Macroeconomic Fundamentals and Exchange Rate Dynamics: A No-Arbitrage Multi-Country Model

Weiwei Yin*

Bocconi University, Milan, Italy.

Abstract

This paper investigates the joint dynamics of multiple nominal exchange rates in a multi-country framework. Using a no-arbitrage macro-finance approach, information regarding macroeconomic fundamentals is employed to model exchange rate dynamics. Macroeconomic fundamentals are assumed to be determined by global (common) factors as well as by country-idiosyncratic factors. The empirical study focuses on an open economy including four countries, i.e. Germany, the UK, Japan and the US (the US dollar being the numeraire currency). Empirical evidence shows that the model is able to well characterize the joint dynamics of exchange rates, with 57%, 66% and 33% of the variations in the observed movements of the USD/DEM (EUR), the USD/GBP and the USD/JPY being explained. The model implied foreign risk premia satisfy the Fama conditions (1984) and they are counter cyclical with respect to the US economy. Moreover, global and country-idiosyncratic macroeconomic factors do exist and play very different roles in driving exchange rate dynamics and foreign risk premia.

Keywords: Multi-Country Model, Exchange Rate Dynamics, Macroeconomic Fundamentals, Global and Country-idiosyncratic Factors

JEL: F31, G12, E43

---

[*] E-mail: *weiwei.yin@phd.unibocconi.it.* First version: January, 2011; this version: September, 2011. I am very grateful for useful suggestions and comments on this project to Francesco Giavazzi, Carlo Favero, Lucio Sarno, Claudio Tebaldi, Massimo Guidolin and M. Hashem Pesaran. I would like to thank Li Junye, Andrew Meldrum as well as people who attended the Alesina winter seminar on January 19, 2011 at Bocconi University for very helpful comments.

I. Introduction

The floating nominal exchange rate is the market price of one currency converted into another. It is one of the most important factors in international economic activities, such as international trade and international investment. The question about whether exchange rate dynamics are driven by macroeconomic fundamentals or not has puzzled a great number of researchers after the seminal work by Meese and Rogoff (1983).

A large amount of models have been proposed to try to explain exchange rate dynamics by macroeconomic fundamentals. For instance, monetary models (Frenkel (1976, 1979), Mussa (1976), Bilson (1978), Dornbusch (1976)) state that the existence of a long-run equilibrium for the nominal exchange rate is a function of the differentials of money supplies and income levels between home and foreign countries. Recent studies proposing new open economy macroeconomic models (Obstfeld and Rogoff (2003)) investigate exchange rate movements using the dynamic stochastic general equilibrium approach. In particular, they solve the optimization problem in an open economy framework. However, these models cannot find empirical evidence of a close relationship between short-run exchange rate movements and macroeconomic fundamentals (Meese (1990), Frankel and Rose (1995), Engel and West (2005)). It is also worth mentioning that most of the studies on nominal exchange rates are under a two-country setting which is the simplest setting in open economy studies.

The model presented in this paper has three main differences with respect to traditional exchange rate models. First of all, this paper investigates exchange rate movements using macroeconomic fundamental information under a no-arbitrage macro-finance approach. Under this approach, the bilateral nominal exchange rate changes are endogenously determined by the ratio of stochastic discount factors between the two countries. The stochastic discount factor, also named marginal rate of substitution, is modeled through a factor representation under the no-arbitrage condition. Output, inflation and short-term interest rates represent the macroeconomic fundamentals. Real output growth directly influences the aggregate consumption of a country and thus is a key element in determining the stochastic discount factor. Inflation can also enter the stochastic discount factor via its dynamic interactions with real production (Piazzesi and Schneider (2006)). The short-term interest rate is typically viewed as a macro variable reflecting monetary policy (Duffee (2007)). This paper adopts the common specification for the stochastic discount factor used in macro-finance term structure models (Ang and Piazzesi (2003), Diebold, Rudebusch, and Aruoba (2005), Ang, Dong, and Piazzesi (2007)) and extends it to a multi-country framework. Term structure information on interest rates is included in order to help identify the time-varying market prices of risk, which in turn determine the foreign risk premium and amplify the role played by macroeconomic innovations on exchange rate changes. This is important since, ignoring the foreign risk premium or assuming it is constant, may mislead to the conclusion that exchange rate dynamics are not linked to macroeconomic fundamentals.

Secondly, the model is built under a multi-country setting. Thus, it is able to investigate the dynamics of multiple exchange rates simultaneously. Dollar exchange rates are positively correlated according to the data. Hodrick and Vassalou (2002) point out that in affine term structure models multi-country models can better explain the dynamics of exchange rates compared to two-country models. Two-country models are only able to study single exchange rate movements. Additionally, in order to study more than one exchange rate at a time, each exchange rate for each two-country case has to be separately analyzed. Therefore, inconsistency issues concerning the parameters related to the numeraire country may potentially arise.

Thirdly, in this paper, global and country-idiosyncratic macroeconomic factors setting are used to model the correlated macroeconomic fundamentals across countries. The global and local factors have been used by Ahn (2004) in the study of exchange rates dynamics as well, but these factors are latent and do not have any economic meaning. The importance of the existence of global factors in modeling exchange rate dynamics has been mentioned by Litterman and Scheinkman (1991), Hodrick and Vassalou (2002), and Sarno, Schneider and Wagner (2011). Additionally, since the prices of risks are notoriously difficult to estimate from a statistical standpoint, through this setting the number of risk price parameters in the model can be significantly reduced so that the estimation becomes tractable when compared to the setting involving country-level macroeconomic factors. Moreover, this setting allows to distinguish the different roles played by global and country-idiosyncratic macroeconomic factors in driving exchange rate dynamics and foreign risk premia.

There are some recent studies on multiple exchange rates using international term structure models with different focuses or using different methodologies. Adopting the endogenous foreign exchange risk premia arising from the no-arbitrage condition, Sarno, Schneider and Wagner (2011) focus on the properties of foreign premia and Ang and Chen (2010) concentrate on yield curve predictors on foreign premia. Graveline and Joslin (2011) concentrate on the returns of currencies as a portfolio. Bauer and Diez (2011) assume that the law of motion of exchange rate movements is exogenous.

The empirical study of this paper focuses on an open economy including four countries, i.e. Germany, the UK, Japan and the US, where the US is taken as the home country. This multi-country no-arbitrage term structure model is able to explain 57%, 66% and 33% of the variations of the observed exchange rate changes of the USD/DEM (EUR), the USD/GBP and the USD/JPY, respectively. The model-implied foreign risk premia satisfy the Fama conditions (1984) and they are counter cyclical with respect to the US economy. The macroeconomic innovations, or "news", are important in determining the exchange rate dynamics. Moreover, global and country-idiosyncratic macroeconomic factors do exist and play very different roles in driving exchange rate dynamics and foreign risk premia. Global factors drive foreign risk premia almost exclusively and account for more than half of the forecast error variance of exchange rate dynamics. In the short-run global interest rate is the dominant factor, while in the long-run global output becomes dominant in driving both exchange rate dynamics and foreign risk premia. Even though country-idiosyncratic macroeconomic factors are less important compared to global ones, they do play some role in determining the short-run exchange rate dynamics, especially the US and German interest rate, and the UK and Japanese output.

The rest of this paper is organized as follows. Section II describes the data. Section III introduces a multi-country no-arbitrage exchange rate model from a macro-finance perspective. Section IV proposes the econometric methodology, the likelihood-based estimation combined with the unscented Kalman filter. Section V presents the empirical results and discusses their economic implications. Section VI concludes.

II. Data and Preliminary Analysis

A. Data

Consider a multi-country world, with $(N+1)$ countries. The last country (the $(N+1)^{th}$ country) is the domestic country and the first N countries are the foreign countries. A $(3+1)$-country open economy case will be analyzed in the empirical study of this paper. The countries are Germany, the UK, Japan and the US. The first three are taken as the foreign countries, while the US is taken as the home country. The data is in the monthly time frequency and the sample period goes from January 1985 to May 2009.

The macroeconomic fundamentals taken into account are output growth, inflation rates and short term interest rates. The nominal exchange rates are the end-of-the-period market rates. Both exchange rates and macroeconomic data are coming from the International Financial Statistics (IFS) database, provided by the International Monetary Fund (IMF).

Output growth rates and inflation rates are the one-year percentage changes of seasonal adjusted Industrial Production Indices (line *66*) and Consumer Price Indices (line *64*), respectively. Exchange rate data are the US dollar per national currency (line *ag*). The exchange rate for the German mark after 1999 is replaced by the exchange rate of the Euro, following Corte, Sarno and Tsiakas (2009).

— Figure 1 around here —

Moreover, yield data are included in order to better identify the parameters that determine the market prices of risk, since the market prices of risk are important in modeling exchange rate dynamics. The zero-coupon bond yield data for Germany, the UK, Japan and the US are taken from the International Zero Coupon Yield Curve Dataset used by Wright (2011). We take yields with different maturities: 3 months (the shortest one in this dataset), 24 months and 60 months, which stand for the short, medium and long term yields, respectively. These three yields are commonly used to get the empirical "level", "slope" and "curvature" components, which are sufficient to capture the term structure of interest rates. In addition, short-term interest rates are proxied by 3-month zero-coupon bond yields. In order to match the unit of monthly exchange rate movements, both the macroeconomic and yield data are divided by 12 so as to get monthly equal quantities.

B. Preliminary Analysis

It is important to clarify the cross-country relationships of macroeconomic fundamentals. The correlation matrix of these variables is presented in Table 8.

— Table 8 around here —

The diagonal sub-matrices of this correlation matrix point out relevant correlations among the three groups of macroeconomic variables (output growths, inflations and short-term interest rates). The first 4×4 triangular matrix on the diagonal of this correlation matrix shows that the output growths are all positively correlated across these four countries. Among them, the highest two correlations are between Germany and Japan (53%), and between the UK and the US (53%). The second 4×4 triangular matrix on the diagonal of this matrix shows that the inflation rates are positively correlated as well. The highest correlation is between the UK and the US (78%), followed by the one between the UK and Japan (70%). The third 4×4 triangular matrix on the diagonal of the same matrix shows that short-term interest rates are also positively correlated, with the highest two correlations, equal to 84% and 80%, between the UK and Japan and between the UK and the US, respectively. The fact that macroeconomic variables are positively correlated across countries suggests that there may exist some global macroeconomic factors driving the cross-country comovements of output growth, inflation and short-term interest rates.

— Table 9 around here —

The above result is a starting point to explore deeper into the question of whether some common factors exist which drive the comovement of macroeconomic fundamentals across countries. Principle components analysis is conducted for each group of macroeconomic fundamentals (output growth, inflation and short-term interest rates). The results are reported in Table 9. In each group of macroeconomic fundamentals, the first principle component associated with the highest eigenvalue is able to explain 76%, 71% and 84% of the variations, respectively. This result implies that there should exist a global (common) factor in each group, determining the comovement of these macroeconomic variables across countries. These results provide clear evidence that global and country-idiosyncratic macroeconomic factors should be included in the setting of our model.

III. A Multi-Country No-Arbitrage Exchange Rate Dynamic Model

Consider a (*N+1*)-country world, with *N* foreign countries and *1* domestic country. Because of the important role of the US economy and its currency in the global economy activities after the Bretton Woods system collapse, the US is chosen as the home country and, correspondingly, the US dollar is the numeraire currency. Among these *N+1* countries' currencies, the sufficient amount of bilateral nominal exchange rate relationships is N, since the others can be deduced by the triangle relationship among these N bilateral exchange rates. Therefore, this paper focuses on the N bilateral exchange rates, which are the rates of the N foreign currencies against the US dollar.

The exchange rate dynamics are determined by the ratio of the stochastic discount between the home and the foreign country, following the no-arbitrage assumption and the law of one price. In this section, we first discuss the global and country-specific factor setting in a multi-country economy in subsection *A*. Next, we explain how to model stochastic discount factors which are determined by macroeconomic fundamentals in subsection *B*. After that, we proceed to model the exchange rate dynamics in subsection *C*. Finally, we present the recursive relationship which characterizes the bond pricing for each country under the affine term structure modeling framework in subsection *D*.

A. A Global and Country-Idiosyncratic Macro Factor Setting in a Multi-Country Economy

The choice of the state dynamics driving a multi-country economic system is a tradeoff. On one side, it is better to include as much macroeconomic information as possible. On the other side, it is necessary to keep the amount of the parameters as low as possible in order to be able to carry out the estimation. For this reason the macroeconomic global and country-idiosyncratic factors are introduced, since they are able to balance these two points well.

Suppose there exist three global macroeconomic factors in a (*N+1*)-country economy, output growth g_t^G, inflation π_t^G, and short-term interest rate r_t^G, which drive the comovement of macroeconomic fundamentals across countries. We construct a global factor vector G_t, where $G_t = \left(g_t^G,\ \pi_t^G,\ r_t^G\right)^T$.

For each economy i ($i = 1, 2, ..., N+1$), we assume that its underlying macroeconomic fundamental vector $X_{i,t} = \left(\tilde{g}_{i,t},\ \tilde{\pi}_{i,t},\ \tilde{r}_{i,t}\right)^T$ loads on the global factor vector $G_t = \left(g_t^G,\ \pi_t^G,\ r_t^G\right)^T$, as well as on its country-idiosyncratic factor vector $F_{i,t} = \left(f_{i,t}^g,\ f_{i,t}^\pi,\ f_{i,t}^r\right)^T$. Note that the tilde is used to distinguish between the unobserved underlying fundamentals from the observed data with the measurement errors between them. Hence for country i, its underlying macroeconomic fundamentals $\tilde{g}_{i,t},\ \tilde{\pi}_{i,t}$ and $\tilde{r}_{i,t}$ are,

$$\begin{aligned} \tilde{g}_{i,t} &= \alpha_i^g + \beta_i^g g_t^G + f_{i,t}^g, \\ \tilde{\pi}_{i,t} &= \alpha_i^\pi + \beta_i^\pi \pi_t^G + f_{i,t}^\pi, \\ \tilde{r}_{i,t} &= \alpha_i^r + \beta_i^r r_t^G + f_{i,t}^r, \end{aligned} \tag{1}$$

where $\{\alpha_i^g, \alpha_i^\pi, \alpha_i^r\}_{i=1,\dots,N+1}$ are constant terms, and $\{\beta_i^g, \beta_i^\pi, \beta_i^r\}_{i=1,\dots,N+1}$ are loadings on global factors ($g_t^G,\ \pi_t^G,\ r_t^G$) for country i; $\{f_{i,t}^g, f_{i,t}^\pi, f_{i,t}^r\}_{i=1,\dots,N+1}$ are country-idiosyncratic factors in country i. Rewriting the above equations into matrix form,

$$X_{i,t} = \alpha_i + \beta_i G_t + F_{i,t}, \tag{2}$$

where $\{\alpha_i\}_{i=1,\dots,N+1}$ are constant 3×1 vectors, and $\{\beta_i\}_{i=1,\dots,N+1}$ are diagonal matrices of the loading on global factor G_t.

G_t and $F_{i,t}$ are two different types of state vectors and together they determine the underlying macroeconomic fundamentals. The global factor vector G_t is assumed to follow a Gaussian vector autoregression process,

$$G_t = \Phi^G G_{t-1} + \Sigma^G \upsilon_t^G, \tag{3}$$

where Φ^G is a constant 3×3 matrix; υ_t^G is an i.i.d. Gaussian white noise, with zero mean and an identity variance-covariance matrix; Σ^G is a diagonal matrix. In order to identify the global factors, two sets of assumptions are needed. Firstly, since the magnitudes of global factors and their loadings cannot be separately identified, we assume that the innovations related to global factors have a standard deviation of 0.001, which means $\Sigma^G = 0.001 \times I_3$. Secondly, we assume that the US loadings on the global factors are positive, in order to identify the signs of global factors and their loadings.

The country-idiosyncratic factor vector $F_{i,t}$ is assumed to have a Gaussian vector autoregression process,

$$F_{i,t} = \Phi^{F_i} F_{i,t-1} + \Sigma^{F_i} \upsilon^F_{i,t}, \tag{4}$$

where Φ^{F_i} is a constant 3×3 diagonal matrix; $\upsilon^F_{i,t}$ is country-idiosyncratic shock vector, with zero mean and an identity variance-covariance matrix. We assume that shocks in this equation are independent, hence the variance-covariance matrix of $\Sigma^{F_i}(\Sigma^{F_i})^T$ is diagonal. A similar setting with global and country-idiosyncratic factors associated to "level" and "slope" is used by Diebold, Li, and Yue (2008) to investigate the global yield curve in a multi-country economy.

B. Relating Macroeconomic Fundamentals to Stochastic Discount Factors

In this multi-country world, we assume that the no-arbitrage condition holds. Then there exists at least one almost surely positive process M_t with $M_0 = 1$ denominated in each currency, such that the discounted gain process associated with any admissible trading strategy is a martingale (Harrison and Kreps (1979)). M_t is called the stochastic discount factor (*SDF*). We denote the country i's SDF as $M_{i,t}$, for $i = 1, 2, ..., N+1$.

Since there is no a widely accepted general equilibrium model for asset pricing, we follow those studies that choose a partial equilibrium approach to model the financial market and use flexible factor models with the no-arbitrage condition (Cochrane, 2004). In this paper, we use a factor representation for the SDF's, which allows us to model both exchange rates and term structures of interest rates. Under the complete market assumption, there exists one unique stochastic discount factor $M_{i,t}$, associated with each country i's currency, for $i = 1, 2, ..., N+1$. Given that the dynamics of country i's economy are jointly determined by the global factor as well as by its country-idiosyncratic factor, we assume that the SDF for country i has the following exponential form,

$$
\begin{aligned}
M_{i,t+1} &= \exp(m_{i,t+1}) \\
&= \exp\Big(-\tilde{r}_{i,t} - \frac{1}{2}(\lambda^G_{i,t})^T\lambda^G_{i,t} - \frac{1}{2}(\lambda^F_{i,t})^T\lambda^F_{i,t} - (\lambda^G_{i,t})^T\upsilon^G_{t+1} - (\lambda^F_{i,t})^T\upsilon^F_{i,t+1}\Big), \qquad (5)
\end{aligned}
$$

where $\tilde{r}_{i,t}$ is the short-term interest rate of country i; $\lambda^G_{i,t}$ and $\lambda^F_{i,t}$ are the time-varying market prices of global and country-idiosyncratic risks assigned by investors to those assets denominated in country i's currency; υ^G_{t+1} and $\upsilon^F_{i,t+1}$ are the global and country-idiosyncratic "uncertainties" related to the country i's economy at time t, defined by equations (3) and (4).

This specification for the SDF process is similar to the one commonly used in macro finance term structure literature (Ang and Piazzesi (2003), Duffee (2002) and Duffee (2007)). The only difference from the standard ones is that in this paper there are two types of market prices of risks and innovations associated with two types of state vectors, global and country-idiosyncratic ones. The stochastic discount factor is also named the intertemporal marginal rate of substitution in a Lucas-type exchange economy (Lucas (1982)), which is derived by solving the representative agent's optimization problem.

Note that the market prices of global and country-idiosyncratic risks related to country i's currency are $\lambda^G_{i,t}$ and $\lambda^F_{i,t}$, respectively. The country i's state vectors G_t and $F_{i,t}$ summarize the uncertainties in country i's economy. We assume that the market prices of global and country-idiosyncratic risk related to each country i's currency are affine functions of their corresponding state vectors, G_t and $F_{i,t}$, for each country $i = 1, ..., N+1$, (Dai and Singleton (2002); Duffee (2002)),

$$
\begin{aligned}
\lambda^G_{i,t} &= \lambda^G_{i,0} + \lambda^G_{i,1}G_t, \qquad (6)\\
\lambda^F_{i,t} &= \lambda^F_{i,0} + \lambda^F_{i,1}F_{i,t}, \qquad (7)
\end{aligned}
$$

where $\lambda^G_{i,0}$ and $\lambda^F_{i,0}$ are constant 3×1 vectors, and $\lambda^G_{i,1}$ and $\lambda^F_{i,1}$ are constant 3×3 matrices. It is crucial to make some reasonable restrictions on the coefficient matrices, $\lambda^G_{i,1}$ and $\lambda^F_{i,1}$, in order to be able to estimate the model. Here we simplify $\lambda^G_{i,1}$ and $\lambda^F_{i,1}$ to be diagonal matrices. By doing so, we are able to reduce the amount of parameters in this multi-country model without loss of generality and efficiency when modeling market prices of risk using macroeconomic information.

C. Exchange Rate Dynamics

Let $\mathcal{S}_{j,t}$ $(j = 1, ..., N)$ be the exchange rate between the foreign country j and the US, which is defined as the price of the US dollar per one unit of the foreign country j's currency. The no-arbitrage assumption and the law of one price imply that the ratio of the stochastic discount factors between the home and the foreign country determines the dynamics of their exchange rate (Bachus, Foresi, and Telmer (2001); Bekaert (1996); Brandt and Santa-Clara (2002); Brandt, Cochrane, and Santa-Clara (2006)). Thus we have,

$$\frac{\mathcal{S}_{j,t+1}}{\mathcal{S}_{j,t}} = \frac{M_{j,t+1}}{M_{N+1,t+1}}. \tag{8}$$

The above relation formally defines the link between the stochastic discount factors of two economies and the exchange rate movements between them. In complete markets, the stochastic discount factors in both economies are unique, therefore they uniquely determine the dynamics of their exchange rate.

Taking natural logarithms of both sides of equation (8) and using the specification of the SDF (equation (5)), we obtain the following equation for exchange rate changes,

$$\begin{aligned}\Delta s_{j,t+1} \quad = \quad & \Big(\tilde{r}_{N+1,t} - \tilde{r}_{j,t}\Big) + \frac{1}{2}\Big((\lambda^G_{N+1,t})^T\lambda^G_{N+1,t} - (\lambda^G_{j,t})^T\lambda^G_{j,t} + (\lambda^F_{N+1,t})^T\lambda^F_{N+1,t} - (\lambda^F_{j,t})^T\lambda^F_{j,t}\Big) \\ & + \Big((\lambda^G_{N+1,t})^T\upsilon^G_{t+1} - (\lambda^G_{j,t})^T\upsilon^G_{t+1} + (\lambda^F_{N+1,t})^T\upsilon^F_{N+1,t+1} - (\lambda^F_{j,t})^T\upsilon^F_{j,t+1}\Big),\end{aligned} \tag{9}$$

which shows that the global factor G and the two country-idiosyncratic factors, F_j for foreign country j and F_{N+1} for the home country, determine the exchange rate changes $\Delta s_{j,t+1}$, via market prices of risk in a nonlinear form. This is in contrast to the traditional models that often get a linear relation between the exchange rate dynamics and macroeconomic fundamentals or to other models that only use latent factors which do not have any economically meaningful interpretations.

The exchange rate changes can be divided into two components, the expected and the unexpected component. The expected change of the exchange rate is,

$$\begin{aligned}
\Delta s_{j,t+1}^{exp.} &\equiv E_t\Big(\Delta s_{j,t+1}\Big) \qquad (10)\\
&= \Big(\tilde{r}_{N+1,t} - \tilde{r}_{j,t}\Big) + \frac{1}{2}\Big((\lambda_{N+1,t}^G)^T \lambda_{N+1,t}^G - (\lambda_{j,t}^G)^T \lambda_{j,t}^G\Big) + \frac{1}{2}\Big((\lambda_{N+1,t}^F)^T \lambda_{N+1,t}^F - (\lambda_{j,t}^F)^T \lambda_{j,t}^F\Big),
\end{aligned}$$

It captures the predictable variation of returns in foreign exchange markets. We can see that market prices of risks are important in modeling the expected component of exchange rate changes. The uncovered interest rate parity does not hold for this model. Since the expected exchange rate changes are determined not only by the interest rate differentials between the two countries $(\tilde{r}_{N+1,t} - \tilde{r}_{j,t})$, but also by a time varying foreign exchange risk premium term, $rp_{j,t+1}$,

$$rp_{j,t+1} \equiv \frac{1}{2}\Big((\lambda_{N+1,t}^G)^T \lambda_{N+1,t}^G - (\lambda_{j,t}^G)^T \lambda_{j,t}^G\Big) + \frac{1}{2}\Big((\lambda_{N+1,t}^F)^T \lambda_{N+1,t}^F - (\lambda_{j,t}^F)^T \lambda_{j,t}^F\Big), \qquad (11)$$

The above equation shows that the foreign exchange risk premium is determined by two components, the one driven by global factors and the other driven by country-idiosyncratic factors.

The unexpected change of the exchange rate is,

$$\begin{aligned}
\Delta s_{j,t+1}^{unexp.} &\equiv \Delta s_{j,t+1} - E_t\Big(\Delta s_{j,t+1}\Big) \qquad (12)\\
&= \Big((\lambda_{N+1,t}^G)^T \upsilon_{t+1}^G - (\lambda_{j,t}^G)^T \upsilon_{t+1}^G\Big) + \Big((\lambda_{N+1,t}^F)^T \upsilon_{N+1,t+1}^F - (\lambda_{j,t}^F)^T \upsilon_{j,t+1}^F\Big),
\end{aligned}$$

It implies that the unexpected change of the exchange rate is composed by the products of state vector shocks times their corresponding market prices of risk. Similarly to the foreign risk premium in equation (11), the unexpected change of the exchange rate also has two components, the global component and the country-idiosyncratic component. It can be noticed that the market prices of risk are time-varying, which are driven by the dynamics of the global factor and the country-idiosyncratic factor. This implies the exchange rate changes are heteroskedastic.

Summing up, the exchange rate dynamic equation (9) can be write in the following way as well,

$$\Delta s_{j,t+1} = \left(\tilde{r}_{N+1,t} - \tilde{r}_{j,t}\right) + rp_{j,t+1} + \Delta s_{j,t+1}^{unexp.}, \tag{13}$$

$$= \Delta s_{j,t+1}^{exp.} + \Delta s_{j,t+1}^{unexp.}, \tag{14}$$

D. Bond Pricing

In the last part of our model, we introduce the recursive relationship which characterizes the bond pricing for each country i $(i = 1, ..., N+1)$ under the affine term structure modeling framework.

For each country i, having specified the stochastic discount factor $M_{i,t}$ (equation (5)) and its state dynamics (equation (3) and (4)), we can price its zero-coupon bonds. Introducing bond information in our model is important to identify market prices of risk.

Each country's short rate $\tilde{r}_{i,t}$ is a function of the global factor as well as its country-idiosyncratic factor, as equation (1) shows. We can write the short rate equation as an affine function of the global factor G_t and its country-idiosyncratic factor $F_{i,t}$,

$$\tilde{r}_{i,t} = \delta_{i,0} + (\delta_{i,1}^{G})^T G_t + (\delta_{i,1}^{F})^T F_{i,t}, \tag{15}$$

with $\delta_{i,0} = \alpha_i^r$, $\delta_{i,1}^G = \left(0, 0, \beta_i^r\right)^T$, and $\delta_{i,1}^F = \left(0, 0, 1\right)^T$.

In each country i, the no-arbitrage condition guarantees that a zero-coupon bond with n-period maturity at time t can be priced according to the following Euler equation,

$$\tilde{P}_{i,t}^{(n)} = E_t\left[M_{i,t+1}\tilde{P}_{i,t+1}^{(n-1)}\right] \tag{16}$$

with the initial condition $\tilde{P}_{i,t}^{(0)} = 1$. As before, tilde indicates the true value.

By combining equations (3) and (4), which describe the dynamics of the factor vectors, together with equation (15) which defines the short rate and equation (5) which specifies the SDF, we can show that the country i's bond price is an exponential linear function of the global factor G_t and of the country-idiosyncratic factors $F_{i,t}$,

$$\tilde{P}_{i,t}^{(n)} = \exp\Big(A_{i,n} + (B_{i,n})^T G_t + (C_{i,n})^T F_{i,t}\Big), \tag{17}$$

where $A_{i,n}, B_{i,n}$ and $C_{i,n}$ solve the following difference equations,

$$\begin{aligned} A_{i,n+1} &= A_{i,n} - (B_{i,n})^T \Sigma^G \lambda_{i,0}^G - (C_{i,n})^T \Sigma^{F_i} \lambda_{i,0}^F + \frac{1}{2}(B_{i,n})^T \Sigma^G (\Sigma^G)^T B_{i,n} + \frac{1}{2}(C_{i,n})^T \Sigma^{F_i} (\Sigma^{F_i})^T C_{i,n} - \delta_{i,0}, \\ B_{i,n+1} &= \Big(\Phi^G - \Sigma^G \lambda_{i,1}^G\Big)^T B_{i,n} - \delta_{i,1}^G, \\ C_{i,n+1} &= \Big(\Phi^{F_i} - \Sigma^{F_i} \lambda_{i,1}^F\Big)^T C_{i,n} - \delta_{i,1}^F, \end{aligned} \tag{18}$$

with $A_{i,1} = -\delta_{i,0}$, $B_{i,1} = -\delta_{i,1}^G$, and $C_{i,1} = -\delta_{i,1}^F$ being the initial conditions. Accordingly, the yield is also an affine function of the state

$$\tilde{y}_{i,t}^{(n)} \equiv -\frac{\log P_{i,t}^{(n)}}{n} = a_{i,n} + (b_{i,n})^T G_t + (c_{i,n})^T F_{i,t}, \tag{19}$$

where $a_{i,n} = -A_{i,n}/n$, $b_{i,n} = -B_{i,n}/n$, and $c_{i,n} = -C_{i,n}/n$.

From the difference equations (18), we can see that the constant coefficients of market price of risk $\lambda_{i,0}^G$ and $\lambda_{i,0}^F$ only affect the constant yield coefficient $a_{i,n}$; on the contrary, the parameters $\lambda_{i,1}^G$ and $\lambda_{i,1}^F$ affect the loadings on the global and the country-idiosyncratic factors, $b_{i,n}$ and $c_{i,n}$, respectively. This implies that the parameters $\lambda_{i,0}^G$ and $\lambda_{i,0}^F$ affect average term spreads and average expected bond returns, whereas the parameters $\lambda_{i,1}^G$ and $\lambda_{i,1}^F$ determine time variation in term spreads and expected bond returns.

IV. Econometric Methodology

We have assumed that the macroeconomic factors $X_{i,t}$, yields $y_{i,t}$ and exchange rate changes $\Delta s_{j,t}$ are unobservable and that the econometrician observe the corresponding ones, $X_{i,t}^{obs.}$, $y_{i,t}^{obs.}$ and $\Delta s_{j,t}^{obs.}$, with measurement errors, $\eta_{i,t}^X$, $\eta_{i,t}^y$ and $\eta_{j,t}^{\Delta s}$. We can first transform the model into a state-space representation and then use a Bayesian filtering approach to estimate it.

A. State-Space Model Representation

At each period t, we can observe the exchange rate changes, the macroeconomic variables and the zero-coupon bond data. We assume that each of these variables is collected with normal i.i.d measurement errors. Thus, we have the following measurement equations

$$
\begin{aligned}
\Delta s_{j,t}^{obs.} &= \Big(\tilde{r}_{N+1,t-1} - \tilde{r}_{j,t-1}\Big) + \frac{1}{2}\Big((\lambda_{N+1,t-1}^{G})^{T}\lambda_{N+1,t-1}^{G} - (\lambda_{j,t-1}^{G})^{T}\lambda_{j,t-1}^{G} \\
&\quad +(\lambda_{N+1,t-1}^{F})^{T}\lambda_{N+1,t-1}^{F} - (\lambda_{j,t-1}^{F})^{T}\lambda_{j,t-1}^{F}\Big) + (\lambda_{N+1,t-1}^{G} - \lambda_{j,t-1}^{G})^{T}(\Sigma^{G})^{-1}(G_t - \Phi^{G}G_{t-1}) \\
&\quad +\Big((\lambda_{N+1,t-1}^{F})^{T}(\Sigma^{F_{N+1}})^{-1}(F_{N+1,t} - \Phi^{F_{N+1}}F_{t-1}) - (\lambda_{j,t-1}^{F})^{T}(\Sigma^{F_j})^{-1}(F_{j,t} - \Phi^{F_j}F_{t-1})\Big) + \eta_{j,t}^{\Delta s}, \\
&\quad for\ j = 1,\ \ldots\ , N; \qquad (20) \\
X_{i,t}^{obs.} &= \alpha_i + \beta_i G_t + F_{i,t} + \eta_{i,t}^{X},\ for\ i = 1, ..., N+1; , \qquad (21) \\
y_{i,t}^{obs.} &= a_i + (b_i)^{T} G_t + (c_i)^{T} F_{i,t} + \eta_{i,t}^{y},\ \ for\ i = 1,\ \ldots\ , N+1. \qquad (22)
\end{aligned}
$$

where in the exchange rate changes equation (20), we use $\upsilon_t^G = (\Sigma^G)^{-1}(G_t - \Phi^G G_{t-1})$ and $\upsilon_{i,t} = (\Sigma^{F_i})^{-1}(F_{i,t} - \Phi^{F_i} F_{t-1})$ (for $i = 1, ..., N+1$), from equations (3) and (4); the market prices of risk are linear functions of the state vectors, $\lambda_{i,t-1}^G = \lambda_0^G + \lambda_1^G G_t$ and $\lambda_{i,t-1}^F = \lambda_0^F + \lambda_1^F F_{i,t}$. Measurement errors ($\eta_t$) are with distinct variances for different variables/series and are assumed to be mutually independent.

The state vectors in this multi-country system, which are the global factor G_t and the country-idiosyncratic factor $F_{i,t}$ (for $i = 1, 2, 3, 4$), follow a first-order VAR and their dynamics are described by equations (3) and (4). From the measurement equations we can notice that observations depend on both current and lagged values of the global and the country-idiosyncratic factors. Hence all of them should be taken as states and the state equations are the following,

$$
\begin{pmatrix} G_t \\ G_{t-1} \end{pmatrix} = \begin{pmatrix} \Phi^G & 0_{3\times3} \\ I_3 & 0_{3\times3} \end{pmatrix} \begin{pmatrix} G_{t-1} \\ G_{t-2} \end{pmatrix} + \begin{pmatrix} I_3 \\ 0_{3\times3} \end{pmatrix} \Sigma^G \upsilon_t^G, \qquad (23)
$$

$$
\begin{pmatrix} F_{i,t} \\ F_{i,t-1} \end{pmatrix} = \begin{pmatrix} \Phi^{F_i} & 0_{3\times3} \\ I_3 & 0_{3\times3} \end{pmatrix} \begin{pmatrix} F_{i,t-1} \\ F_{i,t-2} \end{pmatrix} + \begin{pmatrix} I_3 \\ 0_{3\times3} \end{pmatrix} \Sigma^{F_i} \upsilon_{i,t}^F,\ for\ i = 1, ..., N+1. \qquad (24)
$$

Therefore, the set of parameters of this multi-Country model which we have to estimate is,

$$\Theta = \Big(\{\alpha_i,\ \beta_i;\ \Phi^{F_i},\ \Sigma^{F_i};\ \lambda^G_{i,0}\,\lambda^G_{i,1},\ \ \lambda^F_{i,0}\,\lambda^F_{i,1};\ \Sigma^{\eta^{X_i}},\ \Sigma^{\eta^{y_i}}\}_{i=1,2,3,4};\ \{\sigma^{\eta^{\Delta s_j}}\}_{j=1,2,3};\ \Phi^G,\ \Sigma^G\Big), \tag{25}$$

B. Quasi-Maximum Likelihood Estimation and Unscented Kalman Filter

Given that our state-space model representation (equations (20) to (24)) has Gaussian noises, we can implement the model estimation using Bayesian filtering approaches. The exchange rate dynamic equations are highly non-linear functions of the states, which makes the standard Kalman filter inapplicable. Instead, we can use the nonlinear Kalman filters. The most commonly used nonlinear Kalman filter is the extended Kalman filter, which linearizes the nonlinear system around the current state estimate using a Taylor approximation. However, for the highly nonlinear system, the extended Kalman filter is computationally demanding and performs very poorly. An alternative is the unscented Kalman filter (UKF), recently developed in the field of engineering (Julier and Ulman (1997, 2004)). The idea behind this approach is that in order to estimate the state information after a nonlinear transformation, it is better to approximate the probability distribution directly instead of linearizing the nonlinear functions. The unscented Kalman filter overcomes a large extent pitfalls inherent to the extended Kalman filter and improves estimation accuracy and robustness without increasing computational cost.

In order to implement the unscented Kalman filter, we firstly concatenate the state variables $x_{t-1} = [G_{t-1}, F_{1,t-1}, ..., F_{4,t-1}, G_{t-2}, F_{1,t-2}, ..., F_{4,t-2},]'$, the observation noises η_{t-1} and the state noises $\varepsilon_{t-1} = [\upsilon^G_{t-1}, \upsilon^F_{1,t-1}, ..., \upsilon^F_{4,t-1}]'$ at time $t-1$,

$$x^e_{t-1} = \begin{bmatrix} x'_{t-1} & \eta'_{t-1} & \varepsilon'_{t-1} \end{bmatrix}', \tag{26}$$

whose dimension is $L = L_x + L_\eta + L_\varepsilon$ and whose mean and covariance are

$$\hat{x}^e_{t-1} = \Big[E[x_{t-1}] \quad 0 \quad 0\Big]', \qquad P^e_{t-1} = \begin{bmatrix} P^x_{t-1} & 0 & 0 \\ 0 & \Sigma^2_\eta & 0 \\ 0 & 0 & I_{15} \end{bmatrix}.$$

We then form a set of $2L+1$ sigma points

$$\chi^e_{t-1} = \left[\begin{array}{ccc} \hat{x}^e_{t-1} & \hat{x}^e_{t-1} + \sqrt{(L+\lambda)P^e_{t-1}} & \hat{x}^e_{t-1} - \sqrt{(L+\lambda)P^e_{t-1}} \end{array}\right] \tag{27}$$

and the corresponding weights

$$\begin{aligned} w^{(m)}_0 &= \frac{\lambda}{L+\lambda}, \quad w^{(c)}_0 = \frac{\lambda}{L+\lambda} + (1-\alpha^2+\beta), \qquad (28) \\ w^{(m)}_i &= w^{(c)}_i = \frac{1}{2(L+\lambda)}, \; i = 1, 2, \ldots, 2L, \qquad (29) \end{aligned}$$

where superscripts (m) and (c) indicate that the weights are for construction of the posterior mean and of the covariance, respectively; $\lambda = \alpha^2(L+\bar{\kappa}) - L$ is a scaling parameter; the constant α determines the spread of sigma points around $\bar{x}$ and is usually set to be a small positive value; $\bar{\kappa}$ is a second scaling parameter with value set to 0 or $3-L$; β is a covariance correction parameter and is used to incorporate prior knowledge of the distribution of x.

With these sigma points, we implement the UKF as follows: for the time update

$$\begin{aligned} \chi^x_{t|t-1} &= F(\chi^x_{t-1}, \chi^\varepsilon_{t-1}), \quad \hat{x}^-_t = \sum_{i=0}^{2L} w^{(m)}_i \chi^x_{i,t|t-1}, \\ P^-_{x_t} &= \sum_{i=0}^{2L} w^{(c)}_i (\chi^x_{i,t|t-1} - \hat{x}^-_t)(\chi^x_{i,t|t-1} - \hat{x}^-_t)', \end{aligned}$$

and for the measurement update

$$\begin{aligned} \mathcal{Y}_{t|t-1} &= H(\chi^x_{t|t-1}, \chi^\eta_{t|t-1}), \quad \hat{Y}^-_t = \sum_{i=0}^{2L} w^{(m)}_i \mathcal{Y}_{i,t|t-1}, \\ P^-_{Y_t} &= \sum_{i=0}^{2L} w^{(c)}_i (\mathcal{Y}_{i,t|t-1} - \hat{Y}^-_t)(\mathcal{Y}_{i,t|t-1} - \hat{Y}^-_t)', \\ P_{x_t Y_t} &= \sum_{i=0}^{2L} w^{(c)}_i (\chi^x_{i,t|t-1} - \hat{x}^-_t)(\mathcal{Y}_{i,t|t-1} - \hat{Y}^-_t)', \\ \hat{x}_t &= \hat{x}^-_t + P_{x_t Y_t}(P^-_{Y_t})^{-1}(Y_t - \hat{Y}^-_t), \\ P_{x_t} &= P^-_{x_t} - (P_{x_t Y_t}(P^-_{Y_t})^{-1})P^-_{Y_t}(P_{x_t Y_t}(P^-_{Y_t})^{-1})', \end{aligned}$$

where Y_t is the observation vector containing all the observed variables, $\hat{Y}_t^-$ its predicted values, $P_{Y_t}^-$ its conditional variance-covariance matrix, $\hat{x}_t$ the filtered state vector, and P_{x_t} its variance-covariance matrix.

Assuming that the predictive errors are normally distributed, we can construct the log likelihood function at time t as follows

$$\mathcal{L}_t(\Theta) = -\frac{1}{2}\ln|P_{Y_t}^-| - \frac{1}{2}(Y_t - \hat{Y}_t^-)'(P_{Y_t}^-)^{-1}(Y_t - \hat{Y}_t^-), \tag{30}$$

where Θ is a parameter vector of the model. Parameter estimates can be obtained by maximizing the joint log likelihood

$$\hat{\Theta} = \arg\max_{\Theta \in \Xi} \sum_{t=1}^{T} \mathcal{L}_t(\Theta), \tag{31}$$

where Ξ is a compact parameter space, and T is the length of total observations of the data. Since the log likelihood function is misspecified for the non-Gaussian model, a robust estimate of the variance-covariance matrix of parameter estimates can be obtained using the approach proposed by White (1982)

$$\hat{\Sigma}_\Theta = \frac{1}{T}\Big[AB^{-1}A\Big]^{-1}, \tag{32}$$

where

$$A = -\frac{1}{T}\sum_{t=1}^{T}\frac{\partial^2 \mathcal{L}_t(\hat{\Theta})}{\partial\Theta\partial\Theta'}, \qquad B = \frac{1}{T}\sum_{t=1}^{T}\frac{\partial \mathcal{L}_t(\hat{\Theta})}{\partial\Theta}\frac{\partial \mathcal{L}_t(\hat{\Theta})}{\partial\Theta'}. \tag{33}$$

With these parameter estimates $\hat{\Theta}$, the latent global and country-idiosyncratic factors, $\hat{G}_t$ and $\hat{F}_{i,t}$ $(i = 1, 2, 3, 4)$, can be extracted using the unscented Kalman filter.

The number of parameters in our model is large. Maximization of the likelihood (30) may involve a large number of likelihood evaluations. Therefore, we adopt a sophisticated quasi-Newton approach with the inverse Hessian matrix of the likelihood function updated by the BFGS algorithm. The initial values are carefully selected in the following way. We first run the Nelder-Mead optimization algorithm for 100 feasible sets of starting values and stop them after 100 iterations. Then the best 10 parameter estimate sets (in terms of the likelihood) are selected among these 100 runnings as the initial values for the quasi-Newton algorithm. The parameter estimates are those comiing from the largest likelihood among these 10 runnings of the quasi-Newton method.

V. Empirical Results and Discussions

A. Model Performance on Exchange Rate Dynamics

Exchange rates dynamics are the main focus of this paper. The summary of statistics from the observed and the model-implied data exchange rates dynamics are reported in Table 4. The model is able to capture the statistic moments of the observed exchange rate movements. However model-implied exchange rate changes are less volatile than the observed ones.

— Table 4 around here —

The above results are also confirmed by Figure 2, where both the model implied exchange rates dynamics and the observed ones are plotted. Generally speaking, the model implied exchange rates dynamics are able to reproduce the dynamics of the observed ones very well along the sample period. The first panel in Table 5 shows that the model-implied exchange rate dynamics are able to capture 57%, 66% and 33% of the variations of the observed exchange rate dynamics for the USD/DEM (EUR), the USD/GBP and the USD/JPY, respectively. Comparing with linear models of exchange rate dynamics that use macroeconomic fundamental information, this no-arbitrage multi-country model represents a big improvement. This finding is quite consistent with other studies based on alternative approaches. For instance, Evans and Lyons (2002) adopt a micro-market structure approach and they obtain the R^2 statistics equal to 64% and 45% for daily Deutsch mark/dollar and Japanese yen/dollar log changes between May 1 to August 31, 1996.

— Figure 2 around here —

B. Macroeconomic Shocks and the Exchange Rate Dynamics

Previous studies have found out that exchange rate movements are largely disconnected with macroeconomic fundamentals. In monetary models and/or new open economy macroeconomic models, the exchange rate is a linear function of contemporaneous macroeconomic variables. Since the residuals are usually serially correlated in these models, the estimation is implemented using the first-order differences of relevant variables,

$$\Delta s_t = \beta_0 + \beta_1^{(h)} \Delta r_t^{(h)} + \beta_1^{(f)} \Delta r_t^{(f)} + \beta_2^{(h)} \Delta g_t^{(h)} + \beta_2^{(f)} \Delta g_t^{(f)} + \beta_3^{(h)} \Delta \pi_t + \beta_3^{(f)} \Delta \pi_t^{(f)} + u_t, \quad (34)$$

where u_t is a noise term. In these models, coefficients are typically constrained by $\beta_k^{(h)} = -\beta_k^{(f)}$, for $k = 1, 2, 3$. When estimating this linear model for the three types of exchange rate changes used in this paper, we find R^2 equal to 3.3%, 5.7% and 4.5% for the unconstrained regressions and R^2 equal to 1.4%, 1.2% and 1.5% for the constrained regressions. Even though macroeconomic fundamentals in our model are able to account for 57%, 66% and 33% of the variation of exchange rate movements for the three dollar exchange rate dynamics respectively, the linear model in equation (34) cannot capture this link between macroeconomic fundamentals and exchange rates.

What exact roles do macroeconomic fundamentals play in our model? Recall the exchange rate dynamic equation (13),

$$\Delta s_{j,t+1} \quad = \quad \left(\tilde{r}_{N+1,t} - \tilde{r}_{j,t}\right) + rp_{j,t+1} + \Delta s_{j,t+1}^{unexp.},$$

where we decompose the exchange rate dynamics into three components, the short-term interest differential, the foreign risk premium and the unexpected exchange rate changes.

— Figure 3 around here —

Figure 3 presents these three components of the exchange rate changes as well as their sum, the model-implied exchange rate changes. For each exchange rate change, the first component, $(\tilde{r}_{N+1,t} - \tilde{r}_{j,t})$, which is the only concern in the UIP model, is very smooth for all the three dollar exchange rates. The second one, foreign risk premium $rp_{j,t+1}$, becomes volatile in comparison to the first term, but it still has much smaller variation than the model-implied exchange rate changes. This implies that the third component, $\Delta s_{j,t+1}^{unexp.}$, must be more volatile and should play a more important role in explaining exchange rate movements. Figure 3 shows that $\Delta s_{j,t+1}^{unexp.}$ is very volatile and reproduces fluctuations of exchange rate changes for each exchange rates dynamics.

From equation (12), the unexpected exchange rate change $\Delta s_{j,t+1}^{unexp.}$ has two parts: the one of them is driven by global innovations $(\lambda_{N+1,t}^{G})^{T}\upsilon_{t+1}^{G} - (\lambda_{j,t}^{G})^{T}\upsilon_{t+1}^{G}$; the other is driven by country-idiosyncratic innovations $(\lambda_{N+1,t}^{F})^{T}\upsilon_{N+1,t+1}^{F} - (\lambda_{j,t}^{F})^{T}\upsilon_{j,t+1}^{F}$, both of which are macro-dependent. The regression of the data on the unexpected exchange rate changes and a constant results in the R^2 of 48% (57% or 21%), which equal to 84% (86% or 64%) of the total explained variance for the USD/DEM (the USD/GDP or the USD/JPY) by our model. However, when we regress the data on the model-implied macroeconomic innovations (or macroeconomic "news") $\hat{\upsilon}_{t+1}^{G}, \hat{\upsilon}_{N+1,t+1}^{F}, \hat{\upsilon}_{j,t+1}^{F}$ with a constant, the R^2 reduce into 30%, 36%, and 13%. This is very close to several other studies. For instance, Evans (2010) using weekly USD/EUR exchange rate finds that 23% of the variance in excess currency returns over one-month horizon can be explained by macroeconomic "news", from GDP, CPI, and M1. Evans and Lyons (2008) show that the arrival of intraday macro "news" can account for more than 30% of the variance in the daily price of the DEM/USD. Furthermore, in our model, the role of the macro innovations is further amplified by the time-varying market prices of risk, and hence the exchange rate dynamics are heteroskedastic. The importance of macroeconomic "news" macroeconomic has been investigated by Engel, Mark and West (2007) and Andersen et al. (2003) as well.

C. Model Performance on Macroeconomic and Yield Variables

This no-arbitrage macro-finance model is able not only to model exchange rate dynamics but also to model macroeconomic and financial variables. Table 6 and Table 7 show the observed and model-implied statistic summaries for macroeconomic and yield data. It is different with respect to the results for exchange rate dynamics. The model-implied macroeconomic and yield variables capture the statistics of observed ones very well, such as the mean, the standard error, the skewness, the kurtosis and the autocorrelation.

— Table 6 and 7 around here —

Moreover, Figure 6 and 7 show that the model implied macroeconomic and financial variables move tightly with the observed ones. The good model fit of these two types of variables can be also deduced from the estimates of the standard deviation of measurement errors in Table 3. They are very small, with values between 0.2 and 10.8 basis points.

— Figure 6 and 7 around here —

D. Foreign Exchange Risk Premium and Forward Premium Anomaly

One of the most notable puzzles in foreign exchange markets is the forward premium anomaly. It implies that the high interest rate currencies tend to appreciate. Fama (1984) attributes this departure from uncovered interest parity (UIP) to a time-varying risk premium. Our model also suggests that the expected exchange rate change is equal to the sum of the interest rate differentials and the time-varying foreign risk premium, which is constructed using the market prices of risk.

— Table 2 around here —

Table 2 provides the estimates of market prices of risk, where more than half of the parameters are statistically significant. Most of the estimates in $\lambda_{US,0}$ and $\lambda_{GM/UK/JP,0}$ ($\lambda_{US,1}$ and $\lambda_{GM/UK/JP,1}$) not only have the same signs, but also have very close values with respect to each other. This implies that the SDFs of the three foreign currencies should be highly correlated with the US SDF. Indeed, the correlations of the model-implied SDFs are as high as 99%. Brandt et al. (2006) show that the volatility of the exchange rate and the volatility of the SDFs from asset markets imply that the SDFs must be highly correlated across countries. The parameters in $\lambda^G_{i,0}$ and in $\lambda^F_{i,0}$ are negative, which is consist with previous findings (Backus et al., 1998). In addition, the parameters in $\lambda^G_{i,1}$ are much larger than in $\lambda^F_{i,1}$. The two parts of the foreign exchange risk premia (driven by global factors and by country-idiosyncratic factors) can been seen in Figure 4, and Figure 3 shows the sum of these two parts.

— Figure 4 around here —

Figure 4 shows that for each of these three dollar foreign risk premia, the component driven by the global factors is the dominant one, since the magnitude is 100 times larger with respect to the component driven by the country-idiosyncratic factors. The component driven by the global factors have similar patterns among the three dollar exchange rates. They have three positive peaks around three monetary/financial crises, i.e., the European monetary mechanism crisis in 1992, the Asian financial crisis in 1997 and the recently financial crisis that started in 2008. Along the sample period, European monetary mechanism crisis generates the biggest effect on the Germany mark and on the British pound against the US dollar, while the Asian financial crisis creates the highest peak of risk premia for the Japanese yen against the US dollar. On the contrary, the parts of foreign risk premia induced by country-idiosyncratic factor have very idiosyncratic dynamic patterns among these three dollar exchange rates.

Fama (1984) argues that the implied risk premium should be negatively correlated with and have larger variance than the interest rate differentials. They are usually termed as the Fama conditions. For each of the three dollar exchange rates, our model implied risk premium (rp_t) does negatively correlate with the interest rate differentials ($r^{(h)} - r^{(f)}$) with correlations of -9%, -58%, and -22%, and has a larger variance (0.82 vs. 0.04, 0.62 vs. 0.02, and 0.84 vs. 0.03). These results are presented in Panel B, Table 5.

— Table 5 around here —

Moreover, the estimated foreign risk premia are counter-cyclical to the US economy. The last panel in Table 5 it shows that foreign risk premia are negatively correlated with respect to output growth differentials between the US and foreign countries. This negative correlation implies that when the foreign output growth is higher than the domestic one, people in the market anticipate that the foreign currency will appreciate with respect to the domestic currency. When one country is in a better economic situation than the other, the market becomes more confident in that country's currency and thus people would like to hold it, leading to the appreciation of that currency.

Similarly to the output growth differentials, the inflation rate differentials ($\pi^{(h)} - \pi^{(f)}$) are negatively correlated with respect to the foreign risk premia , as the last panel in Table 5 shows. This means that when foreign inflation is relatively high, the foreign currency tends to appreciate with respect to the domestic currency. According to the Taylor rule, if the current inflation of the foreign country is high, people may expect the central bank to increase its interest rate in the future. This results in a decreased interest rate differential and an increased risk premium. This is consistent with the finding of Engel and West (2006) that high German inflation is associated with a strong mark with respect to the dollar. However, traditional monetary models (Frankel, 1979; Engel and Frankel, 1984) tend to predict the opposite fact, stating that high inflation is associated with a weak currency.

E. What Drives Exchange Rate Dynamics and Foreign Risk premia, Global or Country-Idiosyncratic Factors?

In order to know which factors are important in driving exchange rate dynamics and foreign risk premia, we implement the variance decomposition for this nonlinear exchange rate dynamics model. According to Harris and Yu (2010), given that $\Delta s_{i,t}$, $rp_{i,t}$, and $\Delta s_{i,t}^{unexp.}$ are nonlinear functions of the state vectors G_t, $F_{i,t}$, ..., $F_{N+1,t}$, the variance decompositions can be computed by using Monte Carlo simulation conditional on filtered state factors in the sample period. First, we simulate the model by drawing random shocks v_{t+h}^G, $v_{i,t+h}^F$, ..., $v_{N+1,t+h}^F$, (for $h = 1, 2, ..., 60$) from $N(0, I)$. Then the evolution of the state vectors can be computed by using state dynamic equations (23) and (24), and the corresponding values of Δs_i, rp_i and $\Delta s_i^{unexp.}$ can be obtained by equations (9), (11) and (12). Finally, we numerically compute the variances of forecast errors following Harris and Yu (2010) by repeating the whole process for 1000 times, in order to get the nonlinear variance decompositions.

The result of the variance decomposition for each type of foreign exchange rate is reported in Tables 10, 11 and 12, respectively.

— Table 10, 11 and 12 around here —

These three tables imply that global factors are much more important comparing to country-idiosyncratic factors in driving dynamics of exchange rates, foreign risk premia as well as the unexpected exchange rate changes.

For exchange rate changes Δs of all the three types of foreign exchange rate changes (Panel A in Tables 10, 11 and 12), the global factors explain around 60% to 70% of the short-run (1-month) forecast error variances. Their importance increases as the forecast horizon increases, and in the long-run (60-month) they explain more than 90% of forecast error variances. Among the global factors, the output growth and the interest rate are the two most important ones. In the short-run, the global interest rate is the dominant factor for the USD/GBP and the USD/JPY, and is almost equally important as the global output growth for the USD/DEM (EUR). However, in the long-run, global output growth is the most important factor for all the three dollar exchange rate dynamics. Even though country-idiosyncratic factors are less important, they do play a certain role in the short-run, accounting for 30% to 40% of the forecast error variances. For the USD/DEM (EUR), the interest rate factors of the US and Germany are the two most important country-idiosyncratic factors, and the US interest rate factor explains around twice the forecast variance of German rate factor does. For the USD/GBP (USD/JPY), the US interest rate factor and the UK (Japanese) output growth factor are the two most important country-idiosyncratic factors.

Foreign risk premia are almost exclusively explained by the three global factors, as the results in Panel B of Tables 10, 11 and 12 show. This is consistent with the information provided by Figure 4, which shows that the magnitude of the component driven by global factors is about 100 times larger as the one driven by country-idiosyncratic factors. For each of the three types of foreign risk premia, more than two thirds of the short-run forecast error variance is driven by the global interest rate factor. However, the role of the global interest rate factor decreases, while the role of global output and inflation factors increases, as the forecast horizon increases. Almost half of the forecast variance is explained by output factor and one fourth by inflation factor in the long-run. This finding is consistent with other studies as well. For instance, Bauer and Diez (2011) find that global output growth and inflation account for about 40% of the variation in USD/EUR risk premium in 1-year forecasting horizon, while the same value is around 50% in our model.

The variance decompositions for unexpected exchange changes $\Delta s^{unexp.}$ have similar patterns as the ones for exchange rate changes. This is consistent with the above finding that the macroeconomic shocks play a very important role in driving exchange rate dynamics. Even though foreign risk premia are exclusively driven by global factors, the role of the country-idiosyncratic factors on exchange changes cannot be ignored at all.

F. Global and Country-Idiosyncratic Macroeconomic Factors

The above section shows that global and country-idiosyncratic macroeconomic factors play very different roles in driving exchange rate dynamics and foreign risk premia. Hence it is worth to investigate them.

— Table 1 around here —

Table 1 reports the estimates of parameters related to the global and country-idiosyncratic factors. In the upper panel, the factor loadings for underlying macroeconomic fundamentals on global factors are reported. Most coefficients of the global loadings are significantly different from zero. The middle panel reports the global factor dynamics. From the coefficient matrix Φ^G, we can see that each global macroeconomic factor is highly persistent, with the diagonal values being close to one. The statistic t-ratios in these two panels imply that global macroeconomic factors do exit.

The bottom panel in Table 1 presents the dynamics of country-idiosyncratic factors. Diagonal values of matrix Φ^{F_i} $(i = 1, 2, 3, 4)$ are significantly different from zero. Hence the role of the country-idiosyncratic factors in determining the underlying macroeconomic fundamentals X_i cannot be ignored. In addition, these values are close to one, especially the values of the coefficients for output growth. This implies that country-idiosyncratic factors are very persistent.

— Figure 8 around here —

Figure 8 draws the three global macroeconomic factors, i.e. output growth, inflation and interest rate. There are three big jumps for global output growth factor around the years of 1992, 1997 and 2008, when there were monetary or financial crises, for instance, the European monetary mechanism crisis, the Asian financial crisis and the recently crisis that started in the US. The global inflation factor is positive along the sample period except for the time around 2009. The global short-term interest rate factor has the highest peak around 1991, which is exactly the same time of the peak of short-term interest rates (Figure 1).

— Figure 9 around here —

Figure 9 plots country-idiosyncratic output growth, inflation, interest rate factors in the top, middle and bottom panel, respectively. In each panel, there are four country-idiosyncratic factors for Germany, the UK, Japan and the US, respectively. Comparing to the macroeconomic fundamentals in Figure 1, country-idiosyncratic factors show much less cross-country comovement.

VI. Conclusion

This paper simultaneously investigates the dynamics of multiple bilateral nominal exchange rates using a multi-country framework. Macroeconomic fundamental information is introduced to model exchange rate dynamics and a no-arbitrage macro-finance approach is adopted. Macroeconomic fundamentals are assumed to be determined by both global (common) factors and country-idiosyncratic factors.

The empirical study focuses an open economy including four countries, i.e. Germany, the UK, Japan and the US, where the US is taken as the home country. The empirical results show that this multi-county model is able to capture 57%, 66% and 33% of the variations of the observed changes of the USD/DEM (EUR), the USD/GBP and the USD/JPY, respectively. The model-implied foreign risk premia satisfy the Fama conditions (1984) and they are counter cyclical with respect to the US economy. The macroeconomic innovations, or “news” are important in determining the exchange rate dynamics. Moreover, global and country-idiosyncratic macroeconomic factors do exist and play very different roles in driving exchange rate dynamics and foreign risk premia. Global factors drive foreign risk premia almost exclusively and account for more than half of the forecast error variance of exchange rate dynamics. The dominant factor in the short-run is global interest rate, while in the long-run global output becomes dominant, in driving both exchange rate dynamics and foreign risk premia. Even though country-idiosyncratic macroeconomic factors are less important, they do play some roles in the short-run exchange rate dynamics, especially the US and German interest rate, and the UK and Japanese output growth.

REFERENCES

[1] Ahn, D.H. "Common Factors and Local Factors: Implications for Term Structures and Exchange Rates." *Journal of Financial and Quantitative Analysis*, 39 (2004), 69-102.

[2] Ang, A and M. Piazzesi. "A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables." *Journal of Monetary Economics*, 50 (2003), 745-787.

[3] Ang, A and J. Chen. "Yield Curve Predictors of Foreign Exchange Returns." Working paper (2010).

[4] Backus, D.K., A. Gregory, and C.I. Telmer. "Accounting for Forward Rates in Markets for Foreign Currency." *Journal of Finance*, 48 (1993), 1887-1908.

[5] Bekaert, G. "The Time Variation of Risk and Return in Foreign Exchange Markets: A General Equilibrium Perspective." *Review of Financial Studies*, 9 (1996), 427-70.

[6] Bilson, F. O. John. "The Current Experience with Floating Exchange Rates: An Appraisal of the Monetary Approach." *American Economic Review*, 68 (1978), 392-397.

[7] Bordo, Michael, and Thomas Helbling. "International Business Cycle Synchronization in Historical Perspective." NBER Working Paper, No. 16103.

[8] Brandt, M.W., J.H. Cochrane, and P. Santa-Clara. "International Risk Sharing is Better Then You Think, or Exchange Rates are Too Smooth." *Journal of Monetary Economics*, 53 (2006), 671-91.

[9] Brandt, M.W. and P. Santa-Clara. "Simulated Likelihood Estimation of Diffusions with an Application to Exchange Rate Dynamics in Incomplete Markets." *Journal of Financial Economics*, 63 (2002), 161-210.

[10] Buttler, H-J, and B. Schips. "Equilibrium exchange rates in a multi-country model: An econometric study." *Journal Review of World Economics*, 123 (1987), 1-23.

[11] Cochrane, J. *Asset Pricing (Revised).* Princeton: Princeton University Press (2005).

[12] Dai, Q., and K.J. Singleton. "Specification Analysis of Affine Term Structure Models." *Journal of Finance*, 55 (2000), 1943-78.

[13] Della Corte, P., L. Sarno, and I. Tsiakas. "An Economic Evaluation of Empirical Exchange Rate Models." *Review of Financial Studies*, 22 (2009), 3491-3530.

[14] Diebold, F.X., C. Li, and V. Z. Yue. "Global Yield Curve Dynamics and Interactions: A Dynamic Nelson-Siegel Approach." *Journal of Econometrics*, 146 (2008), 351-363.

[15] Dong, S. "Macro Variables Do Drive Exchange Rate Movements: Evidence from a No-Arbitrage Model." Working paper (2006).

[16] Duffee, G.R. "Term Premia and Interest Rate Forecasts in Affine Models." *Journal of Finace*, 57 (2002), 406-43.

[17] Dornbusch, R. "Expectations and Exchange Rate Dynamics." *Journal of Political Economics*, 84 (1976), 1161-1176.

[18] Duffee, R.G. "Are Variations in Term Premia Related to The Macroeconomy?" Working paper (2007).

[19] Duffie, D., and R. Kan. "A Yield-Factor Model of Interest Rate." *Mathematical Finance*, 6 (1996), 379-406.

[20] Engel, C., and K.D. West. "Exchange Rates and Fundamentals." *Journal of Political Economy*, 113 (2005), 485-517.

[21] Fama, E. "Forward and Spot Exchange Rates. " *Journal of Monetary Economics*, 14 (1984), 319-38.

[22] Frenkel, A.J. "A Monetary Approach to the Exchange Rate: Doctrinal Aspects and Empirical Evidence." *Scandinavian Journal of Economics*, 78 (1976), 200-224.

[23] Frenkel, A.J., and M.L. Mussa. "Asset Markets, Exchange Rates, and the Balance of Payments." *Handbook of International Economics*, vol. II (1985), 679-747.

[24] Geigerm, F.. "International Interest Rate Risk Premia in a Affine Term Structure Models." Working paper.

[25] Graveline, F., and S. Joslin. "G10 Swap and Exchange Rates." Working paper (2011).

[26] Harris, T.J., and W. Yu. "Variance decompositions of nonlinear-dynamic stochastic system." *Journal of Process Control*, 20 (2010), 195-205.

[27] Harrison, M., and D. Kreps. “Martingales and Arbitrage in Multiperiod Securities Markets.” *Journal of Economic Theory*, 20 (1979), 381-408.

[28] Hodrick, R.J. “Risk, Uncertainty, and Exchange Rates.” *Journal of Monetary Economics*, 23 (1989), 433-59.

[29] Hodrick, R.J., and E.C. Prescott. “Postwar US Business Cycles: An Empirical Investigation.” *Journal of Money, Credit and Banking*, 29 (1997), 1-16.

[30] Hodrick, R.J., and M. Vassalou. “Do we need multi-country models to explain exchange rate and interest rate and bond return dynamics?” *Journal of Economic Dynamics and Control*, 26 (2002), 1275-1299.

[31] Hooper, P., and J.E. Morton. “Fluctuations in the Dollar: A Model of Nominal and Real Exchange Rate Determination.” *Journal of International Money and Finance*, 1 (1978), 39-56.

[32] Hordahl, P., O. Tristani, and D. Vestin. “A Joint Econometric Model of Macroeconomic and Term Structure Dynamics.” *Journal of Econometrics*, 131 (2006), 405-44.

[33] Julier, J.S. and J.K. Uhlmann. “A New Extension of the Kalman Filter to Nonlinear Systems.” *Proceedings of AeroSense: The 11th International Symposium on Aerospace/Defense Sensing, Simulation and Controls* (1997), 182-193.

[34] Julier, J.S. and J.K. Uhlmann. “Unscented Filtering and Nonlinear Estimation.” *Proceedings of the IEEE*, 92 (2004), 401-422.

[35] Kamin, S.B. “A multi-country comparison of the linkages between inflation and exchange rate competitiveness.” Working paper (1997).

[36] Lucas, R. “Interest Rates and Currency Prices in a Two-Country World.” *Journal of Monetary Economics*, 10 (1982), 335-359.

[37] Masson, P.R. “Strategies for Modeling Exchange Rates and Capital Flows in Multi-Country Macroeconomic Models.” IMF Working Paper (1987).

[38] Meese, A.R., and K. Rogoff. “Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?” *Journal of International Economics*, 14 (1983), 3-24.

[39] Nelson, C.R., and A.F. Siegel. "Parsimonious Modeling Of Yield Curves," *Journal of Business*, 60 (1987), 473-489.

[40] Modugno, M. and K. Nikolaou. "The Forecasting Power of International Yield Curve Linkages." ECB working paper (2009).

[41] Obstfeld, M., and K.S. Rogoff. *Foundations of International Macroeconomics*. Cambridge, MA: MIT Press (1996).

[42] Obstfeld, M., and K.S. Rogoff. "Risk and Exchange Rates." In *Economic Policy in the International Economy: Essays in Honor of Assaf Razin*, E. Helpman and E. Sadka, eds. Cambridge: Cambridge University Press (2003).

[43] Sarno, L., Schneider P., and C. Wagner. "Properties of Foreign Exchange Rate Premia." *Journal of Financial Economics*, forthcoming.

[44] White, H. "Maximum likelihood estimation of misspecified models." *Econometrica*, 50 (1982), 1-25

[45] Wright, J. "Term Premia and Inflation Uncertainty: Empirical Evidence from an International Panel Dataset." *American Economic Review*, forthcoming.

Table 1: **Estimates of the Global and Country-Idiosyncratic Factor Parameters**

Factor Loadings ($X_{i,t} = \alpha_i + \beta_i G_t + F_{i,t}$)						
	$\alpha_i(\times 10^3)$			β_i		
	α_i^g	α_i^π	α_i^r	β_i^g	β_i^π	β_i^r
GM	4.59	-0.14	-0.08	0.62	0.16	0.72
	(2.26)	(2.83)	(2.20)	(1.86)	(1.98)	(4.46)
UK	-0.10	0.18	0.14	0.31	0.26	0.85
	(1.81)	(2.68)	(2.74)	(2.61)	(3.74)	(4.01)
JP	0.15	-0.59	0.23	0.94	0.22	0.57
	(3.09)	(2.99)	(3.28)	(2.96)	(2.59)	(5.27)
US	0.46	0.26	0.01	0.18	0.25	0.76
	(1.94)	(2.28)	(3.02)	(3.67)	(2.70)	(4.03)
Global Factor Dynamics ($G_t = \Phi^G G_{t-1} + \Sigma^G \upsilon_t^G$)						
	Φ^G			$\Sigma^G(\times 10^3)$		
	g^G	π^G	r^G	g^G	π^G	r^G
g^G	0.98	-0.01	-0.02	1	0	0
	(48.45)	(3.64)	(2.82)	–	–	–
π^G	0.15	0.90	0.07	0	1	0
	(2.40)	(27.40)	(1.64)	–	–	–
r^G	-0.16	0.08	0.89	0	0	1
	(2.53)	(4.90)	(16.16)	–	–	–
Country-Idiosyncratic Factor Dynamics ($F_{i,t} = \Phi^{F_i} F_{i,t-1} + \Sigma^{F_i} \upsilon_{i,t}^F$)						
	Φ^{F_i} (*diagonal*)			Σ^{F_i} ($\times 10^3$, *diagonal*)		
	f_i^g	f_i^π	f_i^r	f_i^g	f_i^π	f_i^r
GM	0.98	0.99	0.94	1.07	0.26	0.25
	(16.70)	(73.16)	(160.39)	(3.74)	(7.47)	(6.43)
UK	0.99	0.99	0.87	0.73	0.33	0.42
	(54.54)	(66.79)	(62.35)	(5.37)	(9.31)	(6.39)
JP	0.98	0.96	0.99	1.34	0.28	0.15
	(52.73)	(32.75)	(145.85)	(1.85)	(5.99)	(5.14)
US	0.99	0.98	0.93	0.65	0.26	0.33
	(93.88)	(38.83)	(133.83)	(3.80)	(2.66)	(5.90)

Note: This table reports the estimates of the global and country-idiosyncratic factor parameters. In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 2: **Estimates of Market Price of Risk Parameters**

	$\lambda^{G}_{i,0}$ $(\times 10^{2})$			$\lambda^{F}_{i,0}$ $(\times 10^{2})$		
	g^{G}	π^{G}	r^{G}	f^{g}_{i}	f^{π}_{i}	f^{r}_{i}
GM	-4.45	-7.81	-6.09	-0.39	-0.14	-0.82
	(2.76)	(4.45)	(3.13)	(1.92)	(1.22)	(1.29)
UK	-4.37	-9.51	-2.95	-1.10	-0.14	0.48
	(3.35)	(3.90)	(2.85)	(1.84)	(3.00)	(3.12)
JP	-3.84	-9.55	-2.04	-1.84	-0.41	-0.25
	(5.45)	(2.77)	(2.03)	(5.09)	(1.42)	(3.30)
US	-1.61	-7.75	-4.56	-0.60	-0.40	-1.33
	(2.28)	(2.42)	(3.83)	(4.20)	(2.27)	(4.14)
	$\lambda^{G}_{i,1}$ *(diagonal)*			$\lambda^{F}_{i,1}$ *(diagonal)*		
	g^{G}	π^{G}	r^{G}	f^{g}_{i}	f^{π}_{i}	f^{r}_{i}
GM	39.37	-25.65	-34.44	1.46	1.98	-6.62
	(7.30)	(5.96)	(8.68)	(1.74)	(1.83)	(2.69)
UK	39.89	-23.63	-37.71	3.63	-1.12	-6.90
	(7.90)	(5.17)	(9.49)	(2.38)	(1.36)	(1.70)
JP	38.53	-22.79	-39.11	2.69	-1.79	-4.44
	(7.27)	(3.98)	(7.68)	(2.68)	(1.32)	(3.41)
US	42.18	-24.10	-40.96	1.44	-2.86	-2.77
	(7.59)	(6.16)	(9.35)	(1.89)	(2.02)	(1.96)

Note: This table reports the estimates of market price of risk parameters. In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 3: **Estimates of Standard Deviation of Measurement Error Parameters** ($\times 10^4$)

	g_i	π_i	r_i	$y_i^{(24)}$	$y_i^{(60)}$	Δs_j
GM	10.81	0.34	3.75	0.26	1.77	212.27
	(4.65)	(2.87)	(7.84)	(3.49)	(3.10)	(2.48)
UK	6.56	0.67	6.62	0.09	0.88	202.52
	(3.70)	(2.59)	(4.60)	(1.91)	(1.95)	(3.64)
JP	9.70	1.31	1.56	0.79	0.99	271.85
	(2.12)	(3.73)	(2.82)	(3.60)	(4.52)	(7.58)
US	0.55	0.28	5.64	0.66	2.80	
	(2.89)	(3.57)	(6.73)	(2.18)	(2.94)	

Note: This Table reports the estimates of standard deviation of measurement error parameters. In parentheses, the absolute value of t-ratio of each estimate is reported. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 4: **Model Performance: Exchange Rate Dynamics**

	Mean(%)	*Std. Dev.(%)*	*Skewness*	*Kurtosis*	*Autocorr.*
			1. USD/GEM (EUR)		
Data	0.28	3.22	-0.24	3.92	0.07
Model	0.02	2.59	-0.27	4.93	0.09
			2. USD/GBP		
Data	0.12	3.05	-0.28	5.95	0.11
Model	-0.09	2.54	-0.41	5.89	0.14
			3. USD/JPY		
Data	0.33	3.24	0.30	4.43	0.08
Model	0.17	2.49	-0.97	8.57	0.13

Note: This table reports model performance for exchange rate dynamics. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 5: **Model-implied Exchange Rate Dynamics and Foreign Risk Premia**

Panel A. Δs and $\hat{\Delta s}$			
	USD/DEM	*USD/GDP*	*USD/JPY*
Explained Variation (R^2,%)	57	66	33
Corr(Δs,$\hat{\Delta s}$)(%)	75	81	58
Panel B. Fama Conditions			
	USD/DEM	*USD/GDP*	*USD/JPY*
Corr(rp,$r^{(h)} - r^{(f)}$)(%)	-9	-58	-22
Var(rp) ($\times 10^4$)	0.82	0.62	0.84
Var($r^{(h)} - r^{(f)}$) ($\times 10^4$)	0.04	0.02	0.03
Panel C. Foreign Risk Premia and Macro Differentials			
	USD/DEM	*USD/GDP*	*USD/JPY*
Corr(rp,$g^{(h)} - g^{(f)}$)(%)	-11	-32	-4
Corr(rp,$\pi^{(h)} - \pi^{(f)}$)(%)	-13	-59	-27

Note: This table reports model fitting for exchange rate dynamics in Panel A, the Fama Conditions in Panel B and the correlations between foreign risk premia and macroeconomic differentials in Panel C. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 6: **Model Performance: Macroeconomic Variables**

		Mean(%)	*Std. Dev.(%)*	*Skewness*	*Kurtosis*	*Autocorr.*
1. Germany						
output growth	Data	0.14	0.42	-2.07	10.51	0.86
	Model	0.15	0.40	-2.24	10.65	0.92
inflation	Data	0.16	0.11	0.85	4.34	0.96
	Model	0.16	0.11	0.85	4.34	0.96
interest rate	Data	0.38	0.18	0.84	2.93	0.98
	Model	0.38	0.18	1.02	3.21	0.99
2. UK						
output growth	Data	0.07	0.27	-1.02	6.85	0.88
	Model	0.07	0.26	-1.09	6.99	0.93
inflation	Data	0.31	0.17	1.24	4.79	0.97
	Model	0.31	0.17	1.24	4.79	0.97
interest rate	Data	0.61	0.28	0.76	2.73	0.98
	Model	0.59	0.25	0.57	2.65	0.97
3. Japan						
output growth	Data	0.07	0.54	-2.32	11.82	0.92
	Model	0.07	0.53	-2.37	12.01	0.93
inflation	Data	0.06	0.10	0.73	2.71	0.95
	Model	0.06	0.10	0.72	2.67	0.96
interest rate	Data	0.18	0.21	0.93	2.49	0.99
	Model	0.17	0.20	0.90	2.41	0.99
4. US						
output growth	Data	0.19	0.29	-1.41	6.76	0.94
	Model	0.19	0.29	-1.41	6.76	0.94
inflation	Data	0.25	0.10	0.01	3.63	0.93
	Model	0.25	0.10	0.01	3.63	0.93
interest rate	Data	0.37	0.17	-0.21	2.41	0.98
	Model	0.41	0.19	-0.15	2.47	0.97

Note: This table reports the statistic summary of observed and model-implied macroeconomic variables. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 7: **Model Performance: Yield Data**

Maturities		*Mean(%)*	*Std. Dev.(%)*	*Skewness*	*Kurtosis*	*Autocorr.*
1. Germany						
24-m	Data	0.40	0.16	0.79	2.88	0.98
	Model	0.40	0.16	0.78	2.88	0.98
60-m	Data	0.44	0.14	0.50	2.48	0.98
	Model	0.44	0.15	0.49	2.31	0.98
2. UK						
24-m	Data	0.57	0.22	0.44	2.32	0.98
	Model	0.57	0.22	0.44	2.32	0.98
60-m	Data	0.58	0.20	0.33	1.86	0.98
	Model	0.59	0.20	0.31	1.86	0.98
3. Japan						
24-m	Data	0.19	0.19	0.84	2.34	0.99
	Model	0.19	0.19	0.81	2.28	0.99
60-m	Data	0.23	0.18	0.61	1.98	0.99
	Model	0.23	0.18	0.66	2.08	0.99
4. US						
24-m	Data	0.44	0.18	-0.12	2.41	0.97
	Model	0.44	0.18	-0.11	2.42	0.97
60-m	Data	0.49	0.16	0.18	2.49	0.97
	Model	0.49	0.17	-0.03	2.07	0.98

Note: This table reports the statistic summary of observed and model-implied yield variables. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 8: **Data Correlations: Macro and Exchange Rate Data**

	01	*02*	*03*	*04*	*05*	*06*	*07*	*08*	*09*	*10*	*11*	*12*	*13*	*14*	*15*
01	1.00														
02	0.15	1.00													
03	0.53	0.32	1.00												
04	0.14	0.53	0.30	1.00											
05	-0.25	-0.19	-0.22	-0.16	1.00										
06	0.49	0.01	0.37	-0.17	0.20	1.00									
07	0.20	-0.07	0.11	-0.08	0.55	0.70	1.00								
08	0.34	0.04	0.49	-0.12	0.35	0.78	0.58	1.00							
09	-0.03	-0.15	-0.12	-0.24	0.76	0.56	0.73	0.56	1.00						
10	0.36	0.12	0.24	-0.18	0.19	0.88	0.72	0.68	0.65	1.00					
11	0.08	-0.01	0.09	-0.27	0.44	0.71	0.77	0.61	0.82	0.84	1.00				
12	0.48	0.35	0.36	0.14	-0.04	0.67	0.47	0.60	0.42	0.80	0.54	1.00			
13	0.03	0.03	-0.02	-0.05	-0.02	0.07	0.03	-0.04	0.03	0.10	0.10	-0.00	1.00		
14	0.02	0.01	0.03	0.02	-0.06	0.01	-0.01	-0.03	-0.06	0.02	0.01	-0.02	0.72	1.00	
15	-0.05	0.10	-0.01	0.00	-0.03	0.01	0.03	-0.02	0.03	0.05	0.12	-0.01	0.54	0.44	1.00

Note: This table reports the correlations of original monthly macroeconomic variables and exchange rate changes. There are four time series of output growth rates (index of *1-4*), inflation rates (index of *5-8*) and short-term interest rates (index of *9-12*), which are for Germany, the UK, Japan, and the US, repectively. Exchange rate changes data (index of *13-15*) are respectively the Germany mark/euro, the British proud and the Japanese yen, against the US dollar. The exchange rate changes are the monthly changes of log nominal exchange rate (the end of period market rate). The output growth rates and inflation rates are the annualized changes of IP and CPI, respectively. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 9: **Principal Component Analysis for Macroeconomic Fundamentals**

I. Output Growth				
	PC1	*PC2*	*PC3*	*PC4*
Variance prop.	76	12	8	4
Cumulative prop.	76	88	96	100
II. Inflation Rates				
	PC1	*PC2*	*PC3*	*PC4*
Variance prop.	71	18	7	3
Cumulative prop.	71	89	97	100
III. Short-Term Interest Rates				
	PC1	*PC2*	*PC3*	*PC4*
Variance prop.	84	10	4	2
Cumulative prop.	84	94	98	100

Note: This table reports the preliminary analysis of principle component analysis for macroeconomic fundamentals. For each group of output growth rates, inflation rates, and short-term interest rates, I report the variance proportions and cumulative variance proportions in percentage associated with the four principal components, which are positioned with a descending order according to associated eigenvalues. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 10: **Variance Decompositions: USD/DEM (EUR)**

	Global: G			*US*:F_4			*Germany*: F_1		
N	g^G	π^G	r^G	f_4^g	f_4^π	f_4^r	f_1^g	f_1^π	f_1^r
Panel A. Exchange rate changes, Δs									
1	31.47	4.41	27.81	2.29	4.35	15.80	3.80	0.39	9.67
3	31.82	4.36	29.57	2.28	4.15	14.95	3.77	0.41	8.68
12	39.33	4.35	28.64	2.32	3.20	11.58	3.74	0.41	6.42
24	48.44	5.46	29.87	1.83	1.55	6.60	2.70	0.30	3.25
60	51.69	10.33	30.83	0.98	0.62	2.80	1.26	0.14	1.36
Panel B. Foreign risk premium, rp									
1	22.59	4.41	72.98	0.00	0.00	0.00	0.00	0.00	0.01
3	25.69	3.26	71.04	0.00	0.00	0.00	0.00	0.00	0.01
12	41.89	11.47	46.63	0.00	0.00	0.00	0.00	0.00	0.00
24	49.43	18.07	32.50	0.00	0.00	0.00	0.00	0.00	0.00
60	49.59	21.28	29.13	0.00	0.00	0.00	0.00	0.00	0.00
Panel C. Unexpected changes, $\Delta s^{unexp.}$									
1	31.46	4.42	27.64	2.30	4.37	15.88	3.82	0.39	9.71
3	31.83	4.38	29.22	2.30	4.19	15.10	3.80	0.42	8.76
12	38.85	4.09	28.20	2.42	3.35	12.05	3.91	0.44	6.69
24	48.20	2.26	29.76	2.24	1.90	8.01	3.28	0.37	3.98
60	52.74	1.59	33.54	1.66	1.05	4.76	2.13	0.24	2.31

Note: This table reports variance decompositions for the model-implied exchange rate changes, the risk premium and the unexpected changes of the USD/DEM (EUR). The forecast horizons (N) are in months. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 11: **Variance Decompositions: USD/GBP**

	Global: G			*US*:F_4			*UK*: F_2		
N	g^G	π^G	r^G	f_4^g	f_4^π	f_4^r	f_2^g	f_2^π	f_2^r
Panel A. Exchange rate changes, Δs									
1	20.22	11.22	47.11	1.29	2.55	8.88	5.69	0.50	2.53
3	20.69	11.28	46.70	1.35	2.58	8.81	5.85	0.50	2.24
12	24.61	11.12	42.36	1.54	2.28	8.09	7.46	0.50	2.04
24	36.37	11.13	32.80	1.65	1.45	5.92	8.37	0.35	1.96
60	45.64	13.70	27.81	1.22	0.76	3.40	6.17	0.19	1.12
Panel B. Foreign risk premium, rp									
1	20.24	1.95	77.78	0.00	0.00	0.00	0.02	0.00	0.01
3	23.09	1.58	75.30	0.00	0.00	0.00	0.02	0.00	0.01
12	37.81	14.90	47.25	0.00	0.00	0.00	0.03	0.00	0.00
24	47.68	20.43	31.87	0.00	0.00	0.00	0.01	0.00	0.00
60	48.90	22.74	28.35	0.00	0.00	0.00	0.00	0.00	0.00
Panel C. Unexpected changes, $\Delta s^{unexp.}$									
1	20.23	11.23	47.06	1.29	2.56	8.89	5.71	0.50	2.52
3	20.67	11.32	46.61	1.36	2.59	8.83	5.88	0.51	2.24
12	24.03	11.10	42.49	1.58	2.33	8.26	7.64	0.51	2.06
24	34.37	9.47	33.81	1.88	1.67	6.73	9.47	0.40	2.21
60	44.41	7.57	29.00	1.80	1.14	5.08	9.08	0.28	1.64

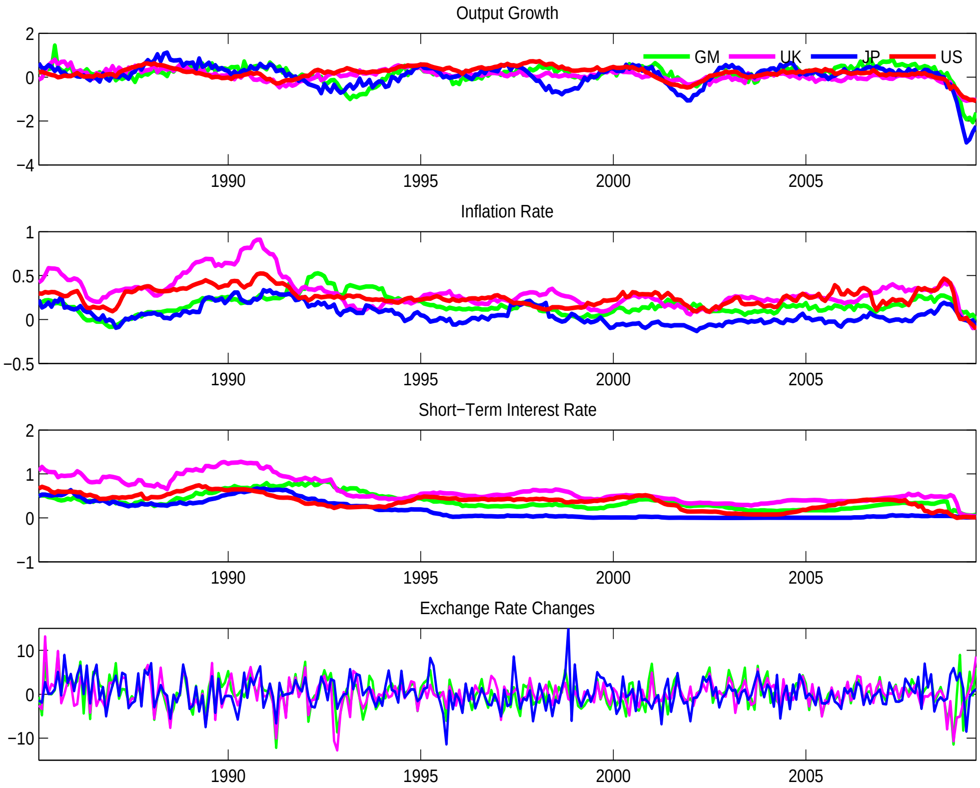
Note: This table reports variance decompositions for the model-implied exchange rate changes, the risk premium and the unexpected changes of the USD/GBP. The forecast horizons (N) are in months. The sample period is from 1985m01 to 2009m05 (293 observations).

Table 12: **Variance Decompositions: USD/JPY**

	Global: G			*US*:F_4			*Japan*: F_3		
N	g^G	π^G	r^G	f_4^g	f_4^π	f_4^r	f_3^g	f_3^π	f_3^r
Panel A. Exchange rate changes, Δs									
1	18.10	6.77	51.50	1.09	2.30	8.25	9.76	0.89	1.34
3	19.51	6.57	50.31	1.13	2.28	7.95	10.12	0.88	1.25
12	27.51	6.79	42.02	1.29	1.87	6.83	11.87	0.77	1.05
24	44.26	9.08	28.66	1.20	1.02	4.29	10.44	0.46	0.59
60	54.18	13.42	22.23	0.75	0.48	2.15	6.29	0.23	0.28
Panel B. Foreign risk premium, rp									
1	30.01	0.60	69.33	0.00	0.00	0.00	0.06	0.00	0.00
3	32.36	1.46	66.12	0.00	0.00	0.00	0.06	0.00	0.00
12	45.55	15.55	38.82	0.00	0.00	0.00	0.07	0.00	0.00
24	51.28	21.23	27.47	0.00	0.00	0.00	0.02	0.00	0.00
60	50.57	24.00	25.42	0.00	0.00	0.00	0.00	0.00	0.00
Panel C. Unexpected changes, $\Delta s^{unexp.}$									
1	18.07	6.79	51.44	1.10	2.31	8.28	9.78	0.89	1.35
3	19.40	6.60	50.25	1.14	2.30	7.98	10.18	0.89	1.26
12	26.99	6.58	42.06	1.33	1.93	7.02	12.23	0.80	1.07
24	44.27	5.96	28.42	1.42	1.23	5.09	12.36	0.55	0.69
60	60.30	4.57	18.84	1.19	0.77	3.46	10.05	0.38	0.44

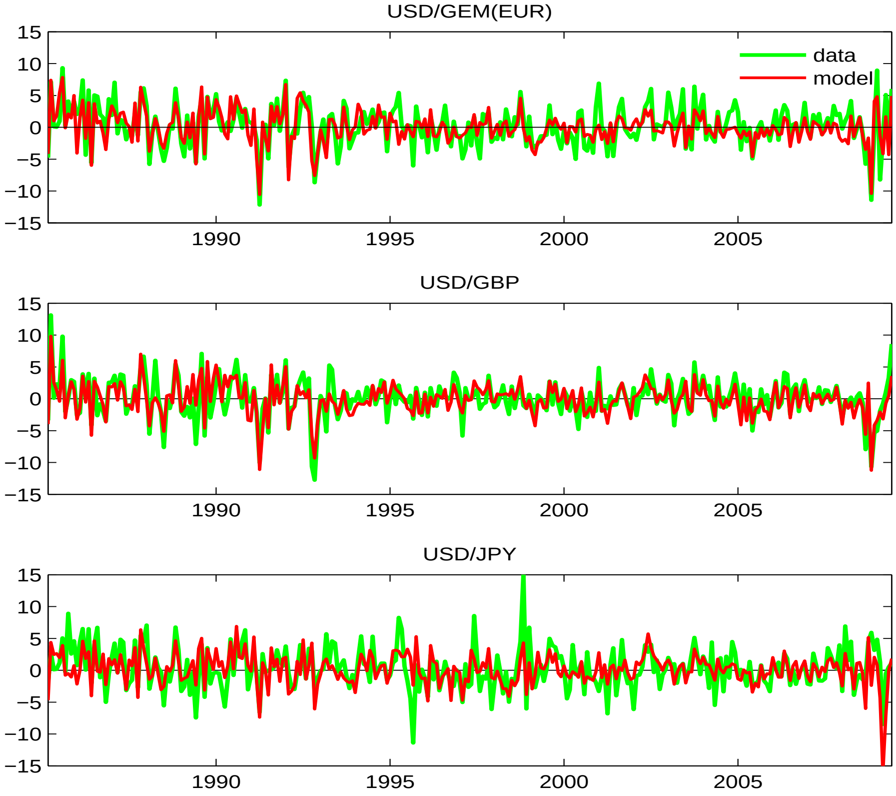
Note: This table reports variance decompositions for the model-implied exchange rate changes, the risk premium and the unexpected changes of the USD/JPY. The forecast horizons (N) are in months. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 1: Macroeconomic Data and Exchange Rates Changes



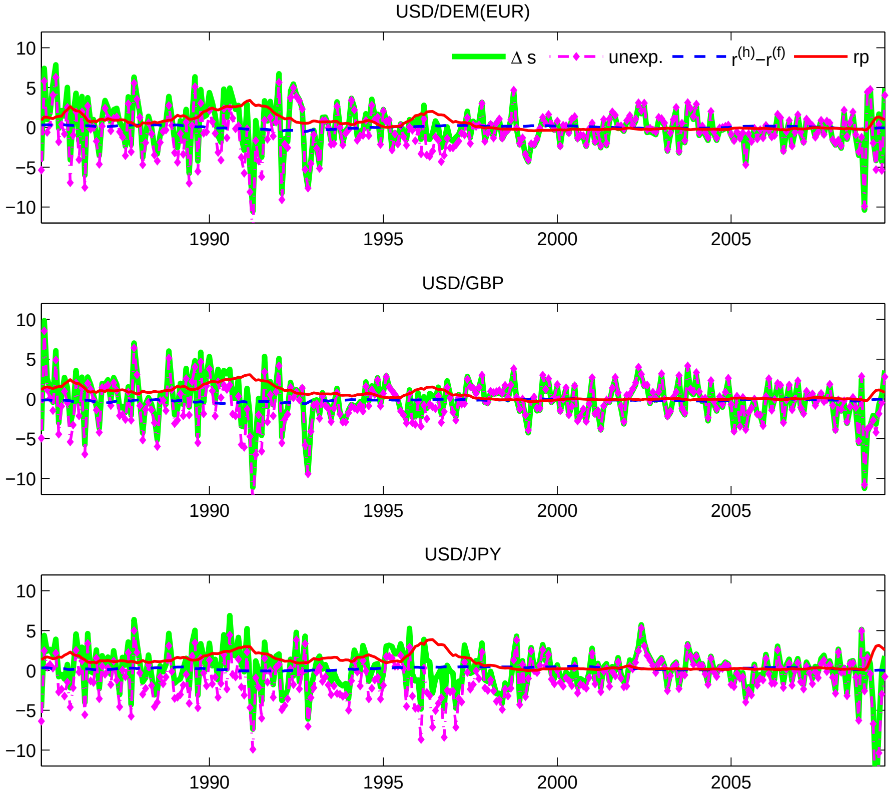
Note: This figure plots the macroeconomic fundamentals and monthly changes of exchange rates used in the estimation, for Germany, the UK, Japan, and the US. The panels are for output growth rates, inflation rates, short-term interest rates and exchange rate changes, respectively. Both macroeconomic variables and exchange rates changes are in monthly equalized quantities. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 2: Exchange Rate Dynamics



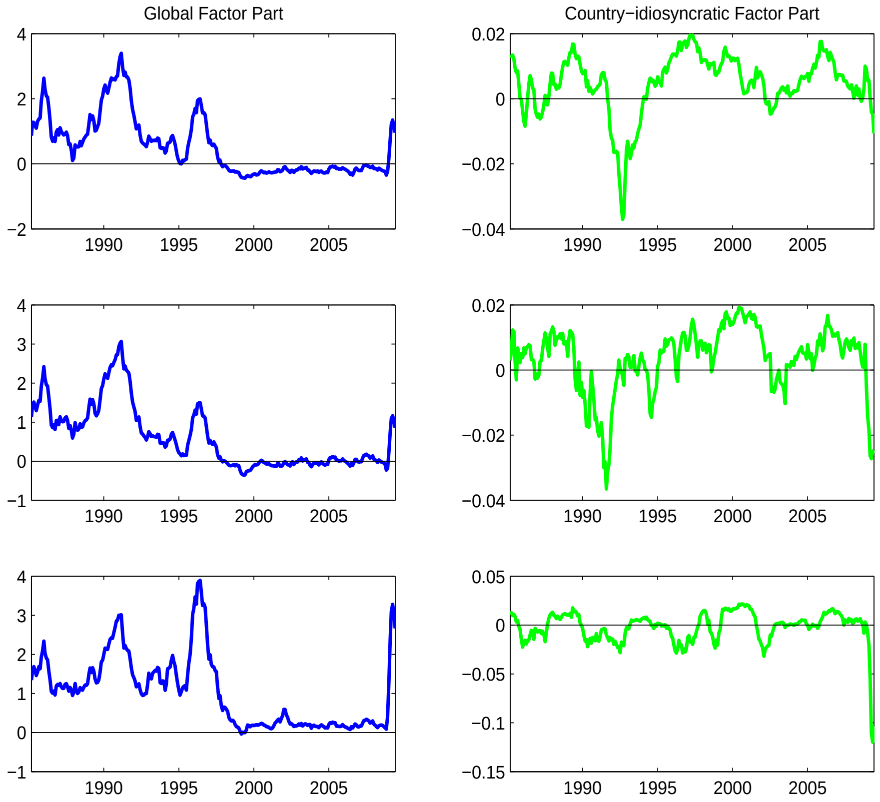
Note: This figure plots the monthly exchange rate changes in percentage of the foreign currencies, such as, Germany, the UK, and Japan, against the US dollar. The thick lines are observed data, while the thin lines are model-implied ones. The model is able to capture 57%, 66% and 33% of the observed exchange rates variances for the USD/GEM(EUR), the USD/GBP and the USD/JPY, respectively. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 3: Model-implied Exchange Rate Dynamics and Their Components



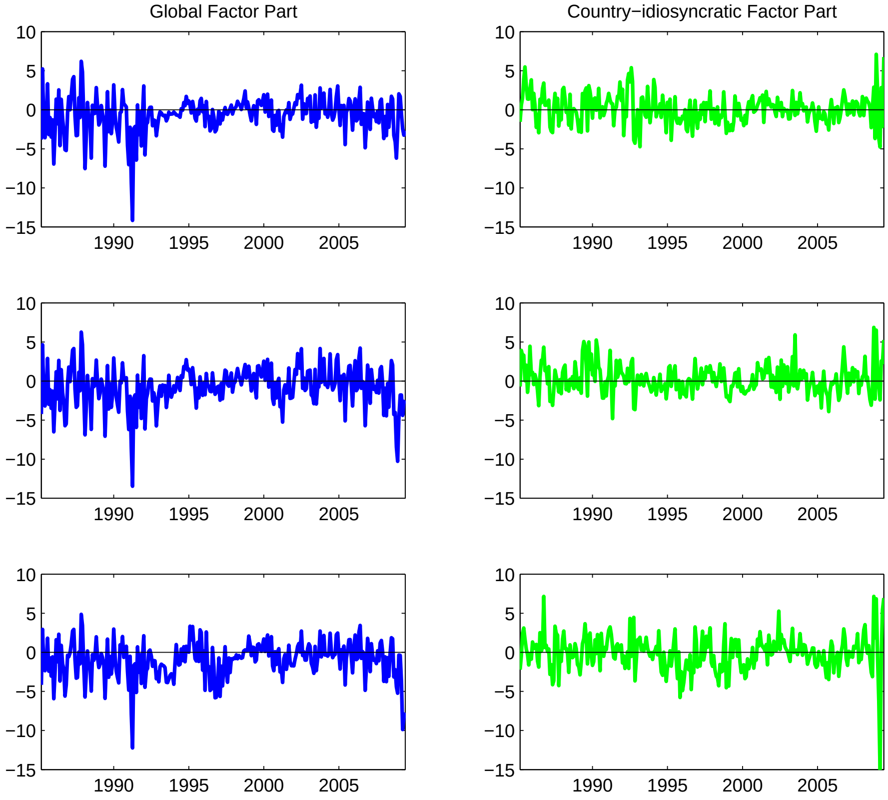
Note: This figure plots the model-implied monthly exchange rate changes, unexpected exchange rate changes, short-term interest rate differentials as well as foreign risk premia, in percentage for the foreign currencies, such as, Germany, the UK, and Japan, against the US dollar. The thick light lines are model-implied exchange rate changes, the thin dark lines are model-implied foreign risk premia, while the dash-dot lines are short-term interest rate differentials. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 4: Model-implied Foreign Risk premia by Global and by Country-idiosyncratic Factors



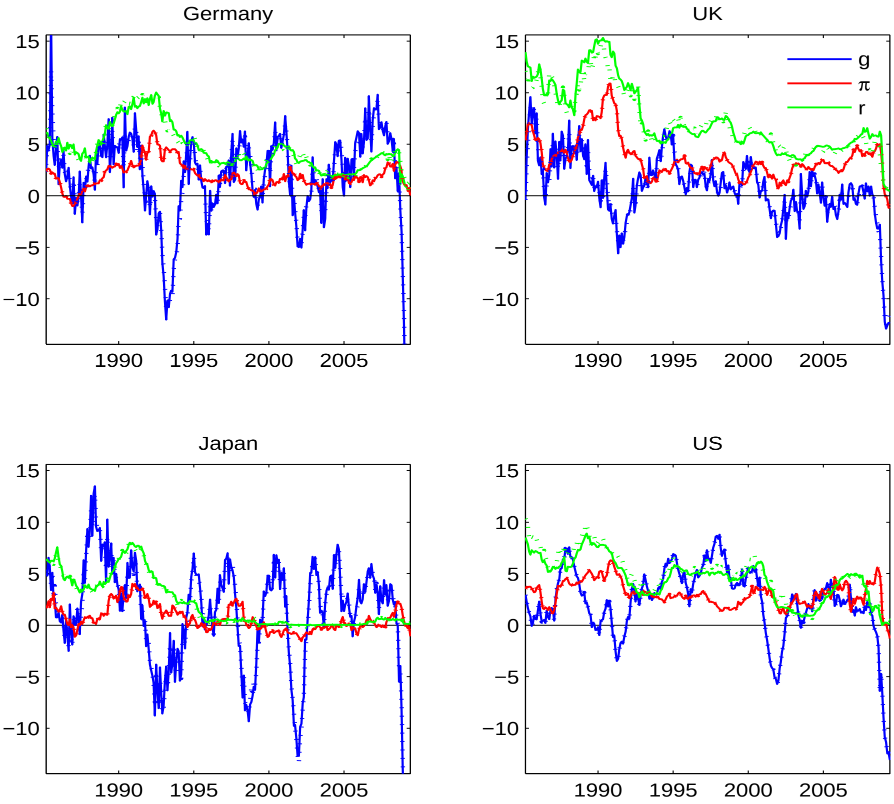
Note: This figure plots the model-implied foreign risk premia driving by global factors (the left panels) and country-idiosyncratic factors (the right panels) in percentage. Pictures from the top to the bottom panels are for the USD/DEM(EUR), the USD/GBP and the USD/JPY. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 5: Model-implied Unexpected Exchange Rate Dynamics by Global and by Country-idiosyncratic Factors



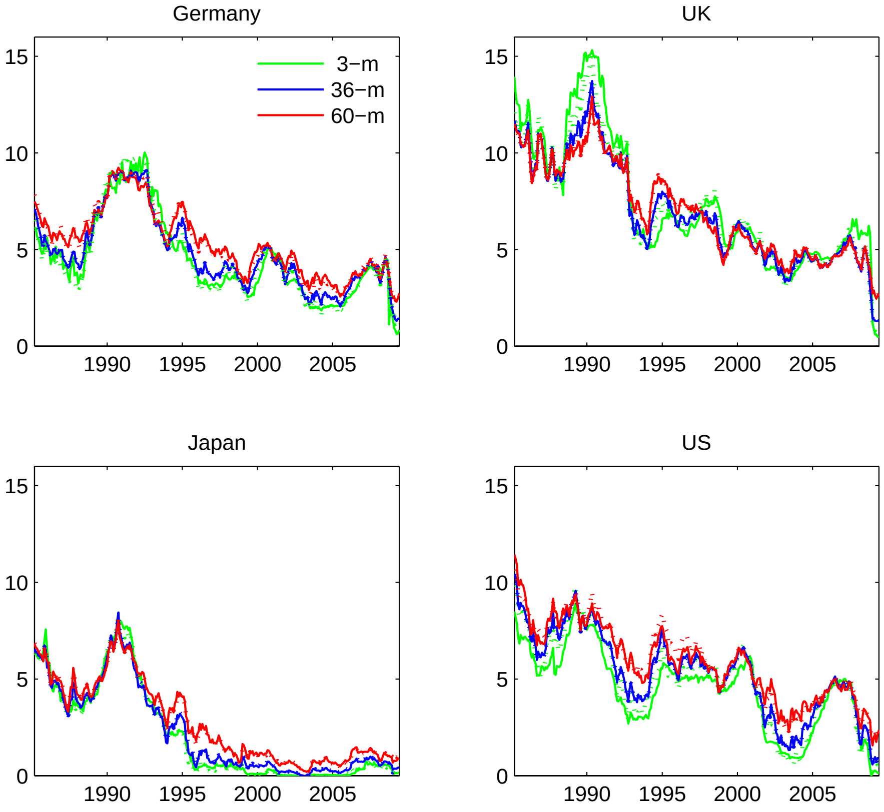
Note: This figure plots the model-implied unexpected exchange rate dynamics driving by global factors (the left panels) and country-idiosyncratic factors (the right panels) in percentage. Pictures from the top to the bottom panels are for the USD/DEM(EUR), the USD/GBP and the USD/JPY. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 6: Observed and Model-implied Macroeconomic Fundamentals



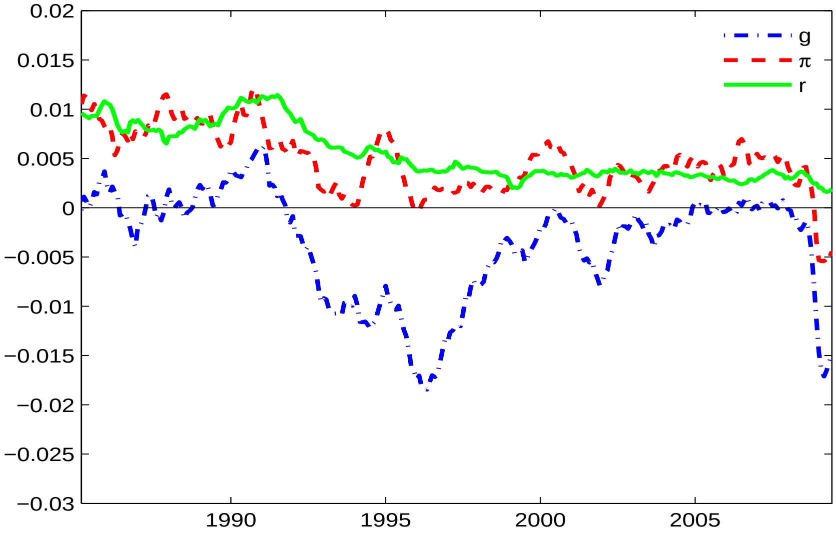
*Note:*This figure plots the monthly observed (solid lines) and model-implied (dotted lines) macroeconomic fundamentals, i.e. output growth, inflation rate, short-term interest rate, for Germany, the UK, Japan and the US. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 7: Observed and Model-implied Yield Data



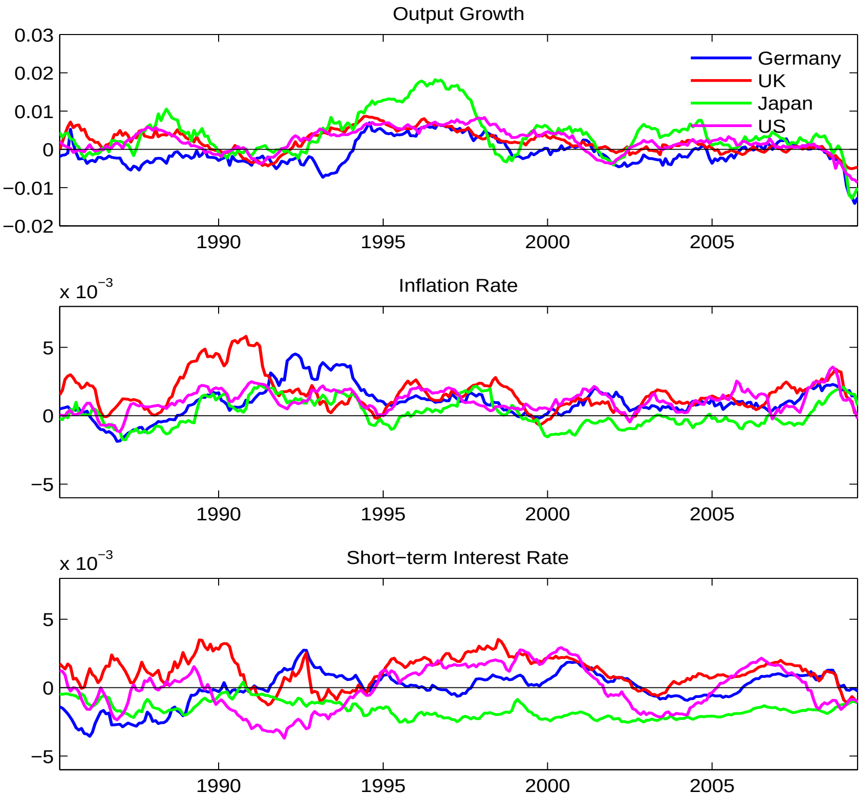
*Note:*This figure plots the monthly observed (solid lines) and model-implied (dotted lines) yield data, with maturities of 3-month, 24-month and 60-month, for Germany, the UK, Japan and the US. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 8: Global Macroeconomic Factors



Note: This figure plots the global macroeconomic factors filtered from the no-arbitrage multi-country model. The sample period is from 1985m01 to 2009m05 (293 observations).

Figure 9: Country-Idiosyncratic Macroeconomic Factors



Note: This figure plots the country-idiosyncratic macroeconomic factors filtered from the no-arbitrage multi-country model, for Germany, the UK, Japan and the US, respectively. The sample period is from 1985m01 to 2009m05 (293 observations).