Rewarding Idleness

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Abstract

Market wages reflect expected productivity, making use of signals of past performance and past experience. These signals are generated at least partially on the job, creating incentives for agents to choose high profile and highly visible tasks. When agents have private information about the profitability of different tasks, firms may wish to prevent over-investment in those that entail signaling, by increasing opportunity cost for these activities, for instance using employee perks. Introducing heterogeneity in employee types induces substantial diversity in organizational and contractual choices, in particular regarding the extent to which signaling activities are tolerated or encouraged, the use of employee perks, and contingent wages. Organizational choices in turn affect the shape of the payoff function, and thus incentives to signal in earlier periods.

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1 Introduction

Some companies provide their employees with sizable non-monetary benefits, especially when the tasks involved require creativity and originality. Software developers are a case in point: for example, employees at Google have at their disposal a wide variety of on site services and sports facilities, such as tennis courts and a swimming pool, free catering in a high profile restaurant and cafeterias, various entertainment facilities, such as table football, and are allowed to use one workday per week for personal projects.[1] Similarly, computer

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[1] One should mention that Microsoft and Yahoo! also give access to substantial perks, including free cafeterias, a game room, massage service, or lake access.

game developers are known not only to provide their employees with free catering, but also with their own products at work, and in some instance arcade games on site.[2]

The usage of employee perks has been attributed to productive characteristics of the perks, such as nice office furniture or access to corporate jets (see Marino and Zábojník, 2008, Rajan and Wulf, 2006). While the provision of free catering may be attributable to such concerns, other perks like video gaming or tennis courts seem rather to complement leisure activities. Perks have also been interpreted as non-monetary remuneration substituting for cash payments (see e.g. Rosen, 1986).[3] This is likely when perks are an efficient means to transfer utility, for instance when awarding status, when there are tax reasons, or when perks come in the form of public goods. Generally, such perks will differentially affect an employees' marginal utility from different task and thus the optimal choice of task. This will be relevant in jobs where workers have a large discretion over what activity to pursue (i.e. creative jobs). Finally, perks have been attributed to managerial discretionary power over free cash flow (see e.g. Jensen, 1986, Bebchuk and Fried, 2004). Yet most of the perks mentioned above are employee perks, in the sense that those who benefit do not have the authority to introduce them.

This paper argues that employee perks that seem to encourage idleness do precisely this. Idleness can be desirable when, because of career concerns, employees have an incentive to over-invest in tasks that appear productive, in order to generate a payoff-relevant signal. This is likely when agents' productivity types are not observed before the work begins, but a signal about previous work experience is available. Then a good signal will command a market premium in later stages, and therefore create incentives for the agent to excel in a task that permits signaling, beyond the monetary incentives specified in a wage contract.

This may lead to excessive signaling, decreasing the firm's profits. As a consequence, contracts and organizational form will respond to workers' incentive to signal, biasing firms' investment toward employee benefits that are complementary to leisure rather than to productive activities. When agents of different productivity levels differ in their propensity to signal, organizational heterogeneity will arise in order to cater to agents' types taking into account signaling incentives. As a result organizations will vary in the extent to which they tolerate and reward idleness, for instance by means of corporate culture or by offering

[2] The company Blizzard is widely supposed to outfit its employees with digital equipment for its online game World of Warcraft.

[3] In a similar vein Holmström and Milgrom (1991) state that allowing for over-investment in less productive tasks in a multi-task environment can be optimal in the presence of risk aversion when the agent's participation constraint binds.

employee perks. Diversity of organizational form is linked to the wage distribution, as differentials in compensation based on observable signals affect agents' incentives to signal.

We address these issues formally in a model of multi-task agency, where an agent has private information about the profitability of each task and there are career concerns. Agents live for two periods and produce output in firms. When employed by a firm an agent chooses to perform one of two tasks, a or b. Task a is a complex task that induces a utility cost to the agent, and which may succeed or fail. The probability of success depends on the agent's type and the outcome of task a is publicly observable as in Harris and Holmström (1982). Revenue generated by task a is uncertain and may result in a loss for the firm. The agent has private information about the expected value of a success. That is, task a corresponds to a high-profile, visible activity that commits a sizable portion of the principal's assets to a project, for instance preparing a merger, developing a new product, or launching a research project. Task b is costless for the agent and pays off 0 with certainty; it is best interpreted as staying idle or routine work. That is, the model best describes professions that value creativity or expert knowledge, and employees that seek to demonstrate they have what it takes.

Hence, performing task a when young creates an opportunity for agents to signal their ability to the market. This induces a bias toward task a despite the fact that an agent incurs lower effort cost for task b.[4] The value of the signal is determined by the period 2 wage distribution. Firms respond by adequately choosing contractual and organizational form. Choice of organization is modeled as an investment decision on providing corporate public goods to agents, which reduces the cost of performing task a, so that lower investment effectively subsidizes performing task b. Contracts can condition on the task performed, and, in case of a, on success or failure.

In a market equilibrium, contracts for old agents implement the efficient outcome, as signaling is no longer a concern. Old agents who succeeded in task a when young obtain a premium, which determines the incentive to signal. Firms offer young agents either a separating contract that induces the agent to choose the task with higher expected profit, or a pooling contract that ignores the agent's private information and implements a task.

[4] This mirrors findings of distortions in principal-agent settings due to career concerns, such as excessive or too little risk taking (Hermalin, 1993, Hirshleifer and Thakor, 1992), over-investment in or under-usage of information (Scharfstein and Stein, 1990, Milbourn et al., 2001), over-provision of effort (Holmström, 1999), or distorted project choice (Holmström and Ricart i Costa, 1986, Narayanan, 1985). This paper is concerned with the firm's organizational response.

The optimal type of contract depends on the value of signaling for young agents. For agents who have low return from signaling, the principal will implement a separating contract that pays a bonus for task a. For agents who have an intermediate return from signaling, the principal will implement a separating contract that discourages the agent from task a by rewarding task b with a monetary payment, or by adjusting investment to increase the opportunity cost of a. For sufficiently high values of signaling discouraging the agent from signaling becomes too costly, so that the optimal contract is pooling, paying a flat wage and inducing the agent to choose task a.

The value of signaling is endogenous and is determined both by the convexity of old agents' wages and the informativeness of the signal. The more convex old agents' payoffs are in their productivities, the more desirable is it to gamble for a success when young, as the wage premium in case of success overcompensates the wage discount in case of failure. Bayesian updating implies that a signal is most informative for neutral priors, hence agents of intermediate productivity have highest informational gains from signaling. We assume that the wage function has a constant second derivative, so that the variation in signaling incentives depends exclusively on the agents' expected productivity. We show that organizational and contractual choices cater to the agents' type: for low productivity agents task a is rewarded (corresponding to low-powered incentives for clerical work) and as productivity grows organizational form rewards idleness (specialists or staff in advisory capacity). For intermediate productivity types organizations punish idleness (middle management) using pooling contracts. As productivity increases further employee perks are used again (creative professionals), and for highest expected productivity employers rely on high-powered monetary incentives (executives, key professionals).

Optimal organizational choice is governed by productivity threshold levels; hence, firms may choose substantially different organizational forms (tolerating excessive signaling or rewarding idleness) although being observationally quite similar. Second, organizational and contractual form is related to turnover: agents who obtain a pooling contract are more likely to generate new information, resulting in a job change.

Finally, organizational response to employees' incentives for signaling may induce flatter payoffs for young agents than for old agents, in particular when using pooling contracts. This in turn suggests that incentives for signaling in earlier periods are muted, and the value of signaling in a labor market equilibrium is non-monotonic in time for a given productivity level. This differs from much of the career concerns literature, notably Gibbons and Murphy (1992), as we take into account organizational responses to signaling with contracts are short term. An exception is Kaarbøe and Olsen (2006) who analyze effects of career concerns on optimal contracts in a multi-task setting where the tasks' productivities are known to the principal. This does not allow to distinguish between separating and pooling contracts and the associated organizational choice, an issue this paper emphasizes. Harstad (2007) analyzes a similar setting where the firms' choice of organizational form affects the transparency of the managers' signals. By design firms can extract the full value of signaling and therefore finds it profitable to increase transparency and charge the manager a premium. In our model some firms may discourage signaling, while others may encourage it, depending on the value of signaling for employees. Raith (2008) examines an agency setting with private information of the agent on the productivity of tasks and determines the optimal use of input and output monitoring without career concerns.

This paper is also related to the literature on delegation and experts. Closest is probably Prat (2005) who examines a setting where an expert may have an incentive to report untruthfully, if this coincides with the prior and therefore signals that the expert is of high quality (see also Prendergast, 1993), and concludes that avoiding full transparency on the agent's action in agency settings may be desirable. This paper is concerned rather with using perks, or investment complementary to less productive tasks, to remedy distortions of incentives by signaling; observability of actions plays no major role our model.

The remainder of the paper is organized as follows. Section 2 introduces the theoretical framework, and Section 3 derives the properties of the basic model. Section 4 considers the general model and derives properties of the stationary distribution of wages and organizational choices. Section 5 concludes.

2 A Simple Model

2.1 Agents

The economy is populated by a continuum of agents $i \in [0,1]$ and a continuum of homogeneous principals $j \in [0,1]$. Both agents and principals are endowed with a measure 1. Agents are born with zero wealth, live for two periods, and are heterogeneous in their productivity type $p_i \in \{\underline{p};\overline{p}\}$, with $0 < \underline{p} < \overline{p} < 1$. Productivity is unobservable to both agents and principals. In the first period, agents are heterogeneous in their expected productivity:

$$\tilde{p_i} = E[p_i].$$

2.2 Production

Principals and agents jointly generate output in firms of size 2. Solitary individuals obtain a payoff of 0. In a firm (i, j) an agent decides on a task $d \in \{a, b\}$ to work on. Task b is a routine task that yields revenue 0 to the principal.[5] In contrast, task a is complex and may succeed (S) or fail (F). The probability of success is the agent's productivity type p, the one of failure $1-p$. In case of success revenue $R(s)$ accrues for the principal, with $s \in \{A, B\}$ denoting the state of the world and $R(A) > R(B)$. In case of failure the revenue is 0. Task a is thus best interpreted as starting a new project, for instance, developing a new product, which may succeed or fail. In case the product development succeeds, the product is launched. Its profitability, however, depends on the state of the world s. In particular the case $R(A) > 0 > R(B)$ may arise, for instance if the product flops and fails to break even, quality problems hurt the firm's reputation, or design flaws trigger legal action and fines. The state s is specific to a match (i, j) and drawn independently, assigning probability q to A and $1-q$ to B. Performing task a or b the agent incurs a utility cost.

2.3 Corporate investments in infrastructure

The agent's cost of performing task a or b can be affected by the principal's investments in corporate infrastructure denoted by k_a and k_b:

$$c_b(k_b) = -k_b \text{ and } c_a(k_a) = c - k_a.$$

We interpret k_a as investment in perks that are complementary to production, and k_b as investment in perks complementary to leisure. Investment cost is convex in total infrastructure investment with cost function given by $(k_a^2 + k_b^2)/2$. [6] We impose the following restriction:

$$c > q$$

The above assumption will guarantee that, in equilibrium, task a is costly for the agent.

[5] The case of positive revenue is a straightforward extension generating similar results but assigning a role to pooling on task b.

[6] Assuming substitutability in the cost function is for expositional simplicity, implying all infrastructure investment is complementary to only one task in optimum, greatly facilitating the computation of value functions. The only difference is that otherwise conditions on the technology to ensure convex value functions are stricter.

2.4 Contractual and Informational Environment and Payoffs

In a firm (i,j) contracts specify investment by the principal (k_a, k_b) and payments w_I, w_F and w_S, which are contingent on the events that the agent chooses task b, chooses task a and a failure occurs, chooses task a and a success occurs, respectively. Of course, payments w_S and w_F can be interpreted as a wage w_F for task a and a success bonus $w_S - w_F$; as effort choice of the agent is absent this will have no role here. Since agents are wealth constrained, contracts must respect a limited liability condition and induce positive payments for agents. Task choice and the outcome of task a, that is success or failure, are publicly observable by all firms, but revenue is not.[7] Individuals can only sign two one-period contracts (equivalently, the parties can renegotiate any long-term contract signed in period 1).

In each period, the payoff of an agent is thus given by $u = w_I + k_b$ if task b was chosen, and by $u = w - c + k_a$ if task a was chosen with $w = w_F$ in case of failure and $w = w_S$ in case of success. Correspondingly the principal obtains payoffs $\pi = -w_I - \kappa$, $\pi = -w_F - \kappa$, and $\pi = R(s) - w_S - \kappa$ with $\kappa = (k_a^2 + k_b^2)/2$, respectively. There is no discounting.

2.5 Timing of Events

In each period events in this economy unfold as follows.

1. At the beginning of a period a labor market matches principals and agents, who sign a binding contract.

2. Principals invest as specified in the contract.

3. Within each match (i,j) a state of nature $s \in \{A, B\}$ realizes.

4. The agent chooses a task $d \in \{a, b\}$.

5. Success or failure realizes if the task is a, revenue accrues and payments are made and consumed.

An equilibrium in the labor market for agents is an individually rational stable allocation of pairs of principals and agents, such that there is no pair of principal and agent, not

[7] That revenue is unobservable is not crucial for our results, observability of the signal of success or failure is, however. Whether the contract signed in a match is publicly observable is not important when the task choice is observable.

matched in equilibrium, who could obtain strictly higher joint payoff $u + \pi$ if they match and use a contract of the form $(k_a, k_b, w_I, w_F, w_S)$.

3 Partial Equilibrium

To ease exposition, we begin by deriving the optimal behavior of a principal and an agent in a partial setting, treating market payoffs accruing to an agent of expected productivity $\tilde{p}_i$ in period t of his life $v_t(\tilde{p}_i)$ as exogenous. Given these payoffs the principal is residual claimant of the output.

3.1 Old Agents

Examine the case of an old agent ($t = 2$) first. The main characteristic of an old agent is that signaling has no value. A match (i, j) can use a pooling or a separating contract. The separating contract implements task a in state A and task b in state B. Incentive compatibility requires the agent to be indifferent between tasks a and b, that is

$$\tilde{p}_i w_S + (1 - \tilde{p}_i) w_F - c + k_a = w_I + k_b.$$

The participation constraint is

$$q(\tilde{p}_i w_S + (1 - \tilde{p}_i) w_F - c + k_a) + (1 - q)(w_I + k_b) = v_2(\tilde{p}_i).$$

Therefore $w_I + k_b = v_2(\tilde{p}_i)$. With incentive compatibility the principal's payoff is

$$\pi = q\tilde{p}_i(R(A) - c) - w_I - qk_b + qk_a - (k_a^2 + k_b^2)/2,$$

which decreases in w_I and k_b. Therefore $\tilde{p}_i w_S + (1-\tilde{p}_i) w_F = c - k_a + v_2(\tilde{p}_i)$. Since principal and agent are risk neutral any combination $w_S = (c - k_a + v_2(\tilde{p}_i))/\tilde{p}_i - w_F(1 - \tilde{p}_i)/\tilde{p}_i$ with $w_S, w_F \geq 0$ is feasible and maximizes the principal's payoff while giving $v_2(\tilde{p}_i)$ to the agent. Since $c \geq q$ it is optimal and feasible to set $k_a = q$. The choice of k_b will depend on $v_2(\tilde{p}_i)$ since $w_I + k_b = v_2(\tilde{p}_i)$ with $w_I \geq 0$. Hence, $k_b = v_2(\tilde{p}_i)$ if $v_2(\tilde{p}_i) < 1 - q$ and $k_b = 1 - q$ otherwise.

Note how the investment in k_a is used to change the relative cost of the two actions, while the investment in k_b is used to reward the agent. Also, the non-negativity constraint on w_I may be binding: the principal may want to set a large k_b and then charge a negative wage, but this in not feasible. The optimal investment will have a corner solution at $k_b = v_2(\tilde{p}_i)$.

Intuitively, in a separating contract, workers with a low market value receive a lower level of perks complementary to leisure than workers with a high market value. These perks could be a corporate climbing wall or a swimming pool. On the other hand, perks complementary to production - such as office space or powerful computers - are the same for all workers.

Alternatively, a pooling contract may be used, implementing a task independently of the state of the world. Obviously, there can be a pooling contract only on task a, since pooling on task b is equivalent to not producing, or not hiring the agent. In order to implements a, incentive compatibility requires

$$\tilde{p_i} w_S + (1 - \tilde{p_i}) w_F - c + k_a \geq w_I + k_b.$$

Individual rationality requires

$$\tilde{p_i} w_S + (1 - \tilde{p_i}) w_F - c + k_a = v_2(\tilde{p_i}).$$

Therefore $k_b = 0$ since the principal's payoff decreases in k_b and $w_I \leq v_2(\tilde{p_i})$. Hence, the principal's payoff is

$$\pi = \tilde{p_i}(qR(A) + (1-q)R(B)) - c - v_2(\tilde{p_i}) + k_a - k_a^2/2.$$

Since $w_S, w_F \geq 0$ and $c > q$, optimally $k_a = c + v_2(\tilde{p_i})$ if $v_2(\tilde{p_i}) < 1 - c$ and $k_a = 1$ otherwise. Note that, also here, the non negativity constraint on wage can be binding. However, in this case, k_a is used to satisfy the participation constraint. It follows that, in a pooling equilibrium, the lower worker's market value the lower the amount of perks complementary to production - such as office space or nice office furniture - a worker receives.

The following lemma sums up the results.

Lemma 1. *Contracts for Old Agents:*
A separating *contract specifies* $w_I = v_2(\tilde{p_i}) - k_b$, $w_S, w_F \geq 0$ *with* $\tilde{p_i} w_S + (1 - \tilde{p_i}) w_F = c - q + v_2(\tilde{p_i})$, *and* $k_a = q$ *and* $k_b = \min\{1 - q, v_2(\tilde{p_i})\}$. *The principal has payoff*

$$\pi_2^{sep} = q(\tilde{p_i} R(A) - c + q/2) - v_2(\tilde{p_i}) + (1-q)k_b - k_b^2/2.$$

A pooling *contract implementing* a *has* $w_I \leq v_2(\tilde{p_i})$, $w_S, w_F \geq 0$ *with* $\tilde{p_i} w_S + (1 - \tilde{p_i}) w_F = c - k_a + v_2(\tilde{p_i})$, *and* $k_a = \min\{1, c + v_2(\tilde{p_i})\}$ *and* $k_b = 0$. *The principal has payoff*

$$\pi_2^{pool} = \tilde{p_i}(qR(A) + (1-q)R(B)) - c - v_2(\tilde{p_i}) + k_a - k_a^2/2.$$

If $R(B) \leq 0$, *a separating contract induces higher surplus than a pooling contract for all* $v_2(\tilde{p_i})$.

The proof of the last statement is in the appendix. Both contracts can be interpreted as using perks to transfer rents from the principal to the agents. A pooling contract focuses on productive perks while a separating uses nonproductive perks. By assuming $R(B) < 0$ separating contracts are preferable to pooling contracts, so that production occurs if

$$q(\tilde{p}_i R(A) - c + q/2) - q v_2(\tilde{p}_i) - (v_2(\tilde{p}_i))^2/2 \geq 0.$$

3.2 Young Agents

The contracting problem for young agents is complicated by the possibility to signal by choosing task a. Failing or succeeding in task a provides an informative signal about the agent's productivity p_i, while remaining idle - either choosing task b or remaining unmatched - does not. Correspondingly, denote the posterior expectation of an old agent's productivity by $p_I(\tilde{p}_i)$ if the agent remained idle in period 1, by $p_F(\tilde{p}_i)$ if the agents failed at task a, and by $p_S(\tilde{p}_i)$ if the agent succeeded. Applying Bayes' formula (see appendix for details) yields the following statement.

Lemma 2. *Expected productivity when old is* $p_S(\tilde{p}_i) = \overline{p} + \underline{p} - \frac{\overline{p}\underline{p}}{\tilde{p}_i}$ *after observing a success in task* a, $p_F(\tilde{p}_i) = \frac{\tilde{p}_i(1-\underline{p}-\overline{p})+\overline{p}\underline{p}}{1-\tilde{p}_i}$ *after observing a failure in task* a, *and* $p_I(\tilde{p}_i) = \tilde{p}_i$ *otherwise.*

Clearly, $p_F(\tilde{p}_i) < p_I(\tilde{p}_i) = \tilde{p}_i < p_S(\tilde{p}_i)$. This will determine an agent's market payoff when old, $v_2(p_h(\tilde{p}_i))$, depending on history $h = I, S, F$.

A separating contract for a young agent of expected productivity $\tilde{p}_i$ has to satisfy incentive compatibility

$$\tilde{p}_i w_S + (1 - \tilde{p}_i) w_F + s(\tilde{p}_i) - c + k_a = w_I + k_b,$$

where $s(\tilde{p}_i)$ denotes the value of signaling for a young agent with expected productivity $\tilde{p}_i$ defined as

$$s(\tilde{p}_i) = \tilde{p}_i v_2(p_S(\tilde{p}_i)) + (1 - \tilde{p}_i) v_2(p_F(\tilde{p}_i)) - v_2(\tilde{p}_i).$$

Individual rationality requires

$$q(\tilde{p}_i w_S + (1 - \tilde{p}_i) w_F + s(\tilde{p}_i) - c + k_a) + (1 - q)(w_I + k_b) = v_1(\tilde{p}_i)$$

where $v_1(\tilde{p}_i)$ denotes a young agent's payoff in period 1. This implies that $w_I + k_b = v_1(\tilde{p}_i)$. With incentive compatibility this is

$$\tilde{p}_i w_S + (1 - \tilde{p}_i) w_F + s(\tilde{p}_i) - c + k_a = w_I + k_b = v_1(\tilde{p}_i).$$

Since $s(\tilde{p}_i)$ can be positive, $w_S, w_F \geq 0$ may imply that $k_a < q$. Hence, $k_a = q$ if $v_1(\tilde{p}_i) \geq s(\tilde{p}_i) + q - c$ and $k_a = v_1(\tilde{p}_i) + c - s(\tilde{p}_i)$ otherwise. As above, $k_b = v_2(\tilde{p}_i)$ if $v_2(\tilde{p}_i) < 1 - q$ and $k_b = 1 - q$ otherwise.

The separating contract offered to young agents looks similar to the one offered to old agents, except with one respect: the investment in productive perks k_a may be agent specific. The reason is that each agent has a different incentive to signal, biasing the agent choice of task toward the visible task a. When monetary incentives are at their lower bound, the principal may decrease the investment in k_a in order to mnatain separation. In other words, the principal makes work relative more costly than leisure: idleness is rewarded.

Similar to the case of old agents an optimal contract implementing a for young agents can be derived. Incentive compatibility requires

$$\tilde{p}_i w_S + (1 - \tilde{p}_i) w_F + s(\tilde{p}_i) - c + k_a \geq w_I + k_b.$$

Individual rationality requires

$$\tilde{p}_i w_S + (1 - \tilde{p}_i) w_F + s(\tilde{p}_i) - c + k_a = v_1(\tilde{p}_i).$$

Therefore $k_b = 0$ and $w_I \leq v_1(\tilde{p}_i)$. Analogously to the old-agent case, optimally $k_a = c + v_2(\tilde{p}_i)$ if $v_2(\tilde{p}_i) < 1 - c$ and $k_a = 1$ otherwise.

The following lemma sums up these results.

Lemma 3. *Contracts for Young Agents:*
(i) a separating *contract specifies* $k_a = \min\{q, v_1(\tilde{p}) - s(\tilde{p}) + c\}$, $k_b = \min\{1 - q, v_1(\tilde{p})\}$, $\tilde{p} w_S + (1 - \tilde{p}) w_F = v_1(\tilde{p}) + c - s(\tilde{p} - k_a$*, and* $w_I = v_1(\tilde{p}) - k_b$*. The principal's payoff is*

$$\pi_1^{sep} = q(\tilde{p} R(A) - c) + q k_a + (1 - q) k_b + q s(\tilde{p}) - v^1(\tilde{p}) - (k_a^2 + k_b^2)/2.$$

(ii) A pooling *contract implementing a specifies* $w_I \leq v^1(\tilde{p})$ *and* $k_b = 0$, $k_a = \min\{1, v_1(\tilde{p}) - s(\tilde{p}) + c\}$*, and* $\tilde{p} w_S + (1 - \tilde{p}) w_F = v_1(\tilde{p}) + c - s(\tilde{p}) - k_a$*. The principal has payoff*

$$\pi_1^{pool} = \tilde{p}(q R(A) + (1 - q) R(B)) + s(\tilde{p}) - c + k_a - k_a^2/2 - v_1(\tilde{p}).$$

Determine now whether a pooling or a separating contract is more profitable given that an agent of expected productivity $\tilde{p}$ obtains a payoff of $v_1(\tilde{p})$ and has signaling value $s(\tilde{p})$.

Proposition 4. *For every* $\tilde{p}_i$*, there is a cutoff* $\hat{v}(\tilde{p}_i)$ *such that a young agent will obtain a separating contract if* $v_1(\tilde{p}_i) \leq \hat{v}(\tilde{p}_i)$*,*

$$\hat{v}(\tilde{p}_i) = \begin{cases} (1 - q)/2 - \tilde{p}_i R(B) & \text{if } -2\tilde{p}_i R(B) \geq 1 - q \\ \sqrt{2(1 - q)(-\tilde{p}_i R(B))} & \text{otherwise.} \end{cases}$$

If $v_1(\tilde{p}_i) > \hat{v}(\tilde{p}_i)$, there is a function $\hat{s}(v)$ such that a young agent obtains a separating contract if $s(\tilde{p}) \leq \hat{s}(v_1(\tilde{p}))$ and a pooling contract otherwise. $\hat{s}(v)$ is decreasing in its argument and approaching $c - q - \tilde{p}R(B)$ as v grows out of bounds.

Figure 1 shows the possible contractual and organizational regimes. A minimum rent for the agent is needed to make pooling viable. The reason for this is that the rent from signaling goes exclusively to the agent, while the cost, in form of foregone revenue $R(A) - E[R(s)]$ is born by the principal. Limited liability and liquidity constraints of the agents prevent agents to pay for the opportunity to signal by means of negative wages. Hence, to ensure that the principal gets payoff π the joint surplus has to be at least $\pi - (1-q)\tilde{p}R(B)$. If this is the case pooling will dominate separating contract whenever the benefit of signaling $s(\tilde{p})$ is higher than the cost $\hat{s}(.)$ consisting of foregone revenue and, possibly, investment distortions (if there is under-investment in k_a in pooling but efficient investment in separating contracts).

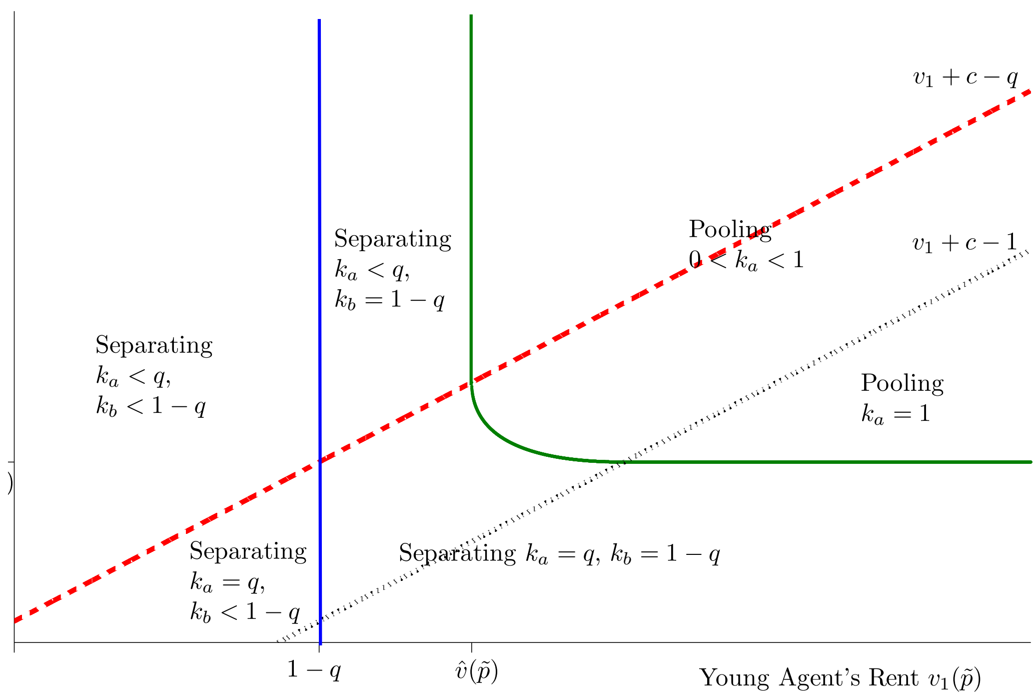

Fig. 1: Contractual and organizational regimes depending on signaling value $s(\tilde{p})$ and agent's market payoff when young $v^1(\tilde{p})$.

4 Labor Market Equilibrium

To close the model, we need to derive the value of signaling $s(\tilde{p}_i)$ and market rents $v_t(\tilde{p}_i)$ endogenously. In the labor market a continuum of principals of unit measure compete for a continuum of agents of measure 1, with measure 1/2 of young and old agents each. Old agents are characterized by the conditional expectation of their productivity given the work history when young: as above an agent i born with prior $\tilde{p}_i$ has expected productivity when old $p_S(\tilde{p}_i)$ in case of a success in task a when young, $p_F(\tilde{p}_i)$ in case of a failure in task a, and $\tilde{p}_i$ in case i stayed idle when young. Hence, the distribution of old agents' expected productivities depends on the degree of experimentation induced by organizational and contractual choice in the labor equilibrium.

Under our assumption that $R(B) \leq 0$ the optimal contract for an old agent is separating. Focusing on separating contracts for old agents facilitates exposition but does not drive the results. Using Lemma 1 a principal and an old agent with expected productivity $\tilde{p}_i$ have positive expected surplus from production under a separating contract if

$$q(\tilde{p}_i R(A) - c + q/2) - qv - v^2/2 \geq 0 \text{ for } v \geq 0.$$

That is, with an old agent production occurs only if

$$\tilde{p}_i \geq (c - q/2)/R(A) := p^o.$$

Assume that for high productivity $\overline{p}$ agents production using task a is profitable, but not for low productivity $\underline{p}$ agents:

$$\underline{p} < p^o < \overline{p}.$$

Hence, all old agents with $\tilde{p}_i \leq p^o$ are unemployed obtaining zero payoff, and principals compete for agents who enable strictly positive expected surplus in a match. This in turn implies that principals obtain zero payoff in equilibrium, the same as their outside option of not matching. By Lemma 1 equilibrium payoffs for old agents and principals are thus

$$\pi = 0 \text{ and } v_2(\tilde{p}_i) = \begin{cases} 0 & \text{if } \tilde{p}_i \leq p^o \\ \sqrt{2q(\tilde{p}_i R(A) - c + q)} - q & \text{if} p^o < \tilde{p}_i < \frac{1+2q(c-q)}{2qR(A)} \\ q(\tilde{p}_i R(A) - c + q/2) + (1-q)^2/2 & \text{otherwise.} \end{cases}$$

Equilibrium payoffs for old agents then determine the value of signaling for young agents. Note that that for $\tilde{p}_i$ sufficiently high so that $p_F(\tilde{p}_i) > p^o$ the value of signaling is zero as

$$p_F(\tilde{p}_i) = \frac{\tilde{p}_i}{1 - \tilde{p}_i}(1 - p_S(\tilde{p}_i))$$

by Lemma 2. The value of signaling for young agents is given by

$$s(\tilde{p}_i) = \begin{cases} \tilde{p}_i v_2(p_S(\tilde{p}_i)) & \text{if } \tilde{p}_i < p^o < p_S(\tilde{p}_i) \\ \tilde{p}_i v_2(p_S(\tilde{p}_i)) - v_2(\tilde{p}_i) & \text{if } p_F(\tilde{p}_i) < p^o < \tilde{p}_i \\ 0 & \text{otherwise.} \end{cases}$$

Differentiating $v_2(\tilde{p}_i)$ implies that $s(\tilde{p}_i)$ strictly increases on $\tilde{p}_i < p^o < p_S(\tilde{p}_i)$ and strictly decreases on $p_F(\tilde{p}_i) < \tilde{p}^o < \tilde{p}_i$.

Define the cut-off productivity below which a young agent is not hired, and above which a young agent is hired as p_0, and note that $v_1(p_0) = 0$: the marginal agent hired receives no payment. Since principals receive $\pi = 0$ and transfers cannot be negative, in a labor market equilibrium a young agent with expected productivity $\tilde{p}_i$ is hired if he generates non-negative expected output. We also know by Lemma 3 that the output-maximizing contract is separating. Therefore the optimal contract at productivity p_0 is separating with $k_b = 0$ since, if this was not the case, the principal could switch to separating and increase her profit. Hence, the agent's payoff under such a contract satisfies

$$q(p_0 R(A) + s(p_0) - c) - q v_1(p_0) - v_1(p_0)^2/2 + q k_a - k_a^2/2 = \pi = 0,$$

with $k_a = q$ if $s(\tilde{p_0}) \leq c - q$ and $k_a = c - s(\tilde{p_0})$ otherwise. From this expression the break even productivity for young agents p_0 can be calculated, as stated in the following proposition.

Proposition 5. *There is p_0 such that all young agents with $\tilde{p}_i \geq p_0$ are hired by a principal. For $\tilde{p}_i$ close to p_0 this contract is separating. Moreover, $p_0 < p^o$, i.e. young agents are hired by principals at lower productivity than old agents.*

Proof: Establish first that $v_1(\tilde{p}_i)$ strictly increases in $\tilde{p}_i$ when implementing task a at least some of the time. For this we need that $\frac{\partial s(\tilde{p})}{\partial \tilde{p}} > -R(A)$, which is easily verified using the definitions of $s(\tilde{p}_i)$ and $v_2(\tilde{p}_i)$, which increases in $\tilde{p}_i$ as does $p_S(\tilde{p}_i)$. For separating contracts,

$$v_1(\tilde{p}_i) = \begin{cases} \frac{1}{2}[\sqrt{4\tilde{p}_i q R(A) - (c - s(\tilde{p}_i))^2} - (c - s(\tilde{p}_i))] & \text{if } v_1(\tilde{p}_i) < 1 - q, q + s(\tilde{p}_i) - c \\ \sqrt{2q(\tilde{p}_i R(A) + s(\tilde{p}_i) - c + q)} - q & \text{if } q + s(\tilde{p}_i) - c < v_1(\tilde{p}_i) < 1 - q \\ \sqrt{2[(1-q)(c - s(\tilde{p}_i)) + q\tilde{p}_i R(A)]} - (1 - q + c - s(\tilde{p}_i)) & \text{if } 1 - q < v_1(\tilde{p}_i) < q + s(\tilde{p}_i) - c \\ q(\tilde{p}_i R(A) - c + s(\tilde{p}_i) + q/2) + (1-q)^2/2 & \text{if } v_1(\tilde{p}_i) > 1 - q, q + s(\tilde{p}_i) - c. \end{cases}$$

For pooling contracts,

$$v_1(\tilde{p}_i) = \begin{cases} \sqrt{2\tilde{p}_i R(A)} - (c - s(\tilde{p}_i)) & \text{if } v_1(\tilde{p}_i) < 1 + s(\tilde{p}_i) - c \\ \tilde{p}_i R(A) - c + s(\tilde{p}_i) + 1/2 & \text{if } v_1(\tilde{p}_i) > 1 + s(\tilde{p}_i) - c. \end{cases}$$

In all cases the first derivative with respect to $\tilde{p}_i$ is positive. For the cut off productivity p_0 it must hold that $0 = v_1(p_0) < 1 - q$ and $0 = v_1(p_0) < \hat{v}(\tilde{p}_i)$. Hence, either $v_1(\tilde{p}_i) < 1 - q, q + s(\tilde{p}_i) - c$ or $q + s(\tilde{p}_i - c < v_1(\tilde{p}_i) < 1 - q$ must be the case. Setting $v_1(p_0) = 0$ then yields

$$p_0 R(A) = (c - s(p_0))^2/2q \text{ if } s(p_0) > c - q \text{ and}$$
$$p_0 R(A) = (c - s(p_0) - q/2) \text{ otherwise.}$$

This immediately implies $p_0 \leq p^o = (c - q/2)/R(A)$, with a strict inequality if $s(p^o) > 0$. This also implies that $s(p_0) = p_0 v_2(p_S(p_0))$, which ensures that $s(\tilde{p}_i)$ is increasing at p_0. Using the definition of $s(.)$ yields the cutoff productivity in terms of the primitives. □

That is, young agents are employed for lower productivity than are old agents. The reason for this is that young agent derive signaling value from task a, which partly compensates their cost of effort c put in task a. Therefore young agents are prepared to work for less remuneration of their effort than are old agents, making them less costly at the same productivity. The second observation is less obvious: young agents who are just productive enough to obtain a contract that allows them to signal, are employed using a separating contract rather than a pooling contract, which would maximize the opportunity to signal. The reason for this is the same as in the partial setting: to obtain a pooling contract a young agent needs to have a high value of signaling and to obtain sufficient payoff $v_1(\tilde{p}_i)$. The reason for the latter is that the principal needs to be compensated for the decrease in expected revenue (since $E[R(s)] < qR(A)$).

Turn now to the question of whether agents find it optimal to use pooling contracts at all. This depends on their value of signaling. It is possible to translate the cutoff value $\hat{s}(v_1(\tilde{p}_i))$ from Proposition 4 into cutoff values depending directly on $\tilde{p}_i$, $\hat{s}(\tilde{p}_i)$. The following Lemma summarizes its properties, its proof uses relatively standard arguments and is in the appendix.

Lemma 6. *There exists a function $\hat{s}(\tilde{p}_i)$ such that a pooling contract is preferred to a separating contract whenever $s(\tilde{p}_i) > \hat{s}(\tilde{p}_i)$. $\hat{s}(\tilde{p}_i)$ strictly decreases for $\tilde{p}_i \in [p_0, p_1]$ for some $p_1 > p_0$ and strictly increases for $v_1(\tilde{p}_i) \geq 1 - q - \tilde{p}_i R(B)$. If $q/2 > (2q - 1)(c - q)$ then $\hat{s}(\tilde{p}_i)$ is convex and has a unique minimum.*

A pooling contract is preferable to a separating contract for agents with productivities $\tilde{p}_i$ such that $v_1(\tilde{p}_i) > \hat{v}(\tilde{p}_i)$ and $s(\tilde{p}_i) > \hat{s}(v_1(\tilde{p}_i))$. Using Lemma 6 and going through the cases yields the following proposition on the labor market outcome. Its proof can be found in the appendix.

Proposition 7 (Labor Market Outcome)**.** *In a labor market equilibrium, there are values* $\underline{p} < \tilde{p}_0 < \tilde{p}^o < \overline{p}$ *such that old agents obtain a separating contract if* $\tilde{p}_i \geq p^o$ *and no contract otherwise, while young agents obtain some contract if* $\tilde{p}_i \geq \tilde{p}_0$*.*

If $R(B)$ *is sufficiently close to* 0 *and* c *sufficiently close to* q *there are thresholds* $p_0 < p_1 < p_2 \leq p_3 < p_4 < \overline{p}$ *such that the optimal contract for a young agent is*

(i) separating for $p_0 < \tilde{p}_i < p_1$ *with* $\tilde{p}_i w_S + (1 - \tilde{p}_i) w_F = 0$ *and* $0 < k_a \leq q$ *and* $0 < k_b \leq 1 - q$*,*

(ii) pooling for $p_1 < \tilde{p}_i < p_2$ *and* $p_3 < \tilde{p}_i < p_4$ *with* $\tilde{p}_i w_S + (1 - \tilde{p}_i) w_F \geq 0$ *and* $q < k_a \leq 1$ *and* $k_b = 0$*, or*

(iii) separating for $p_3 < \tilde{p}_i < \overline{p}$ *with* $\tilde{p}_i w_S + (1 - \tilde{p}_i) w_F > 0$ *and* $k_a = q$ *and* $0 < k_b \leq 1 - q$*.*

If $q/2 > (2q - 1)(c - q)$ *then* $p_2 = p_3$*.*

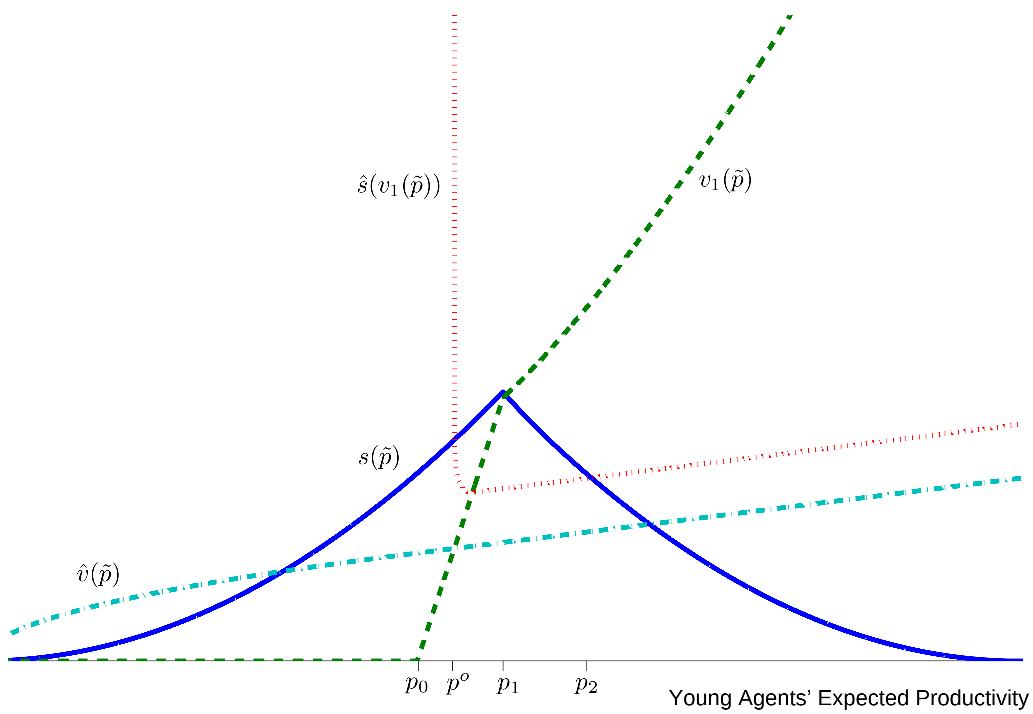

Fig. 2: Contractual and organizational regimes depending on expected productivity $\tilde{p}$.

That is, firms that hire agents may find it optimal to react to the agent's signaling incentive in one of three ways. If signaling incentives are strong but the associated market payoff is low $p_0 < \tilde{p}_i < p_1$ a separating contract is used, with under-investment in k_a to curb the agent's enthusiasm for task a. For productivities associated to high signaling incentives and sufficient market payoff ($p_1 < \tilde{p}_i < p_2$) the agent is able to transfer enough of his rent to the principal to compensate for the shortfall in expected revenue $qR(A) - E[R(s)]$ by reducing the wage $\tilde{p}_i w_S + (1 - \tilde{p}_i) w_F$. Agents with weak signaling incentives and sufficient market payoff obtain a separating contract with efficient investment in k_a, and possibly in k_b. This is summarized in Figure 2, which shows the regimes in the labor market equilibrium for young agents.

The intuition is that k_b serves as payment to the agent whereas k_a can be used to discourage choosing task a. Note further that contracts for agents with $p_0 < \tilde{p}_i < p^o$ can also be interpreted as contacts that threaten to fire an agent who chooses task a. This is particularly true when $v_1(p_1) > 1 - q$ since in this case $w_I > 0$ whereas $w_S = w_F = 0$. Taking task a the agent obtains no wage but a signal, and either will not get no contract in case of failure, or a different contract with possibly a different employer in case of success.

Efficiency

Turn now to the efficiency of investments and contractual choice. To determine the first best allocation note that, under a separating contract, joint surplus of an agent and a principal is given by

$$q(\tilde{p}_i R(A) - c + s(\tilde{p}_i)) + qk_a - k_a^2/2 + (1 - q)k_b - k_b^2/2$$

implying that the efficient investment is $k_a = q$ and $k_b = 1 - q$. Using a pooling contract, joint surplus is

$$\tilde{p}_i(E[R(s)] - c + s(\tilde{p}_i)) + k_a - k_a^2/2$$

implying that the efficient investment is $k_a = 1$. Hence, in any cohort, for low values of $\tilde{p}_i$ the equilibrium investments will be lower than the efficient investments. This implies a distortion in the incentive to signal, since investment when old affects the incentive to signal when young. A pooling contract is efficient if

$$s(\tilde{p}_i) \geq c - q - \tilde{p}_i R(B).$$

with strict inequality for $v_1(\tilde{p}_i) < 1 - q - \tilde{p}_i R(B)$. Otherwise a separating contract is efficient.

Note that, for old agents, under the assumption $R(B) < 0$, a separating contract maximizes joint surplus. It follows that old agents will be matched if

$$\tilde{p}_i \geq \frac{qc - 1/2 + q(1-q)}{qR(A)} := p^* < p^o.$$

The first-best value of signaling is given by

$$s(\tilde{p}_i) = \tilde{p}_i[q(p_S(\tilde{p}_i)R(A) - c + q/2) + (1-q)^2/2] \text{ for } \tilde{p}_i \leq p^*,$$
$$s(\tilde{p}_i) = (1-\tilde{p}_i)(qc - q^2/2 - (1-q)^2/2) - \tilde{p}_i(1 - p_S(\tilde{p}_i)qR(A) \text{ for } \tilde{p}_i > p^*.$$

That is, the first-best value of signaling exceeds the decentralized value of signaling for low productivities and falls short of it for high productivities. However, the distortion in the signaling value comes exclusively from the distortion in the investment in k_b: in the decentralized equilibrium the private value of signaling coincides with the social value of signaling.

Hence, when looking at young agents and comparing the equilibrium allocation with the first best allocation, there are two elements to keep into consideration. First, taking the value of signaling as given, the contract offered to young agents will see less pooling than efficient. The reason is that the principal cannot charge the agent for the right to signal, and may prevent the agent from choosing action a even when pooling on a is efficient. Second, the first-best value of signaling is different than the equilibrium value of signaling: compared to the decentralized allocation, the first best value of signaling is higher for low productivity types, and lower for high productivity types. This has implication for the measure of young agents that will be hired, and for the types of contract offered.

Proposition 8 (Efficiency)**.** *Comparing the labor market equilibrium allocation with the first best, for old agents investment in k_a is efficient, but there is under-investment in k_b for low productivity types. Too few old agents are hired. For young agents there is under-investment both in k_a and k_b. Some young agents obtain a separating contract with efficient k_b but too low k_a. High productive young agents get separating contracts with efficient investment. There is too little signaling for young agents with $\tilde{p}_i$ around p_1, there is too much signaling around p_2.*

Figure 3 shows the differences between first-best allocation and decentralized allocation. First best outcomes are marked by stars.

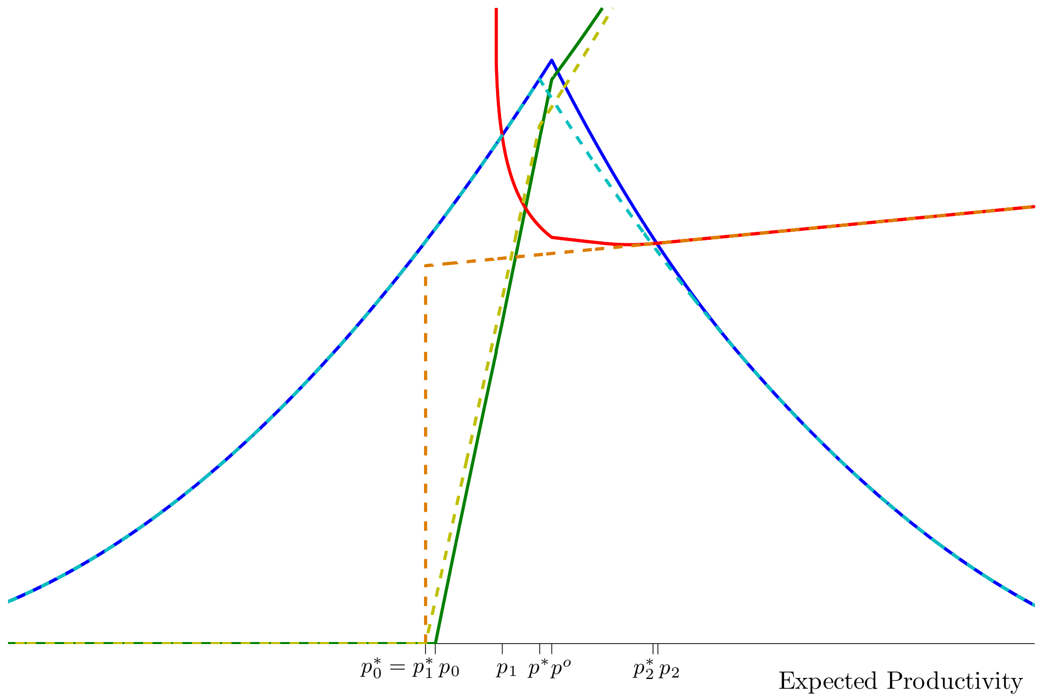

Fig. 3: Equilibrium (solid) versus first best (dashed) outcomes.

Dynamics of Signaling

The pattern described above holds as more generations are added. In this case the option value of signaling becomes greater the younger the agent, so that the younger an agent the greater the signaling value. This has the obvious consequence for participation and contractual and organizational choice. Figure 4 shows an example.

A Mathematical Appendix

Proof of Lemma 1

To establish the last statement suppose $v_2(\tilde{p} < 1 - c$ first. This implies $v_2(\tilde{p}) < 1 - q$. A separating is profitable if

$$(1-q)c + q^2/2 + (1-q)v_2(\tilde{p}) - (v_2(\tilde{p}))^2/2 > (1-q)pR(B) + c + v_2(\tilde{p}) - (c + v_2(\tilde{p}))^2/2.$$

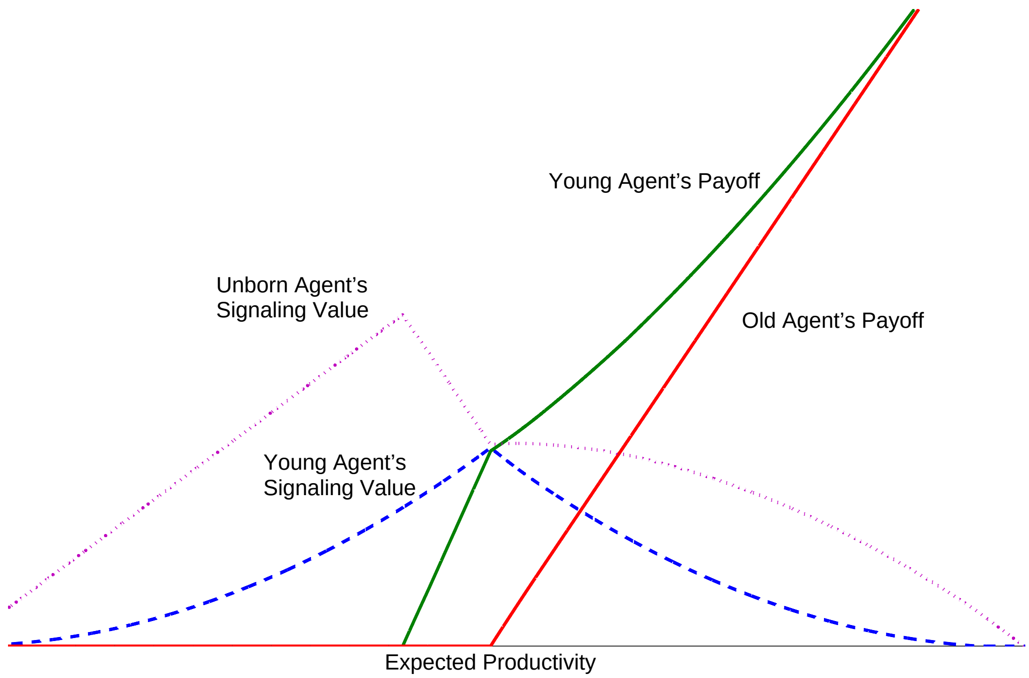

Fig. 4: Payoffs and signaling values for different generations.

After some rearranging this becomes

$$(c-q)^2/2+(c-q)v_2(\tilde{p})>(1-q)pR(B),$$

where the LHS is strictly positive. Let now $1-c<v_2(\tilde{p}<1-q$. Then separating is profitable if

$$(1-q)c+q^2/2+(1-q)v_2(\tilde{p})-(v_2(\tilde{p}))^2/2>(1-q)pR(B)+1/2.$$

This becomes

$$c-(1+q)/2+v_2(\tilde{p})(1-v_2(\tilde{p})/(2(1-q)))>pR(B).$$

Since $1-c<v_2(\tilde{p}<1-q$ by assumption, the LHS is bounded below by $(c-q)/2>0$. Finally, in case $v_2(\tilde{p}>1-q$ the condition becomes

$$(1-q)c+q^2/2+(1-q)^2/2>(1-q)pR(B)+1/2\Leftrightarrow c-q>pR(B).$$

his establishes the statement.

Proof of Lemma 2

Denote by π the prior belief over the distribution of $\underline{p}$ and $\overline{p}$, so that $\tilde{p} = \pi\overline{p} + (1-\pi)\underline{p}$. Then

$$\tilde{p}_S = \frac{\pi\overline{p}}{\pi\overline{p} + (1-\pi)\underline{p}}\overline{p} + \left(1 - \frac{\pi\overline{p}}{\pi\overline{p} + (1-\pi)\underline{p}}\right)\underline{p}.$$

Using $\tilde{p} = \pi\overline{p} + (1-\pi)\underline{p}$ yields the expression in the lemma. An analogous argument yields $\tilde{p}_F$. Since all agents are ex ante identical both if an agent chose b in the first period or remained unmatched generates no new information, hence $\tilde{p}_I = \tilde{p}$.

Proof of Proposition 4

Given optimal investments in Lemma 3 we need to distinguish several cases. Start by supposing that $v_1(\tilde{p}) \leq s(\tilde{p}) + q - c$. A separating contract is more profitable than a pooling contract given that an agent of expected productivity $\tilde{p}$ obtains a payoff of $v_1(\tilde{p})$ and has signaling value $s(\tilde{p})$ if

$$q\tilde{p}R(A) - (1-q)v_1(\tilde{p}) + (1-q)k_b - k_b^2/2 > \tilde{p}(qR(A) + (1-q)R(B)).$$

This can be rearranged, using $k_b = \min\{1-q, v_1(\tilde{p})\}$, to yield the condition

$$v_1(\tilde{p}) < \hat{v} =: \begin{cases} (1-q)/2 - pR(B) & \text{if } -2\tilde{p}R(B) \geq 1-q \\ \sqrt{2(1-q)(-\tilde{p}R(B))} & \text{otherwise.} \end{cases}$$

Turn now to the case $s(\tilde{p}) + q - c \leq v_1(\tilde{p}) \leq s(\tilde{p}) + 1 - c$. Surplus is higher under a separating than under a pooling contract if

$$q(s(\tilde{p}) - c) + \frac{q^2}{2} - v_1(\tilde{p}) + (1-q)k_b - \frac{k_b^2}{2} > \tilde{p}(1-q)R(B) - \frac{(v_1(\tilde{p}) - s(\tilde{p}) + c)^2}{2}.$$

Solving for $s(\tilde{p})$ this yields a quadratic equation. Its determinant is positive if and only if $v_1(\tilde{p}) \geq \hat{v}$. Supposing this is the case the condition becomes

$$s(\tilde{p}) < \hat{s}(v_1(\tilde{p})) := v_1(\tilde{p}) + c - q - \begin{cases} \sqrt{2(1-q)(v_1(\tilde{p}) + \tilde{p}R(B) - (1-q)/2)} & \text{if } v_1(\tilde{p}) \geq 1-q \\ \sqrt{(v_1(\tilde{p}))^2 + 2(1-q)\tilde{p}R(B))} & \text{otherwise.} \end{cases}$$

This defines $\hat{s}(v)$ for $v \geq \hat{v}$ and $\hat{s}(v) + q - c \leq v \leq \hat{s}(v) + 1 - c$. Since $\hat{s}(v) \leq v + c - q$ holds for the expression above, only the upper bound has a bite and becomes

$$(1-q)^2 - 2(1-q)\tilde{p}R(B) \geq \begin{cases} 2(1-q)(v - (1-q)/2) & \text{if } v \geq 1-q \\ v^2 & \text{otherwise.} \end{cases}$$

Since $(1-q)^2 - 2(1-q)\tilde{p}R(B) > (1-q)^2$ the condition $\hat{s}(v) + q - c \leq v \leq \hat{s}(v) + 1 - c$ holds if and only if $\hat{v} \leq v \leq 1 - q - \tilde{p}R(B)$. Differentiating yields that $\hat{s}(v)$ is strictly decreasing on this interval.

Finally, let $v_1(\tilde{p}) > s(\tilde{p}) + 1 - c$. The condition for a separating contract to be profitable is now

$$q(s(\tilde{p}) - c) + \frac{q^2}{2} + (1-q)k_b - \frac{k_b^2}{2} > \tilde{p}(1-q)R(B) - c + s(\tilde{p}) + 1/2.$$

That is,

$$s(\tilde{p}) < c - \tilde{p}R(B) - \begin{cases} q & \text{if } v_1(\tilde{p}) \geq 1 - q \\ ((1+q)/2 - v_1(\tilde{p})(1 - v_1(\tilde{p})/(2(1-q)))) & \text{otherwise.} \end{cases}$$

This defines $\hat{s}(v)$ for $v > 1 - q - \tilde{p}R(B)$, since by assumption $\hat{s}(v) - v - c + 1 < 0$, which in turn becomes $1 - v - \tilde{p}R(B) < q$ since $(1-q)^2 - 2(1-q)\tilde{p}R(B) < v^2 < (1-q)^2$ yields a contradiction. That is,

$$\hat{s}(v_1(\tilde{p})) = c - q - \tilde{p}R(B) > 0$$

for $v_1(\tilde{p}) > 2(1 - q - \tilde{p}R(B))$. This establishes the proposition.

Proof of Lemma 6

Note that $\hat{v}(\tilde{p}) \leq v_1(\tilde{p})$ implies that $\tilde{p} \geq p_1$ for some $p_1 > 0$ since $\frac{\partial v_1(\tilde{p})}{\partial \tilde{p}} > \frac{\partial \hat{v}(\tilde{p})}{\partial \tilde{p}}$.

Suppose that $\hat{v}(\tilde{p}) \leq v_1(\tilde{p}) < 1 - q$ first. Then

$$\frac{\partial \hat{s}(v_1(\tilde{p}))}{\partial \tilde{p}} = \frac{\partial v_1(\tilde{p})}{\partial \tilde{p}} - \frac{v_1(\tilde{p})\frac{\partial v_1(\tilde{p})}{\partial \tilde{p}} + (1-q)R(B)}{\sqrt{(v_1(\tilde{p}))^2 + 2(1-q)\tilde{p}R(B)}}. \tag{1}$$

Note that $\frac{\partial v_1(\tilde{p})}{\partial \tilde{p}} = \frac{qR(A)}{v_1(\tilde{p})+q} > 0$ (see proof of Proposition 5). If $s(\tilde{p})$ is convex, $\frac{\partial^2 v_1(\tilde{p})}{\partial \tilde{p}^2} < 0$. $\frac{\partial \hat{s}(v_1(\tilde{p}))}{\partial \tilde{p}} < 0$ as $v_1(\tilde{p})$ approaches $\hat{v}(\tilde{p})$ if

$$\frac{\hat{v}(\tilde{p})}{v_1(\tilde{p}) + q} qR(A) + (1-q)R(B) > 0,$$

as the nominator tends to zero. Plugging in the expressions for $\hat{v}(\tilde{p})$ and $v_1(\tilde{p})$ this condition becomes

$$\tilde{p}E[R]R(A) > (1-q)R(B)(c - q - s(\tilde{p})),$$

which must hold under the assumption $R(B) \leq 0$ and $v_1(\tilde{p}) > \hat{v}(\tilde{p})$. Therefore $\hat{s}(v_1(\tilde{p}), \tilde{p})$ is strictly decreasing in $\tilde{p}$ for $\tilde{p}$ such that $v_1(\tilde{p})$ close to $\hat{v}(\tilde{p})$ not requiring convexity of $s(\tilde{p})$.

Differentiating (1) once, the second derivative is positive if

$$\frac{\left(v_1(\tilde{p})\frac{\partial v_1(\tilde{p})}{\partial \tilde{p}} + (1-q)R(B)\right)^2}{(v_1(\tilde{p}))^2 + 2(1-q)\tilde{p}R(B)} - \left(\frac{\partial v_1(\tilde{p})}{\partial \tilde{p}}\right)^2 > \left(v_1(\tilde{p}) - \sqrt{(v_1(\tilde{p}))^2 + 2(1-q)\tilde{p}R(B)}\right)\frac{\partial^2 v_1(\tilde{p})}{\partial \tilde{p}^2}.$$

If $s(\tilde{p})$ is convex, the right hand side is negative and a sufficient condition for convexity of $\hat{s}$ in $\tilde{p}$ is

$$\left(v_1(\tilde{p})\frac{\partial v_1(\tilde{p})}{\partial \tilde{p}} + (1-q)R(B)\right)^2 > \left((v_1(\tilde{p}))^2 + 2(1-q)\tilde{p}R(B)\right)\left(\frac{\partial v_1(\tilde{p})}{\partial \tilde{p}}\right)^2.$$

Using $\frac{\partial v_1(\tilde{p})}{\partial \tilde{p}} = \frac{qR(A)}{v_1(\tilde{p})+q}$ and noting that $R(B) \leq 0$ yields a sufficient condition,

$$\tilde{p}qR(A) > v_1(\tilde{p})(v_1(\tilde{p}) + q).$$

Since $q + s(\tilde{p} - c \leq v_1(\tilde{p}) \leq 1-q$ this holds whenever $q > 1/2$. Hence, the second derivative of $\hat{s}$ with respect to $\tilde{p}$ exists and is positive.

Turn now to the case $1-q \leq v_1(\tilde{p}) < 1-q-\tilde{p}R(B)$. Differentiating $v_1(\tilde{p})$ in this case yields

$$\frac{\partial v_1(\tilde{p})}{\partial \tilde{p}} = q\left(R(A) + \frac{\partial s(\tilde{p})}{\partial \tilde{p}}\right).$$

Differentiating $\hat{s}(v_1(\tilde{p}))$ with respect to $\tilde{p}$ yields

$$\frac{\partial \hat{s}(v_1(\tilde{p}))}{\partial \tilde{p}} = q\left(R(A) + \frac{\partial s(\tilde{p})}{\partial \tilde{p}}\right) - \sqrt{1-q}\frac{q\left(R(A) + \frac{\partial s(\tilde{p})}{\partial \tilde{p}}\right) + R(B)}{\sqrt{2(v_1(\tilde{p}) + \tilde{p}R(B) - (1-q)/2)}}. \tag{2}$$

The second derivative is clearly positive as $s(\tilde{p})$ is convex:

$$\frac{\partial^2 \hat{s}(v_1(\tilde{p}))}{\partial \tilde{p}^2} = q\frac{\partial^2 s(\tilde{p})}{\partial \tilde{p}^2}\left(1 - \frac{\sqrt{1-q}}{\sqrt{2(v_1(\tilde{p}) + \tilde{p}R(B) - (1-q)/2)}}\right) + \sqrt{1-q}\frac{\left(q\left(R(A) + \frac{\partial s(\tilde{p})}{\partial \tilde{p}}\right) + R(B)\right)^2}{2(v_1(\tilde{p}) + \tilde{p}R(B) - (1-q)/2)} > 0. \tag{3}$$

Note that as $v_1(\tilde{p})$ approaches $1-q$ both $\hat{s}(.)$ and its first derivative converge both from below and from above.

That is, $\hat{s}$ is strictly convex for $\tilde{p}$ such that $\hat{v}(\tilde{p}) \leq v_1(\tilde{p}) \leq 1-q-\tilde{p}R(B)$, initially decreasing and increasing at the upper bound as can be quickly verified using (2), $\frac{\partial \hat{s}(1-q-\tilde{p}R(B))}{\partial \tilde{p}} = -R(B) > 0$.

In case $v_1(\tilde{p}) > 1-q-\tilde{p}R(B)$ $\hat{s}(.)$ is linear, increasing function of $\tilde{p}$ with slope $-R(B)$, which does not require convexity of $s(\tilde{p})$. This establishes the lemma.

Proof of Proposition 7

The statement on p_0 follows from Proposition 5. A pooling contract is preferable for some productivity $\tilde{p}$ if and only if $s(\tilde{p}) \geq \hat{s}(\tilde{p})$. By Proposition 5 the optimal contract for young agents with $\tilde{p} \geq p_0$ close to p_0 is separating. Hence, there is $p_1 > p_0$ such that separating contracts are optimal for $p_0 \leq \tilde{p} \leq p_1$. Clearly, $\lim_{\tilde{p}\to\overline{p}} s(\tilde{p}) = 0$. Therefore there is $p_4 < \overline{p}$ such that separating contracts are optimal for $p_4 \leq \tilde{p} \leq \overline{p}$.

Derive a sufficient condition for existence of pooling contracts next. To do so focus on $\tilde{p}^o$. By definition $\tilde{p}^o R(A) = c - q/2$ and therefore

$$v_1(\tilde{p}^o) = \begin{cases} qs(\tilde{p}^o) + (1-q)^2/2 & \text{if } s(\tilde{p}^o) \geq (1-q^2)/(2q) \\ \sqrt{2qs(\tilde{p}^o) + q^2} - q & \text{otherwise.} \end{cases}$$

A pooling contract is desirable for $\tilde{p}^o$ if $v_1(\tilde{p}^o) > \hat{v}(\tilde{p}^o)$ and $s(\tilde{p}^o) > \hat{s}(\tilde{p}^o)$. Using the definition of $\hat{s}(v_1(\tilde{p}))$, a sufficient condition for the first is $s(\tilde{p}^o) > v_1(\tilde{p}^o) + c - q$, which translates into

$$s(\tilde{p}^o) > \begin{cases} (1-q)/2 + (c-q)/(1-q) & \text{if } s(\tilde{p}^o) \geq (1-q^2)/(2q) \\ c - q + \sqrt{2q(c-q)} & \text{otherwise.} \end{cases} \tag{4}$$

Note that $s(\tilde{p}^o) \geq (1-q^2)/(2q)$ implies the first condition and the second condition implies $s(\tilde{p}^o) \geq (1-q^2)/(2q)$, if $c < (1+q^2)/(2q)$. Otherwise $s(\tilde{p}^o) > (1-q)/2+(c-q)/(1-q)$ implies $s(\tilde{p}^o) > (1-q^2)/(2q)$. Note that $s(\tilde{p}^o) < (1-q^2)/(2q)$ implies $s(\tilde{p}^o) < c - q + \sqrt{2q(c-q)}$ whenever $c > (1+q^2)/(2q) - (1-q) > 1$. $v_1(\tilde{p}^o) > \hat{v}(\tilde{p}^o)$ translates into

$$s(\tilde{p}^o) > \begin{cases} (1-q)/2 - \tilde{p}^o R(B)/q & \text{if } s(\tilde{p}^o) \geq (1-q^2)/(2q) \\ \frac{1-q}{q}\tilde{p}^o R(B) + \sqrt{2(1-q)(-\tilde{p}^o R(B))} & \text{otherwise.} \end{cases} \tag{5}$$

Note that the first condition implies $s(\tilde{p}^o) \geq (1-q^2)/(2q)$ as $\hat{v}(\tilde{p}) > 1-q$ implies $1-q < -2\tilde{p}R(B)$. Otherwise $s(\tilde{p}^o) \geq (1-q^2)/(2q)$ implies $s(\tilde{p}^o) > (1-q)/2 + \tilde{p}^o R(B)/q$. Summarizing pooling will occur in case

(i) $c < \frac{(1+q)^2}{2q} - (1-q)$ if $s(\tilde{p}^o) > (1-q)/2 - \tilde{p}^o R(B)/q$ for $1-q < -2\tilde{p}^o R(B)$, or $s(\tilde{p}^o) > \frac{1-q}{q}\tilde{p}^o R(B) + \sqrt{2(1-q)(-\tilde{p}^o R(B))}$ and $s(\tilde{p}^o) > c - q + \sqrt{2q(c-q)}$ for $1-q \geq -2\tilde{p}^o R(B)$,

(ii) $\frac{(1+q)^2}{2q} - (1-q) < c < \frac{(1+q)^2}{2q}$ if $s(\tilde{p}^o) > (1-q)/2 - \tilde{p}^o R(B)/q$ if $1-q < -2\tilde{p}^o R(B)$ and $s(\tilde{p}^o) > (1-q^2)/(2q)$ if $1-q \geq -2\tilde{p}^o R(B)$,

(iii) $c > \frac{(1+q)^2}{2q}$ and $s(\tilde{p}^o) > (1-q)/2 + (c-q)/(1-q)$ and, if $1 - q < -2\tilde{p}^o R(B)$, $s(\tilde{p}^o) > (1-q)/2 - \tilde{p}^o R(B)/q$.

All these conditions become slacker as $R(B)$ increases and c decreases. A single sufficient condition for pooling is for instance $s(\tilde{p}^o) > (1-q)/2 + -\tilde{p}^o R(B)/q$ and $c < (1+q)^2/(2q)$. Since $s(\tilde{p}^o)$ has a maximum in $R(A)$ at $(c - q/2)/\sqrt{\overline{p}\underline{p}}$ pooling contracts are favored by higher $R(A)$ if $\tilde{p}^o > \sqrt{\overline{p}\underline{p}}$.

$\hat{s}(\tilde{p})$ is first strictly decreasing then strictly increasing by Lemma 6. Since $s(\tilde{p})$ is first strictly increasing then strictly decreasing and has a unique maximum, existence of pooling contracts implies there are $\underline{p} < p_a < p_b < \overline{p}$ such that pooling contracts are optimal for $p_a < \tilde{p} < p_b$. If $q/2 > (2q-1)(c-q)$ $\hat{s}(\tilde{p})$ has a unique minimum by Lemma 6, which implies that $\hat{s}(\tilde{p})$ and $s(\tilde{p})$ intersect twice at most and therefore $p_a = p_1$ and $p_b = p_4$. Otherwise, there may be more intersection points. Optimality of separating contracts for $p_0 \leq \tilde{p} \leq p_1$ and $p_4 \leq \tilde{p} \leq \overline{p}$ implies then existence of $p_2 \leq p_3$ such that pooling contracts are preferred for $p_1 \leq \tilde{p} \leq p_2$ and $p_3 \leq \tilde{p} \leq p_4$.

For separating contracts $k_b = \min\{1-q; v_1(\tilde{p})\}$. Therefore $0 < k_b \leq< 1-q$ for $\tilde{p} > \tilde{p}_0$. For pooling contracts $k_b = 0$. Pooling is optimal only if $s(\tilde{p}) > \hat{s}(\tilde{p})$. This implies $v_1(\tilde{p}) + c - s(\tilde{p}) > q$ (see proof of Lemma 6). This means $q < k_a \leq 1$ in pooling contracts. For $s(\tilde{p}) < \hat{s}(\tilde{p})$ a separating contract is optimal, with $k_a = \min\{v_1(\tilde{p}) + c - s(\tilde{p})\}$. Since $v_1(\tilde{p}) + c - s(\tilde{p}) > q$ for $p_1 < \tilde{p} < p_2$ and $\frac{\partial v_1(\tilde{p})}{\partial \tilde{p}} > \frac{\partial s(\tilde{p})}{\partial \tilde{p}}$ for $q > 1/2$, $v_1(\tilde{p}) + c - s(\tilde{p}) > q$ and $k_a = q$ for $\tilde{p} > p_2$.

Finally, a sufficient condition for pooling contracts not to occur is $s(\tilde{p}) < c - q - \tilde{p}R(B)$. This is implied by $s(\tilde{p}^o) < c - q - \tilde{p}^o R(B)$.

References