Vickrey Auction with Interval Values and Bids

Prasenjit Banerjee[a] and Jason F. Shogren[b]

15 October, 2011

Abstract

Psychologists and behavioral economists have studied that people have sets of values instead of one specific value for a good and they might prefer to state their WTP in intervals. This paper studies human behavior in expressing WTP given people have point or interval values in an induced value Vickrey auction experiment. Results suggest, given the opportunity, people prefer to report true values in intervals. They select their interval bids taking induced point value as mean of those intervals. With interval values, bidders bid their expected value. Given the option of reporting bids in interval with interval values, they sincerely bid in intervals in that expected bid and value coincides.

Key Words: Auction, Interval values, Interval bids, Risk Neutrality, Incentive Compatible.

[a] University of Manchester, Economics, Oxford Road, Manchester, M13 9PL, UK
[b] University of Wyoming, Economics, Laramie, Wyoming, 82072, USA

We thank Bugas fund to support this work. We also thank Pritam Gupta and Amrita Mukherjee for laboratory assistance.

1. **Introduction**

Economists have, over the years, devoted considerable resources to studying markets and to investigate how value is generated through social exchange. A market can be defined as an institution, “in which buyers and sellers are in such free intercourse with one another that the prices of the same goods tend to equality easily and quickly” (Cournot, 1838). The ability of assigning a monetary value to costs and benefits of any good—market and non-market—leads to an efficient resource allocation as a good can be valued accurately and an exact WTP revealed accordingly[1].

The problem is that the desire and the anticipated satisfaction from the possession of a particular good are hard to capture (see, e.g., Marshall, 1890; Pigou, 1903). Even when these subjective elements are clearly understood, it is hard to estimate an exact dollar value that captures the amount an individual is willing to sacrifice for securing some additional desired satisfaction. Kahneman and Sugden (2005) propose that a true value of a good may be hard to obtain before using it and unknown “anchors” may be used guide us to value the good in such instances. Some researchers suggest preference may exhibit arbitrariness with some form of logical ordering; this means individuals’ choices are coherent but starting points are arbitrary (Ariely et al. 2003). Literature also suggests individuals may have an interval of values in mind, but they round it up in one single value while reporting WTP (Manski and Molinari, 2010; see also Belyeav and Kristrom, 2010).

Economists have attempted to incorporate these sources of behavioral failure in valuation of environmental resources. For example, recognizing the problems faced by respondents while eliciting preferences in a discrete choice format, contingent valuation have adopted multiple

[1] For example, in case of a contingent valuation study, respondents should be interviewed in such a way that they can reveal their WTP for marginal changes in air quality strictly in terms of Dollars; and this information can then be used to derive a demand curve (see, e.g., Ciriacy-Wantrup, 1947).

bounded a discrete choice questionnaire format (see, e.g., Welsh and Poe, 1998, Broberg and Brannlund, 2008)[2]. In another stated preference experiment, people were given the opportunity to choose whether to report their WTP to improve water quality (for a coastal town in Scotland) in a point value or an interval of values (Hanley et al., 2009). Supporting the idea of "coherent arbitrariness", it concluded people prefer to express their WTP in intervals as they might have interval values in their mind to begin with. In contrast, another survey revealed people might have a tendency to report extreme values as they just want to round up (or narrow) their intervals to point values (Dominitz and Manski, 1997). But both of these studies agree on the idea that people may have an interval value rather than a point estimate.

These finding are based on how people state their preference according to the questionnaire of the experimenter. But none of these actually test respondents' behavior in expressing their WTP given they have point or interval values. Also, in case of a public good, it is often difficult to report a point value because: (i) information about the true importance of the good might be incomplete; (ii) the choice is made in a hypothetical context; and (iii) people lack incentives to reveal their true WTP in a stated preference valuation experiment. Therefore, it remains an open question whether people can express WTP in single number given they have point value or interval values in their mind.

The purpose of this paper is to explore how people behave in revealing their WTP in an induced value experiment, where they would have a choice in expressing their WTP between a point and an interval value, given they have a point or an interval value for a hypothetical good. To test this, this paper follows a Vickrey auction mechanism as it is incentive compatible to state

[2] Some economists still prefer exact question/answer in dichotomous choice questionnaire in stated preference valuation method (e.g., NOAA recommendation, see Arrow et al., 1993). They argue that open-ended questionnaires would create instances of free riding and respondents would strategically overstate WTP. Also, people are relatively more comfortable with exact dichotomous questionnaires than open-ended ones. There are counter arguments too, e.g., people could be biased toward saying "yes" (Kanninen 1995).

true values (see, e.g., Coursey et al. 1987; Hoffman et al. 1993; Shogren et al. 1994). In one laboratory experiment, participants have to submit bids in the auction set up under four treatments – (i) point value – point bid; (ii) point value – interval bid; (iii) interval value – point bid; and (iv) interval value – interval bid.

From a policy maker's perspective, it would be an important issue to consider if people are unable to state their WTP in one single number when they have point value or interval values in their mind. This paper studies this problem. Results suggest we cannot reject demand reveling nature of Vickrey auction with point value and point bids. But, given the opportunity, people prefer to report true values in intervals. They select their interval bids taking induced point value as mean of those intervals. With interval values, bidders bid their expected value. Given the option of reporting bids in interval with interval values, they sincerely bid in intervals in that expected bid and value coincides.

Next section discusses the theoretical underpinning of human behavior under Vickrey auction with interval values and interval bids. Section 3 explains the experimental design. Results are discussed in Section 4 and Section 5 concludes.

2. **Bidding Behavior with and without intervals**

Now consider the logic of bidding for the four cases we consider: Case I: Point value-point bid; Case II: Point value-interval bid; Case III: Interval value-point bid; and Case IV: Interval value-interval bid.

2.1. Case I: Point Value and Point Bid

Case I is the classic Vickrey second-price auction. We repeat the logic here so we can discuss how it might or might not change when ranges are introduced. If we have point private value and we express our WTP in point through sealed bid, the second price auction mechanism is weakly demand revealing.

Assume nature selects private value v for each of k bidders. Following second price auction procedure, bidders submit their bid, b, and the bids will be ranked from the highest to lowest. The highest bidder buys the good and pays the second highest bid. Suppose the market price (i.e., the second highest bid) is p. The bidder k can buy the good if his bid, b_k, exceeds the market price. The difference between his value, v_k, and the market price, p, would be his profit. The payoff, π, is,

$$\pi_k = \begin{cases} v_k - p & if \quad b_k > p \\ 0 & if \quad b_k \leq p \end{cases} \qquad (1)$$

Following Vickrey (1961), if the bidder has point value and bids in a single number the dominant strategy is to bid the value (i.e., $v_k = b_k$). Suppose, the second-highest bid under the dominant strategy is b^*, i.e., if $b_k = v_k$ then $b = b^*$. If π_k^* is the payoff from this strategy, given the moves of the $k-1$ bidders and nature, then the expression (1) implies, $\pi_k^* = \max\{0, v_k - b*\}$, since if $vk<b*$ then if $bk<b*$ (see, Milgrom and Weber 1982).

Here we explain why it would be optimal for a rational bidder to bid the value. First, consider the situation where the bidder k overbids, i.e., $b_k > v_k$. If $p < v_k$, then the bidder k would win and make a positive profit. It does not matter to him if he overbids or sincerely bids because the two strategies, overbidding and sincere bidding, have same payoff in this case. If $p > b_k$, then the bidder cannot win, then both the strategies would lead to the same payoff – zero profit. If $v_k < p < b_k$, then the bidder would win by overbidding. The payoff would be negative for overbidding because the bidder paid more than the value of the item, while the payoff for a sincere bidding would be zero. The strategy of bidding higher than one's true valuation is dominated by the strategy of sincere bidding.

With similar logic, underbidding is dominated by sincere bidding. Assume that the bidder k bids $b_k < v_k$. If $p > v_k$, then there is no way that the bidder k can win. In that case, his payoff would be same, zero, if he bids his value or underbids. If $p < b_k$, the bidder would win the good either way – if he bids sincerely or underbids. The payoff from the strategies of underbidding and sincere bidding would be the same. But, if $v_k > p > b_k$, then the bidder would lose the good due to underbidding – he could win by bidding sincerely. The payoff for the sincere bidding would be positive as they paid less than their value of the item, while the payoff for an underbid bid would be zero. It suggests underbidding is dominated by sincere bidding.

This implies bidding sincerely would dominate over underbidding and overbidding. Rational bidders should bid their value.

2.2. Case II: Point Value and Interval Bid

When people are asked to submit bids in intervals given point value, rational bidders have two choices—bid a range or a point; sincere bidding or not. They should bid in one single number which is their given private point value.

Assume nature selects a private value v for each of k bidders and bidders submit their bid in a range, $b \in (b_{min}, b_{max})$. The bids will be ranked from the highest to lowest on the basis of one random number drawn from the submitted range of bids. Bidders can submit their bids in one single number too. In that case, the upper and lower bound of interval coincide. The highest bidder buys the good and pays the second highest bid, p. The bidder k can buy the good if a random number drawn from his range, $\hat{b}_k$, exceeds the market price. The difference between his value, v_k, and the market price, p, would be his profit. His payoff depends on the number randomly drawn from his range of bids, $(b_{k_{min}}, b_{k_{max}})$. If the random number drawn from his interval bids is greater than the market price, then he can win the auction. If his value is greater than $\hat{b}_k$, he will incur a positive profit. The payoff, π, conditional on the draw $\hat{b}_k$, is same as expressed in equation (1).

Following the logic discussed in Case – I, it is not optimal strategy for a rational bidder to choose his interval bid higher $(b_{k_{min}} > v_k)$ or lower $(b_{k_{max}} < v_k)$ than the value. Now we explain why bidding true point value is optimal strategy. Suppose bidder *k* bids in interval, $(b_{k_{min}}, b_{k_{max}})$, and this interval bid contains his value, v_k. If $p < v_k$, and $v_k < \hat{b}_k$, then the bidder *k* would win and get a positive profit. The payoff from selecting the interval taking the value within it would be equal to the payoff from the strategy of bidding the true point value. If $p > \hat{b}_k > v_k$, then the bidder cannot win, here also both the strategies would lead to the same payoff. If $v_k < p < \hat{b}_k$, then the bidder would win by bidding in interval. But, his payoff would be negative because the bidder paid more than the value of the item, while the payoff for a sincere bid (i.e., bid the point value) would be zero. If $p > v_k$, and $v_k > \hat{b}_k$, then the bidder *k* cannot win and his payoff would be zero. There will be no change in his payoff if he bids his value. But, if $v_k > p > \hat{b}_k$, then the bidder would lose the good because the random draw from his interval bid is less than the market. He could win by bidding in one single number equal to the given value. The payoff for a sincere bidding would be positive as they paid less than their value of the item. The strategy of bidding the true value dominates over bidding interval, even if the interval bid contains the true value. Also, given the bidder bids in point, overbidding and underbidding are dominated by sincere bidding (as we discuss in Case – I).

2.3. Case III: Interval Value and Point Bid

There is no straight forward prediction of rational bidding behavior when people have interval values and they have to bid in one single number. Now risk preferences could play a role.

Assume nature selects an interval of private values $v \in (v_{i_{min}}, v_{i_{max}})$ for *i*th bidder, $i = 1, 2, \ldots, k$. Bidders submit their point bid, *b*, and the highest bidder buys the good and pays the second highest bid, *p*. But the payoff of the highest bidder depends on a number randomly drawn from his range of values. If the number randomly drawn from his values is higher than the market price, then only he will incur a positive profit. If $\hat{v}_k$ is the random number drawn from *k*th bidder's interval value, $(v_{k_{min}}, v_{k_{max}})$, and b_k is his point bid, then the payoff, π, conditional on the draw $\hat{v}_k$, is same as expression (1). Assuming the values are from a uniform distribution, any number within the range of values is equally likely to be drawn from the range.

A bidder has two choices to make – whether to bid within $(v_{k_{min}}, v_{k_{max}})$, or bid outside that value range. It would be irrational to bid outside the range. Suppose the bidder *k* bids lower than the interval value, $b_k < v_{k_{min}}$, and the second highest bid is p. If $p > v_{k_{max}}$, then the bidder loses the auction. In that case, he would not be better off by choosing his point bid within the value range. If $p < b_k$, the bidder would win – it would not be affected by his choice of bid within or below the value range. The payoff from those two strategies would be the same. But, if $v_{k_{max}} > p > b_k$, then the bidder would lose the good due to bidding below the interval value. There could be a possibility of winning the good in auction by bidding within the value range. It suggests bidding below the value range is weakly dominated by bidding within the range.

Suppose the bidder k bids $b_k > v_{k_{max}}$. If $p < v_{k_{max}}$, then the bidder k would win and it would not affect his payoff due to choosing bid higher the value range, it would be same as bidding inside the range of values, i.e., $v_{k_{max}} > b_k > p$. He could make a positive profit if $\hat{v}_k < p$. The two strategies have same payoff in this case, provided $\hat{v}_k < p$. But, he could lose if he selects hid bid within the interval values such that $v_{k_{max}} > p > b_k$. If $p > b_k$, then the bidder cannot win, here also both strategies would lead to the same payoff. If $\hat{v}_k < p < b_k$, then the bidder would win by bidding higher than the interval value. The payoff would be negative because the bidder paid more than the value of the item, while the payoff for a bid within the interval value could be zero. The strategy of bidding higher than interval value is weakly dominated by the strategy of bidding within the value range.

The next question is where inside the range he would bid. It depends on the risk preference of the bidder. Assume bidders are risk neutral. Also, the values are selected from a uniform distribution – any range of values is equally likely to be selected. By definition, risk neutrality would induce bidders to bid the expected value. Given uniform distribution of values and risk neutrality, the bidder k would select his bid where his expectation of given interval values equates the bid, $E[v_{k_{min}}, v_{k_{max}}] = b_k = v_{k_{median}}$. To study bidders' bidding behavior, we test this hypothesis that bidders would bid the expected value.

2.4. Case – IV: Interval Value and Interval Bid

Bidders face a few choices when they have interval values and they bid in a range – whether to bid a point or a range; bid outside or inside the interval values; if inside, where in the interval values. It would always be weakly dominant strategy for rational bidders to bid the interval values.

Assume nature selects a range of private values $v \in (v_{min}, v_{max})$ for each of *k* bidders. Bidders submit their interval bids, $b \in (b_{min}, b_{max})$. The bids will be ranked from the highest to lowest on the basis of one random number drawn from the range. The highest bidder buys the good and pays the second highest bid. Suppose the market price (i.e., the second highest bid) is *p*. The highest bidder, say the *k*th bidder, can buy the good if the random number drawn from his range of bids, $\hat{b}_k$, exceeds the market price. His payoff depends on numbers randomly drawn from his interval values and bids. If a random number drawn from his interval bids is greater than the market price and the number randomly drawn from his values is higher than the market price, then only he will incur a positive profit. If $\hat{v}_k$ and $\hat{b}_k$ are drawn randomly from $(v_{k_{min}}, v_{k_{max}})$ and $(b_{\ k_{min}}, b_{\ k_{max}})$, then the payoff conditional on realized $\hat{v}_k$ and $\hat{b}_k$ would be the same as described in expression (1).

First we discuss the situation where the bidder *k* chooses interval bid. We explain here why it would be optimal for the rational bidder to bid the interval values. Then we argue the rationale behind bidding an interval rather than a point.

Suppose the bidder *k* chooses interval bids, $(b_{\ k_{min}}, b_{\ k_{max}})$, little higher than the given interval values such that $b_{\ k_{max}} = v_{k_{max}} + \varepsilon$, where $\varepsilon > 0$, and $b_{\ k_{min}} = v_{k_{min}}$. If the market price, $p > b_{\ k_{max}}$, then the bidder cannot win the auction and his payoff is zero. He could not do any better if he chooses $b_{\ k_{max}} = v_{k_{max}}$. The payoff from both the strategies would be the same. If $b_{\ k_{max}} > p \geq v_{k_{max}}$, he could win the auction if the number randomly drawn from his interval bid, $\hat{b}_k$, is the highest bid, i.e., $\hat{b}_k > p$. But, even if he wins, it is not certain that he could make a positive profit – it would depend on the number randomly drawn from his interval value, $\hat{v}$. In this case, he could get zero return if $\hat{v} = v_{k_{max}} = p$. Then, both the strategies – choosing the interval little higher than the value and choosing the upper bound of interval value – would generate the same payoff. But, if $b_{\ k_{max}} \geq \hat{b}_k > p > v_{k_{max}}$, he would incur a loss. He could get zero if he chooses $b_{\ k_{max}} = v_{k_{max}}$. The strategy of choosing the upper bound of interval bid little higher than the upper bound of interval value is weakly dominated by the strategy of choosing $b_{\ k_{max}} = v_{k_{max}}$.

Now consider the situation when the bidder chooses the upper bound of interval bid little lower than the upper bound of interval value. Suppose the bidder *k* chooses interval bids, $(b_{\ k_{min}}, b_{\ k_{max}})$, such that $b_{\ k_{max}} = v_{k_{max}} - \varepsilon$, where $\varepsilon > 0$, and $b_{\ k_{min}} = v_{k_{min}}$. If the market price, $p > v_{k_{max}}$, then the bidder cannot win the auction and his payoff is zero. He would not gain anything more even if he chooses $b_{\ k_{max}} = v_{k_{max}}$. If $v_{k_{max}} > p > b_{\ k_{max}}$, he cannot win the good. There could be a possibility of winning the auction if he chooses $b_{\ k_{max}} = v_{k_{max}}$. It would, then, depend on, $\hat{b}_k$ – he could win if $\hat{b}_k > p$. In that case, he could have made a positive profit, whereas selecting upper bound of interval bid less than the upper bound of interval value would generate zero profit. But, he might get zero return too if $\hat{v} = p$. Then, the strategy of choosing $b_{\ k_{max}} = v_{k_{max}} - \varepsilon$ is weakly dominated by the strategy of choosing $b_{\ k_{max}} = v_{k_{max}}$. Similar logic would lead to the conclusion that the strategy $b_{\ k_{min}} = v_{k_{min}}$ weakly dominates over the strategies $b_{\ k_{min}} = v_{k_{min}} + \varepsilon$ and $b_{\ k_{min}} = v_{k_{min}} - \varepsilon$. Combining these arguments, it would be optimal for a rational bidder to choose an interval bid equal to the given interval values.

Now consider the logic behind bidding in interval rather than a point. Suppose bidder *k* bids a point, b_k. Following the discussion in Case – III, it is not optimal for the bidder to bid higher ($b_k > v_{k_{max}}$) or lower ($b_k < v_{k_{min}}$) than the given interval values. Then, assume his point bid is within the interval values, i.e., $v_{k_{max}} > b_k > v_{k_{min}}$. If the second highest bid, $> v_{k_{max}}$, the bidder cannot win the auction. If he bids the given interval value, then also he would get the same payoff (zero). If $v_{k_{max}} > p > b_k$, then he would lose the good by bidding a point. There could be a possibility of winning the good by bidding interval values. If he bids interval values and the random number drawn from his interval, $\hat{b}_k$, where, $v_{k_{max}} > \hat{b}_k > p$, he could win and could make a positive profit provided $\hat{v} > p$. Selecting interval bids equal to the given interval value would give the bidder higher possibility of winning and gaining positive profit than bidding a point within the interval values. The strategy of bidding interval value weakly dominates over the strategy of bidding a point.

To test whether bidders bid their interval values or not, we would examine in our statistical analysis whether the expected value of given private interval values equates the expected bid of their selected interval bids. This is because the values are selected from a uniform distribution. If bidders choose interval values, then the interval bids also follow a uniform distribution. Testing the hypothesis that $b_{mean} = E[b_{\ min}, b_{\ max}] = E[v_{\ min}, v_{\ max}] = v_{mean}$, we can actually test whether they choose interval bids sincerely given interval values (i.e., interval values = interval bids).

3. **Experimental Design**

Following the second-price auction setting, our design provides the subjects with an opportunity to reveal their WTP in a point bid and in a range of values. The induced private values of the good are given in point and in a range of values as well. Four treatment combinations were used in 2x2 design: (i) point private value and point bid, (ii) point value and interval bid, (iii) interval values and point bid, and (iv) interval values and interval bid (see Table 1).

Seventy students at the University of Wyoming – both undergraduate and graduate – participated in eight sessions of the experiment. At the beginning of every session, the experiment conductor read the instruction of the game aloud and addressed any questions that the participants asked. Participants then answered a short quiz which ensured they understood the game. No verbal communication was allowed throughout the experiment. Each treatment but the treatment one was used in two sessions. Treatment one was employed in one session only. In each session, there were 10 bidders and they played for 10 rounds. In each round, participants had to reveal their WTP to buy one unit of a hypothetical good by submitting bids in a sealed envelope. The induced private value of the good for each player was given to them before they bid. Then the experiment conductor ranked those bids from highest to lowest. The highest bidder only could buy one unit in each round and paid second highest bid. The experiment conductor bought back that unit of the good from the highest bidder and paid him the private value he was given at the beginning of the round. His profit was the difference between resale value (i.e., the private value) and the market price (i.e., the second highest bid). This structure was followed in all rounds and in all sessions, but the private values and the bidding process was different in each treatment. Also, all the bidders get different values in each round.

In session one, the treatment one – point private value and point bid – was introduced. Participants were given exact private value at the beginning of each round and they were asked to submit their bid in exact point value. The sets of specific private values were drawn from a uniform distribution of [$0.10, $10] in $0.10 increments: [$0.40, $1.80, $3.20, $5.30, $6.10, $6.50, $6.80, $7.10, $7.60, $8.40] (similar to Shogren et al. 2001). Treatment two – subjects were given point private value and they were asked to bid in range of values – was used in sessions three and four. The private values were same as treatment one. The experiment conductor ranked the bids on the basis of one randomly drawn number from each of their interval bid. The rest of game was same as before.

In the rest of the sessions, subjects were given interval private values. The interval private values were drawn from a uniform distribution of [$0.10, $10]: [{$0.10, $0.40}, {$0.40, $1.80}, {$1.80, $3.20}, {$3.20, $5.30}, {$5.30, $6.10}, {$6.10, $6.50}, {$6.50, $6.80}, {$6.80, $7.10}, {$7.10, $7.60}, {$7.60, $8.40}]. Under treatment three, they were given interval private values and asked to submit point bid. To calculate the highest bidder's profit in each round, one single private value was randomly determined from their range of private values. In last two sessions, subjects had to submit interval bid with interval private values (treatment four). One single private value and single bid were randomly determined from each player's interval private values and interval bids. The bids were ranked from highest to lowest on the basis of the random number drawn from the interval bids and the resale value was the random number drawn from the interval values.

Based on second-price Vickrey auction mechanism, this experimental set up tests following hypotheses

Hypotheses 1: Given point private value, when subjects are asked to bid in single numbers, they bid their private value.

Hypotheses 2: Given point private value and the opportunity to bid in interval, subjects bid their true point value.

Hypotheses 2A: Given point private value and opportunity to bid in interval, if bidders bid in intervals, they choose their expected bid corresponding to their true point value.

Hypotheses 3: Given interval private values, when subjects are asked to bid in single numbers subjects bid expected values.

Hypotheses 4: Given interval private values, when subjects are asked to bid in interval subjects bid their interval values.

Hypotheses 4A: Given interval private values and opportunity to bid in interval, if bidders bid in single number, expectation of chosen bids are the expected private values.

4. **Results**

4.1. Case I: Point Value and Point Bid

Result 1. The hypothesis that people bid sincerely cannot be rejected when they have point value and they have to bid in one single number.

Support. 10 participants took part under Treatment one – point value and point bid – and they played for 10 rounds. Table 2 shows people bid sincerely as the mean of their bids is 5.37 and mean of values is 5.31. They bid just their values in 48% of cases; they bid conservatively in 30% of cases and overbid in 22% of cases (see Table 3). Figure 1 and 2 also support this inspection. The graph of point bid is mostly matches with the point values as seen in Figure 1. Although there are few huge outliers of bids, but the scatter plot of bid and value gives an almost 45^{o} line (Figure 2).

This unconditional result of summary statistics is verified by a regression analysis of point bid on point value incorporating two sources of random effects due to rounds and individual effects. Result of the panel regression analysis shows the estimate of coefficient, β, is 1.01 and estimate of constant, α, is -0.07 (see Table 4). It implies one unit increase in value increases bids by one unit. The fitted line between the values and chosen bids is not significantly different from a 45^{o} line as we cannot reject the joint hypothesis, $\alpha = 0, \beta = 1$ (with $chi^2 = 0.62$ and Prob> $chi^2 = 0.73$). This result indicates sincere bidding.

4.2. Point value and Interval Bid

Result 2. We reject hypothesis 2 that bidders prefer to submit their bids in one single number equal to their private values. We are unable to reject hypothesis 2A that bidders form their expected bid corresponding to the given values.

Support. Subjects are more inclined to bid in intervals (in 56% of cases, see Table 5). They bid just their point values in only 22.5% of cases. Although expected values match with mean of interval bids in almost 23% of cases, the difference between mean bid and point value lies between +0.5 and -0.5 in almost 60% of cases. It shows some indication that subjects choose their interval bids based on expected values. Table 6 shows the mean of the expected interval bids (5.89) is not very far from the mean of point values (5.32). The scatter plot and the graph of the means of interval bid show a close match of means of interval with the point private values except a few outliers (see Figure 3 and 4).

A logit model of a binary variable – whether they choose to submit bids in one single number or not – shows one additional unit increase in their value decreases the probability that they report their bids in one single number by 0.02(Table 7). But, the result is not statistically significant. It indicates people are less likely to bid a point. A probit model of whether bidders bid exact values or not shows a negative but insignificant impact of pointvalue on the choice of bids equals to values (Table 8) It indicates bidders are less likely to bid their point values.

Now we investigate the results from summary statistics that bidders choose the means of intervals close to the values using conditional regression analysis. Given panel data set, we perform a two-way random effect model of expected bid to capture the randomness due to different rounds and individual player. A two way random effect (panel) regression of expected bid on values gives the estimate of coefficient, β, is 0.94 and the estimate of constant term, α, is 0.26 (Table 9). It implies one unit increase in value increases the mean bid (means of interval bids) by 0.94. The fitted line between the means of interval bids and values has a slope of 0.94

and intercept of 0.26. This fitted line is not significantly different from a 45^0 line as we cannot reject the joint hypothesis $\alpha = 0, \beta = 1$ (Table 9). It implies bidders choose expected bids sincerely given point values.

4.3. Interval Values and Point Bids

> *Result 3*. We cannot reject the hypothesis 3. Given interval values, bidders choose their point bids sincerely in that they bid expected values.

Support. Table 3 shows participants bid within the given private interval values in 67.5% of cases. The mean of bids (5.09) is very close to the mean of the expected interval values (4.90) (see Table 10). A graph of point bids mostly match with the medians of interval values (see Figure 5) and a scatter diagram also shows the same (Figure 6). To observe how close the bids are to the values we see the difference between point bids and means of interval values. This difference falls in the region of ± 0.05 from zero in 48 % of cases. It seems the bids are chosen close to the mid points of the given interval values, which suggests bidders bid expected values.

We investigate the bidding behavior to verify the results from summary statistics by performing regression analysis. A two way random effect regression of point bids on mean of interval values shows the estimates of slope, β, and intercept, α, are 0.96 and 0.071 (Table 13). It means one unit change in medians of interval values raises bids by 0.96. The fitted line of bids with median-values is not significantly different from a 45^o line as the joint hypothesis of, $\alpha = 0, \beta = 1$, cannot be rejected (with $chi^2 = 0.97$, Prob.> chi^2=0.61). It suggests bidders choose their bids sincerely corresponding to the mean of interval values.

4.4. Interval Values and Interval Bids

Result. *4.* We reject the hypothesis that people can sincerely bid their true interval values. We cannot reject the hypothesis that bidders choose their expected bids sincerely corresponding to their expected values.

Support. Table 14 shows subjects submit point bid (i.e., upper bound of bid matches with the lower bound) in only 23% of cases. They bid exact private value range in 4.5% of cases. They overbid (lower bound of bid range exceeds the upper bound of value range) and underbid (upper bound of bid range lies below the lower bound of value range) in 22% and 15% of cases. Summary statistics also shows the mean of the expected of interval bids (6.44) is, on average, higher than the mean of the interval values (4.90, see Table 15). A graph and a scatter diagram of the means of the interval bids reveal a close match with the means of the interval values (see, Figure 7 and Figure 8). This indicates expected bids are selected sincerely given expected values.

We now verify the unconditional results of summary statistics using conditional regression analysis. We first examine whether people prefer to bid in one single number by conducting a logit regression. A logit model of point bid on median-value illustrates there is almost zero and insignificant impact of median-value on point bids (Table 16). So we cannot conclude that people are likely to report their bids in one single number.

We investigate the results from summary statistics about the choice of expected bids corresponding to the expected values by conducting conditional regression analysis. A panel regression (two way random effect) of means of interval bids on medians of interval values is performed. Result shows the estimates of slope, β , and intercept, α , are 0.87 and 0.05 (Table 17). It suggests people raise their mean-bids by 0.87 with 1 unit increase in mean-values, provided intervals are represented by the medians. The fitted line between the median s of interval bids and values has a slope of 0.87 and an intercept of 0.05. Also, we cannot reject that the values and bids have a one-to-one relation as the joint hypothesis, $\alpha = 0, \beta = 1$, cannot be rejected. This result indicates bidders choose their expected bids sincerely given expected values.

5. **Concluding Remarks:**

The foundation of welfare economics assumes public policy to consider the preferences of the people who will be affected by the policy. In case of environmental goods, where markets are mostly missing, assigning monetary values are important for designing polices (Hanemann 1994). We rely on valuation as people can make their choices for their benefits or costs accurately from environmental goods assuming they have exact values in their mind. Psychologists and behavioral economists have studied that people have sets of values instead of one specific value for a good and they might prefer to state their WTP in intervals. From a policy maker's perspective, it would be an important issue to consider if people are unable to state their WTP in one single number when they have point value or interval values in their mind. This paper addresses this problem by studying human behavior in expressing their WTP given they have point or interval values in a demand revealing auction experiment.

Results suggest[3], given the opportunity to state their WTP in intervals, people prefer to report true value in interval rather than in a one single number with point or interval value in mind. When they have point value, they prefer to express their WTP in interval taking true point value as mid point of the interval – they choose expected bids corresponding to the point values. Given the option of reporting bids in interval with interval values, they are good at revealing their true interval values – bidders sincerely chose their expected bid corresponding to the

[3] We adopt different statistical methodologies to capture the underlying preferences in choosing bids in points and in intervals given point and interval values. We carry out the following statistical analysis: (i) Classical regression method with panel data (OLS and Logit model); (ii) Interval regression method; (iii) Quantile regression; (iv) Cluster analysis; and (v) Latent class analysis. The summary of the results under different methods are listed in Table 18. Overall, these results support the findings of classical regression method.

expected value. This result is similar to finding of Hanley et al. (2009) that people prefer to express their WTP in intervals. But, they bid sincerely their values when they have to bid in one single number, given point value. With interval values, they tend to reveal their WTP in single number at and above the mid points of the interval values.

References:

Aldrich, G. A., K. M.Grimsrud, J. A. Thacher, and M. J. Kotchen (2007), "Relating Environmental Attitudes and Contingent Values: How Robust are Methods for Identifying Preference Heterogeneity?", Environmental and Resource Economics, Vol. 37, pp. 757-775.

Anderberg, M.R. (1973), "Cluster Analysis for Applications", Academic Press, New York.

Ariely, D., G. LoeInstein, and D. Prelec (2003), "Coherent Arbitrariness": Stable Demand Curves Without Stable Preferences", *Quarterly Journal of Economics*, Vol. 118, pp. 73-105.

Arrow, K., R. Solow, P. R. Portney, E. E. Leaner, R. Rander, and H. Schuman (1993), "Report of the NOAA Panel on Contingent Valuation", Fed. Reg., pp. 4601-4614.

Broberg, T. and R. Brannlund (2008), "An Alternative Interpretation of Multiple Bounded WTP Data – Certainty Dependent Payment Card Intervals", *Resource and Energy Economics*, Vol. 30, pp. 555-567.

Belyeav, Y. and B. Kristrom (2010), "Approach to Analysis of Self Selected Interval Data", CERE Working Paper.

Calinski, T., and J. Harabasz (1974), "A Dendrite Method for Cluster Analysis", *Communications in Statistics – Simulation and Computation*, Vol. 3 (1), pp. 1188-1190.

Cameron, T. A., and D. D. Huppert (1989), "OLS versus ML Estimation of Non-Market Resource Values with Payment Card Interval Data", *Journal of Environmental Economics and Management*, Vol. 17, pp. 230-246.

Ciriacy-Wantrup, S. V. (1947), "Capital Returns from Soil-Conservation Practices", *Journal of Farm Economics*, Vol. 29, pp. 1188-90.

Cournot, A. A. (1838), "Recherches sur les Principes Mathématiques de la Théorie des Richesses", chapter IV (1897, English translation by N.T. Bacon).

Coursey, D., J. Hovis, W. Schulze (1987), "The Disparity between Willingness to Accept and Willingness to Pay Measures of Value", *Quarterly Journal of Economics*, Vol. 102, pp. 679-690.

Dominitz, J. and C. Manski (1997), "Perceptions of Economic Insecurity: Evidence from the Survey of Economic Expectations", *Public Opinion Quarterly*, Vol. 61, pp. 261-287.

Everitt, R., S. Landau, and M. Leese (2001), "Cluster Analysis", Oxford University Press, New York.

Hanley, N., B. Kristrom, and J. Shogren (2009), "Coherent Arbitrariness: On Value Uncertainty for Environmental Goods, *Land Economics*, Vol. 85 (1), pp. 41-50.

Hanemann, W. M. (1994), "Valuing the Environment through Contingent Valuation", *Journal of Economic Perspectives*, Vol. 8 (4), pp. 19-43.

Hasselblad, V., A. G. Stead, and W. Galke (1980), "Analysis of Coarsely Grouped Data from the Lognormal Distribution", *Journal of the American Statistical Association*, Vol. 75 (372), pp. 771-778.

Hoffman, E., D. Menkhaus, D. Chakravarty, R. Field, G. Whipple (1993), "Using Laboratory Experimental Auctions in Marketing Research: A Case Study of New Packaging for Fresh Beef", *Marketing Science*, Vol. 12, pp. 318-338.

Jacquemet, N., R. Joule, S. Luchini, and J. F. Shogren (2009), "Preference Elicitation under Oath", *CES Working Paper*.

Kahneman, D., and R. Sugden (2005), "Experienced Utility as a Standard Policy Evaluation", *Environmental and Resource Economics*, Vol. 32, pp. 161-181.

Kanninen, B. J. (1995) ,"Bias in Discrete Response Contingent Valuation", *Journal of Environmental Economics and Management*, Vol. 28 (1), pp. 114-125.
Koenker, R., and K. F. Hallock (2001), "Quantile Regression", *Journal of Economic Perspectives,* Vol. 15 (4), pp. 143-156.
Koenker, R., and Jr. G. Bassett (1978), "Regression Quantiles", *Econometrica*, Vol. 46 (1), pp. 33-50.
Manski, C. F. and F. Molinari (2010), "Rounding Probabilistic Expectations in Surveys", *Journal of Business and Economic Statistics*, Vol. 28, pp. 219-231.
Marshall, A. (1890), "Principles of Economics", 8th Edition, London: Macmillan and Co., Ltd.
McCutcheon, A. L. (1987), "Latent Class Analysis", Sage University Paper Series on Quantitative Application in the Social Sciences, Beverly Hills and London: Sage Publications.
Milgrom, P. R. and Weber, R. J. (1982), "A Theory of Auctions and Competitive Bidding", *Econometrica*, Vol. 50 (5), pp. 1089-1122.
O'Garra, T, and S. Mouratto (2007), "Public Preferences for Hydrogen Buses: Comparing Interval Data, OLS, and Quantile Regression Approaches, *Environmental and Resource Economics*, Vol. 36, pp. 389-411.
Pigou, A. C. (1903), "Some Remark on Utility", *Economic Journal*, Vol. 13 (49), pp. 58-68.
Rabe-Hesketh, S., A. Skrondal, and A. Pickles (2004), "GLLAMM Manual", U. C. Berkeley Division of Biostatistics Working Paper Series, University of California, Berkeley, Paper # 160.
Romesberg, H. (1984), "Cluster Analysis for Researchers", Lifetime Learning Publications, Belmont SAS Institute Inc.(1987).
Shogren, J., S. Shin, D. Hayes, J. Kliebenstein (1994), "Resolving Differences in Willingness to Pay and Willingness to Accept", *American Economic Review*, Vol. 84, pp. 255-270.
Shogren, J. F., M. Margolis, C. Koo, and J. A. List, (2001), "A Random nth-price Auction", *Journal of Economic Behavior & Organization*, Vol. 46, pp. 409-421.
Shogren, J. F., G. M. Parkhrust, and P. Banerjee (2010), "Behavioral Economics and the Environment", Working Paper.
Skrondal, A. and S. Rabe-Hesketh (2004), "Generalized Latent Variable Modeling: Multilevel, Longitudinal, and Structural Equation Models", Chapman & Hall/CRC.
Smith, A. (1776), "An Inquiry into the Nature and Causes of the Ialth of Nations", Vol. 1, W. Strahan and T. Cadell, London.
Vickrey, W. (1961), "Counterspeculation, auctions, and competitive sealed tenders", *Journal of Finance*, Vol. 16, pp. 8-37.
Ward, Jr., J. H. (1963), "Hierarchical Grouping to Optimize an Objective Function", *Journal of the American Statistical Association*, Vol. 58 (301), pp. 236-244.
Welsh, M. and G. L. Poe (1998), "Elicitation Effects in Contingent Valuation: Comparisons to a Multiple Bounded Discrete Choice Approach", *Journal of Environmental Economics and Management*, Vol. 36, pp. 170-185.

Appendix 1

Figures

Figure 1. The Graph of Point Value and Point Bid (Case I)

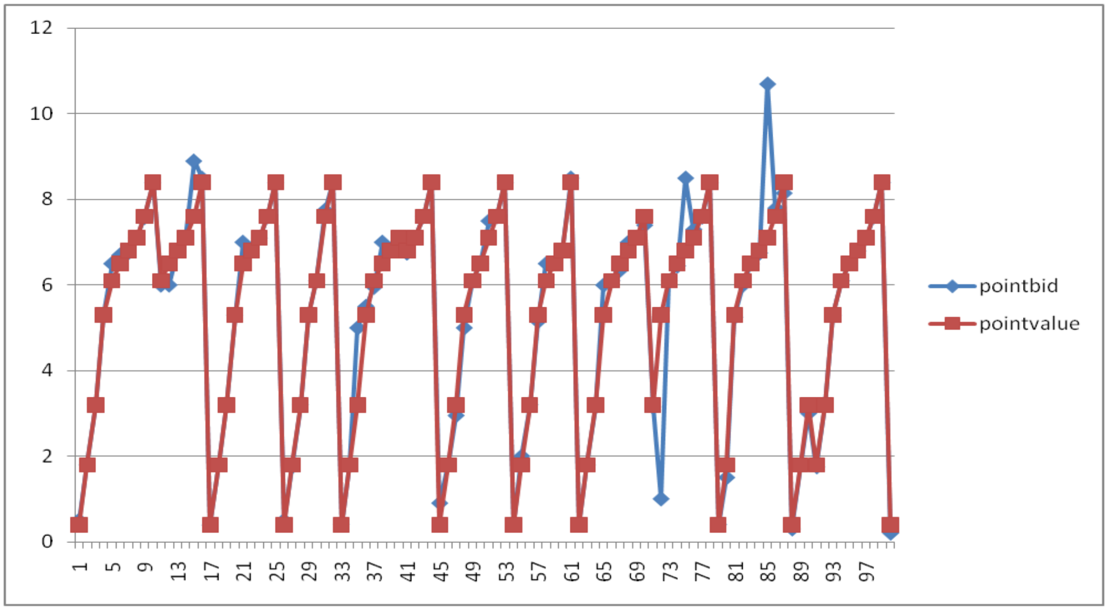


Figure 2. Plots of Point Bids corresponding to different levels of Point Values

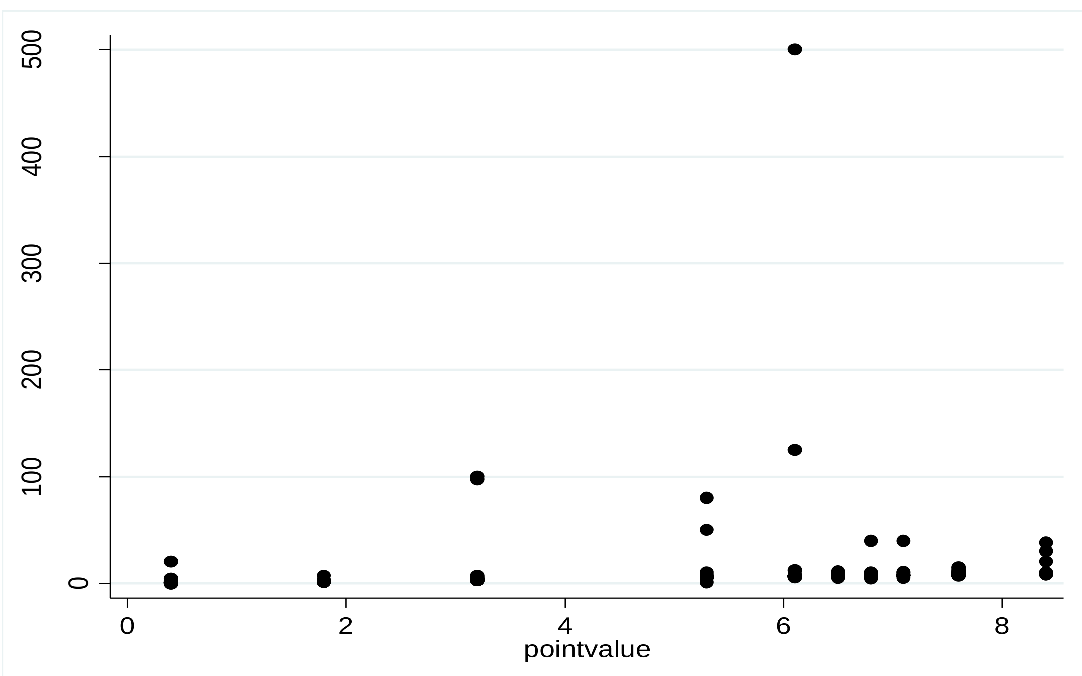


Figure 3. The Scatter Plots of Means of Interval Bid and Point Value (Case II)

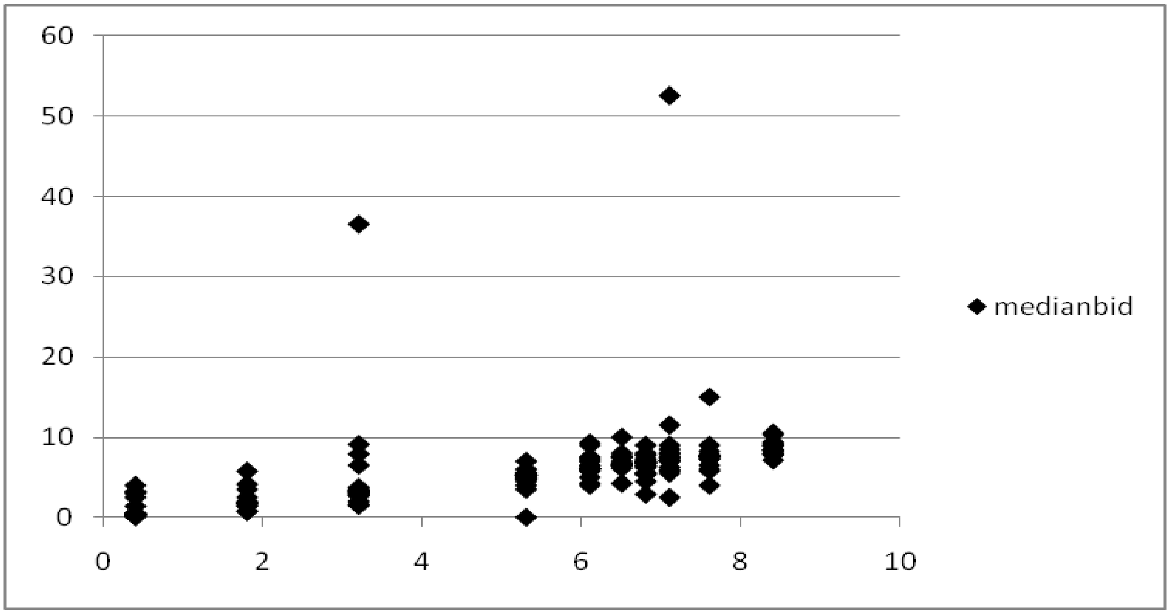


Figure 4. The Graph of Point Value and Means of Interval Bid

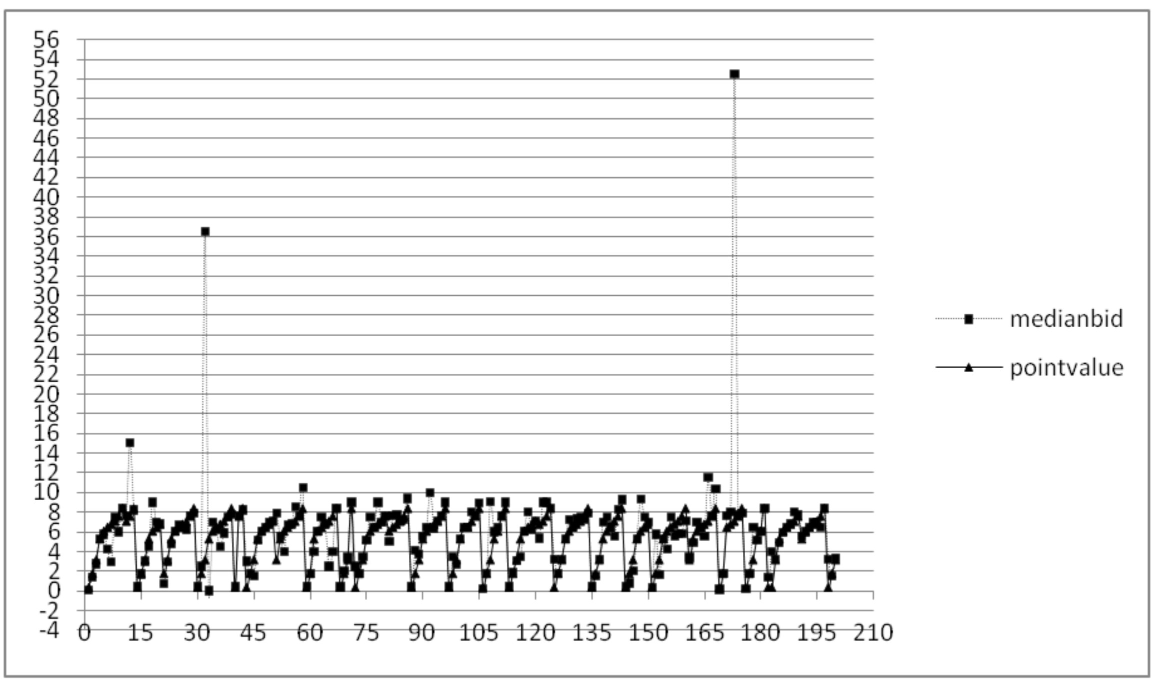


Figure 5. Graph of Median of Interval Value and Point Bid (Case III)

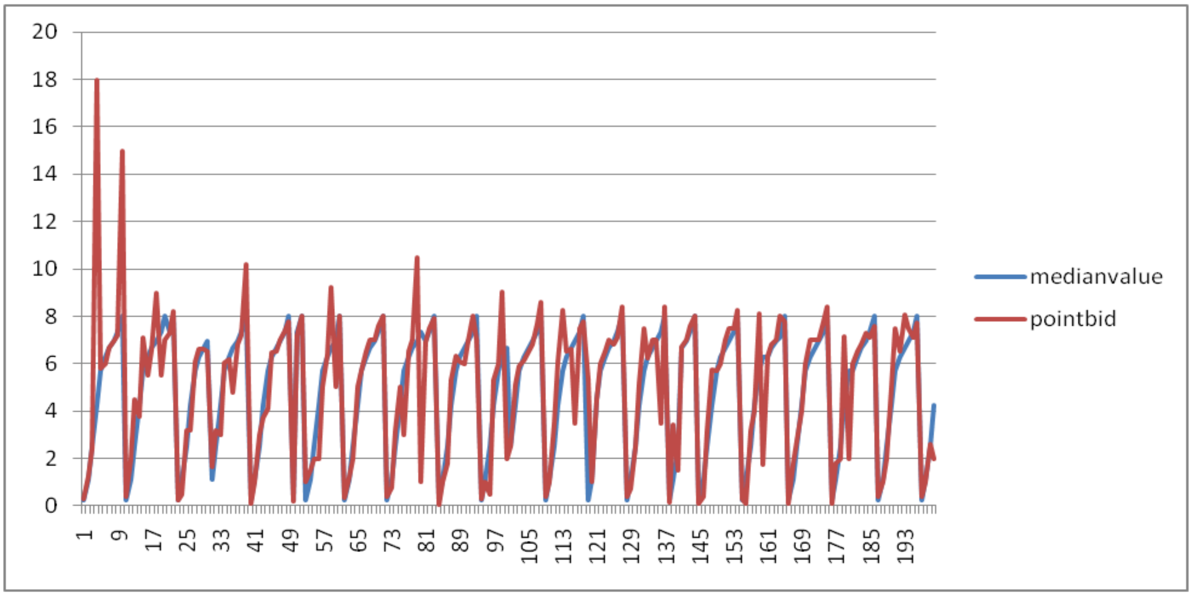


Figure 6. Scatter Plots of Point Bid and Medians of Interval Value (Case III)

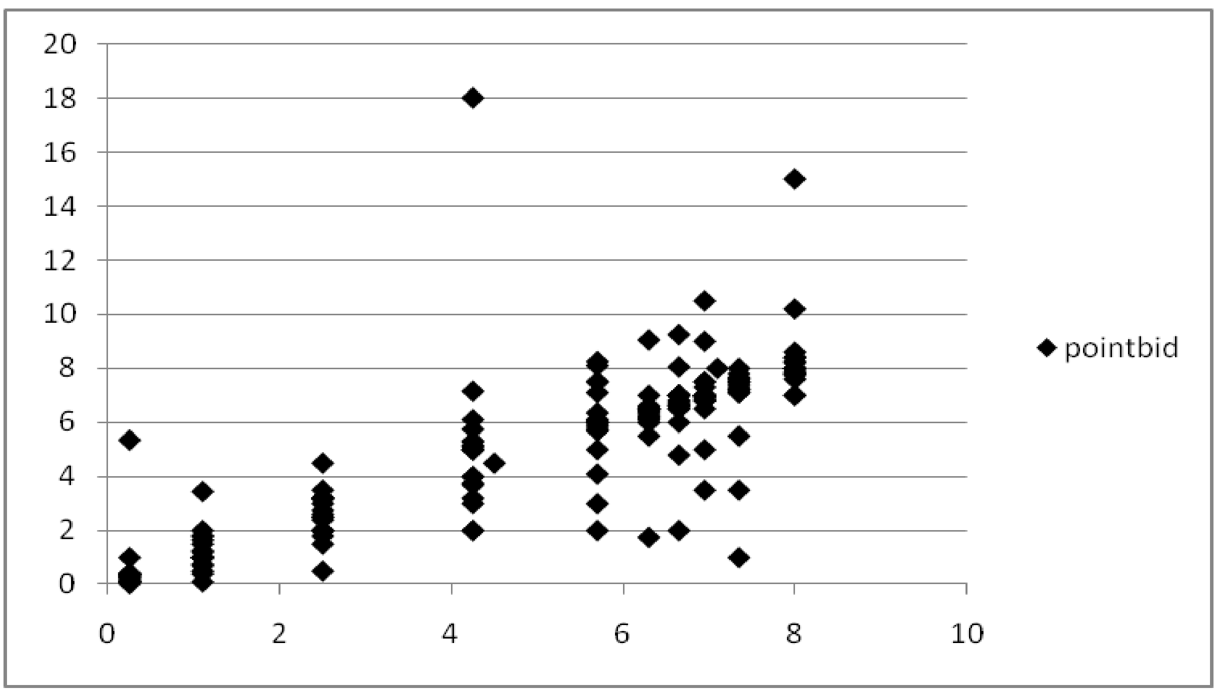


Figure 7. Graph of the Means of Interval Bids and the Means of Intervals of Values

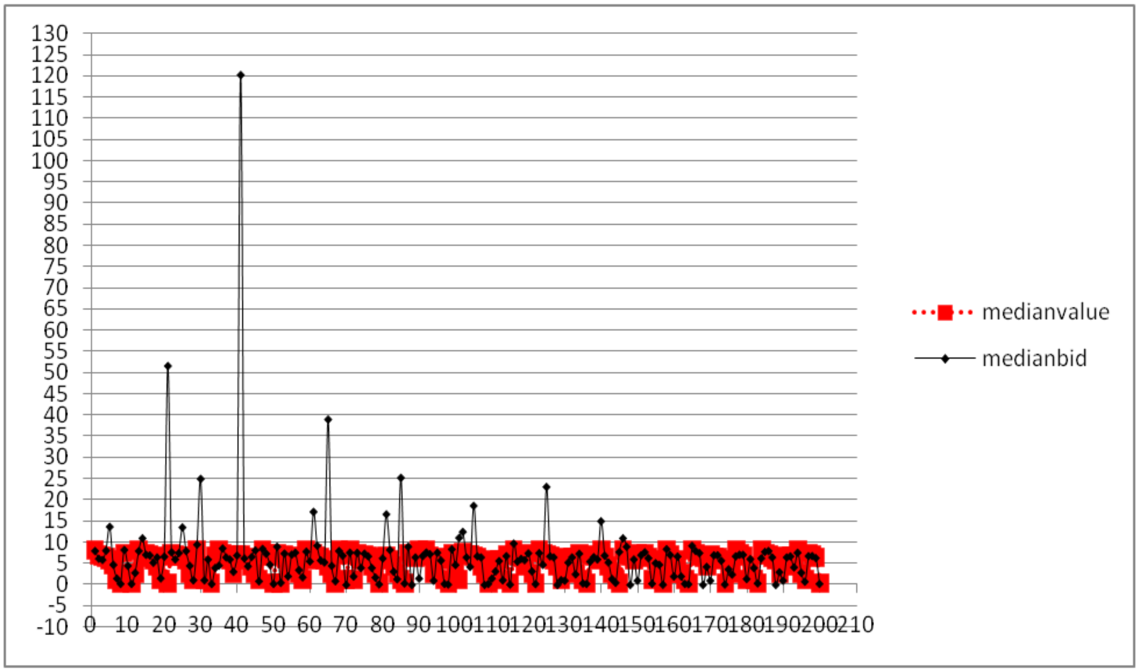
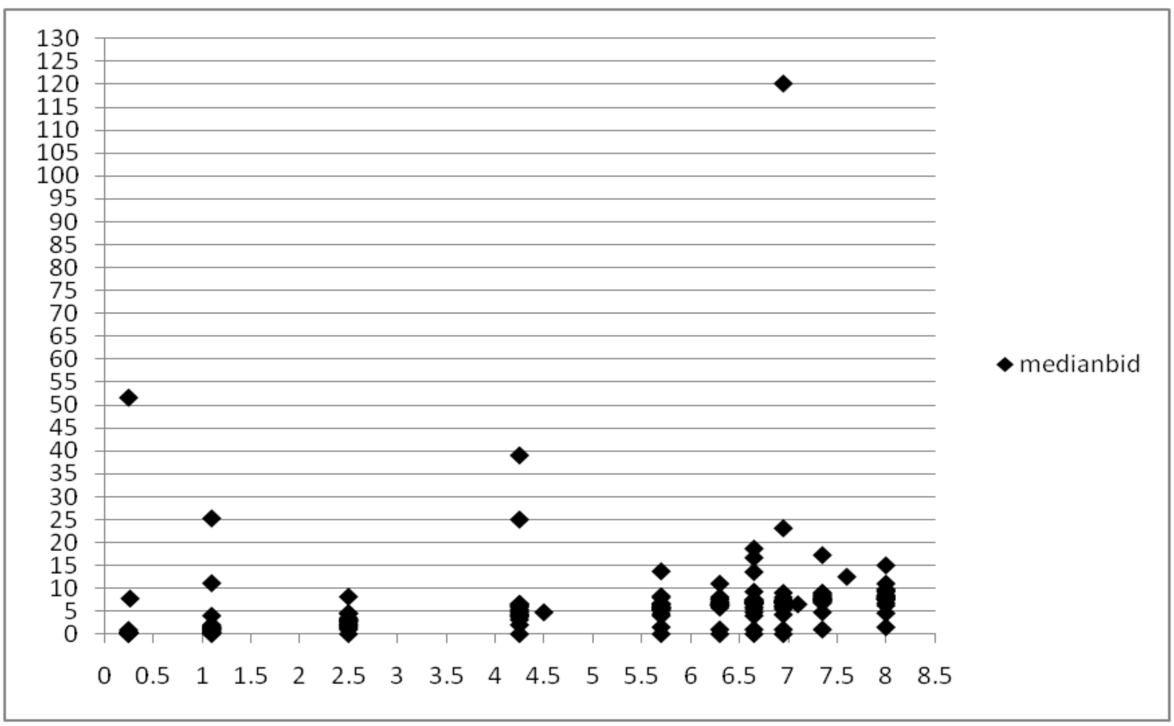


Figure 8. Scatter Diagram of the Medians of Interval Bids and the Medians of Interval Values



Appendix 2

Tables of Statistical Results

Table 1. Treatment Combination with Point and Range of Values and Bids under Vickrey Auction

		Induced Values: Point Value	Induced Values: Interval Values
WTP	Point Bids	*(Point, Point)*	*(Point, Range)*
	Interval Bids	*(Range, Point)*	*(Range, Range)*

Table 2: Summary Statistics of Case I

	Mean	Std. Dev.	Min.	Max.
Value	5.31	2.52	0.4	8.4
Bid	5.37	2.64	0.20	10.7

Table 3: Number of cases where people overbid, underbid and sincerely Bid under Case I

	Exact Bid	Over Bid	Under Bid
Treatment 1	48%	22%	30%
Treatment 3	67.5%	19.5%	13%

Table 4: Two-way Random Effect Regression Results of the Model

Point Bid = Cons. + Point Value + Sum Round + Sum Individual + Error

	Coefficient	Std. Error	Z	P>z
Value	1.01	0.022	47.91	0.00
Constant	-0.07	0.131	-0.55	0.583

We *cannot* reject the joint hypotheses, $\alpha = 0, \beta = 1$, with $F = 1.03, P = 0.59$. chi2=0.62, P=0.73.

Table 5: Number of cases where people overbid, underbid and sincerely Bid in Case II

	Over Bid (*i.e., Lower Bound of Bid > Point Value*)	Under Bid (*i.e., Upper Bound of Bid < Point Value*	Single Bid (*i.e., Upper Bound of Bid = Lower Bound of Bid*)	Exact Bid (*i.e., Upper Bound of Bid = Lower Bound of Bid = Point Value*)
Treatment 2	20.5%	18.5%	44%	22.5%

Table 6: Summary Statistics of Case II

	Mean	Std. Dev.	Min.	Max.
Value	5.31	2.52	0.4	8.4
Lower bound of Bid-Range	4.83	4.20	0	50
Upper bound of Bid-range	6.95	6.59	0.03	70
Median of bid range	5.89	4.83	0.02	52.5

Table 7: Logit Model: Point Bid = Cons + Point Value + Round + Individual + Error

	Coefficient	Std. Error	z	P>z
Value	-0.023	0.108	-0.21	0.830
Constant	-1.82	1.26	-1.44	0.150

Table 8: Probit Model: Exact Bid = Cons + Point Value + Round + Individual + Error

	Coefficient	Std. Error	z	P>z
Value	-0.114	0.071	-1.60	0.110
Constant	-7.05	83366.42	-0.00	0.999

Table 9: Two way Random Effect Model:

Mean of Interval Bid = cons. + point-value + Sum of rounds + Sum Individual + error

	Coefficient	Std. Error	Z	P>z
Value	0.94	0.11	8.52	0.00
Constant	0.26	0.67	0.40	0.691

We *cannot reject* the joint hypotheses, $\alpha = 0, \beta = 1$, with chi2=0.31, P=0.85

Table 10: Summary Statistics of Case III

	Mean	Std. Dev.	Min.	Max.
Lower bound of Value Range	4.49	2.72	0.1	7.6
Upper bound of Value Range	5.32	2.52	0.4	8.4
Median of Value Range	4.90	2.60	0.25	8
Point Bid	5.04	2.99	0.03	18

Table 13: Two way Random Effect Model:

Point Bid = cons. + Median of Interval values + Sum of rounds + Sum Individual + error

	Coefficient	Std. Error	Z	P>z
Value	0.96	0.039	24.55	0.00
Constant	0.071	0.357	0.20	0.843

We *cannot reject* the joint hypotheses, $\alpha = 0, \beta = 1$, with $chi2 = 0.97, Porb. > chi2 = 0.61$.

Table 14: Number of cases where people overbid, underbid and sincerely Bid in Case IV

	Over Bid (*i.e., Lower Bound of Bid > Upper Bound of Value*)	Under Bid (*i.e., Upper Bound of Bid < Lower Bound of Value*	Upper Bound Bid > Upper Bound Value	Lower Bound Bid < Lower Bound Value	Single Bid (*i.e., Upper Bound of Bid = Lower Bound of Bid*)	True Upper Bid (*i.e., Upper Bund of Bid = Upper Bound of Value*)	True Lower Bid (*i.e., Lower Bund of Bid = Lower Bound of Value*)	Exact Range Bid (*i.e., Bid Range = Value Range*)
Treatment 4	22%	15%	46.5%	34.5%	23%	9%	12.5%	4.5%

Table 15: Summary Statistics of Case IV

	Mean	Std. Dev.	Min.	Max.
Lower bound of Value Range	4.49	2.72	0.1	7.6
Upper bound of Value Range	5.32	2.52	0.4	8.4
Median of Value Range	4.90	2.60	0.25	8
Lower bound of Bid Range	5.46	9.41	0	120.17
Upper bound of Bid Range	7.43	11.60	0	120.17
Median of Bid Range	6.44	9.97	0	120.17

Table 16: Logit Model: Point Bid = Cons + Median Value + Round + Individual + Error

	Coefficient	Std. Error	z	P>z
Value	0.05	0.097	0.52	0.606
Constant	2.70	1.51	1.78	0.074

Table 17: Two way Random Effect Model

Expected Interval Bid = cons + Expected Interval Value + Sum Rounds +Sum Individual + error

	Coefficient	Std. Error	z	P>z
Value	0.87	0.223	3.91	0.000
Constant	0.052	1.31	0.04	0.968

We *cannot reject* the joint hypotheses, $\alpha = 0, \beta = 1$, with $chi2 = 0.95, Porb. > chi2 = 0.62$.

Table 18. Summary of Results of four cases under different methods

	OLS	Interval	Quantile	Cluster	Latent Class
Case I	$\alpha = -0.07$ $\beta = 1.01*$	____	$\alpha = -0.14, \beta = 1*$ Q=0.25		$\alpha = 4.57$ $\beta = 1.21$
			$\alpha = -0.1, \beta = 1*$ Q=0.50		
			$\alpha = -.05, \beta = 1*$ Q=0.75		
	Joint Hypothesis – **Not Rejected**	Joint Hypothesis – **Not Rejected**	Joint Hypothesis – **Not Rejected**		Joint Hypothesis – **Rejected**
Case II	$\alpha = 0.26$ $\beta = 0.94*$	$\alpha = 3.44*$ $\beta = 1*$	$\alpha = 1.09$ $\beta = 0.96*$ Q=0.25		$\alpha = 0.09$ $\beta = 0.99*$
			$\alpha = 1.02$ $\beta = 0.99*$ Q=0.50		
			$\alpha = 1.61$ $\beta = 0.98*$ Q=0.75		
	Joint Hypothesis – **Not Rejected**	Joint Hypothesis – **Rejected**	Joint Hypothesis – **Not Rejected**		Joint Hypothesis – **Not Rejected**
Case III	$\alpha = 0.07$ $\beta = 0.96*$	___	$\alpha = -0.38$ $\beta = 1*$ Q=0.25		$\alpha = 0.27$ $\beta = 0.98*$
			$\alpha = 0.23$ $\beta = 0.99*$ Q=0.50		
			$\alpha = 0.64$ $\beta = 0.97*$ Q=0.75		
	Joint Hypothesis – **Not Rejected**	Joint Hypothesis – **Not Rejected**	Joint Hypothesis – **Not Rejected**		Joint Hypothesis – **Not Rejected**
Case IV	$\alpha = 0.05$ $\beta = 0.87*$	$\alpha = 0.48$ $\beta = 0.91*$	$\alpha = 0.26$ $\beta = 0.98*$ Q=0.25		$\alpha = 2.14$ $\beta = 0.87*$
			$\alpha = 0.53$ $\beta = 1*$ Q=0.50		
			$\alpha = 0.65$ $\beta = 0.99*$ Q=0.75		
	Joint Hypothesis – **Not Rejected**	Joint Hypothesis – **Not Rejected**	Joint Hypothesis – **Not Rejected**		Joint Hypothesis – **Rejected**