Liquidity, Term Spreads and Monetary Policy*

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Abstract

The slope of the yield curve and the term spread have important implications for macroeconomic outcomes being amongst the key variables in predicting output growth (Estrella and Hardouvelis (1991)). We propose a model that delivers endogenous variations in term spread driven primarily by changes in banks' expected profitability and their appetite to bear the risk of maturity transformation. We show that fluctuations of the future profitability of banks portfolios affect their ability to cover for any liquidity shortage and hence affect the premium they require to carry maturity risk. During a boom, profitability is increasing and hence spreads are low, while during a recession profitability is decreasing and hence spreads are high, in accordance with the cyclical properties of term spreads in the data. We also document empirical evidence showing the relevance of financial business profitability in explaining real output growth and the linkages between yield spreads and expected profitability. Finally, we use the model to look at monetary policy and show that allowing banks to sell long term assets to the Central Bank after a liquidity shock leads to a sharp decrease in long term rates and term spreads. Furthermore, such interventions have significant impact on long term investment, decreasing the amplitude of output responses after a liquidity shock. The short term rate does not need to be decreased as much and inflation turns out to be much higher than if no QE interventions were done.

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1. Introduction

The presence of term spreads has implications on financial transactions, macroeconomic outcomes and policy design[1]. The existing literature finds that the slope of the yield curve has significant predictive power in explaining US business cycle fluctuations (see, for instance, Estrella and Hardouvelis (1991) or more recently Rudebusch and Williams (2009)). This is not only linked to the fact the slope is related to the future path of short term interest rates but also due to the changes in the term spread. Moreover, recent large scale purchase programmes adopted by the Federal Reserve Bank, the Bank of England and the European Central Bank have had significant impacts on the shape of the yield curve, increasing the importance of understanding and modeling the fluctuations in term spreads. While there is a growing literature on term spreads and macroeconomic outcomes, there is no consensus as to which are the determinants of time varying risk premium, one of the key components behind the variations in the shape of the yield curve. In this paper we propose a model that provides an explanation for endogenous variations in term spread driven primarily by changes in banks' balance sheets, their expected profitability and their appetite to bear the risk of maturity transformation.

The first structural models that focused on the term structure of interest rates relied on the expectations hypothesis limiting the analysis to cases of either no or constant risk premium. In these models, a difference between current long (equivalently expected future short rate) and short rates will occur when rational agents anticipate a change in future inflation and/or output gap. In their strong form long term interest rates are just averages of expected short term interest rates priced at the stochastic discount factor such that the term premium is non-existent. In their weak form researchers assume a constant term premium. There is evidence[2], however, that the term premium is time varying; therefore these models are far from satisfactory. In a natural development from these earlier contributions, structural models were developed focusing on the variability of the stochastic discount factor and its links with macroeconomic variables (macro-finance or affine term structure models). The literature is extensive. Recent work by Piazzesi and Schneider (2007) and by Rudebusch and Swanson (2008a) model risk premium as an outcome of the negative covariance between inflation and consumption growth. Epstein-Zin preferences break the link between the intertemporal elasticity of substitution and the coefficient of risk aversion, allowing for the covariance to have a more significant impact on agent's decisions. In this framework financial investors demand a higher risk premium as a hedge against (long-term) inflation risk. Other contributions to the literature focused on the effects of learning about long term inflation targets of the Central Bank (Kozicki and Tinsley (2005)) or on the possible segmentation of short and long term bond markets (Vayanos and Vila (2009)) identifying other drivers of time varying risk premium.

Although supporting the view that long run inflation risk is an important determinant of term spread fluctuations, three main empirical facts motivate the search for additional factors external to monetary policy. First, dynamics of short-run rates and inflation expectations do not explain all the variability of long term rates, particularly in the last decade (De Graeve, Emiris, and Wouters (2009)). Second, Benati and Goodhart (2008) observe that during the 2000's the marginal predictive content of term spreads to future output increased, although monetary policy uncertainty remained low. Finally, as stressed by Gürkaynak and Wright (2010), the US treasury inflation

[1] We will use term spreads and term premia interchangeably.

[2] See for instance Piazzesi and Swanson (2008) and references therein.

protected securities (TIPS) forward rate dynamics have not been that different than their nominal counterparts, indicating the term premia are also influenced by real factors. Hence, in this paper, we develop a DSGE model with endogenous term spreads derived from bank's risk assessment of potential liquidity shortages impacting their profitability and balance sheets. The importance of bank balance sheets and bank risk taking has been recently stressed by Adrian and Shin (2008, 2009), and Adrian, Shin, and Moench (2010). They show that financial intermediary balance sheets contain strong predictive power for future excess returns on a broad set of equity, corporate, and Treasury bond portfolios. Adrian, Estrella, and Shin (2010) focus particularly on the link between bank balance sheets, spread and economic activity. They show that "the term spread forecasts recessions because it forecasts lower future net interest margin, lower future asset growth, and lower future GDP growth". While they focus more on leveraging and banking asset growth rates we will be looking particularly at the variability of profits and the maturity transformation risk. We show that empirically financial sector profits contain additional information to explain variation in real output next to term spreads. Furthermore, we document a statistically significant link between yield spreads and changes in expected financial sector profitability, giving support to our main channel.

The funding/banking structure of our model, which focuses on potential liquidity risks, relies on the contributions[3] of Holmstrom and Tirole (1998) and Diamond and Rajan (2001, 2005). We assume that firms' long-term projects may suffer from potential liquidity shortages during their first period, thus these are only completed if a liquidity injection, ultimately made by the bank, is done. Banks fund their portfolio of equities (mature investments), short and long-term lending with short term borrowing, thus banks bear the risk of maturity transformation. Term spreads are then determined by the volatility of future short term rates, as usual in the macro finance literature, plus the new element introduced here; the premium for bearing the maturity risk. Another relevant feature of our framework, which is not present in macro finance models, is that the term spreads have an explicit impact on the fundamental variables given that long term rates influence investment on (long-term) capital formation.

Confirming the stylized facts we obtain counter-cyclical term spread movements. Furthermore, we observe that term premia are a good predictor of future activity. As the economy approaches the peak of the business cycle spreads tend to be at their lowest and will tend to increase thereafter. On the other hand, as the economy approaches the bottom of the business cycle spreads tend to be at their highest. The main driver of these endogenous movements in long term rates and term spreads is the fluctuations of the future profitability of banks' portfolios. Banks rely on the overall profitability of their portfolio to assess their ability to cover for any liquidity shortages. Hence, future profitability relates directly to their appetite to bear maturity risk or the risk premia they require to commit to provide long term funding to firms. As output is increasing, profitability is expected to be high and hence spreads are low. During recessions, profitability is low and hence spreads tend to be high. We also show that our model delivers considerably more volatile term spreads comparing to the affine term structure models although we did not assume higher degree of risk aversion and neither introduce implausibly high variance for the exogenous disturbances. The main reason for this result is that spread movements in our model are driven by bank profits

[3]Although we simplify the model structure excluding moral hazard problems and the potential for bank failures to focus particularly on term spreads.

which are more volatile than consumption, the determinant of spread movements in affine models such as Rudebusch and Swanson (2008b).

Another interesting feature of the model that could help explain the sharp decline in term spreads before the crisis both in the UK (inverted yield curve) and the US ("conundrum" of 2004-2006) is the effect of potential future liquidity shortages. As the yield curve is on average positively sloped, we assume that banks at the steady state would face a need for providing liquidity injections, charging a positive premia. However, during the recent period of boom in securitization and the development of structure finance activities, long term commitments could have actually delivered additional gains offsetting these potential shortages. Banks were able to re-package and sell portions of their long term asset portfolio at advantageous terms (see, for instance, Ivashina and Sun (2011)). Therefore, bank balance sheet conditions could be the external (to monetary policy) factor that determines spread movements and in fact may have led to the increase in the marginal predictive content of spreads during the beginning of the decade as identified by Benati and Goodhart (2008).

Finally, we use the model to consider the impact of both conventional and unconventional monetary policies. Although we find that the base rate dynamics is implicitly influenced by movements in term spreads due to their impact on output and inflation we do not find that responding explicitly to their movements (altering the standard (Taylor) monetary rule to include term spreads) leads to greater stabilization. We then look at the impact of unconventional monetary policy similar to the recent quantitative easing (QE) adopted in the US and the UK. Note that the channel through which QE affects the economy in our framework is distinct to the one stressed in some the recent theoretical papers (see, among others, Gertler and Kiyotaki (2009)). In the latter the mechanism is generally through a direct replenishing of banking capital, covering for current shortages. In contrast, unconventional monetary policy in our framework aims at protecting banks from potential liquidity shortages in the future, increasing banks' willingness to carry maturity transformation risk, or in other words, reducing long term spreads. We find that allowing banks to sell long term assets to the Central Bank after a liquidity shock leads to a sharp decrease in long term rates and term spreads matching the results found by several empirical studies done on the recent QE policies in the US, UK and the Eurozone (see Gagnon, Raskin, Remache, and Sack (2010), Joyce, Lasaosa, Stevens, and Tong (2010) and Beirne, Dalitz, Ejsing, Grothe, Manganelli, Monar, Sahel, Suec, Tapking, and Vong (2011)). Furthermore, such interventions have significant impact on long term investment, decreasing the amplitude of output responses after a significant liquidity shock. The base rate does not need to decrease as much and inflation turns out to be higher than if no QE intervention was done.

The paper is organized as follows. Section 2 presents a simple model to highlight the relationship between banks profitability, balance sheets and term spreads. The empirical evidence on the link between financial business profitability, output and spreads in support of our channel is presented in Section 3. Section 4 presents the model of endogenous term spreads. We start presenting our results in Section 5 focusing on the main drivers of the endogenous movements in term spreads, its potential to predict future output growth and its amplification effects. Section 6 presents the volatility in term spreads at different points of the yield curve, comparing to the data and the figures obtained by Rudebusch and Swanson (2008b). Section 7 discusses conventional and unconventional monetary policies. Finally, Section 8 concludes.

2. A Simple Model of Bank's Balance Sheet Composition

This section focuses on explaining the link between bank's balance sheet (portfolio), bank's profits and term spreads by presenting a simple two period model of banks portfolio decision. We later embed a similar decision problem into a general equilibrium model to explore the effects of endogenous fluctuations of spreads on economic activity and monetary policy.

We assume the bank decides its portfolio at time zero selecting the demand for the three assets available in the economy, namely, a long term asset (X_L), a short term asset (X_S) and equity (or a portfolio of the rest of risky short term assets available in the economy), denoted Z. The short term asset pays out a certain return of R_S at period 1. Equities also pay out in period 1 but their return R_E is uncertain; we assume $R_E \sim N(\bar{R}_E, \sigma_E^2)$. Finally, long term assets pay out a certain return of R_L in period 2, but in period 1 the bank might be forced to make an injection of liquidity (ρ) to keep the asset in the portfolio; we assume $\rho \sim N(\bar{\rho}, \sigma_\rho^2)$. We denote $co_{E,\rho}$ the correlation index between the two disturbances. The bank fully funds this portfolio with deposits that provide the holder a gross return of R_D (for simplicity we assume deposits are in infinite supply at the equilibrium short term rate and R_D is exogenously set; in the general equilibrium model in section 4 this rate is endogenously determined).

As it is well known, if banks have no regard to risk, being risk neutral and/or not having to abide by any constraint on risk taking, term spreads would be constant. Hence, in order to study term spread fluctuations we assume banks care about risk in two possible ways. In the first case, also used in our general equilibrium model, we assume banks are risk averse, maximizing a constant relative risk aversion function of profits. In the second case, we assume banks are risk neutral, thus maximizing expected profits, but are subject to a value-at-risk constraint. Both cases deliver a similar association between expected portfolio returns and term spread fluctuations. The bank problem, formally, is

$$
\begin{aligned}
\max_{\{X_S, X_L, Z\}} \quad & E\left[\Pi^B\right] \\
s.t. \quad & \Pi^B = \frac{(\Pi_1^B)^{1-\sigma_B}}{1-\sigma_B} + \beta \frac{(\Pi_2^B)^{1-\sigma_B}}{1-\sigma_B} \\
& \Pi_1^B = (R_E - 1)Z + (R_S - 1)X_S - \rho X_L - (R_D - 1)D_0 \\
& \Pi_2^B = (R_L - 1)X_L - (R_D - 1)D_1 \\
& D_0 = Z + X_L + X_S \\
& D_1 = X_L \\
& VaR(\Pi^B) \geqslant \Lambda \text{ iff } \sigma_B = 0
\end{aligned} \tag{1}
$$

where $VaR(\Pi^B)$ is the value at risk of the banks portfolio defined as the expected minimum portfolio return over the two periods within a 1% confidence interval.

It is straightforward to see that at equilibrium short term rates will be equal to the return on deposits. The key equations to determine banks portfolio and in turn rates of return on risky assets are, therefore, given by

$$
\begin{aligned}
-E\left[(\Pi_1^B)^{-\sigma_B}(R_D-1+\rho)\right]+\beta E\left[(\Pi_2^B)^{-\sigma_B}(R_L-R_D)\right]+\Im\frac{\partial VaR}{\partial X_L}\varsigma &= 0 \qquad (2)\\
-E\left[(\Pi_1^B)^{-\sigma_B}(R_E-R_D)\right]+\Im\frac{\partial VaR}{\partial Z}\varsigma &= 0 \qquad (3)
\end{aligned}
$$

where $\Im$ is an indicator function that takes the value of 1 when $\sigma_B = 0$ and zero otherwise[4] and ς is the lagrange multiplier of the value-at-risk constraint. In effect, these two equations above determine the demand for equity and long term assets of the bank. In order to obtain an equilibrium for these two markets (again, these will be endogenously set in the general equilibrium model) we assume that[5]

$$
\begin{aligned}
X_L &= \frac{\gamma_S}{R_L+E[\rho]} \qquad (4)\\
\bar{R_E} &= \alpha_E-\gamma_E Z. \qquad (5)
\end{aligned}
$$

That way, the higher the long term rate, the lower the supply of long term assets to the bank will be (or the investors demand for long term loans the bank provides) and the higher the demand for equity, the higher prices would be or the lower expected returns would be.

Case 1 - $\sigma_B > 1$

If banks are risk averse (for simplicity we drop the VaR constraint in this case), the equilibrium condition that determines the long term rate is simply given by

$$
E\left[(\Pi_1^B)^{-\sigma_B}(R_D-1+\rho)\right]=\beta E\left[(\Pi_2^B)^{-\sigma_B}(R_L-R_D)\right]. \qquad (6)
$$

It is easy to see that when expected profitability in period 1 increases relative to the profitability in period 2, due to higher expected return on equity, then the left-hand side term of (6) decreases, and in equilibrium, long term rates will also decrease and the bank's demand for long term assets will increase. Note that the covariance between profitability (Π_1^B) and future liquidity shortages (ρ) will also be important to determine the extent to which an increase in profitability affects the decrease in the LHS of (6). We will discuss the role of this covariance in more detail in the general equilibrium model. Also note that in this simple model we assume the deposit rate remains constant in period 1. In the general equilibrium model presented below this will also be uncertain, affecting the term premia.

Case 2 - $\sigma_B = 0$

Both in case 1 and in the general equilibrium model that will be presented later we assume banks are risk averse. That delivers a straightforward link between expected profits and term premia; banks require lower premia in periods in which profits are expected to be high. However, we only require that banks care about some measure of risk in order to establish a link between profitability and risk premium. As such, in this case we assume that banks are risk neutral but are

[4]The VaR constraint binds since expected profit increases and VaR decreases as X_L and Z increase.

[5]The bank takes $\bar{R_E}$ as given while selecting Z.

subject to a value-at-risk constraint. Our main interest is to verify how term spreads (measure in basis points), defined as

$$tp = \frac{1}{2}\left(\left(R_L - 1\right) - \left(R_D - 1\right) - \left(R_D - 1\right)\right) 10000,$$

move as the overall profitability (expected returns) of the bank's portfolio varies. In doing so we aim at establishing a link between banks appetite to accumulate long term assets in the balance sheet, incurring the risk of maturity transformation, the equilibrium long term rates and the expected performance of bank investments. The details of the solution of the model in this case is given in the appendix with the discussion provided here.

As α_E increases, the return on equity, holding demand for Z constant, rises. That implies that if banks were to hold the same portfolio, their VaR (expected minimum return) would increase above the constraint. That would allow banks to increase the demand for both equity and long term assets, increasing expected profit, until the constraint becomes binding again (akin to an income effect) and/or increase demand for equity and decrease the demand for long term assets, as equity became the relatively better asset (substitution effect). As long as there is a gain in asset diversification in the bank's portfolio (or $\rho_{E,\rho} < 0$), then the income effect dominates and the demand for long term assets will increase as expected return on equity, or profitability, increases[6]. Consequently, as portfolio returns increase, long term rates decrease, leading to a flatter yield curve or narrower term spreads. Thus, as the expected profitability of banks increase they are willing to charge a lower premium to bear the risk of maturity transformation, increasing their balance sheet position until reaching the constraint on expected minimum return (or maximum expected losses). The additional expected profits can then be used to cover potential liquidity injections needed to maintain long term assets in the balance sheet.

Thus, in both Case 1 and Case 2, during periods in which the assets held in their balance sheets are expected to pay higher returns, banks are willing to increase exposure to maturity transformation risk, charging a lower risk premium. Higher profits on investments allow banks to cover for potential liquidity shortages. These fluctuations of expected bank profitability, therefore, lead to endogenous movements in term spreads. We now look at some empirical evidence on the relevance of financial business profitability in explaining real output growth and the linkages between spreads and expected profitability.

3. Empirical Evidence

As argued in the introduction there is strong evidence that US term spreads help predicting US real output growth (see, for instance, Rudebusch and Williams (2009)). Furthermore, Adrian, Shin, and Moench (2010) highlight the importance of financial sector variables, particularly the growth in financial intermediary asset holdings, in predicting several asset prices and risk measures. Adrian, Estrella, and Shin (2010) show that "the term spread forecasts recessions because it forecasts lower future net interest margin, lower future asset growth, and lower future GDP

[6]The strength of the substitution effect is directly related to the covariance between the asset returns. As the covariance decreases so does the substitution effect and more likely it is that the demand for long term assets will increase with α_E.

growth", linking financial sector variables to term spread movements and economic activity. We complement this empirical evidence by looking particularly at the linkages between financial sector profitability, term spreads and output growth, as suggested by our model.

We use quarterly US data covering the period 1980Q1-2008Q2. Our data consists of seasonally adjusted US real GDP expressed in billions of chained 2005 Dollars as reported by the Bureau of Economic Analysis and financial business undistributed corporate profits as reported by the Flow of Funds Statistics of the US Federal Reserve. This measure is for all financial businesses, including not only commercial banks but also for instance mutual funds and security brokers and dealers (see the Appendix for further details on the data used). Term spreads are computed using the US Treasury Bill rate in percent per annum and 10 year government bond rate in percent per annum as reported by the IMF/IFS.

We first investigate the additional marginal predictive content in movements of financial business profits to explain real output movements next to autoregressive components of real output, past term spreads and the T-Bill rate (which controls for the level of interest rates in the economy). We focus explicitly on correlation structures as the preliminary test of statistical connectedness between variables. The reduced form/information value approach is immune to questions of causality and exogeneity issues as first advocated by Sims (1972, 1980) and Friedman and Kuttner (1992) among others. We also study linkages between term spreads and the changes in expected financial sector profitability. Term spreads are inherently forward looking variables reflecting the future path of short term rates plus the risk premia from holding long term positions, and hence if the channel explained above exists, variation in spreads should contain information on the expected financial business profitability.

3.1. Information Content of Financial Business Profitability

Our primary interest is to investigate whether variations in financial sector profits contain exploitable information that will help predict variations in real output beyond those already predictable by using past variations in real output, term spreads and the short term interest rates, similar in nature to the analysis conducted by Benati and Goodhart (2008).

Our specification for real output changes is given by[7]

$$\Delta y_t = \alpha + \sum_{i=1}^{8} \beta_i \Delta y_{t-i} + \sum_{i=1}^{8} \gamma_i \left(r_{t-i}^L - r_{t-i}^S \right) + \sum_{i=1}^{8} \delta_i \Delta \pi_{t-i} + \sum_{i=1}^{8} \zeta_i \Delta r_{t-i}^S + \varepsilon_t \qquad (7)$$

The terms Δy, $\left(r^L - r^S\right)$, $\Delta\pi$ and ε represent the annualized changes in output, the spread between 10 year government bond (r^L) and 3-months Treasury Bill (r^S), the annualized changes in financial business profits and an error term, respectively. We include 8 lags for independent variables as an inverted yield curve is found to contain predictive power for recessions within a 12 to 18 months period (see for instance, Estrella and Hardouvelis (1991) and Rudebusch and Williams (2009)). We test whether the lagged coefficients of each variable are jointly significant (Wald test). In the first estimation (A) we include all four explanatory variables with lags (as in equation (7)) and in

[7] We have also included inflation in the benchmark estimation but all lags were shown to be jointly insignificant. Furthermore, we estimate a similar equation for financial profits and spreads and find that term spreads significantly explain changes in profits and vice versa. Results are available upon request.

Table 1: Measuring the Marginal Predictive Content of Financial Business Profits for Output Growth

	Estimation A		Estimation B	
	1980Q1-2008Q2			
	$\chi-Square$	$p-value$	$\chi-Square$	$p-value$
β_i	45.82	0.00	34.63	0.00
γ_i	15.20	0.06	11.97	0.15
δ_i	20.50	0.01	-	-
ζ_i	19.93	0.01	19.37	0.01
R^2	0.58		0.51	

a second estimation (B) we include variations in lagged output, lagged short term interest rates and lagged spreads only, excluding financial sector profits (setting $\delta_i = 0$ in equation (7)). Table 1 reports the $\chi - Square$ statistics and the $p - values$ of the estimated coefficients for each given lag structured[8].

Results for Estimation A reveal that the marginal predictive content of both spreads and variations in financial business profits are significant[9]. That is variations in financial business profits add significant information in predicting real output movements next to variations in past real output, term spreads and short term interest rates (see Estimation B).

3.2. Spreads and Expected Financial Business Profitability

In the previous section we argue that both spreads and financial business profitability are closely linked and are important to predict future output growth. Furthermore, term spreads are inherently forward looking variables reflecting the future path of short term rates plus the risk premia from holding long term positions, and hence variation in spreads should contain information on the expected profitability. For this purpose we follow Wright (2006) closely and analyze whether spreads today are associated with future decreases in profitability.

Let the yearly financial business profits to be equal to $\pi_t^Y = \frac{\sum_{i=0}^{3}\pi_{t-i}}{4}$. The change in profits at period t is given by $\Delta\pi_t^Y = \frac{\pi_t^Y - \pi_{t-1}^Y}{\pi_{t-1}^Y}$. We then construct a binary variable ($D\pi_t^Y$) that takes the value 1 when financial business profitability is decreasing and zero otherwise. Formally,

$$D\pi_t^Y = \begin{cases} 1 & \text{if } \Delta\pi_t^Y < \Delta\pi_{t-1}^Y \\ 0 & \text{otherwise} \end{cases}$$

This variable is used in the same way as NBER recession was used in Wright (2006). We

[8]We use White heteroskedasticity consistent standard errors.

[9]In order to gauge an idea of the stability of the information content of changes in financial business profits to explain output, we have recursively estimated Equation 7 in a rolling fashion; first estimation being for the period 1970Q1-2000Q1 and rolling estimation sample one period forward at a time. Wald tests indicate the association between financial business profits changes and real output is reasonably stable. Results are available upon request

Table 2: Probit Results - Spreads and Future Profitability

	Coefficient	$p-value$
Term Spread (γ_1)	0.222	0.0187
Mc Fadden R^2	0.037	

employ the following probit model to assess whether spreads are linked to future profitability, estimating[10]

$$P(D\pi^{Y}_{t+6}=1)=\Phi\left(\gamma_0+\gamma_1\left(r^L_t-r^S_t\right)\right),$$

where $\Phi(.)$ denotes the standard normal cumulative distribution function. If γ_1 is significant then spreads are relevant in predicting the future (negative) movements in profitability. Table 2 shows the results. The estimated coefficient for the term spread is significant and positive. This suggests that high term spreads are associated with a higher probability of observing decreasing financial business profitability in the future. We also run an augmented probit model with spreads and the 3 month T-Bill. Results remain qualitatively the same.

This evidence presented here indicates that banks balance sheets and expected profitability may influence term spreads, as suggested in our simple two period model. Next, we present a DSGE model with an explicit bank/financial sector where profitability and term spreads affect macroeconomic activity.

4. Model

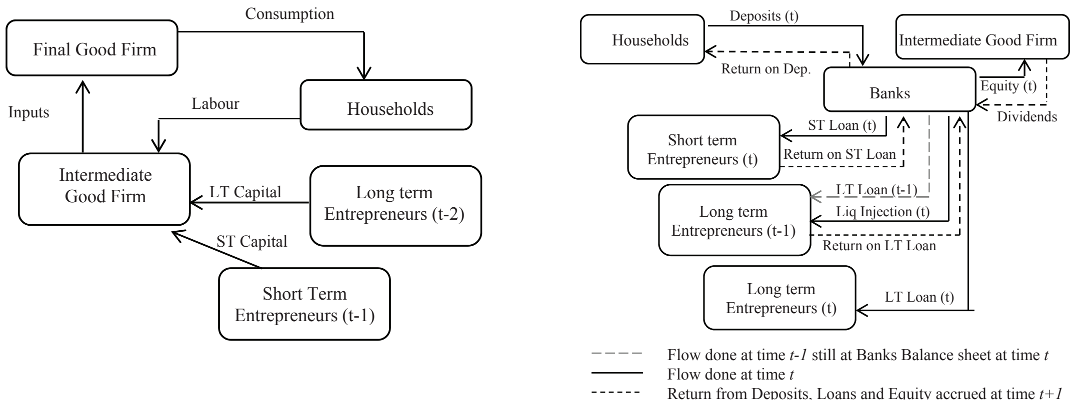
The model economy is populated by a continuum $i\in[0,1]$ of intermediate good firms, a final good producer, a continuum of households, banks, entrepreneurs and the Central Bank. Entrepreneurs borrow funds from a bank and transform consumption goods into capital. There are two types of entrepreneurs, one with access to a short-run investment project and one with a long-run investment opportunity available. This introduce a segmentation of short and long term funding requirements, similar to the one stressed by Vayanos and Vila (2009). Firm i hires labour from the households, produces a differentiated input using labour and the current capital stock, and at the end of the period sells the inputs to the final good firm and buys new capital from the entrepreneurs. The final good firm combines all inputs to produce consumption goods that are then sold to households and entrepreneurs. We assume the households (workers) receive the profits from banks and entrepreneurs, which are all of mass unit. Thus, only households consume. An equivalent alternative would be to follow a similar model structure of Gertler and Kiyotaki (2009), where a family is split into banks and consumers but consumption is done at the family level.

The bank receives deposits from the households, provides loans to both entrepreneur types and buys equity from the intermediate firms. Note that long term loans are issued at every period but

[10] We estimate $D\pi^{Y}_{t+6}$ on term spreads determined at time t, which reflect the slope of yield curve from quarter $t+1$ to quarter $t+40$ (3 month to 10 year points), since that binary variable depends on financial business profits from $t+1$ until $t+6$.

do last for two periods, thus banks' balance sheets will contain three loan agreements. These are: a short term loan, a long term loan and another long term loan issued in the previous period. Finally, we assume that during the current period the bank might need to provide a liquidity injection to long term entrepreneurs who borrowed in the previous period. Figure 1 shows the production and financial flows of the model.

Figure 1: Model Structure



(a) Production Flows (b) Financial Flows

4.1. Households

The household maximizes its expected discounted lifetime utility given by

$$\max_{C_t, D_t} E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{H_t^{1+\eta}}{1+\eta} \right), \quad \beta \in (0,1) \quad \sigma, \eta > 0$$

where C_t denotes the household's total consumption and H_t denotes the composite labour index. The curvature parameters σ, η are strictly positive. β is the discount factor. The household faces the following budget constraint

$$C_t + \frac{D_t}{P_t} \leqslant \frac{W_t H_t}{P_t} + \frac{R_{t-1,CB} D_{t-1}}{P_t} + \frac{\tilde{\Pi}_t}{P_t}$$

where W_t is the wage index, $R_{t,CB}$ is the rate of return on deposits D_t. We assume the Central Bank sets $R_{t,CB}$ directly according to a monetary policy rule to be specified. We assume the only asset available to the "worker" are deposits made directly to the financial intermediary (workers make deposits on banks owned by other households), thus only banks invest in equities issued by the intermediate good firms and lend to entrepreneurs. Although not modeled here, one reason for that would be the existence of higher household-firm agency costs relative to bank-firm agency costs.

Finally, $\tilde{\Pi}_t = \tilde{\Pi}_t^{ES} + \tilde{\Pi}_t^{EL} + \Pi_t^B P_t$ is the sum of the nominal profits realized at period t for the entrepreneurs with short-term projects, with long-term projects and the bank, respectively, which is passed on to the household.

4.1.1. Optimal Wage Setting

Households supply a continuum of labour types $j \in [0,1]$. The composite labour index H_t is then given by

$$H_t = \left[\int_0^1 H_{j,t}^{\frac{\varepsilon_w - 1}{\varepsilon_w}}\right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}}.$$

From the firms minimization problem we have that the demand for each labour type and the wage index are given by

$$H_{j,t} = \left(\frac{W_{jt}}{W_t}\right)^{-\varepsilon_w} H_t,$$

$$W_t = \left[\int_0^1 W_{j,t}^{1-\varepsilon_w}\right]^{\frac{1}{1-\varepsilon_w}}.$$

The households, when allowed (Calvo scheme with parameter ω_w), set wages $W_{j,t}$ to maximize expected utility subject to the budget constraint and the labour demand equation. The main reason to include both price and wage rigidity is to ensure firm's real profits are pro-cyclical after a productivity shock (see Carlstrom and Fuerst (2007)).

4.2. Entrepreneurs

Entrepreneurs are responsible for capital formation. We assume a set of mass unit of entrepreneurs has a short-term investment opportunity available, at each period. Another set of mass unit of entrepreneurs has a long-term (two periods) investment opportunity available, at each period. Thus, there are always three mass units of active entrepreneurs in the economy.

Short-term entrepreneurs borrow funds from the bank ($X_{S,t}$), buy consumption goods and transform it into capital next period with the following production function

$$yk_{t+1}^S = \gamma_S \ln(1 + X_{S,t}).$$

The profits of these entrepreneurs are given by

$$\tilde{\Pi}_{t+1}^{ES} = P_{t+1} q_{t+1}^S \gamma_S \ln(1 + X_{S,t}) - R_{t,S} P_t X_{S,t},$$

where $R_{t,S}$ is the interest rate on short term borrowing and q_t^S is the price of short term capital denominated in consumption goods.

Long-term entrepreneurs also borrow from the bank ($X_{L,t}$), buy consumption goods and transform it into capital after two periods with the following production function

$$yk_{t+2}^L = \gamma_L \ln(1 + X_{L,t}),$$

where $\gamma_L > \gamma_S$.

The production functions for short and long term capital output (yk^m) takes the form $\gamma_m \ln(1 + X_{m,t})$ for $m = \{S, L\}$ since we need (*i*) capital production to have decreasing returns (concave function) such that movements in borrowing rates influence the marginal propensity to invest[11]; (*ii*) long term capital investment to be more productive than short term capital due to the liquidity shock explained below (thus, $\gamma_L > \gamma_S$); and (*iii*) one unit of consumption invested to return more than one unit of capital goods, thus we normalize the production function to be the log of one plus the amount invested. That way each unit of consumption good invested ($X_{m,t}$) is turned into one unit of capital plus an increment, which decreases as the amount invested increases, and whose overall size depends on the parameter γ_m (this interpretation holds as long as $X_{m,t}$ is small and γ_m close to one, which will be the case in our calibration).

Following the contributions of Holmstrom and Tirole (1998) and Diamond and Rajan (2001, 2005) we assume long-term projects suffer from a potential liquidity shortage during its first period, thus it only gets completed if an injection of $\rho_{t+1}X_{L,t}$ is done[12]. As entrepreneurs have no income at period 1, this injection is provided by the bank. Entrepreneurs profits are then given by

$$\tilde{\Pi}^{EL}_{t+2} = P_{t+2}q^L_{t+2}\gamma_L \ln(1+X_{L,t}) - R_{t,L}P_tX_{L,t} - \rho_{t+1}X_{L,t}P_{t+1},$$

where $R_{t,L}$ is the interest rate on long term borrowing and q^L_t is the price of long term capital denominated in consumption goods.

Note that although it is natural to think of $\rho > 0$, or potential liquidity shortages at steady state, we could also have liquidity gains with $\rho_t < 0$, for instance due to the increased in the potential for securitization of long term assets in the banks' balance sheets. That would imply instead of liquidity shortages, we would have excess of liquidity, which would imply an inverted yield curve. That might have been the case in the UK during the few years preceding the 2007 crisis. An interesting extension to the model left for future research is to make ρ endogenous based on potential for securitization versus expected share of non-performing assets and consequent need for liquid funds/ provisions.

4.3. Banks

At every period t a bank, representing all financial business in the economy, acquires three types of nominal assets, a short term debt ($P_tX_{S,t}$), a long-term debt ($P_tX_{L,t}$) and equity (Z_t). Furthermore, it has a long-term asset it carries over from last period ($P_{t-1}X_{L,t-1}$). Banks funds these investments with deposits (D_t) from households[13]. Equities are acquired from the intermediate good producers. The investment in equity made at time t, Z_t, pays off a gross dividend at period $t+1$, denoted by DIV_{t+1} (see detail below). Short term entrepreneurs pay back the loan made at time t in period $t+1$, providing a return to the bank of $(R_{S,t}-1)P_tX_{S,t}$, where $R_{S,t}$ is the nominal short term rate. Long term entrepreneurs pay back the loan made at time $t-1$ in period $t+1$, providing a return to the bank of $(R_{L,t-1}-1)P_{t-1}X_{L,t-1}$ where $R_{L,t}$ is the nominal long term rate. Finally, long term entrepreneurs that borrowed at time t may require a liquidity injection at time $t+1$ of $\rho_{t+1}P_{t+1}$ per unit invested. Hence, banks real profits at period $t+1$ are given by

[11] This assumption allows us to have the original Keynesian investment equation dependant on the interest rate.

[12] As in Holmstrom and Tirole (1998), the injection is not added as an input to the production function, being just an additional cost ("money") needed to complete the project.

[13] In this paper we do not consider banking capital requirements.

$$\Pi^B_{t+1} = \frac{1}{P_{t+1}}(DIV_{t+1} + (R_{L,t-1} - 1)P_{t-1}X_{L,t-1} + (R_{S,t} - 1)P_tX_{S,t} - D_t(R_{t,CB} - 1) - \rho_{t+1}X_{L,t}P_{t+1}).$$

We assume banks are risk averse. The only risk involved in the banking business in our model is the maturity transformation risk since banks must commit to lend money to long term investment opportunities having to acquire funds next period to re-finance this balance sheet commitment plus any additional liquidity injection needed. Risk aversion here implies that banks do not only care about the return on short and long term assets, requiring them simply to pay the same expected return on average. Banks will weight these returns according to the expected profitability of the overall portfolio, requiring higher return when that is low and lower when that is high. Effectively, the bank will care about the covariance between the return of each asset and the return of the overall portfolio.

Note that even if banks were in fact risk neutral, the limits on Value-at-Risk (VaR) banks normally abide to would effectively imply that overall profitability of assets would influence the banks required premium to bear maturity risk through the VaR constraint (as discussed in section 2). Hence, the assumption that banks are risk averse reflects the fact that some measure of overall riskiness and expected profitability affects banks long term rate setting decision or the premium they required for bearing maturity transformation risk. The bank maximization profit problem is given by

$$\begin{aligned} \max_{\{X_{S,t},X_{L,t}\}_{t=0}^{\infty}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \frac{{\Pi^B_t}^{1-\sigma_b}}{1-\sigma_b} \\ s.t. \quad & D_t = P_tX_{S,t} + P_tX_{L,t} + Z_t + P_{t-1}X_{L,t-1}, \end{aligned}$$

where σ_b controls the degree of risk aversion.

4.4. Firms

The final good representative firm combines a continuum of intermediate inputs $i \in [0,1]$ with the following production function

$$Y_t = \left[\int_0^1 y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}}.$$

As standard this implies a demand function given by

$$y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\varepsilon} Y_t, \tag{8}$$

where the aggregate price level is

$$P_t = \left[\int_0^1 P_{i,t}^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}.$$

The intermediate sector is constituted of a continuum of firms $i \in [0,1]$ producing differentiated inputs with the following constant returns to scale production function

$$Y_i = A_t {K_{i,t}^S}^{\alpha\zeta} {K_{i,t}^L}^{\alpha(1-\zeta)} H_{i,t}^{1-\alpha}, \tag{9}$$

where A_t denotes the productivity at time t, $K_{i,t}^S$ is the capital stock originated from short term projects, $K_{i,t}^L$ is the capital stock originated from long term projects, and $H_{i,t}$ is the household composite labour used in production. Each firm hires labour and invests in both stocks of capital. Implicit here is the assumption that short term and long term capital are not perfect substitutes, which reflect the fact that long-term projects might have a distinct technological enhancement compared to capital based on short-run investments.

To characterize the problem of intermediate firms, we split their decision into a pricing decision given their real marginal cost and the production decision to minimize costs. Following the standard Calvo pricing scheme (ω), firm i, when allowed, sets prices $P_{i,t}$ according to

$$\max_{P_{i,t}} E_t \left\{ \sum_{s=0}^{\infty} P_{t+s} Q_{t,t+s} \omega^s Y_{i,t+s} \left[\frac{P_{i,t}}{P_{t+s}} - \Lambda_{t+s,i} \right] \right\},$$

subject to the demand function (8), where $Q_{t,t+s}$ is the economy's stochastic discount factor, defined in the next section and $\Lambda_{t+s,i}$ is the firm's i real marginal cost at time $t+s$. To obtain the real marginal cost, we need to solve the firm's intertemporal cost minimization problem. That is

$$\min_{K_{i,t+1}^S, K_{i,t+1}^L, H_{i,t}} E_t \left\{ \sum_{t=0}^{\infty} Q_{0,t} \left(W_t H_{i,t} + P_t q_t^S I_{i,t}^S + P_t q_t^L I_{i,t}^L \right) \right\},$$

subject to the production function (9) and investment equation $I_{i,t}^m = K_{i,t+1}^m - (1-\delta) K_{i,t}^m$ for $m = S, L$.[14]

Finally, dividends[15], which are paid one period after production takes place, are given by

$$DIV_{i,t+1} = P_{i,t} Y_{i,t} - W_t H_{i,t} - P_t (q_t^S I_{i,t}^S + q_t^L I_{i,t}^L) + P_t (q_t^S K_{t+1}^S + q_t^L K_{t+1}^L) - Z_t,$$

where $Z_t = P_{t-1}(q_{t-1}^S K_t^S + q_{t-1}^L K_t^L)$. The first three terms comprise the profits (flow) and the last two the capital gains (due to changes in amount and price of capital held in the firm). Note that K_t^S is the capital used in production at time t, which is set at time $t-1$ and hence Z_t is the price of buying all the capital of the firm at the beginning of period t before production and investment in (new) capital takes place.

4.5. Market Clearing Conditions

The capital market clearing conditions are given by

[14] Note that the demand for each type of labour stated in the household wage setting problem can be obtained by minimizing the total cost of labour $\int_0^j W(j) H_{i,j,t} dj$ subject to the labour composite index.

[15] Dividends here are in fact profits plus capital gains and represent the gross return on equity.

$$I_t^S = yk_t^S = \gamma_S \ln(1 + X_{S,t-1}), \text{ and} \quad (10)$$
$$I_t^L = yk_t^L = \gamma_L \ln(1 + X_{L,t-2}). \quad (11)$$

The good market clearing condition, or the aggregate demand, is given by

$$Y_t = C_t + X_{S,t} + X_{L,t} + \rho_t X_{L,t-1}. \quad (12)$$

Furthermore, capital and labour markets across firms are aggregated such that

$$K_t^m = \int_0^1 K_{i,t}^m di \text{ for } m = S, L \text{ and } H_t = \int_0^1 H_{i,t} di.$$

Finally, the credit market clearing condition is

$$\frac{D_t}{P_t} = X_{S,t} + X_{L,t} + \frac{Z_t}{P_t} + \frac{X_{L,t-1}}{\pi_t},$$

where $\frac{Z_t}{P_t} = \frac{(q_{t-1}^S K_t^S + q_{t-1}^L K_t^L)}{\pi_t}$.

4.6. Equilibrium and Calibration

The equilibrium of the economy is defined as the Lagrange multiplier $\{\Lambda_t\}$, the allocation set $\{C_t, H_t, K_{t+1}^S, K_{t+1}^L, X_{S,t}, X_{L,t}, Y_t, D_t, I_t^S, I_t^L, DIV_t, \Pi_t^B\}$, and the vector of prices $\{P_{i,t}$, π_t, w_t, $w_{j,t}$, $R_{L,t}$, $R_{t,CB}$, q_t^S,q_t^L, $R_{s,t}$, $tp_t\}$ such that the household, the final good firm, intermediate firms, the entrepreneurs and banks maximization problems are solved, and the market clearing conditions hold.

The household maximization routines yield the following equilibrium conditions

$$\beta E_t \left(\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}} \right) = \frac{C_t^{-\sigma}}{R_{t,CB}} \quad (13)$$

and

$$w_{j,t} = \frac{\varepsilon_w}{\varepsilon_w - 1} \frac{E_t \left\{ \sum_{s=0}^{\infty} \frac{C_{t+s}^{-\sigma}}{C_t^{-\sigma}} (\omega_w \beta)^s \frac{\chi H_{t+s}^{\eta}}{C_{t+s}^{-\sigma}} H_{t+s} \right\}}{E_t \left\{ \sum_{s=0}^{\infty} \frac{C_{t+s}^{-\sigma}}{C_t^{-\sigma}} (\omega_w \beta)^s H_{t+s} \left(\prod_{k=1}^{s} \pi_{t+k} \right)^{-1} \right\}}. \quad (14)$$

This equation can be conveniently expressed in recursive form as such

$$0 = f_{1,t}^w \frac{\varepsilon_w}{\varepsilon_w - 1} - f_{2,t}^w w_{j,t},$$
$$f_{1,t}^w = H_t \frac{\chi H_t^{\eta}}{C_t^{-\sigma}} + E_t \left[\beta \omega_w \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} f_{1,t+1}^w \right],$$
$$f_{2,t}^w = H_t + E_t \left[\beta \omega_w \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \pi_{t+1}^{-1} f_{2,t+1}^w \right],$$

where, $w_{j,t} = W_{j,t}/P_t$ and $w_t = W_t/P_t$, is given by

$$w_t^{1-\varepsilon_w} = (1-\omega_w) w_{j,t}^{1-\varepsilon_w} + \omega_w w_{t-1}^{\varepsilon_w - 1}. \tag{15}$$

We assume firms discount future payoffs using the households stochastic discount factor given by

$$Q_{t,t+1} = \beta E_t \left(\frac{C_{t+1}^{-\sigma}}{\pi_{t+1} C_t^{-\sigma}} \right) = \frac{1}{R_{CB,t}}.$$

Given that the purpose of our analysis is not to look at the effects of firm-specific capital we assume that there exists a capital market within firms. That way all firms will have the same labour-capital ratio and $\Lambda_{t,i} = \Lambda_t$ for all i, as in the case where a capital rental market is available. The net aggregate investment in (new) capital is then acquired from entrepreneurs. Note that, as shown by Sveen and Weinke (2007), the relevant difference of considering firm-specific capital is that the parameter on the marginal cost in the Phillips curve would be lower, increasing effective price stickiness. Our results are not qualitatively affected by this change.

Based on that $p_{i,t}$ is determined by solving the price setting maximization, substituting for the stochastic discount factor and using $\Lambda_{t+s,i} = \Lambda_{t+s}$. That gives

$$p_{i,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t \left\{ \sum_{s=0}^{\infty} \frac{C_{t+s}^{-\sigma}}{C_t^{-\sigma}} (\omega\beta)^s \Lambda_{t+s} Y_{t+s} \left(\prod_{k=1}^{s} \pi_{t+k} \right)^{\varepsilon} \right\}}{E_t \left\{ \sum_{s=0}^{\infty} \frac{C_{t+s}^{-\sigma}}{C_t^{-\sigma}} (\omega\beta)^s Y_{t+s} \left(\prod_{k=1}^{s} \pi_{t+k} \right)^{\varepsilon - 1} \right\}}. \tag{16}$$

The recursive formulation is given by

$$\begin{aligned}
0 &= f_{1,t} \frac{\varepsilon}{\varepsilon - 1} - f_{2,t} p_{i,t}, \\
f_{1,t} &= Y_t \Lambda_t + E_t \left[\beta\omega \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \pi_{t+1}^{\varepsilon} f_{1,t+1} \right], \\
f_{2,t} &= Y_t + E_t \left[\beta\omega \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \pi_{t+1}^{\varepsilon - 1} f_{2,t+1} \right],
\end{aligned}$$

where, $p_{i,t} = P_{i,t}/P_t$ and $\pi_t = P_t/P_{t-1}$, is given by

$$1 = (1-\omega) p_{i,t}^{1-\varepsilon} + \omega \pi_t^{\varepsilon - 1}. \tag{17}$$

From the firm cost minimization problem we obtain the demand for capital and labour. After rearranging the first order conditions and substituting for the stochastic discount factor $Q_{t,t+1}$, we obtain the following equilibrium conditions[16]

[16]Once again we have used the fact that marginal costs are the same across firms.

$$
\begin{aligned}
Y_t &= A_t K_t^{S\zeta\alpha} K_t^{L(1-\zeta)\alpha} H_t^{1-\alpha}, && (18)\\
\Lambda_t &= \frac{w_t H_t}{Y_t(1-\alpha)}, && (19)\\
q_t^S &= \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{\pi_{t+1} C_t^{-\sigma}} \left[\Lambda_{t+1} \frac{\alpha\zeta Y_{t+1}}{K_{t+1}^S} + (1-\delta) q_{t+1}^S \right] \right\}, \text{ and} && (20)\\
q_t^L &= \beta E_t \left\{ \frac{C_{t+1}^{-\sigma}}{\pi_{t+1} C_t^{-\sigma}} \left[\Lambda_{t+1} \frac{\alpha(1-\zeta) Y_{t+1}}{K_{t+1}^L} + (1-\delta) q_{t+1}^L \right] \right\}. && (21)
\end{aligned}
$$

Using the capital aggregation conditions, investment evolves according to

$$
\begin{aligned}
I_t^S &= K_{t+1}^S - (1-\delta) K_t^S, \text{ and} && (22)\\
I_t^L &= K_{t+1}^L - (1-\delta) K_t^L. && (23)
\end{aligned}
$$

From the entrepreneurs maximization problems we obtain

$$
\begin{aligned}
X_{S,t} &= \frac{\gamma_S E_t[q_{t+1}^S \pi_{t+1}]}{R_{S,t}} - 1, \text{ and} && (24)\\
X_{L,t} &= \frac{\gamma_L E_t[q_{t+2}^L \pi_{t+1}\pi_{t+2}]}{R_{L,t} + E_t[\rho_{t+1}\pi_{t+1}]} - 1. && (25)
\end{aligned}
$$

Finally, from the bank maximization problem we have that

$$
\begin{aligned}
R_{S,t} &= R_{t,CB}, \text{ and} && (26)\\
E_t \left[{\Pi_{t+1}^B}^{-\sigma_B} \left(\frac{(R_{t,CB}-1)}{\pi_{t+1}} + \rho_{t+1} \right) \right] &= \beta E_t \left[\frac{1}{\pi_{t+1}\pi_{t+2}} {\Pi_{t+2}^B}^{-\sigma_B} (R_{L,t} - R_{t+1,CB}) \right], && (27)
\end{aligned}
$$

where

$$
\begin{aligned}
\Pi_t^B &= \frac{DIV_t}{P_t} + \frac{(R_{t-2,L}-1)}{\pi_t \pi_{t-1}} X_{L,t-2} + (R_{t-1,CB}-1) \left(X_{S,t-1} - \frac{D_{t-1}}{P_{t-1}} \right) \frac{1}{\pi_t} - \rho_t X_{L,t-1}, && (28)\\
\frac{DIV_t}{P_t}\pi_t &= Y_{t-1} - w_{t-1}H_{t-1} - q_{t-1}^S I_{t-1}^S - q_{t-1}^L I_{t-1}^L \\
&\quad + q_{t-1}^S K_t^S + q_{t-1}^L K_t^L - \frac{q_{t-2}^S K_{t-1}^S + q_{t-2}^L K_{t-1}^L}{\pi_{t-1}}, \text{ and} && (29)\\
\frac{D_t}{P_t} &= X_{S,t} + X_{L,t} + \frac{(q_{t-1}^S K_t^S + q_{t-1}^L K_t^L)}{\pi_t} + \frac{X_{L,t-1}}{\pi_t}. && (30)
\end{aligned}
$$

We define the term spread (annual rate in percentage points) between long and short term rate as

$$
tp = \frac{1}{2} \left((R_{L,t}-1) - (R_{t,CB}-1) - (R_{t+1,CB}-1) \right) 400. \tag{31}
$$

Finally, the central bank sets monetary policy according to

$$\frac{R_{t,CB}}{\bar{R}} = \left[\left(\frac{\pi_t}{\bar{\pi}} \right)^{\varepsilon_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\varepsilon_Y} \right]. \tag{32}$$

Therefore, the recursive equilibrium is determined as the solution to equations (10) - (32).

Details of the steady state of the model are shown in the Appendix. Note that the steady state long term rate is given by $R_L = \frac{1}{\beta^2} + \frac{\rho}{\beta}$ and thus depends on the liquidity shortage at steady state (ρ). Before discussing the main results we quickly present the main parameter values used. As standard we set the goods market mark-up to 20%, thus $\varepsilon = 6$. The labour market mark-up[17] is set to 7.5% or $\varepsilon^w = 14$. As standard the discount factor is $\beta = 0.99$, the intertemporal elasticity of substitution in consumption, $\sigma = 1$; and the Frisch elasticity of labour supply $\eta = 1$. The Calvo price and wage parameters[18] are $\omega = 0.5$ and $\omega^w = 0.6$. The depreciation rate is set to $\delta = 0.05$, the share of capital in production to $\alpha = 0.36$, and the share of short run capital to $\zeta = 0.4$. That ensures that at the steady state the share of long term loans in total loans are 60%. Fan, Titman, and Twite (2010) report that the debt maturity ratio, (that is, long term interest bearing debt over total debt) is about 80% in the US, 60% in the UK, 55% in Germany and 40% in Japan during the period 1991-2006. They found that the median long term debt ratio across 39 different countries is estimated to be around 60%. We set the degree of risk aversion of banks to $\sigma^B = 1$, which is the same as the one for the household and we set $\bar{\rho} = 0.0025$, such that the 10 year term premium is roughly 100 basis points matching the US data (Rudebusch and Swanson (2008b)). We initially assume that the Central Bank follows a simple Taylor Rule with inflation parameter $\varepsilon_\pi = 2.5$ and output gap parameter $\varepsilon_Y = 0.125$. Note that higher values of ε_Y and lower values of ε_π easily lead to indeterminacy issues in models with cost channels (See Aksoy, Basso, and Coto-Martinez (2009)). Finally, we set the persistence of technology shocks $\rho^A = 0.9$ and the persistence of liquidity shocks $\rho^L = 0.9$, while setting their standard deviation to $v_a = 0.01$ and $v_l = 0.0001$, respectively. The model is solved to a third order approximation using Dynare++.

5. Term Spreads and Economic Activity

In this section we analyze the mechanism that drives term spread fluctuations in our model, look at their effects on economic activity and the link between spread movements and future output growth.

5.1. Endogenous Term Spreads

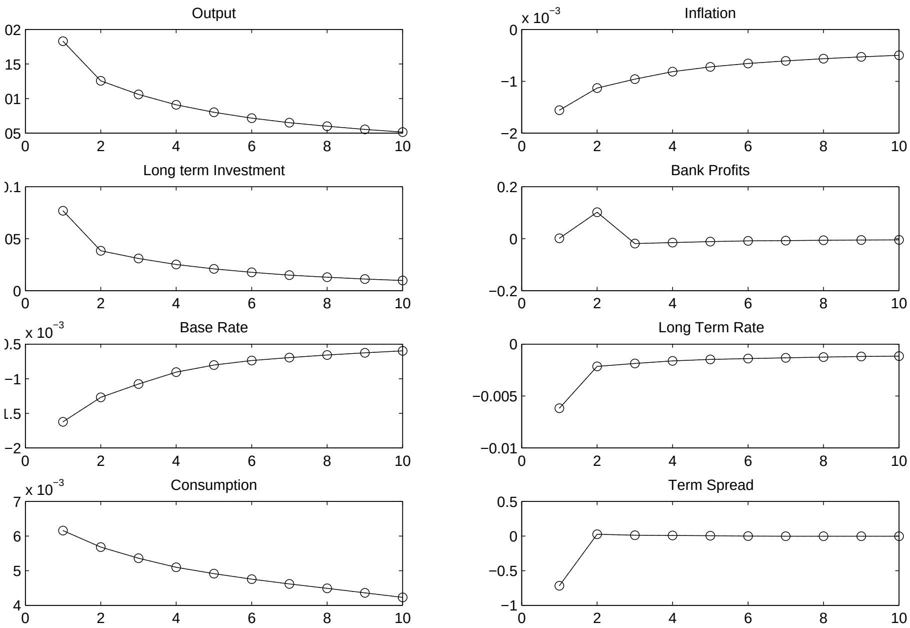
We start by focusing on the benchmark model and the fluctuation of term spreads after a positive technology shock. Figure 2 shows the impulse responses of the main variables of interest. For all variables the percentage deviation from steady state is shown apart from for term spreads

[17] While some contributions to the DSGE literature set $\varepsilon^w = 21$ others set $\varepsilon^w = 2$. Our results are unchanged when we vary ε^w within this range.

[18] These are a bit smaller than the ones obtained in DSGE-based Bayesian estimations. However, all these studies have assumed wage and price indexation decreasing the effect of nominal rigidity on economic activity, while here for simplicity we do not. Our results are unchanged when higher degrees of price and wage rigidity are assumed.

movements where the change in the percentage rate is reported (thus a 0.5 deviation implies a 50 basis point change in term spreads).

Figure 2: Benchmark - Productivity Shocks



As expected, a one standard deviation positive technology shock leads to an increase in output, a decrease in inflation, higher consumption and long term investment in the benchmark model with steady state liquidity shortages ($\bar{\rho} > 0$). The term spread evolves counter-cyclically[19], while both long term and the base rate decline. The decline in the long rate is about 3 times larger than the base rate leading to an overall decline in term spreads.

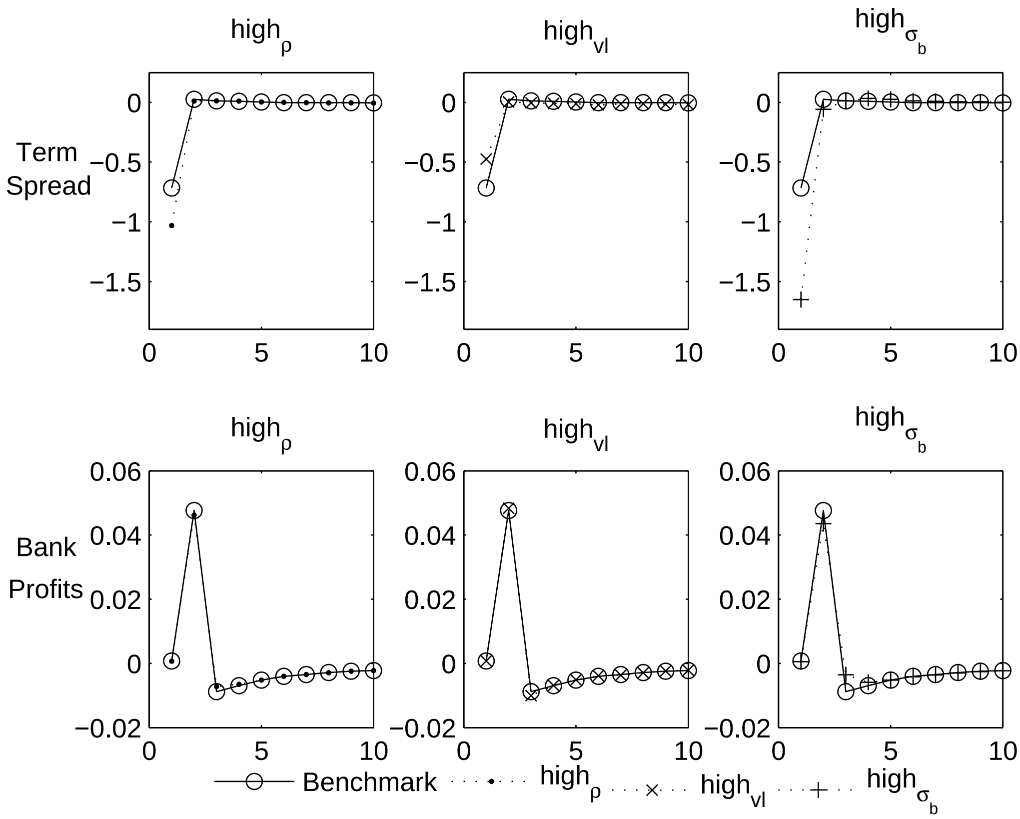
This is mainly due to the response of bank profits after the shock. Banks set a term premium according to the potential liquidity shortage they may face and the fluctuations of profits. In periods of increasing profits banks are less likely to suffer balance sheet problems from liquidity shortages in long term projects and hence bearing maturity risk becomes relatively cheaper such that long term rates fall significantly resulting in lower term spreads. When profits are decreasing the opposite occurs. Note that even though the economy is still subject to persistent liquidity shortages on long term investment funding, these shortages occur when banks have higher expected profits and hence we observe an expansion of long term credit supply. Our empirical results highlight exactly that relationship between spreads and financial sector profitability. Higher term spreads indicate higher probability of lower future profits.

In order to further investigate the drivers of the endogenous movements in term spreads after productivity shocks we run three variants of our benchmark model. In the first variant we set a

[19]The correlation between output and spread is negative when we simulate the model with random persistent shocks.

higher steady state liquidity shortage ($\bar{\rho} = 0.0175$), and denote it $high_{\rho}$. In the second we set a high variance of liquidity shock ($v_l = 0.04$), denoting it $high_{vl}$ and finally, in the third, we assume banks are more risk-averse setting ($\sigma_B = 3$), denoting it $high_{\sigma_b}$. Figure 3 shows the results.

Figure 3: Fluctuations in Term Spreads



Firstly, note that, in all three cases bank profits move in a very similar way, hence the key to distinguish the three cases is to uncover how the same response in profits lead to different dynamics of term spreads. In the first variant we observe that spread movements are amplified under higher steady state liquidity shortages. The main intuition for this result is as follows. Initially, banks set long term rates higher than short term rates to offset potential liquidity shortages and hence, higher ρ at the steady state or higher the average need for liquidity injection, higher will the steady state long term rates be. Given that long term rate are relatively high, an equivalent increase in bank profits induce a stronger adjustment in long term rates by the banks, which in turn imply long term rates fall by a greater amount than under the benchmark case.

Secondly, the opposite occurs when the variance of the liquidity shock is high. Banks are willing to bear more maturity risk under periods of high profits since they know that high profits can be used to offset liquidity shortages. However, the more volatile are these shortages, the less certain the bank will be that high profits will be enough to offset them. Therefore, an equivalent movement in bank profits leads to dampened or smoother movements in long term rates and term spreads after a productivity shock.

Finally, the third variant illustrates that term spreads are more responsive to productivity shocks the higher the degree of bank risk aversion. σ_B effectively determines how fluctuations of bank profits influence the banks long term rates decision. When $\sigma_B \to 0$ banks will set long term rates to be a discounted sum of short term rates and term spreads will be constant. This mechanism is the same as the one explored in the macro-finance literature where Epstein-Zin preferences are used to increase risk aversion in order to match volatility of risk/term premia (see Rudebusch and Swanson (2008a)).

The second order approximation of the long-term rate decision by the bank, equation (27), derived in the appendix and repeated below can also be used to illustrate the main determinants of term spread movements. As easily verified, setting $\sigma_b = 0$, or assuming banks utility is linear on profits, eliminates all the effects of movements of profits on term spread decisions. Looking at the covariance terms we observe that higher the covariance between profits and the liquidity shortage (first term), lower term spreads will be, with the strength of the effect being positively associated with σ_b and $\bar{\rho}$. Hence, as we increase $\bar{\rho}$, movements in spreads are amplified. Finally, the increase in the variance of the liquidity shock v_l has two opposing effects. Firstly, it increases spreads since $(\widehat{\rho_{t+1}})^2$ has a positive impact on spreads. Secondly, it becomes a stronger driver of the expected covariance between profits and ρ_{t+1}. That implies that the positive movement in profits due to the productivity shock will have little effect on the covariance term and one of the key drivers of the endogenous movements of spreads loses its significance. As a result of second effect, higher volatility of liquidity shocks dampens the impact of productivity shocks on term spreads.

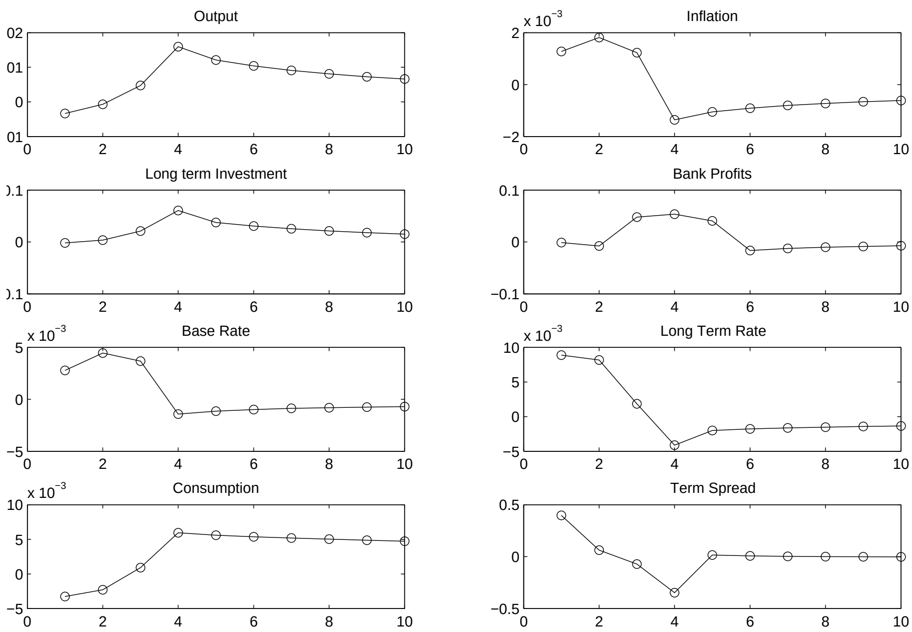
$$
\begin{aligned}
E_t\left[\widehat{tp}+0.5\left(\widehat{tp}\right)^2\right] &= E_t\left[\sigma^b\left(\widehat{\Pi^B_{t+2}}-\widehat{\Pi^B_{t+1}}\right)+0.5\sigma^b\left(\left(\widehat{\Pi^B_{t+2}}\right)^2-\left(\widehat{\Pi^B_{t+1}}\right)^2\right)+\widehat{\pi_{t+2}}-0.5\left(\widehat{\pi_{t+2}}\right)^2+\tfrac{\bar{\rho}}{\Gamma}\left(\widehat{\rho_{t+1}}+0.5\left(\widehat{\rho_{t+1}}\right)^2\right)+CovTerms\right] \\
CovTerms &= -\sigma^b\widehat{\Pi^B_{t+1}}\tfrac{\bar{\rho}}{\Gamma}\widehat{\rho_{t+1}}-\sigma^b\widehat{\Pi^B_{t+1}}\tfrac{1}{\beta\Gamma}\widehat{R_{CB,t}}+\sigma^b\widehat{\Pi^B_{t+1}}\widehat{\pi_{t+1}}-\left(\tfrac{1}{\beta\Gamma}\widehat{R_{CB,t}}+\tfrac{\rho}{\Gamma}\widehat{\rho_{t+1}}\right)\widehat{\pi_{t+1}} \\
&\quad +\sigma^b\widehat{\Pi^B_{t+2}}\left(\frac{\frac{1}{\beta}+\bar{\rho}}{\Gamma}\widehat{R_{L,t}}-\tfrac{1}{\beta\Gamma}\widehat{R_{CB,t+1}}-\widehat{\pi_{t+1}}-\widehat{\pi_{t+2}}\right)+\left(\frac{\frac{1}{\beta}+\bar{\rho}}{\Gamma}\widehat{R_{L,t}}-\tfrac{1}{\beta\Gamma}\widehat{R_{CB,t+1}}\right)(\widehat{\pi_{t+1}}+\widehat{\pi_{t+2}})-\widehat{\pi_{t+2}}\widehat{\pi_{t+1}}
\end{aligned}
$$

5.2. Yield Spreads and Output Growth

As reported by Rudebusch, Sack, and Swanson (2007), the Congressional Budget Office output gap and the 10Y term premium during the period 1960 and 2005 seem to be negatively related (see Figure 6 in their paper). Hence, the counter-cyclical movement of spreads obtained in our model after both a productivity and a monetary shocks (not reported here) match the overall characteristic of the US data. As stressed by Gürkaynak and Wright (2010), term structure models should generate a high slope of the yield curve at the beginning of recoveries from recessions, feature which is related to the predictive power of yield spreads. Hamilton and Kim (2002) conclude that lower term premiums predict slower GDP growth, although this effect appear to be strong only in the short-run, while Wright (2006) shows that lower term premium raises the odds of a recession.

In order to verify if the dynamics of term spreads in our model is consistent with this feature we study the impact of a three period anticipated technology shock (Figure 4). That way, based on the information at time t banks form an expectation of future growth and profits which will affect long term rates and thus term spreads. These will feed back to the economy influencing long term investment and output. We observe that output and long term investment increase from t until $t+3$ (time of the realization of the productivity shock). Therefore, if an econometrician is looking at output gains $\widehat{y}_{t+3} - \widehat{y}_t$ and regressing on $\widehat{tp}_t$ (a variant of Hamilton and Kim (2002) estimation), obtaining a positive estimate, we must have that $\widehat{tp}_t > 0$, which is what we obtain. The main driver of this result is the future path of bank profits. Bank profits will initially decrease, making it more costly to bear maturity risk, and hence long term rates and spreads increase. Spreads are at the highest when output is at its lowest and expected to increase in the future. As we approach $t+3$, bank profits will be increasing and spreads decreasing given that the premium banks charge to bear maturity risk is lower when banks portfolio have higher returns. That leads to increasing long term capital investment and consumption. At $t+3$, when productivity is at its peak, output is at its highest and spreads at their lowest point. Hence, high slope of yield curve indicates future output is increasing while a flat yield curve indicates output is at its peak.

Figure 4: Anticipated Productivity Shock

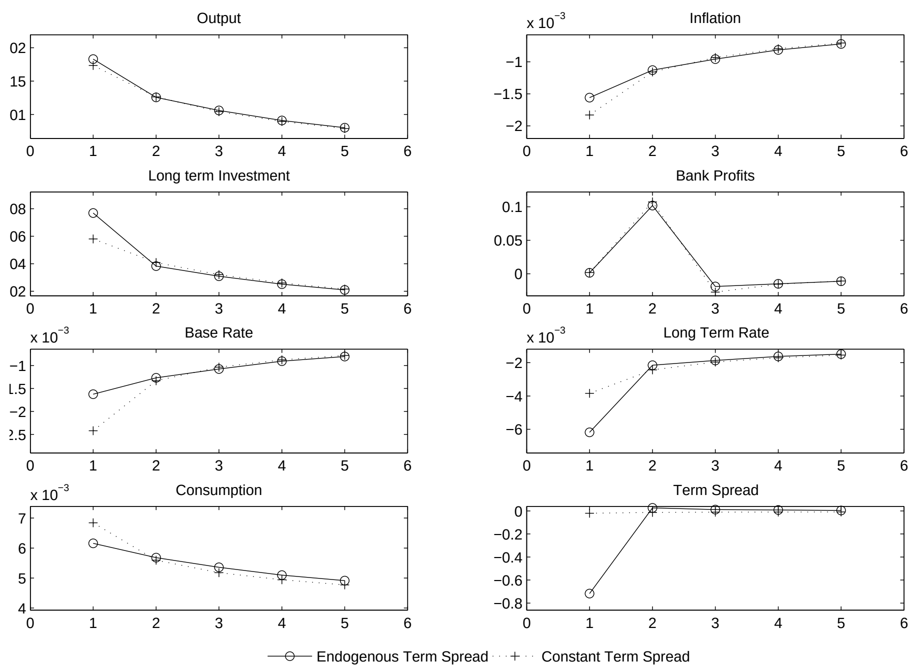


Note that as Rudebusch, Sack, and Swanson (2007) discuss, a declining term premium would suggest additional stimulus to the economy (simple Keynesian view) and hence one could expect the opposite results than the one observed here and obtained by the literature cited above and in their paper. As opposed to the macro-finance literature where the yield curve is build based on the stochastic discount factor, the term premium here has a direct effect on long term investment and output and hence this mechanism is in place. As the term premium falls, long term investment increases and output also increases, hence a regression of output differences $\widehat{y}_{t+1} - \widehat{y}_t$ on spread differences $\widehat{tp}_t - \widehat{tp}_{t-1}$ results in a negative parameter being obtained, as suggested by the last specification estimated in Rudebusch, Sack, and Swanson (2007). However, at the time the anticipated shock is known, period t, bank profits are expected to remain low for the next two periods forcing banks to initially charge more for long term commitments. Thus, term spreads are high but decreasing.

5.3. Term Spreads and Macroeconomic Dynamics

As we mentioned earlier, in the DSGE asset-pricing models, which are now standard in the structural model of the yield curve literature (e.g. Rudebusch and Swanson (2008b)), current output is determined by the expected path of short term rates, and hence, term premia or long term rates have no effect on economic activity. In our model long term rates are determined by banks, being a function of expected future short term rates and potential liquidity shortages in banks' balance sheets, and most importantly, they affect firm's long term capital investment decisions. That way, we purposely open a channel through which term premia fluctuations affect economic activity. The effect of this endogenous fluctuations of term spreads on output in our model can be highlighted by looking at the impulse responses after a productivity shock with $\sigma_b = 0$, thus spreads are constant, plotted against the benchmark case. Figure 5 shows the results.

Figure 5: Endogenous vs Constant Term Spreads



We find that countercyclical term spread movements lead to an amplification of output responses after a productivity shock, although that seems fairly small comparing to the movement in spreads and long term rates. The main reason for that is the shift in the composition of output. While after a sharp decrease in long term rates, long term investment increases significantly relative to the case with constant spreads, the response of consumption moves in the opposite direction[20]. Given that long term rates have decreased significantly, the base rate set by the Central Bank does not need to fall as much. As a result, the demand channel is dampened and consumption and inflation do not move as much as in the constant spread case. On the other hand, when spreads are constant, the Central Bank moves the base rate more aggressively, pushing consumption up and leading to higher inflation deviations. In our model consumption is determined by the standard Euler equation dependent on the short-term (base) rate. If part of the consumption is financed by long-term borrowing (durable consumption) then the endogenous movements in term spreads would have a much stronger effect on output given that both consumption and investment would expand further relative to the constant spreads case. Moreover, monetary policy would be forced to be considerably less aggressive to control output and inflation volatility.

Finally, in all the analysis so far we kept ρ constant. While we presented the model we have focused on potential liquidity shortages affecting banks' balance sheets, setting the steady state value of ρ to be positive, since the slope of the yield curve is positive on average. In section 7.2. we will use the model to replicate a feature of the recent crisis when banks faced significant liquidity shortages to study the impact of different policies interventions. However, there could be periods of increased liquidity in the banking sector when ρ actually decreases, leading to lower long-term rates and narrowing term spreads and consequently expanding economic activity. That could be a potential description of the US and UK economies during the period 2003 - 2007 where long term rates fell significantly (the yield curve in the UK actually inverted). De Graeve, Emiris, and Wouters (2009) decompose this fall in long term rates in the US and shows that its main drivers were declining term spreads. During the same period we observed a boom in securitization or development of structured finance activities. These activities actually meant that long-term commitment/assets could be re-packaged and sold; increasing profits through fees or advantageous balance sheet operations. Effectively, banks found themselves operating in a market in which bearing maturity transformation was relatively cheap, or ρ was significantly smaller or even negative, bringing term spreads down.

In fact, Benati and Goodhart (2008) show that during the post-war period the marginal predictive content of spreads increased during periods where current (and future) monetary policy regimes were uncertain. They conjecture that the additional predictive power of the yield curve was due to higher long term rates, which reflected this uncertainty, depressing output. On the other hand, the same authors also observe that during the early 2000 the marginal predictive content of spreads also increased while monetary regimes had been successfully established. They then conjecture that external forces (to monetary policy) were holding down long-term yields, relating the predictive content of spreads to the real yield curve which reflects structural conditions of the economy. The fluctuations of parameter ρ given the conditions in the banking sector can be one potential avenue to explain some of these external factors.

[20]Note that including variable capital utilization might dampen these movements in consumption.

6. Yield Curve and Volatility

In our basic model, long term funding is made for two periods while short term funding is for one period. In order to compare term spreads in our model to the data, we ought to consider longer maturities. One of the main challenges is that in our framework, due to the liquidity premium, the expectations hypothesis does not hold (see Dewachter, Iania, and Lyrio (2011) for new evidence on the rejection of the expectation hypothesis in term structure models).

In order to obtain the zero coupon rate for a three period bond we need the zero coupon rate for two periods $R_{L,t}$ and the forward rate between period $t+2$ and $t+3$, denominated ${}_{t+2}f_{t+3}$. There are two possible ways, one is to use $R_{S,t+2}$, or the short term rate of period $t+2$, such that it is equivalent to invest in an three period zero bond or first investing in a two period bond and then in a one period asset. However, that implies that the liquidity risk will only affect the investment from period 1 to 2. The alternative is to obtain what would be the premium from investing in long term maturity assets from $t+2$ till $t+3$. That is setting ${}_{t+2}f_{t+3} = \frac{R_{L,t+1}}{R_{S,t+1}}$ (note that the rate $R_{L,t+1}$ is for holding a long term asset from period $t+1$ until $t+3$). We will set forward rates following the latter and hence ${}_{t+n}f_{t+n+1} = \frac{R_{L,t+n-1}}{R_{S,t+n-1}}$, such that the future liquidity risk impacts the entire yield curve. We will therefore build the yield curve obtaining the forward rates using the forward looking long term rates against the short term rates at each period they are set. Let the price of a n quarters zero coupon bond be $E_t[p_t^{(n)}]$. Let the risk neutral and liquidity risk free zero coupon bond be $E_t[\hat{p}_t^{(n)}]$. The pricing of each of these assets are given by

$$E_t[p_t^{(n)}] = \exp\left(-(R_{L,t}-1) - \sum_{i=2}^{n-1}({}_{t+i}f_{t+i+1}-1)\right), \text{ and} \tag{33}$$

$$E_t[\hat{p}_t^{(n)}] = \exp\left(-\sum_{i=0}^{n-1}(R_{S,t+i}-1)\right). \tag{34}$$

The term premium will therefore be

$$tp_t^{(n)} = \frac{1}{n}\left(\ln(\hat{p}_t^{(n)}) - \ln(p_t^{(n)})\right) = (R_{L,t}-1) + \sum_{i=2}^{n-1}({}_{t+i}f_{t+i+1}-1) - \sum_{i=0}^{n-1}(R_{S,t+i}-1).$$

Note that the standard measure used in the macro-finance literature (see Rudebusch and Swanson (2008b)) is effectively given by

$$\tilde{tp}_t^{(n)} = \sum_{i=0}^{n-1}\left(\frac{1}{m_{t+i}}-1\right) - \sum_{i=0}^{n-1}(R_{S,t+i}-1),$$

where m_t is the stochastic discount factor obtained from the household Euler equation. As Rudebusch and Swanson (2008b) stress this measure invariably generates term spreads movements that do not match the data unless one assumes shocks with high standard deviations, which in turn worsens the model's ability to match standard business cycle facts.

We start by comparing the volatility of the 1 and 2 years term spreads of this standard measure and the one in our model. We then compare the 5 years term spreads obtained in our model

Table 3: Volatility of Term Premium

	Banking Channel against Macro-Finance			
	Term Premium - tp	MF Premium - $\tilde{tp}$		
1Y	0.3868	0.00042		
2Y	0.1884	0.0053		
	Banking Channel against the Data			
	Data (1990 - 2011)	Benchmark	High σ_b	$RS_{Baseline}$
5Y	0.657	0.0745	0.1938	0.0013

against the data (collected from Kim and Wright (2005)) and the volatility obtained by the baseline model[21] of Rudebusch and Swanson (2008b) (denoted $RS_{Baseline}$). We simulate our model without liquidity shocks, thus all variability in spreads are due to endogenous movements after productivity and monetary policy shocks. We provided the results of two simulations, one with the benchmark parameters and one with a higher bank risk aversion parameter $\sigma_b = 3$. Table 3 shows the results (0.5 denotes 50 basis points).

Firstly, as clearly seen the banking balance sheet/profitability channel explored in our model is able to generate considerably more volatile term spreads than the standard macro-finance measure. This occurs despite the fact that we kept the standard deviation of shocks to be in the order of 1% and the risk aversion parameter (benchmark) to be equal to 1, lower than the one used by Rudebusch and Swanson (2008b). Bank profits are considerably more volatile than consumption, delivering greater volatility of term spreads for the same degree of risk aversion and variance of exogenous shocks. Finally, when we compare our benchmark case with the data we come closer than the standard macro-finance model. One important feature of the data, which is not replicated in our benchmark model, is the fact that both the mean and the standard deviation of term spreads tend to increase with maturity. In our model, while the mean increases, the standard deviation decreases. This is because we introduce only two securities in the banks' balance sheet, a one period and a two periods security, hence a movement in spreads today affect the short end of the curve more heavily than the long end. Banks would have a variety of securities with different maturity and risk profiles, including assets bearing long term inflation risks. Extending the model to consider a richer bank portfolio and including long term inflation risks may be, therefore, fruitful areas for further research.

Rudebusch and Swanson (2008a) have extended the standard macro-finance model to include long-term inflation shocks and adopt Epstein-Zin preferences to break the link between intertemporal elasticity of substitution and coefficient of risk aversion. They are able to deliver volatile term spreads and match the dynamics of the main macroeconomic variables. We see the bank balance sheet channel explored here as a potential complement to variations in term spreads due to inflation risk premia. In fact, given that banks would be exposed to inflation risk while bearing maturity risk, a potential long term inflation shock would also generate increased volatility of term spreads in our model without the need of increasing the degree of risk aversion.

[21] We use their code to calculate the 5Y term spread volatility since they only report the 10Y point.

Although acknowledging the importance of long term inflation risk as an important driver of term premium, as discussed by Gürkaynak, Levin, and Swanson (2006) while covering the regime changes in the UK in the last 20 years, there is also evidence that the dynamics of short-run rates and inflation expectations do not explain all the variability of long term rates (De Graeve, Emiris, and Wouters (2009)) or its output predictive power (Benati and Goodhart (2008)). More importantly, as stressed by Gürkaynak and Wright (2010), the US treasury inflation protected securities (TIPS) forward rate dynamics have not been that different than their nominal counterparts (see Figure 5 in their paper), indicating the term premia are also influenced by real factors.

7. Monetary Policy

In this section we firstly look at conventional monetary policies in the presence of endogenous term spreads, focusing on different short term rate policy rules. We then look at unconventional policies during periods of large shocks to liquidity shortages.

7.1. Conventional

Our interest here is to provide an answer to the following question: Should the Central Bank directly change short term rates given the fluctuations in term spreads? This direct adjustment of short term rates could be relevant in our model since, as opposed to the other models of term structure, spreads here feed back to the macroeconomic variables of the model through their effects on long term investment. A similar question is tackled by Aksoy, Basso, and Coto-Martinez (2010) in the case of banking spreads movements. Their finding is that welfare increases when central banks explicitly take spreads into account when setting base rates.

In order to calculate welfare under different policy rules we first obtain a third order approximation solution of the main variables of the model and then simply approximate the unconditional mean of the expected utility function of the household using a standard Monte Carlo Method, thus

$$E_0\Sigma_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\sigma}}{1-\sigma}-\chi\frac{H_t^{1+\eta}}{1+\eta}\right)=\int_{-\infty}^{\infty}\left[\Sigma_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\sigma}}{1-\sigma}-\chi\frac{H_t^{1+\eta}}{1+\eta}\right)\right]f(z^t)dz^t\approx\frac{1}{M}\Sigma_i^M\left[\Sigma_{t=0}^{\infty}\beta^t\left(\frac{C_t^{i\,1-\sigma}}{1-\sigma}-\chi\frac{H_t^{i\,1+\eta}}{1+\eta}\right)\right]$$

where z^t is a series of shocks, $f(z^t)$ the probability distribution of these shocks and x_t^i the realization of the endogenous variables under a specific series of shocks z_i^t for $i = 1$ to M.

We set the monetary rule to be

$$\frac{R_{t,CB}}{\bar{R}}=\left[\left(\frac{\pi_t}{\bar{\pi}}\right)^{\varepsilon\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\varepsilon_Y}\left(\frac{tp_t}{\bar{tp}}\right)^{\varepsilon_{tp}}\right].$$

We observe that setting the parameter $\varepsilon_{tp} > 0.002$, the degree to which the Central Bank moves the base rate given fluctuations of term spreads, causes model indeterminacy (no matter how strongly the Central Bank responds to inflation). Nonetheless, for our benchmark case we observe that setting $\varepsilon_{tp} = 0.001$ in fact leads to higher welfare. This is due to the fact that as monetary policy responds to spread fluctuations it is able to compensate for the distortion created by the liquidity shortages which is assumed to be positive at steady state. When we modify our benchmark specification setting $\bar{\rho} = 0$, hence on average banks no longer face liquidity shortages in long term investments, then setting $\varepsilon_{tp} = 0.001$ does not increase welfare relative to the standard monetary rule where the base rate depends only on output and inflation deviations. Hence, we conclude that although fluctuation of term spreads are important for economic activity, and monetary policy should take these fluctuations into account, the Central Bank should not respond explicitly to term spread movements, unless it wants to correct for steady state distortions. However, note that fluctuations in term spreads do affect output and inflation, and hence, even when relying on a standard monetary policy rule, the Central Bank is taking spread movements implicitly into account (see Figure 5).

7.2. Unconventional

The Federal Reserve Bank (FED) conducted two purchase programmes of long-term Treasuries and other long term bonds, known as QE1 in 2008-2009 and QE2 in 2010-2011. These quantitative easing policies comprised of purchase of mortgage backed securities, Treasuries and "Agencies" from private sector. Krishnamurthy and Vissing-Jorgensen (2011) find that these policies lead to a significant decline in nominal rates on long term safe assets (Treasuries and "Agencies") and only a small effect on, more relevant, less safe assets such as corporate rates and mortgage rates. Their results suggest that the effects of asset purchases on the duration of risk premium are small, while effects on liquidity-safety premium are substantial.

Gagnon, Raskin, Remache, and Sack (2010) analyze the effectiveness of the Large-Scale Assets Purchases conducted by the FED. They find that the purchase programme lead to reductions in long-term interest rates on a range of securities, including some securities that were not included in the purchase programme, indicating that portfolio balancing effects were in play. They argue that the reductions in interest rate primarily reflect lower risk/liquidity premiums rather than lower expectations of future short term rates. Beirne, Dalitz, Ejsing, Grothe, Manganelli, Monar, Sahel, Suec, Tapking, and Vong (2011) report on the effectiveness of the Covered Bond Purchase Programme (CBPP), which started in July 2009 for a period of 12 months in the Eurozone, and show that covered bond yields decreased by 12 basis points, that the programme increased the liquidity of secondary market and that it managed to encourage lending. Overall, a constant theme in these studies is the effect on long term rates through lower term spreads being crucial for the effectiveness of the interventions and allowing the financial market to continue funding economic activity. Finally, Joyce, Lasaosa, Stevens, and Tong (2010) report that the QE interventions in the UK led to a 100 basis point decrease in gilt yields. Given the effects on other asset classes, although the purchase programme has been overwhelmingly of government securities, they also stress the importance of portfolio balancing effects. Borio and Disyatat (2010) provide a survey of different forms of possible unconventional monetary policies and argue that main balance sheet channel operates by central banks ability to reduce yields and ease financing constraints by altering the risk profile of private portfolios.

Two main characteristics of our model are particularly important to try and formalize in a DSGE framework this type of interventions. First, fluctuations in term spreads are a relevant factor in determining output as long term rates influence long term investment decision, and hence, an intervention aimed at lowering long term rates affect economic activity. Second, given term spreads or long-term rate decisions are directly determined by fluctuations in future bank profits or changes in their balance sheets, our model provides a new channel through which the effects occur. An important caveat, which highlights a possible extension of the model for future research, is the fact that our bank portfolios are fairly simple, with only three assets. They do not include, for instance, housing debt/mortgages, thereby restricting the analysis of some of the portfolio balancing effects mentioned.

In order to study the main effects of QE policies in our model economy we first introduce two types of unconventional monetary policies and then analyze their impact after a liquidity shortage shock. The first is a simple liquidity injection (QE_t) to banks financed by a lump-sum tax collected from the households. Liquidity injection, which is costless to the bank, is set such that $QE_t = \xi_t X_{L,t-1} P_{t-1} \rho_t$, where $\xi_t = \phi_\xi \left(\frac{\bar{X}_L}{\bar{X}_S} - \frac{\tilde{X}_{L,t}}{\tilde{X}_{S,t}} \right) \frac{\bar{X}_S}{\bar{X}_L}$, where $\frac{\tilde{X}_{L,t}}{\tilde{X}_{S,t}}$ is the ratio of long to short run funding that would be in place without QE intervention. Hence, the liquidity injection is a proportion ξ_t of the current long term asset exposure of the bank, and its intensity depends on how skewed current investment funding is towards short term relative to long term funding. Note that in our model this relative difference will be a direct function of future liquidity conditions.

The second unconventional policy is the existence of favorable conditions for banks to borrow funds from the Central Bank using their long term asset positions as collateral. Favorable conditions in our context imply a lower rate of borrowing relative to the short term funding currently available for banks. Banks now decide the fraction of long term assets (Θ_t) they want to pledge as collateral to get funds from the Central Bank (CB). Effectively, at time t, banks make a two period investment. At period $t+1$ they sell a portion (Θ_t) of these assets to the CB to get additional funds, promising to buy them back at $t+2$ before they mature. The total cost of Central Bank funding is $\Theta_t X_{L,t-1} P_{t-1} (R_{t,QE} - 1) + \frac{\phi_{re}}{2} \Theta_t^2$, where $R_{t,QE}$ is the borrowing rate. The term $\frac{\phi_{re}}{2} \Theta_t^2$ is included such that the marginal cost of this type of funding is increasing as usage increases. The bank problem now becomes

$$\begin{aligned} \max_{\{X_{S,t}, X_{L,t}, \Theta_t\}_0^t} \quad & E_0 \sum_{t=0}^{\infty} \beta_t \frac{{\Pi_t^B}^{1-\sigma_b}}{1-\sigma_b} \\ s.t. \quad & D_t = P_t X_{S,t} + P_t X_{L,t} + Z_t + P_{t-1} X_{L,t-1} - \Theta_t X_{L,t-1} P_{t-1} \end{aligned}$$

where

$$\begin{aligned} \Pi_{t+1}^B &= \frac{1}{P_{t+1}} (Div_{t+1} + (R_{L,t-1} - 1) P_{t-1} X_{L,t-1} + (R_{S,t} - 1) P_t X_{S,t} - D_t (R_{S,t} - 1) - \\ & \quad -\rho_{t+1} X_{L,t} P_{t+1}) - \Theta_t X_{L,t-1} P_{t-1} (R_{QE,t} - 1) - \frac{\phi_{QE}}{2} \Theta_t^2. \end{aligned}$$

The first order conditions in this case are

$$\begin{aligned} R_{S,t} &= R_{t,CB}, \\ E_t \left[{\Pi_{t+1}^B}^{-\sigma_B} \left(\frac{(R_{S,t} - 1)}{\pi_{t+1}} + \rho_{t+1} \right) \right] &= E_t \left[\frac{\beta {\Pi_{t+2}^B}^{-\sigma_B}}{\pi_{t+1} \pi_{t+2}} (R_{L,t} - R_{t+1,CB} + \Theta_{t+1} (R_{S,t+1} - R_{QE,t+1})) \right], \\ 0 &= {\Pi_{t+1}^B}^{-\sigma_B} \left((R_{S,t} - R_{QE,t}) \frac{X_{L,t-1}}{\pi_t \pi_{t+1}} - \phi_{QE} \Theta_t \right). \end{aligned}$$

Finally, we assume the Central Bank sets $R_{QE,t} = R_{S,t}\left(1-\phi_{re}\left(\frac{\bar{X}_L}{\bar{X}_S}-\frac{\tilde{X}_{L,t}}{\tilde{X}_{S,t}}\right)\frac{\bar{X}_S}{\bar{X}_L}\right)$. Thus, lower the long term funding relative to short term without intervention, more favorable Central Bank funding will be.

Figure 6: QE Policies

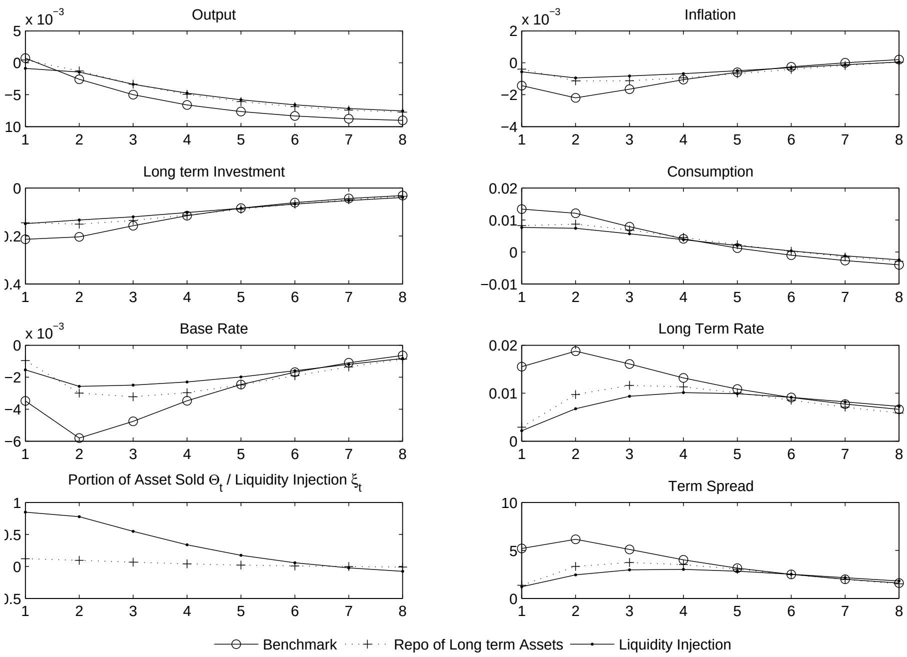
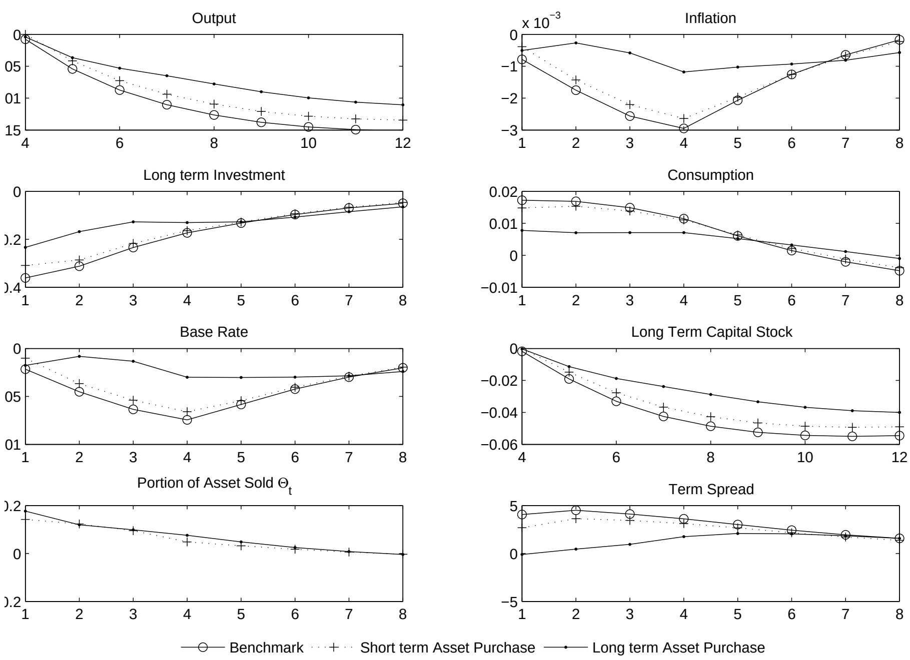


Figure 6 illustrates the results for a liquidity shock of 0.04 with $\phi_\xi = 3$, $\phi_{QE} = 0.35$ and $\phi_{re} = 1$. That translates in the Central Bank covering roughly 90% of the liquidity shortage under the first intervention after the shock or, in the second case, buying roughly 10% of long term assets from the bank's balance sheets (see the bottom left graph in Figure 6). We observe that both QE policies do have a significant impact on long term rates and term spreads. That translates in dampened responses of long term investment and output relative to the case where only conventional monetary policy is used. The main difference between these two interventions is that asset purchases have a stronger initial impact since they immediately free up the balance sheet of banks which are then able to maintain long term funding despite the liquidity shock. Liquidity injections do affect term spreads and eventually help sustain higher output; however, these require a longer period to work through the bank long term rate setting as it protects banks from future liquidity shortages. Note that we allow base rates to move freely in both cases, hence, short term rates do not need to decrease as much while asset purchases are conducted. An important result is that inflation turns out to be significantly higher after QE interventions, thus even if nominal rates are close to 1, QE interventions will lead to lower real rates.

One of the important debates at Central Banks in the UK, the US and the Eurozone is at which point to unwind the large-scale purchases. Although not completely suited to give a definite answer to such a question, we can use our model to verify the effectiveness of short term asset purchase agreements, which sell securities back to banks after one period, and the interventions that allow banks to move long term assets away from balance sheets for longer periods. In order to do that we modify the model such that long term investments now require one year (4 quarters) commitments from firms and hence from banks. The appendix shows the details of the model and of each of the two QE interventions: one period asset purchases and three periods asset purchases. Figure 7 shows the results (impulse responses for output and long term capital stock are shown after the fourth period since that is the point changes to long term investment done at time $t = 1$ start having effects). We set ϕ_{QE} for each of these two interventions such that the portion of long term assets bought by the Central Bank are matched (roughly 20%, see graph at the bottom left corner).

Figure 7: Short versus Long term Asset Purchases



We observe that when the Central Bank holds assets for longer periods the same intervention in terms of assets purchases leads to lower levels of term spreads / long term yields and to a lower decrease in long term investment after a liquidity shock. There is a gain for the Central Bank to hold the securities bought in such interventions for longer periods of time since they are more effective in freeing up the balance sheet of banks, fomenting long term funding. Obviously, these securities remain in the Central Bank balance sheet for longer and thus the monetary authority is taking significantly more risks then when it keeps securities for only one period. Finally, the long holding period interventions allow Central Banks to bring short term rates back to their steady state levels sooner.

8. Conclusions

Term spread fluctuations have important implications for macroeconomic outcomes and predict output growth. Undoubtedly, inflation expectations or more generally long term inflation risks are an important determinant of these fluctuations. However, the observation that nominal and real yield curves move together in many instances suggest that other factors are in play. We propose a model that delivers endogenous variations in term spread driven primarily by changes in banks' expected profitability and their appetite to bear the risk of maturity transformation. We show that fluctuations of the future profitability of banks portfolios affect their ability to cover for any liquidity shortage and hence affect the premium they require to carry maturity risk. Additionally, we also present empirical evidence on the relevance of financial business profits in explaining output dynamics and on the link between yield spreads and expected profitability.

While we present a model in which bank portfolios are fairly simple we are able to match important features of the data. Our model suggests that factors external to monetary policy may contribute not only to the marginal predictive power of spreads but also to the understanding of the linkages between banks, spread movements and the macroeconomy.

Embedding this simple banking sector framework into a DSGE New Keynesian model allows us to analyze the interaction between these spread movements and conventional and unconventional policies. Spread movements effectively imply tighter or looser monetary conditions forcing the Central Bank to adjust short term rates accordingly. Once again, spreads between different interest rates in the economy are shown to be crucial and should be explicitly included in models that analyze optimal policies. Unconventional policies are shown to have a strong impact on spread movements fomenting long term investment and helping reduce output losses after negative liquidity shocks, matching the general view on the effects of recent asset purchases programmes. Finally, we show that asset purchases programmes that keep the assets in the Central Bank balance sheet for longer are more effective in offsetting a liquidity shock and allow the Central Bank to re-store short term rates to steady state levels more quickly. This result indicates that the initial decision of the ECB to hold asset purchased under the CBPP programme until maturity give more strength to this type of intervention.

The present work highlights three areas in which further research may be fruitful. First, increasing the complexity of banks portfolios will provide a better understanding of this important channel, most notably, including (workers) housing investment funded by banks. That would mean term spread fluctuations would not only influence investment but also consumption, potentially amplifying the effects of spread movements, since as we observe, consumption and investment move in opposite directions compensating each other. Moreover, including other long term asset classes may potentially allow us to study portfolio balancing effects after QE interventions. Second, making liquidity shortages endogenous based on the potential for securitization of long term assets may be crucial to fully understand those factors behind spread movements and their marginal predictive power. Finally, final investors (after securitization) and bank sentiment or risk assessment could also be time varying affecting the linkage between long term funding risks and economic activity.

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Appendix A Simple Model - Case 2

Given the assumption for the two stochastic processes (R_E, ρ) and setting $\sigma_B = 0$, total profits Π^B are also normally distributed with mean $\mu_\Pi = (\bar{R_E} - R_D)Z + (\beta(R_L - R_D) - R_D - \bar{\rho})X_L$ and variance $\sigma_\Pi^2 = Z^2\sigma_E^2 + \beta^2 X_L^2 \sigma_\rho^2 + 2co_{E,\rho}X_L Z\sigma_E\sigma_\rho$.

Using an approximation for the first percentile of the profit probability density function we then obtain

$$0.01 = \frac{1}{2}\left[1 + erf\left(\frac{VaR - \mu_\Pi}{\sigma_\Pi\sqrt{2}}\right)\right] \tag{35}$$

where erf is the error function. We can differentiate the equation above with respect to X_L and Z to obtain

$$\begin{aligned}
\frac{\partial VaR}{\partial X_L} &= (\beta(R_L - R_D) - R_D - \bar{\rho})\sigma_\Pi\sqrt{2} \\
&\quad + \left[VaR - \mu_\Pi\right](\sigma_\Pi)^{-1}\sqrt{2}(\beta^2 X_L\sigma_\rho^2 + 2co_{E,\rho}Z\sigma_E\sigma_\rho) \\
\frac{\partial VaR}{\partial Z} &= (\bar{R_E} - R_D)\sigma_\Pi\sqrt{2} \\
&\quad + \left[VaR - \mu_\Pi\right](\sigma_\Pi)^{-1}\sqrt{2}(Z\sigma_E^2 + 2co_{E,\rho}X_L\sigma_E\sigma_\rho)
\end{aligned}$$

We then substitute these conditions into (1) - (5) and (35) to determine the equilibrium. Our main interest is to verify how term spreads (measure in basis points), defined as

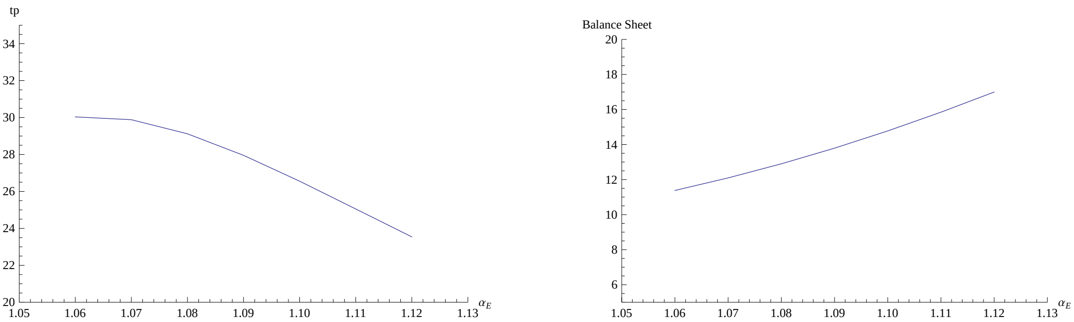
$$tp = \frac{1}{2}\left((R_L - 1) - (R_D - 1) - (R_D - 1)\right)10000,$$

move as the overall profitability (expected returns) of the bank's portfolio varies. In doing so we aim at establishing a link between banks appetite to accumulate long term assets in the balance sheet, incurring the risk of maturity transformation, the equilibrium long term rates and the expected performance of bank investments. As such we look at the equilibrium level of term premium as α_E (which controls expected returns on equity) varies. Figure 8 shows the results for the following parameter values $R_D = 1.01$ (base rate equal to a 4% annual), $\bar{\rho} = 0.005$ (annual spread of roughly 100 basis points) , $\sigma_E = 0.03$, $\sigma_\rho = 0.01$, $\gamma_L = 6$, $\gamma_E = 0.0006$, $\Lambda = -0.3$ (the VaR limit implies a loss of roughly 2.5 standard deviations). The qualitative implications are unchanged when these are altered. Finally, the important parameter to determine the results is the correlation between the asset returns. We set it to -0.1 (allowing for gains of diversification). The impact on term spreads reverse when this correlation is positive, since in this instance the substitution effect will be greater than the income effect (see the discussion in the text). Note that the increase in banks balance sheet occur since both Z and X_L holding increase, although equity holding increase more sharply.

Appendix B Data

This provides a description of the data used in the empirical study.

Figure 8: Assets and Term spreads as expected bank profits increase



(a) Term Spreads

(b) Assets in Banks Balance Sheet

- Treasury Bill Rate (Units: Percent per Annum), (Series ID: 60C..ZF) Source: International Financial Statistics/IMF
- Government Bond Yield: 10 year (Units: Percent per Annum), (Series ID: 61...ZF) Source: International Financial Statistics/IMF
- CPI All Items City Average (Units: Index Number), (Series ID: 64...ZF), Source: International Financial Statistics/IMF
- Real Gross Domestic Product, Seasonally Adjusted Annual Rate , (Series ID: GDPC96) Source: U.S. Department of Commerce: Bureau of Economic Analysis
- Financial Business; undistributed corporate profits excluding CCAdj, (FOF Code: FA796006403.Q), Source: Flow of Funds Accounts, Board of Governors of the Federal Reserve

Appendix C Steady State

From pricing equation (normalizing prices at steady state to 1) we have that

$$\Lambda = \frac{\varepsilon - 1}{\varepsilon}.$$

From wage pricing equation we have that

$$\bar{w} = \frac{\varepsilon_w}{\varepsilon_w - 1}\frac{\chi \bar{H}^{\eta}}{\bar{C}^{-\sigma}}.$$

From the firm problem we have that

$$\Lambda = \frac{\bar{w}\bar{H}}{\bar{Y}(1-\alpha)}$$

$$\bar{q}_s = \frac{\beta\Lambda\alpha\zeta\bar{Y}}{\bar{K}^S(1-\beta(1-\delta))}$$

$$\bar{q}_L = \frac{\beta\Lambda\alpha(1-\zeta)\bar{Y}}{\bar{K}^L(1-\beta(1-\delta))}.$$

From entrepreneurs problems we have that

$$\bar{X}_S = \gamma_S \beta \bar{q}_S - 1$$

$$\bar{X}_L = \frac{\gamma_L \bar{q}_L}{R_L + \rho} - 1.$$

From the bank problem we have that

$$\left(\frac{1}{\beta} - 1\right) + \rho = \beta\left(R_L - \frac{1}{\beta}\right) \text{ or } R_L = \frac{1}{\beta^2} + \frac{\rho}{\beta}.$$

The term spread at the steady state is given by

$$t\bar{p} = \frac{1}{2}((R_L - 1) - (1/\beta - 1) - (1/\beta - 1))400.$$

Clearing conditions and investment flow equation determine that

$$\bar{Y} = \bar{C} + \bar{X}_S + (1+\rho)\bar{X}_L$$

$$\bar{Y} = \bar{H}^{(1-\alpha)}\bar{K}_S^{\alpha\zeta}\bar{K}_L^{\alpha(1-\zeta)}$$

$$\delta\bar{K}_S = \gamma_S \ln(1+\bar{X}_S)$$

$$\delta\bar{K}_L = \gamma_L \ln(1+\bar{X}_L).$$

Appendix D Second Order Approximation of Long Term Rate Decision

Bank equilibrium condition is given by

$$E_t \frac{1}{\pi_{+1}}\left[\left(\Pi^B_{t+1}\right)^{-\sigma_B}\left((R_{CB,t} - 1) + \rho_{t+1}\right)\right] = \beta E_t \frac{1}{\pi_{+1}\pi_{+2}}\left[\left(\Pi^B_{t+2}\right)^{-\sigma_B}\left(R_{L,t} - R_{CB,t+1}\right)\right],$$

and at the steady state

$$\left(\frac{1}{\beta} + \bar{\rho}\right)\frac{1}{\beta} = R_L \text{ and } R_{CB} = \frac{1}{\beta}.$$

Let $W_1 = (R_{CB,t} - 1 + \rho_{t+1})$ and $W_2 = (R_{L,t} - R_{CB,t+1})$,

Approximation of Left-Hand Side (LHS)

$$
\begin{aligned}
&E_t \frac{1}{\pi_{+1}} \left[\left(\Pi^B_{t+1}\right)^{-\sigma_B} \left((R_{CB,t}-1)+\rho_{t+1}\right) \right] = \\
&E_t \frac{1}{\pi_{+1}} \left[\left(\Pi^B_{t+1}\right)^{-\sigma_B} W_1 \right] \approx \\
&E_t \left[-\sigma^b \widehat{\Pi^B_{t+1}} + 0.5\sigma^b \left(\widehat{\Pi^B_{t+1}}\right)^2 + \widehat{W_1} + 0.5\left(\widehat{W_1}\right)^2 - \widehat{\pi_{t+1}} + 0.5\left(\widehat{\pi_{t+1}}\right)^2 - \sigma^b \widehat{\Pi^B_{t+1}}\widehat{W_1} + \sigma^b \widehat{\Pi^B_{t+1}}\widehat{\pi_{t+1}} - \widehat{W_1}\widehat{\pi_{t+1}} \right].
\end{aligned}
$$

From the definition of W_1

$$
\begin{aligned}
\widehat{W_1} &= \frac{1}{\beta\Gamma}\widehat{R_{CB,t}} + \frac{\rho}{\Gamma}\widehat{\rho_{t+1}}, \text{ where } \Gamma = \left(\frac{1}{\beta} - 1 + \bar{\rho}\right), \text{ and} \\
\widehat{W_1} + 0.5\left(\widehat{W_1}\right)^2 &= \frac{1}{\beta\Gamma}\left(\widehat{R_{CB,t}} + 0.5\left(\widehat{R_{CB,t}}\right)^2\right) + \frac{\bar{\rho}}{\Gamma}\left(\widehat{\rho_{t+1}} + 0.5\left(\widehat{\rho_{t+1}}\right)^2\right).
\end{aligned}
$$

Hence, LHS becomes

$$
E_t \left[\begin{array}{c} -\sigma^b \widehat{\Pi^B_{t+1}} + 0.5\sigma^b \left(\widehat{\Pi^B_{t+1}}\right)^2 + \frac{1}{\beta\Gamma}\left(\widehat{R_{CB,t}} + 0.5\left(\widehat{R_{CB,t}}\right)^2\right) + \frac{\bar{\rho}}{\Gamma}\left(\widehat{\rho_{t+1}} + 0.5\left(\widehat{\rho_{t+1}}\right)^2\right) - \widehat{\pi_{t+1}} + 0.5\left(\widehat{\pi_{t+1}}\right)^2 - \\ -\sigma^b \widehat{\Pi^B_{t+1}}\left(\frac{1}{\beta\Gamma}\widehat{R_{CB,t}} + \frac{\bar{\rho}}{\Gamma}\widehat{\rho_{t+1}}\right) + \sigma^b \widehat{\Pi^B_{t+1}}\widehat{\pi_{t+1}} - \left(\frac{1}{\beta\Gamma}\widehat{R_{CB,t}} + \frac{\rho}{\Gamma}\widehat{\rho_{t+1}}\right)\widehat{\pi_{t+1}}. \end{array} \right]
$$

Approximation of Right-Hand Side (RHS)

$$
\begin{aligned}
&\beta E_t \frac{1}{\pi_{+1}\pi_{+2}} \left[\left(\Pi^B_{t+2}\right)^{-\sigma_B} \left(R_{L,t} - R_{CB,t+1}\right) \right] = \\
&\beta E_t \frac{1}{\pi_{+1}\pi_{+2}} \left[\left(\Pi^B_{t+2}\right)^{-\sigma_B} W_2 \right] \approx \\
&E_t \Big[-\sigma^b \widehat{\Pi^B_{t+2}} + 0.5\sigma^b \left(\widehat{\Pi^B_{t+2}}\right)^2 + \widehat{W_2} + 0.5\left(\widehat{W_2}\right)^2 - \widehat{\pi_{t+1}} + 0.5\left(\widehat{\pi_{t+1}}\right)^2 - \widehat{\pi_{t+2}} + 0.5\left(\widehat{\pi_{t+2}}\right)^2 \\
&-\sigma^b \widehat{\Pi^B_{t+2}}\widehat{W_2} + \sigma^b \widehat{\Pi^B_{t+2}}\widehat{\pi_{t+1}} + \sigma^b \widehat{\Pi^B_{t+2}}\widehat{\pi_{t+2}} - \widehat{W_2}\widehat{\pi_{t+1}} - \widehat{W_2}\widehat{\pi_{t+2}} + \widehat{\pi_{t+2}}\widehat{\pi_{t+1}} \Big].
\end{aligned}
$$

From the definition of W_2

$$
\begin{aligned}
\widehat{W_2} &= \left(\frac{\frac{1}{\beta} + \rho}{\Gamma}\widehat{R_{L,t}} - \frac{1}{\beta\Gamma}\widehat{R_{CB,t+1}}\right) \text{ where } \Gamma = \left(\frac{1}{\beta} - 1 + \bar{\rho}\right) \text{ and} \\
\widehat{W_2} + 0.5\left(\widehat{W_2}\right)^2 &= \frac{\frac{1}{\beta} + \bar{\rho}}{\Gamma}\left(\widehat{R_{L,t}} + 0.5\left(\widehat{R_{L,t}}\right)^2\right) - \frac{1}{\beta\Gamma}\left(\widehat{R_{CB,t+1}} + 0.5\left(\widehat{R_{CB,t+1}}\right)^2\right)
\end{aligned}
$$

Hence, RHS becomes

$$
E_t \left[\begin{array}{c} -\sigma^b \widehat{\Pi^B_{t+2}} + 0.5\sigma^b \left(\widehat{\Pi^B_{t+2}}\right)^2 + \frac{\frac{1}{\beta}+\bar{\rho}}{\Gamma}\left(\widehat{R_{L,t}} + 0.5\left(\widehat{R_{L,t}}\right)^2\right) - \frac{1}{\beta\Gamma}\left(\widehat{R_{CB,t+1}} + 0.5\left(\widehat{R_{CB,t+1}}\right)^2\right) - \widehat{\pi_{t+1}} + 0.5\left(\widehat{\pi_{t+1}}\right)^2 - \widehat{\pi_{t+2}} + 0.5\left(\widehat{\pi_{t+2}}\right)^2 - \\ -\sigma^b \widehat{\Pi^B_{t+2}}\left(\frac{\frac{1}{\beta}+\bar{\rho}}{\Gamma}\widehat{R_{L,t}} - \frac{1}{\beta\Gamma}\widehat{R_{CB,t+1}}\right) + \sigma^b \widehat{\Pi^B_{t+2}}\widehat{\pi_{t+1}} + \sigma^b \widehat{\Pi^B_{t+2}}\widehat{\pi_{t+2}} - \left(\frac{\frac{1}{\beta}+\bar{\rho}}{\Gamma}\widehat{R_{L,t}} - \frac{1}{\beta\Gamma}\widehat{R_{CB,t+1}}\right)\widehat{\pi_{t+1}} - \left(\frac{\frac{1}{\beta}+\bar{\rho}}{\Gamma}\widehat{R_{L,t}} - \frac{1}{\beta\Gamma}\widehat{R_{CB,t+1}}\right)\widehat{\pi_{t+2}} + \widehat{\pi_{t+2}}\widehat{\pi_{t+1}}. \end{array} \right]
$$

From the definition of term premium we have that $tp = 0.5(R_{L,t} - R_{CB,t+1} - R_{CB,t} + 1)$, hence[22]

$$\widehat{tp} + 0.5\left(\widehat{tp}\right)^2 \approx \frac{\frac{1}{\beta}+\rho}{\Gamma}\left(\widehat{R_{L,t}} + 0.5\left(\widehat{R_{L,t}}\right)^2\right) - \frac{1}{\beta\Gamma}\left(\widehat{R_{CB,t+1}} + 0.5\left(\widehat{R_{CB,t+1}}\right)^2\right) - \frac{1}{\beta\Gamma}\left(\widehat{R_{CB,t}} + 0.5\left(\widehat{R_{CB,t}}\right)^2\right).$$

We can now combine the LHS and RHS to get

$$E_t\left[\widehat{tp}+0.5\left(\widehat{tp}\right)^2\right] = E_t\left[\sigma^b\left(\widehat{\Pi^B_{t+2}}-\widehat{\Pi^B_{t+1}}\right)+0.5\sigma^b\left(\left(\widehat{\Pi^B_{t+2}}\right)^2-\left(\widehat{\Pi^B_{t+1}}\right)^2\right)+\widehat{\pi_{t+2}}-0.5\left(\widehat{\pi_{t+2}}\right)^2+\frac{\bar{\rho}}{\Gamma}\left(\widehat{\rho_{t+1}}+0.5\left(\widehat{\rho_{t+1}}\right)^2\right)+CovTerms\right]$$

where,

$$\begin{aligned} CovTerms = & -\sigma^b\widehat{\Pi^B_{t+1}}\left(\frac{1}{\beta\Gamma}\widehat{R_{CB,t}}+\frac{\bar{\rho}}{\Gamma}\widehat{\rho_{t+1}}\right)+\sigma^b\widehat{\Pi^B_{t+1}}\widehat{\pi_{t+1}}-\left(\frac{1}{\beta\Gamma}\widehat{R_{CB,t}}+\frac{\rho}{\Gamma}\widehat{\rho_{t+1}}\right)\widehat{\pi_{t+1}} \\ & +\sigma^b\widehat{\Pi^B_{t+2}}\left(\frac{\frac{1}{\beta}+\bar{\rho}}{\Gamma}\widehat{R_{L,t}}-\frac{1}{\beta\Gamma}\widehat{R_{CB,t+1}}-\widehat{\pi_{t+1}}-\widehat{\pi_{t+2}}\right)+\left(\frac{\frac{1}{\beta}+\bar{\rho}}{\Gamma}\widehat{R_{L,t}}-\frac{1}{\beta\Gamma}\widehat{R_{CB,t+1}}\right)(\widehat{\pi_{t+1}}+\widehat{\pi_{t+2}})-\widehat{\pi_{t+2}}\widehat{\pi_{t+1}}. \end{aligned}$$

Appendix E Long Term Investment with 1Y maturity

If we assume long term investments are done at period t but mature at $t+4$ then $X_{L,t}$ becomes

$$X_{L,t} = \frac{\gamma_L E_t[q^L_{t+4}\pi_{t+1}\pi_{t+2}\pi_{t+3}\pi_{t+4}]}{R_{L,t}+E_t\left[\rho_{t+1}\pi_{t+1}+\rho_{t+2}\pi_{t+1}\pi_{t+2}+\rho_{t+3}\pi_{t+1}\pi_{t+3}\pi_{t+4}\right]} - 1.$$

And the long term rate is set such that

$$\begin{aligned} \beta^3 E_t\left[\frac{1}{\pi_{t+1}\pi_{t+2}\pi_{t+3}\pi_{t+4}}{\Pi^B_{t+4}}^{-\sigma_B}(R_{L,t}-R_{t+3,CB})\right] = & E_t\left[{\Pi^B_{t+1}}^{-\sigma_B}\left(\frac{(R_{t,CB}-1)}{\pi_{t+1}}+\rho_{t+1}\right)\right]+ \\ & \beta E_t\left[{\Pi^B_{t+2}}^{-\sigma_B}\left(\frac{(R_{t+1,CB}-1)}{\pi_{t+1}\pi_{t+2}}+\rho_{t+2}\right)\right]+ \\ & \beta^2 E_t\left[{\Pi^B_{t+3}}^{-\sigma_B}\left(\frac{(R_{t+2,CB}-1)}{\pi_{t+1}\pi_{t+2}\pi_{t+3}}+\rho_{t+3}\right)\right]. \end{aligned}$$

Where

$$\begin{aligned} \Pi^B_t = & div_t + \frac{(R_{t-4,L}-1)}{\pi_t\pi_{t-1}\pi_{t-2}\pi_{t-3}}X_{L,t-4} + (R_{t-1,CB}-1)(X_{S,t-1}-d_{t-1})\frac{1}{\pi_t} \\ & -\rho_t(X_{L,t-1}+X_{L,t-2}+X_{L,t-3}), \text{ and} \\ d_t = & X_{S,t}+X_{L,t}+\frac{X_{L,t-1}}{\pi_t}+\frac{X_{L,t-2}}{\pi_t\pi_{t-1}}+\frac{X_{L,t-3}}{\pi_t\pi_{t-1}\pi_{t-2}}+z_t. \end{aligned}$$

[22]Note that the approximated signed is also used here since the denominator should be $\left(\frac{1}{\beta}-1+\rho\right)+\beta-1$ and not $\Gamma = \left(\frac{1}{\beta}-1+\rho\right)$.

We define the term premium (annual rate in percentage points) between long and short term rate as

$$tp = \frac{1}{4}\left(R_{L,t} - R_{t,CB} - R_{t+1,CB} - R_{t+2,CB} - R_{t+3,CB} + 3\right)400.$$

Finally, the good market clearing condition is

$$Y_t = C_t + X_{S,t} + X_{L,t} + \rho_t\left(X_{L,t-1} + X_{L,t-2} + X_{L,t-3}\right).$$

Short term asset purchase agreements

We assume banks can only repo the long term asset that is about to mature.

Profits and deposits are given by

$$\begin{aligned}
\Pi_t^B &= div_t + \frac{(R_{t-4,L} - 1)}{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}} X_{L,t-4} + (R_{t-1,CB} - 1)(X_{S,t-1} - d_{t-1}) \frac{1}{\pi_t} \\
&\quad - \rho_t (X_{L,t-1} + X_{L,t-2} + X_{L,t-3}) + \frac{\Theta_{t-1} X_{L,t-4}}{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}} (R_{QE,t} - 1) - \frac{\phi_{QE}}{2} \Theta_{t-1}^2, \text{ and} \\
d_t &= X_{S,t} + X_{L,t} + \frac{X_{L,t-1}}{\pi_t} + \frac{X_{L,t-2}}{\pi_t \pi_{t-1}} + \frac{X_{L,t-3}}{\pi_t \pi_{t-1} \pi_{t-2}} + z_t - \Theta_t \frac{X_{L,t-3}}{\pi_t \pi_{t-1} \pi_{t-2}}.
\end{aligned}$$

Which implies

$$\begin{aligned}
\beta^3 E_t \left[\frac{{\Pi_{t+4}^B}^{-\sigma_B} \left(R_{L,t} - R_{t+3,CB} + \Theta_{t+3}(R_{t+3,CB} - R_{QE,t+3})\right)}{\pi_{t+1} \pi_{t+2} \pi_{t+3} \pi_{t+4}} \right] &= E_t \left[{\Pi_{t+1}^B}^{-\sigma_B} \left(\frac{(R_{t,CB} - 1)}{\pi_{t+1}} + \rho_{t+1} \right) \right] \\
&\quad + \beta E_t \left[{\Pi_{t+2}^B}^{-\sigma_B} \left(\frac{(R_{t+1,CB} - 1)}{\pi_{t+1} \pi_{t+2}} + \rho_{t+2} \right) \right] \\
&\quad + \beta^2 E_t \left[{\Pi_{t+3}^B}^{-\sigma_B} \left(\frac{(R_{t+2,CB} - 1)}{\pi_{t+1} \pi_{t+2} \pi_{t+3}} + \rho_{t+3} \right) \right] \\
0 &= {\Pi_{t+1}^B}^{-\sigma_B} \left((R_{CB,t} - R_{QE,t}) \frac{X_{L,t-3}}{\pi_{t+1} \pi_t \pi_{t-1} \pi_{t-2}} - \phi_{QE} \Theta_t \right).
\end{aligned}$$

Long term asset purchase agreements

We assume banks can sell the long term asset with the longest maturity and buy back before maturity.

Profits and deposits are given by

$$\begin{aligned}
\Pi_t^B &= div_t + \frac{(R_{t-4,L} - 1)}{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}} X_{L,t-4} + (R_{t-1,CB} - 1)(X_{S,t-1} - d_{t-1}) \frac{1}{\pi_t} \\
&\quad - \rho_t (X_{L,t-1} + X_{L,t-2} + X_{L,t-3}) + \frac{\Theta_{t-3} X_{L,t-4}}{\pi_t \pi_{t-1} \pi_{t-2} \pi_{t-3}} (R_{QE,t} - 1) - \frac{\phi_{QE}}{2} \Theta_{t-3}^2, \text{ and} \\
D_t &= P_t X_{S,t} + P_t X_{L,t} + P_{t-1} X_{L,t-1} + P_{t-2} X_{L,t-2} + P_{t-3} X_{L,t-3} + Z_t - \Theta_t P_{t-1} X_{L,t-1} - \Theta_{t-1} P_{t-2} X_{L,t-2} - \Theta_{t-2} P_{t-3} X_{L,t-3}.
\end{aligned}$$

Which implies

$$\begin{aligned}\beta^3 E_t\left[\frac{{\Pi^B_{t+4}}^{-\sigma_B}(R_{L,t}-R_{t+3,CB}+\Theta_{t+1}(R_{t+3,CB}-R_{QE,t+1}))}{\pi_{t+1}\pi_{t+2}\pi_{t+3}\pi_{t+4}}\right] &= E_t\left[{\Pi^B_{t+1}}^{-\sigma_B}\left(\frac{(R_{t,CB}-1)}{\pi_{t+1}}+\rho_{t+1}\right)\right] \\ &+\beta E_t\left[{\Pi^B_{t+2}}^{-\sigma_B}(1-\Theta_{t+1})\left(\frac{(R_{t+1,CB}-1)}{\pi_{t+1}\pi_{t+2}}+\rho_{t+2}\right)\right] \\ &+\beta^2 E_t\left[{\Pi^B_{t+3}}^{-\sigma_B}(1-\Theta_{t+1})\left(\frac{(R_{t+2,CB}-1)}{\pi_{t+1}\pi_{t+2}\pi_{t+3}}+\rho_{t+3}\right)\right]\end{aligned}$$

$${\Pi^B_{t+1}}^{-\sigma_B}\frac{(R_{CB,t}-1)X_{L,t-1}}{\pi_t\pi_{t+1}}+{\Pi^B_{t+2}}^{-\sigma_B}\frac{(R_{CB,t+1}-1)X_{L,t-1}}{\pi_t\pi_{t+1}\pi_{t+2}}+{\Pi^B_{t+3}}^{-\sigma_B}\left((R_{CB,t+3}-R_{QE,t})\frac{X_{L,t-1}}{\pi_t\pi_{t+1}\pi_{t+2}\pi_{t+3}}-\phi_{QE}\Theta_t\right)=0.$$