11.1.1. Comparison to universal banking

As before, with universal banking the risk premium will be zero: $R_t^h = R_t^C$. Moreover, in the absence of a risk premium, there will always be some banks in default and government support defined in (11.2) will always be positive. In this sense, $u^* = +\infty$ and constant (11.4) is defined as

$$\Lambda_\infty = \int_0^\infty \frac{1}{u\Omega\Delta - gG_N^u(u)} dF_A(u). \quad (11.6)$$

So equilibrium labour is given by

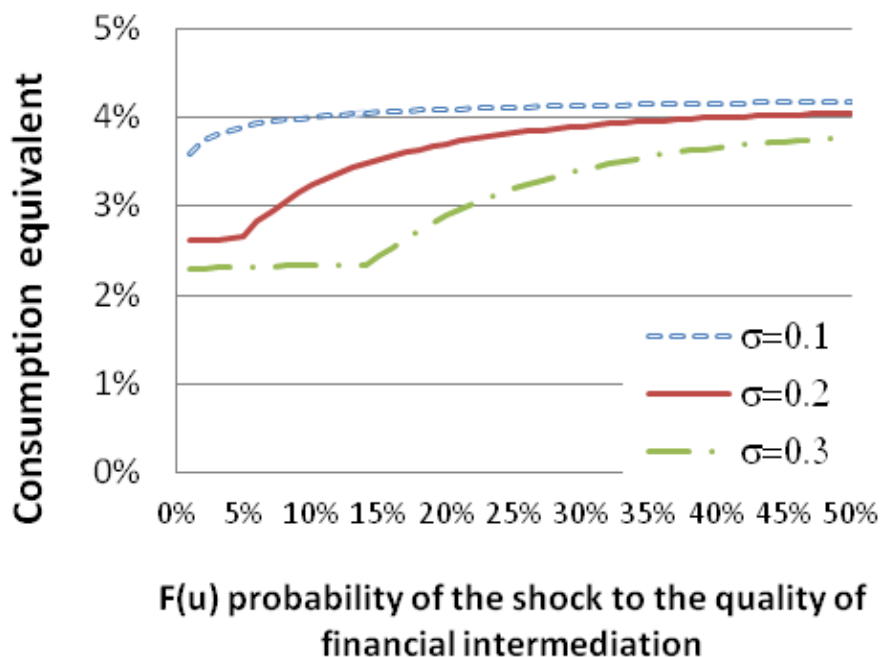
$$N_t^u = \Lambda_\infty \frac{\Omega}{\lambda} \Delta^{1/\eta} [\varepsilon^D]^{\frac{\eta-1}{\eta}} \frac{(\theta-1)}{\theta} \exp\left(\frac{1}{2}\sigma_A^2\right).$$

Using the calibration described previously, and assuming that the variance of $\log(u) = 0.2$, the results for the case of shocks to the quality of capital is presented in Table 3.

Table 3: quality shocks ¹	R^h	R^C	C	G	G/C	N	Welfare Gain
$g = 20\%$							
$u = 1, F(u) = 0.5$							
Separate	2.0%	13.6%	0.61	0	0%	0.60	
Universal	1.6%	1.6%	0.69	0	0.2%	0.68	4.0%
$u = 0.78, F(u) = 0.1$							
Separate	4.6%	16.4%	0.47	0	0%	0.60	
Universal	5.0%	5.0%	0.53	0.02	3.9%	0.68	3.3%
$u = 0.63, F(u) = 0.01$							
Separate	8.1%	20.4%	0.38	0.02	6.2%	0.60	
Universal	9.3%	9.3%	0.42	0.06	13.5%	0.68	2.6%

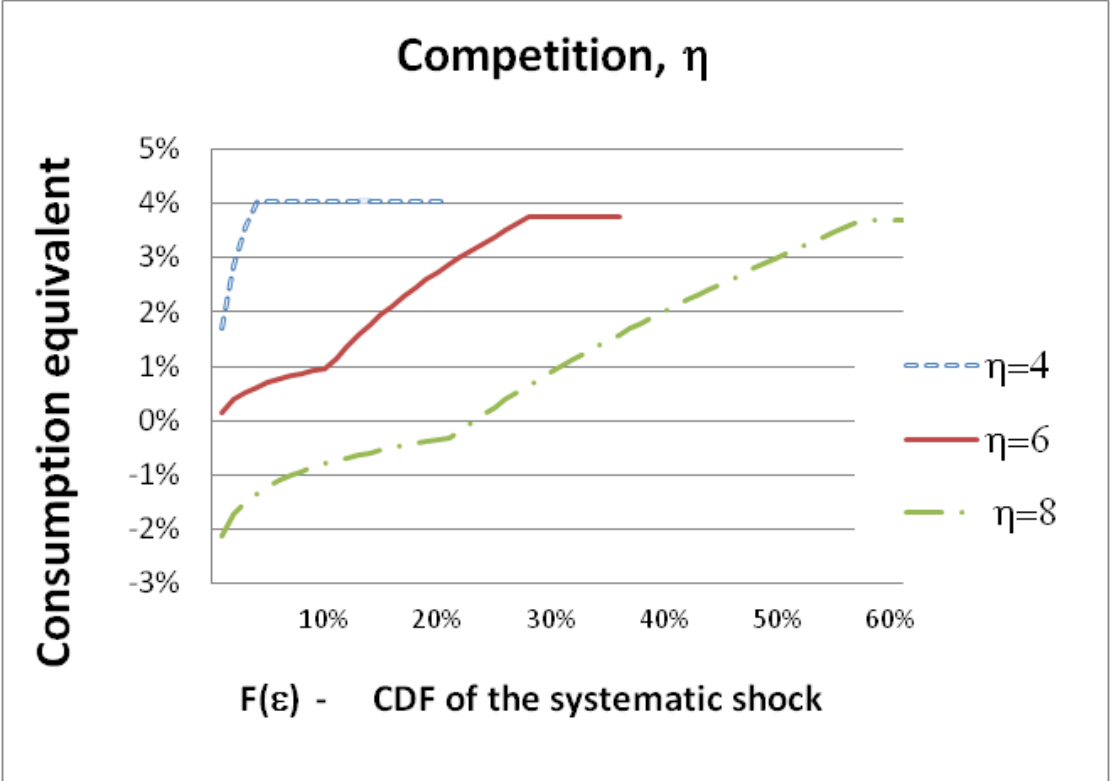
Although in this particular case both production and consumption are higher under universal banking, when the shock to capital quality is unfavorable, the impact of the government bail out may be significant, especially when the cost of fund raising is high. Welfare increases from universal banking are negatively related to the realization of the shock to capital quality as it is shown on figure 2. The intuition for this observation seems to be that a low draw on financial efficiency is relatively costly in welfare terms since the output of the financial sector is larger under universal banking; it is as if final goods producers over-invested in financial capital. It is interesting to see that as the volatility of the shocks increase, the welfare gain from universal banking is smaller.

Figure 2. Volatility of the quality shock and the welfare gain from UB



It is also worth noting that universal banking is more desirable the lower is competition in the banking sector. This can be seen in Figure 3. When competition is weak, mark-ups are high and government intervention is required less often, therefore the relative costs of universal banking is lower. As competition increases, ceteris paribus, universal banking may be less attractive than separated banking in the case where the systematic shock is unfavorable.

Figure 3. Competition and the welfare gain from UB



11.2. The welfare effect of competition

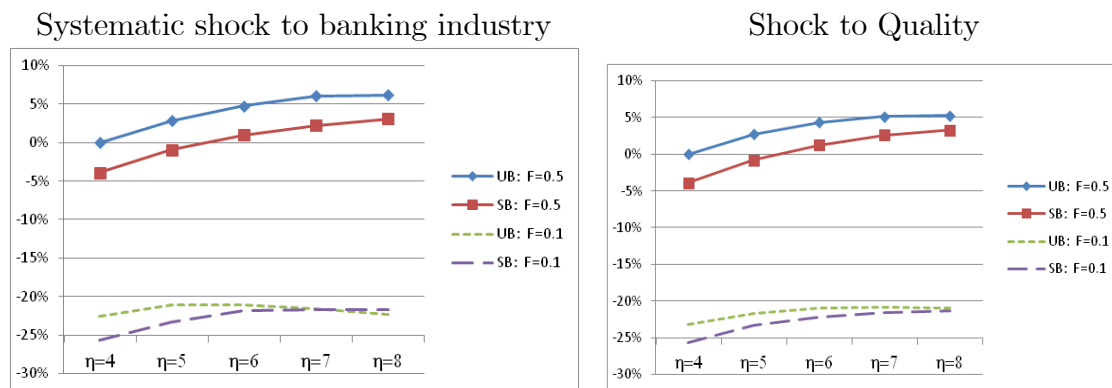
In this section we analyse the welfare effect of competition. First we consider the case when the banking sector is subject to a systematic shock. When that shock is moderate, increased competition still leads to welfare improvement. However, when the shock is particularly unfavorable, the net gain from competition is smaller and may even be negative, especially for a universal banking structure. Figure 4 shows¹² that for a median shock ($F(\epsilon) = 0.5$), an increase in competition leads to higher welfare for both universal and separated banking structures. However when the shock is relatively low ($F(\epsilon) = 0.1$), the relation between welfare and competition is U-shaped and the optimal competition level is lower for UB than

¹²UB denotes Universal Banking, SB denotes Separated, Banking and F is the CDF value of the shock. Welfare is given in consumption equivalent.

for SB. On average, the gain from competition in good times looks like it dominates the losses in bad times.

We get a very similar result for the shock to capital quality. Although the optimal level of competitiveness may be higher in this case.

Figure 4. Competition and the welfare



12. Summary and conclusion

A basic framework has been developed within which certain key policy questions can begin to be addressed: What factors determine the riskiness of financial intermediation? How does increased competition in the financial sector impact on the risk profile of financial intermediaries? When might universal banking prove to be welfare improving? How do these factors impact macroeconomic outcomes?

Our preliminary findings suggest that the risk profile of investment strategies has a noncontinuous dependence on the expected return to risky investment: it may switch quite markedly when the expected return declines.

We investigated the determinants of the interest spread in an economy where default may occur and where governments stand ready to bail out retail depositors. We found that the desirability or otherwise of universal banking turns on a key trade-off; the eradication of a double marginalization problem (including the premium) in the financial sector, versus larger government bail-outs. That basic trade-off occurs whether shocks to the investment banks trading strategies are idiosyncratic or common. We tentatively concluded that universal banking was often preferable; double marginalization is often a more costly distortion than government bail-outs when the common shock is not too negative.

We also find that a higher degree of competition is often welfare improving but the relation may be U-shaped when an economy experiences a negative common shock. The

optimal level of competitiveness seems to be higher for the separated banking sector than for the universal one.

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13. Appendix

The following theorem provides a useful inequality concerning integrals of certain functions. This theorem will be used later in the appendix.

Theorem 13.1. Consider $I = \int_a^b u(x)v(x)f(x)dx$. where $0 \leq a < b$, and for any $x \in [a, b]$ where

i) $f(x) \geq 0$; ii) $v(x)$ is a decreasing and positive function and iii) $u(x)$ is an increasing function, negative on $[a, x_0]$, positive on $[x_0, b]$ and where $a \leq x_0 \leq b$. Then the following is true

$$I = \int_a^b u(x)v(x)f(x)dx \leq v(x_0) \int_a^b u(x)f(x)dx. \quad (13.1)$$

Proof. If $x \in [a, x_0]$, then $u(x) < 0$, and $v(x) > v(x_0)$, and therefore,

$$u(x)v(x)f(x) \leq v(x_0)u(x)f(x). \quad (13.2)$$

However, if $x \in [x_0, b]$, then $u(x) > 0$, and $v(x) < v(x_0)$, and therefore (13.2) is also true. Inequality (13.2) implies (13.1). ■

13.1. The investment bank's problem

Here the solution is analyzed to optimization problem (5.11)-(5.15). The Kuhn-Tucker conditions are used following the maximizing of the Lagrangian (13.3) with respect to three endogenous variables: the default threshold, ε_t^D , the investment in the safe strategy, s_t , and the revenue of the threshold investor, D_t . As a matter of notation, in what follows when we write ε we really mean $\varepsilon(j)$.

$$\begin{aligned} L = & D_t^{\eta-1} \left(\int_{\varepsilon^D}^{\varepsilon^{\max}} \left[\frac{(1-s_t)\Omega_t\varepsilon_t + s_t\alpha_t}{D_t} \right]^{(\eta-1)/\eta} f(\varepsilon) d\varepsilon - [1 - F(\varepsilon_t^D)] \right) \\ & + \mu [(1-s_t)\Omega_t\varepsilon_t^D + s_t\alpha_t - D_t] \\ & + \lambda_d \varepsilon_t^D + \lambda_0 s_t + \lambda_1 (1-s_t). \end{aligned} \quad (13.3)$$

The first order condition with respect to ε_t^D is

$$\begin{aligned} \frac{\partial L}{\partial \varepsilon^D} &= D_t^{\eta-1} \left(\left[\frac{(1-s_t)\Omega_t\varepsilon_t^D + s_t\alpha_t}{D_t} \right]^{(\eta-1)/\eta} f(\varepsilon_t^D) - f(\varepsilon_t^D) \right) \\ +\mu(1-s_t)\Omega_t + \lambda_d &= 0, \end{aligned}$$

which in equilibrium is equivalent to

$$\frac{\partial L}{\partial \varepsilon^D} = D^{\eta-1} (f(\varepsilon_t^D) - f(\varepsilon_t^D)) + \mu(1-s_t)\Omega_t + \lambda_d = 0. \quad (13.4)$$

This implies that either $\lambda_d > 0$ and $\varepsilon_t^D = 0$, or $\lambda_d = 0$. When $\lambda_d > 0$, formula (13.4) implies that $s_t \neq 1$ and $\mu < 0$. If $\lambda_d > 0$ then $\varepsilon_t^D = 0$ and there is no expected default; it is expected that *any* realization of ε will be consistent with nonnegative profits. On the other hand, when there is positive default probability $\varepsilon_t^D > 0$ it follows that $\mu = 0$.

Consider the case of positive expected default probability. The first order condition with respect to D_t is

$$(\eta-1)D_t^{\eta-2} \left[\frac{\eta-1}{\eta} \int_{\varepsilon^D}^{+\infty} \left(\frac{(1-s_t)\Omega_t\varepsilon_t + s_t\alpha_t}{D_t} \right)^{(\eta-1)/\eta} f(\varepsilon) d\varepsilon - [1 - F(\varepsilon_t^D)] \right] - \mu = 0. \quad (13.5)$$

When $\mu = 0$, this implies that

$$\int_{\varepsilon^D}^{\varepsilon^{\max}} \left(\frac{(1-s_t)\Omega_t\varepsilon_t + s_t\alpha_t}{(1-s_t)\Omega_t\varepsilon_t^D + s_t\alpha_t} \right)^{(\eta-1)/\eta} f(\varepsilon) d\varepsilon = \frac{\eta}{\eta-1} [1 - F(\varepsilon_t^D)] \quad (13.6)$$

and says that expected marginal revenue should be equal to a constant mark up over marginal costs.

The first order condition with respect to s_t is

$$\begin{aligned} \frac{\partial L}{\partial s_t} &= D_t^{(\eta-1)\eta-1} \left[\int_{\varepsilon^D}^{+\infty} \frac{\alpha_t - \Omega_t\varepsilon_t}{D_t} \left(\frac{(1-s_t)\Omega_t\varepsilon_t + s_t\alpha_t}{D_t} \right)^{-1/\eta} f(\varepsilon) d\varepsilon \right] \\ +\mu(\alpha_t - \Omega_t\varepsilon_t^D) + (\lambda_0 - \lambda_1) &= 0. \end{aligned} \quad (13.7)$$

13.2. Proof of Proposition 5.1

Proposition 5.1 asserts the nonexistence of an internal solution to the investment banking firm's optimization problem. It is now demonstrated that at least one of the Lagrange multipliers associated with the inequality constraints $(\lambda_d, \lambda_0, \lambda_1)$ is positive. The argument proceeds by contradiction.

Assume that an internal solution exists. Then it should satisfy the following equations

$$\begin{aligned} \int_0^{\varepsilon^{\max}} \frac{\alpha_t - \Omega_t\varepsilon_t}{(1-s_t)\Omega_t\varepsilon_t + s_t\alpha_t} \left(\frac{(1-s_t)\Omega_t\varepsilon_t + s_t\alpha_t}{D_t} \right)^{(\eta-1)/\eta} f(\varepsilon) d\varepsilon &= 0; \\ \int_0^{\varepsilon^{\max}} \left(\frac{(1-s_t)\Omega_t\varepsilon_t + s_t\alpha_t}{(1-s_t)\Omega_t\varepsilon_t^D + s_t\alpha_t} \right)^{(\eta-1)/\eta} f(\varepsilon) d\varepsilon - \frac{\eta}{\eta-1} [1 - F(\varepsilon_t^D)] &= 0; \\ (1-s_t)\Omega_t\varepsilon_t^D + s_t\alpha_t - D_t &= 0. \end{aligned} \quad (13.8)$$

To check the necessary second order conditions, one first constructs the Hessian by differentiating (13.4), (13.5) and (13.7) :

$$\begin{aligned}
\frac{\partial^2 L}{\partial \varepsilon^D \partial s_t} &= D_t^{\eta-2} (\alpha_t - \Omega_t \varepsilon_t^D) \frac{\eta-1}{\eta} f(\varepsilon_t^D) - \mu \Omega_t; \\
\frac{\partial^2 L}{(\partial \varepsilon^D)^2} &= \frac{\eta-1}{\eta} D_t^{\eta-2} f(\varepsilon_t^D) (1-s_t) \Omega_t > 0; \\
\frac{\partial^2 L}{\partial \varepsilon^D \partial D_t} &= -\frac{\eta-1}{\eta} D_t^{\eta-2} f(\varepsilon_t^D); \\
\frac{\partial^2 L}{\partial D_t^2} &= -(\eta-1) D_t^{\eta-3} \left(\frac{\eta-1}{\eta} \right) [1 - F(\varepsilon_t^D)] < 0; \\
\frac{\partial^2 L}{\partial D_t \partial s_t} &= 0; \\
\frac{\partial^2 L}{\partial s_t^2} &= -D_t^{\eta-1} \frac{\eta-1}{\eta^2} \int_{\varepsilon^D}^{\varepsilon^{\max}} \left(\frac{\alpha_t - \Omega_t \varepsilon_t}{D_t} \right)^2 \left(\frac{(1-s_t)\Omega_t \varepsilon_t + s_t \alpha_t}{D_t} \right)^{-1/\eta-1} f(\varepsilon) d\varepsilon < 0.
\end{aligned}$$

This problem has only one binding constraint (13.8) with Jacobian

$$J = \begin{bmatrix} (1-s_t)\Omega_t \\ -1 \\ \alpha_t - \Omega_t \varepsilon_t^D \end{bmatrix}.$$

The necessary second order conditions for a constrained local maximum require the Hessian, H , to be negative definite on the null space of the Jacobian of the binding constraints¹³. However it can be shown that that $x'Hx > 0$ and $J'x = 0$, for

$$x := \begin{pmatrix} \alpha_t - \Omega_t \varepsilon_t^D \\ 0 \\ -(1-s_t)\Omega_t \end{pmatrix}.$$

Therefore, every internal solution which satisfies the first order conditions, violates the second order necessary conditions for a local maximum. This completes the proof of Proposition 5.1.

13.3. Proof of Proposition 5.2

Following Proposition 5.1 only two possible solutions are of relevance: First, either there is no expected default, $\varepsilon^D = 0$, or second everything is invested in the risky strategy, $s_t = 0$. Sometimes both of those solutions provide for local maxima, and one needs to compute the value function in order to find which is the global maximum. Thus, first consider:

Case 1: $\lambda_d > 0$, $\varepsilon^D = 0$, $\lambda_0 = \lambda_1 = 0$.

It follows from (13.4) that $\mu \leq 0$. The optimal investment in the safe strategy solves the equations

$$D_t^{\eta-2} \frac{\eta-1}{\eta} \left[(\eta-1) \int_0^\infty \left(\frac{1-s_t}{s_t} \phi \varepsilon + 1 \right)^{(\eta-1)/\eta} f(\varepsilon) d\varepsilon - \eta \right] = \mu < 0; \tag{13.9}$$

$$D_t^{(\eta-2)} \frac{\eta-1}{\eta} \left[\int_0^\infty (1-\phi\varepsilon) \left(\frac{1-s_t}{s_t} \phi \varepsilon + 1 \right)^{-1/\eta} f(\varepsilon) d\varepsilon \right] = -\mu > 0; \tag{13.10}$$

$$D_t = s_t \alpha_t, \tag{13.11}$$

¹³See for example Cornuejols and Tutuincu (2007), page 102.

where $\phi \equiv \Omega_t/\alpha_t$. (13.9) is a straightforward implication of (13.5). (13.10) follows from (13.7) using (13.11). Combining (13.9) and (13.10) gives

$$(\eta - 1) \int_0^\infty \left(\frac{1 - s_t}{s_t} \phi \varepsilon + 1 \right)^{(\eta-1)/\eta} f(\varepsilon) d\varepsilon - \eta + \int_0^\infty (1 - \phi \varepsilon) \left(\frac{1 - s_t}{s_t} \phi \varepsilon + 1 \right)^{-1/\eta} f(\varepsilon) d\varepsilon = 0. \quad (13.12)$$

This equation provides an expression for the optimal share of the safe investment. We label this solution: $s_t^b(\phi)$.

Next consider:

Case 2: $\lambda_d = 0$.

If $\lambda_d = 0$, then $s_t = 0$ since we rule out $s_t = 1$ ¹⁴. The equilibrium default threshold can then be found from (13.6)

$$\int_{\varepsilon^D}^\infty \left(\frac{\varepsilon_t}{\varepsilon_t^D} \right)^{(\eta-1)/\eta} f(\varepsilon) d\varepsilon = \frac{\eta}{\eta - 1} [1 - F(\varepsilon_t^D)]. \quad (13.13)$$

We now turn to the analysis of (13.12) and (13.13).

13.3.1. Existence of "safe local maximum"

First the case is considered when there is a maximum at the zero default probability border, $\lambda_d > 0$, $\varepsilon_t^D = 0$. As noted above, a safe local maximum can only exist when $\mu < 0$ and therefore the solution of (13.12) satisfies the following inequality (13.9) which is equivalent to

$$h(s_t, \phi) := \int_0^\infty \left(\frac{1 - s_t}{s_t} \phi \varepsilon + 1 \right)^{(\eta-1)/\eta} f(\varepsilon) d\varepsilon - \frac{\eta}{\eta - 1} < 0. \quad (13.14)$$

Let s_t^* be a solution of $h(s_t^*, \phi) = 0$. It is easy to verify that $h(1, \phi) = -\frac{1}{\eta-1} < 0$, and that $\frac{\partial h}{\partial s_t} < 0$. Also, let s_t^b be a solution to the first order condition (13.12). By continuity, s_t^b satisfies (13.14) if and only if $s_t^b > s_t^*$. Note that s_t^b is a solution to (13.12) which we rewrite as¹⁵

$$\begin{aligned} h^1(s_t^b, \phi) &: = \frac{\eta}{\eta - 1} \int_0^{+\infty} \left(\frac{1 - s_t^b}{s_t^b} \phi \varepsilon + 1 \right)^{1-1/\eta} f(\varepsilon) d\varepsilon - \\ &\quad \frac{\eta}{\eta - 1} - \frac{1}{\eta - 1} \frac{\phi}{s_t^b} \int_0^{+\infty} \varepsilon \left(\frac{1 - s_t^b}{s_t^b} \phi \varepsilon + 1 \right)^{-1/\eta} f(\varepsilon) d\varepsilon = 0 \end{aligned} \quad (13.15)$$

First, note that (13.15) is negative for $s_t = 1$, and that it is a strictly decreasing function with respect to s_t for $\eta > 2$. Thus, if $h^1(s_t^*, \phi)$ is positive then $s_t^b > s_t^*$, and the first order condition (13.9) is satisfied. However, if $h^1(s_t^*, \phi) < 0$, then $s_t^b < s_t^*$ and the safe maximum does not exist. Thus, evaluating (13.15) at s_t^* , gives

$$(\eta - 1) h^1(s_t^*, \phi) = \frac{\eta}{\eta - 1} - \frac{\phi}{s_t^*} \int_0^\infty \varepsilon \left(\frac{1 - s_t^*}{s_t^*} \phi \varepsilon + 1 \right)^{-1/\eta} f(\varepsilon) d\varepsilon. \quad (13.16)$$

It is useful to define s_t^* in an alternative way. In particular $s_t^* = \frac{\phi}{\phi + k^*}$ where k^* is defined as a solution to (13.17)

$$\int_0^\infty (k^* \varepsilon + 1)^{(\eta-1)/\eta} f(\varepsilon) d\varepsilon = \frac{\eta}{\eta - 1}. \quad (13.17)$$

¹⁴When $s = 1$, a safe investment strategy is pursued. The expected probability of default is either zero or one. In the latter case profit is zero which cannot be optimal.

¹⁵We use the fact that $(1 - \phi \varepsilon) = ((1 - s)/s)\phi \varepsilon + 1 - (\phi \varepsilon/s)$.

It is apparent that k^* is a positive number which does not depend on ϕ . Then (13.16) can, in turn, be rewritten as

$$(\eta - 1) h^1(s_t^*, \phi) = \frac{\eta}{\eta - 1} - (\phi + k^*) \int_0^{\infty} \varepsilon (k^* \varepsilon + 1)^{-1/\eta} f(\varepsilon) d\varepsilon.$$

It follows that $h^1(s_t^*, \phi)$ declines in ϕ , it is positive for $\phi = 0$ and for any $\phi < \phi^h$. It is negative for $\phi > \phi^h$, where ϕ^h solves $h^1\left(\frac{\phi}{\phi + k^*}, \phi\right) = 0$

$$\phi^h = \frac{\eta}{\eta - 1} \left(\int_0^{+\infty} \varepsilon (k^* \varepsilon + 1)^{-1/\eta} f(\varepsilon) d\varepsilon \right)^{-1} - k^* > 0.$$

To complete the proof, the second order conditions are verified.

In this case, the constraint is $\varepsilon^D = 0$, $D_t - (1 - s_t) \Omega_t \varepsilon^D - s_t \alpha_t = 0$. The Jacobian is $(\varepsilon^D, D_t, s_t)$:

$$J = \begin{bmatrix} 1 & 0 & 0 \\ -(1 - s_t) \Omega_t & 1 & -\alpha \end{bmatrix}.$$

There is only one null vector of the Jacobian, $x' = (0, \alpha, 1)$. To check whether $x' H x < 0$, note that

$$x' H x = \frac{\partial^2 L}{\partial D_t^2} \alpha^2 + 2\alpha \frac{\partial^2 L}{\partial D_t \partial s_t} + \frac{\partial^2 L}{\partial s_t^2} < 0.$$

Lemma 13.2. *There exists a threshold level of relative returns ϕ^h , such that for $\frac{\Omega_t}{\alpha_t} < \phi^h$, there is a unique local maximum at which the default threshold is zero. As the relative return of the risky strategy rises above ϕ^h , such a safe local maximum no longer exists.*

Finally, note that s_t in this case may be quite large. In fact, in our simulation example s increases with $\frac{\Omega_t}{\alpha_t}$ and is larger than 0.6. That is reflected in the diagram we provide in the body of the paper.

13.3.2. Existence of risky local maximum

Now, assume the existence of a local maximum such that $F(\varepsilon^D) > 0$. It has been shown that in this case it necessarily follows that $s_t = 0$, and ε^D satisfies (13.13) which comes from the first order necessary condition for an optimum with respect to D_t . Note, that the condition is independent of relative profitability of the risky strategy.

Also the first order necessary condition with respect to s_t requires the following condition

$$\int_{\varepsilon^D}^{\infty} (1 - \phi \varepsilon) \left(\frac{\varepsilon}{\varepsilon^D} \right)^{-1/\eta} f(\varepsilon) d\varepsilon < 0. \quad (13.18)$$

We label ϕ^l , the solution of (13.18) with strict equality. The inequality holds if and only if $\phi > \phi^l$, and it is in this region where the risky strategy can be a local maximum.

Now we need to verify the second order conditions at $s = 0$. Thus,

$$\begin{aligned}
\frac{\partial^2 L}{\partial \varepsilon^D \partial s_t} &= D_t^{\eta-2} (\alpha_t - \Omega_t \varepsilon_t^D) \frac{\eta-1}{\eta} f(\varepsilon_t^D); \\
\frac{\partial^2 L}{(\partial \varepsilon^D)^2} &= \frac{\eta-1}{\eta} D_t^{\eta-2} f(\varepsilon_t^D) (1-s_t) \Omega_t \geq 0; \\
\frac{\partial^2 L}{\partial \varepsilon^D \partial D_t} &= -\frac{\eta-1}{\eta} D_t^{\eta-2} f(\varepsilon_t^D); \\
\frac{\partial^2 L}{\partial D_t^2} &= -(\eta-1) D_t^{\eta-3} \left(\frac{\eta-1}{\eta} \right) [1 - F(\varepsilon_t^D)] < 0; \\
\frac{\partial^2 L}{\partial D_t \partial s_t} &= 0; \\
\frac{\partial^2 L}{\partial s_t^2} &= -D_t^{\eta-1} \frac{\eta-1}{\eta^2} \int_{\varepsilon^D}^{\infty} \left(\frac{\alpha_t - \Omega_t \varepsilon_t}{D_t} \right)^2 \left(\frac{(1-s_t) \Omega_t \varepsilon_t + s_t \alpha_t}{D_t} \right)^{-1/\eta-1} f(\varepsilon) d\varepsilon < 0.
\end{aligned}$$

In this case there are two constraints: $\{s_t = 0; D - (1-s_t) \Omega_t \varepsilon^D - s_t \alpha_t = 0\}$, and the Jacobean with respect to $(\varepsilon^D, D_t, s_t)$ is

$$J = \begin{bmatrix} 0 & 0 & 1 \\ -(1-s_t) \Omega_t & 1 & -a \end{bmatrix}.$$

There is only one null vector of the Jacobean, $x' = (1, \Omega_t, 0)$ and it is readily verified that $x' H x < 0$:

$$\begin{aligned}
x' H x &= \frac{\partial^2 L}{(\partial \varepsilon^D)^2} + 2\Omega_t \frac{\partial^2 L}{\partial D_t \partial \varepsilon^D} + \Omega_t^2 \frac{\partial^2 L}{(\partial D_t)^2} < 0 \\
&= \frac{\eta-1}{\eta} D_t^{\eta-2} \left[f(\varepsilon_t^D) \Omega_t - 2\Omega_t f(\varepsilon_t^D) + \Omega_t^2 \frac{\partial^2 L}{(\partial D_t)^2} \right] < 0.
\end{aligned}$$

Hence, the following proposition may be stated:

Lemma 13.3. *There exists a threshold level of relative returns ϕ^l such that, for $\frac{\Omega_t}{\alpha_t} > \phi^l$, there is a local maximum corresponding to $s_t = 0$ at which the probability of default is zero defined from (13.13) and where*

$$\phi^l = \frac{1}{\varepsilon_t^D [1 - F(\varepsilon_t^D)]} \frac{\eta-1}{\eta} \int_{\varepsilon^D}^{\infty} \left(\frac{\varepsilon_t}{\varepsilon^D} \right)^{-1/\eta} f(\varepsilon) d\varepsilon.$$

When $\frac{\Omega_t}{\alpha_t} < \phi^l$, there is no local maximum such that the probability of default is strictly positive, $F(\varepsilon^D) > 0$.

Since the expected profit function is continuous, the maximum exists for any value of relative returns. So, one would expect that $\phi^l < \phi^h$, since otherwise, expected profit would have no maximum for $\phi \in [\phi^h, \phi^l]$, which cannot be the case for a continuous function on a compact domain.

Therefore, it has been proved that there exists $\phi^l < \phi^h$ such that when:

1. $\frac{\Omega_t}{\alpha_t} < \phi^l$, there is only one type of solution, which corresponds to a strategy with zero probability of default;
2. $\phi^l < \frac{\Omega_t}{\alpha_t} < \phi^h$, there are two local maxima, one corresponds to zero default and another corresponds to the risky strategy;
3. $\frac{\Omega_t}{\alpha_t} > \phi^h$, there is only one solution to the optimization problem and it corresponds to investment in the risky strategy only.

13.4. Strategy with zero expected default: proof of Corollary 5.5

The Lagrangian for the problem (5.18, 5.19) is

$$\begin{aligned} L(N, s) &= \int_0^{+\infty} E_t \left[Q_{t+1} (X_{t+1})^{1/\eta} \left([(1-s_t)\Omega_t\varepsilon + s_t\alpha_t] N_t \right)^{(\eta-1)/\eta} \right] f(\varepsilon) d\varepsilon - W_t R_t^C N_t \\ &\quad + \mu_s E_t \left[Q_{t+1} (X_{t+1})^{1/\eta} (s_t\alpha_t N_t)^{(\eta-1)/\eta} - W_t R_t^C N_t \right] + \mu_1(1-s) + \mu_0 s \end{aligned}$$

It is easy to see that $\mu_0 = 0$, (otherwise constraint (5.19) is never satisfied). The first order conditions are

$$\begin{aligned} \frac{\partial L(N, s)}{\partial s} &= -E_t Q_{t+1} (X_{t+1})^{1/\eta} N_t^{1-1/\eta} \frac{\eta-1}{\eta} \int_0^{+\infty} (\Omega_t\varepsilon - \alpha_t) [(1-s_t)\Omega_t\varepsilon + s_t\alpha_t]^{-1/\eta} f(\varepsilon) d\varepsilon \\ &\quad + \frac{\eta-1}{\eta} \frac{\mu_s}{s} E_t \left[Q_{t+1} (X_{t+1})^{1/\eta} (s_t\alpha_t N_t)^{(\eta-1)/\eta} \right] - \mu_1; \end{aligned} \quad (13.19)$$

$$\begin{aligned} \frac{\partial L(N, s)}{\partial N} N_t &= E_t Q_{t+1} (X_{t+1})^{1/\eta} N_t^{1-1/\eta} \frac{\eta-1}{\eta} \int_0^{+\infty} [(1-s_t)\Omega_t\varepsilon + s_t\alpha_t]^{1-1/\eta} f(\varepsilon) d\varepsilon - W_t R_t^C N_t \\ &\quad + \mu_s E_t \left[\frac{\eta-1}{\eta} Q_{t+1} (X_{t+1})^{1/\eta} (s_t\alpha_t N_t)^{(\eta-1)/\eta} - W_t R_t^C N_t \right]. \end{aligned} \quad (13.20)$$

There are three cases to examine:

Case 1. Consider $s = 1$, $\mu_1 > 0$. The constraint will satisfy

$$\alpha_t E_t Q_{t+1} (X_{t+1}/\alpha_t N_t)^{1/\eta} > W_t R_t^C.$$

The first order conditions with respect to N is

$$\begin{aligned} \frac{\partial L(N, s)}{\partial N} N_t &= E_t Q_{t+1} (X_{t+1})^{1/\eta} (\alpha_t N_t)^{1-1/\eta} \frac{\eta-1}{\eta} - W_t R_t^C N_t \\ &\quad + \mu_s \frac{\eta-1}{\eta} E_t \left[Q_{t+1} (X_{t+1})^{1/\eta} (\alpha_t N_t)^{(\eta-1)/\eta} - W_t R_t^C N_t \right]; \end{aligned} \quad (13.21)$$

and with respect to s is

$$\frac{\partial L(N, s)}{\partial s} \frac{\eta}{\eta-1} (\alpha_t N_t)^{-(\eta-1)/\eta} = -(\phi_t - 1) + \mu_s - \mu_1 \frac{\eta}{\eta-1} (\alpha_t N_t)^{-(\eta-1)/\eta} E_t \left[Q_{t+1} (X_{t+1})^{1/\eta} \right]^{-1} = 0. \quad (13.22)$$

It is easy to check if μ_s is zero. If $\mu_s > 0$, then

$$E_t Q_{t+1} (X_{t+1})^{1/\eta} (\alpha_t N_t)^{(\eta-1)/\eta} = W_t R_t^C N_t \quad (13.23)$$

and (13.21) is violated. Since $\mu_s = 0$, condition (13.22) is equivalent to $\phi_t \leq 1$. Now one may conclude that $s = 1$ and zero probability of default is optimal if and only if $\phi_t \leq 1$.

Case 2. Consider $\mu_s > 0$, then we already know that $\mu_1 = 0$. The constraint (5.19) is binding and the first order conditions (13.19, 13.20) become

$$\int_0^{+\infty} (\phi_t \varepsilon - 1) \left[\left(\frac{1-s_t}{s_t} \right) \phi_t \varepsilon + 1 \right]^{-1/\eta} f(\varepsilon) d\varepsilon = \mu_s > 0 \quad (13.24)$$

$$(\eta-1) \int_0^{+\infty} \left[\left(\frac{1-s_t}{s_t} \right) \phi_t \varepsilon + 1 \right]^{1-1/\eta} f(\varepsilon) d\varepsilon - \eta = \mu_s > 0 \quad (13.25)$$

These relations can be rewritten as:

$$\int_0^{+\infty} (\phi_t \varepsilon - 1) \left[\left(\frac{1-s_t}{s_t} \right) \phi_t \varepsilon + 1 \right]^{-1/\eta} f(\varepsilon) d\varepsilon = (\eta - 1) \int_0^{+\infty} \left[\left(\frac{1-s_t}{s_t} \right) \phi_t \varepsilon + 1 \right]^{1-1/\eta} f(\varepsilon) d\varepsilon - \eta, \quad (13.26)$$

which solves for optimal s .

Note that conditions (13.24) and (13.25) are completely opposite to (13.9), (13.10). Above we have shown that (13.9) and (13.10) are satisfied when $\phi_t < \phi^h$. Using a similar approach (13.24) and (13.25) are shown to be satisfied when $\phi_t > \phi^h$. Moreover, since equation (13.26) is the same as (13.12) we can conclude that constraint (5.19) is binding if and only $\phi_t > \phi^h$.

Case 3. Consider the remaining case $\phi_t \in [1, \phi^h]$. In this case $\mu_1 = \mu_s = 0$ and there is an internal solution, which can be found from the first order conditions

$$\int_0^{+\infty} (\phi \varepsilon - 1) \left[\left(\frac{1-s}{s} \right) \phi \varepsilon + 1 \right]^{-1/\eta} f(\varepsilon) d\varepsilon = 0; \quad (13.27)$$

$$E_t (X_{t+1}/N_t s_t \alpha_t)^{1/\eta} \int_0^{+\infty} \left[\left(\frac{1-s}{s} \right) \phi \varepsilon + 1 \right]^{1-1/\eta} f(\varepsilon) d\varepsilon = \frac{\eta}{\eta-1} E_t \frac{W_t R_t^C}{Q_{t+1}}. \quad (13.28)$$

That completes the proof.

13.5. Proof that (13.27) implies $ds/d\phi < 0$

Define a function Λ as follows

$$\Lambda(s, \phi) = \int_0^{+\infty} (\phi \varepsilon - 1) \left[\left(\frac{1-s}{s} \right) \phi \varepsilon + 1 \right]^{-1/\eta} f(\varepsilon) d\varepsilon = 0. \quad (13.29)$$

Now use the implicit function theorem $\frac{ds}{d\phi} = -\frac{\partial \Lambda / \partial \phi}{\partial \Lambda / \partial s}$, where

$$\frac{\partial \Lambda}{\partial s} = \frac{1}{s^2} \frac{1}{\eta} \int_0^{+\infty} (\phi \varepsilon - 1) \phi \varepsilon \left[\left(\frac{1-s}{s} - 1 \right) \phi \varepsilon + 1 \right]^{-1/\eta-1} f(\varepsilon) d\varepsilon.$$

That equation may be rewritten as

$$\frac{\partial \Lambda}{\partial s} = \frac{1}{s(1-s)} \frac{1}{\eta} \int_0^{+\infty} (\phi \varepsilon - 1) \left[\left(\frac{1-s}{s} - 1 \right) \phi \varepsilon + 1 \right]^{-1/\eta} f(\varepsilon) d\varepsilon; \quad (13.30)$$

$$- \frac{1}{s(1-s)} \frac{1}{\eta} \int_0^{+\infty} (\phi \varepsilon - 1) \left[\left(\frac{1-s}{s} \right) \phi \varepsilon + 1 \right]^{-1/\eta-1} f(\varepsilon) d\varepsilon. \quad (13.31)$$

The first line (13.30) is zero by definition of (13.29). Now one may apply theorem 13.1 to the second line, (13.31) with $u(\varepsilon) = (\phi \varepsilon - 1)$, and $v(\varepsilon) = \left[\left(\frac{1-s}{s} \right) \phi \varepsilon + 1 \right]$, to find that

$$\begin{aligned} \frac{\partial \Lambda}{\partial s} s(1-s)\eta &= - \int_0^{1/\phi} (\phi \varepsilon - 1) \left[\left(\frac{1-s}{s} \right) \phi \varepsilon + 1 \right]^{-1/\eta-1} f(\varepsilon) d\varepsilon \\ &> - \int_0^{1/\phi} (\phi \varepsilon - 1) \left[\left(\frac{1-s}{s} \right) \phi \varepsilon + 1 \right]^{-1/\eta} d\varepsilon = 0. \end{aligned} \quad (13.32)$$

That proves that $\frac{\partial \Lambda}{\partial s} > 0$. The other derivative, $\frac{\partial \Lambda}{\partial \phi}$, is also positive, since

$$\begin{aligned} \frac{\partial \Lambda}{\partial \phi} &= \int_0^{+\infty} \varepsilon \left[\left(\frac{1-s}{s} \right) \phi \varepsilon + 1 \right]^{-1/\eta} f(\varepsilon) d\varepsilon \\ &\quad - \frac{1}{\eta} \int_0^{+\infty} (\phi \varepsilon - 1) \left(\frac{1-s}{s} \right) \varepsilon \left[\left(\frac{1-s}{s} \right) \phi \varepsilon + 1 \right]^{-1/\eta-1} f(\varepsilon) d\varepsilon. \end{aligned}$$

The first term is positive, while the second term is proved to be positive in (13.32). This completes the proof that when $\phi > 1$, investment in the safe trading strategy is declining in the relative rates of return, $\frac{ds}{d\phi} < 0$.

13.6. Switching between strategies

Now compare expected profits, $E_t \Pi^i$ (i stands for *risk*, *mixed* or *safe*) under alternative trading strategies: "risky" means $s = 0$, "mixed" means $0 < s < 1$ and "safe" means $s = 1$. We have shown that in general expected profit is proportional to expected cost

$$E_t \Pi^i = \frac{1}{\eta-1} W_t R_t^C N_t^i \left[1 - F(\varepsilon_t^D) \right],$$

where N_t^i is the demand for labour for the corresponding trading strategy, as in (5.9). Therefore, expected profits for the various strategies are:

$$E_t \Pi^{safe} = \frac{1}{\eta-1} \left(\frac{\alpha}{W_t R_t^C} \right)^{\eta-1} \left[E_t X_{t+1}^{1/\eta} (Q_{t+1}) \right]^\eta; \quad (13.33)$$

$$E_t \Pi^{risk} = \frac{1}{\eta-1} \left[1 - F(\varepsilon_t^D) \right] \left(\frac{\varepsilon_t^D \alpha}{W_t R_t^C} \right)^{\eta-1} \phi^{\eta-1} \left[E_t X_{t+1}^{1/\eta} (Q_{t+1}) \right]^\eta; \quad (13.34)$$

$$E_t \Pi^{mixed} = \frac{1}{\eta-1} \left[\frac{s_t \alpha_t}{W_t R_t^C} \right]^{\eta-1} \left[\frac{\eta-1}{\eta} \int_0^{+\infty} \left[\left(\frac{1-s}{s} \right) \phi_t \varepsilon + 1 \right]^{1-1/\eta} f(\varepsilon) d\varepsilon \right]^\eta \left[E_t X_{t+1}^{1/\eta} (Q_{t+1}) \right]^\eta \quad (13.35)$$

It has been established that when $\phi > \phi^h$ the risky strategy dominates both the safe and mixed strategies. Now consider the case where $\phi \leq \phi^h$.

For what follows, it is helpful to prove the following corollaries:

Corollary 13.4. *There exists a unique $\phi^{sw1} \in (0, \phi^h)$, such that if $\phi < \phi^{sw1}$, then $E_t \Pi^{risk} < \Pi^{safe}$, and if $\phi > \phi^{sw1}$, then $E_t \Pi^{risk} > \Pi^{safe}$ where ϕ^{sw1} is defined as*

$$\phi^{sw1} = \left[1 - F(\varepsilon_t^D) \right]^{-1/(\eta-1)} / \varepsilon_t^D. \quad (13.36)$$

Proof. Consider the ratio

$$\Lambda_1(\phi) = E_t \Pi^{risk}(\phi) / \Pi^{safe} = \left[1 - F(\varepsilon_t^D) \right] (\phi \varepsilon_t^D)^{\eta-1}. \quad (13.37)$$

It is an increasing function with $\Lambda_1(0) = 0$, and $\Lambda_1(\phi^h) > 1$. ■

Corollary 13.5. *There exists a unique $\phi^{sw2} \in (0, \phi^h)$ such that if $\phi < \phi^{sw2}$, then $E_t \Pi^{risk} < E_t \Pi^{mixed}$, and if $\phi > \phi^{sw2}$, then $E_t \Pi^{risk} > E_t \Pi^{mixed}$.*

Proof. Consider the ratio

$$\begin{aligned}\Lambda_2(\phi) &= E_t \Pi^{risk} / E_t \Pi^{mixed} \\ &= [1 - F(\varepsilon_t^D)] \left(\frac{\varepsilon_t^D}{s_t} \right)^{\eta-1} \left(\frac{\eta-1}{\eta} \right)^{-\eta} \left[\int_0^{+\infty} \left[\left(\frac{1}{s} - 1 \right) \varepsilon + \frac{1}{\phi} \right]^{1-1/\eta} f(\varepsilon) d\varepsilon \right]^{-\eta}.\end{aligned}$$

The relation between s and ϕ is given in (13.27). First, one can show that that $\Lambda_2(\phi)$ is an increasing function of ϕ . By direct differentiation, it is easy to prove that $\frac{\partial \Lambda}{\partial \phi} > 0$, and $\frac{\partial \Lambda}{\partial s} < 0$. It is proved in section (13.5) that $\frac{ds}{d\phi} < 0$. Therefore $\frac{d\Lambda_2}{d\phi} = \frac{d\Lambda_2}{\partial \phi} + \frac{d\Lambda_2}{\partial s} \frac{ds}{d\phi} > 0$. It is also known that $\Lambda(\phi^h) > 1$ and that $\Lambda(\phi^l) < 1$. Hence, by continuity there exists a unique ϕ^{sw2} which solves $\Lambda(\phi^{sw2}) = 1$ and moreover $\phi^{sw2} \in [\phi^l, \phi^h]$. ■

To continue we will also need the following two short lemmas.

Lemma 13.6. If $\phi^{sw1} < \phi^{sw2}$. Then $\phi^{sw1} > 1$. And if $\phi < \phi^{sw2}$, then zero default is expected, while if $\phi > \phi^{sw2}$, the risky strategy is more profitable.

Proof. By contradiction. Let $\phi^{sw1} < 1$. $E_t \Pi^{case3}(\phi^{sw1}) < \Pi^{s1} = E_t \Pi^{risk}(\phi^{sw1})$. But since $\phi^{sw1} < \phi^{sw2}$, this contradicts to Corollary 13.5. Therefore, $\phi^{sw2} > \phi^{sw1} > 1$. ■

Furthermore, when $\phi^{sw1} < \phi^{sw2}$, one can conclude that if $\phi > \phi^{sw2}$, the profit maximizing strategy entails $s = 0$, whilst for $\phi \in [1, \phi^{sw2}]$ the optimal strategy is $0 < s < 1$, with zero expected default.

Lemma 13.7. If $\phi^{sw1} > \phi^{sw2}$, then $\phi^{sw1} < 1$. That implies that only two strategies are possible: i) $s = 0$, for $\phi > \phi^{sw1}$, and ii) $s = 1$, for $\phi < \phi^{sw1}$.

Proof. If $\phi^{sw1} > \phi^{sw2}$ then Corollary 13.4 implies that $\Pi^{s1} = E_t \Pi^{risk}(\phi^{sw1}) > E_t \Pi^{case3}(\phi^{sw1})$. But from Corollary 5.5 we know that $\Pi^{s1} > E_t \Pi^{case3}(\phi)$, if and only if $\phi < 1$. ■

Now define $\phi^{sw} = \max(\phi^{sw1}, \phi^{sw2})$. Then we can prove that ϕ^{sw} is a threshold such if $\phi < \phi^{sw}$, there is no default and if $\phi > \phi^{sw}$, then $s = 0$. That complete the proof of the Proposition.

13.7. Proof that Ψ_{t+1} declines in ε^A

In the main text, the relationship between aggregate net assets and the default threshold was examined. It was asserted that Ψ_{t+1} declines in ε^A and that commercial banks will have negative net assets only when the actual default rate is greater than some threshold value, $\varepsilon^A > \varepsilon^C$. The basis for that statement is now examined. Recall from the text that

$$\Psi_{t+1}(\varepsilon^A) = Q_{t+1} X_{t+1} \left[\int_0^{\varepsilon^A} \left[\frac{\varepsilon}{\Delta} \right]^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon + \left(\frac{\varepsilon^A}{\Delta} \right)^{\frac{\eta-1}{\eta}} \left(1 - F(\varepsilon^A) - \frac{R_t^h}{R_t^c} \right) \right]. \quad (13.38)$$

First, the existence and uniqueness of the solution to

$$\Psi_{t+1}(\varepsilon^C) = 0, \quad \varepsilon^C > 0 \quad (13.39)$$

is established. To prove uniqueness, differentiate this expression with respect to ε^A to find that

$$\frac{\partial \Psi_{t+1}(\varepsilon^A)}{\partial \varepsilon^A} = \frac{\eta-1}{\eta} \frac{1}{\varepsilon^A} \left(\frac{\varepsilon^A}{\Delta} \right)^{\frac{\eta-1}{\eta}} \left(1 - F(\varepsilon^A) - \frac{R_t^h}{R_t^c} \right).$$

It is ascertained that when $\Psi_{t+1}(\varepsilon^A) < 0$, expression $\left(1 - F(\varepsilon^A) - \frac{R_t^h}{R_t^c}\right)$ is also negative and therefore $\frac{\partial \Psi_{t+1}(\varepsilon^A)}{\partial \varepsilon^A} < 0$. Moreover, it is clear that at its maximum ε' , we have that

$$F(\varepsilon') = 1 - \frac{R_t^h}{R_t^c}.$$

Note, we assume $\frac{R_t^h}{R_t^c} < 1$. Expression (13.38) is positive and for any $\varepsilon^A < \varepsilon'$, $\frac{\partial \Psi_{t+1}(\varepsilon^A)}{\partial \varepsilon^A} > 0$. One can also see that $\Psi_{t+1}(0) = 0$. Therefore, there is no solution on $(0, \varepsilon']$ and $\frac{\partial \Psi_{t+1}(\varepsilon^A)}{\partial \varepsilon^A} < 0$ for any $\varepsilon^A > \varepsilon'$. Any strictly monotone function can not have more than one zero point and so uniqueness is established.

Existence is not guaranteed and depends on the distribution. However, existence *is* guaranteed if and only if $\Psi_{t+1}(\varepsilon^{\max}) < 0$. Below we establish existence for the specific case of infinite domain, when $\varepsilon^{\max} = +\infty$.

Recall the formula for the deposit-loan spread:

$$R_t^C = \frac{\delta}{\delta - 1} \frac{1}{\Gamma} R_t^h, \quad (13.40)$$

where

$$\Gamma_t := \int_0^{\varepsilon_t^D} \left(\frac{\varepsilon}{\varepsilon^D}\right)^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon + [1 - F(\varepsilon_t^D)] < 1. \quad (13.41)$$

Using these expressions in (13.38) one obtains

$$\begin{aligned} \frac{\Psi_{t+1}(\varepsilon^A)}{Q_{t+1}X_{t+1}} [\Delta]^{\frac{\eta-1}{\eta}} &= \left[\begin{aligned} &\int_0^{\varepsilon^A} \varepsilon^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon + (\varepsilon^A)^{\frac{\eta-1}{\eta}} \times \\ &\left(1 - F(\varepsilon^A) - \frac{\delta-1}{\delta} \left[\int_0^{\varepsilon_t^D} \left(\frac{\varepsilon}{\varepsilon^D}\right)^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon + [1 - F(\varepsilon_t^D)] \right] \right) \end{aligned} \right]. \quad (13.42) \\ &= -(\varepsilon^A)^{\frac{\eta-1}{\eta}} \left[\int_0^{\varepsilon_t^D} \left(\frac{\varepsilon}{\varepsilon^D}\right)^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon - \int_0^{\varepsilon^A} \left(\frac{\varepsilon}{\varepsilon^A}\right)^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon \right] \\ &\quad - \frac{1}{\delta} (\varepsilon^A)^{\frac{\eta-1}{\eta}} \left[(1 - F(\varepsilon^A)) - \int_0^{\varepsilon_t^D} \left(\frac{\varepsilon}{\varepsilon^D}\right)^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon \right]. \end{aligned}$$

It follows then, that

$$\frac{\Psi_{t+1}(\varepsilon^A)}{Q_{t+1}X_{t+1}} \left[\frac{\Delta}{\varepsilon^A}\right]^{\frac{\eta-1}{\eta}} < - \left[\int_0^{\varepsilon_t^D} \left(\frac{\varepsilon}{\varepsilon^D}\right)^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon - \frac{\Delta}{(\varepsilon^A)^{\frac{\eta-1}{\eta}}} \right] - \frac{1}{\delta} \int_0^{\varepsilon_t^D} \left[1 - \left(\frac{\varepsilon}{\varepsilon^D}\right)^{\frac{\eta-1}{\eta}}\right] f(\varepsilon) d\varepsilon. \quad (13.43)$$

Therefore it is always possible to find ε^A so that expression (13.43) is negative. That completes the proof of existence for the case of infinite domain.

If the domain is finite and $\Psi_{t+1}(\varepsilon^{\max}) > 0$. Then the economy is safe and the government intervention is never required when commercial and investment banking sectors are separated.

It is also useful to prove that $\Psi_{t+1}(\varepsilon^D) > 0$, such that government intervention is not required when actual default is equal to expected default. The proof follows immediately from (13.42)

$$\frac{\Psi_{t+1}(\varepsilon^D)}{Q_{t+1}X_{t+1}} \left[\frac{\Delta}{\varepsilon^D}\right]^{\frac{\eta-1}{\eta}} = \frac{1}{\delta} \left[\int_0^{\varepsilon_t^D} \left(\frac{\varepsilon}{\varepsilon^D}\right)^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon + [1 - F(\varepsilon_t^D)] \right] > 0.$$

14. Aggregate equations and numerical analysis

To analyze the implications of the model for universal banking we compare welfare in the stochastic steady state of the model with and without universal banking. In what follows, unless otherwise indicated in the tables of the text, the parameterization of the steady state uses the following values:

$$\pi = 1, \beta = 0.96, \lambda = 1, A = 1, \Omega_t = 1.02, a = 1, \theta = 9, \eta = 4, g = 0.2, \sigma = 0.2, \delta = 10.$$

A key parameter is σ , the standard deviation of the shock to the trading strategies. In order to get a rough guide on a reasonable value for this parameter we proceeded as follows. In the numerical analysis we employ the lognormal distribution $f(\varepsilon)$ with parameters σ and μ . Parameter μ is naturally chosen by normalization of the mean so that $E(\varepsilon) = 1$ implies $\mu = -\sigma^2/2$. The following data looks at the empirical volatility of the log returns to banking equity. The data, taken from yahoo finance is the volatility of quarterly log returns.

Table UK	St. Dev	Time period
BARCLAYS	0.250	Q1 2003-Q2 2011
HSBC	0.106	Q3 2000-Q2 2011
Santander	0.170	Q3 2006-Q2 2011
RBS	0.343	Q1 2003-Q2 2011
Lloyds	0.200	Q3 2000-Q2 2011

Table USA	St. Dev	Time period
Bank of America Corporation	0.208	Q3 1986-Q2 2011
The Goldman Sachs Group	0.168	Q3 1999-Q2 2011
JPMorgan Chase	0.166	Q1 1984-Q2 2011
Morgan Stanley	0.170	Q2 1993-Q2 2011
Credit Suisse Group	0.161	Q1 1999-Q2 2011
Citigroup	0.190	Q1 1977-Q2 2011

As our base case, therefore, we set $\sigma = 0.2$.

14.1. Idiosyncratic shocks with separated banks

In steady state $\varepsilon^A = \varepsilon^D$. When commercial banks are separated from investment banks, it is possible to diversify the risk completely. All commercial banks have the same positive profit $\Psi_{t+1} > 0$, there is no default in the commercial banking industry and $\varepsilon^C = 0$. Furthermore, no government intervention is required, $G_t = 0$. Thus, the equations of the steady state of the model are

$$\frac{\beta}{\pi} = \frac{1}{R^h} \quad (14.1)$$

$$C = \frac{w}{\lambda}; \quad (14.2)$$

$$q = \frac{(\theta - 1)}{\theta} A \quad (14.3)$$

$$Y = AX \quad (14.4)$$

$$X = \Omega N \Delta \quad (14.5)$$

$$\left(\frac{\pi w R^C}{q \Omega} \right)^{-\eta} = \Delta [\varepsilon_t^D]^{\eta-1} \quad (14.6)$$

$$R^C = \frac{\delta}{\delta - 1} \frac{1}{\Gamma} R^h \quad (14.7)$$

$$C = Y \quad (14.8)$$

14.2. Systematic shocks and separate banking

The full set of equations are

$$E_t \left\{ \frac{\beta C_t}{C_{t+1}} \frac{1}{\pi_{t+1}} \right\} = \frac{1}{R_t^h} \quad (14.9)$$

$$C_t = \frac{w_t}{\lambda}; \quad (14.10)$$

$$q_t = \frac{(\theta - 1)}{\theta} A_t \quad (14.11)$$

$$Y_t = A_t X_t \quad (14.12)$$

$$X_{t+1} = \Omega_t N_t \varepsilon_{t+1} \quad (14.13)$$

$$E_t X_{t+1} = \Omega_t N_t \quad (14.14)$$

$$N_t = E_t X_{t+1} \left[\Omega_t \varepsilon_{t+1}^D \right]^{\eta-1} \left(\frac{w_t R_t^C}{\pi_{t+1} q_{t+1}^c} \right)^{-\eta} \quad (14.15)$$

$$R_t^C = \frac{\delta}{\delta - 1} \frac{1}{\Gamma_s} R_t^h \quad (14.16)$$

$$C_t = Y_t - g G_t \quad (14.17)$$

$$G_t = R_t^h W_t N_t \max \left[\left(1 - \left(\frac{\varepsilon}{\varepsilon^C} \right) \right), 0 \right] \quad (14.18)$$

$$\left(\frac{\varepsilon_{t+1}^A}{\varepsilon^D} \right)^{\frac{\eta-1}{\eta}} = \frac{E_t Q_{t+1}}{Q_{t+1}} \quad (14.19)$$

$$\varepsilon^C = \frac{R_t^h}{R_t^C} \varepsilon^A \quad (14.20)$$

14.3. Idiosyncratic shocks and shocks to the efficiency of capital

Here we have additional state variable, A_{t-1} , which we set to be equal to 1 when simulating results of Table 3.

$$E_t \left\{ \frac{\beta C_t}{C_{t+1}} \right\} = \frac{1}{R_t^h} \quad (14.21)$$

$$A_t^\rho = A_{t-1}^\rho u_t \quad (14.22)$$

$$E_t q_{t+1} = \frac{(\theta - 1)}{\theta} A_t^\rho \exp\left(\frac{1}{2} \sigma_A^2\right) \quad (14.23)$$

$$q_t = \frac{(\theta - 1)}{\theta} A_t \quad (14.24)$$

$$X_t = \Omega N_{t-1} \Delta \quad (14.25)$$

$$\lambda C_t R_t^C = \Omega \Delta^{1/\eta} [\varepsilon^D]^{\frac{\eta-1}{\eta}} \frac{(\theta - 1)}{\theta} A_t^\rho \exp\left(\frac{1}{2} \sigma_A^2\right) \quad (14.26)$$

$$R_t^C = \frac{\delta}{\delta - 1} \frac{1}{\Gamma} R_t^h \quad (14.27)$$

$$C_t = A_t \Omega N_{t-1} \Delta - g G_t \quad (14.28)$$

$$G_t = \max(-\Psi_t(\varepsilon^A), 0) \quad (14.29)$$

$$\Psi_t(\varepsilon_t^A) = \frac{(\theta - 1)}{\theta} A_t \Omega N_{t-1} \Delta \frac{1}{\eta} \left[\int_0^{\varepsilon_t^A} \varepsilon^{\frac{\eta-1}{\eta}} f(\varepsilon) d\varepsilon + (\varepsilon_t^A)^{\frac{\eta-1}{\eta}} \left(1 - F(\varepsilon_t^A) - \frac{R_t^h}{R_t^c} \right) \right] \quad (14.30)$$

$$\varepsilon_t^A = \left(\frac{\exp(\frac{1}{2} \sigma_A^2)}{u_t} \right)^{\frac{\eta}{\eta-1}} \varepsilon^D. \quad (14.31)$$