

# Bank Competition, Securitization and Risky Investment

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October 8, 2011

## Abstract

We build a general equilibrium model of bank competition in which securitization is banks' optimal choice. A symmetric capacity-constrained Bertrand competition equilibrium exists as in the directed search literature, e.g. Burdett, Shi and Wright (2001). A key feature of the model is that banks face heterogeneous projects and they can use their lending rate as a tool to compete for good projects. The competition of banks lowers the lending rate, which in turn results in a low deposit rate. Consequently, a low level of credit supply and some uninvested high-return projects coexist. The shortage of credit supply resulting from bank competition naturally motivates banks to sell their assets through securities in order to raise more fund to invest in the projects being rationed.

JEL: E44

Keywords: bank competition, directed search, capital requirement, securitization, risky investment

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# 1 Introduction

Securitization is the financial practice of pooling mortgages, bank loans, credit-card debts, and other financial assets into securities that are sold to investors. As a financial innovation securitization appeared in the late 1970's and the volume of securitization experienced a rapid expansion thereafter, especially during the 2000's. In 2006 the outstanding securitized assets by financial intermediaries in U.S. has raised to more than 2.5 trillion US dollars (Duffie, 2007). Along with the burgeoning of the volume of securitization, two questions have been raised: what is the incentive of securitization and what are the consequences of the surge of securitization?

These questions have been broadly discussed. First, securitization activity is closely related to "credit risk transfer" (Gordon and Pennacchi, 1995; Calstrom and Samolyk, 1992, 1993; Allen and Carletti, 2006). However, securitization fails to achieve in practice what is sought in theory: it fails to shift the burden of defaults on securitized assets from financial institutions to final investors. Alan Greenspan stated that "(securitization can) spread risk over a broader spectrum of financial markets". The similar argument of risk sharing role of securitization is also supported in Allen and Gale (2005), Wagner and March (2006), etc.. But recent evidence tends to show that securitization increases financial risk to the originators of securities (Purnanandam, 2009; Keys et al., 2010; Greenlaw et al., 2008; Gordon, 2008; Berndt and Gupta, 2009; Michalak and Uhde, 2009). The evidence does not support risk-sharing as the first motive of securitization. Using US bank holding company data from 2001 to 2007, Sarkisyan et al. (2009) also draw a conclusion that banks view securitization as a financing rather than a risk management mechanism.

Another theory of bank loan sales appeal to regulatory constraints to motivate this off-balance-sheet activity (Pennacchi, 1988; Duffee and Zhou, 2001; Calomiris and Mason, 2004). Asset sales may allow a bank to avoid "regulatory taxes," i.e., reserve requirements and capital requirements. The evolution of capital requirement regulation could have encouraged securitization, but the timing of the Basel accords are not consistent with the timing of the securitization boom. An international rule on banks' lending activities, the Basel i accord, took place in 1988, which specifies 8% of capital requirement for banks' assets. The 2004

Basel ii accord enriched the details of capital requirement. The U.S. employed the Basel ii accord in 2005, while the most dramatic increase of the volume of securitization was in the beginning of 2000's. Moreover, although the empirical evidence shows that regulatory taxes have an important impact on loan sales, it also shows that more primary factors that may affect loan sales can be a bank's comparative advantage in originating loans (Pavel and Phillis, 1987; Demsetz, 2000).

A third theory is related to recycling bank funds: more loan originating opportunities of a bank may motivate the bank to securitize his assets in order to raise funds to invest in the available good projects (Gordon and Pennacchi, 1995; Parlour and Plantin, 2008). This theory is supported by empirical evidence. E.g., Demsetz (2000) finds that banks with ample loan origination opportunities more likely to sell loans. However, rare papers have paid attention to what might have caused the increased loan originating opportunities during the period when securitization booms. To our knowledge, the existing models did not endogenize the loan originating opportunity of banks. We propose that fierier bank competition might have distorted the loan market, generating excess demand for credit.

It is not new that the bank competition may cause excess demand for credit and it is supported by both theory and empirical evidence. Petersen and Rajan (1995) finds that it is easier for firms to get funding in the concentrated financial market than in the more competitive financial markets. It means that bank competition does not make borrowers easier, as a first instinct may tell, but rather harder to get loans. So wherever lending market is more competitive, there might be excess demand for credit.

There has been increasing evidence showing the competition among financial intermediaries has become fierier during the period when the scale of securitization has surged due to financial liberalization and deregulation. The quarterly Senior Loan Officer Opinion Survey (1997-2011) on bank lending practices released by the board of governors of the Federal Reserve System states that almost all domestic and foreign respondents (investment and commercial banks and other financial intermediaries) cited more aggressive competition from other banks or non-bank lenders as the most important reason for easing their lending standards and terms. The survey also revealed that 50% of those responding believe that increased competitive pressure reflected a permanent shift in the loan market. More than

20% of banks have reported eased lending standards for Commercial and Industrial loans and around 50% of banks have reported decreasing spreads on loan pricing. There are also some papers that document the increased competition in the financial sector. For example, Dell’Arrica et. al. (2008) find that lending standards declined more in areas that experienced large credit booms and that kept a higher volume of securitization and this change was triggered by entry of new and large lenders. A few papers (Boot and Schmeits, 2006; Hakenes and Schnabel, 2010, HS thereafter; Ahn and Breton, 2010, AB thereafter; Ahn, 2010) also remarked that interbank competition has been dramatically increasing.

We build a dynamic general equilibrium model of bank competition to show that bank competition could lead to excess demand for credit and motivate securitization. A key feature of the model is that banks face heterogenous projects and they can use lending rate as a tool to compete for good projects. Under the commonly applied capital requirement rules, the equilibrium features capacity-constrained Bertrand competition. The competition of banks makes them to face a trade-off between lending rate and number of borrowers. The competition induces a low lending rate, and therefore a low deposit rate, which in turn restricts the aggregate amount of credit supply. As a consequence, some profitable projects will be rationed, which motivates financial intermediaries to innovate in order to extend the size of their profitable investment. Securitization is such an innovation. Consistent with the financial liberalization during the period of securitization boom, we make two key assumptions in the model – capital requirement and free lending rate.

Our baseline bank competition framework could be considered as one application of the capacity-constrained Bertrand competition (or directed search) as in Peters (1984) and Burdett, Shi and Wright (2001, BSW thereafter). As shown in Peters (1984) and BSW, there is always a unique symmetric mixed strategy equilibrium, where all the sellers (banks in our model) post an identical price that is lower than the monopolistic price and higher than the perfect competitive price, and all the buyers (entrepreneurs in our model) choose an identical mixed strategy (a probability profile) on which island to attend.<sup>1</sup>

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<sup>1</sup>Besides a symmetric mixed strategy equilibrium, there could be other equilibria. For example, there could be an equilibrium of price dispersion (Arnold 2000, Shi 2009). We will assume that the banks can commit to the contracts that they post. If banks could renegotiate the terms of contract after they meet, their could also be price dispersion (See Camera and Selcuk 2009). Moreover, we allow an entrepreneur meets only one bank rather than multiple meetings, although the later may be also very interesting. For

Our model is most closely related to a few recent papers that paid attention to two ignored factors in the previous theory. First, bank competition could be a motive of asset sales (HS, 2010; AB, 2010; Ahn, 2010). Second, securitization could be motivated by endogenous credit supply (Shin, 2009). The theoretical models related to the incentive for securitization are mostly partial equilibrium analysis, while Shin (2009) advanced the focus to endogenous credit supply.

Besides the important difference in endogeneity of credit supply between our model and HS (2010), AB (2010) and Ahn (2010), we also explore different mechanism and deliver different welfare and risk implications. In terms of mechanism, HS uses a Salop spatial competition (Salop, 1979), and AB uses a two period duopoly competition model with switching cost as in (Gehrig and Stenbacka, 2007), while we use a capacity-constrained Bertrand competition (Kreps and Scheinkman 1983, and Peters 1984). Ahn (2010) focuses on a special form of loan sales — the single name loan sales, rather than securitization. The entrepreneurs in HS face a trade-off between interest rate and distance, and in AB they face a trade-off between interest rate and switching cost, but in our model they face a trade-off between interest rate and probability of being financed. The trade-off in our model arises because banks' lending capacity is restricted by the endogenous supply of funding. The probability of being financed depends on the loan market tightness (the number of borrowers relative to the number of lenders). While the switching cost in AB and the "iceberg" cost in HS are reasonable, their roles in the bank competition are diminishing as the information technology (including electronic banking) advances and transportation becomes less costly.

The consequences of securitization in our model are different than AB's model. AB finds that the equilibrium without monitoring, caused by securitization, entails the banks higher profit and leads to a worse quality of invested projects, which reduces the welfare. In our model, bank competition restricts credit supply, while securitization creates credit supply. In the equilibrium, output, profit of banks, and welfare are higher, but the quality of invested projects is lower. So securitization increases both welfare and aggregate risk in our model.

Securitization can cause a higher risk for an economy, but it is not fair to judge secu-

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reference, Albrecht, Gautier and Vroman (2006) has explored a directed search equilibrium with multiple job applications.

ritization by a higher risk alone. If the banks are effective in selecting good projects, the best projects should have been invested first and what is left should have lower quality in general. The extended lending through raising funds from securitization should be subprime loans. These loans increase the aggregate risk of default in an economy. While this increase in aggregate risk caused by securitization is easy to predict, it is often confused with "risk transfer" associated with securitization. Securitized assets are often repackaged and split into tranches. Each tranche has a different level of risk exposure than another: there is generally a senior ("A") class of securities and one or more junior subordinated ("B," "C," etc.) classes. The senior classes have first claim on the cash that the Special Purpose Vehicle (SPV) receives, and the more junior classes only start receiving repayment after the more senior classes have been repaid. After the repackaging the assets, the final investors may find their more favorable assets. For example, highly risk-aversion investors may favorite AAA rated securities rather than a private car loan, while some risk-love investors may be attracted by a high interest and choose a B or C class of securities. If the securities are priced reasonably, the transfer of risk does not affect the aggregate risk of assets, but influences the distribution of risk across different investors.

In addition, when we try to interpret the consequences of securitization, it will not be enough to ask whether there are higher default risks related to securitization. Rather we should ask whether those projects with higher risks should be invested or not. If those projects with higher risk should be invested, the consequences is a higher risk (which will be compensated by the return), no matter the projects are funded by selling loans or any other ways. Similar argument in Murray (2001) also points out that we should determine if there are risks unique to securitization as distinct from risks related to changes in overall market conditions. To get these comprehensive understanding of the consequences of securitization, we should use a general equilibrium model.

To understand how to price securities with different levels of risks is also very important. However, in the current paper, we are not going to model risk management by a bank, but focus on the motivation of securitization. It is true that after selling the assets as securities, the banks may have also changed the level of risk they face. Sometimes the banks may even strategically manage their risk by designing securities. There could also be some asymmetric

information problems in the process of securitization. Influential works on the designing of securities include Demarzo and Duffie (1999) and Rahi and Duffie (1995) among others. In this paper, we leave out this complication of security pricing. Instead, we simply assume that the banks sell proportionately their assets of different levels of risks. We also assume that the pricing of securitized assets is efficient. So there will be no aggregate risk generated in the process of securitization. What we focus will be why the economy generates excess demand for funding, or what causes some profitable projects not being invested in the first place, so that banks need to raise more money through securitization.

Our paper also contributes to constructing a tractable framework with many banks and a large number of diversified projects. In the model households make optimal portfolio choice, so the model could be extended to study pricing securities with different tranches. The model could be calibrated to data and simulated to analyze macroeconomic policy.

The rest of the paper is organized as follows. Section 2 describes the model environment; section 3 gives the optimal decisions of different agents: the households, the banks and the entrepreneurs; section 4 defines and characterizes the equilibrium without securitization; section 5 defines and characterizes the equilibrium with securitization; section 6 gives an example, and section 7 concludes.

## 2 Environment

Time is infinite in the forward direction and is divided into discrete periods indexed by  $t$ ,  $t = 0, 1, 2, 3, \dots$ . The economy consists of  $B$  ( $\geq 2$ ) islands indexed by  $i$ ,  $i = 1, 2, 3, \dots B$ . In each island, there is one bank. A bank may operate for multiple periods until it defaults. There are overlapping generations of two-period lived households (and an initial "old" generation in period zero). Each generation has a unit measure. A representative young household is endowed with one unit of labor, which is supplied inelastically to produce a canned good. The representative young household consumes a part of his labor income and saves the rest for consumption when he becomes old. The canned good can be saved either by a storage technology or through deposit at a bank. A household has access to all the banks. A bank can lend the deposit to entrepreneurs. There is also a large number  $N$  ( $\gg B$ ) of entrepreneurs,

each of them is endowed with a one-period project in every period  $t$ ,  $t = 1, 2, 3, \dots$ . A project takes the canned good as input and produce an intermediate good as output. We assume that the old households own the entrepreneurs in the sense that they get all the profits from entrepreneurs' projects.

The canned good is produced by a constant returns to scale technology using intermediate good and labor. Since the labor supply is fixed, we may write the production function in per young-household terms. For any period  $t$ , the production function of the canned good  $y_t$  is  $y_t = z_t f(m_{t-1})$ , where  $m_{t-1}$  is the amount of intermediate-good per young household (the production of  $m$  will be defined later) and  $z_t$  is an aggregate productivity shock. We take the random variable  $z_t$  to be i.i.d. over time, to be distributed continuously over a finite positive support, and to have a mean equal to  $\tilde{z}$ . The canned good can be consumed, stored, or invested in intermediate-good production.

One unit of the canned good in period  $t$  can still be one unit of the canned good in period  $t + 1$  through a storage technology. The canned good in period  $t$  can also be transformed into period  $t + 1$  intermediate good (without the use of labor) by means of an investment technology. This investment technology comes in discrete, indivisible units, called "projects". Each entrepreneur is endowed with one of these projects (and we assume that it is too costly to trade or transfer a project away from the original owner). A project takes exactly  $x$  unit of the canned good as input. With less than  $x$  units of the canned good, nothing is produced, and the marginal product of increments of the canned good to a project that already has its requisite quantity of input is zero.

Any project that is undertaken in period  $t$  produces a quantity of intermediate-good available for use in period  $t + 1$ . The amount of intermediate-good produced by a given project is a discrete random variable with possible outcomes  $\xi_j$ ,  $j = 1, 2$ . We focus on the case of only two outcomes: a good outcome  $\xi_1 = 1$  with probability  $\theta$ , and a bad outcome  $\xi_2 = 0$  with probability  $1 - \theta$ . The entrepreneur's type  $\theta$  obeys an i.i.d distribution with a Cumulative Distribution Function (CDF)  $G(\theta)$  and a Probability Density Function (PDF)  $g(\theta)$  on the support of  $[\underline{\theta}, \bar{\theta}]$  with  $0 < \underline{\theta} < \bar{\theta} \leq 1$ . The intermediate-good cannot be consumed but it can be used in the production of the canned good. The intermediate good is assumed to depreciate fully in one period (this is for an expositional reason only).



Although the entrepreneurs are endowed with the intermediate-good production technology, they do not have the canned good as input in the production. So the entrepreneurs need to borrow from the banks. Banks are assumed to have the expertise to screen and monitor entrepreneurs, since the later may report falsely their type if there is no screening and hide the intermediate good if there is no monitoring. But for simplicity, we assume that the banks can screen and monitor the entrepreneurs with zero cost while the households cannot screen and monitor the entrepreneurs.

Besides the bank's traditional intermediation function as delegated monitors (Diamond, 1984), banks are also assumed to have a new function: originate-to-distribute (O&D) (including securitization). The bank's new business of securitization is to fully exploit their special expertise of analyzing the credit worthiness of borrowers. That is, the banks originate a larger pool of loans and resell some of their loans to other investors. In order to avoid banks' moral hazard problem of investing in bad projects and reselling them to other investors, the banks are often required to keep a proportion of their securities. This is usually called "skin in the game".

An authority (central bank) is assumed to regulate the banks' behavior. First, the central bank sets a capital requirement for loans. That is, for a certain amount of loan,  $k$  ( $1 > k \geq 0$ ) proportion of it has to be financed by bank's capital, and only  $1 - k$  proportion of it could be financed by households' deposit. Second, if a bank securitizes his assets, the bank has to hold at least  $\lambda$  proportion of the securities (the original assets).

In each period, given the central bank's regulation, the banks make an investment plan. According to their plan, they choose a quantity of bank capital  $K_i$ . Given  $K_i$ , the banks raise deposit in a competitive market. All the banks take deposit rate  $r$  as given, the total volume of deposit that the bank  $i$  will get,  $S_i(r)$ , has to satisfy  $\frac{S_i(r)}{K_i + S_i(r)} \leq 1 - k$ . We assume that the depositors have full insurance, so the volume of deposits depends only on the interest rate, and it does not depend on any risk.

After the funds have been raised, the bank lends them to entrepreneurs. The procedure of applying for funds is as follows. (1) The banks post their loan contract conditions which are observable to other banks and all the entrepreneurs; (2) Given the posted contracts, every entrepreneur chooses a strategy (a probability profile) on which island to attend. This

strategy is a public information; (3) Every entrepreneur visits an island to apply for funds; (4) The banks evaluate the risk of each project that comes to their own island and discover the quality of the project,  $\theta$ . The quality of a project  $\theta$  is a common information in the island where the project is evaluated, but other banks do not know it. (5) The banks decide which entrepreneurs to finance. (6) The entrepreneurs stay in the island: the ones that have been financed produce the intermediate goods, while the ones that have no funding do nothing but stay in the island. Some of the entrepreneurs who have relatively good quality may have the chance to be invested later if the banks could raise more money through securitization. Note that, during the period, the entrepreneurs cannot move to other islands (or during the period, these entrepreneurs will not be evaluated by other banks).

After the banks have invested in their selected projects, they can resell a proportion of their loans to the young households as securities. Let  $\hat{\theta}$  denote the average success probability of the securities in the market. The security will entail a return  $r^a$  with probability  $\hat{\theta}$ . Both the banks and the households will take the contract of securities  $(r^a, \hat{\theta})$  as given. The total volume of securities from bank  $i$  is denoted by  $S_i^a(r^a, \hat{\theta})$ . If  $r < 1$ , then the representative young household stores all the canned goods that he will save for old consumption. We denote the storage by  $s$ . The banks can use the fund from selling the securities to invest in new projects. All these invested projects produce the intermediate goods, which will be available for the next period.

## 3 Optimal Decisions

### 3.1 Households

A representative young household in period  $t$  supplies 1 unit of labor inelastically and earns wage income  $w_t$ . He consumes  $c_t^y \geq 0$  and saves the rest through the following means: deposit at a bank  $i$ ,  $S_{i,t} \geq 0$ , securities from a bank  $i$ ,  $S_{i,t}^a \geq 0$ , for all  $i = 1, 2, \dots, B$ , and storage,  $s_t \geq 0$ . The saving decisions are made according to the interest rate of the deposit,  $r_t$ , and the interest rate of the securities,  $r_t^a$ , together with the average probability of getting the returns of the securities,  $\hat{\theta}_t$ .

A representative old household owns the entrepreneurs who produce the intermediate goods and takes all their profit. The old household passes the ownership of the entrepreneurs to the next generation when he dies. The expected profit of entrepreneurs is denoted by  $\tilde{\pi}_{e,t+1}$ . This profit from entrepreneurs and the savings of the representative household will all be used to consume when he becomes old. The consumption of the representative old household in period  $t + 1$  is  $c_{t+1}^o \geq 0$ .

The representative young household's maximization problem is

$$\max U(c_t^y) + \beta EU(c_{t+1}^o)$$

subject to the following budget constraints

$$c_t^y + \sum_{i=1}^B S_{i,t} + \sum_{i=1}^B S_{i,t}^a + s_t = w_t, \text{ and } c_{t+1}^o = r_t \sum_{i=1}^B S_{i,t} + \theta_t r_t^a \sum_{i=1}^B S_{i,t}^a + s_t + \tilde{\pi}_{e,t+1}. \quad (3.1)$$

The utility function satisfies the usual assumptions and the discounting factor satisfies  $0 < \beta < 1$ . We restrict our attention to the case where the deposit at bank  $i$  is non-decreasing in interest rate  $r_t$ , i.e.  $S_{i,t}(r_t)$  weakly increases in  $r_t$ .

## 3.2 Entrepreneurs

The entrepreneurs make one period decision only. So we omit the subscript  $t$ . After observing the loan contract conditions, the entrepreneurs choose a strategy on which island to attend. The contract conditions are described by  $\left\{ \gamma_i, \vec{\theta}_i(n_i) \right\}_{i=1}^B$ , where  $\gamma_i$  is the bank's share of the produced intermediate good by the project,  $\vec{\theta}_i$  is a threshold value of project quality  $\theta$  above which the project will be financed. As in Peters (1984), we let  $\gamma_i$  not contingent on the number of visitors to simplify matters. However, the selection criterion  $\vec{\theta}_i$  will depend on the number of visitors. If the number of visitors is large,  $\vec{\theta}_i$  is going to be large given the bank's fixed lending capacity. We also assume that the banks can commit their  $\gamma_i$ , so we do not consider the possible bargaining after the banks meet with the entrepreneurs as in Camera and Selcuk (2009). Also note that the bank's profit is contingent on the realization of the project, but it does not depend on the evaluated quality  $\theta$  of a project. This contingency

means that a bank  $i$  will get a positive profit (in a  $\gamma_i$  proportion) from the project if and only if the project succeeds.

After an entrepreneur has arrived at his chosen island, he draws a success probability  $\theta$  from the distribution  $G(\theta)$ . The information of  $\theta$  is unknown to anybody. The bank needs to evaluate the project in order to discover the value of  $\theta$ . For simplicity, we ignore any cost associated with evaluation except that the banks can evaluate all the projects only at once in any one period. After the bank's evaluation,  $\theta$  is known to both the bank and the entrepreneur, but it is still a sealed information for other banks. So we shut down the incentive of entrepreneurs to move to other islands once they are evaluated by a bank since they have lost the opportunity to be evaluated by other banks in the same period. According to the revealed  $\theta$ , the bank decides whether to lend fund to the entrepreneur. All the entrepreneurs bear limited liability, that is an entrepreneur pays back to the bank at most the amount  $\xi_i$  if state  $i$  is realized. With limited liability, an entrepreneur always has a positive expected return if he invests in his project, so he will always be willing to borrow from the bank.

If an entrepreneur goes to island  $i$ , he faces a contract  $\{\gamma_i, \vec{\theta}_i(n_i)\}$ . An entrepreneur can expect to be financed with probability  $p_i = 1 - G(\vec{\theta}_i(n_i))$ . Let  $\hat{\theta}_i = \frac{\int_{\vec{\theta}_i(n_i)}^{\bar{\theta}} \theta dG(\theta)}{1 - G(\vec{\theta}_i(n_i))}$  be the expected average probability of success, the expected profit of an entrepreneur that chooses island  $i$  is

$$\pi_{e,i} = p_i \hat{q} (1 - \gamma_i) \hat{\theta}_i, \quad (3.2)$$

where  $\hat{q}$  is the expected price of the intermediate good in the next period.

Not only the conditional contract value  $\vec{\theta}_i(n_i)$  depends on  $n_i$ , but also implicitly on both the profitability of the projects and the available funding ( $K_i + S_i$ ) at the bank  $i$ . First, if the expected price of the intermediate goods  $\hat{q}$  is high, or/and the marginal cost of funding, denoted by  $\eta$  (to be defined later), is low, then the projects are more profitable for the bank. If the projects are more profitable in general, then a project with a lower  $\theta$  might be worth investing. We define  $\hat{\theta}_i$  as the lower bound of project quality, above which projects

are profitable. Then  $\hat{\theta}_i$  is determined by

$$x\eta = \hat{q}\hat{\theta}_i\gamma_i. \quad (3.3)$$

To satisfy the profitability condition, we should have the selection criterion  $\vec{\theta}_i \geq \hat{\theta}_i = \frac{x\eta}{\hat{q}\gamma_i}$ .

Second, the conditional contract value  $\vec{\theta}_i$  also depends on the potential number of projects attracted to island  $i$  and the available funding at bank  $i$ ,  $D_i = K_i + S_i$ . Let  $n_i$  be the total number of entrepreneurs that came to island  $i$ . Given any  $D_i$  and  $n_i$ , there is a  $\tilde{\theta}_i$  that satisfies (3.4),

$$n_i x \left[ 1 - G(\tilde{\theta}_i) \right] = D_i. \quad (3.4)$$

We denote  $\tilde{\theta}_i$  as a function  $\Theta(D_i, n_i)$ . The value of  $\Theta(D_i, n_i)$  decreases in  $D_i$ ,  $\Theta_1(D_i, n_i) < 0$ , and increases in  $n_i$ ,  $\Theta_2(D_i, n_i) > 0$ . If  $\tilde{\theta}_i > \hat{\theta}_i$ , then  $D_i$  is not sufficient to support all the profitable projects. As a consequence, bank  $i$  selects only the good projects that have  $\theta \geq \tilde{\theta}_i$ . The threshold value  $\vec{\theta}_i$  is therefore determined by  $\vec{\theta}_i = \max \left\{ \hat{\theta}_i, \tilde{\theta}_i \right\}$ .

An entrepreneur chooses an island according to  $\max_i \{ \pi_{e,i} \}$  across all  $i$ . Given  $\pi_{e,i}$  determined in (3.2), an entrepreneur faces a trade-off between  $\gamma_i$  and  $\vec{\theta}_i$ : in an island with a lower  $\gamma_i$ , although the profit share of the entrepreneur,  $1 - \gamma_i$ , is higher, the probability of being invested,  $p_i$ , is lower, since the island  $i$  with a lower  $\gamma_i$  may attract a larger number of entrepreneurs (higher  $n_i$ ). Given this trade-off, the entrepreneurs' expected profits in all islands should be equal in an equilibrium. Otherwise, if an island  $j$  offers lower expected profit than other islands, the entrepreneurs would choose not to come to this island.

### 3.3 Banks

We assume that banks are very impatient and risk neutral. A bank  $i$ 's expected utility is

$$u_b = E_0 \sum_{t=0}^{\infty} \frac{1}{\rho^t} c_{b,i,t},$$

where  $c_{b,i,t}$  is the bank  $i$ 's consumption of the canned good in period  $t$ . We assume that the discount factor of a bank,  $1/\rho$ , is much smaller than  $\beta$ . This is a simple way to motivate a high cost of acquiring bank capital: the bank rather consumes everything he has if there is

no capital requirement, and borrow from the household to invest in his available projects. If there is capital requirement, the bank must maintain some bank equity. We assume that every bank is endowed with a large amount of bank equity at period 0. Such that the banks have enough canned good to cover the capital requirement. In the future period, the banks have to save from their profit to maintain the bank equity.<sup>2</sup>

With this linear utility function, a bank's objective is equivalent to maximize the expected present value of his life time profits, which is also called the Franchise value of the bank. The Franchise value of a bank  $i$  in period  $t$  is

$$V_{i,t} = \max \pi_{b,i,t} + \frac{1}{\rho} E_t (V_{i,t+1}), \quad (3.5)$$

where  $\pi_{b,i,t}$  is the expected profit of bank  $i$  in period  $t$ .

Here we do not allow banks to strategically default on the deposits of households. "Strategic default" means that the default plan is made before the aggregate states and the individual states are realized. If a bank strategically defaults, he may earn excess profit in the event of default at the expenses of depositors. Invulnerable default, on the contrary, is due to bad state and the bank earns zero profit when it defaults. Whether to strategically default may depend crucially on the default regulation and the capital requirement rate  $k$ . If any strategic default (when banks earn positive profits) would be caught and severely punished, there is no strategic default. But if not all the strategic defaults will be caught, some speculators may take the chance to default strategically (moral hazard). Capital requirement may reduce the bank's incentive of strategic default.

In every period, a bank  $i$  makes decisions on his capital, deposit, loan contract, and security sale, sequentially. We divide every period into four stages accordingly. Without confusion, we omit the subscript  $t$  below. In the first stage, the bank chooses an amount of capital,  $K_i$ , with the fixed marginal cost  $\rho$ . The banks have to make rational expectation about the optimal decisions in the following stages, in order to decide how much  $K_i$  to hold.

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<sup>2</sup>In order to avoid the case in which bank industry's total capital (equity) is constrained by the total previous period profit in the bank sector, we assume that banks can acquire capital from outside with a fixed cost  $\rho$ . This assumption does not affect steady state analysis. If we take serious the constraint of bank industry's total capital, there might be interesting business cycle dynamics from the bank industry. But that is out of the scope of the current paper.

In the second stage, taking the market rate  $r$  as given, the bank  $i$  raises deposit  $S_i$  from the young households, with a constraint  $k(S_i + K_i) \leq K_i$ . In the third stage, the bank  $i$  posts the contract  $\{\gamma_i, \vec{\theta}_i(n_i)\}$  and lends the fund to entrepreneurs after entrepreneurs' types are discovered. Those entrepreneurs who receive funding have the top tier projects, which are projects with quality  $\theta$  on the right tail of the distribution of  $\theta$ . If the banks are still interested in some second tier projects, which are not invested through funding from bank capital (equity) and deposit (debt), they may look for "out of balance sheet" method to raise fund. In the fourth stage, the bank  $i$  decides how much of his loan should be securitized. The banks use funding through securities to invest in the second tier projects, which we can also call "subprime" loans.

We first consider a case in which banks have not innovated securitization, and leave the securitization analysis to section 5. We solve the bank's problem by backward induction. In the third stage, the bank  $i$  chooses  $\gamma_i$ , given that  $K_i$  and  $S_i$  have been determined. Let  $\pi_{b,i}$  be the bank  $i$ 's profit and  $n_i p_i$  be the total number of projects that the bank  $i$  finances, then the expected profit is

$$\pi_{b,i} = \max_{\gamma_i} \left\{ \hat{q} \hat{\theta}_i \gamma_i n_i p_i - \rho K_i - r S_i \right\}. \quad (3.6)$$

The banks will always ensure this expected profit non-negative. However, if  $n_i$  is a finite number, then it is possible that, ex post, a bank earns a negative profit. To avoid this problem of deposit risk, we assume full insurance in the banking sector. Moreover, the rate of this potential failure of a bank is small if capital requirement  $k$  is large.

Let  $\Phi(\gamma_i)$  be the total loan the bank  $i$  would have if he posts  $\gamma_i$ ,

$$\Phi(\gamma_i) = \min \left[ n_i(\gamma_i) \left[ 1 - G(\hat{\theta}(\gamma_i)) \right] x, D_i \right]. \quad (3.7)$$

The cut-off value of  $\theta$ ,  $\hat{\theta}_i$ , is determined by the bank's profit break-even condition, i.e  $x\eta = \hat{q}\hat{\theta}_i\gamma_i$ . If the bank increases  $\gamma_i$ , more projects become worth investing for the bank, resulting in an decrease in  $\hat{\theta}_i$ ,  $\hat{\theta}'_i(\gamma_i) \leq 0$ . The total number of attracted entrepreneurs, however, decreases in  $\gamma_i$ ,  $n'_i(\gamma_i) < 0$ .

## 4 Symmetric equilibrium without securitization

We first shut down the technology for securitization and restrict our attention to a stationary symmetric strong Nash equilibrium where  $K_i, S_i, D_i, \gamma_i, n_i, p_i$  and  $\vec{\theta}_i$  are identical for all  $i = 1, 2, \dots, B$ , and all the entrepreneurs choose an identical mixed strategy on which banks to attend. As shown in Peters (1984) and BSW (2001), such a capacity-constrained Bertrand equilibrium always exists and it is unique.

The aggregate state variables in the economy are the total quantity of the intermediate good,  $m$ , and the aggregate productivity,  $z$ , at the beginning of each period. The wage rate and the price of the intermediate good are determined by these two state variables, that is,  $q = zf'(m)$  and  $w = y - qm$ . Here we have assumed that the output production function features constant returns to scale.

We are going to compare an equilibrium with only one bank and a symmetric equilibrium with many banks. In the equilibrium with many banks, the banks can compete with each other. In this capacity-constrained Bertrand competition with a larger number of entrepreneurs  $N$ , the number of banks  $B$  does not affect the equilibrium, as long as there are more than two banks and  $N/B$  is always very large. By comparing the two equilibria, we show that the competition across banks lowers the equilibrium lending rate and creates excess demand for funding.

### 4.1 One bank equilibrium

If there is only one bank, the bank can earn the highest possible profit by posting  $\gamma = 1$ . We assume that an entrepreneur always invests in his project as long as he receives fund, even if he earns zero profit. So  $\gamma = 1$  means that the bank gets all the surplus from the invested projects and the entrepreneurs earn zero profits. In this case, the marginal project  $\vec{\theta}$ , above which all projects will be financed, satisfies  $\vec{\theta} = \hat{\theta} = x\eta/\hat{q}$ , where  $\eta = (1 - k)r + k\rho$  is the marginal cost of one unit of fund.

In the equilibrium, the bank lends exactly what he has raised. The bank has no incentive to raise more fund than what is needed for his investment, since there is no benefit from additional funding but there is additional cost associated with additional funding. In lemma



1 We can show that this statement is true for both an equilibrium with only one bank and a symmetric equilibrium with many banks. The proof of lemma 1 is provided in appendix A.

**Lemma 1** *In a symmetric equilibrium, all the banks raise an amount of fund such that*

$$\Phi(\gamma_i) = n_i(\gamma_i) \left[ 1 - G\left(\hat{\theta}(\gamma_i)\right) \right] x = D_i.$$

According to Lemma 1, the funding supply equals the funding demand:

$$N \left[ 1 - G(\hat{\theta}) \right] x = D. \quad (4.1)$$

Given that  $\hat{\theta} = x\eta/\hat{q}$  and  $\eta = (1-k)r + k\rho$ , the equilibrium condition (4.1) can be written as

$$N \left[ 1 - G\left(\frac{x[(1-k)r + k\rho]}{\hat{q}}\right) \right] x = \frac{1}{1-k} S,$$

which gives a fund-demand function  $S^d = S^d(r)$ . It is easy to show that  $S^d(r) < 0$ . Together with the fund-supply function  $S(r)$  from the consumer's problem, we can solve the equilibrium interest rate. Since we assume that  $S'(r) \geq 0$ , a unique  $r$  can be solved. So there is a unique equilibrium with  $\gamma = 1$ .

## 4.2 A symmetric equilibrium with many banks

If there are more than one bank, i.e.  $B \geq 2$ , then banks cannot maintain a symmetric equilibrium with  $\gamma_i = 1$ , for all  $i = 1, 2, \dots, B$ . If all the banks post  $\gamma_i = 1$ , the equilibrium outcome is equivalent to one with only a single bank. However, if all the banks post  $\gamma_i = 1$ , then a bank has an incentive to deviate from it. If a bank  $i$  decreases his  $\gamma_i$  a little bit, so that  $\gamma_i < 1$  and  $\gamma_j = 1$  for all  $j \neq i$ , an entrepreneur can expect a positive profit from visiting bank  $i$ , since  $(1 - \gamma_i)\theta > 0$ . As a consequence, all the entrepreneurs would be attracted to the deviating island  $i$ . If all the entrepreneurs come to island  $i$ , the bank  $i$  can select better projects than before the deviation. Let  $\vec{\theta}_i^d$  be the project selection criterion by the bank  $i$ , let  $\vec{\theta}^*$  be the project selection criterion if all the banks post  $\gamma = 1$ , then  $\vec{\theta}_i^d > \vec{\theta}^*$ . The bank faces all  $N$  potential projects when it deviates from  $\gamma = 1$ , while it faces  $N/B$  if all banks

post  $\gamma = 1$ . The total funding  $D_i$  now can be used to support  $N \left[1 - G(\vec{\theta}_i^d)\right]$  number of projects, that is  $Nx \left[1 - G(\vec{\theta}_i^d)\right] = D_i$ , while before the deviation  $D_i$  can be used to support  $\frac{N}{B} \left[1 - G(\vec{\theta}^*)\right]$  number of projects, that is  $\frac{N}{B}x \left[1 - G(\vec{\theta}^*)\right] = D_i$ . Since  $B \geq 2$ , we have  $\vec{\theta}_i^d > \vec{\theta}^*$ . Therefore, the average success probability of the invested projects is higher. Using the same funding  $D_i$ , now the bank  $i$  can invest in a larger number of projects with a higher average probability of success, the bank  $i$  would deviate from posting  $\gamma_i = 1$ . As a result, it is not an equilibrium if all the banks post  $\gamma = 1$ . If there is a symmetric equilibrium with bank competition, then  $\gamma_i < 1$ .

**Definition 1** *A symmetric equilibrium with bank competition is defined by sequences of quantities  $\left\{\{n_{i,t}, S_{i,t}, K_{i,t}, D_{i,t}\}_{i=1}^B, m_t, y_t, c_t^y, c_t^o\right\}_{t=1}^\infty$ , prices  $\{\gamma_t^c, r_t^c, q_t, w_t\}_{t=1}^\infty$ , an initial value of intermediate good  $m_0$ , an initial value of bank capital  $a_0$ , and a policy parameter  $k$  such that: (1) the representative young household maximizes his expected life-time utility subject to (3.1), taking as given the wage rate, the interest rates, and the expected profit from entrepreneurs; (2) the representative old household consumes everything he gets from his income; (3) taking as given the market deposit rate, the strategy of entrepreneurs and the strategy of other banks, the capital requirement rate  $k$ , and the expected price of the intermediate good  $\hat{q}_{t+1}$ , the banks choose their capital  $K_{i,t}$ , raise deposit  $S_{i,t}$  from the young households, post a profit division rule  $\gamma_t^c$  and a project selection rule (i.e. a threshold value of project quality  $\vec{\theta}_{i,t}$ ) to maximize the expected value (3.5); (4) an entrepreneur chooses a strategy on which islands to attend to maximize his expected profit; (5) the total canned good is produced according to  $y_t = z_t f(m_{t-1})$ ; (6) all the markets clear; (7) all the prices and quantities are identical across islands; and (8) no banks deviate from the equilibrium.*

The markets clearing conditions are apparent in the labor market, the intermediate good market, and the credit market. In the canned good market, it should be

$$c_1^y + c_1^o + S_1 + s_1 + c_{b,1} + K_1 = y_1 + b_1 + a_0, \text{ where } y_1 = z_1 f(m_0),$$

and

$$c_t^y + c_t^o + S_t + s_t + c_{b,t} + K_t + \rho b_{t-1} = y_t + v_{t-1} + s_{t-1} + b_t, \text{ for } t = 2, 3, 4, \dots$$

We assume that the initial intermediate goods  $m_0$  is owned by the old households. The bank's initial capital  $a_0$  is large enough so that the bank does not need to borrow in order to satisfy the capital requirement in the first period, that is  $b_1 = 0$  and  $a_0 > K_1$ , more specifically,  $c_{b,1} + K_1 = a_0$ . The demand for the canned good consists of the total consumption by the young households,  $c_t^y$ , the total consumption by the old households,  $c_t^o$ , the deposit and storage of the young households,  $S_t$  and  $s_t$ , the total consumption by the banks,  $c_{b,t}$ , the total bank capital,  $K_t$ , and the debt repayment,  $\rho b_{t-1}$ . The supply of the canned good consists of the total output  $y_t$ , the total new debt of the banks from outside,  $b_t$ , the storage from last period,  $s_{t-1}$ , and the canned good at the banks that have not been lent out,  $v_{t-1} = D_{t-1} - \Phi_{t-1}$ . All the variables in the above market clearing condition are aggregate variables, for example,  $b_t = \int b_{i,t} di$  and  $v_{t-1} = \int D_{i,t-1} di - \int \Phi_{i,t-1} di$ . In the equilibrium  $v_{t-1} = 0$  according to lemma 1.

In a symmetric equilibrium, we have  $n_i = N/B$ ,  $\hat{\theta}_i^c = x\eta / (\hat{q}\gamma^c)$  and  $n_i x \left[ 1 - G\left(\tilde{\theta}_i^c\right) \right] = D_i$ . Without confusion we have dropped the subscript for  $t$  and  $\hat{q}$  is the expected price of intermediate goods in the next period. If a symmetric competitive equilibrium with  $\gamma^c \in (0, 1)$  exists, we have to ensure that no banks deviate from it. We prove that such a symmetric equilibrium exists and it is unique under certain conditions. This is the most important result in this paper and it is summarized in proposition 1. The proof is provided in the appendix.

**Proposition 1** *There exists a symmetric equilibrium with bank competition. This symmetric equilibrium is unique if the distribution of  $\theta$  satisfies that the term  $\frac{\int_{\vec{\theta}} \theta dG(\theta)}{\vec{\theta} [1 - G(\vec{\theta})]}$  weakly decreases in  $\vec{\theta}$ .*

We have shown that  $\gamma < 1$  in the equilibrium with bank competition. Consequently, some projects that could be profitable if  $\gamma = 1$  are not worth investing from the banks' perspective. To maximize profit, the banks have already made the best use of equity and debt. The banks do not want to use more equity, because it is expensive, i.e., the marginal cost of equity  $\rho$  is high. Given the bank equity, he has already used up the maximum amount of deposit he can get according to the capital requirement. In the equilibrium, the banks cannot get funds from the traditional equity and debt to finance the projects being rationed.

The coexistence of potential profitable projects and shortage of funding may motivate the banks to innovate "out of balance sheet" activities.

Of course, securitization is not just about raising more funds, it is also about how to raise more funds (design securities and their prices, e.g. DeMarzo and Duffie 1999 and Rahi and Duffie 1995), which is not the focus of the current model. It is true and important that through securitization banks can sell illiquid assets to raise funds, and they repackage the assets of different levels of risk to suit the tranches of securities to the final investors's taste for risks. This whole process of securitization increases the supply for funding. But if there is no potentially profitable projects available, banks have no need to raise more "out of balance sheet" funds. Our focus is the creation of demand for funding by the competition across banks. Given the role of making some potentially profitable projects rationed, bank competition could be a trigger for securitization.

Here are some remarks on the bank capital and capacity-constrained Bertrand equilibrium. In an environment of bank competition, capital requirement reduces the potential benefit from deviating the symmetric equilibrium. This is because the deposit at a bank is restricted by the capital requirement rate. With restricted size of deposit, the benefit from attracting additional projects is limited. So capital requirement has an effect of preventing excess bank competition, and therefore banks earn positive profit in the equilibrium. Without this restriction, the equilibrium will be a Bertrand equilibrium with all the banks earning zero profit.

## 5 Symmetric equilibrium with securitization

In this paper, securitization is defined as pooling contractual debts that have different risk levels and selling them to households. To be consistent with the above model environment, we let securitization be the fourth stage of banks' decisions, after the banks post their loan contract in the third stage. Suppose a bank  $i$  is allowed to sell up to  $1 - \lambda$  proportion of his investment in projects to households, the bank could raise additional fund to finance the projects rationed in the third stage.

Recall that the information about the types of entrepreneurs is revealed after they visit

one island. This information is a common knowledge to both the bank and the entrepreneurs in the island, but the information is unknown to other banks. Implicitly, other banks will need to evaluate the projects that could be invested by themselves, but that cannot be done in the same time period. By this assumption, the entrepreneurs do not move after their types have been revealed, because they cannot get funded by other banks in the same period. If the banks can raise fund and invest in the rationed projects, the banks can take the whole profit from them since the entrepreneurs do not have outside options. This assumption simplifies the analysis, but the general result should not be affected.

To solve the problem in the securitization stage, we first find the threshold value of  $\theta$ , above which the projects are going to be financed,  $\vec{\theta}_i^a$ . The value of  $\vec{\theta}_i^a$  depends on the profitability of projects and the availability of securities. We denote  $\hat{\theta}_i^a$  as the threshold value of  $\theta$ , above which projects are profitable, then  $\hat{\theta}_i^a$  should satisfy  $xr^a\hat{\theta}_i = \hat{q}\hat{\theta}_i^a$ . Let  $\tilde{\theta}_i^a$  be the threshold value of  $\theta$  that satisfies

$$xn_i \left[ G(\vec{\theta}_i) - G(\tilde{\theta}_i^a) \right] = (1 - \lambda) \Phi_i, \quad (5.1)$$

where  $\vec{\theta}_i$  is the project quality above which projects were financed in the third stage. Recall that  $\Phi_i = \min \left\{ n_i x \left[ 1 - G(\hat{\theta}_i) \right], D_i \right\}$  is the total lending that had gone to the investments in the third stage. So  $(1 - \lambda) \Phi_i$  is the maximum quantity of assets that could be securitized and sold by bank  $i$ . Then,  $\vec{\theta}_i^a = \max \left\{ \hat{\theta}_i^a, \tilde{\theta}_i^a \right\}$ .

The projects with quality between  $\vec{\theta}_i$  and  $\vec{\theta}_i^a$  are not financed before the banks can sell their assets. Bank competition causes  $\vec{\theta}_i$  to be higher than  $\vec{\theta}_i^*$ , the later is the threshold quality in the one bank equilibrium. In the equilibrium with securitization,  $\vec{\theta}_i^a$  might be higher or lower than  $\vec{\theta}_i^*$ . If the constraint (5.1) is not binding, however, it is for sure that  $\vec{\theta}_i^a < \vec{\theta}_i^*$ . This is because the bank can get enough fund to support all the projects that are profitable, i.e. the revenue from the project covers the cost of the project. Moreover, raising money through securities is less costly than through deposit since there is no capital requirement on securitization. Of course, this might not be true if there is other cost associated with securitization. On the other hand, if the constraint (5.1) is binding, then  $\vec{\theta}_i^a$  might be higher than  $\vec{\theta}_i^*$ , due to insufficient funding.

The demand for funding from new projects is  $xn_i \left[ G(\vec{\theta}_i) - G(\vec{\theta}_i^a) \right]$ , which weakly decreases in  $r^a$  through  $\vec{\theta}_i^a$ . The supply of funding from selling securities is denoted by  $S_i^a(r^a, \hat{\theta})$ , where  $r^a$  is the interest rate for securities and  $\hat{\theta}$  is the market success probability of securities. The supply of funding is eventually the households' spending on securities, which strictly increases in  $r^a$ . Given  $r^c$ ,  $\vec{\theta}_i$ ,  $\hat{\theta}$ , and  $\Phi_i$  determined in the third stage, there exists an  $r^a$  such that the demand of funding equals the supply of funding in the fourth stage,

$$xn_i \left[ G(\vec{\theta}_i) - G(\vec{\theta}_i^a) \right] = S_i^a(r^a, \hat{\theta}). \quad (5.2)$$

Through securitization the economy can extend the number of financed projects from  $N \left[ 1 - G(\vec{\theta}_i) \right]$  to  $N \left[ 1 - G(\vec{\theta}_i^a) \right]$ . As a result, the economy is going to have a higher level of the intermediate good,  $m' = \hat{\theta}_i^a N p_i^a$ , where  $\hat{\theta}_i^a = \frac{\int_{\vec{\theta}_i^a}^{\vec{\theta}} \theta dG(\theta)}{1 - G(\vec{\theta}_i^a)}$  and  $p_i^a = 1 - G(\vec{\theta}_i^a)$ . Recall that  $\hat{q} = \hat{z} f'(m')$  and  $w = y - \hat{z} f'(m') m'$ , so the expected price of the intermediate good  $\hat{q}$  will decrease, while the total output  $y$  and wage rate  $w$  will increase.

In the fourth stage, the bank's problem is to maximize the total profit subject to a "skin in the game" constraint

$$xn_i \left[ G(\vec{\theta}_i) - G(\vec{\theta}_i^a) \right] \leq (1 - \lambda) \Phi_i. \quad (5.3)$$

Knowing the problem the bank is going to face in the fourth stage, he chooses a pair of  $(\gamma_i, \vec{\theta}_i)$  in the third stage to maximize the total profit, including the profit in the third stage and the profit in the fourth stage.

In the third stage, the indifference curve over  $(\gamma_i, \vec{\theta}_i)$  for the entrepreneurs is still the same as in the case of no securitization, since we have assumed that the banks get all the profits in the securitization stage so that the entrepreneurs only need to consider their profits in the third stage. However, the banks may post a  $\gamma_i$  higher or lower than in the case of no securitization, since  $\gamma_i$  affects not only the bank's profit in the third stage, but also the bank's profit in the fourth stage. Given the interest rate  $r^c$ , the expected interest rate  $r^a$ , the total funding  $D_i$ , the indifference curve of the entrepreneurs, and the strategy of other banks, the bank  $i$  chooses a pair of  $(\gamma_i, \vec{\theta}_i)$  to maximize the following profit from the third

and the fourth stage:

$$\pi_{b,i} = \max_{\gamma_i} n_i \frac{\gamma_i \pi_e^c}{(1 - \gamma_i)} - \eta D_i + n_i \hat{q} \int_{\vec{\theta}_i^a}^{\vec{\theta}_i} \theta dG(\theta) - \hat{\theta}_i r^a S_i^a(r^a, \hat{\theta}). \quad (5.4)$$

Here,  $n_i = \frac{D_i}{x[1-G(\vec{\theta}_i)]}$ , and  $\vec{\theta}_i = \tilde{\theta}_i$ , since the banks still have no need to raise more funding than necessary.

However, it is possible that  $x\eta > \hat{q}\gamma_i\hat{\theta}_i$ , i.e., the bank may invest in some projects with negative expected return in the third stage. The purpose of investing in these "non-profitable" projects is that the banks can sell their loans through securitization. The fund raised through securitization can be used to invest in new projects to earn more profit, which may cover the loss incurred by the projects with  $x\eta > \hat{q}\gamma_i\theta_i$ .

The possibility of investing "non-profitable" projects highlights the bank's motive of reselling their loans: they have good originating opportunity. This originating opportunity is caused by excess competition across banks: a low  $\gamma_i$  increases the lending standard and cuts off funding for some good projects. The banks put themselves in a difficult situation: if a bank increases his  $\gamma_i$ , he loses a pool of potential projects to his competitors; if the bank keeps a low  $\gamma_i$ , then only some very good projects can give the bank enough return to cover the cost of funding and many good projects cannot be financed. The banks may choose to post a relatively low  $\gamma_i$  to attract the potential projects to them, and then get additional funding from selling their loans to finance the projects that could not get financed in the first run.

Potentially, there is another interesting angle of investing in non-profitable projects. If the final investors are unaware of a high risk of getting negative return when they buy securities backed by these projects with negative expected return, the banks can gain information premium from securitization by charging a higher than "should-be" price from the unsuspecting final investors. In that case, the banks may have an additional incentive for securitization: to get an excess premium from asymmetric information. However, we do not explore this information rent in the current paper, rather we try to clarify the framework of bank competition and securitization in a simple way. Of course, extensions could be made to handle the design and pricing of securities in a general equilibrium framework with bank

competition.

However, even in this simple framework, the bank  $i$  may have an incentive to lower his paying rate of securities  $\hat{\theta}_i$ , taking as given the market success probability of securities,  $\hat{\theta}$ . The bank  $i$  will not consider the externality he imposes on the market when he increases his  $\gamma_i$ . When  $\gamma_i$  increases,  $\hat{\theta}_i$  decreases, and the bank pays the securities  $\hat{\theta}_i r^a S_i^a(r^a, \hat{\theta})$ .

Whether we have some "non-profitable" projects being invested or not depends on whether the "skin in the game" constraint (5.3) is binding. If the constraint is binding, i.e.,

$$xn_i \left[ G(\vec{\theta}_i) - G\left(\vec{\theta}_i^a\right) \right] = (1 - \lambda) \Phi_i,$$

then the bank may invest in some projects that have negative expected return, i.e.  $x\eta > \hat{q}\gamma_i\hat{\theta}_i$ . Since a binding "skin in the game" constraint makes selling additional assets valuable and the banks have an incentive to increase the volume of their loans. On the other hand, if the bank expects that the constraint (5.3) will not be binding, then he has no need to invest in "non-profitable" projects, and in that case  $x\eta = \hat{q}\gamma_i\hat{\theta}_i$ .

Depending on whether the constraint (5.3) is binding, we have two possible cases. In the first case, if the constraint (5.3) is binding, then not all projects with  $\theta > \hat{\theta}_i^a = xr^a\hat{\theta}_i/\hat{q}$  are invested, so  $\vec{\theta}_i^a = \tilde{\theta}_i^a$ , where  $\tilde{\theta}_i^a$  is determined by  $xn_i \left[ G(\tilde{\theta}_i) - G\left(\tilde{\theta}_i^a\right) \right] = (1 - \lambda) \Phi_i$  from (5.1). In the second case, if the constraint (5.3) is not binding, then  $\vec{\theta}_i^a = \hat{\theta}_i^a$  and  $xr^a\hat{\theta}_i = \hat{q}\hat{\theta}_i^a$ .

We first consider the case in which the constraint (5.3) is binding. The binding case is more general in the real world. The reasons could be that the technique of securitization is restricted or the cost of securitization is high. In this case, the binding "skin in the game" constraint gives

$$G(\tilde{\theta}_i) - G\left(\tilde{\theta}_i^a\right) = (1 - \lambda) \left[ 1 - G(\tilde{\theta}_i) \right]. \quad (5.5)$$

We differentiate completely the equation (5.5) to get

$$\frac{d\tilde{\theta}_i^a}{d\tilde{\theta}_i} = \frac{(2 - \lambda) g(\tilde{\theta}_i)}{g(\tilde{\theta}_i^a)}. \quad (5.6)$$

The term  $\frac{d\tilde{\theta}_i^a}{d\tilde{\theta}_i} > 0$  in (5.6), indicating that if  $\tilde{\theta}_i$  increases, then  $\tilde{\theta}_i^a$  increases. It means that a higher quality of assets (a smaller volume of assets) in the third stage would cause a lower



amount of securities in the fourth stage, due to the binding "skin in the game" constraint.

In the first case, a bank's problem is to choose a  $\gamma_i$  to maximize the total profit, both from the investment in the third stage and from the investment in the fourth stage,

$$\pi_{b,i} = \max_{\gamma_i} n_i \frac{\gamma_i \pi_e^c}{(1 - \gamma_i)} - \eta D_i + n_i \hat{q} \int_{\tilde{\theta}_i^a}^{\tilde{\theta}_i} \theta dG(\theta) - \hat{\theta}_i r^a S_i^a(r^a, \hat{\theta}), \quad (5.7)$$

where  $n_i = \frac{D_i}{x[1-G(\tilde{\theta}_i)]}$ . We can show that there is a unique symmetric Nash equilibrium given some conditions. This result is summarized in proposition 2 and the proof is provided in the appendix.

**Proposition 2** *In the case where the constraint (5.3) is binding, if the distribution of  $\theta$  and  $\lambda$  satisfy the condition  $\frac{g(\tilde{\theta}_i^a)}{g(\tilde{\theta}_i)} \leq \frac{2-\lambda}{1-\lambda}$  for any  $0 < \tilde{\theta}_i^a \leq \tilde{\theta}_i$  with  $1 - G(\tilde{\theta}_i^a) = (2 - \lambda) [1 - G(\tilde{\theta}_i)]$ , there exists a unique value of  $\gamma_i$  that maximizes the profit of the bank  $i$ .*

Note that in proposition 2 the conditions for the unique symmetric equilibrium are sufficient conditions.

In the case where the "skin in the game" constraint (5.3) is not binding, an equilibrium does not exist. We will show that in lemma 2 and the proof is provided in the appendix.

**Lemma 2** *If the constraint (5.3) is not binding, there is no stationary symmetric equilibrium.*

As we show in the appendix, the banks will always have incentive to lower  $\gamma_i$  to attract more potential projects, if the constraint (5.3) is not binding. This process will lead us to go back to the first case where  $\gamma_i$  is low enough and the constraint (5.3) is binding.

## 6 Example

In this example we solve a symmetric equilibrium with a uniform distribution of  $\theta$ , i.e.,  $G(\theta) = \frac{\theta - \underline{\theta}}{\theta - \underline{\theta}}$ . With a uniform distribution of  $\theta$ , the term  $\frac{\int_{\tilde{\theta}_i^a}^{\tilde{\theta}_i} \theta dG(\theta)}{\tilde{\theta}_i [1 - G(\tilde{\theta}_i)]}$  decreases in  $\tilde{\theta}_i$  according to proposition 2, so there is a unique symmetric equilibrium in the case of no securitization. For any  $0 < \tilde{\theta}_i^a \leq \tilde{\theta}_i$  with  $1 - G(\tilde{\theta}_i^a) = (2 - \lambda) [1 - G(\tilde{\theta}_i)]$ , it is also true that the condition

$\frac{g(\tilde{\theta}_i^a)}{g(\tilde{\theta}_i)} \leq \frac{2-\lambda}{1-\lambda}$  is satisfied. According to proposition 3, we have a unique symmetric equilibrium with securitization.

## 6.1 Equilibrium without securitization

We first derive the entrepreneur's indifference curve. Given the total funding  $D_i$ , the threshold value of  $\theta$  above which the projects can be financed is

$$\tilde{\theta}_i = \bar{\theta} - (\bar{\theta} - \underline{\theta}) \frac{D_i}{n_i x}. \quad (6.1)$$

Without securitization,  $\hat{\theta}_i = \tilde{\theta}_i$  according to lemma 1. So we can have

$$r^c = \left[ \hat{q} \gamma_i \tilde{\theta}_i / x - (1 - k) \rho \right] / k. \quad (6.2)$$

Using (6.1), the entrepreneur's expected profit becomes

$$\pi_e^c = \frac{\hat{q}(1 - \gamma_i)}{2} \left[ 2\bar{\theta} \frac{D_i}{n_i x} - (\bar{\theta} - \underline{\theta}) \left( \frac{D_i}{n_i x} \right)^2 \right]. \quad (6.3)$$

Equation (6.3) gives an indifference curve over the choices of  $(\gamma_i, n_i)$  for an entrepreneur.

We express  $\gamma_i$  in terms of  $n_i$ ,

$$\gamma_i = 1 - \frac{\pi_e^c}{\frac{\hat{q}}{2} \left[ 2\bar{\theta} \frac{D_i}{n_i x} - (\bar{\theta} - \underline{\theta}) \left( \frac{D_i}{n_i x} \right)^2 \right]}. \quad (6.4)$$

Taking as given the indifference curve of the entrepreneurs (6.3) and their expected profit  $\pi_e^c$ , the expected profit of a bank is

$$\pi_{b,i}(n_i) = n_i \frac{\gamma_i \pi_e^c}{(1 - \gamma_i)} - \eta D_i.$$

Here we have made  $n_i$  the choice variable, instead of  $\gamma_i$ , just for convenience. We substitute

$\gamma_i$  from (6.4) into the above profit function to get

$$\pi_{b,i}(n_i) = n_i \left\{ \frac{\hat{q}}{2} \left[ 2\bar{\theta} \frac{D_i}{n_i x} - (\bar{\theta} - \underline{\theta}) \left( \frac{D_i}{n_i x} \right)^2 \right] - \pi_e^c \right\} - \eta D_i. \quad (6.5)$$

The first order condition is

$$\psi(n_i) = \frac{\hat{q}}{2} (\bar{\theta} - \underline{\theta}) \left( \frac{D_i}{n_i x} \right)^2 - \pi_e^c = 0. \quad (6.6)$$

We can solve for  $n_i^c$ ,

$$n_i^c = \frac{D_i}{x \sqrt{\frac{\pi_e^c}{\frac{\hat{q}}{2}(\bar{\theta} - \underline{\theta})}}}. \quad (6.7)$$

In the symmetric equilibrium, we should have  $n_i^c = N/B$ . Using (6.7) we can solve for  $\pi_e^c$ ,

$$\pi_e^c = \frac{\hat{q}}{2} (\bar{\theta} - \underline{\theta}) \left( \frac{BD_i}{Nx} \right)^2. \quad (6.8)$$

Using (6.8) and (6.4), we can solve for  $\gamma_i$  given the amount of funding  $D_i$ ,

$$\gamma_i = 1 - \frac{(\bar{\theta} - \underline{\theta}) \left( \frac{BD_i}{Nx} \right)}{\left[ 2\bar{\theta} - (\bar{\theta} - \underline{\theta}) \left( \frac{BD_i}{Nx} \right) \right]}. \quad (6.9)$$

We have solved the problem of banks as credit suppliers and the entrepreneurs' problem. Given the total projects being invested, the quantity of the intermediate goods is  $m = \hat{\theta}_i N p_i$ . The corresponding wage rate and the price of the intermediate good are

$$w = (1 - \alpha) \hat{z} \left( \hat{\theta}_i N p_i \right)^\alpha \quad (6.10)$$

and

$$\hat{q} = \alpha \hat{z} \left( \hat{\theta}_i N p_i \right)^{\alpha-1}. \quad (6.11)$$

Next, we are going to solve the problem of banks as credit demanders and the problem of households.

The representative young household takes the value of  $w$ ,  $\hat{q}$ , and  $r^c$  as given. If  $r^c < 1$ , then the representative young household does not deposit. Instead, he stores his canned

good to save. His choice is

$$-u'(w - s) + \beta E u'(s) = 0 \text{ and } S_i = 0 \text{ for all } i = 1, 2, \dots, B. \quad (6.12)$$

If  $r^c \geq 1$ , then the representative young household will deposit the canned goods at banks.

The solution comes from the following first order conditions:

$$-u' \left( w - \sum_{i=1}^B S_i - s \right) + \beta r^c E u' \left( r^c \sum_{i=1}^B S_i + s + \tilde{\pi}_e^c \right) \geq 0, \text{ if } 0 \leq S_i \leq S_i^h, \quad (6.13)$$

where  $S_i^h$  is the limit of deposit contract that bank  $i$  could provide since the bank  $i$  is restricted by the bank capital  $K_i$  through  $\frac{K_i}{K_i + S_i^h} = k$ . In the symmetric equilibrium, if  $r^c > 1$ , then  $s = 0$ ,  $S_i = S_i^h$  and

$$u'(w - BS_i) = \beta r^c E u'(r^c BS_i + \tilde{\pi}_e^c); \quad (6.14)$$

if  $r^c = 1$ , then  $s \geq 0$ ,  $S_i \geq 0$  and

$$u'(w - BS_i - s) = \beta E u'(BS_i + s + \tilde{\pi}_e^c). \quad (6.15)$$

The equations (6.14) and (6.15) give the deposit supply function  $S_i(r^c)$ .

In the equilibrium, we should have the total funding supply equals the total funding demand, that is

$$xNp_i = \frac{BS_i(r^c)}{1 - k}. \quad (6.16)$$

When  $r^c = 1$ , we assume that the household deposits as much as the banks would need, which will be  $S_i = xNp_i(1 - k)/B$ , and stores the rest.

In a steady state, we can solve equations (6.2), (6.8), (6.9), (6.10), (6.11), (6.14) and (6.16) to get the equilibrium  $\hat{q}$ ,  $w$ ,  $r^c$ ,  $S_i$ ,  $\gamma_i$ ,  $\tilde{\pi}_e^c$ , and  $\tilde{\theta}_i$ .

## 6.2 Equilibrium with securitization

With securitization, an entrepreneur's expected profit is still (6.3), since he does not get any profit in the securitization stage. Since a unique symmetric equilibrium exists only in the case where the "skin in the game" constraint is binding according to lemma 2, the following relationship between  $\tilde{\theta}_i^a$  and  $\tilde{\theta}_i$  should hold,

$$\tilde{\theta}_i^a = (2 - \lambda) \tilde{\theta}_i - (1 - \lambda) \bar{\theta}. \quad (6.17)$$

So,  $\frac{d\tilde{\theta}_i^a}{d\tilde{\theta}_i} = (2 - \lambda)$ , which means that a 1% decrease in  $\tilde{\theta}_i$  relaxes  $\tilde{\theta}_i^a$  by  $(2 - \lambda)$  %. Recall that equilibrium symmetry and lemma 1 imply that  $N/B = \frac{D_i}{x[1-G(\theta_i)]}$ , so

$$\tilde{\theta}_i = \bar{\theta} - (\bar{\theta} - \underline{\theta}) \frac{BD_i}{xN}. \quad (6.18)$$

According to (6.17) and (6.18)

$$\tilde{\theta}_i^a = \bar{\theta} - (2 - \lambda) (\bar{\theta} - \underline{\theta}) \frac{BD_i}{xN}. \quad (6.19)$$

The first order derivative of the bank's profit has the same sign as  $\phi(\gamma_i)$ . Using (6.17) and (6.18), and  $\phi(\gamma_i)$  defined in (A.6), we have

$$\phi(\gamma_i) = \frac{(1 - \gamma_i)}{2} \left[ 2\bar{\theta} \frac{BD_i}{xN} - (\bar{\theta} - \underline{\theta}) \left( \frac{BD_i}{Nx} \right)^2 \right] - \left[ (2 - \lambda)^2 - \frac{xr^a}{\hat{q}} (1 - \lambda) \right] \frac{(\bar{\theta} - \underline{\theta})}{2} \left( \frac{BD_i}{Nx} \right)^2.$$

So the first order condition is equivalent to  $\phi(\gamma_i^a) = 0$ , which gives

$$\gamma_i^a = 1 - \frac{\left[ (2 - \lambda)^2 - \frac{xr^a}{\hat{q}} (1 - \lambda) \right] (\bar{\theta} - \underline{\theta}) \left( \frac{BD_i}{Nx} \right)}{\left[ 2\bar{\theta} - (\bar{\theta} - \underline{\theta}) \left( \frac{BD_i}{Nx} \right) \right]}. \quad (6.20)$$

Using a uniform distribution of  $\theta$ , we can analytically show some properties of the equilibrium with securitization. One important result is summarized in Lemma 3.

**Lemma 3** *If  $\lambda < 1$ , there is securitization in the equilibrium. The lending rate is lower compared to an economy without securitization, i.e.  $\gamma_i^a < \gamma_i$ , given the same size of deposit*

$S_i$ , with a uniform distribution of  $\theta$ .

The proof of Lemma 3 is apparent by comparing (6.20) to (6.9). Define

$$\varphi = \left[ (2 - \lambda)^2 - \frac{xr^a}{\hat{q}} (1 - \lambda) \right],$$

all we need to show is that  $\varphi > 1$  for all  $\lambda < 1$ . Recall that  $r^a = \frac{\hat{q}}{x} \frac{\hat{\theta}_i^a}{\theta_i}$ , so  $\frac{xr^a}{\hat{q}} = \frac{\hat{\theta}_i^a}{\theta_i} < 1$ . As a result,  $\varphi > (2 - \lambda)^2 - (1 - \lambda) > 1$ .

A lower lending rate in the equilibrium with securitization comes from the fact that the banks have more incentive to compete for potential projects if they have access to securitization. Moreover, the smaller the value of  $\lambda$  (the looser the "skin in the game" constraint), the smaller the value of  $\gamma_i^a$ , indicating a more intensive competition among banks.

Since the "skin in the game" constraint is binding, it is possible that  $\hat{\theta}_i^a < \tilde{\theta}_i^a$ , i.e. not all the projects can get funding even if they could make profit. So we have  $\vec{\theta}_i^a = \tilde{\theta}_i^a \geq \hat{\theta}_i^a$ . It is difficult to determine whether  $\hat{\theta}_i > \tilde{\theta}_i$  or  $\hat{\theta}_i \leq \tilde{\theta}_i$ . It is possible that  $\hat{\theta}_i > \tilde{\theta}_i$ , i.e. some projects invested in the third stage make negative profit. The banks may have incentive to invest in these projects with negative profit, because they can relax the "skin in the game" constraint such that they can invest in more projects by selling more loans. The negative profit should be compensated by the profit from increased investment in the fourth stage. Moreover, if a project is invested in the third stage, the bank gets a share of  $\gamma_i^a$ ; if the project is invested in the fourth stage, the bank gets a share of 1. The loss from this change should also be compensated by the additional profits from additional investment due a relax of the "skin in the game" constraint.

The total amount of intermediate goods is  $m = \hat{\theta}_i^a N p_i^a$ , where  $\hat{\theta}_i^a = \frac{\int_{\hat{\theta}_i^a}^{\tilde{\theta}_i^a} \theta dG(\theta)}{1 - G(\hat{\theta}_i^a)}$  and  $p_i^a = 1 - G(\hat{\theta}_i^a)$ . The wage rate and the price of the intermediate good are

$$w = (1 - \alpha) \hat{z} \left( \hat{\theta}_i^a N p_i^a \right)^\alpha \quad (6.21)$$

and

$$\hat{q} = \alpha \hat{z} \left( \hat{\theta}_i^a N p_i^a \right)^{\alpha-1}. \quad (6.22)$$

We focus on the equilibrium with  $r^c \geq 1$  ( $r^a \geq r^c$ ). If  $r^c < 1$  the household is going to store all the canned goods and no intermediate goods will be produced. When  $r^c = 1$ , we assume that the representative household deposits as much as the banks would need and stores the rest, so  $s \geq 0$ . If  $r^c > 1$ , then  $s = 0$ . The saving  $S_i$  satisfies

$$\begin{aligned} & u'(w - BS_i - BS_i^a - s) \\ &= \beta r^c E \left\{ \theta'_i u'(r^a BS_i^a + r^c BS_i + s + \tilde{\pi}_e^c) + (1 - \theta'_i) u'(r^c BS_i + s + \tilde{\pi}_e^c) \right\}, \end{aligned} \quad (6.23)$$

and the supply of securities  $S_i^a$  satisfies

$$u'(w - BS_i - BS_i^a - s) = \beta r^a E \left[ \theta'_i u'(r^a BS_i^a + r^c BS_i + s + \tilde{\pi}_e^c) \right]. \quad (6.24)$$

Finally, we have the market clearing conditions. The total funding supply equals the total funding demand, that is

$$\frac{BS_i(r^c)}{1 - k} = (1 - G(\tilde{\theta}_i)) Nx, \quad (6.25)$$

and

$$BS_i^a(r^a) = (G(\tilde{\theta}_i) - G(\tilde{\theta}_i^a)) Nx. \quad (6.26)$$

The canned goods market clears,

$$w + r^c S + r^a S^a + \tilde{\pi}_e^c + B\pi_b = y. \quad (6.27)$$

In a stationary symmetric equilibrium, we can solve equations (6.19) - (6.27) to get the equilibrium  $\hat{q}$ ,  $w$ ,  $r^c$ ,  $r^a$ ,  $S_i$ ,  $S_i^a$ ,  $\gamma_i$ ,  $\tilde{\theta}_i$  and  $\tilde{\theta}_i^a$ .

### 6.3 Numerical example

In order to see the relationship between bank competition and securitization we do a numerical exercise. For simplicity, we demonstrate a static model here. We use an isoelastic utility function  $\frac{c^{1-\sigma}}{1-\sigma}$  and some plausible values of parameters:  $\alpha = 0.4$ ,  $x = 0.01$ ,  $\hat{z} = 1$ ,  $\bar{\theta} = 1$ ,

$\underline{\theta} = 0$ ,  $B = 2$ ,  $k = 0.08$ ,  $\lambda = 0.8$ ,  $\beta = 0.9$  and  $\rho = 1.25$ . The value of risk aversion parameter has to be less than 1 in order to have the deposit supply function increases in interest rate, we let  $\sigma = 0.3$ . The value of  $N$ , the number of entrepreneurs, is a parameter that measures the tightness of the loan market, which will vary when we do comparative statics.

### 6.3.1 Bank competitiveness

First, we need to find an index to measure the competitiveness in the banking sector. The setup of the model makes the equilibrium result independent of the number of banks,  $B$ , as long as  $B \geq 2$  so that the banks will compete with each other. But the equilibrium does vary with respect to the number of potential projects. Although the thickness of the loan market,  $N/B$ , is not a good measure of bank competition given that the equilibrium is not sensitive to  $B$  for  $B > 2$ , it is very important in the equilibrium. So we start with varying  $N$  to find a measure of bank competition. We will see that, in the example, when  $N$  increases,  $\gamma$  increases. This is intuitive. As the potential projects increase, the banks become less aggressive to steal from others, yet get good projects.

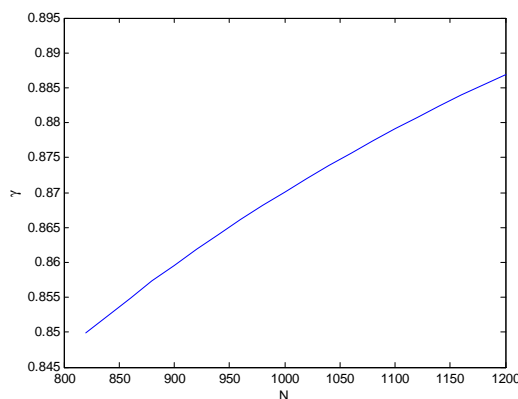


Figure 1.  $\gamma$  and  $N$

When  $N$  increases, there are two forces to increase the total investment. First, the banks face less competition from each other and therefore their share of profit  $\gamma$  increases. Second, the banks invested projects will have a higher average quality. To separate the second effect of  $N$  from its role in decreasing bank's competition, we define the deviation of total investment from a social planner's world (or a world with only one bank) as a measure



of the imperfection of the banking industry. Specifically, we define a measure of bank's competitiveness,  $\mu$ , by

$$\mu = \frac{\text{investment (third stage) in bank competition equilibrium}}{\text{investment in one bank equilibrium}}.$$

We can see from figure 2 that  $\mu$  is also increasing in  $N$ .

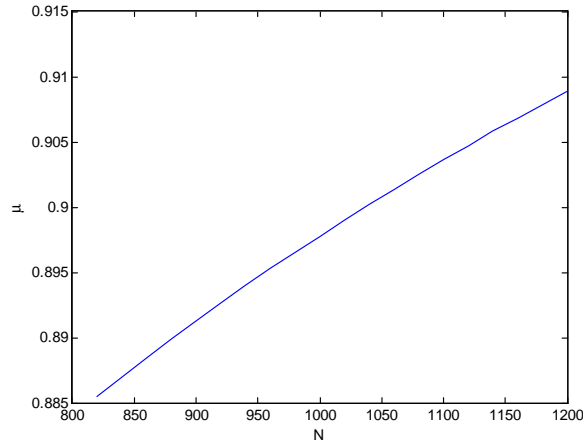


Figure 2.  $\mu$  and  $N$

### 6.3.2 Optimal volume of securitization

We have assumed that  $\lambda$  is a policy parameter. Actually, if there is no constraint from the central bank, the banks may choose an optimal level of securitization. We here endogenize  $\lambda$  and define the relative size of securitization by

$$\kappa = \frac{\text{total volume of securities}}{\text{total investment (third stage)}}.$$

The following figure shows how the share of securitization responds to market competitiveness.

Figure 3.  $\kappa$  and  $\mu$

## 7 Conclusion

We have built a dynamic general equilibrium model with bank competition. The framework is a directed search model. Capital requirement imposes a short-run capacity constraint on banks' lending. Given the capacity constraint, the banks compete for projects using lending rate. The model is a Bertrand competition with capacity-constraint as in Peters (1984) and BSW (2001). We focus on a stationary symmetric mixed strategy equilibrium. We find that bank's competition could be one reason to cause a low equilibrium lending rate and excess demand for funding. As a consequence, banks may seek for fund through selling their loans. We have show that securitization could be motivated with a purpose other than avoiding capital requirement in a model absent of asymmetric information.

# A Appendix

## A.1 Proof of lemma 1.

**Proof.** In a symmetric equilibrium, knowing that the total loan is  $\Phi(\gamma_i)$  given by (3.7), a bank will not raise more fund than needed for his planed investment since additional funding is costly. As a result,

$$D_i = S_i + K_i \leq n_i(\gamma_i) \left[ 1 - G\left(\hat{\theta}(\gamma_i)\right) \right] x. \quad (\text{A.1})$$

Moreover, since bank equity is expensive, the bank will let  $K_i$  and  $S_i$  satisfy  $\frac{K_i}{K_i+S_i} = k$ . Notice that it will not be optimal if  $D_i < n_i(\gamma_i) \left[ 1 - G\left(\hat{\theta}(\gamma_i)\right) \right] x$ , because some profitable projects would not be financed. The foresighted banks would increase bank equity  $K_i$  and deposit  $S_i$  in the first step until  $S_i + K_i = n_i(\gamma_i) \left[ 1 - G\left(\hat{\theta}(\gamma_i)\right) \right] x$ . ■

## A.2 Proof of Proposition 1.

**Proof.** Suppose there is a stationary symmetric equilibrium in which all the banks post  $\{\gamma^c, \bar{\theta}^c\}$  for every period, where  $\gamma^c \in (0, 1)$ . We need to show that no banks are willing to post a  $\gamma_i^d \neq \gamma^c$ . So we focus on a strong Nash equilibrium.

In the third stage after  $K_i$  and  $S_i$  are determined, if a bank  $i$  posts a contract  $(\gamma_i^d, \bar{\theta}_i^d)$  and  $\gamma_i^d < \gamma^c$ , then the corresponding selection rule of the threshold value of project quality  $\bar{\theta}_i^d$  should satisfy  $\bar{\theta}_i^d > \bar{\theta}_i^c$ . This is because a larger number of projects,  $n_i^d$ , will be attracted by the new contract,  $n_i^d > n_i^c$ , such that the bank can select better projects in a larger pool. So the average quality of projects,  $\hat{\theta}_i^d = \frac{\int_{\bar{\theta}_i^d}^{\theta} \theta dG(\theta)}{1-G(\bar{\theta}_i^d)}$  is also higher. The bank's profit is

$$\pi_{b,i}^d = n_i^d p_i^d \hat{q} \hat{\theta}_i^d \gamma_i^d - \eta D_i.$$

The probability of being financed  $p_i^d = 1 - G(\bar{\theta}_i^d)$  is lower. Since the bank faces a better pool of projects, he uses up all of his funding to finance the projects, we have  $n_i^d x \left[ 1 - G(\bar{\theta}_i^d) \right] = D_i$ .

When a bank varies his contract, he faces a trade-off between the share of the expected output from a project and the number of potential projects attracted,  $n_i^d$ . It is crucial to

figure out how  $n_i^d$  moves in response to  $\gamma_i^d$ . Observing  $\gamma_i^d$ , an entrepreneur will visit island  $i$  if the expected profit from borrowing at island  $i$  is higher than or equal to what he could get from other islands. If we consider an economy with a large number of banks and entrepreneurs, the last visitor (marginal visitor) will have the same profit as if he visits any other islands, that is

$$p_i^d \hat{q}(1 - \gamma_i^d) \hat{\theta}_i^d = \hat{q}(1 - \gamma_i^d) \int_{\bar{\theta}_i^d}^{\bar{\theta}} \theta dG(\theta) = \pi_e^c. \quad (\text{A.2})$$

Equation (A.2) gives an indifference curve over the choices of  $(\gamma_i^d, \bar{\theta}_i^d)$  for an entrepreneur. The expected profit of an entrepreneur in the initial symmetric equilibrium with  $\gamma^c$  is

$$\pi_e^c = \left[1 - G(\bar{\theta}_i^c)\right] \hat{q}(1 - \gamma^c) \hat{\theta}_i^c = \hat{q}(1 - \gamma^c) \int_{\bar{\theta}_i^c}^{\bar{\theta}} \theta dG(\theta).$$

Differentiating equation (A.2) completely, we can get

$$\frac{d\bar{\theta}_i^d}{d\gamma_i^d} = - \frac{\int_{\bar{\theta}_i^d}^{\bar{\theta}} \theta dG(\theta)}{(1 - \gamma_i^d) \bar{\theta}_i^d g(\bar{\theta}_i^d)}. \quad (\text{A.3})$$

Given the indifference curve of the entrepreneurs, the expected profit of the deviating bank is

$$\pi_{b,i}^d(\gamma_i^d) = \frac{D_i}{x \left[1 - G(\bar{\theta}_i^d)\right]} \frac{\gamma_i^d \pi_e^c}{(1 - \gamma_i^d)} - \eta D_i.$$

Here we have used  $n_i^d = \frac{D_i}{x[1-G(\bar{\theta}_i^d)]}$ . Define  $\psi(\gamma_i^d) \equiv \frac{\partial \pi_{b,i}^d(\gamma_i^d)}{\partial \gamma_i^d}$ , we have

$$\psi(\gamma_i^d) = \frac{\left[1 - G(\bar{\theta}_i^d)\right] - \int_{\bar{\theta}_i^d}^{\bar{\theta}} \theta dG(\theta) \frac{\gamma_i^d}{\bar{\theta}_i^d}}{\left[1 - G(\bar{\theta}_i^d)\right]^2} \frac{1}{(1 - \gamma_i^d)^2} \frac{\pi_e^c D_i}{x}.$$

Here we have used (A.3) from the entrepreneurs' indifference curve.

If  $\psi(\gamma_i^d) < 0$ , in the local area of the initial assumed equilibrium, there is a lower lending

rate  $\gamma_i^d$  that can increase the profit of a bank. The condition for  $\psi(\gamma_i^d) < 0$  is equivalent to

$$1 < \frac{\int_{\bar{\theta}_i^d}^{\bar{\theta}} \theta dG(\theta)}{\bar{\theta}_i^d [1 - G(\bar{\theta}_i^d)]} \gamma_i^d. \quad (\text{A.4})$$

Since it is always true that  $\frac{\int_{\bar{\theta}_i^d}^{\bar{\theta}} \theta dG(\theta)}{\bar{\theta}_i^d [1 - G(\bar{\theta}_i^d)]} > 1$ , we should have  $\lim_{\gamma_i^d \rightarrow 1} \frac{\int_{\bar{\theta}_i^d}^{\bar{\theta}} \theta dG(\theta)}{\bar{\theta}_i^d [1 - G(\bar{\theta}_i^d)]} \gamma_i^d > 1$ . So (A.4) is true for  $\gamma_i^d \rightarrow 1$ . As a result,  $\gamma_i^c = 1$  (or  $\gamma_i^c$  is large enough) cannot be an equilibrium. As  $\gamma_i^d \rightarrow 0$ , however, the opposite of (A.4) should be true. Moreover, the RHS of (A.4) is increasing in  $\gamma_i^d$  given the assumption that  $\frac{\int_{\bar{\theta}_i^d}^{\bar{\theta}} \theta dG(\theta)}{\bar{\theta}_i^d [1 - G(\bar{\theta}_i^d)]}$  is weakly decreasing in  $\bar{\theta}_i^d$ . So there exists a unique  $\gamma_i^{d*}$  such that  $\psi(\gamma_i^{d*}) = 0$ . This  $\gamma_i^{d*}$  is the unique stable symmetric equilibrium, i.e.,  $\gamma_i^{d*} = \gamma^c$ . ■

### A.3 Proof of Proposition 2.

**Proof.** Given the profit function of a bank  $i$  from (5.7),

$$\pi_{b,i} = \max_{\gamma_i} \frac{D_i}{x [1 - G(\tilde{\theta}_i)]} \frac{\gamma_i \pi_e^c}{(1 - \gamma_i)} - \eta D_i + \frac{D_i}{x [1 - G(\tilde{\theta}_i)]} \hat{q} \int_{\tilde{\theta}_i^a}^{\tilde{\theta}_i} \theta dG(\theta) - \hat{\theta}_i r^a S_i^a(r^a, \hat{\theta}).$$

As in the case without securitization, we have taken  $\pi_e^c$  as given, this is because when the bank varies  $\gamma_i$ , he must change  $\tilde{\theta}_i$  such that the expected profit of an entrepreneur is unaffected. The first order derivative of  $\pi_{b,i}$  with respect to  $\gamma_i$  is

$$\begin{aligned} \psi(\gamma_i) = & \left[ 1 - \gamma_i \frac{\int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta)}{\tilde{\theta}_i [1 - G(\tilde{\theta}_i)]} \right] \frac{D_i \pi_e^c}{x(1 - \gamma_i)^2} \frac{1}{[1 - G(\tilde{\theta}_i)]} \\ & - \left[ 1 - (2 - \lambda) \frac{\tilde{\theta}_i^a}{\tilde{\theta}_i} + \frac{\int_{\tilde{\theta}_i^a}^{\tilde{\theta}_i} \theta dG(\theta)}{\tilde{\theta}_i [1 - G(\tilde{\theta}_i)]} \right] \frac{D_i \pi_e^c}{x(1 - \gamma_i)^2} \frac{1}{[1 - G(\tilde{\theta}_i)]} \\ & + \left[ -1 + \frac{\int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta)}{\tilde{\theta}_i [1 - G(\tilde{\theta}_i)]} \right] \frac{r^a S_i^a \pi_e^c}{\hat{q}(1 - \gamma_i)^2 [1 - G(\tilde{\theta}_i)]}. \end{aligned} \quad (\text{A.5})$$

The first line in (A.5) is the same as in the case without securitization, which is the change in the bank's profit in the third stage; the second line in (A.5) is the decline in revenue in

the securitization stage; the third line in (A.5) is the decreased cost from the reduction in the security paying rate.

Using  $\frac{d\tilde{\theta}_i^a}{d\theta_i}$  from (5.6), the entrepreneur's trade-off between  $\gamma_i$  and  $\tilde{\theta}_i$  given by (A.3), and  $\pi_e^c$  from (A.2), we have

$$\begin{aligned} \psi(\gamma_i) = & \left[ (2 - \lambda) \tilde{\theta}_i^a - \gamma_i \frac{\int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta)}{[1 - G(\tilde{\theta}_i)]} - \frac{\int_{\tilde{\theta}_i^a}^{\bar{\theta}} \theta dG(\theta)}{[1 - G(\tilde{\theta}_i^a)]} \right] \frac{D_i \pi_e^c}{x(1 - \gamma_i)^2} \frac{1}{\tilde{\theta}_i [1 - G(\tilde{\theta}_i)]} \\ & - \left[ \tilde{\theta}_i - \frac{\int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta)}{[1 - G(\tilde{\theta}_i)]} \right] \frac{x r^a (1 - \lambda)}{\hat{q}} \frac{D_i \pi_e^c}{x(1 - \gamma_i)^2} \frac{1}{\tilde{\theta}_i [1 - G(\tilde{\theta}_i)]}. \end{aligned}$$

From the "skin in the game" constraint, the value of  $\tilde{\theta}_i$  and  $\tilde{\theta}_i^a$  satisfy

$$1 - G(\tilde{\theta}_i^a) = (2 - \lambda) [1 - G(\tilde{\theta}_i)],$$

so we have

$$\psi(\gamma_i) = \left[ \begin{aligned} & \tilde{\theta}_i^a [1 - G(\tilde{\theta}_i^a)] - \gamma_i \frac{\int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta)}{[1 - G(\tilde{\theta}_i)]} - \frac{\int_{\tilde{\theta}_i^a}^{\bar{\theta}} \theta dG(\theta)}{[1 - G(\tilde{\theta}_i^a)]} \\ & - \left[ \tilde{\theta}_i [1 - G(\tilde{\theta}_i)] - \frac{\int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta)}{[1 - G(\tilde{\theta}_i)]} \right] \frac{x r^a (1 - \lambda)}{\hat{q}} \end{aligned} \right] \frac{D_i \pi_e^c}{x(1 - \gamma_i)^2} \frac{1}{\tilde{\theta}_i [1 - G(\tilde{\theta}_i)]^2},$$

where  $r^a = \frac{\hat{q} \tilde{\theta}_i^a [1 - G(\tilde{\theta}_i)]}{x \int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta)}$ . We denote

$$\begin{aligned} \phi(\gamma_i) = & \left[ \tilde{\theta}_i^a [1 - G(\tilde{\theta}_i^a)] - \gamma_i \int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta) - \int_{\tilde{\theta}_i^a}^{\bar{\theta}} \theta dG(\theta) \right] \\ & - \left[ \tilde{\theta}_i [1 - G(\tilde{\theta}_i)] - \int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta) \right] \frac{x r^a (1 - \lambda)}{\hat{q}}. \end{aligned} \quad (\text{A.6})$$

The sign of  $\phi(\gamma_i)$  determines the sign of  $\psi(\gamma_i)$ . We first look at the sign of  $\phi(\gamma_i)$  when  $\gamma_i \rightarrow 1$ ,

$$\lim_{\gamma_i \rightarrow 1} \phi(\gamma_i) = - \int_{\tilde{\theta}_i^a}^{\bar{\theta}} (\theta - \tilde{\theta}_i^a) dG(\theta) + \left[ \int_{\tilde{\theta}_i}^{\bar{\theta}} (\theta - \tilde{\theta}_i) dG(\theta) \right] (1 - \lambda) \frac{\tilde{\theta}_i^a [1 - G(\tilde{\theta}_i)]}{\int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta)}.$$

Since  $\frac{\hat{\theta}_i^a [1-G(\tilde{\theta}_i)]}{\int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta)} = \frac{\hat{\theta}_i^a}{\tilde{\theta}_i} \leq 1$ ,  $(1-\lambda) \leq 1$ , and  $\tilde{\theta}_i^a \leq \tilde{\theta}_i$ , then

$$\lim_{\gamma_i \rightarrow 1} \phi(\gamma_i) < - \int_{\tilde{\theta}_i^a}^{\bar{\theta}} (\theta - \tilde{\theta}_i^a) dG(\theta) + \int_{\tilde{\theta}_i}^{\bar{\theta}} (\theta - \tilde{\theta}_i) dG(\theta) < 0.$$

Second, we look at the value of  $\phi(\gamma_i)$  as  $\gamma_i \rightarrow 0$ . Since the sign of  $\lim_{\gamma_i \rightarrow 0} \phi(\gamma_i)$  is the same as the sign of  $\lim_{\gamma_i \rightarrow 0} \frac{\phi(\gamma_i)}{\tilde{\theta}_i^a [G(\tilde{\theta}_i) - G(\tilde{\theta}_i^a)]}$ , we can look at the sign of the latter,

$$\begin{aligned} & \lim_{\gamma_i \rightarrow 0} \frac{\phi(\gamma_i)}{\tilde{\theta}_i^a [G(\tilde{\theta}_i) - G(\tilde{\theta}_i^a)]} \\ = & \lim_{\gamma_i \rightarrow 0} \left\{ \left[ \frac{2-\lambda}{1-\lambda} - \frac{\int_{\tilde{\theta}_i^a}^{\tilde{\theta}_i} \theta dG(\theta)}{\tilde{\theta}_i^a [G(\tilde{\theta}_i) - G(\tilde{\theta}_i^a)]} \right] + \left[ 1 - \frac{\tilde{\theta}_i [1-G(\tilde{\theta}_i)]}{\int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta)} \right] \frac{\hat{\theta}_i^a}{\tilde{\theta}_i^a} \right\} \geq 0. \quad (\text{A.7}) \end{aligned}$$

The inequality in (A.7) holds since  $\lim_{\gamma_i \rightarrow 0} \frac{\int_{\tilde{\theta}_i^a}^{\tilde{\theta}_i} \theta dG(\theta)}{\tilde{\theta}_i^a [G(\tilde{\theta}_i) - G(\tilde{\theta}_i^a)]} = 1$  and  $\left[ 1 - \frac{\tilde{\theta}_i [1-G(\tilde{\theta}_i)]}{\int_{\tilde{\theta}_i}^{\bar{\theta}} \theta dG(\theta)} \right] \geq 0$ . Since  $\phi(\gamma_i)$  is continuous, there exists a  $\gamma_i$  such that  $\phi(\gamma_i) = 0$ . We have proved the existence of a symmetric equilibrium.

Should the equilibrium be unique? A sufficient condition for uniqueness is that the function  $\phi(\gamma_i)$  is monotonically decreasing in  $\gamma_i$ . We have assumed that  $\frac{g(\tilde{\theta}_i^a)}{g(\tilde{\theta}_i)} \leq \frac{2-\lambda}{1-\lambda}$  for any  $\tilde{\theta}_i^a$  and  $\tilde{\theta}_i$  that satisfy  $\tilde{\theta}_i^a \leq \tilde{\theta}_i$  and  $[G(\tilde{\theta}_i) - G(\tilde{\theta}_i^a)] = (1-\lambda) [1 - G(\tilde{\theta}_i)]$ . Given these assumptions, the sign of  $\phi'(\gamma_i)$  is negative as shown below:

$$\phi'(\gamma_i) = \left[ - \frac{(2-\lambda) g(\tilde{\theta}_i) \int_{\tilde{\theta}_i^a}^{\tilde{\theta}_i} \theta dG(\theta)}{g(\tilde{\theta}_i^a) \tilde{\theta}_i [1 - G(\tilde{\theta}_i^a)]} + \frac{(1-\lambda) \hat{\theta}_i^a}{\tilde{\theta}_i} \right] \frac{[1 - G(\tilde{\theta}_i)]^2}{g(\tilde{\theta}_i)(1-\gamma_i)} < 0,$$

since  $\frac{\int_{\tilde{\theta}_i^a}^{\tilde{\theta}_i} \theta dG(\theta)}{\tilde{\theta}_i [1 - G(\tilde{\theta}_i)]} > 1$  and  $\hat{\theta}_i^a \leq \tilde{\theta}_i^a \leq \tilde{\theta}_i$ . As a result, the solution of  $\phi(\gamma_i) = 0$  is unique. We have a unique symmetric equilibrium with the condition  $\frac{g(\tilde{\theta}_i^a)}{g(\tilde{\theta}_i)} \leq \frac{2-\lambda}{1-\lambda}$  for any  $\tilde{\theta}_i^a \leq \tilde{\theta}_i$  and  $1 - G(\tilde{\theta}_i^a) = (2-\lambda) [1 - G(\tilde{\theta}_i)]$ . ■

## A.4 Proof of Lemma 2.

**Proof.** If the constraint (5.3) is not binding, the banks only invest in profitable projects in the third stage, since they do not need to worry about a shortage of assets to back their securities. In this case, the marginal project breaks even, i.e.,  $x\eta = \hat{q}\gamma_i\hat{\theta}_i$ . In the fourth stage, the non-binding constraint (5.3) implies that all the profitable projects will be financed, that is,  $xr^a\hat{\theta}_i = \hat{q}\hat{\theta}_i^a$ , where  $\hat{\theta}_i = \frac{\int_{\hat{\theta}_i^a}^{\bar{\theta}} \theta dG(\theta)}{[1-G(\hat{\theta}_i)]}$ . We can therefore derive  $\frac{d\hat{\theta}_i^a}{d\hat{\theta}_i}$ ,

$$\frac{d\hat{\theta}_i^a}{d\hat{\theta}_i} = \left[ -1 + \frac{\int_{\hat{\theta}_i}^{\bar{\theta}} \theta dG(\theta)}{\hat{\theta}_i [1-G(\hat{\theta}_i)]} \right] \frac{xr^a}{\hat{q}} \frac{\hat{\theta}_i g(\hat{\theta}_i)}{[1-G(\hat{\theta}_i)]}.$$

The bank's maximization problem becomes:

$$\pi_{b,i} = \max_{\gamma_i} n_i \frac{\gamma_i \pi_e^c}{(1-\gamma_i)} - \eta D_i + n_i \hat{q} \int_{\hat{\theta}_i^a}^{\bar{\theta}} \theta dG(\theta) - r^a \hat{\theta}_i S_i^a(r^a, \hat{\theta}), \quad (\text{A.8})$$

where  $n_i = \frac{D_i}{x[1-G(\hat{\theta}_i)]}$ . The first order derivative of  $\pi_{b,i}$  with respect to  $\gamma_i$  is

$$\psi(\gamma_i) = \left[ \begin{array}{c} -\gamma_i \frac{\int_{\hat{\theta}_i}^{\bar{\theta}} \theta dG(\theta)}{\hat{\theta}_i [1-G(\hat{\theta}_i)]} - \frac{\int_{\hat{\theta}_i^a}^{\bar{\theta}} \theta dG(\theta)}{\hat{\theta}_i [1-G(\hat{\theta}_i)]} \\ + \frac{\hat{\theta}_i^a}{\int_{\hat{\theta}_i}^{\bar{\theta}} \theta dG(\theta)} \frac{\int_{\hat{\theta}_i}^{\bar{\theta}} (\theta - \hat{\theta}_i) dG(\theta)}{\hat{\theta}_i [1-G(\hat{\theta}_i)]} \left[ \hat{\theta}_i^a g(\hat{\theta}_i^a) + \frac{S_i^a}{D_i} [1-G(\hat{\theta}_i)] \right] \end{array} \right] \frac{\hat{q} D_i \int_{\hat{\theta}_i}^{\bar{\theta}} \theta dG(\theta)}{x(1-\gamma_i) [1-G(\hat{\theta}_i)]}.$$

Since the distribution of  $\theta$  is continuous,  $g(\hat{\theta}_i^a) = 0$ . Since the "skin in the game" constraint is not binding,  $S_i^a < (1-\lambda) D_i$ . So, we define  $\phi(\gamma_i)$  by the terms in the brackets in  $\psi(\gamma_i)$ , then

$$\begin{aligned} \phi(\gamma_i) &< -\gamma_i \frac{\int_{\hat{\theta}_i}^{\bar{\theta}} \theta dG(\theta)}{\hat{\theta}_i [1-G(\hat{\theta}_i)]} - \frac{\int_{\hat{\theta}_i^a}^{\bar{\theta}} \theta dG(\theta)}{\hat{\theta}_i [1-G(\hat{\theta}_i)]} + \frac{\hat{\theta}_i^a}{\int_{\hat{\theta}_i}^{\bar{\theta}} \theta dG(\theta)} \frac{\int_{\hat{\theta}_i}^{\bar{\theta}} (\theta - \hat{\theta}_i) dG(\theta)}{\hat{\theta}_i} (1-\lambda) \\ &< -\gamma_i \frac{\int_{\hat{\theta}_i}^{\bar{\theta}} \theta dG(\theta)}{\hat{\theta}_i [1-G(\hat{\theta}_i)]} - \frac{\hat{\theta}_i^a (G(\hat{\theta}_i) - G(\hat{\theta}_i^a))}{\hat{\theta}_i [1-G(\hat{\theta}_i)]} + \frac{\hat{\theta}_i^a}{\hat{\theta}_i} \frac{\int_{\hat{\theta}_i}^{\bar{\theta}} (\theta - \hat{\theta}_i) dG(\theta)}{\int_{\hat{\theta}_i}^{\bar{\theta}} \theta dG(\theta)} (1-\lambda) \\ &< -\gamma_i \frac{\int_{\hat{\theta}_i}^{\bar{\theta}} \theta dG(\theta)}{\hat{\theta}_i [1-G(\hat{\theta}_i)]}. \end{aligned}$$



As a result,  $\psi(\gamma_i) < 0$  for all  $\gamma_i > 0$ , so the bank is going to lower  $\gamma_i$  until the constraint (5.3) becomes binding. Then, we go back to the case with the constraint (5.3) binding. There is no equilibrium when the "skin in the game" constraint is not binding. ■

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