Competitive Dynamic Pricing with Alternating Offers:
Theory and Experiment

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Abstract

We propose a model of duopolistic dynamic pricing in which a buyer alternates between two sellers for price offers over a finite season. The game ends when the buyer accepts a price offer or the season is over, whichever comes first. Previous research (Granot, Granot, and Mantin 2007) shows that there are successive markdowns in equilibrium when the buyer is myopic; our analysis reveals that a strategic buyer leads to a constant price offer over time. Moreover, lengthening the season increases (generally decreases) both sellers’ profits when the buyer is myopic (strategic). An experimental study largely supports the theoretical predictions when buyer is myopic but to a lesser extent when she is strategic.

Keywords: Dynamic pricing, competitive duopoly, strategic buyer, myopic buyer, experimental economics

JEL Classification: C72, C78, C92, D82
1. Introduction

In contrast with many economic models, it often happens in naturally occurring situations that the market is not instantaneously closed and all demand instantaneously realized upon the posting of prices by competing firms. Rather, potential buyers of a firm’s products may wait strategically for the best offer or may enter/exit the market at different times, while competitors’ prices may change over time along with a host of other contextual factors. How, then, should a firm adjust its prices temporally in order to maximize profit? The study of this problem, commonly known as dynamic pricing, has engendered a family of revenue management strategies that are widely used by firms and retailers in the airline industry, car industry, hotels, rental cars, and sales of perishable goods. Starting largely with Stokey (1979, 1981), an extensive academic literature on dynamic pricing has developed in economics and operations management. Models of dynamic pricing differ from one another on multiple dimensions that include, among others, the number of firms and the rules governing their interaction, the number of customers, the sources of demand uncertainty, and the customer’s degree of strategic sophistication. Research approaches include optimization and game-theoretical models, empirical examination of field data, and laboratory experiments. Surveys appear in Bitran and Caldentey (2003), Elmaghraby and Keskinocak (2003), Chan et al. (2004), Shen and Su (2007), and Talluri and van Ryzin (2004).

In this study, we depart from most previous research by incorporating duopolistic competition between the sellers in a dynamic pricing model. We contribute to the literature by examining the complex influence of buyers’ strategic sophistication and the length of selling season in such a setting; in addition, we test our theoretical claims experimentally. For our purpose, we first propose a stylized model of finite-horizon duopolistic selling in which a buyer
alternates between two competing sellers. We provide both theoretical and experimental results showing that buyers’ strategic sophistication and the length of season interact to produce non-intuitive effects on the pricing and profits of firms engaged in this form of competition.

Both buyer’s strategic sophistication and length of season are known to be crucial factors affecting firms engaged in dynamic pricing. To start with, it is well known that consumers often actively evaluate alternatives, compare prices, and delay purchase in anticipation of markdowns. The retail industry has been concerned with strategically sophisticated buyers in the consumer population, to the point of hoping to “weed out” these buyers using data analysis techniques (Kadet 2004, Arndorfer and Creamer 2005). However, some researchers (e.g. Talluri and van Ryzin 2004, Cachon and Swinney 2009) have argued that consumer behavior in reality should better be approximated as myopic. A comparison of model results for myopic vs. strategic buyer can therefore present important economic implications. On the other hand, the gradual lengthening of the holiday shopping season over the years has led to debate regarding whether a longer season will hurt or enhance profits (Knowledge@Wharton 2006). As we shall show, in the context of our model, the answer to this question depends significantly on the buyer’s strategic sophistication.

1.1. Literature Review

Much of the earlier revenue management research only studies myopic consumers (see Talluri and van Ryzin 2004) who make purchases whenever the current price is not higher than their valuations of the relevant product. Empirical studies have supported the notion that consumers do wait strategically for expected markdowns (e.g. Holak et al. 1987). More recent research on monopolistic dynamic pricing suggests that retailers could suffer significantly if they overlook strategic waiting (Aviv and Pazgal 2008; Besanko and Winston 1990). In contrast, the economic
literature on intertemporal price discrimination typically does assume strategic consumers, but otherwise the assumptions and focus of previous studies in this literature render them not directly comparable with our work.¹

Competing sellers is another major feature of our model. Most studies on dynamic pricing use a monopolistic setting; a number of exceptions address firm competition under various assumptions of demand uncertainty and inventory constraints. These include Dudey (1992), who studied competition between two firms that offer identical products so that a consumer always chooses the one with the lowest price – but unlike our model, each consumer “lives” for only one period in the market. Chintagunta and Rao (1996) proposed a differential game model for duopolistic competition in which brand choice probabilities are described by a logit function. Dana (1998) examined competitive pricing when there is demand uncertainty over time and advance-purchase discounts are possible. Perakis and Sood (2006) employed a robust optimization approach to study multi-period, fixed inventory dynamic pricing in which the sellers pre-commit to price paths at the beginning of the season. Lin and Sidbari (2009) examined dynamic price competition between firms facing consumers who arrive according to a Bernoulli process; delay in purchasing is not allowed. In Xu and Hopp (2006), retailers set inventory levels in the first stage and then compete on prices in the second. Arrival times of their homogenous consumers are either deterministic or stochastic. Lastly, in what is possibly the most comprehensive model among recent works, Levin, McGill, and Nediak (2009) present a

¹ For example, Stokey (1979) assumes that the firm can pre-commit to a price schedule over time. Stokey (1981) turns to subgame perfect rational expectations equilibrium, as in our case, but focuses on capacity constraints as a strategic variable. The models of Conlisk, Gerstner, and Sobel (1984) and Sobel (1984, 1991) assume new consumers arrive in every period in the context of durable goods pricing over an infinite time horizon. Of other major branches of intertemporal pricing research, the advance-purchase discount literature is concerned with demand uncertainty over time (e.g. Gale and Holmes 1993, Dana 1998, Courty 2003), while the clearance sales literature (e.g. Lazear 1986, Nocke and Peitz 2007) assume myopic consumers, pre-committed price schedule, inventory constraints, or other model settings that are not directly relevant to the present study.
multi-period, finite-horizon dynamic pricing model for oligopolistic firms that sell differentiated perishable goods to multiple finite segments of strategic consumers. We follow a similar approach in a more restricted model that considers only two sellers, a single buyer, fixed capacities, but look at sequential rather than simultaneous price offers.

Our study may alternatively be viewed as a development from Sobel and Takahashi (1983)’s multistage two-party bargaining model, in which the buyer, whose valuation is unknown to the seller, may only accept or reject price offers. The link between dynamic pricing and sequential bargaining with incomplete information has been noticed by previous writers (e.g. Ausubel and Deneckere 1989). However, our model involves a three-player game with two competing sellers, while the sequential bargaining literature focuses on the one seller-one buyer scenario (cf. the surveys by Kennan and Wilson 1993 and Ausubel, Cramton, and Deneckere 2002). The difference is substantive, since competition between sellers adds to the difficulty of each seller to use price offers to gauge the buyer’s valuation.

1.2. Overview

In the following sections, we first introduce our model, in which a buyer alternates between two sellers who do not know the buyer’s valuation. Each time the buyer visits a seller, the seller posts a price offer and the buyer either accepts or rejects the offer. The game ends when the buyer accepts an offer or the season is over, whichever comes first. Equilibrium pricing when the buyer is myopic was obtained by Granot, Granot, and Mantin (GGM, 2007); we analyze the more realistic case when the buyer is strategic, which increases the complexity of the model as it is now about gaming interaction between three rather than two persons. While GGM shows that there are successive markdowns in equilibrium when the buyer is myopic, our analysis reveals that a strategic buyer leads to a constant, low price offer over time (Proposition 1). While the
case of myopic buyer is motivated by the firms’ attempt to intertemporally price discriminate (albeit under duopolistic competition), the case of a strategic buyer is driven by completely different factors: since the buyer forms expectations about future price offers had she given up the present one, the present offer must be driven down to the same level as any of the rival’s expected offers in the future if the seller hopes to have any sale at all.

Comparison between the myopic and strategic buyer cases shows that, controlling for the length of the season, the Sellers earn less in the latter than in the former case (Proposition 2). Furthermore, lengthening the season increases both sellers’ profits when the buyer is myopic (Proposition 3), but generally decreases profits when she is strategic (Proposition 4). The reason is that, when the Buyer is strategic, a shorter selling horizon allows both Sellers to gain more bargaining power at the expense of the Buyer who has “less time left” to wait strategically for the best offer (in the limit when the season has only one period, the Seller has monopolistic power over the Buyer). When the Buyer is myopic, strategic waiting is not an issue, and selling is like skimming off one segment (of valuations) after another for the Sellers, so that a shortened selling horizon means fewer segments to be skimmed and thus lower profits. We discuss three model extensions to complete our theoretical section.

We next report an experiment in which subjects were assigned roles as buyers and sellers in a trading environment that operationalized our model. We find evidence that the effect of longer season on seller profits is as predicted: positive when the buyer is myopic, but negative when she is strategic. However, the observed price offers provided support to equilibrium predictions only when the buyer was myopic. When the buyer was strategic, sellers’ pricing deviated substantially from equilibrium predictions, and the sellers managed to earn higher profits than predicted. It is as if the sellers in our experiment were able to rein in their competition earlier in the season,
sometimes posting price offers that were almost certainly going to be rejected by the buyer, and only began to compete more intensely nearer the end of the season. Such behavior did not occur in our experiment when the buyer was myopic, suggesting that sellers are capable of reacting differently to different buyer types to their advantage.

2. The Model

Consider the following non-cooperative game involving three agents called Seller 1, Seller 2, and Buyer. Over the duration of the game, which consists of $T$ periods ($T \geq 2$), the Buyer demands at most one unit of an indivisible good. Each Seller has a single unit of the good, which is of zero value to both. The marginal cost of selling is zero for both Sellers. The Buyer’s valuation of the good, $v$, is known by the Buyer but not by the two Sellers. Rather, the Sellers are assumed to hold a prior belief of $v$, which is commonly known by all players and is assumed to be a uniform distribution over $[0, \bar{v}]$ ($\bar{v} > 0$). The number of periods in the selling season, $T$, is commonly known. In period 1, Seller 1 offers to sell the good at price $p_1$. The Buyer either accepts or rejects the price offer. If she rejects the offer, then the season proceeds to the next period. If she accepts the offer, then the season terminates.

Period 2 is structured in the same way with the exception that Seller 2 posts the price offer $p_2$. The game proceeds in this way until the buyer accepts the price offer $p_t$ that is posted in period $t$, or the season ends at period $T$ with no trade, whichever comes first. If it ends at period $t$, then: (a) if $t$ is odd, then the payoffs are $v-p_t$ to the Buyer, $p_t$ to Seller 1, 0 to Seller 2; (b) if $t$ is even, then the payoffs are $v-p_t$ to the Buyer, 0 to Seller 1, $p_t$ to Seller 2. If $t=T$, and the Buyer rejects the final price offer, then every player receives zero payoff.

Importantly, we make the “zigzagging” assumption in the model that the Buyer responds to price offers from different Sellers in any two consecutive visits, starting from Seller 1 in period
1. While this assumption makes for convenient exposition and experimentation, we discuss in Section 5.1 how zigzagging may arise endogenously, when a strategic Buyer not only can decide when to *buy* but also when to *visit* which Seller. Another simplifying feature is that there is only a single Buyer. Formally, as far as the Sellers’ pricing decisions are concerned, one Buyer with uncertain valuation to the Sellers is identical to a continuum of vertically differentiated consumers whose total population is normalized to one and who have the same distribution of valuation as the Buyer’s prior; the ex ante probability of making a transaction in the single-Buyer model then corresponds to realized demand in the “continuum” interpretation. However, with the “continuum” interpretation, our model assumes that the whole market zigzags in the same way. In Section 5.1, we present generalization of the model which relaxes this assumption and show that our major insights remain unchanged with this relaxation.

In the next section, we present a theoretical analysis of our model. While we always assume that both Sellers are strategic in our analysis, we distinguish between the cases when the Buyer is myopic vs. when she is strategic.

### 3. Theoretical Results

#### 3.1. Myopic Buyer

If the Buyer is myopic, then the equilibrium solution for the price offers is known to decrease in an approximately exponential fashion (GGM). In more detail, GGM’s equilibrium prices $p_1^*, p_2^*, \ldots, p_T^*$ (where $p_t^*$ is the equilibrium price posted at period $t$) satisfy:

$$p_t^* = B_t \cdot p_{t-1}^*,$$

where $p_0^* = \bar{v}$ and the ratios $\{B_t\}$ can be computed recursively as follow:

$$B_1 = \frac{1}{2} \text{, and } B_t = \frac{1}{2 - (B_{t+1})^2 B_{t+2}} \text{ for } t = 1, 2, 3 \ldots T-2.$$
GGM establish that \( \{B_t\} \) monotonically decreases as \( t \) increases, and that \( 0.55 > B_t \geq 0.5 \) for all \( t = 1, 2, 3, \ldots T \). Hence, the equilibrium prices decrease with \( (0.55)^t \bar{v} > p_t^* \geq (0.5)^t \bar{v} \).

3.2. Strategic Buyer

We next present equilibrium results for the case when the Buyer (as well as the two Sellers) is strategic (forward looking). Our first result applies to pure-strategy pricing equilibrium, in which the price offer in any period \( t \) in- or out-of-equilibrium is deterministically conditioned on \( t \) and the previous history of offers (all proofs are in Appendix A):

LEMMA 1. In a pure strategy pricing equilibrium with strategic Buyer, price offers are identical in all periods in which there is positive ex ante probability that trade occurs. Moreover, prices in such periods must not be higher than prices in periods in which there is zero ex ante probability that trade occurs in equilibrium.

It is apparent from the proof of Lemma 1 that, in fact, it applies to any prior for \( v \), not only the uniform distribution. The lemma indicates that, when the Buyer is strategic, price matching emerges endogenously as an equilibrium strategy at least over periods in which trade may occur. The result is due to a strategic Buyer effectively putting the two Sellers in competition with each other, even though they offer prices sequentially rather than simultaneously. This outcome is in stark contrast with the myopic Buyer of GGM in which there is an approximately exponential drop in prices over the season with positive ex ante demand in each period.

We now present our major theoretical result. We focus on pure strategy equilibria in which the Sellers and the Buyer play pure strategies in- or out-of-equilibrium: the Sellers employ pure strategies to decide their price offers as in a pure strategy pricing equilibrium, while the Buyer’s purchase decision in period \( t \) (if she has not purchased yet) is deterministically conditioned on \( t \), her valuation, and the current and previous price offers.
PROPOSITION 1. When the Buyer is strategic, there exists a pure strategy equilibrium for any $T$. In any such pure strategy equilibrium, the price offer is $p^* = \frac{\bar{v}}{2T}$ in every period.

Moreover:

1) The ex ante probability that trade occurs in period $t$ is:
   
   (a) $\frac{1}{T}$ if $1 \leq t < T$, and
   
   (b) $\frac{1}{2T}$ if $t = T$;

2) (a) Seller 1’s ex ante profit is $\frac{\bar{v}}{4T}$;
   
   (b) Seller 2’s ex ante profit is $\frac{\bar{v}(T-1)}{4T^2}$.

Proposition 1 implies that, across all pure strategy equilibria: (1) corroborating Lemma 1, equilibrium pricing is always the same and depends only on $\bar{v}$ and $T$; (2) trade may occur in any period; (3) there is always some form of endogenous separation among Buyers of different valuations, and (4) although Seller 1 has the advantage over Seller 2, the ratio of the ex ante profits $T/(T-1)$ approaches one as the duration of the season, $T$, increases. Conclusion (3) obtains because of two facts: first, for any period $t$ there is a positive probability that trade occurs in that period; second, the Buyer employs pure strategy. Combining both facts, Buyers of different valuations must have “sorted themselves” into buying in different periods in any pure strategy equilibrium. The implication that separation is feasible despite the price being constant over time has to do as much with what the players do in- as well as out-of-equilibrium. As detailed in the proof in Appendix A, a Buyer who would have bought in a certain period in equilibrium may turn out not to buy if the Seller who makes an offer in that period deviates from equilibrium and posts a higher price than $p^*$. What if the Seller posts a lower price than $p^*$? Then, he might get a higher ex ante demand than in equilibrium in that period, but there will be two negative consequences. First, his profit in that period may decrease if the increase in ex ante demand does
not compensate for the price decrease profit-wise. Second, the other Seller is expected to match his price in the next period, so that the impact on ex ante demand is lessened.

An important implication of Proposition 1 is that the pure strategy equilibrium of the game is unique as far as the price offers are concerned. Sellers’ price offers are independent of Buyer strategy in equilibrium as long as the Buyer plays pure strategy. In fact, it suffices that the Sellers commonly believe that the Buyer must be playing a pure strategy (without having to be sure which particular equilibrium it is), since any deviation from equilibrium play will not be detected by the Sellers as the season progresses. This presents a considerably general prediction that avails itself of experimental testing.

3.3. Effects of Buyer’s Strategic Sophistication

A comparison between GGM’s results and ours for pure strategy equilibria leads to the following:

PROPOSITION 2. Controlling for T, when the Buyer is myopic, compared with when the Buyer is strategic:

(1) Both Sellers earn higher individual ex ante profits;
(2) Equilibrium price in period 1 is higher;
(3) Equilibrium price in period 2 is higher when T > 2 and the same when T = 2.

Moreover, when T>2, there is a period $t_c$ with $T \geq t_c > 2$ such that the equilibrium price with myopic Buyer is lower than that with strategic Buyer in all periods $t \geq t_c$.

Proposition 2 states that the equilibrium price with myopic Buyer starts out higher than with strategic Buyer but in general dips below that in future periods. These higher initial prices lead to both Sellers earning more—and occasionally considerably more— from a myopic than a strategic Buyer. To demonstrate this point, consider the conditions in our experiment (see Section 4), in
which either $T=4$ or $T=8$. When $T=4$, the ex ante profits of Seller 1 and Seller 2 with a strategic Buyer are respectively 23.2% and 60.7% of what they earn with a myopic Buyer. When $T=8$, these two ratios reduce by a factor of approximately two to 11.5% and 34.1%.

### 3.4. Effects of Season Length

The effects of the length of season, $T$, on Sellers’ (expected) profits helps in answering whether Sellers may benefit from starting the season earlier. To begin with, we state the following results from GGM:

**PROPOSITION 3** (adapted from GGM Proposition 2.10 and Figure 2(a)). *When the Buyer is myopic:*

(a) Every Seller’s profit increases with $T$ controlling for whether the Seller is first visited by the Buyer;

(b) Given any $T \geq 2$ and $T' \geq 2$, Seller 1’s profit in a season with $T$ periods is higher than that of Seller 2 in another season with $T'$ periods.

The implication is that when the Buyer is myopic both Sellers earn higher expected profit when the season is longer if their role (as Seller 1 or Seller 2) remains the same after the season is lengthened. There is also a strong “first Seller advantage”, so that both Sellers always prefer to first be visited by the Buyer. To put it differently, any individual Seller has an incentive to start the season ahead of his competitor.

Next, consider the case when the Buyer is strategic. Define the following:

$$\pi_{iT} = \frac{\overline{v}}{4T}, \quad \pi_{2T} = \frac{\overline{v}(T-1)}{4T^2}.$$  

That is, $\pi_{iT}$ is the expected profit of Seller $i$ when the season has $T$ periods. It is then straightforward to verify that:
(1) \( \frac{\partial \pi_{1T}}{\partial T} < 0 \);

(2) \( \frac{\partial \pi_{2T}}{\partial T} < 0 \) when \( T > 2 \) and \( \frac{\partial \pi_{2T}}{\partial T} = 0 \) when \( T = 2 \);

(3) \( \pi_{1T} > \pi_{2T} \) when \( T \geq 2 \) and \( T' \leq T + 1 \) (this includes the case \( T' = T \) and highlights a “first Seller advantage”);

(4) \( \pi_{1T} < \pi_{2T} \) when \( T > 2 \) and \( T' > T + 1 \);

(5) \( \pi_{1T} = \pi_{2T} \) when \( T = 2 \) and \( T' = 4 \), while \( \pi_{1T} < \pi_{2T} \) when \( T = 2 \) and \( T' > 4 \).

These results imply that:

**PROPOSITION 4.** When \( T > 2 \) and the Buyer is strategic:

1. Every Seller earns less expected profit when \( T \) is increased by at least two periods regardless of who is first visited by the Buyer after the increase;

2. Seller 2 in a season with \( T \) periods earns lower expected profit than Seller 1 in a season with \( T + 1 \) periods.

Therefore, when \( T > 2 \) and the season is lengthened by at least two periods, both Sellers earn higher (lower) expected profits when the Buyer is myopic (strategic). The reason is that, when the Buyer is strategic, a shorter selling horizon allows both Sellers to gain more bargaining power at the expense of the Buyer as she has “less time left” to wait strategically for the best buy. When the Buyer is myopic, strategic waiting is not an issue, and selling is like skimming off one segment (of valuations) after another for the Sellers, so that a shortened selling horizon means fewer segments to be skimmed and thus lower profits. Indeed, the last-period price in equilibrium, according to GGM’s result, is decreasing in \( T \), so that a longer selling horizon allows for capturing Buyers with lower valuations. Thus, a longer season generally has an adverse effect on profits only when consumers are strategic but not when they are myopic.
Astute Sellers who compete in the market repeatedly may implicitly collude to shorten the selling season, most likely by initially offering relatively high prices that most buyers would reject, in an attempt to maximize profit.

While Proposition 4(1) highlights that profits generally decrease with the length of the season, Proposition 4(2) reveals an important exceptional case. That is, if Seller 2 were allowed to “jump the gun” and start the game ahead of Seller 1 by just one period, he would have an incentive to do so, as his new “first Seller advantage” could overcome the negative effect on profits caused by a slightly lengthened season. But note also that (as is already implied by Proposition 4(1)), when the Buyer is strategic, \( \pi_{1T} < \pi_{2T} \) if \( T > 2 \) and \( T' > T + 1 \). To sum up, Sellers always have an incentive to jump each other’s gun regardless of the Buyer’s strategic sophistication (similar phenomena have been observed in very different settings, cf. Roth and Xing 1994); moreover, iterations of Sellers mutually jumping each other’s gun will lead to a considerable lengthening of the season that is harmful (beneficial) to both of them if the Buyer is strategic (myopic).

4. Experiment

In this section, we describe a laboratory experiment designed to test our major theoretical predictions about the effects of the Buyer’s strategic sophistication and length of season on prices and profits. Our primary objective is to examine how the Buyer’s level of strategic sophistication and the length of the season lead to changes in behavior as predicted.

4.1. Method

4.1.1. Subjects. One hundred and thirty two subjects, in approximately equal proportions of men and women, participated in the experiment. The subjects were primarily undergraduate students of business administration who volunteered to take part in a computerized decision making experiment for payoff contingent on performance.
4.1.2. Design. The experiment used a 2×2 between-subject fixed-group design. One factor was the number of periods in the selling season (T=4 vs T=8) and the other the type of Buyer (myopic vs. strategic). We shall refer to the four experimental conditions as T4M, T8M, T4S, and T8S (M for “myopic” and S for “strategic”). There were 10, 11, 11, and 12 three-person groups in Conditions T4M, T4S, T8M, and T8S, respectively. The competitive pricing game that we described in Section 2 was iterated for 60 identical selling seasons. The only difference was in the assignment of subjects to player roles. The three roles of Seller 1, Seller 2, and Buyer were randomly determined on each season under the constraint that each subject was assigned each role 20 times. Group composition remained unchanged during the 60 seasons. Subjects were run together in cohorts of 12 to 24 members (four to eight 3-person groups).

The Buyer’s reservation value \( v \) in each season was randomly drawn with equal probability from the set \{1, 2, 3, ..., 1000\} at the beginning of that season. Thus, the prior of \( v \) can be approximated as a uniform distribution over \([0,1000]\) (i.e. \( \bar{v} = 1000 \)). As stated above, the distribution of \( v \) was commonly known to all three group members, whereas the actual valuation was private knowledge to the Buyer. Different random sequences of valuations were separately drawn for each Buyer.

4.1.3. Procedure. The experiment was conducted at a large computerized laboratory with networked PC terminals separated from one another by partitions. Any form of communication between the subjects was strictly forbidden. Once seated in their cubicles, the subjects proceeded to read the subject instructions (see Appendix B for Condition T4S) at their own pace.

Each period was structured as follows. In period \( t \), the Sellers were presented with a Decision Screen (see Appendix B) that included information about the season number, the cumulative score from period 1 to period \( t-1 \) (called “total score”), the subject role for the season, the value
of \( t \), and the value of \( T \). Seller 1 (if \( t \) is odd) or Seller 2 (if \( t \) is even) was then probed to post his price offer for period \( t \). When it was his turn to play, each Seller was informed of the posted price offer in the previous period.

The Buyer was then presented with the Buyer Screen that displayed the season number, total score, Buyer’s reservation value \( v_t \), the period number \( t \), the value of \( T \), the Seller’s posted price for period \( t \) (i.e. \( p_t \)), and the Buyer’s potential profit were she to purchase the good (i.e. \( v_t - p_t \)). In the two myopic conditions T4M and T8M, the Buyer could only accept the offer (by pressing “Yes”) if \( v_t - p_t \geq 0 \), and reject it (by pressing “No”), otherwise.\(^2\) In Conditions T4S and T8S, the Buyer was not subject to this restriction.

Once the selling season ended, each subject was presented with a summary screen that described the transaction, if there was one, and listed the payoff for the season. The Buyer’s valuation was also revealed to all players at that point. Once all the groups in the cohort completed the stage game, the experiment proceeded to the next season. During the experiment, each subject could access a History Screen that described the sequence of trades completed up to this point. At the end of the session, the cumulative payoff in tokens was converted to US dollars at the rate 600 tokens = $1.00. Subjects received their earnings plus a $5.00 show-up bonus and dismissed from the laboratory.

4.1.4. Predictions. Table 1 lists the equilibrium predictions in all four experimental conditions. To illustrate, consider Condition T4S. The equilibrium price for all four periods is \( p^* = 125 \) (bottom line), and the associated probability of no trade is therefore 0.125. In

\(^2\) The Buyer’s decision in Conditions T4M and T8M was completely determined by \( v \) and \( p_t \); she had no decision to make. Because our objective was to compare the myopic and strategic conditions to each other, we opted to use real Buyers instead of pre-programmed robots and thereby maintained the same experimental design and set-up in all four conditions.
equilibrium, the probability that a transaction occurs in period 1 is 0.25; this (ex ante) probability remains the same in periods 2 and 3, but in period 4 the probability that a transaction occurs decreases to 0.125. For the two myopic conditions, the two right-hand columns in Table 1 show that price offers decrease in an approximately exponential fashion starting with 538.3 in Condition T4M and 543.6 in Condition T8M. Regardless of the value of $T$, strategic Buyers have a considerably larger probability of ending the season with no trade than myopic Buyers (line 2 from bottom): compare 0.125 vs. 0.063 for Conditions T4S and T8S to 0.072 vs. 0.006 for Conditions T4M and T8M.

— Insert Table 1 about here —

4.2. Results

The presentation of the results is organized in two sections. Section 4.2.1 reports the results of the two conditions with myopic Buyers and Section 4.2.2 the two conditions with strategic Buyers. All the analyses use the equilibrium solutions as benchmarks.

4.2.1. Myopic Buyers. Figure 1 displays the mean observed price offers in different periods separately for seasons 1-30 (called Block 1) and seasons 31-60 (called Block 2). The upper panel displays the results for Condition T4M and the lower panel for Condition T8M. The mean price offer in each block is calculated by first taking the mean for each group across all the 30 seasons, and then averaging over all groups (10 and 11 in Conditions T4M and T8M, respectively).

— Insert Figure 1 about here —

The first main finding in both conditions is a steep decline in price offers across the $T$ periods. This finding supports the equilibrium analysis of GGM. The mean price offers over all seasons in Condition T4M are 456.0, 284.3, 159.2, and 78.4 in periods 1, 2, 3, and 4, respectively. With no exception, mean price offers decline over periods in each of the two
blocks. In Condition T8M, the overall mean group price offers similarly decline sharply from 523.8 in period 1 and 349.5 in period 2 to 20.4 in period 8. Apart from an outlier in period 7 in Block 1, the same monotonically decreasing trend is observed in each block.\(^3\) There is a strong selection bias in each condition, more so in Condition T8M than Condition T4M, because the number of price offers, and not only their values, also declines steeply across the \(T\) periods.

Another finding (compare upper and lower panels of Figure 1) is higher mean observed price offers in Condition T8M than Condition T4M i.e., the Sellers priced higher if they knew the selling season had more periods. This is consistent with the equilibrium predictions in Table 1. Because of the selection bias, a direct statistical comparison between the two conditions is only valid for period 1. As it turns out, the mean price offers in period 1 between Conditions T4M and T8M are not statistically significant (Wilcoxon rank-sum test \(U=92, p>.2\)). To sum up, there is directional evidence that the first period prices increase with \(T\) in accordance with the GGM solutions, although the evidence is not statistically significant. We also find that: (1) mean observed price offers are below equilibrium in period 1 in T4M but not in T8M according to Wilcoxon signed-rank test \((T=6, p=.03, \text{ and } T=29, p>.7, \text{ respectively})\); (2) in all other periods, mean observed prices apparently exceed the equilibrium prices, particularly in Condition T8M.

Note, however, that direct comparisons between observed and predicted price offers or between the observed price offers in different periods are invalid except for period 1 because of selection bias i.e., examination of a price offer data point in any period after period 1 has to take into account previous price offers in the same season. It is difficult to apply existing learning

\(^3\) Only one percent of all the 660 games in Condition T8M were predicted in equilibrium to reach period 8. In the first 10 seasons, when there was much experimentation, there were only six price offers in period 7, and one of them (presumably an error) was equal to 1000, which happened in Season 4. Exclusion of this outlier observation resulted in a mean price offer of 46.5 (instead of 182.7) in Block 1 for period 7, which is in line with the general decreasing trend across periods.
models for games (cf. Camerer 2003, Chapter 6) to analyze prices beyond period 1 because of
the complexity of our setting, which involves incomplete information with non-discrete prior,
multiple sequential moves, and continuous strategy space. Therefore, we proceed with an
analysis that compares the Seller’s price offer in any period after period 1 to his best response
price \textit{conditional} on the observed price history in the current selling season. The best response is
calculated by assuming that it is common knowledge that both Sellers follow strategies
conforming to the theoretical equilibrium construction – which are in-equilibrium (out-of-
equilibrium) strategies if previous prices all follow (do not all follow) equilibrium – from period
\( t \) onwards, regardless of any deviations from equilibrium in previous periods. While
acknowledging that this assumption may not be behaviorally valid, we point out that the
resulting best response price serves as a convenient benchmark for detailed analysis. Moreover,
if it occurs (as shall be seen) that the Sellers largely follow best responses from a certain period
onward in many seasons, it is indeed reasonable for a Seller to keep using best response pricing
whenever a season progresses beyond that period.

GGM’s solutions imply that a Seller’s best response in period \( t \), which we call the
\textit{equilibrium-to-go price} and denote it by \( p_{t}^{**} \), can be computed using the formula:
\[
p_{t}^{**} = B_{t} \cdot \min\{p_{0}, p_{1}, p_{2}, \ldots, p_{t-1}\},
\]
where \( p_{0}^{**} = \bar{v} \) and \( p_{1}, p_{2}, \ldots, p_{t-1} \) denote the observed price offers from period 1 through period
\( t - 1 \). The ratios \( \{B_{t}\} \) are computed recursively from:
\[
B_{t} = B_{t-1} = \frac{1}{2} \text{ and } B_{t} = \frac{1}{2 - (B_{t+1})^{2}(B_{t+2})} \text{ for } t = 1, 2, \ldots, T-2.
\]

\textit{Example.} Consider Condition T4M and assume that the Buyer’s valuation is \( v=200 \). Assume that
Seller 1 deviates from equilibrium play ( \( p_{1}^{*}=538.3 \) ) by posting the higher price offer \( p_{1}=700 \).
Then, the equilibrium-to-go price offer in period 2 is $p_2^{**}=373.33$ (which is subsequently rejected by the Buyer). To continue this example, assume that the observed price offers in periods 1, 2, and 3 are, respectively, $p_1=700, p_2=400$, and $p_3=300$. Then, the equilibrium-to-go price in period 4 is $p_4^{**}=150$ (which is subsequently accepted by the Buyer). In period 1, which is not preceded by any price offer, $p_1^{**}=p_1^*$. 

For every season played by every group, we computed separately the values of $p_t^{**}, t = 1, 2, \ldots, t'$, where $t'$ is the period in which a transaction took place in the season if there was a transaction at all, and otherwise $t'=T$. To measure the deviation from equilibrium-to-go prices, we calculated the deviation scores:

$$\Delta_t = p_t - p_t^{**}, t = 1, 2, \ldots, t'.$$

Then, for each period $s$ in each condition, we computed the mean deviation score over all groups by block (30 seasons each). Figure 2 displays the results. The upper panel exhibits the results for Condition T4M and the lower panel for Condition T8M.

— Insert Figure 2 about here —

Figure 2 (and later Fig. 4) is critical to our analysis. The two panels essentially display the same finding. There is some deviation from equilibrium best responses in period 1, especially in the earlier seasons; deviations in subsequent periods decline sharply with the period (except for one outlying data point in period 7 of Block 1 of T8M; cf. footnote 3) and decline across the blocks overall. As the Sellers progressed in each season, no matter what the price history was in earlier periods, the Sellers learned to respond to previous price offers in a very similar way. They also were able to best respond in the last period from the early seasons onwards. In Condition T4M, their price offers in period 4 – the final period in the season – converge to 4.2 tokens.
below the best response prices on average over all seasons, whereas in Condition T8M they converge to only 0.6 token below the best response price on average in the final period.

We turn next from the price offers to payoffs. Previous analysis already shows that, while the offers in period 1 tended to deviate from equilibrium (sometimes significantly), offers in subsequent periods were largely best responses (particularly in T8M). As a result, subjects assigned the role of Seller 1 earned, on average, less than predicted, and Buyers earned more than their equilibrium share. Table 2 presents the mean payoff for each group in each of the two conditions. Each mean was computed across all the 60 selling seasons. On average, in Condition T4M, subjects assigned the role of Seller 1 earned 13.6 percent less than predicted (compare 232.3 to the equilibrium payoff of 269.0). The deviation from equilibrium payoff is significant according to Wilcoxon signed-rank test (T=5, \( p = .02 \)). Seller 1 in Condition T8M earned, on average, 7.4 percent less than predicted, a deviation that is only marginally significant according to Wilcoxon signed-rank test (T=12, \( p = .07 \)). Mostly because of the under-pricing in period 1, Buyers earned more than predicted: 32.2 percent and 19.7 percent in Conditions T4M and T8M, respectively, which are significant according to Wilcoxon signed-rank test (T4M: T=3, \( p < 0.01 \), T8M: T=12, \( p = .07 \)). No significant differences between observed and equilibrium payoffs for Seller 2 were found in either Condition T4M or T8M according to Wilcoxon signed-rank test (T=16, \( p > .2 \), and T=32, \( p > .9 \), respectively).

The effect of dynamic pricing on profits is that Seller 1 mostly lost revenue because of under-pricing in period 1, and the Buyer benefitted accordingly. Approximately 50 percent of the transactions were concluded in period 1. Because price offers declined sharply and largely followed best responses after period 1, the percentage of seasons that ended with no trade did not
differ significantly from our predictions: 6.67 percent compared to the equilibrium percent of 7.18 in T4M (the equilibrium percent is equal to the equilibrium price in the final period divided by $\bar{v} = 1000$), and 1.21 percent compared to the equilibrium percent of 0.62 in Condition T8M.

To sum up, although there were directional deviations from equilibrium pricing in the myopic Buyer conditions, only the under-pricing in period 1 in Condition T4M is statistically significant. Moreover, price offers after period 1 were by and large best responses given previous history of offers. Because of this, there are significant differences in earnings from equilibrium only with Seller 1 and the Buyer in Condition T4M; otherwise, the payoffs largely conform to equilibrium predictions. This indicates that, overall, our experimental price offers support equilibrium predictions for the two myopic Buyer conditions.

4.2.2. Strategic Buyers. Figure 3 exhibits the observed mean price offers for Conditions T4S and T8S. The same format is used as in Figure 1 to display the means. The upper panel displays the results for Condition T4S and the lower panel for Condition T8S. The equilibrium price offers (horizontal lines in the bottom) are the same for each of the $T$ periods: 125 and 62.5 for Conditions T4S and T8S respectively.

All the major findings concerning price offers in Conditions T4S and T8S reject equilibrium play. Our first major finding, which is inconsistent with the price-matching predictions from our theoretical results, is a steady decline in the observed mean price offers across periods in both blocks. The mean price offers in Condition T4S across all the 60 selling seasons are 356.8, 233.8, 176.0, and 214.4 in periods 1, 2, 3, and 4, respectively; the slopes of decline from period 1 to period 3 are not as steep as with Condition T4M (see Figure 1). When the Buyer is strategic rather than myopic, mean prices during the selling season dropped more slowly, revealing a
“leveling effect” due to the Buyer being prone to strategic waiting – an effect that results in constant price across periods in equilibrium. Also in violation of equilibrium predictions is the finding that the overall mean prices increase by 21.8 percent (from 176.0 to 214.4) as the season proceeds from period 3 to period 4 in Condition T4S. Apparently, Seller 2 exercised his monopolistic power in the final period over the Buyer, who rejected all previous price offers because of either strategic delay or low valuation. It is as if Seller 2, who might have attributed the Buyer’s delays to strategic reasons, had finally the opportunity of “calling the Buyer’s bluff.”

We observe the same pattern of behavior in Condition T8S. Once again, for each block of 30 periods the mean price offers decline from period 1 to period 7. The overall means decline significantly from 472.6 in period 1 and 417.3 in period 2 to 155.7 in period 7. As in Condition T4S, subjects assigned the role of Seller 2 exercised their monopolistic power over the Buyer in the final period: mean price offers increase significantly from 155.7 in period 7 to 199.0 in period 8, a rise of 28 percent. Overall, 16.36 percent of the selling seasons in Condition T4S and 11.53 of the seasons in Condition T8S concluded with no trade compared to only 6.67 percent and 1.21 percent in Conditions T4M and T8M.

Our second major finding is that for strategic Buyers we find higher observed mean price offers in period 1 in Condition T8S than in Condition T4S. As in the myopic Buyer conditions, Sellers tended to start at higher prices when the selling season was longer – but while this is consistent with equilibrium predictions (Table 1) when the Buyer is myopic, it contradicts predictions when the Buyer is strategic. Note, however, that the observed difference is not

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4 Note that there were indeed considerable instances of strategic waiting among Buyer subjects. For example, among the experimental seasons in Conditions T4S and T8S in which the price in period 1 was lower than the Buyer’s valuation, the Buyer did not purchase in period 1 73.5 percent of the time. Among the experimental seasons in these two conditions in which the Buyer did not buy in period 1 while prices in periods 1 and 2 were both lower than the Buyer’s valuation, the Buyer did not purchase in period 2 69.5 percent of the time.
significant (Wilcoxon rank-sum test U=112, \( p > 0.2 \)), probably because of the very large between-group variations.

The observed pattern of pricing behavior raise a similar question to the one posed in Section 4.2.1, namely, whether Sellers learned with experience to best respond to the previous price offers in their selling season (see also the discussion in that section on the use of best response pricing as benchmark in our analysis). Although the question is the same as in the previous section, the computation of best responses is entirely different. As before, denote by \( p_1, p_2, \ldots, p_t \) the observed price path in a season, where \( t' \) is the period in which a transaction took place if there was a transaction at all, and otherwise \( t' = T \). Assume that all the prices before \( t' \) are positive (this assumption is valid with our data). If Buyer behavior follows a pure strategy equilibrium, then the equilibrium-to-go price \( p_t^{**} \) of the Seller whose turn it is to post an offer in period \( t \) (cf. Proposition A1 in Appendix A) is given by:

\[
p_t^{**} = \frac{1 - \sum_{s=0}^{t-1} D_s}{2(T-t+1)}, \text{ where}
\]

\[
D_0 = 0, \text{ and } D_s = \max \left\{ 0,1 - \sum_{r=1}^{t-1} D_r - \frac{2(T-s)p_s}{V} \right\} \text{ for any } 1 \leq s \leq T - 1.
\]

**Example.** Consider Condition T4S and assume that \( p_1 = 700 \) (rather than the equilibrium price of 125). Suppose that the Buyer rejects the first offer. Then, Seller 2’s best response is to set \( p_2^{**} = 166.7 \). If Seller 2 sets the price in period 2 at \( p_2 = 400 \) tokens (rather than 166.7 tokens), then Seller 1’s best response in period 3 is \( p_3^{**} = 250 \). This example illustrates the computation of the Seller’s best response. It also illustrates that the best response prices may not always decrease over time even when the observed prices decrease over time. Note that \( p_1^{**} = p_1^* = 125 \), the equilibrium price.
We followed similar procedures as with the myopic Buyer conditions in Section 4.2.1, and for each period in each condition we computed the mean deviation score over all groups by block. Figure 4 displays the results. Similar to Figure 2, the upper part of Figure 4 displays the mean value of the deviation score $\Delta_i$ for each of the two blocks in Condition T4S, and the bottom panel for each block in Condition T8S. Comparison of Figures 2 and 4 shows both similarities and dissimilarities. Overall, observed pricing in the myopic Buyer conditions followed equilibrium best response relatively closely, while that in the strategic Buyer conditions deviated much more significantly, especially in Condition T8S. The deviations in the strategic Buyer conditions remain large in magnitude even in later seasons in the session, despite signs of a slow decline across the blocks. The major similarity between the myopic and strategic Buyer conditions is a sharp decline in the mean deviation scores across the selling season. In both Conditions T4S and T8S, by periods $T-1$ and $T$ Sellers were behaving as if they were best responding to the previous offers in the selling season.5

We turn next to mean payoffs. Using the same format as Table 2, Table 3 presents the mean payoff for each player in Conditions T4S and T8S. Focusing first on Condition T4S, Table 3 shows that Seller 1 earned, on average, 6.89 percent less than predicted, but this difference is not significant according to Wilcoxon signed-rank test ($T=16, p=.15$). Meanwhile, Seller 2 earned 34.1 percent more than predicted, which is a significant deviation according to Wilcoxon signed-

--- Insert Figure 4 about here ---

5 It might be surmised that the trend is an artifact because the best response prices decreased across periods while the Sellers followed some heuristics that involved systematic markdowns; when both best response and observed prices became smaller and smaller, the fact that they were bounded below by zero means their differences could only become smaller. However, it turned out that best response prices very often increased across periods in the strategic Buyer conditions. For example, in Condition T4S, in 42 percent of the times when a transaction took place in periods 3 or 4 (so that it would be meaningful to calculate a best response price for period 4), the best response price increased from period 3 to 4. The corresponding percentage in Condition T8S for period 7 to 8 is 73 percent.
rank test (T=5, p<.01). Finally, the Buyer earned 12.90 percent more than predicted, a marginally significant difference according to Wilcoxon signed-rank test (T=13, p=.08). Overall, Seller 2 benefitted significantly from the Sellers’ deviation from equilibrium pricing while the Buyer did marginally better than equilibrium at the expense of Seller 1, who suffered slightly (in fact non-significantly) from his deviation from equilibrium. In Condition T8S, on average (Wilcoxon signed-rank test results in parentheses), Seller 1 earned 81.79 percent more than predicted (T=1, p<.001), Seller 2 earned 41.0 percent more than predicted (T=11, p<.05), while the Buyer earned 2.96 percent less than predicted (T=26, p>.3). In the longer selling season with eight periods, the Buyer earned, on average, four times as much as the combined earnings of the two Sellers – but she should have earned seven times more on average under equilibrium play. We thus conclude that deviations from equilibrium play led to higher profits for both Sellers at the expense of the Buyer in Condition T8S relative to equilibrium predictions; the same conclusions applied in a weaker sense to Condition T4S.

— Insert Table 3 about here —

While observed and predicted price offers in Conditions T4M and T8M largely followed equilibrium, price offers in Conditions T4S and T8S did not. This difference between the myopic and strategic Buyer conditions can hardly be explained by the basic design of the experiment, such as the reassignment of roles to subjects across seasons, since it is controlled across all conditions. We surmise that the deviations from equilibrium predictions in Conditions T4S and T8S are caused by the Sellers attempting to postpone price competition by tacit cooperation, so that prices are generally high in the early periods. This allowed the Sellers to earn higher-than-equilibrium profits by effectively decreasing $T$, despite the fact that an individual Seller could have priced lower in the early periods to attract the Buyer to make an early purchase. In
agreement with this explanation, it was not uncommon for a Seller to post a price that was higher than the one-period optimal monopolistic price of 500 (and frequently even as high as $\bar{v} = 1000$) in an early or even intermediate period, virtually informing the Buyer (and the other Seller) that the focal Seller was currently unwilling to trade.

The data also suggest that trade was often delayed to periods $T-1$ or $T$, when a Seller could exercise high bargaining power over the Buyer – an observation confirmed by Table 4, which shows that there is a shift in the distribution of time of transaction towards later periods (only) in the strategic Buyer conditions, compared with equilibrium predictions. Overall, Table 4 presents the transaction distributions by condition and period. The results for Conditions T4M and T8M are only presented for completion, as the time of transaction is solely determined by the Buyer’s valuation. In fact, there are no significant differences between the observed and predicted distributions of time of transaction exhibited in each of the top two panels of Table 4 (T4M: $\chi^2(3) = 3.15$; T8M: $\chi^2(7) = 1.39$; $p > .1$ in both cases). This is not the case when the Buyer is strategic. Table 4 shows that in period $T-1$ transactions occurred 31.5 percent in Condition T4S and 26.5 percent in Condition T8S. These figures exceed the equilibrium predictions of 25.0 and 12.5 percent. A chi-square test rejects the null hypothesis that the observed and predicted percentage of transactions stem from the same distribution both for Condition T4S ($\chi^2(3) = 10.92$, $p < .05$) and T8S ($\chi^2(7) = 40.19$, $p < 0.001$). The Sellers apparently succeeded to a considerable extent in “cornering” the Buyer into delaying her purchase. Realizing that Seller 2 would possess monopolistic power in period $T$ but at the same time “cornered” by high prices in early periods, many of the Buyers (with a sufficiently high valuation) delayed their purchasing decision until the last moment before the Seller could exercise his power. Note that there is no place for any of such behavior under equilibrium play. While only Seller 2 managed to earn
significantly higher profits through this exercise in Condition T4S, both Sellers succeeded in earning significantly higher profits than predicted in Condition T8S—apparently, when the season has a larger number of periods, the “cornering” tactics had a much more pronounced effect on profits.

— Insert Table 4 about here —

Because of the significant deviations from equilibrium pricing in the strategic Buyer conditions, it is not meaningful to verify statistically the statements about prices in Proposition 2, which compares equilibria under myopic and strategic Buyers at the same $T$. It suffices to observe that Figures 1 and 3 do suggest that the data satisfy all those statements. In the next subsection, we focus on analyzing Sellers’ profits across conditions.

4.2.3. Comparison of Sellers’ Profits across Conditions. A key objective of our experiment is to verify if the Buyer’s strategic sophistication and the length of the season influence Sellers’ profits in the lab in accordance with Propositions 2 to 4. Even if the observed price offers deviated from equilibrium in the strategic Buyer conditions, the forces that drive the differences in equilibrium profits across conditions may still lead to similar differences in observed profits, for which we seek relevant evidence. To start with, we point out that the overall mean Seller payoffs in Tables 2 and 3 support the prediction that both Sellers earned significantly more when the Buyer was myopic than when she was strategic (Proposition 2(1)). To verify this statistically, we combined the overall payoffs of all the subjects who were assigned the role of Seller 1 in Conditions T4M and T8M, and the overall payoffs of the subjects who were assigned the role of Seller 1 in Conditions T4S and T8S. We then compared the two aggregate variables using Wilcoxon rank-sum test, which reveals a significant difference ($U=714, p<0.001$). We repeated the same comparison for the overall payoffs of Seller 2 in the myopic and strategic conditions.
and obtained a similar result (U=580, \( p<0.02 \)). Further analysis shows that these comparisons remain statistically significant in the predicted directions (with \( p<0.001 \)) at every level of \( T \) for both Sellers, except for Seller 2 at \( T = 4 \) (when the difference is non-significant, \( p > 0.9 \)). We conclude that, overall, in agreement with the theoretical results, both Sellers in our experiment benefitted if the Buyer was deprived of the option to strategically delay her purchase decision.

Tables 2 and 3 also support the predicted effect of increasing \( T \) on Sellers’ payoffs (Propositions 3 and 4), which should be positive (negative) when the Buyer is myopic (strategic) in our experimental parameters. To verify these observations statistically, we first average the mean payoffs of Sellers 1 and 2 for each group to form a new average profit variable (note that any statistical test must use group as the unit of analysis). We then apply ANOVA on a 2(\( T=4 \) vs. \( T=8 \)) × 2(Buyer type: myopic vs. strategic) design to test if there is an interaction effect. Indeed, there is a significant interaction (\( F(1,40)=7.04, p=0.011 \)), as well as a main effect in Buyer type (\( F(1,40)=331.11, p<0.0001 \)) that corroborates with the earlier Wilcoxon rank-sum test. Also, as expected, the main effect of \( T \) is non-significant (\( p>0.7 \)). The simple effect of \( T \) on the mean profit is significant when the Buyer is myopic (\( F(1,19)=2.91, p=0.10 \)) and marginally significant when she is strategic (\( F(1,21)=5.28, p<0.05 \)). Repeating the two simple effect comparisons using the Wilcoxon rank-sum test shows similar results (U=85, \( p<0.01 \) when the Buyer is myopic; U=166, \( p<0.05 \) when the Buyer is strategic). But Wilcoxon rank-sum test comparing each Seller’s payoffs at different values of \( T \) (controlling for Buyer type) produces significant effect only for Seller 2 when the Buyer is strategic (U=184, \( p<0.01 \)); otherwise, the differences are non-significant at the \( p>0.2 \) level. Despite the last results, the directional observations from Tables 2 and 3 and the tests on the average profit variable support our predictions. Thus, our
experiment provides evidence that lengthening the season has a positive effect on Sellers’ profits when the Buyer is myopic, but a negative effect when she is strategic.

5. Concluding Remarks

This paper proposes a model of competitive dynamic pricing characterized by a Buyer who alternates between two Sellers over a finite and commonly known horizon. We contribute to the literature on dynamic pricing by examining non-intuitive effects of Buyers’ strategic sophistication and length of season on prices and profits; additionally, we test our theoretical predictions experimentally. Our experimental results largely support game-theoretic predictions – but with the significant exception of pricing strategies in the strategic Buyer conditions, which in fact led to the Sellers earning more profits than theoretically predicted.

It needs be emphasized that competition is a key driver of our results. If the model has only a single Seller to which a strategic Buyer visits repeatedly over a maximum of $T$ periods, then the equilibrium price would be $\bar{v}/2$, the one-period monopolistic selling price, regardless of $T$. Thus, without competition, the negative effect of a longer season on profits in Proposition 4(1) will not obtain, since in that case a strategic Buyer cannot leverage a longer competitive horizon to push down prices and Seller profits.

5.1. Model Extensions and Future Directions

In this section, we briefly discuss three extensions of the standard setting of our model that generalize our theoretical results, and suggest other future directions.

5.1.1. Multiple Buyers. Consider the case when there are $N > 1$ Buyers all of whom zigzag between Sellers 1 and 2 over the duration of the season (i.e., $T$ periods) as in the single-Buyer model, with $N_1$ starting with Seller 1 and $N_2$ starting with Seller 2 in period 1 ($N_1+N_2=N$). To the Sellers, every Buyer’s valuation is an i.i.d. random variable distributed uniformly on $[0,\bar{v}]$. 

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In Section 4.1 of Appendix A we provide an argument showing that our theoretical results for strategic Buyer remain unchanged with this extension (see also GGM Section 4.2). Stated differently, when all the Buyers are myopic (strategic), pure-strategy equilibrium pricing and results concerning the incentives to postpone sales and competition are the same as when there is a single myopic (strategic) Buyer.

5.1.2. Endogenizing the Buyer’s Route. Next, consider the case when the Buyer is strategic and is free to schedule her visits. To be specific, assume that the season consists of $T$ periods or “weekends” and thus allows for at most $T$ shopping trips; the Buyer can choose whether to go on a shopping trip in any particular period, and if she decides to go on a shopping trip, she can choose which Seller to visit. In Section 4.2 of Appendix A, we argue that the Buyer zigzagging between the Sellers all through the $T$ periods is a feasible equilibrium outcome, so that our results concerning equilibrium and postponement of sales and competition remain applicable. The fact that zigzagging may arise endogenously has an intuitive appeal, as strategic consumers are expected to be constantly on the watch for “good deals” through visiting different retailers in frequent shopping trips.

5.1.3. Endogenizing the Start of the Season. Consider a modified game in which each Seller can put his good up for sale with an accompanying price offer at any period over $T$ periods. That is, the $T$ periods only (in principle) define the temporal boundaries over which trade might occur and the Sellers might start the season. For simplicity, assume that putting up a good for sale is costless and the Buyer is a habitual shopper who keeps zigzagging between the Sellers from period 1 – starting with a random choice between the Sellers – regardless of whether any Sellers have put up goods for sale. It then follows that the Sellers would start the season from period 1 onwards posting the equilibrium prices of previously discussed solutions with $T$ periods.
regardless of the Buyer’s strategic sophistication. This is because not putting up a good for sale is equivalent to pricing the good at $\bar{v}$ or above; therefore, any equilibrium in the modified game in which a Seller puts up a good for sale at period $t > 1$ should also be reflected as an equilibrium in previously discussed settings with $T$ periods in which the Seller posts price offers that are $\bar{v}$ or above before $t$. As we have seen from GGM’s results and Proposition 1, no pure strategy equilibrium supports such an outcome whether the Buyer is myopic or strategic. In fact, if, for example, the equilibrium in the modified game is such that the first time Seller $i$ puts up a good for sale is in period $t > 1$, followed by his rival Seller $j$ in the next period, then Seller $j$ always has an incentive to jump the gun (cf. Section 3.4) and put up a good for sale in period $t – 1$, thus destroying the equilibrium. This conclusion is still valid upon other variations of this modification, such as when a Seller may withhold a good from sale after putting it on sale earlier – since not putting up a good for sale or withholding it from sale are equivalent to pricing at $\bar{v}$ or above. As a result, the season will become maximally long.

These extensions are also intended to serve as directions for future experimental research. Another major future direction is to incorporate intermediate time discounting into the players’ preferences; we have effectively considered in this study the benchmark cases $\delta \to 0^+$ (myopic Buyer) and $\delta \to 1^-$ (strategic Buyer), where $\delta$ is the per-period time discount factor of the Buyer, while assuming no time discounting for the Sellers. Other possibilities for theoretical and experimental development include introducing demand uncertainty in the number of buyers, capacity constraints, and Sellers’ uncertainty over whether the Buyer is myopic or strategic.
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Table 1. Equilibrium Predictions by Condition when $v \sim U[0,1000]$

<table>
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<th>Period</th>
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<th>Condition T8S</th>
<th>Condition T4M</th>
<th>Condition T8M</th>
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</table>

NA: Not applicable
Table 2. Mean Payoff by Group and Condition: Myopic Buyers

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<th>Buyer</th>
<th>Seller 1</th>
<th>Seller 2</th>
<th>Buyer</th>
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<tbody>
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Table 3. Mean Payoff by Group and Condition: Strategic Buyers

<table>
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<tr>
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<th>Condition T4S</th>
<th>Seller 1</th>
<th>Seller 2</th>
<th>Buyer</th>
<th>Condition T8S</th>
<th>Seller 1</th>
<th>Seller 2</th>
<th>Buyer</th>
</tr>
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<tbody>
<tr>
<td></td>
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<td>Buyer</td>
<td></td>
<td>Seller 1</td>
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Table 4. Observed (absolute counts in parenthesis) and Predicted Percent of Transactions by Period and Condition

**Condition T4M**

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**Condition T8M**

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<tbody>
<tr>
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**Condition T4S**

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**Condition T8S**

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<th>8</th>
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<td>(35)</td>
<td>(64)</td>
<td>(191)</td>
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<td><strong>12.50</strong></td>
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<td><strong>6.25</strong></td>
<td><strong>93.75</strong></td>
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Figure 1. Mean Price Offers in the Myopic Buyer Conditions
Figure 2. Mean Deviations of Observed Price from Equilibrium-to-go Price in the Myopic Buyer Conditions
Figure 3. Mean Price Offers in the Strategic Buyer Conditions
Figure 4. Mean Deviations of Observed Price from Equilibrium-to-go Price in the Strategic Buyer Conditions
APPENDICES

APPENDIX A

1. Proof of Lemma 1
Consider the first statement. Suppose that the statement is not true in a certain pure strategy pricing equilibrium. Then, under that equilibrium, there must exist $t$ and $t+r$ ($r > 0$) such that: (1) the ex ante probability that trade occurs is positive for both periods $t$ and $t+r$, while $p_t \neq p_{t+r}$; (2) either $r = 1$ or there is zero probability that a trade occurs in all periods between $t$ and $t+r$; (3) over all periods (if any) after $t + r$ in which there is a positive ex ante probability that a trade occurs, the price offers are equal to $p_{t+r}$.

Assume for the moment that $p_t < p_{t+r}$. Then, if the Buyer has not yet made a purchase up to period $t-1$, she certainly will buy in period $t$ as long as $v > p_t$, since she knows that if she rejects in period $t$, the prices in all future periods in which she might buy at all (i.e., periods in which there is positive ex ante probability that a trade occurs in equilibrium) will be higher. This means that a Buyer who rejects all offers up to and including period $t$ must have $v < p_t$, so that price offers in any period from $t+1$ onwards cannot be higher than $p_t$ if there is a positive ex ante probability that a trade occurs in that period. This contradicts the assumption that $p_t < p_{t+r}$.

Next, we proceed to consider the possibility that $p_t > p_{t+r}$. Then, a Buyer who has not yet made a purchase up to period $t-1$ will definitely not buy in period $t$ either, since she knows that if she rejects the offer of $p_t$, then she can at least buy at a lower price in period $t+r$. This contradicts the assumption that there is positive ex ante probability that a trade occurs in period $t$, and hence completes the proof of the lemma.

The second statement of Lemma 1 can be proved straightforwardly since a Buyer only buys in periods in which the price is lowest in equilibrium. If the second statement is not true, then a Buyer only buys in period(s) that are supposed to have no trade in equilibrium, which is a contradiction.

It is obvious from the proof just described that Lemma 1 applies to any prior of $v$, not only to the uniform distribution.

2. Proof of Proposition 1
2.1. Preliminaries
Any equilibrium must include complete specifications of strategies in- or out-of-equilibrium. Moreover, it needs to fulfill rational expectations requirements i.e., the players form mutually
consistent beliefs (expectations) of what each other will do in all future periods – which must be
best responses to all the beliefs – conditioned on the information they hold at every stage of the
game. As stated in the text, we focus on equilibria in which all players employ pure strategies at
any point in- or out-of-equilibrium. We first state two lemmas that are useful for the proof of
Proposition 1.

LEMMA A1. In any pure strategy pricing equilibrium, in any period t in- or out-of-
equilibrium, if the posted offer is \( p_t \):

1. If there is positive probability that trade occurs in period t, price offers must also be \( p_t \) in all
   subsequent periods in which there is positive ex ante probability that trade occurs. Moreover, prices in periods in which there is zero ex ante probability that trade occurs must not be lower than \( p_t \);

2. Regardless of the probability that trade occurs in period t, price offers must not be higher
   than \( p_t \) in all subsequent periods in which there is positive ex ante probability that trade
   occurs.

Proof. The logic is very similar to that employed in the proof of Lemma 1 and is omitted here.

LEMMA A2. In any pure strategy equilibrium, in any period t in- or out-of-equilibrium
(\( T > t \geq 1 \)) where all previous price offers (if any) have been positive, if the posterior of \( v \) at the
beginning of that period is \( G(\cdot) \), define:

\[
p^m(G) = \min \arg \max_p p[1 - G(p)].
\]

Then:

1. \( G(\cdot) \) is a uniform distribution function over \([0, p^m(G) + \varepsilon]\) for some \( \varepsilon > 0 \);

2. The probability of trade occurring in period t is zero if the price offered in that period is
   higher than or equal to \( p^m(G) \).

Proof. \( p^m(G) \) is the minimum one-period monopolistic price when \( v \) is distributed according to
\( G(\cdot) \). Given the premise of the lemma, it must be that \( p^m(G) > 0 \), \( 1 - G(p^m(G)) > 0 \), and \( G(\cdot) \)
must have positive mass at valuations that are sufficiently low. We first prove (1). First of all, \( G(\cdot) \) must have mass immediately above \( p^m(G) \); otherwise \( p^m(G) \) is dominated by some higher
price in one-period monopolistic selling with \( v \sim G(\cdot) \). This means that \( G(\cdot) \) is a uniform
distribution function over \([p^m(G), p^m(G) + \varepsilon]\) for some \( \varepsilon > 0 \). If (1) is not true, then there exist
Consider the out-of-equilibrium scenario in which all price offers from \( t \) to \( T-1 \) are \( v_2 \). If there is a positive probability that trade occurs in any of these periods, then, according to Lemma A1, the optimal price in period \( T \) given the posterior that results at the beginning of \( T \) must be \( v_2 \). But this cannot be true, because: (a) if the posterior at \( T \) has positive mass above \( v_1 \), \( v_2 \) must be dominated by pricing at \( v_1 \); (b) if not, \( v_2 \) must be dominated by some lower price. Therefore, there is zero probability that trade occurs from \( t \) to \( T-1 \) when the offers in those periods are all \( v_2 \). The posterior at \( T \) is then \( G(\cdot) \) and the optimal price that will be offered in that period is at least \( p''(G) > v_2 \), so that a Buyer with \( v > v_2 \) could have done better by purchasing in period \( T-1 \) at price \( v_2 \) in anticipation of what would happen in period \( T \). This implies that there must be a positive probability that trade occurs in period \( T-1 \), which leads to a contradiction. Therefore (1) must be true. We now proceed to prove (2). Denote the price offered at \( t \) as \( p_t \). By definition:

\[
p''(G)[1-G(p''(G))] \geq p_t[1-G(p_t)] \text{ for any } p_t.
\]

If (2) is not true, there exists \( p_t \geq p''(G) \) such that there is a positive probability that trade occurs in period \( t \). Consider first the case \( p_t > p''(G) \). By Lemma A1, if both Sellers follow best responses after \( t \), the price offer in any period after \( t \) with positive probability that trade occurs must be \( p_t \). That should include period \( T \) too; note that there must be a positive probability that the Buyer will not buy before period \( T \), since all prices would have been positive up to then by Lemma A1, and a Buyer with sufficiently low valuation must not have bought at all those prices. Moreover, if any Buyer buys in a period from \( t \) to \( T-1 \), her valuation cannot be lower than \( p_t \), so that the posterior of \( v \) at \( T \) can only be different from \( G(\cdot) \) at values of \( v \) that are at least \( p_t \). Lastly, \( p_t \) must be the one-period monopolistic price with respect to the posterior of \( v \) at \( T \). Therefore we can write:

\[
p_t[1-G(p_t)-d] \geq p''(G)[1-G(p''(G))-d],
\]

for some \( d > 0 \) that measures the “depletion” of the posterior of \( v \) at \( T \) from \( G(\cdot) \) confined to values of \( v \) that are at least \( p_t \). But this implies:

\[
p_t[1-G(p_t)] \geq p''(G)[1-G(p''(G))] + (p_t - p''(G))d > p''(G)[1-G(p''(G))],
\]
which contradicts $p^m(G)[1 - G(p^m(G))] \geq p_t[1 - G(p_t)]$. Hence (2) must be true for $p_t > p^m(G)$. For $p_t = p^m(G)$, again suppose (2) is not true, and re-define $d > 0$ as the “depletion” of ex ante probability of trade when $p_t = p^m(G)$ relative to $G(\cdot)$ itself. Then by Lemma A2(1), which we have just proved, it must be that:

$$p^m(G) \in \arg \max_{p \in [0, p^m(G)]} p[1 - \Delta - d - p / \vec{v}]$$

where

$$\Delta = 1 - \left[ \frac{p^m(G)}{\vec{v}}, \frac{1}{G(p^m(G))} \right]$$

measures the “depletion” of $G(\cdot)$ from the prior (i.e. $U[0, \vec{v}]$), which, by Lemma A2(1), can only come from valuations that are above $p^m(G)$. The solution of the optimization problem is:

$$\min \left\{ p^m(G), \frac{1 - \Delta - d}{2} \cdot \vec{v} \right\}.$$ 

Meanwhile, we know that:

$$p^m(G) \in \arg \max_{p \in [0, p^m(G) + \epsilon]} p[1 - \Delta - (p / \vec{v})],$$

and so

$$p^m(G) = (1 - \Delta)(\vec{v} / 2),$$

so that

$$p^m(G) > \frac{1 - \Delta - d}{2} \cdot \vec{v}.$$ 

That is, the optimal price in period $T$ cannot be $p^m(G)$, which leads to a contradiction. Thus, the probability of trade in period $t$ must be zero when $p_t = p^m(G)$.

**Lemma A3.** If a zero price offer is posted in a period, trade occurs with probability one in that period.

*Proof.* This must be true in period $T$. Suppose this is not true for some period $t < T$. This means there exists an equilibrium specification with which $p_t = 0$ and yet the posterior of $v$ in $t+1$, say $G(\cdot)$, has positive mass over some values of $v$ that together have positive measure over $[0, \vec{v}]$. This means that it must be possible to offer a positive price in some period within $t+1$ to $T$ to get a positive expected demand (and hence positive expected profit), because: (1) this is obviously true when $t+1 = T$; (2) when $t+1 < T$, if this is not true in periods $t+1$, $t+2$ ... $T-1$, then the posterior of $v$ remains $G(\cdot)$ in period $T$ and we are effectively back to case (1). That is, the best response prices in periods after $t$ must include positive prices. Therefore, by Lemma A1, all best
response prices in periods after $t$ with positive probability of trade must be positive and identical. If trade occurs with positive probability in $t$ at $p_t = 0$, this contradicts Lemma A1. If trade occurs with zero probability in $t$, the Buyer, knowing that she will only buy at a positive price (if at all) afterwards, can do better buying in $t$, which leads to a contradiction.

Lemma A3 means that, once a zero price is posted, both Sellers earn zero expected profits from that point onwards, because the posted price is zero while deviation by the Buyer (i.e. the Buyer does not purchase) is a zero probability event.

Note that, in most cases, if a Buyer deviates from her equilibrium behavior, the Sellers cannot detect it, since the Buyer’s valuation is unknown to the Sellers. Therefore, at any point in- or out-of-equilibrium, the Sellers always update their (commonly known) posterior of $v$ by means of what they believe – as part of their rational expectations – to be the Buyer’s responses to the history of price offers. The only exceptions to this conclusion are when there is probability zero or one that the Buyer should buy in a period (in response to the current price and the history of offers). In the case of the Buyer purchasing with probability zero according to equilibrium specifications, a deviation means that the Buyer decides to purchase instead; but since the game ends with the deviation, the Sellers have no more decisions to make and we can effectively ignore the scenario. In the case of the Buyer purchasing with probability one according to equilibrium specifications, the Buyer decides not to purchase instead. Purchasing with probability one can happen only if the current price and/or one of the previous prices is zero; otherwise, there is a positive probability that $v$ is less than the minimum price offer so far, and a Buyer with such $v$ will not buy. While out-of-equilibrium specifications in such scenarios need be tackled in principle e.g. using the notions of sequential equilibrium (Kreps and Wilson 1982), the observation through Lemma A3 that an offer of zero price leads to zero expected profits for both Sellers is sufficient for the following proof of Proposition 1.

2.2. The Case $T=2$
In this subsection, we consider the case $T=2$ as an illustration of the basic idea behind the central part of our proof. Note that only Lemma A1, but not Lemmas A2 or A3, will be assumed as given in the following discussion.

Given a price in period 1, say $p_1$, denote as $D_1(v; p_1)$ the probability that a transaction occurs with a Buyer whose valuation is at least $v$. In the “continuum” interpretation discussed at the end of Appendix A Section 2, $D_1(v; p_1)$ is the part of the demand given $p_1$ that comes from consumers with valuations that are at least $v$. The probability that a transaction occurs (i.e., the total demand in the “continuum” interpretation) in period 1 is $D_1(p_1; p_1)$, since a Buyer whose valuation is less than $p_1$ will definitely not buy.

Denote as $p_2**(p_1)$ the best response of Seller 2 in period 2 given price $p_1$ in period 1. In period 2, which is the last period, the Buyer (if she has not yet purchased) behaves like a myopic Buyer and the Seller in that period prices like a monopolist.

Consider first the possible case of $p_1 > 0$ and $1 > D_1(p_1; p_1) > 0$. The latter means that there is positive expected demand in both periods and we must have $p_1 = p_2**(p_1)$ by Lemma A1, or:

$$p_1 \in \arg \max_{p_2} p_2 \left[ \frac{\bar{v} - p_2}{\bar{v}} - D_1(p_2; p_1) \right].$$

Note that $D_1(v; p_1)$ must be a constant within a sufficiently small neighborhood of $v = p_1$. The only case when this is not true is that a Buyer whose valuation is at or immediately higher than $p_1$ buys in period 1. But this would mean that the posterior of $v$ in period 2 has a “gap” just above $p_1$, which makes $p_1$ a dominated strategy for Seller 2, so that $p_2**(p_1)$ cannot be $p_1$. Therefore, a Buyer whose valuation is at or immediately higher than $p_1$ will certainly not buy in period 1, which means that $D_1(v; p_1)$ is unchanged when $v$ increases through $p_1$ over a sufficiently small interval. Therefore,

$$p_1 \in \arg \max_{p_2} p_2 \left[ \frac{\bar{v} - p_2}{\bar{v}} - D_1(p_2; p_1) \right] \text{ implies }$$

$$\bar{v} - 2p_1 D_1(p_1; p_1)\bar{v} = p_1 D_1'(p_1; p_1)\bar{v} = 0,$$

where

$$D_1'(p_1; p_1) = \frac{\partial D_1(p_2; p_1)}{\partial p_2} \bigg|_{p_2=p_1} = 0,$$
and hence we have:

$$D_1(p_1; p_i) = \frac{v - 2p_1}{v}.$$

It needs be re-emphasized that the argument above is applicable only when \(1 > D_1(p_1; p_i) > 0\).

We now map out \(D_1(p_1; p_i)\) over the whole range of \(p_i \in [0, v]\) to see when the argument is indeed applicable, and to derive the equilibrium price.

When \(p^m = \frac{v}{2} > p_i > 0\), where \(p^m\) is the one-period monopoly price with respect to the prior, we must have \(1 > D_1(p_1; p_i) > 0\) and thus \(D_1(p_1; p_i) = (\frac{v - 2p_1}{v})\). The reasoning is as follows: (1) \(D_1(p_1; p_i) < 1\) because a Buyer with \(v \in [v, p_i]\) will definitely not buy from Seller 1; (2) but if \(D_1(p_1; p_i) = 0\), so that the Buyer waits regardless of her valuation, Seller 2 will price at \(p^m\) in period 2 and the Buyer can do better by buying in period 1, leading to a contradiction.

When \(p_1 = 0\), we must have \(D_1(p_1; p_i) = 1\), because otherwise Seller 2’s price in period 2 must be positive, and any Buyer who does not buy in period 1 can do better by buying in period 1 (this is in fact a special case of Lemma A3).

When \(p_1 \geq p^m\), we must have \(D_1(p_1; p_i) < 1\) since the Buyer will certainly not buy if \(v < p^m\). But if \(1 > D_1(p_1; p_i) > 0\), we would have:

$$D_1(p_1; p_i) = \frac{v - 2p_1}{v} \leq 0,$$

which is a contradiction. Hence \(D_1(p_1; p_i) = 0\) when \(p_1 \geq p^m\) (a special case of Lemma A2). To sum up:

(1) When \(p_1 = 0\), \(D_1(p_1; p_i) = 1\);
(2) When \(p^* = \frac{v}{2} > p_1 > 0\), \(1 > D_1(p_1; p_i) = (\frac{v - 2p_1}{v}) > 0\);
(3) When \(p_1 \geq p^*\), \(D_1(p_1; p_i) = 0\).

It can be seen that the function \(D_1(p_1; p_i)\) is continuous over \(p_i \in [0, v]\), and Seller 1’s profit is positive only when \(p_i \in (0, \frac{v}{2})\). Therefore, Seller 1’s optimal \(p_1\) maximizes:

$$p_1D_1(p_1; p_i) = p_1 \cdot \frac{v - 2p_1}{v},$$
over \([0, \overline{v}/2]\), which gives the equilibrium price \(p^* = \overline{v}/4 = \overline{v}/2T\). The ex ante probability that trade occurs is \((\overline{v} - 2p^*)/\overline{v} = 1/2\) in period 1 and \([(\overline{v} - p^*)/\overline{v}] - [(\overline{v} - 2p^*)/\overline{v}] = 1/4\); the ex ante profit is \(p^*/2 = \overline{v}/8\) for Seller 1 and \(p^*/4 = \overline{v}/16\) for Seller 2. All of these results, which hold in any pure strategy equilibrium, corroborate with Proposition 1.

To prove that a pure strategy equilibrium exists, it suffices to construct one with the following Buyer behavior: a Buyer purchases in period 1 if and only if her valuation lies in the interval \([v_i, \overline{v}]\) where \(v_i = \overline{v}[1 - D_1(p_i; p_i)]\). It is obvious that this is consistent with all our previous results, including the Sellers’ best responses and equilibrium pricing; note that the Buyer behaves as if in a price skimming scenario (even though there is no pricing skimming in the actual equilibrium). Thus we have completed the proof of Proposition 1 for \(T=2\).

2.3. The Complete Proof

We now generalize the results in the last subsection to \(T > 2\). We shall use the notation \(P_t\) to denote a \(t\)-tuple of price offers up to period \(t\) in which the \(n\)th entry, denoted as \(p_{n}^t\), is the price in period \(n\). Define \(P_0\) to be the empty set for notational convenience. We shall abuse notation and use \((P_{t-1}, p_{t})\) to denote a \(P_t\) in which the history of offers before period \(t\) are described by \(P_{t-1}\) while the price in period \(t\) is \(p_{t}\). In any well-defined pure strategy equilibrium, given \(P_t\) – even if it is an out-of-equilibrium price path – all the demand (i.e. ex ante probability of trade occurring) up to period \(t\) and all the (best response) prices and demand in any period after \(t\) can be precisely calculated with respect to players’ in- or out-of-equilibrium strategies. Therefore, it is legitimate to denote as \(D_{t}(v; P_{t})\) the ex ante probability that a transaction occurs in any period \(t'\) with a Buyer whose valuation is at least \(v\) given \(P_{t}\). In the “continuum” interpretation, \(D_{t}(v; P_{t})\) is the part of the demand in period \(t'\), given \(P_{t}\), that comes from consumers with valuations that are at least \(v\), and \(D_{t}(p_{t}; P_{t})\) is the total demand in period \(t'\) (because in any period there cannot be any demand from valuations that are lower than the currently posted price). Note that \(D_{t}(v; P_{t})\) depends only on the first \(\max\{t', t\}\) entries of \(P_{t}\) (i.e. prices up to period \(t'\) or \(t\), whichever is earlier). Define \(D_{0}(v; P_{t}) = 0\) for any \(P_{t}\) for notational convenience.

The major portion of this subsection will be devoted to proving the following results:
PROPOSITION A1. In any pure strategy equilibrium, for any \((P_{t-1}, p_t)\) that consist of only positive prices:

\(1\) \(D_t(p_t; (P_{t-1}, p_t)) = \max \left\{ 0, 1 - \sum_{s=0}^{t-1} D_s(p_s; P_{t-1}) - \frac{2p_t}{\bar{v}} (T - t) \right\} \) for any \(1 \leq t \leq T - 1;\)

\(2\) \(p_t^{**}(P_{t-1})\), the best response price in period \(t\) given \(P_{t-1}\), is:

\[
p_t^{**}(P_{t-1}) = \frac{1 - \sum_{s=0}^{t-1} D_s(p_s; P_{t-1})}{2(T - t + 1)} \cdot \bar{v}.
\]

It is straightforward to check that Proposition A1 directly implies most of the claims in Proposition 1, since: (1) Proposition 1 is about a special case of Proposition A1 when all prices are along the equilibrium path; (2) although Proposition A1 is valid only when prices are restricted to be positive, the expected profit for any Seller is positive in the equilibrium thus derived, while, by Lemma A3, the expected profits of both Sellers once a zero price is posted must be zero. Therefore, zero prices are inferior strategies and need not be considered. The only claims in Proposition 1 that is not directly implied by Proposition A1 are the existence of a pure strategy equilibrium and Proposition 1(1b). However, the latter can be deduced straightforwardly from the other claims of Proposition 1. We shall prove existence of pure strategy equilibrium at the end of this subsection.

2.3.1. Proof of Proposition A1. By Lemma A2, at the beginning of any period \(t\) with \(T > t \geq 1\), regardless of whether previous prices are in- or out-of-equilibrium, if the posterior of \(v\) is \(G(\cdot)\), the Seller effectively only needs to consider prices in \([0, p^m(G)]\), while \(G(\cdot)\) is a uniform distribution function over \([0, p^m(G) + \varepsilon]\) for some \(\varepsilon > 0\). Moreover, we have shown in the proof of Lemma A2 that:

\[
p^m(G) = \frac{1 - \Delta}{2} \cdot \bar{v}, \text{ where }
\]

\[
\Delta = 1 - \left[ \frac{p^m(G)}{\bar{v}} \cdot \frac{1}{G(p^m(G))} \right]
\]

measures the “depletion” of \(G(\cdot)\) from the prior (i.e. \(U[0, \bar{v}]\)). By Lemma A2(1),

\[
\Delta = \sum_{s=0}^{t-1} D_s(p_s; P_{t-1}),
\]

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so:

\[ p^m(G) = \frac{1 - \sum_{i=0}^{r-1} D_i(p_s; P_{T-1})}{2} \cdot \bar{v}. \]

Now look at the decision problem of the Seller who posts a price in period \( T \). If:

\[ p_{T-1} \geq \frac{1 - \sum_{i=0}^{r-2} D_i(p_s; P_{T-2})}{2} \cdot \bar{v}, \]

\[ D_{T-1}(P_{T-1}; (P_{T-2}, P_{T-1})) = 0 \]

because of Lemma A2, so that the best response price in \( T \) must be:

\[ p_T^{**}(P_{T-1}) = \frac{1 - \sum_{i=0}^{r-1} D_i(p_s; P_{T-1})}{2} \cdot \bar{v}, \]

which is consistent with Proposition A1.

If:

\[ p_{T-1} < \frac{1 - \sum_{i=0}^{r-2} D_i(p_s; P_{T-2})}{2} \cdot \bar{v}, \]

we must have \( D_{T-1}(P_{T-1}; (P_{T-2}, P_{T-1})) > 0 \), because otherwise the price in period \( T \) will be:

\[ \frac{1 - \sum_{i=0}^{r-2} D_i(p_s; P_{T-2})}{2} \cdot \bar{v}, \]

which is higher than \( P_{T-1} \) and the Buyer can do better by buying in \( T-1 \). By Lemma A1, this means that \( p_T^{**}(P_{T-1}) = p_{T-1} \), or:

\[ p_{T-1} \in \arg \max_{p_T} p_T D_T(p_T; (P_{T-1}, p_T)), \]

or

\[ p_{T-1} \in \arg \max_{p_T < p_{T-1} + \varepsilon} p_T \left[ 1 - \sum_{i=0}^{r-1} D_i(p_s; P_{T-1}) - \frac{p_T}{\bar{v}} \right] \text{ for some } \varepsilon > 0, \]

because of another application of Lemma A2. This means that:

\[ 1 - \sum_{i=0}^{r-1} D_i(p_s; P_{T-1}) - D_{T-1}(P_{T-1}; (P_{T-2}, P_{T-1})) = \frac{2p_{T-1}}{\bar{v}}, \]

\[ D_{T-1}(P_{T-1}; (P_{T-2}, P_{T-1})) = 1 - \sum_{i=0}^{r-2} D_i(p_s; P_{T-2}) - \frac{2p_{T-1}}{\bar{v}}. \]
We can re-write the above as:

\[ p_T **(P_{T-1}) = p_{T-1} = \frac{1 - \sum_{s=0}^{T-1} D_s(p_s; P_{T-1})}{2} \cdot \bar{v}. \]

We next consider the decision problem of the Seller who posts a price in period \( T-1 \). We know that:

\[ D_{T-1}(p_{T-1}; (P_{T-2}, p_{T-1})) = 1 - \sum_{s=0}^{T-2} D_s(p_s; P_{T-2}) - \frac{2p_{T-1}}{\bar{v}} \text{ if } p_{T-1} < \frac{1 - \sum_{s=0}^{T-2} D_s(p_s; P_{T-2})}{2}, \]

\[ D_{T-1}(p_{T-1}; (P_{T-2}, p_{T-1})) = 0 \text{ otherwise}; \]

so:

\[ p_{T-1} **(P_{T-2}) \in \arg \max_{p_{T-1} < p^n} p_{T-1} \left[ 1 - \sum_{s=0}^{T-2} D_s(p_s; P_{T-2}) - \frac{2p_{T-1}}{\bar{v}} \right], \]

where:

\[ p^n = \frac{1 - \sum_{s=0}^{T-2} D_s(p_s; P_{T-2})}{2} \cdot \bar{v}. \]

That is:

\[ p_{T-1} **(P_{T-2}) = \frac{1 - \sum_{s=0}^{T-2} D_s(p_s; P_{T-2})}{4} \cdot \bar{v}. \]

The above results thus prove Proposition A1 for \( t = T-1 \) and Proposition A1(2) for \( t = T \).

Now, suppose Proposition A1 is true for periods \( t+1 \ldots T \), where \( 1 \leq t < T \). If we can prove that this implies Proposition A1 is also true for period \( t \), we have completed the proof of the proposition by induction. By an application of Lemma A2, we can write the decision problem of the Seller who posts a price in period \( t \) as:

\[ p_t **(P_{t-1}) \in \arg \max_{p_t < p^n} p_t \left\{ 1 - \frac{p_t}{\bar{v}} - \left[ \sum_{s=0}^{t-1} D_s(p_s; P_{t-1}) \right] - D_{t+1}(p_t; (P_{t-1}, p_t)) - D_{t+3}(p_t; (P_{t-1}, p_t)) - \ldots \right\}, \]

where \( D_{t+1}(p_t; (P_{t-1}, p_t)), D_{t+3}(p_t; (P_{t-1}, p_t)) \) etc. are ex ante probabilities of trade assuming all prices are best response from period \( t+1 \) onwards, and:
By applying Proposition A1 to periods $t+1$, $t+3$ ... under best response pricing from period $t$ onwards, we know that:

$$D_t(p_t;(P_{t-1}, p_r)) = \max \left\{ 0, \frac{2p_r **(P_{t-1}, p_r)}{\bar{v}} \right\} = \frac{2p_r **(P_{t-1}, p_r)}{\bar{v}}.$$

for $r = t+1, \ldots T-1$, where $p_r **(P_{t-1}, p_r)$ is the best response price in period $r$ given that the prices up to $t$ are described by $(P_{t-1}, p_t)$ and all prices afterwards are best responses. Meanwhile, by Proposition A1:

$$D_T(p_t;(P_{t-1}, p_r)) = 1 - \sum_{s=0}^{T-1} D_s(p_s;(P_{s-1}, p_r)) = \frac{p_T **(P_{t-1}, p_r)}{\bar{v}} = \frac{p_T **(P_{t-1}, p_r)}{\bar{v}}.$$

Next we show that:

$$D_t(p_t;(P_{t-1}, p_r)) > 0 \text{ if } p_t < \frac{1 - \sum_{s=0}^{t-1} D_s(p_s;P_{s-1})}{2(T-t) \bar{v}},$$

$$D_t(p_t;(P_{t-1}, p_r)) = 0 \text{ otherwise.}$$

The first line is true because:

$$p_{t+1} **(P_{t-1}, \bar{v}) = \frac{1 - \sum_{s=0}^{t-1} D_s(p_s;P_{s-1})}{2(T-t) \bar{v}} \bar{v}$$

is the best response price in period $t+1$ if $D_t(p_t;(P_{t-1}, p_r)) = 0$ (which is ensured if $p_t = \bar{v}$ and hence the notation). Therefore, if $p_t$ is less than $p_{t+1} **(P_{t-1}, \bar{v})$ and $D_t(p_t;(P_{t-1}, p_r)) = 0$, prices from the next period onwards will be higher than $p_t$, which means the Buyer should have bought in $t$, leading to a contradiction. Now suppose the second line is not true; by Lemma A2, this means that there exists $p^m > p_t \geq p_{t+1} **(P_{t-1}, \bar{v})$ such that $D_t(p_t;(P_{t-1}, p_r)) > 0$. All subsequent best response prices must also be $p_t \geq p_{t+1} **(P_{t-1}, \bar{v})$ and the total loss of probability mass (i.e. total ex ante probability of a trade occurring) from $t$ to $T-1$ is
\[ D_t(p; (P_{t-r}, p_t)) + 2(T-t-1)p_t > 2(T-t-1)p_{r+1} \]
\[ = \sum_{s=t}^{T-1} D_s(p_{t+1}**(P_{t-1}, \bar{v}); (P_{t-1}, \bar{v})) \geq \sum_{s=t}^{T-1} D_s(p_t; (P_{t-1}, \bar{v})). \]

Define:
\[ d' = D_t(p; (P_{t-1}, p_t)) + 2(T-t-1)p_t - \sum_{s=t}^{T-1} D_s(p_t; (P_{t-1}, \bar{v})) > 0, \]

which measures the “depletion” of ex ante probability of trade for valuations above \( p_t \) and from \( t \) to \( T-1 \) when the price in period \( t \) is \( p_t \) compared with when it is \( \bar{v} \). Meanwhile, \( p_t \) must be the one-period monopolistic price in period \( T \) with respect to the posterior at the beginning of that period. Using the same argument as that in the proof of Lemma A2(2), with \( d' \) taking the role of \( d \) in that proof, we can then show that this leads to a contradiction, and so it must be that \( D_t(p; (P_{t-1}, p_t)) = 0 \) in this case.

If \( D_t(p; (P_{t-1}, p_t)) > 0 \), we must have \( p_t**(P_{t-1}, p_t) = p_t \) for \( r = t+1, t+3 \ldots \) and substitution gives:
\[ p_t**(P_{t-1}) \in \arg \max_{p_t < p_t^m} p_t \left[ 1 - \frac{p_t}{\bar{v}} - \sum_{s=0}^{T-t-1} D_s(p_t; P_{t-1}) - \frac{(T-t)p_t}{\bar{v}} \right] \text{ if } T-t \text{ is even, and} \]
\[ p_t**(P_{t-1}) \in \arg \max_{p_t < p_t^m} p_t \left[ 1 - \frac{p_t}{\bar{v}} - \sum_{s=0}^{T-t-1} D_s(p_t; P_{t-1}) - \frac{(T-t-1)p_t}{\bar{v}} - \frac{p_t}{\bar{v}} \right] \text{ if } T-t \text{ is odd.} \]

In both cases, the objective function is:
\[ p_t \left[ 1 - \sum_{s=0}^{T-t-1} D_s(p_t; P_{t-1}) - \frac{(T-t+1)p_t}{\bar{v}} \right]. \]

Both lead to the best response expression:
\[ p_t**(P_{t-1}) = \frac{1}{2(T-t+1)} \cdot \bar{v} < p_{t+1}**(P_{t-1}, \bar{v}) \leq p_t^m. \]

We also know that, if \( D_t(p; (P_{t-1}, p_t)) = 0 \), the Seller’s profit will be the above objective function with \( p_t = p_{t+1}**(P_{t-1}, \bar{v}) \), which we already know cannot yield a higher profit than the \( p_t**(P_{t-1}) \) that we have obtained. Hence Proposition A1(2) is true for period \( t \) too.

From the objective function and the fact the Proposition A1 is true for periods \( t+2, t+4 \ldots \), we can derive that:
where:

$$D_t(p_t; (P_{t-1}, p_t)) = 1 - \sum_{s=0}^{t-1} D_s(p_s; P_{t-1}) - \frac{(T-t+1)p_t}{\bar{v}} - \frac{(T-t-1)p_t}{\bar{v}} = 1 - \sum_{s=0}^{t-1} D_s(p_s; P_{t-1}) - \frac{2(T-t)p_t}{\bar{v}},$$

which becomes non-positive when $p_t \geq p_{r+1}^{**}(P_{r+1}, \bar{v})$, for which we have shown that $D_t(p_t; (P_{t-1}, p_t)) = 0$. This implies that Proposition A1(1) is true for period $t$. Thus we have completed the proof of Proposition A1.

### 2.3.2. Existence of pure strategy equilibrium

So far we have shown that, if there exists a pure strategy equilibrium, then its pricing and other properties must follow Proposition A1 and thus Proposition 1. To complete the proof of Proposition 1, we need to show that there does exist a pure strategy equilibrium by construction. One possibility is to specify Buyer behavior to be as if in a price skimming scenario (even though there is no actual price skimming in the equilibrium). That is, we introduce the following feature: in period $t$, a Buyer purchases if and only if her valuation lies in the interval $[v_0, v_{t-1}]$, where:

$$v_t = \bar{v} \left[ 1 - \sum_{s=1}^{t} D_s(p_s; P_s) \right] \text{ for } t = 1, 2, ..., T, \text{ and define } v_0 = \bar{v}.$$

This feature implies that the posterior of $v$ in any period $t > 1$ is always a uniform distribution over $[0, v_{t-1}]$, whatever the previous price path. It is obvious that this construction is consistent with Lemmas A1, A2, A3, and Proposition A1, and hence enables us to complete the proof of Proposition 1.

### 3. Proof of Proposition 2

#### 3.1. Proposition 2(1)

With myopic Buyer, Seller 1’s ex ante profit, say $\pi_1^m$, is at least $p_1^* \times [1 - (p_1^*/\bar{v})]$. Since $0.55 > p_1^*/\bar{v} \geq 0.5$, we have $\pi_1^m \geq \bar{v} \cdot 0.5 \cdot (1 - 0.55) = 0.225\bar{v}$. Meanwhile, Seller 1’s ex ante profit with strategic Buyer, say $\pi_1^s$, is $\bar{v} / (4T)$. Since $0.225 > 1/(4T)$ whenever $T \geq 2$, we know that Seller 1’s ex ante profits is less with strategic Buyers.

Next, when $T=2$ it is easy to work out that Seller 2’s ex ante profit is the same whether the Buyer is myopic or strategic. When $T = 3$, Seller 2’s ex ante profit with myopic Buyer is:

$$p_2^* \times (p_1^* - p_2^*) / \bar{v} = (4/15) \times [(8/15) - (4/15)] \cdot \bar{v} = 0.0711\bar{v}.$$

Meanwhile, Seller 2’s ex ante profit with strategic Buyer is $\bar{v} (T - 1) / (4T^2)$, which is $0.0556\bar{v}$ when $T = 3$, and which is lower than that with myopic Buyer. For $T > 3$, observe that Seller 2’s
ex ante profit with myopic Buyer, $\pi_m^2$, is at least $p_t^* (p_t^* - p_2^*) / \bar{\nu}$. Since
$(0.55)^T > p_t^* / \bar{\nu} \geq (0.5)^T$, we have $\pi_m^2 \geq \bar{\nu} \cdot (0.5)^2 \cdot [0.5 - (0.55)^2] = 0.049375 \bar{\nu}$. Meanwhile, Seller 2’s ex ante profit with strategic Buyer, $\pi_2' = \bar{\nu}(T - 1)/(4T^2)$, decreases with $T$ when $T = 4, 5, \ldots$. At $T = 4$, $\pi_2^s = 3\bar{\nu}/64 < 0.049375\bar{\nu}$. Hence, we conclude that Seller 2’s ex ante profit is less when the Buyer is strategic.

3.2. Propositions 2(2) and 2(3)
With myopic Buyer, $0.55 > p_t^* / \bar{\nu} \geq 0.5$ and $(0.55)^2 > p_2^* / \bar{\nu} \geq (0.5)^2$. Meanwhile, with strategic Buyer, the equilibrium price is $\bar{\nu}/2T \leq (0.5)^2 \bar{\nu} < 0.5\bar{\nu}$. In fact, it is only when $T=2$ that $\bar{\nu}/2T = (0.5)^2 \bar{\nu}$, and it is straightforward to see that when $T=2$, the equilibrium prices in period 2 are indeed the same. Hence we have Propositions 2(2) and 2(3).

3.3. The Last Statement of Proposition 2
Given Proposition 2(2) and the monotonically decreasing property of the equilibrium prices with myopic Buyer, it suffices to show that the last period price with myopic Buyer, $p_T^*$, is less than $\bar{\nu}/(2T)$. Since $(0.55)^T > p_T^* / \bar{\nu} \geq (0.5)^T$, this is achieved if we show that $1/(2T) > (0.55)^T$ when $T > 2$ i.e. $-\ln 2 - \ln T - T \ln 0.55 > 0$ when $T > 2$. Now,
$$
\frac{\partial}{\partial T} (-\ln 2 - \ln T - T \ln 0.55) = -\frac{1}{T} - \ln 0.55 > 0 \text{ when } T > 2,
$$

hence $-\ln 2 - \ln T - T \ln 0.55$ is increasing in $T$. It is straightforward to show that $-\ln 2 - \ln 3 - 3\ln 0.55 > 0$. This completes the proof. Note that when $T=2$, the equilibrium prices in period 2 are the same whether the Buyer is strategic or myopic, and are equal to $\bar{\nu}/4$, while the equilibrium price in period 1 with myopic Buyer is $\bar{\nu}/2$.

4. Model Extensions

4.1. Multiple Buyers
This is the case (Section 5.1.1 in the main text) when there are $N > 1$ Buyers all of whom zigzag between Sellers 1 and 2 over the duration of the season (i.e. $T$ periods), with $N_1$ starting with Seller 1 and $N_2$ starting with Seller 2. To the Sellers, every Buyer’s valuation is an i.i.d. random variable distributed uniformly over $[0, \bar{\nu}]$. GGM proves that equilibrium results remain the same as with the single-Buyer case when the Buyer is myopic (Proposition 2.13). In the following, we provide an argument for a similar result that is applicable to when all the Buyers are strategic.
First, Seller $i$’s price in period $t$ should have no influence on his own prices in period $t+1$, $t+3$ ..., and Seller $j$’s ($j \neq i$) prices in period $t$, $t+2$ ..., since the zigzagging routes are pre-determined and the Buyers visiting these Sellers in these periods must be different from those who visit Seller $i$ in period $t$. That is, we can consider equilibrium along the two zigzagging routes in the game (one starting with Seller 1 and the other with Seller 2) independently, as any Seller can maximize his combined profits from the two routes by employing independent pricing strategies for each route.

Next, because every Buyer’s valuation is i.i.d., Sellers cannot infer the Buyer’s valuation based on another Buyer’s behavior. That is, the number of Buyers making purchases in a period should not be an input on the Sellers’ pricing strategy. This, in turn, means that any Buyer’s purchase decision in any period (which may be partly based on her expectation of future prices) should not be influenced by how many other Buyers have bought before that period. Therefore, the posterior distribution of $v$ for any Buyer who visits Seller $i$ in period $t$ is: (a) identical to other Buyers who visit Seller $i$ in period $t$; (b) conditioned only on previous prices along her zigzagging route. Now, suppose $N_{i,t}$ Buyers visit Seller $i$ in period $t$. By definition, $N_{i,1} = N_i$, and we must have $N_{i,t} \geq N_{2-i,t+1}$ ($t < T$) in any realized play of the game. Then, Seller $i$ needs to post price $p_t$ in period $t$ so as to maximize $N_{i,t} \pi_{i,t}(p_t; p_1, p_2 ... p_{t-1})$, where $\pi_{i,t}(p_t; p_1, p_2 ... p_{t-1})$ is the expected profit from period $t$ onwards that can be earned by $i$ from each one of the $N_{i,t}$ Buyers, given the previous price offers $p_1, p_2 ... p_{t-1}$ along the relevant zigzag route and assuming that all pricing decisions of both Sellers follow equilibrium strategies from $t+1$ onwards, given that $i$ posts $p_t$ in period $t$. Since $\pi_{i,t}(p_t; p_1, p_2 ... p_{t-1})$ is identical among all $N_{i,t}$ Buyers, $N_{i,t}$ only appears as an irrelevant scaling factor. The optimal $p_t$ should be the same whether $N_{i,t} = 1$ or larger. This reasoning applies in all periods along all histories of play. Therefore, analysis of equilibrium should be independent of the number of Buyers.

4.2. Endogenizing the Buyer’s Route

This is the case (Section 5.1.2 in the main text) in which the Buyer is strategic and is free to schedule her visits at Sellers. In detail, assume that the season consists of $T$ periods or “weekends” and thus allows for at most $T$ shopping trips; the Buyer can choose whether to go on a shopping trip in any particular period, and if she decides to go on a shopping trip, she can
choose which Seller to visit. The Buyer’s valuation $v$ is unknown to the Sellers, who hold the common prior that it is uniformly distributed over $[0, \overline{v}]$.

The introduction of route choice as a decision variable potentially leads to a great variety of equilibria. This is because, in principle, a Buyer cannot commit to her future route choices, and the Sellers can form any expectations on the Buyer’s future route choices given any history of play. The only qualification regarding equilibrium route choices is that the Buyer does not deviate from her expected (i.e. as expected by the Sellers) equilibrium route, because her expected out-of-equilibrium route after a deviation will drive the Sellers to offer prices that are not lower than in equilibrium.

We shall show that it is a feasible equilibrium outcome to have “complete zigzagging” i.e., if the Buyer visits Seller $i$ in period $t$ ($t < T$) but does not make a purchase, then she visits Seller $3-i$ in period $t+1$; moreover, we shall show that complete zigzagging as an equilibrium outcome satisfies a number of appealing properties.

Specifically, we look for equilibria in which, in- or out-of-equilibrium:

(1) All players employ pure strategies in all decisions;

(2) There is pooling in route choice among Buyers with different valuations;

(3) The Buyer’s future route choices are independent of current and previous price offers;

(4) At any point in time in- or out-of-equilibrium, the Sellers’ expectation of the Buyer’s future route choices leads to the Sellers offering the lowest equilibrium price among the equilibrium prices of all possible expected future route choices satisfying (1) to (3).

Condition (2) means that, given that a Buyer has not yet purchased at the beginning of period $t$, her route choice in period $t$ is independent of her valuation, so that there is no signaling of $v$ by route choice (note that even if there are equilibria with signaling, there could not be any difference in pricing for Buyers with different valuations in those equilibria, because if the contrary is true, Buyers of all valuations would imitate those who get the best offer and destroy the equilibrium). Condition (3) is reasonable because the Buyer cannot commit to her future route choices after all. Condition (4) means that the Sellers expect the Buyer to behave in a way that will bring the Buyer the most benefits, and is consistent with (in fact stronger than) the notion of a rational expectations equilibrium in which the Buyer acts in accordance with the Sellers’ expectations.
Given the above, it is straightforward to adapt the proof of Lemma 1 to deduce that the equilibrium prices must be the same in all periods with positive ex ante probability of a trade occurring.

We next show that complete zigzagging satisfies (1) to (4) and is feasible as part of an equilibrium. It is trivial to see that it satisfies (1) to (3). We now show that it satisfies (4). We first propose the following for the extended model:

**LEMMA A4.** *Given any prior for v, the minimum among all pure strategy equilibrium prices is non-increasing in T.*

*Proof.* Suppose $T$ increases from $T_1$ to $T_1 + \Delta$ ($\Delta > 0$). Suppose the lemma is not true. Then it is a pure strategy equilibrium for the Buyer to wait (i.e. not visit any Seller) over the first $\Delta$ periods regardless of her valuation, and “kick start” a pure strategy equilibrium over the remaining $T_1$ periods which has a lower equilibrium price than any equilibrium when $T = T_1 + \Delta$. This leads to a contradiction.

Now observe that, if part of the equilibrium route choices involves the Buyer visiting the same Seller, say Seller $j$, in any two consecutive periods (or two periods separated by a number of periods in which the Buyer does not go on any shopping trips), then Seller $j$, expecting this, would post the same price offers in the two periods. That is, the two periods effectively become one. In general, any expected route is identical to a zigzagging route with possibly fewer total periods than $T$. But by Lemma A4, the lowest equilibrium price that can ever arise from any expected route is attained when the expected route has the largest possible number of total periods i.e., complete zigzagging over all the $T$ periods. A similar argument can be applied to show that complete zigzagging satisfies the out-of-equilibrium part of (4). Hence, complete zigzagging satisfies (1) to (4). Moreover, any deviation from equilibrium under this specification cannot lead to the Sellers posting lower price offers than in equilibrium, since deviation will effectively lead to zigzagging over fewer periods. Therefore, the only equilibria satisfying (1) to (4) in the extended model involve complete zigzagging over all $T$ periods (there are only two such equilibria which differ in which Seller to visit first).
APPENDIX B: Subject Instructions (Condition T4S)

Welcome to this decision making experiment. You are about to participate in a computer-controlled experiment designed to study behavior in a small market with two sellers who try to sell a single good to a single buyer. Please read the instructions carefully at your own pace. If you follow them closely and make sensible decisions, you may earn a considerable amount of money.

The experiment consists of 60 identical decision making sequences that are called seasons. The unit of exchange in all the trades is called token. At the end of the experiment, your cumulative earnings in tokens will be converted into money at the exchange of 600 tokens = $1.0, and paid to you in cash. A research foundation has contributed the funds to finance this experiment.

We ask that you refrain from communicating with one another. If you do communicate, the experiment will be terminated.

**General Description of the Zigzag Trading Game**

Basically, this experiment is about a market comprised of two sellers and a single buyer who alternates between them in search of a good price. Each of the 60 seasons consists of 4 periods. All the seasons are structured in exactly the same way. At the beginning of each season, the computer will randomly assign you to one of three roles called Seller 1, Seller 2, and Buyer. Your role will not change during the season. Throughout the 60 seasons in this experiment you will be assigned each role an equal number of time. However, the order in which the roles will be assigned to you will randomly be determined.

**Stages of the trade**

Each season will proceed as follows. Seller 1 will be presented with the following decision screen (see screen on next page). On the right-hand corner of this screen there is a window showing the season number (1 in this example) and your total score in the experiment (0 in this example). In the middle of the decision screen you may see your role for the season (Seller #1 in this example), the current period (1 in this example), and the maximum number of periods in this season (4 in this example). Lastly, you have a keypad which will be used to enter an asking price.

Seller 2 will be presented with a similar screen.

The computer will then probe Seller 1 (the statement Please Enter Your Price will flash) to submit her asking price for the good. Seller 1 can choose any asking price between 1 and 9999. To do so, please type in your asking price, one integer at a time (100 in this example), and then confirm your price by pressing the CONFIRM button. If you wish to change the price before pressing this button, press the clear button “C” and re-type your price.
The Buyer will be presented with a different screen (see next page). At the top of the screen is the **value** of the good for the Buyer. This value (500 in the present example) is the maximum the Buyer should be willing to pay for the good. The value of the good for the Buyer will randomly be chosen from the integers 1 through 1000. Therefore, each integer value between 1 and 1000 is as likely to be chosen as another. *The Buyer will be informed of the value of the good. However, the Sellers will not be informed of this value until the season is over. They only know that each integer between 1 and 1000 is equally likely to be chosen.*

After Seller 1 submits her asking price for period 1, the price will be exhibited on the Buyer’s (as well as Seller 2’s) screen (100 in this example). The computer will display to the Buyer his potential profit (500-100=400 in this example), if he decides to purchase the good. This profit may be positive (if the Buyer’s value exceeds the asking price) or negative (if the asking price exceeds the Buyer’s value). The computer will then probe the Buyer to enter his decision: YES or NO. After choosing either YES or NO, the Buyer will be asked to confirm his decision by pressing the button CONFIRM.

If the Buyer purchases the good, the season is over. If not, then the computer will inform both sellers of the buyer’s decision and probe Seller 2 (now in period 2) to enter her asking price. Once she does so, it is the Buyer’s turn to decide whether he is willing to purchase the good from Seller 2. This process will be repeated with the buyer zigzagging from one seller to the other until either he purchases the good (by choosing YES) or the final period (i.e., period 4) is concluded and the season is over (with no trade taking place), whichever comes first.
Buyer’s Decision Screen

Once the season is over, all the three traders will be presented with an end of season summary screen. The summary screen for Seller 1 is presented below. The ones for the Buyer and Seller 2 are essentially the same and are therefore not displayed here. As you may observe, in this brief example the Buyer purchased the good from Seller 1 on period 1. As explained earlier, Seller 1’s profit is her asking price of 100. The Buyer’s profit (in this example) is his value of 500 minus the 100 he paid for the good, which equals 400 (500-100=400). Seller 2 who did not sell the good in this example ends the season with a profit of zero. If a season ends without a trade, then all three players earn zero.

Once all the participants in the experiment (there are several groups running in parallel) complete a season you will be able to proceed to the next season. Note that some participants may take longer to complete a season than others either because they take more time to calculate their decisions or they play more periods. All groups will start each season simultaneously. Once you are prompted, please press the NEXT button and continue to the next season. This process will continue until all 60 seasons are completed and the experiment is over.

History
Your decision screen contains the button HISTORY. You may press this button at any time during the experiment in order to observe the sequence of trade in all earlier seasons.
Summary

Considered in this experiment is a trading scenario in which two sellers set prices, one at a time, for a single good. The buyer alternates between the two sellers, starting with Seller 1. In this market the sellers do not know the exact value of the good for the Buyer; they only know that the value can assume any value between 1 and 1000 with equal probability. Trading proceeds until either the buyer purchases the good from one of the sellers or the season is over, whichever comes first. The buyer’s profit, if he purchases the good, is simply the difference between his value and the asking price. The profit to the seller, if she sells the good, is simply her asking price. The other seller earns nothing.

Once the experiment is over, the computer will display your cumulative payoff in tokens and in dollars. You will be paid your earnings in cash. In addition, you will also receive a show-up bonus of $5.00. In order to maximize your earning we recommend that you consider your decisions carefully.

Once you are certain you understand the task please place the instructions on the table in front of you to indicate that you have completed reading them. If you have any questions, please raise your hand and one of the supervisors will come to assist you. Thank you for your participation.

Good Luck!