Escaping Expectations Traps: How Much Commitment is Required?*

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Abstract

Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2010) developed a framework that allows us to study a continuum of intermediate cases between commitment and discretion. In this paper we apply this framework to two New Keynesian models and study the properties of existence and uniqueness of policy equilibria under limited commitment policy. We demonstrate the existence of expectations traps under the limited commitment and identify the minimum degree of commitment which is needed to escape from these traps. Despite our models are quite different, the results are similar: The degree of precommitment which is sufficient to generate uniqueness of the Pareto-preferred equilibrium requires the policy maker to stay in the office for the period of two to five years. This is consistent with monetary policy arrangements in many developed countries.

Key Words: Limited Commitment, Commitment, Discretion, Multiple Equilibria

JEL References: E31, E52, E58, E61, C61

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1 Introduction

In this paper we study existence and uniqueness properties of monetary policy with limited commitment in the Blanchard and Kahn (1980) class of infinite-horizon discrete-time non-singular linear dynamic models that is typically used to study aggregate fluctuations in macroeconomics.[1] Building on research in Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2010) we show the existence of multiple equilibria under the policy with limited commitment. Similar to discretion, policy makers cannot manage private sector expectations under limited commitment. As a consequence expectation traps and coordination failures can occur. We investigate the question of how much precommitment is needed to escape the expectations traps and coordinate on the Pareto-preferred equilibrium. We find that the necessary degree of precommitment to eliminate multiplicity is relatively small – from two to five years – which is consistent with tenure terms of monetary policy makers in many countries.

It is well known that in models with rational expectations (RE) commitment and discretion policies may imply very different dynamics of the economy. With full commitment the policy maker has complete control over the private sector's expectations about future policy and steers them in a way that furthers his stabilization goals. The policy maker can coordinate all future actions of consequent policy makers, which allows him to choose once, and apply indefinitely, an intertemporal contingency plan (Kydland and Prescott (1977)). In linear quadratic (LQ) models a commitment policy, if it exists, is always unique (Kwakernaak and Sivan (1972), Backus and Driffill (1986)).

With no commitment at all, i.e. under pure discretion, the policy maker does not control the expectations of the private sector and fails to coordinate the actions of consequent policy makers. Under discretion the policy maker optimizes in each period of time and the private sector knows that future policy makers will implement the same decision process in subsequent periods (Oudiz and Sachs (1985), Backus and Driffill (1986), Currie and Levine (1993)). However, in this framework expectation traps and multiple equilibria can arise, because the expectations of the private sector are shaped by anticipations about future policy behavior. Since the policy maker cannot fully control private sector expectations, those expectations may trap the policy maker into implementing a policy that validates them. The trap is closed if it is less costly for the policy maker to validate the private sector beliefs about future policy than ignoring those

[1] Models with multiple policy equilibria can help to explain the observed volatility of macroeconomic data and can help to suggest how control policies should be improved to avoid these traps (see. e.g. Davig and Leeper (2006)).

expectations.[2]

Our contribution is twofold. First, we demonstrate by example that similar expectations traps exist under the policy with limited commitment.[3] We study two New Keynesian (NK) models. The first model is a simple NK model with government debt accumulation, which describes economic behavior that is familiar from the literature on the fiscal theory of the price level in the spirit of Leeper (1991). The second example is a small open economy model with incomplete financial markets similar to Benigno (2001) and De Paoli (2009). Because of sequential policy making under limited commitment, the policy maker can neither completely control the expectations of the private sector, nor can he coordinate the actions of all future policy makers, similar to what is observed under discretion. Coordination failures between the sequence of policy makers and the private sector result in multiple equilibria and expectations traps.

Second, we obtain the minimum degree of policy precommitment that is required to select the best equilibrium. Although the two models are quite different, the results are surprisingly similar: a tenure of about 2-5 years is sufficient to eliminate all equilibria except the best one in both models.

The paper is organized as follows. In the next Section 2 we introduce the NK model with debt accumulation. We first review properties of discretion and commitment policies for this model and then demonstrate the existence of expectations traps under the quasi-commitment policy. We also discuss the welfare implications of different degrees of fiscal feedback and find the minimum length of precommitment period required to select the best equilibrium. Section open economy model 3 demonstrates the robustness of our results for the model of small open economy with incomplete financial markets. Section 4 concludes. Finally, the Appendix presents a numerical algorithm to find policy with limited commitment.

2 The Model with Government Debt

This Section presents a simple NK model with government debt and proves by example the existence of multiple equilibria under policies with limited commitment. This model is well suited to use it as a laboratory to demonstrate expectations traps and to study the dynamic

[2]Models with multiple discretionary equilibria are presented in Lockwood and Philippopoulos (1994), King and Wolman (2004), Albanesi et al. (2003), Blake and Kirsanova (2008) and Dennis and Kirsanova (2009).

[3]Originally, the framework is based on Roberds (1987). Schaumburg and Tambalotti (2007) term limited commitment 'Quasi-commitment' and Debortoli and Nunes (2010) use 'loose commitment'. In this paper we use these terms interchangeably.

properties of an economy under monetary policy with limited commitment. First, unlike the model in Schaumburg and Tambalotti (2007) this model has an endogenous predetermined state variable, government debt, which is affected by policy. The presence of such a variable is crucial to generate multiple equilibria under discretionary policy, which is a limiting case of quasi-commitment policy (Blake and Kirsanova (2008)). Second, the model is simple enough to derive most of our results analytically.

We adopt the model from Benigno and Woodford (2004).[4] The model describes an economic behavior familiar from the literature on the fiscal theory of the price level.[5] The economy consists of a representative household, a representative firm that produces the final good, a continuum of intermediate goods-producing firms and a monetary and fiscal authority. The intermediate goods-producing firms act under monopolistic competition and produce according to a production function that depends only on labor. Goods are combined via a Dixit and Stiglitz (1977) technology to produce aggregate output. Firms set their prices subject to a Calvo (1983) price rigidity. Households choose their consumption and leisure and can transfer income through time through their holdings of government bonds. All agents can observe and affect the accumulation of the real government debt. The accumulation of government debt must depend on the fiscal stance. Hence, in the model there is a non-optimizing fiscal authority that faces a stream of exogenous public consumption. These expenditures are financed by levying income taxes and by issuing one-period risk-free nominal bonds. We assume that the fiscal authority imposes a simple proportional rule for the tax rate: if the real debt is higher (lower) than in the steady state the tax rate rises (falls). We shall refer to the tax rate as 'taxes' and to the parameter of the proportional rule as the 'fiscal feedback'. The size of the fiscal feedback measures the strength of the fiscal stabilization of debt and, as we shall show, plays an important role in the model. The presence of the non-optimizing fiscal authority in the economy can be captured by this single parameter μ.

We assume that all public debt consist of riskless one-period bonds. The nominal value $\mathcal{B}_t$ of end-of-period public debt then evolves according to the following law of motion:

$$\mathcal{B}_t = (1 + i_{t-1})\,\mathcal{B}_{t-1} + P_t G_t - \tau_t P_t Y_t, \tag{1}$$

where τ_t is the share of national product Y_t that is collected by the government in period t, and government purchases G_t are treated as exogenously given and time-invariant. P_t is the

[4] It was also used in Blake and Kirsanova (2008).

[5] See e.g. Leeper (1991), Woodford (2001), Davig and Leeper (2006), Favero and Monacelli (2005).

aggregate price level and i_t is the interest rate on bonds. The national income identity yields

$$Y_t = C_t + G_t \tag{2}$$

where C_t is private consumption. For analytical convenience we introduce the real value of debt at maturity $B_t = (1+i_{t-1})\mathcal{B}_{t-1}/P_{t-1}$, observed at the beginning of period t, so that (1) becomes

$$B_{t+1} = (1+i_t)\left(B_t \frac{P_{t-1}}{P_t} - \tau_t Y_t + G_t\right). \tag{3}$$

We assume that fiscal authorities operate with simple mechanistic feedback rule that relates the tax rate τ_t and B_t

$$\tau_t = \tau_o \left(\frac{B_t}{B_o}\right)^{\mu \frac{B_o}{Y_o}}, \tag{4}$$

where τ_o and B_o are steady state values of tax rate and real debt correspondingly.

Log-linearizing (3) and (4) yields

$$b_{t+1} = \frac{B_o}{Y_o}\iota_t + \frac{1}{\beta}\left((1-\mu\tau_o)\, b_t - \frac{C_o}{Y_o}\tau_o c_t - \frac{B_o}{Y_o}\pi_t\right), \tag{5}$$

where $b_t = \frac{B_o}{Y_o}\ln\left(\frac{B_t}{B_o}\right)$, $c_t = \ln\left(\frac{C_t}{C_o}\right)$, $\iota_t = \ln\frac{(1+i_t)}{(1+i_o)}$ and the subscript o denotes steady state values of corresponding variables in zero inflation steady state. The private sector's discount factor is $\beta = 1/(1+i_o)$. To make the model particularly simple we assume $B_o = 0$, which eliminates the first-order effects of the interest rate and inflation on debt. Thus the final version of the linearized debt accumulation equation can be written as:

$$b_{t+1} = \rho b_t - \eta c_t \tag{6}$$

where the parameter $\rho = (1-\mu\tau_o)/\beta$ is a function of the tax rate. From the definition of ρ follows that the higher the fiscal feedback parameter μ the faster debt is stabilized. Parameter $\eta = C_o\tau_o/(\beta Y_o)$ describes the sensitivity of debt to the tax base.

The derivation of the appropriate Phillips curve that describes Calvo-type price-setting decisions of monopolistically competitive firms is standard (Benigno and Woodford (2004), Sec. A.5). A log-linearization of the aggregate supply relationship around the zero-inflation steady state yields the following (deterministic) New Keynesian Phillips curve

$$\pi_t = \beta\pi_{t+1} + \zeta\left(\left(\frac{1}{\sigma} + \frac{\theta}{\psi}\right)c_t + \frac{\tau_o}{(1-\tau_o)}\tau_t\right)$$

where ζ is the slope of Phillips curve, $\tau_t = \ln\left(\frac{\tau_t}{\tau_o}\right)$ and σ and ψ are parameters of the private sector utility function, and $\theta = C_o/Y_o$. Note that marginal cost is a function of output and taxes. Substituting in the log-linearized (2) and (4) yields

$$\pi_t = \beta\pi_{t+1} + \kappa c_t + \nu b_t, \tag{7}$$

where $\nu = \mu\kappa\tau_o/\left(1-\tau_o\right)$ and $\lambda = \kappa\left(1/\sigma + \theta/\psi\right)$.

Therefore the model is described by two equations, the debt accumulation equation (6) and Phillips curve (7). The aggregate agents' decision variable is inflation, π_t, and we assume that the policy maker chooses consumption c_t. Debt b_t is the aggregate predetermined state variable in period t. The economy evolves according to (6) and (7) and the initial state $\bar{b}$ is known to all agents. In contrast to the standard NK model policy c_t affects the predetermined state b_{t+1}.

The inter-temporal welfare criterion of the policy maker is defined by the quadratic loss function[6]

$$L = \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^2 + \lambda c_t^2\right). \tag{8}$$

The policy maker knows the laws of motion (6)-(7) of the aggregate economy and takes them into account when formulating policy.

2.1 Preliminaries: Discretion and Commitment

We shall compare the dynamics of the model under quasi-commitment policy with dynamics under the two limiting cases, discretion and commitment.[7] This Section gives all necessary definitions and presents solutions to these two limiting cases in a comparable form.

In this and the next Section we shall work with the deterministic version of the model. After having solved the deterministic version of the LQ RE model it is easy to reinstate the stochastic components. Note that because of the certainty equivalence result the dynamics of the economy is fully determined by its deterministic part.

2.1.1 Discretionary Policy

Under discretion there is a sequence of policy makers: each period a new policy maker arrives in office. The new policy maker chooses the best policy knowing that he stays in office for only

[6] The criterion is derived under the assumption of steady state labour subsidy. Here parameter α is a function of model parameters, $\alpha = \theta\lambda/\epsilon$, and ϵ is the elasticity of substitution between any pair of monopolistically produced goods.

[7] In this section we largely follow the approach and results in Blake and Kirsanova (2008) and in Kirsanova and Wren-Lewis (2011), but present results a the form convenient for our purposes.

one period and the next-period policy maker will re-optimize again.[8] The law of motion of the aggregate economy (6)-(7) is known by the policy maker and taken into account when he formulates the optimal policy. Furthermore, the policy maker finds the best action every period and knows that future policy makers have the freedom to change policy, but will apply the same decision process. At every point in time t the decision rules of each agent are linear functions of the current state

$$c_t = c_b b_t, \tag{9}$$

$$\pi_t = \pi_b b_t. \tag{10}$$

Note that from

$$\pi_{t+1} \overset{eq.(10)}{=} \pi_b b_{t+1} \overset{eq.(6)}{=} \pi_b \left(\rho b_t - \eta c_t\right) \overset{eq.(7)}{=} \frac{1}{\beta}\pi_t - \frac{\kappa}{\beta}c_t - \frac{\nu}{\beta}b_t,$$

it follows that the private sector's decision can also be written as

$$\pi_t = \left(\beta\rho\pi_b + \nu\right) b_t + \left(\kappa - \beta\eta\pi_b\right) c_t. \tag{11}$$

The policy maker moves first within each period and the private sector observes the action of the policy maker. Thus, the private sector takes into account the 'instantaneous' influence of the policy choice measured by $\left(\kappa - \beta\eta\pi_b\right)$.

We can give now a more precise definition of discretionary policy: A policy determined by (9) is *discretionary* if the policy maker *finds it optimal* to follow it every period $s > t$, given the private sector (i) observes the current policy, (ii) knows that future policy makers re-optimize and use the same decision process, (iii) expects policy (9) will be implemented in all future periods.

We can write the criterion for optimality as

$$Sb_t^2 = \min_{c_t} \left(\left(\pi_t^2 + \lambda c_t^2\right) + \beta S b_{t+1}^2 \right), \tag{12}$$

subject to constraints (6) and (11).

One can solve the problem using Lagrange multipliers. The Lagrangian can be written as

$$\mathcal{L}_t^d = \frac{1}{2}\left(\pi_t^2 + \lambda c_t^2\right) + \beta\frac{1}{2}Sb_{t+1}^2 + \xi_{t+1}\left(\rho b_t - \eta c_t - b_{t+1}\right) + \phi_{t+1}\left(\pi_t - \kappa c_t - \nu b_t - \beta\pi_b b_{t+1}\right). \tag{13}$$

[8] Our definition of discretionary policy is standard and follows Oudiz and Sachs (1985), Backus and Driffill (1986), see also Clarida et al. (1999).

This approach exploits the intertemporal representation (6)-(7) together with the underlying assumption that private sector expectations about its own future decisions will be necessarily a function of the future state, which is $\pi_{t+1} = \pi_b b_{t+1}$ for our model.

Only current period constraints matter for the policy maker and the first order conditions can be written as

$$
\begin{aligned}
0 &= \beta S b_{t+1} - \xi_{t+1} - \beta \pi_b \phi_{t+1}, && (14)\\
0 &= \pi_t + \phi_{t+1}, && (15)\\
0 &= \lambda c_t - \eta \xi_{t+1} - \kappa \phi_{t+1}, && (16)\\
0 &= \rho b_t - \eta c_t - b_{t+1}, && (17)\\
0 &= \beta \pi_b b_{t+1} + \kappa c_t + \nu b_t - \pi_t. && (18)
\end{aligned}
$$

These conditions yield the linear decision rule for the policy maker:

$$c_t = -\frac{\left(\left(\beta\rho\pi_b + \nu\right)\left(\kappa - \pi_b\eta\beta\right) - \beta\rho\eta S\right)}{\left(\left(\kappa - \beta\eta\pi_b\right)^2 + \lambda + \beta\eta^2 S\right)} b_t = c_b b_t. \tag{19}$$

The coefficient c_b in (19) determines the optimal policy feedback on the predetermined state variable, b_t, where the feedback coefficient is a function of S. S can be found from the optimality criterion (12):

$$S = \frac{\left(\left(\beta\rho\pi_b + \nu\right) + \left(\kappa - \beta\eta\pi_b\right) c_b\right)^2 + \lambda c_b^2}{\left(1 - \beta\left(\rho - \eta c_b\right)^2\right)}. \tag{20}$$

Hence the first order conditions (11), (19) and (20) define c_b, π_b and S.

We can obtain all solutions in the following way. Suppose the policy maker guesses the response of the private sector to the state, π_b. Then the optimal discretionary policy is given by the pair (19) and (20). Given c_b the optimal response π_b^* of the private sector is given by (11). Therefore, for every – not necessarily optimal – π_b we can compute a unique π_b^* and plot the dependence $\pi_b^*(\pi_b)$, see the first panel in Figure 4. Clearly, if $\pi_b = \pi_b^*$ we have a solution to the discretionary problem.

For our base line calibration the graph of $\pi_b^*(\pi_b)$ intersects the 45^o degree line in three points labelled A, B and C, so we have three discretionary policy equilibria.[9] A moderate inflation,

[9] The benchmark calibration follows Schaumburg and Tambalotti (2007) and Blake and Kirsanova (2008). The model's frequency is quarterly. The subjective discount rate β is set to 0.99, the government share of total output $1-\rho$ is 0.25. The elasticity of intertemporal substitution σ is 1/2, the Frisch elasticity of labor supply $\varphi = 1/2$, an elasticity of demand ϵ of 7. The Calvo parameter $\gamma = 0.75$. Fiscal feedback μ is set to 0.075.

set by the firms in response to a given debt level, π_b, increases the marginal return to a policy decision that increases consumption in response to this level of debt, c_b. Higher consumption raises demand and firms will increase their response to debt, π_b^*. This complementarity ensures steepness of $\pi_b^*(\pi_b)$ and three equilibria arise.

2.1.2 Commitment Policy

Under the full commitment policy the policy maker optimizes only once, in the initial moment. It chooses a contingency plan, which is than applied indefinitely but can be implemented sequentially. If there is a change of policy makers, the subsequent policy maker continues the policy of its predecessor; therefore we can assume that there is only one policy maker which takes office in period zero and stays infinitely.

When optimizing, the policy maker internalizes the effect of its choice on private sector's expectations and solves the following Lagrangian

$$\mathcal{L}^c = \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{2} \left(\pi_t^2 + \lambda c_t^2 \right) + \xi_{t+1} \left(\rho b_t - \eta c_t - b_{t+1} \right) + \phi_{t+1} \left(\pi_t - \kappa c_t - \nu b_t - \beta \pi_{t+1} \right) \right).$$

The corresponding first order conditions are:

$$0 = -\xi_t + \rho\beta\xi_{t+1} - \nu\beta\phi_{t+1}, \quad (21)$$

$$0 = \pi_t + \phi_{t+1} - \phi_t, \quad (22)$$

$$0 = \lambda c_t - \eta\xi_{t+1} - \kappa\phi_{t+1}, \quad (23)$$

$$0 = \rho b_t - \eta c_t - b_{t+1}, \quad (24)$$

$$0 = \beta\pi_{t+1} + \kappa c_t + \nu b_t - \pi_t, \quad (25)$$

for $t \geq 0$; with initial conditions $b_0 = \bar{b}$ and $\phi_0 = 0$, and the transversality condition $\lim_{t\to\infty} b_t < \infty$.

If the system (6)-(7) is controllable, there always exists a unique path $\{c_t, \pi_t, b_t\}_{t\geq 0}$ which (i) satisfies system (21)-(25) and the initial conditions and (ii) all eigenvalues of the resulting transition matrix are less than $1/\sqrt{\beta}$ in modulus (see, e.g. Kwakernaak and Sivan (1972), Backus and Driffill (1986)).[10] For the rest of this paper we use the following definition: The economy is *stabilized* by a policy if all eigenvalues of the transition matrix are inside the unit circle. If the economy is stabilized by a policy we call such a policy *stabilizing*. In general, because $\beta < 1$ a stabilizing commitment policy may not exist for all problems in the LQ RE class.

[10] System (6)-(7) is controllable if $\rho \neq 0$ and $\eta \neq 0$.

One way to solve the system (21)-(25) is to use the Schur decomposition, see e.g. Söderlind (1999). Alternatively, and more convenient for our purpose, we can also solve the system using an iterative scheme. The solution to the LQ commitment problem can be written in the following form

$$\pi_t = \pi_b b_t + \pi_\phi \phi_t, \tag{26}$$

$$c_t = c_b b_t + c_\phi \phi_t, \tag{27}$$

$$\xi_t = \xi_b b_t + \xi_\phi \phi_t. \tag{28}$$

System (21)-(25) yields the following matrix discrete algebraic Riccatti equation[11]:

$$\begin{bmatrix} c_b & c_\phi \\ \pi_b & \pi_\phi \\ \xi_b & \xi_\phi \end{bmatrix} = \begin{bmatrix} -\left(\lambda + \eta^2 \xi_b\right) & \left(\eta \xi_\phi - \kappa\right) & 0 \\ \kappa - \beta \eta \pi_b & \beta \pi_\phi - 1 & 0 \\ -\beta \eta \rho \xi_b & \beta\left(\nu + \rho \xi_\phi\right) & -1 \end{bmatrix}^{-1} \begin{bmatrix} -\eta \rho \xi_b & \eta \xi_\phi - \kappa \\ -\left(\nu + \beta \rho \pi_b\right) & \beta \pi_\phi \\ -\beta \rho^2 \xi_b & \beta\left(\nu + \rho \xi_\phi\right) \end{bmatrix} \tag{29}$$

Therefore, we can guess all feedback coefficients in (26)-(28) and thus in the right hand side of (29). Then, equation (29) gives an update of these coefficients. In the next step we update the right hand side of (29) and iterate until convergence. The algorithm will converge (Lancaster and Rodman (1995)).

This method allows us to compare the solution of the commitment problem with the discretionary solution. Again, suppose the response of the private sector to debt, π_b, is given. We can guess the other feedback coefficients in the system (26)-(28) and iterate the Riccati equation (29), *but do not update* π_b. If the procedure converges, we have obtained the optimal response of the policy maker to the private sector decision, provided that the private sector responds to the Lagrange multiplier (set by the policy maker) in an optimal way. Then, we iterate the Riccati equation *once* to obtain π_b^*. A solution to the commitment problem implies $\pi_b^* = \pi_b$. The graph of $\pi_b^*(\pi_b)$ intersects the 45^o degree line in one point labelled A, see the second panel in Figure 4, and we can verify with standard methods (Söderlind (1999)) that this point is, indeed, a solution. For the base line calibration the economy is stabilized by policy in the unique equilibrium A.

Although the base line calibration delivers a stabilizing solution, note that if the fiscal feedback is weak, $0 < \mu < \mu^*$, where $\mu^* = (1-\tau_o)(1-\beta)\kappa / (\tau_o((1-\tau_o)\kappa - \zeta\theta\tau_o))$, the economy is not stabilized by policy. The optimal monetary policy still delivers a finite value of the loss function (8), but all variables exhibit slow explosion with a rate of explosion less than $1/\sqrt{\beta}$.

[11] See Appendix A.

This solution should be disregarded as it violates the assumption of a finite working week.[12]

Finally, note that equation (22) implies price stability: if $\phi_t = 0$ and $\lim_{t\to\infty} \phi_t = 0$ it follows that $\sum_{t=0}^{\infty} \pi_t = 0$.

2.2 Multiple Equilibria under Quasi-Commitment Policy

This Section studies monetary policy within a limited commitment framework. We discuss the continuum of intermediate cases between commitment and discretion. We want to understand (i) how a 'quasi-commitment bridge' links the economy with a (potentially) non-stabilizing policy under commitment and multiple policy equilibria under discretion, and (ii) whether quasi-commitment helps to eliminate some of the (multiple) equilibria.

2.2.1 Policy Equilibria

The quasi-commitment policy, as introduced in Schaumburg and Tambalotti (2007), also assumes sequential policy making. A new policy maker is appointed with a constant and exogenous probability α every period. When a new policy maker takes office, he reneges on the promises of his predecessor and commits to a new policy plan that is optimal at the time of the change. All agents understand the possibility and the nature of this change and form expectations accordingly. The private sector knows that a new policy maker will re-optimize, therefore it doubts the reliability of outstanding promises.

As in Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2010) we assume that the policy maker's tenure in office depends on a sequence of exogenous i.i.d. Bernoulli signals $\{\eta_t\}_{t\geq 0}$ with $E[\eta_t] = \alpha$. If $\alpha = 1$ the policy authority acts under full discretion and every period a new policy maker arrives in office and re-optimizes the planning problem. If $\alpha = 0$ the policymaker stays in office infinitely long and keeps his promises.

Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2010) demonstrate that the optimization problem under limited commitment can be expressed by the following Lagrangian

$$\begin{aligned} \mathcal{L}^{qc} \;=\; & \sum_{t=0}^{\infty} (\beta(1-\alpha))^t \Big(\frac{1}{2}\left(\pi_t^2 + \lambda c_t^2 + \beta\alpha S b_{t+1}^2\right) + \xi_{t+1}\left(\rho b_t - \eta c_t - b_{t+1}\right) \\ & + \phi_{t+1}\left(\pi_t - \kappa c_t - \nu b_t - \beta(1-\alpha)\pi_{t+1} - \beta\alpha\pi_b b_{t+1}\right)\Big) \end{aligned} \quad (30)$$

[12] This result was shown in a similar model in Schmitt-Grohe and Uribe (2004) and in Kirsanova and Wren-Lewis (2011).

for $0 \leq \alpha < 1$. The first order conditions are

$$0 = \beta\alpha S b_t - \xi_t + \rho\beta(1-\alpha)\xi_{t+1} - \nu\beta(1-\alpha)\phi_{t+1} - \beta\alpha\pi_b\phi_t, \quad (31)$$

$$0 = \pi_t + \phi_{t+1} - \phi_t, \quad (32)$$

$$0 = \lambda c_t - \eta\xi_{t+1} - \kappa\phi_{t+1}, \quad (33)$$

$$0 = \rho b_t - \eta c_t - b_{t+1}, \quad (34)$$

$$0 = \beta(1-\alpha)\pi_{t+1} + \beta\alpha\pi_b b_{t+1} + \kappa c_t + \nu b_t - \pi_t, \quad (35)$$

for $t \geq 0$, initial conditions $b_0 = \bar{b}$ and $\phi_0 = 0$, and the transversality condition $\lim_{t\to\infty} b_t < \infty$.

These first order conditions are similar to those for commitment, but depend on parameters, $\{\pi_b, S\}$, that describe the solution to the corresponding optimization problem under discretion.

We can plot the solutions to this system using the approach as in Section 2.1.2. The solution to (31)-(35) can be written in form of (26)-(28). The corresponding matrix Riccatti equation is similar to (29), but its coefficients depend on $\{\pi_b, S\}$. We can use a similar solution method to find the number of equilibria: suppose we guess the response of the private sector to the state variable, π_b. Then, we can solve the policy maker's problem under discretion and find the 'first guess' of S. In the next step we iterate the Riccati equation, but do not update π_b. If the procedure has converged, we iterate it once to obtain the update π_b^*. Solutions to the system (31)-(35) will be among the points where $\pi_b^* = \pi_b$.

For the base line calibration of $\alpha = 1/2$ (which implies an average regime duration of two quarters) the graph of $\pi_b^*(\pi_b)$ intersects the 45^o degree line in three points labelled A, B and C, see the third panel in Figure 4.[13] Therefore, if we move from pure discretionary policy to a policy maker who stays in office on average for two periods all three equilibria survive.

We now discuss how this result arises. Note that if $\alpha = 1$, the policy maker defaults with certainty every period. Then, the Lagrangian (30) takes the form of (13) and we have the problem of discretionary optimization. However, the first order conditions for the limited commitment optimization problem (31)-(35) are left-discontinuous at point $\alpha = 1$. System (31)-(35) does not collapse to (14)-(18), because the past-period constraints still bind for any $\alpha < 1$. Taking the limit $\alpha \to 1$ in system (31)-(35) does not eliminate ϕ_s – the Lagrange multiplier on the previous-period constraint – in equation (32). Because for any $\alpha < 1$ the private sector does not expect the occurrence of default with certainty in the next period, this property holds at the limit and implies discontinuity of the first order conditions.

[13] Again, we can verify with alternative methods that these are indeed solutions to the optimization problem.

In a next step we investigate the dynamic properties and the uniqueness of the solution at the limit $\alpha \to 1$. By continuity these properties will hold for large enough $\alpha < 1$. After the straightforward substitution of ξ_t, c_t and π_t the first order conditions (31)-(35) collapse to the following system

$$b_{t+1} = \frac{\lambda\rho + (\kappa\rho + \nu\eta)(\kappa - \beta\eta\pi_b)}{\lambda + S\beta\eta^2 + (\kappa - \beta\eta\pi_b)^2} b_t - \frac{\eta(\kappa - \beta\eta\pi_b)}{\lambda + S\beta\eta^2 + (\kappa - \beta\eta\pi_b)^2}\phi_t \tag{36}$$

$$\phi_{t+1} = -\frac{\nu(\lambda + S\beta\eta^2) + \beta\rho(\lambda\pi_b + S\kappa\eta)}{\lambda + S\beta\eta^2 + (\kappa - \beta\eta\pi_b)^2} b_t + \frac{\lambda + S\beta\eta^2}{\lambda + S\beta\eta^2 + (\kappa - \beta\eta\pi_b)^2}\phi_t \tag{37}$$

for $t \geq 0$, where the coefficients depend on the solution to the corresponding discretionary problem, π_b and S.

All variables in system (36)-(37) are predetermined with initial conditions $\phi_0 = 0$ and $b_0 = \bar{b}$. Therefore for a given solution to the discretionary problem $D = \{\pi_b, S\}$ we can construct a unique path $P = \{b_t, \phi_t | D\}_{t\geq 0}$. The limiting case of first order conditions to the quasi-commitment optimization problem will have as many solutions as the corresponding discretionary optimization problem. Because there are three different discretionary equilibria there are three distinct sets $D^j = \left\{\pi_b^j, S^j\right\}$, $j = 1, 2, 3$ for the base line calibration. Therefore, three paths $P^j = \left\{b_t, \phi_t | D^j\right\}_{t\geq 0}$ satisfy the system (36)-(37). We plot the case $\alpha \to 1$ in the fourth panel in Figure 4. The $\pi_b^*(\pi_b)$ line intersects the 45^o degree line in three points, which are the same points as under pure discretion.[14]

However, system (31)-(35) describes the dynamics of the economy in which, although it is expected that new policy makers arrive in office with probability α and renege on the promises of their predecessors, defaults never happen in the realized history and therefore the Lagrange multiplier ϕ_s is never reset to zero for $s > t$. The left-discontinuity of the first order conditions at $\alpha = 1$ arise because for any $\alpha < 1$ the realized reoptimization may never happen, but it happens with certainty for $\alpha = 1$. If the consequent policy makers do reset ϕ_s to zero with probability α, the dynamic properties of the economy are continuous at point $\alpha = 1$.

For a given $\alpha < 1$ the probability of the realized history with no default occurring in the past K periods tends to zero with growing K. In this case the stability properties of system (31)-(35) in each quasi-commitment equilibrium are different from the stability properties of the system that describes the expected evolution of the economy in the same equilibrium. In particular, in the limiting case $\alpha \to 1$ the system (36)-(37) has in two of the three equilibria one eigenvalue outside the unit circle. However, because the policy maker almost surely resets the Lagrange

[14] The shape of $\pi_b^*(\pi_b)$ is different than in Panel I because we take into account the Lagrange multipliers when computing $\pi_b^*(\pi_b)$.

multiplier ϕ_s to zero in every period $t > 0$, the expected evolution of the economy is described by the following system:

$$b_{t+1} = \frac{\lambda\rho + (\kappa\rho + \nu\eta)(\kappa - \beta\eta\pi_b)}{\lambda + S\beta\eta^2 + (\kappa - \beta\eta\pi_b)^2} b_t, \quad (38)$$

$$\phi_t = 0. \quad (39)$$

We obtain equation (38) which also describes the evolution of debt under pure discretion. The evolution of the economy is a stationary process in every discretionary equilibrium D^j.

More generally, the expected evolution of the economy following an initial disturbance is obtained by taking averages of all possible evolutions, integrated over the distribution of the corresponding re-optimization draws. The expected evolution of the economy is described by the following transition matrix: $\alpha \begin{bmatrix} M_{yy} & 0 \\ 0 & 0 \end{bmatrix} + (1-\alpha) M$ in (52), see the Appendix for details. The stability properties of this transition matrix, which represent the dynamic properties of the economy under a limited commitment policy, are different from the stability properties of the system (36)-(37). This system describes only one of many possible re-optimization histories.[15]

This inconsistency between expected and observed paths can destabilize the economy (and ruin the agent's finances). However, this does not happen in all equilibria. If defaults do happen and for every time $t > 0$ there is at least one period $s > t$ when ϕ_s is reset to zero, then there are no issues with dynamic instability of the economy in any equilibrium. These defaults also ensure that if there is less-than-full precommitment, there is no price stability under limited commitment policy.

If no reoptimizations happen, while they are expected by the private sector our numerical analysis shows that the policy maker can control the economy only in equilibrium A for all relevant parameter values. In equilibrium C the absence of re-optimizations implies an unbounded cost of controlling the inflation expectations of the private sector, as we illustrate in the next section. Although this instability can be used as a reason to discriminate against all but equilibrium A, such a criterion would eliminate equilibrium C for all $\mu > 0$, despite its empirical relevance documented in e.g. Davig and Leeper (2006) and Favero and Monacelli (2005). Making the probability of default endogenous might produce a less crude stability-based criterion which would not necessarily rule out equilibrium C for the whole range of relevant parameters. The observed property only suggests that policy in equilibrium A is robust to a (infinitely) long sequence of unexpected events.

[15] Using an analogy with a roulette game, system (36)-(37) describes the history when 'red' never realizes while it is expected – and it is bet on – with probability 1/2.

Finally, the expected evolution of the economy under limited commitment policy is described by a stationary process in all three equilibria, A, B and C in Panel III. Panels II, V and VI in Figure 4 demonstrate that with smaller probability of default only one limited commitment equilibrium survives. Figure 4 plots equilibria for a particular value of the fiscal feedback parameter, $\mu = 0.075$. We shall demonstrate in the next section that multiple equilibria survive for a substantial degree of precommitment only if μ is relatively small.

2.2.2 Impulse Responses

In this Section we look at the impulse response functions to understand the dynamics of our model under a limited commitment policy better. We now use the stochastic version of the model to illustrate the effects of limited commitment on the shock transmission mechanism. As a benchmark we also plot the impulse responses of the two discretionary equilibria, A and C, and under full commitment.

In Figure 1 we show the responses of key variables to a positive unit cost push shock. Under commitment (the blue dotted line) the policy maker engineers a fall in private consumption, which will dampen marginal costs. However, in contrast to discretion, the policy maker keeps consumption for several periods below the steady state. This policy allows the policy maker to lower expected future inflation and ensures price stability in the long run. Government debt initially increases due to the fall in consumption, but is brought back to the steady state with higher taxes.

Discretionary equilibria A and C can be described as policy regimes under active and passive monetary policy. If the fiscal feedback parameter μ is relatively large then an increase in public debt is practically eliminated by fiscal policy within few periods and the policy maker can focus on inflation stabilization. The equilibrium behavior of the discretionary monetary policy maker and of the private sector is, therefore, similar to the one in the standard New Keynesian model. The policy maker cuts consumption to lower marginal costs today and to place downward pressure on inflation. Due to the decrease in consumption, government tax revenues will fall, which leads to a rise in government debt. In subsequent periods the tax rate increases to guarantee fiscal solvency. This is a low-inflation-volatility equilibrium as the firms set relatively low inflation anticipating low consumption in the future. In equilibrium C the fiscal feedback parameter μ is relatively small. Therefore fiscal policy can not ensure fiscal solvency and monetary policy remains passive. In this case monetary policy cannot decrease marginal costs by much in the initial period, because the accompanied fall in consumption would result in a large accumulation of government debt, due to the lower tax base. Because in this equilibrium the monetary policy maker has to ensure fiscal solvency, he rises consumption after the first period and therefore also tax revenues. This policy ensures that government debt will be stabilized. However, this implies a high-inflation-volatility equilibrium as firms set inflation relatively high, reacting to anticipated high demand in the future.

Following Schaumburg and Tambalotti (2007) we plot three different types of impulse responses under quasi-commitment policy. We set $\alpha = 1/2$, which implies an average regime duration of two quarters.

Panel I of Figure 1 shows the impulse response functions of Type (i). These impulse responses demonstrate the evolution of the economy if no reoptimization happens over the horizon of interest, while the private sector expects them to happen every period with probability 1/2. In this scenario a central banker stays in office unexpectedly long, which becomes more and more unrealistic over time. To generate impulse responses we use the transition matrix given by conditions (31)-(35). Similar to discretion we plot the two quasi-commitment equilibria A and C. We use solid and dash-dotted lines correspondingly. Compared to the full commitment policy, quasi-commitment policy in the active monetary policy equilibrium A delivers a stronger and longer lasting decrease in consumption. As reoptimizations are expected to happen the price setters expect future policy makers to increase consumption and therefore expect a high inflation in the future. Therefore, if the policy maker wants to exploit private sector expectations he has to pay a higher cost in from of a stronger recession. In the absence of reoptimizations this results in stronger future deflation and higher debt, compared to commitment.

Type (i) impulse responses under quasi-commitment policy in equilibrium C are explosive. In this case the 'passive' monetary policy is not able to stabilize inflation, while trying to keep debt under control. After the shock occurred the policy maker cannot move consumption by much, since he has to avoid excessive debt accumulation. This is a similar behaviour as in discretionary equilibrium C. Because the private sector expects defaults in the future and hence high future inflation, inflation can only be controlled with low demand. However, lower consumption would result in excessive debt accumulation. Therefore negative consumption unwinds the attempt of the central bank to ensure fiscal solvency and the economy exhibits explosive behavior. As the fourth chart in the first panel shows, the Lagrange multiplier ϕ_s which measures the shadow price of controlling the private sector inflation expectations is much higher in equilibrium C and explodes with time.[16] The result is not surprising, given that the monetary policy maker

[16] This Lagrange multiplier is set on the Phillips curve in the optimization problem of the policy maker.

has to control debt in the passive equilibrium. This task becomes incompatible with inflation stabilization if expected defaults do not happen.

Impulse responses of Type (ii) in Panel II of Figure 1 characterize a more typical behavior of the economy under quasi-commitment. Suppose reoptimizations happen in periods 2, 3, 6 and 8 after the initial shock. In each of these periods the reoptimizing policy maker reneges on the plan of its predecessor. When the policy maker defaults on the promises of his predecessor, he resets the predetermined Lagrange multiplier to zero. The policy maker takes this opportunity to end the promised recession of his predecessor and raises consumption back to its initial level. The increase in consumption also leads to a faster reduction of government debt.

Interestingly, while in Schaumburg and Tambalotti (2007) the quantitative effect of a reoptimization is comparable to the effect of the initial shock, our model generates much smaller effects in both quasi-commitment equilibria; the jumps are much smaller. When the policy maker reneges on previous promises and reoptimizes, it faces the accumulated level of debt. The stock of debt serves as consumption smoothing vehicle and accumulates very slowly under full commitment. The ability of the (quasi-) committing policy maker to manipulate private sector expectations to some extent reduces the need to cut debt abruptly. In other words, the presence of the debt stock works as a commitment device and this results in relatively small values of the Lagrange multiplier, and relatively small costs of its resetting.

Type (iii) impulse responses (Panel III in Figure 1) are the ex ante averages of all the possible conditional IRFs integrated over the distribution of the corresponding reoptimization draws. Therefore they demonstrate the expected evolution of the system following the initial shock. Naturally, they are in between the IRF of the respective discretionary equilibria and the IRF under full commitment.

2.3 Equilibrium Selection and Welfare Analysis

In this section we find the minimum degree of precommitment which is required to select the best equilibrium.

The crucial parameter which generates multiplicity is the fiscal feedback on debt, μ. We first demonstrate this by plotting welfare levels for different values of fiscal feedback parameter μ and for different degrees of precommitment α, see the first panel in Figure 2. Following Schaumburg and Tambalotti (2007) we re-scale the welfare values by normalizing the welfare loss under the best discretionary equilibrium to one. This gives a clear picture of the relative gain in welfare for different values of α and also makes our results directly comparable.

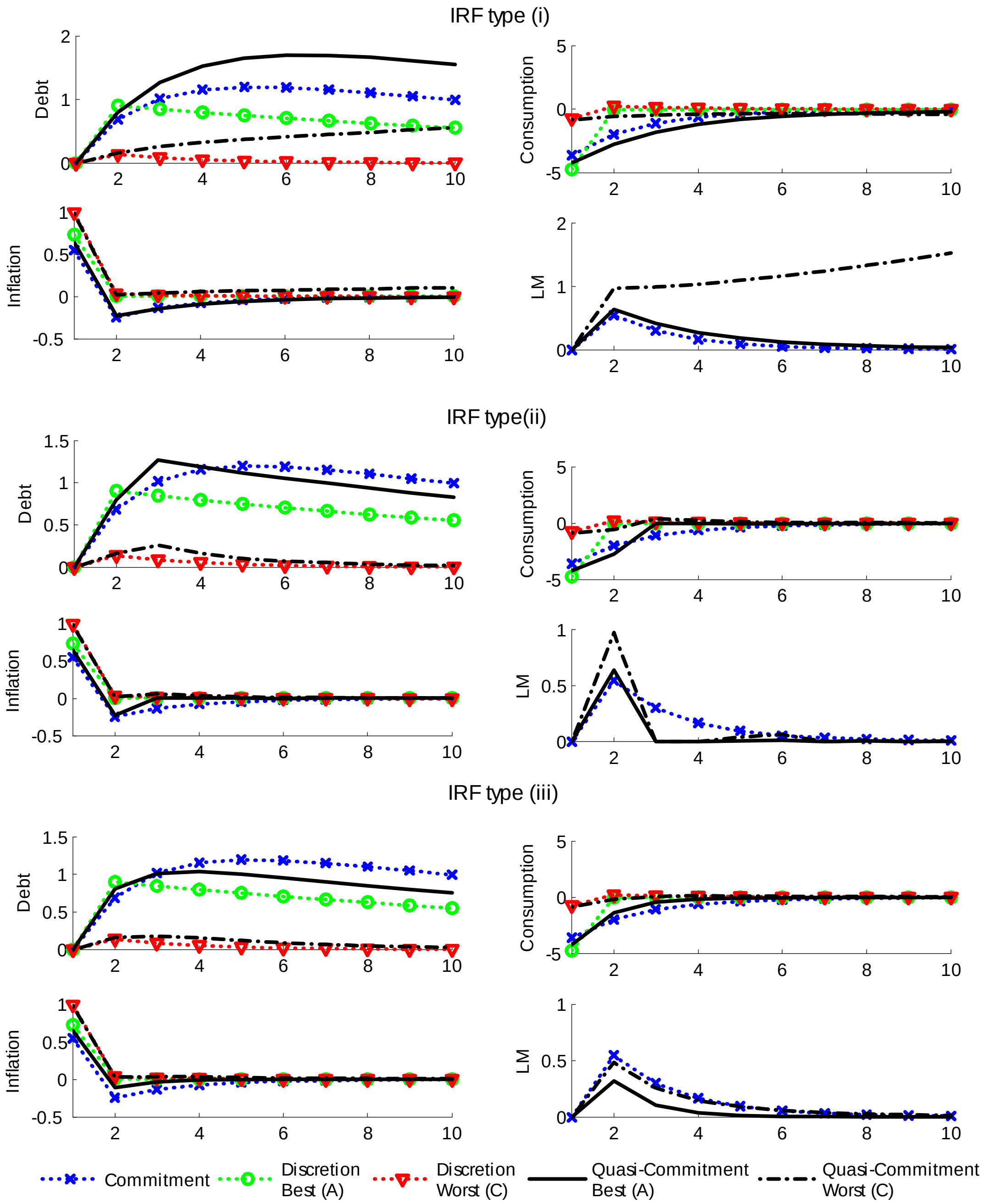


Figure 1: Impulse Responses to a 1% cost push shock in the model with government debt

The line for $\alpha = 1$ demonstrates that for $\mu = 0.075$ we have three discretionary equilibria labelled A^d, B^d and C^d and the loss is the smallest in equilibrium A^d. If μ is greater than some threshold, i.e. fiscal policy controls debt tightly, then only the best equilibrium A survives. If fiscal policy does not control debt or controls it too weakly then only the worst equilibrium C survives (see Blake and Kirsanova (2008)).

Lines for $\alpha = 1/2$, $\alpha = 1/8$ and $\alpha = 1/100$ plot the welfare losses for the respective quasi-commitment regimes. We use solid lines if the quasi-commitment policy is stabilizing and dotted lines otherwise. It is apparent that if the fiscal feedback μ is very small than the economy is not stabilized in equilibria A and B, but stabilized in equilibrium C. Under full commitment, $\alpha = 0$, the economy is non-explosive in $\mu = 0$ and is stabilized for every $\mu > \mu^*$. So, for every degree of precommitment there are at least two determinate regimes, for small and large values of the fiscal feedback parameter μ.

These results demonstrate that with higher degree of precommitment the area of multiplicity is quickly reduced. For a given value of fiscal feedback μ we can look for the minimum amount of precommitment required to select the best equilibrium. The recent empirical evidence suggests that, although the extent of feedback from debt to taxes appears to vary across countries and time, the value $\mu = 0.05$ seems to be on the lower side of such estimates.[17] It is apparent that 8 quarters of precommitment is enough to guarantee the uniqueness of equilibrium in this case. Longer periods of precommitment ensure the uniqueness even for very small values of fiscal feedback.

We can also comment on the size of welfare gain, relative to results found in the literature. Schaumburg and Tambalotti (2007), who solve a standard NK model without government debt, demonstrate that only a small increase in the degree of precommitment leads to a substantial welfare gain. In our model a reduction of the probability of default to $\alpha = 1/2$ does not eliminate any equilibria for $\mu = 0.075$ and their relative welfare-related ranking remains the same. The initial gap between the loss in the best discretionary equilibrium A and commitment is nearly halved. A further reduction in α demonstrates that our result is consistent with Schaumburg and Tambalotti (2007): the gains from even minimal levels of credibility are substantial and the effect of credibility is clearly non-linear. Furthermore, 95% of the gains are produced after about seven years, see panels II and III in Figure 2.

Debortoli and Nunes (2010) solve a non-linear model with flexible prices and find a quali-

[17] Coenen and Straub (2005) and Forni et al. (2009) estimate a tax rule for the Euro Area and find a strength of response of about 0.3 percentage points. Tax and spending rules for the post-1960 period in the US were estimated in Leeper et al. (2010) who find a relatively weak reaction of labour taxes to debt of about 0.05 percentage points.

tatively different result: the loss is reduced only slowly with higher degrees of precommitment. Although our model is quite different from theirs, our results for the *worst* equilibrium C are similar. For example, if $\mu = 0.015$ then all three equilibria still exist for $\alpha = 1/50$. For this degree of precommitment the initial gap between the loss in the *worst* discretionary equilibrium C and under commitment is reduced only by 5%. Panels II and III demonstrate that an increase in the degree of precommitment does not always result in large welfare gains, so the results in Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2010) do not necessary contradict each other.

Finally, we compute the optimal value of the fiscal feedback parameter μ^{opt}. We define μ^{opt} as the value which delivers the minimum social loss if the economy is hit by a cost-push shock. We mark μ^{opt} with stars in Figure 2. Under the full commitment the minimum loss is achieved at the left boundary of the stable regime , $\mu^{opt} = \mu^{*}$. If some (limited) precommitment is possible then μ^{opt} is an interior point of regime A, but only slightly bigger than the point of discontinuity between regimes $\mu^{*}(\alpha)$. Overall, optimal fiscal feedback is small. This extends the results in Kirsanova and Wren-Lewis (2011) to a wider class of policy equilibria.

3 The Model of a Small Open Economy with Incomplete Financial Markets

We demonstrate the robustness of the results on multiplicity and equilibrium selection by presenting an example of a very different model. We use a standard model by Galí and Monacelli (2005), but we allow for a non zero current-account balance by including incomplete financial markets using a framework proposed by Benigno (2001).[18] Since the country is small its economic performance and its domestic policy decisions do not have any impact on the rest of the world. Both economies are populated by a continuum of infinity-living households, which consume two goods. One is produced domestically and the other good is imported from the foreign country. The law of one price holds, but deviations from purchasing power parity arise due to the existence of home bias in consumption. Monopolistically competitive firms produce via a production function that depends only on labor and these goods are combined via a Dixit and Stiglitz (1977) technology to produce aggregate output, $\hat{Y}_t$, which is allocated to either private consumption, $\hat{C}_t$, or government spending $\hat{G}_t$. Households choose their consumption and leisure,

[18] In a very similar model De Paoli (2009) analyzes the welfare effects of incomplete financial markets under Ramsey (precommitment) policy and Erceg and Lopez-Salido (2007) investigate how current account dynamics affects the transmission mechanism of domestic shocks.

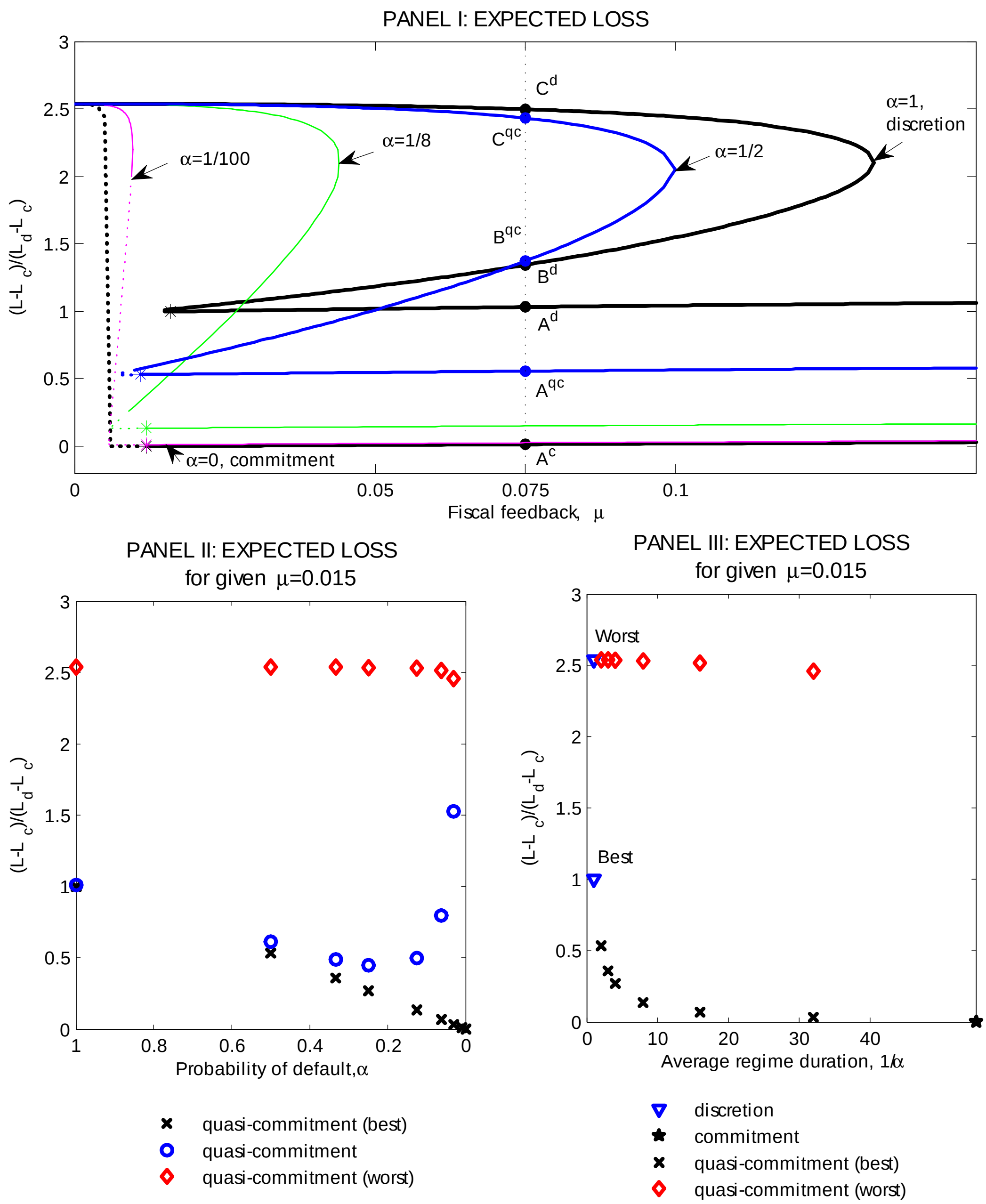


Figure 2: Welfare loss for model with government debt

and can transfer income through time through trading net foreign assets, $\hat{d}_t$. Firms set prices subject to a Calvo (1983) price rigidity and monetary policy uses the nominal interest rate, $\hat{\imath}_t$, as its instrument.

The final log-linearised system of first order conditions consists of the following equations:[19]

$$\hat{C}_t = \hat{C}_{t+1} - \frac{1}{\sigma}\left(\hat{\imath}_t - \mathcal{E}_t\hat{\pi}_{H,t+1} - \alpha\left(\hat{S}_{t+1} - \hat{S}_t\right)\right) \tag{40}$$

$$\pi_{Ht} = \beta\mathcal{E}_t\pi_{Ht+1} + \frac{(1-\theta)(1-\theta\beta)}{\theta}\left(\sigma\hat{C}_t + \phi\hat{Y}_t + \alpha\hat{S}_t - (\phi+1)\hat{A}_t\right) + \eta_t \tag{41}$$

$$\hat{Y}_t = (1-\alpha)(1-\gamma)\hat{C}_t - \eta\alpha(\alpha-2)(1-\gamma)\hat{S}_t + \alpha(1-\gamma)\hat{C}_t^* + \gamma\hat{G}_t \tag{42}$$

$$\hat{\imath}_t = \hat{\imath}_t^* + \hat{S}_{t+1} - \hat{S}_t + \pi_{H,t+1} - \pi_{t+1}^* - \delta\hat{d}_{t+1} \tag{43}$$

$$\beta\hat{d}_{t+1} = \hat{d}_t + \hat{Y}_t - \alpha(1-\gamma)\hat{C}_t - \alpha(1-\gamma)\hat{S}_t - \gamma\hat{G}_{H,t} \tag{44}$$

(40) is the standard Euler equation and describes the consumption smoothing behavior of the private sector in the small open economy. The New Keynesian Phillips curve is given by equation (41) and relates current inflation π_{Ht} to expected future inflation, real marginal costs and a cost push shock. The national income identity equation (42) states that domestic output is positively related to government spending and consumption of the rest of the world, but it is negatively related to improvements in the terms of trade, because $\alpha < 1$ and $\eta > 0$. The described setup allows for deviations from the uncovered interest parity (equation (43)). Owing to the incomplete market setting the Euler equation is not sufficient to determine the dynamics of aggregate demand. We also need equation (44) to pin down the dynamics of the net foreign assets, where the portfolio cost parameter δ influences the evolution of the net foreign assets through its impact on the terms of trade.

We assume that the social welfare function is captured by the following (ad hoc) discounted quadratic loss function:[20]

$$W_t = \frac{1}{2}\sum_{s=t}^{\infty}\beta^{s-t}\left(\pi_{H,s}^2 + \omega y_s^2\right)$$

where all variables are denoted in gap form.

Note that after some straightforward substitution the real marginal cost can be written as

$$mc_t = (\sigma + \phi(1-\alpha)(1-\gamma))\hat{C}_t + \alpha(1+\phi\eta(2-\alpha)(1-\gamma))\hat{S}_t + \phi\alpha(1-\gamma)\hat{C}_t^* + \phi\gamma\hat{G}_t - (\phi+1)\hat{A}_t$$

[19] In line with Benigno (2001) and De Paoli (2009) we assume a symmetric steady state, which implies that the net foreign asset position is zero in the steady state. Note that we define analogous to Benigno (2001) $\hat{d}_t$ as $\frac{d_t - d}{Y}$ and $\delta = -\phi'(d)Y$. For the derivation of the model see e.g. Himmels and Kirsanova (2010).

[20] Note that the form of the loss function is not essiential to generate muliplicity, see e.g. King and Wolman (2004).

(45)

It is apparent that movements in consumption and the terms of trade are substitutable for the control of inflation. Hence, there are multiple paths that bring back inflation on target. These paths are associated with different monetary policies and have different welfare rankings. Monetary policy can have an effect on inflation in equation (45) through two distinct channels. To reduce marginal cost directly the central bank can cut the interest rate, which results in an decrease in consumption via the Euler equation (40) . However, a lower interest rate leads also to an improvement in the terms of trade via the decrease in consumption and equation (43). Therefore, movements in consumption and the terms of trade reinforce each other in having an impact on marginal cost. This is a from of strategic complementarity as e.g. defined in **?**. Note that consumption and the terms of trade solely determine the evolution of net foreign assets. As a consequence, the desirability of each policy from the perspective of the period t policymaker turns on how future policymakers are expected to respond to the evolution of net foreign assets, which is again determined by the evolution of consumption and the terms of trade.

3.1 Multiple Equilibria under Limited Commitment

This model produces multiple equilibria, depending on the expectations of the private sector about the adjustment process of the economy to the shock. Discretionary equilibrium A is the standard result. In discretionary equilibrium B the private sector expects the central bank to be *'passive'*, which implies that the policy maker is not able to control inflation. Even if this equilibrium has a far worse welfare outcome, it is less costly for the central bank to validate equilibrium B than paying the costs of moving the economy to equilibrium A.

We plot impulse response of type (i), (ii) and (iii) for $\alpha = 1/2$, which implies an average regime duration of 2 quarters. Similar to the model with government debt type (i) impulse responses for the worst quasi-commitment equilibrium C are explosive and the impulse responses of type (iii) are in between the IRF of the respective discretionary equilibria and the IRF under full commitment.

When a positive cost-push shock hits the system the policymaker rises the interest rate to reduce marginal costs via a consumption is cut. The fall in output improves the terms of trade. Households use the current account as a risk-sharing tool and sell foreign assets to dampen the decline in consumption. Therefore the country will run a current account deficit. In subsequent periods output and consumption converges to their steady states and the price level converges as well to the steady state through periods of (very small) deflation.

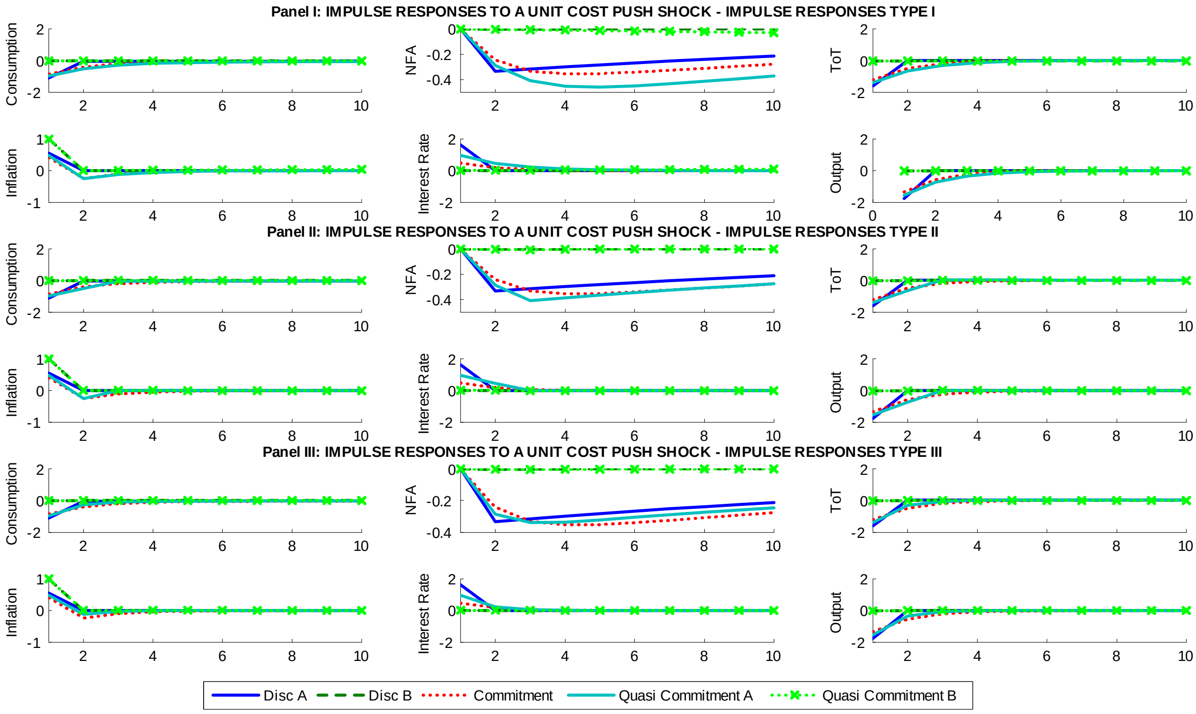


Figure 3: Impulse Responses for a 1% Cost Push Shock in the Open Economy Model

3.2 Equilibrium Selection and Welfare Analysis

The crucial parameter for multiplicity is the intermediation cost δ. In the first panel of figure 3.2 we plot the welfare loss as a function δ for several degrees of precommitment. Again, we re-scale the welfare values by normalizing the welfare loss under the best discretionary equilibrium to one.

The line for $\alpha = 1$ confirmes that for a wide range of δ there are three discretionary equilibria. Lines for $\alpha = 1/2$ and $\alpha = 1/8$ plot the welfare losses for the respective quasi-commitment regimes. Note that δ denote the costs for home agents to borrow bonds in a foreign currency. According to recent studies δ lies between 0.001 and 0.01, which can be interpreted as a 10/100 basis point spread of the domestic rate over the foreign rate. It is apparent that even if this model is very different to the model with government debt, the results are quite similar. A policy maker who is able to commit for 2 years eliminates already multiplicity for $\delta = 0.01$ and and an average regime duration of less than 5 years eliminates multiplicity for $\delta = 0.001$.

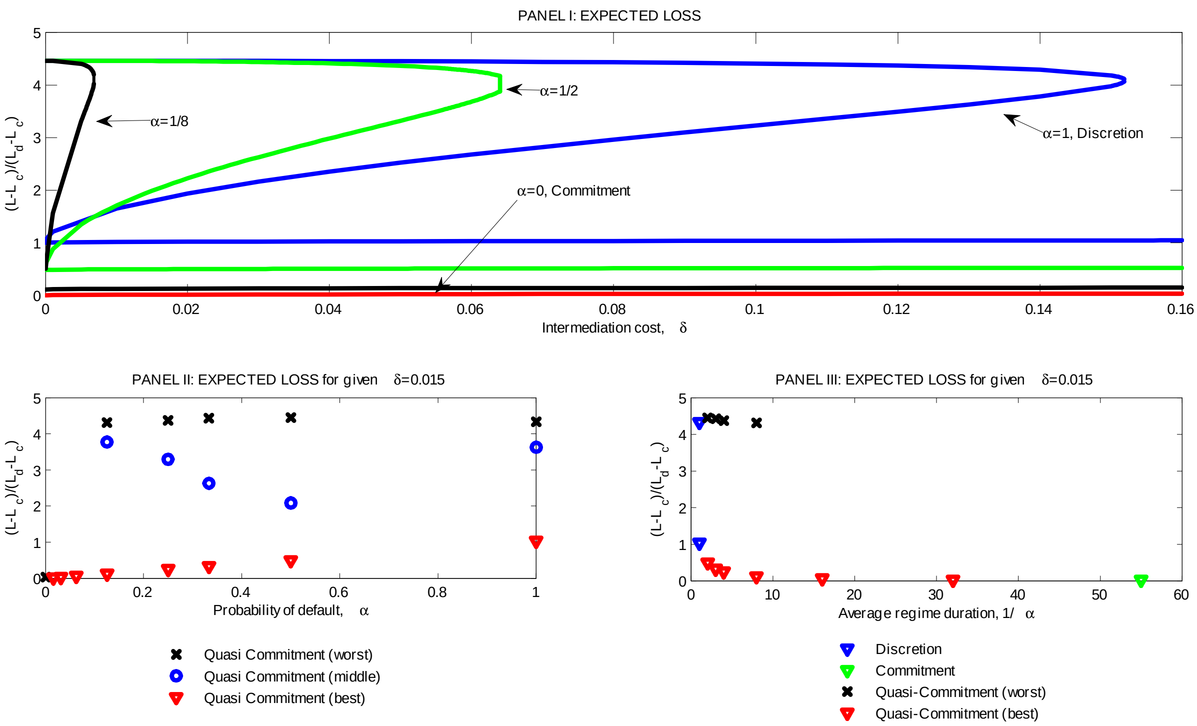
Figure 3.2 is generated analogous to the panels II and III of Figure 2. As in the model with government debt a small increase in the degree of precommitment leads to a substantial welfare gain in the good equilibrium, but the results are less clear in the worst equilibrium

4 Conclusion

In this paper we study monetary policy with limited commitment using two simple New Keynesian models. We demonstrate the existence of expectations traps similar to those existing under pure discretionary policy for a New Keynesian model with government debt and a small open economy model with incomplete financial markets. Because the private sector expects eventual re-optimizations to happen the current policy maker formulates its policy based on the forecast of the private sector about future policy makers' behaviour. We find that there can be at least as many limited commitment policy equilibria as in the corresponding discretionary policy problem.

We demonstrate that although multiple equilibria can survive a substantial degree of precommitment, nearly for all parameter values only a relatively reasonable degree of precommitment is required to select among them and achieve uniqueness.

Depending on which equilibrium prevails, an increase in the degree of precommitment may result in large or small welfare gains. Furthermore, we find that a predetermined state variable reduces the cost of default.


PANEL I: EXPECTED LOSS
(L-L_c)/(L_d-L_c)
α=1/8
α=1/2
α=1, Discretion
α=0, Commitment
Intermediation cost, δ
PANEL II: EXPECTED LOSS for given δ=0.015
Probability of default, α
Quasi Commitment (worst)
Quasi Commitment (middle)
Quasi Commitment (best)
PANEL III: EXPECTED LOSS for given δ=0.015
Average regime duration, 1/ α
Discretion
Commitment
Quasi-Commitment (worst)
Quasi-Commitment (best)

Finally, we find that different equilibria imply different stability properties of the economy if an infinitely long sequence of no re-optimizations realizes. In this case, we demonstrate for both models that the policy maker is only able to control the economy in the Pareto-dominant equilibrium. A further research might generalize this property to the whole class of LQ RE models with multiple equilibria under limited commitment policy.

A Commitment FOCs in Form of Riccati Equation (29)

System (21)-(25) can be written as

$$\begin{bmatrix} 0 & 0 & \eta \\ 0 & \beta & 0 \\ 0 & 0 & \rho\beta \end{bmatrix} \begin{bmatrix} c_{t+1} \\ \pi_{t+1} \\ \xi_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & -\kappa \\ -\nu & 0 \\ 0 & \nu\beta \end{bmatrix} \begin{bmatrix} b_t \\ \phi_t \end{bmatrix} + \begin{bmatrix} \lambda & \kappa & 0 \\ -\kappa & 1 & 0 \\ 0 & -\nu\beta & 1 \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \\ \xi_t \end{bmatrix} \tag{46}$$

$$\begin{bmatrix} b_{t+1} \\ \phi_{t+1} \end{bmatrix} = \begin{bmatrix} \rho & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} b_t \\ \phi_t \end{bmatrix} + \begin{bmatrix} -\eta & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \\ \xi_t \end{bmatrix} \tag{47}$$

Substitute (26)-(28) into both sides of (46) and use (47) to substitute out b_{t+1}, ϕ_{t+1}. We obtain

$$\begin{bmatrix} -\left(\lambda + \eta^2\xi_b\right) & -\left(\kappa - \eta\xi_\phi\right) & 0 \\ \kappa - \beta\eta\pi_b & \beta\pi_\phi - 1 & 0 \\ -\beta\eta\rho\xi_b & \beta\left(\nu + \rho\xi_\phi\right) & -1 \end{bmatrix} \begin{bmatrix} c_t \\ \pi_t \\ \xi_t \end{bmatrix} = \begin{bmatrix} -\eta\rho\xi_b & -\kappa + \eta\xi_\phi \\ -\left(\nu + \beta\rho\pi_b\right) & \beta\pi_\phi \\ -\beta\rho^2\xi_b & \beta\left(\nu + \rho\xi_\phi\right) \end{bmatrix} \begin{bmatrix} b_t \\ \phi_t \end{bmatrix}$$

Substitution of (26)-(28) yields (29).

B Limited Commitment Policy in General LQ RE Framework

We assume a non-singular linear deterministic rational expectations model, augmented by a vector of control instruments. Specifically, the evolution of the economy is explained by the linear system

$$\begin{bmatrix} y_{t+1} \\ E_t x_{t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} [u_t] + C \begin{bmatrix} \xi_{t+1} \\ 0 \end{bmatrix}, \tag{48}$$

where y_t is an n_1-vector of predetermined variables with initial conditions y_0 given, x_t is n_2-vector of non-predetermined (or jump) variables with $\lim_{t\to\infty} x_t = 0$, u_t is a $k-$vector of policy instruments of the policy maker, and ξ_t is a vector of i.i.d. shocks with covariance matrix Σ. For notational convenience we define the n-vector $z_t = (y_t', x_t')'$ where $n = n_1 + n_2$. We assume A_{22} is non-singular.

The inter-temporal policy maker's welfare criterion is defined by the quadratic loss function

$$L_0 = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t g_t' \mathcal{Q} g_t = \frac{1}{2} \sum_{t=0}^{\infty} \beta^{s-t} \left(z_t' Q z_t + 2 z_t' P u_t + u_t' R u_t \right). \tag{49}$$

The elements of vector g_s are the goal variables of the policy maker, $g_t = \mathcal{C}(z_t', u_t')'$. Matrix $\mathcal{Q}$ is assumed to be symmetric and positive semi-definite.[21]

Schaumburg and Tambalotti (2007) and then Debortoli and Nunes (2010) demonstrate that the optimization problem can be written as

$$\min E_0 \sum_{t=0}^{\infty} (\beta\omega)^t \left(z_t' Q z_t + 2 z_t' P u_t + u_t' R u_t + \beta (1-\omega) y_{t+1}' S y_{t+1} \right) \tag{50}$$

where $\omega = 1 - \alpha$, subject to

$$\begin{aligned} y_{t+1} &= A_{11} y_t + A_{12} x_t + B_1 u_t + C \xi_{t+1} \\ \omega E_t x_{t+1} + (1-\omega) H y_{t+1} &= A_{21} y_t + A_{22} x_t + B_2 u_t \end{aligned}$$

where H and S are components of solution to the corresponding discretionary problem, $x_t = H y_t$ and the loss is $L_t(y_t) = \frac{1}{2} y_t' S y_t$.

The first order conditions to the appropriate Lagrangian

$$\begin{aligned} \mathcal{L}^{qc} = & \sum_{t=0}^{\infty} (\beta\omega)^t \left(z_t' Q z_t + 2 z_t' P u_t + u_t' R u_t + \beta (1-\omega) y_{t+1}' S y_{t+1} \right. \\ & + 2 \varphi_{t+1}' \left(A_{21} y_t + A_{22} x_t + B_2 u_t - \omega x_{t+1} - (1-\omega) H y_{t+1} \right) \\ & \left. + 2 \psi_{t+1}' \left(A_{11} y_s + A_{12} x_s + B_1 u_s + \xi_{t+1} - y_{s+1} \right) \right) \end{aligned}$$

can be written as

$$\begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & \beta A_{22}' & 0 & 0 & \beta A_{12}' \\ 0 & B_2' & 0 & 0 & B_1' \\ (1-\omega) H & 0 & 0 & \omega I & 0 \\ 0 & \beta\omega A_{21}' & 0 & 0 & \beta\omega A_{11}' \end{bmatrix} \begin{bmatrix} y_{t+1} \\ \varphi_{t+1} \\ u_{t+1} \\ x_{t+1} \\ \psi_{t+1} \end{bmatrix} \tag{51}$$

$$= \begin{bmatrix} A_{11} & 0 & B_1 & A_{12} & 0 \\ -\beta Q_{12}' & I & -\beta P_2 & -\beta Q_{22} & 0 \\ -P_1' & 0 & -R & -P_2' & 0 \\ A_{21} & 0 & B_2 & A_{22} & 0 \\ -\beta \left(\omega \mathcal{Q}_{11} + (1-\omega) S \right) & (1-\omega) H' & -\beta\omega P_1 & -\beta\omega Q_{12} & I \end{bmatrix} \begin{bmatrix} y_t \\ \varphi_t \\ u_t \\ x_t \\ \psi_t \end{bmatrix}$$

[21] It is standard to assume that R is symmetric positive definite (see Anderson et al. (1996), for example). However, since many economic applications involve a loss function that places no penalty on the control variables, we note that the requirement of $\mathcal{Q}$ being positive definite can be weakened to $\mathcal{Q}$ being positive semi-definite if additional assumptions about other system matrices are met (Clements and Wimmer (2003)). The analysis in this paper is valid for $R \equiv 0$.

Solution to this system (using Schur decomposition, for example, or iteration Riccati equation as we do in the text) can be written in the form

$$\begin{aligned}\begin{bmatrix} u_t \\ x_t \\ \psi_t \end{bmatrix} &= \begin{bmatrix} X_{uy} & X_{u\varphi} \\ X_{xy} & X_{x\varphi} \\ X_{\psi y} & X_{\psi\varphi} \end{bmatrix} \begin{bmatrix} y_t \\ \varphi_t \end{bmatrix}, \quad \begin{bmatrix} y_{t+1} \\ \varphi_{t+1} \end{bmatrix} = \begin{bmatrix} M_{yy} & M_{y\varphi} \\ M_{\varphi y} & M_{\varphi\varphi} \end{bmatrix} \begin{bmatrix} y_t \\ \varphi_t \end{bmatrix}, \qquad (52) \\ W_t\left(y_t, \varphi_t\right) &= \frac{1}{2}\left(\begin{bmatrix} y_t \\ \varphi_t \end{bmatrix}' \begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} \begin{bmatrix} y_t \\ \varphi_t \end{bmatrix} \right).\end{aligned}$$

Equation (50) yields

$$\begin{aligned}\begin{bmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{bmatrix} &= \begin{bmatrix} I & 0 \\ X_{xy} & X_{x\varphi} \\ X_{uy} & X_{u\varphi} \end{bmatrix}' \begin{bmatrix} Q_{11} & Q_{12} & P_1 \\ Q_{12}' & Q_{22} & P_2 \\ P_1' & P_2' & R \end{bmatrix} \begin{bmatrix} I & 0 \\ X_{xy} & X_{x\varphi} \\ X_{uy} & X_{u\varphi} \end{bmatrix} \qquad (53) \\ &+M' \begin{bmatrix} \beta\omega U_{11} + \beta\left(1-\omega\right) S & \beta\omega U_{12} \\ \beta\omega U_{21} & \beta\omega U_{22} \end{bmatrix} M\end{aligned}$$

A possible iterative scheme is (different order of updates is possible):

1. Guess M, X, U, as part of them we have $H = X_{xy}$, $S = U_{11}$
2. Compute an update of U using (53)
3. Solve (51) using Schur decomposition (with stability threshold as $1/\sqrt{\beta\omega}$) to find an update for X and M.

Finally, the loss is found as in Schaumburg and Tambalotti (2007).

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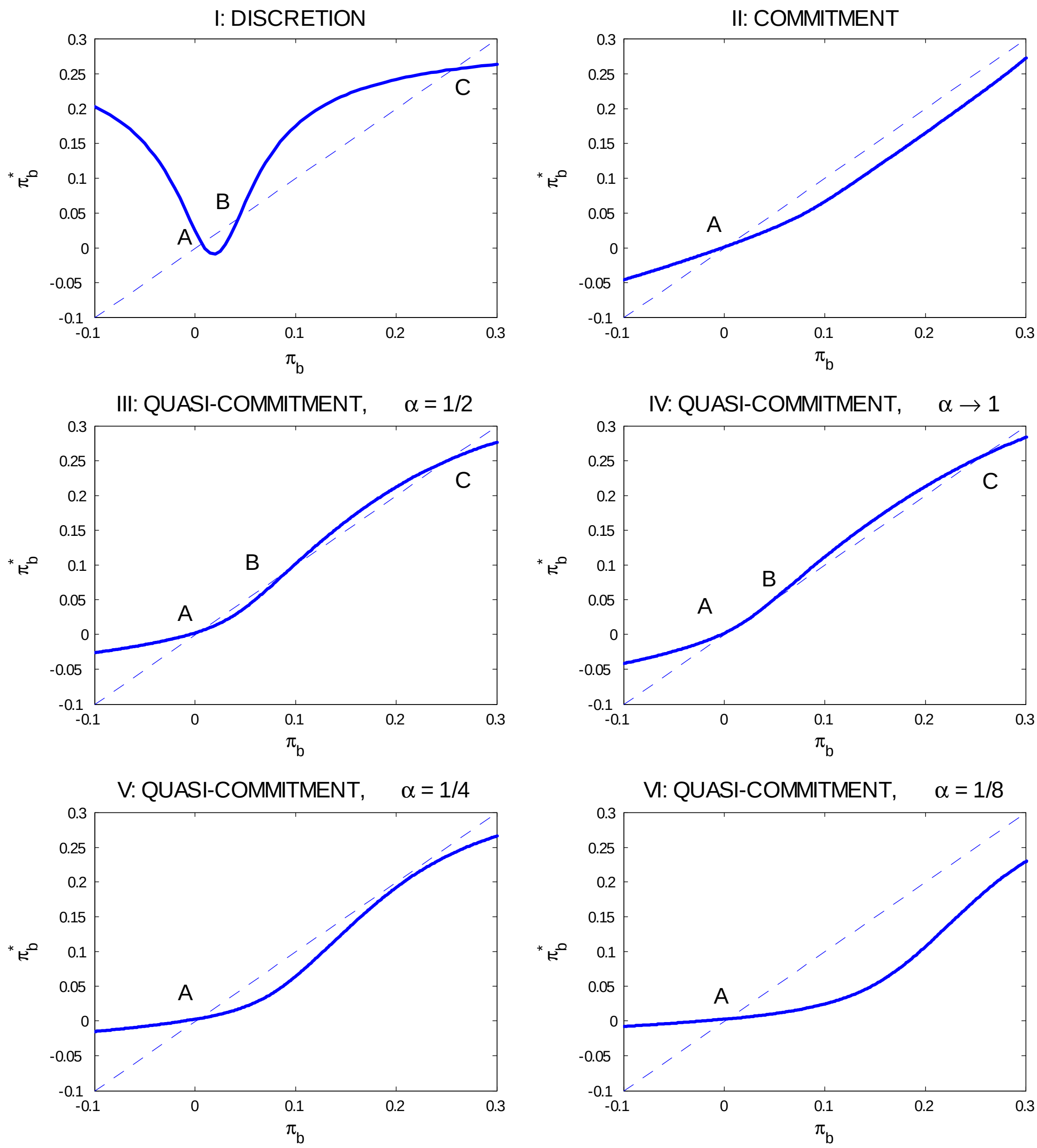

Figure 4: Multiple policy equilibria for different degrees of precommitment