Financial Sector Shocks, External Finance Premium and Business Cycle *

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Abstract

This paper generates a DSGE model with a banking sector maximizing profit subject to a loan production function. The profit maximization in banking sector can predict a relationship between EFP and corporate balance sheet condition as well as factor price in the banking sector. The shock to the technology in loan production resembles the disturbance in the banking sector. Together with the shock to entrepreneurial net wealth, we can see the perturbation in both of the supply and demand side in the financial market. Constructed shock processes show that every post war recession is crashed with the situation that both the two financial shocks are in contraction; either one of them is not strong enough to cause an economy-wide recession. The variance decomposition exercise indicates that the loan productivity shock is a very important source of EFP variation, though playing a minor role in determining the rest of the economy.

**JEL classification**: E32, E44, G10

**Keywords**: DSGE modeling, corporate net wealth shock, loan productivity shock, external financing, shock construction, historical decomposition, variance decomposition

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1 Introduction

The very recent financial crisis and possibly ongoing economic recession demonstrated that
the financial sector should be an important factor which can influence the economic activity.
As stated in Gertler and Kiyotaki (2009), we no longer need to appeal either to the Great
Depression or to the experiences of many emerging market economies to motivate interest on
the role of financial factors in aggregate fluctuations since the worst financial crisis and
economic downturn of the post war era is currently undergoing. The importance of financial
factor in shaping business cycle has been studied extensively in the literature. One of the
most notable contributions in dynamic stochastic general equilibrium (DSGE) context is by
Bernanke, Gertler and Gilchrist (1999), BGG hereafter. They develop the so-called financial
accelerator mechanism and demonstrate that the existence of an optimal financial contracting,
in an environment of information asymmetry between lenders and entrepreneurs, can magnify
and propagate the responses of the economy to some main underlying shocks (e.g., monetary
and total factor productivity (TFP)). Thus the financial markets may unavoidably increase the
volatility of the economy through the endogenous variation of financial frictions. Never-
theless, it is worth noting that the financial sector in BGG solely plays a role of transmitting
shocks originating from other sectors. Thus the framework only captures one branch of the
financial factors which also include the fact that financial structure of the economy can also
be an independent source of volatilities as suggested by recent economic events. For this
reason, the importance of financial sector as a source of aggregate fluctuations is still under
investigation.

Several studies have been carried on to fill this gap. One notable contribution recently is by
Nolan and Thoenissen (2009), henceforth NT. They extract the entrepreneurial net wealth
shock along with the TFP and monetary shocks for US economy from a DSGE model with
financial accelerator mechanism a la BGG and name it as a shock to the efficiency of the
financial sector. They try to distil the contribution to US business cycle of financial shock on
top of the financial friction mechanism. They conclude that their extracted financial shock
process is found to (i) be very tightly linked with the onset of recessions, more so than TFP or
monetary shocks; (ii) remains contractionary after recessions have ended; (iii) account for a
large part of the variance of GDP; (iv) be strongly negatively correlated with the external
finance premium (EFP). Despite these promising findings, the financial shock constructed in NT shouldn’t be considered as a complete description of the disturbance in the financial sector. It is important to recognize that both BGG and NT generally considered the demand side of the financial market. The financial friction developed in BGG is built upon the balance sheet of entrepreneurial sector. Entrepreneurial net worth is crucial to determine the cost of external funds which can influence the demand of external funds by entrepreneurs. Similarly, if the net worth is subject to stochastic disturbance, the shock only affects the entrepreneurial balance sheet and the demand side of the financial market. Regard the recent financial crisis, it seems more appropriate to also think about the effects of financial factor from the supply side of the financial market, the financial intermediaries/banking sector.

Up to date, several studies have been considering the banking sector in determining the financial frictions on the one hand, and the disturbance in financial intermediaries as a source of business fluctuations on the other. Markovic (2006) introduces the bank capital channel in the monetary transmission mechanism on top of the corporate balance sheet channel as in BGG and highlights three sub-channels in the banking sector (supply side): default risk channel, adjustment cost channel and capital loss channel. He concludes that all the three channels in the banking sector reinforce the aggregate credit channel in the monetary transmission mechanism and increase the effects significantly in the event of large shocks to the value of bank capital. Zhang (2009) considers the bank’s balance sheet effect from a different angle where banking sector is assumed to share the risk with entrepreneurial sector. When the economy is subject to large adverse shock, both the entrepreneurial and banking sector balance sheets are exposed to the risk and the deterioration of the two balance sheets reinforce each other and drive the economy down further. Aguiar and Drumond (2009) also emphasize the relevance of bank capital channel in determining the aggregate fluctuations, but from the Basel regulatory perspective. Most of the studies focusing on the bank capital channel in the model’s transmission mechanism also stress the significance of disturbance to bank capital per se, but to a limited extend. They only deal with the impulse response conditional on the bank capital shock, but generally ignore the explicit time series process of the shock and its influence on the business cycle. One notable exception is Hirakata et al. (2010) who estimate a DSGE model with banking sector using Bayesian methodology and extract the shocks to the bank’s net worth. Based on variance decomposition, shocks to the
banking sector are found to be a main source of the spread variations and play a significant role for investment volatility.

All these studies convey an important signal that supply side friction and disturbance are also relevant to credit imperfections and aggregate fluctuations; thus should be dictated for more attention and exploration. This chapter builds on this clue and focuses on the role of supply side disturbance in financial market in shaping the business cycle. Specifically, I generally follow the setup of BGG and NT except that I replace the optimal contracting problem between lender and borrower with explicit profit maximization in banking sector subject to a loan production function. As shown in the previous chapter and this chapter that follows up, the profit maximization in banking sector can predict a relationship between EFP and corporate balance sheet condition as well as factor price in the banking sector. Thus both the demand side (entrepreneurial sector) and supply side (banking sector) contribute to the financial frictions. Moreover, the shock to the technology in loan production resembles the disturbance in the banking sector. Together with the shock to entrepreneurial net wealth, we can see the perturbation in both of the supply and demand side in the financial market. This strategy extends the work of NT by allowing another shock in the banking sector on top of the net worth shock to affect the financial sector and the whole economy on the one hand, and distinguish itself to the work of Hirakata et al. (2010) who designate the disturbance in banking sector to bank net wealth on the other.

The main results of this chapter can be summarized as follows. First of all, use the shock construction procedure discussed in details below, I can extract the four shocks in which TFP, monetary and net worth shocks are close to their counterparts in NT on the one hand, and TFP as well as monetary shocks are observationally similar to the ones constructed with traditional estimation procedure on the other. This can be treated as a robust check that the inclusion of another shock wouldn’t alter the processes of shocks originally generated in NT despite the fact that my model setup is slightly different from theirs. Secondly, subject to the interesting, but also a little bit confusing result in NT that the net worth shock remains contractionary after recessions have ended, it is promising to find that, after add in the loan productivity shock, all the post war recessions happen only when both of the entrepreneurial

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1 Current model is different from that in Hirakata et al. (2010) in other respective as well. In Hirakata et al. (2010), more shocks are used since they apply the Bayesian estimation where I follow the shock construction procedure developed in Benk et al. (2005), (2008).
net worth and the loan productivity shocks are in contraction, implying either one of them is not strong enough to cause an economy-wide recession. Thirdly, both of the extracted loan productivity and net worth shock are negatively correlated with proxies of EFP, despite the fact that the correlation between net worth shock and EFP is higher in absolute value. We can conjecture from it, that the loan productivity shock is also significant in shaping the financial business cycle even if it is not the dominant one. Fourth, consistent with our prediction, the variance decomposition indicates that loan productivity shock is an important source of EFP variation, though the dominant driving force is still net worth shock. This somehow matches the result in Hirakata et al. (2010) in which they predict a quantitatively similar feature of bank capital shock in determining EFP. Even though we assume the shock in the banking sector with different essence, Hirakata et al. (2010) and I reach similar result in this dimension. Finally, net worth shock is still a dominant factor along several other dimensions of the economy after we include the loan productivity shock, more important than TFP and monetary shock in determining output, investment, loan, hours and federal funds rate, while the loan productivity shock plays a minor role.

The rest of this paper is structured as follows. Section 2 derives the model used in this chapter. Section 3 calibrates the model to quarterly data of US economy. The construction of all shock processes and numerical simulation are carried out in section 4. Section 5 concludes with some final remarks.

2 The model

The designated model I develop here is a standard DSGE New Keynesian model largely follows BGG’s original setup and NT’s extension. One noticeable exception is that I develop the financial frictions with a fully micro-founded loan production/management function and a financial intermediaries’ profit maximization problem instead of BGG’s original optimal contracting problem. The other one is that I introduce one more shock from the supply side of financial market on top of the net wealth shock from the demand side as in NT. Besides the banking sector, the model economy is inhabited by households, three types of producers: entrepreneurs, capital producers, and retailers, and a government who conducts monetary policy. Households own differentiated labour service and have the power to set the nominal
wage in the labour market as in Erceg et al. (2000). Entrepreneurs produce intermediate goods and borrow from banks that convert household deposits into business financing for the purchase of capital. The presence of collateral evaluation and labour monitoring costs create the financial friction, which causes loan interest rate higher than deposit interest rate. This makes the entrepreneurial demand for capital depends on their financial position and the supply of external funds depends on the state of the economy. The interaction between the demand and supply equilibrates the credit market. Capital producers purchase investment goods and build new capital to sell to the entrepreneurs. This captures the up and down movement of asset prices. Retailers present because it is more convenient to introduce nominal stickiness this way to keep track of the development in conventional dynamic New Keynesian framework. They set nominal prices in a staggered fashion a la Calvo (1983) and Yun (1996).

2.1 Households

The economy is populated by a continuum of monopolistically competitive households, indexed by $j \in [0,1]$, who consume, work and save. Each of them supplies differentiated labour service to the entrepreneurial and banking sector, which regard each of their labour service as an imperfect substitute for that of others. In this setup, entrepreneurs and banks demand bundles of labour services, which is obtained using the aggregation scheme as in Dixit and Stiglitz (1977)

$$N_t = \left[ \int_0^1 N_t(j) \varepsilon_w^{-1} \varepsilon_w^{-1} dj \right]^{\varepsilon_w \varepsilon_w^{-1}}, \varepsilon_w > 1;$$

The optimal substitution across labour service leads to the following labour demand equation regarding the $j$th labour service

$$N_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} N_t;$$
where \( W_t(j) \) is the nominal wage set by the \( j \)th household, \( W_t \) is the Dixit-Stiglitz aggregate nominal wage given by \( W_t = [\int W_t(j)^{1-\varepsilon} dj]^{1/(1-\varepsilon)} \), and \( \varepsilon \) gives the constant elasticity of substitution across labour service.

To motivate the demand for money, I follow Sidrauski-Brock and include money in the utility function of the representative household. Thus the \( j \)th household derives the expected lifetime utility from consumption (with external habit) of final goods, \( C_t(j) \), real balance holding, \( M_t(j)/P_t \), and leisure, \( 1 - N_t(j) \); with discount factor, \( \beta \in (0,1) \), this is given by

\[
U = E_t \sum_{h=0}^{\infty} \beta^h \left[ \frac{(C_{t+h}(j) - \bar{C}_{t+h})^{1-\eta^c}}{1-\eta^c} + \psi^m \frac{(M_{t+h}(j)/P_{t+h})^{1-\eta^m}}{1-\eta^m} + \psi^x \frac{(1 - N_{t+h}(j))^{1-\eta^x}}{1-\eta^x} \right];
\]

where \( \bar{C}_t \) is aggregate consumption, \( \eta^c \), \( \eta^m \) and \( \eta^x \) measure the intertemporal elasticity of substitution for consumption, real balance and leisure. \( \psi^m \) and \( \psi^x \) represent the weight on real balance and leisure in the utility function.

The \( j \)th household enters period \( t \) with \( P_{t-1}D_t(j) \) units of nominal deposits in a financial intermediary, and nominal money balances, \( M_{t-1}(j) \). While deposits pay a gross nominal interest rate, \( R_{t-1}^n \), between \( t-1 \) and \( t \), money balances bear no interest. During period \( t \), the \( j \)th household supplies labour to the entrepreneur firms and banks, for which he receives total factor payment of \( W_t(j)N_t(j) \). In addition, he receives a lump-sum transfer from the monetary authority, \( T_t(j) \), as well as the dividend payments, \( \Pi_t^F(j) \), from banks and, \( \Pi_t^K(j) \), from retailers, as he owns both of them. All these funds are allocated for consumption, \( P_tC_t(j) \), money holdings, \( M_t(j) \), and nominal deposit holdings, \( P_tD_{t+1}(j) \). Thus the household’s intertemporal budget constraint, in real terms, is

\[
C_t(j) + \frac{M_t(j)}{P_t} + D_{t+1}(j) = \frac{W_t(j)}{P_t} N_t(j) + \frac{M_{t-1}(j)}{P_{t-1}} \frac{P_{t-1}}{P_t} + R_{t-1}^n \frac{P_{t-1}}{P_t} D_t(j) \\
+ T_t(j) \frac{P_t}{P_t} + \frac{\Pi_t^F(j)}{P_t} + \frac{\Pi_t^K(j)}{P_t};
\]
The $j$th household chooses $C_t(j)$, $M_t(j)/P_t$, and $D_{t+1}(j)$ in order to maximize his expected lifetime utility subject to his budget constraint and labour demand constraint. The first order conditions (F.O.Cs) for this optimization problem are:\(^{2}\)

$$U_{Ct} = R^n \beta E_t \left\{ \frac{P_t}{P_{t+1}} \right\};$$

$$U_{Mt} = \frac{R^n - 1}{R^n};$$

Eq. (1) is the usual intertemporal condition, which states that the marginal cost of fore-going a unit of consumption in the current period must be compensated with the marginal benefit in the following period. Eq. (2) is the money demand equation.

Given that the households set nominal wages in staggered contracts with a constant probability, $1 - \theta_w$, of renegotiation in each period, the fraction of households who have the opportunity to reset their wages will set it as a mark-up over the marginal rate of substitution of leisure for consumption ($MRS$) taking account the probability that he cannot reset the wage again. The fraction of households who don’t have the opportunity to reoptimise must apply the wages that was in effect in the preceding period indexed by the steady state gross rate of wage inflation $\omega$. This yields the following maximization problem:

$$\text{Max}_t E_t \sum_{h=0}^{\infty} \left\{ (\beta \theta_w)^h \frac{\lambda_{t+h}^*}{\lambda_t} W_t^*(j) \omega - \frac{MRS_{t+h} P_{t+h}}{P_{t+h}} N_{t+h}(j) \right\};$$

The F.O.C for the maximization problem is

$$W_t^*(j) = \frac{E_t \sum_{h=0}^{\infty} \left\{ (\beta \theta_w)^h \frac{\lambda^*_{t+h+1}}{\lambda_{t+h}} N_{t+h}(j) MRS_{t+h} \right\}}{E_t \sum_{h=0}^{\infty} \left\{ (\beta \theta_w)^h \frac{\lambda^*_{t+h+1}}{\lambda_{t+h}} N_{t+h}(j) \omega / P_{t+h} \right\}};$$

Log-linear approximations of the F.O.C imply the following wage inflation curve:

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\(^{2}\)The omission of households’ index in the F.O.Cs stems from the assumption following Erceg et al. (2000) and Christiano et al. (2005) that the implicit existence of state-contingent securities ensures households’ consumption and asset holding are homogenous.
\begin{align}
\dot{\omega}_t &= \beta \dot{\omega}_{t+1} + \frac{(1 - \beta \theta_w)(1 - \theta_w)}{\theta_w (1 + \eta_w e_w)} \left( \eta_n \frac{N}{1 - N} \hat{h}_t - \hat{\lambda}_t - \hat{\nu}_t \right) \tag{3L}
\end{align}

where \( \omega_t \) is the gross wage inflation and \( \lambda_t \) is the multiplier of households’ budget constraint.

### 2.2 Entrepreneurs

The entrepreneurial sector largely follows the original BGG setup\(^3\). In each period, entrepreneurs combine hired labour and purchased capital to produce intermediate goods in a constant return to scale (CRS) technology. This aggregate production function is given by

\[
Y_t = A_t K_t^\alpha N_t^{1-\alpha} ; \tag{4}
\]

where \( Y_t \) is produced intermediate goods, \( N_t^G \) is hired labour service, \( K_t \) is capital purchased in last period and \( \alpha \) is capital share in production function. \( A_t \) is an exogenous technology measure capturing total factor productivity in goods sector. It follows

\[
\ln A_t = (1 - \rho_a) \ln A + \rho_a \ln A_{t-1} + \varepsilon_{at} ; \tag{5}
\]

with \( \rho_a \in (0,1), \varepsilon_{at} \sim iid (0, \sigma_a^2) \). Consider an entrepreneur’s decision making at the end of period \( t \) as an example. At that moment, the entrepreneur needs to purchase capital, \( K_{t+1} \), that will be used in period \( t+1 \), at the price \( P_t Q_t \) (\( Q_t \) is the relative price of capital goods in terms of the consumption goods). Thus the real cost of capital acquisition is \( Q_t K_{t+1} \). The entrepreneur can only afford part of the expenditure, equalling to his net worth \( NW_{t+1} \), and rely on external funds for the rest. This requires a model of explicit credit market and lender, which is the financial intermediary/bank described in details later. The capital demand of entrepreneurs is determined by the equality of expected marginal external financing cost with expected marginal return of holding capital.

\[
E_t R^d_{t+1} = E_t R^k_{t+1} = E_t \left[ \frac{X_{t+1}(\alpha Y_{t+1}/K_{t+1}) + Q_{t+1}(1 - \delta)}{Q_t} \right] ; \tag{6}
\]

\(^3\) Other similar expositions are Meier and Muller (2006), Gertler et al. (2007) and Christensen and Dib (2008).
where $\delta$ is the depreciation rate of capital, $X_t$ is the price of intermediate goods relative to final goods, and $Q_t$, as described above, is the relative price of capital which varies because of the adjustment cost. Thus the expected return on capital consists of two aspects: the income gain of $X_t\alpha Y_{t+1}/K_{t+1}$ and the capital gain of $Q_{t+1}(1-\delta)$. This return must be equal to the gross loan rate charged by financial intermediary/bank to ensure the optimal holding of capital by entrepreneurs.

Given the existence of credit market imperfections, the gross loan rate $E_tR^l_{t+1}$ will be equal to the multiplication of gross external finance premium $EFP_{t+1}$ and gross deposit rate $E_tR^d_{t+1}$. The determination of $EFP_{t+1}$ is shown in bank’s optimal loan production/management in the next sub-section. As described previously, since the bank promises to pay households a non-state contingent nominal rate of $R^n_t$, the real rate depends on the ex post inflation rate. Thus we would also see a debt deflation effect, a la Fisher (1933), in the credit markets. The key equation to show financial frictions in this model can be written as

$$E_tR^l_{t+1} = E_tR^d_{t+1} = EFP_{t+1}\left(\frac{R^n_t}{E_t(\pi_{t+1})}\right); \quad (7)$$

where $\pi_{t+1} = P_{t+1}/P_t$ is the gross inflation rate. On the other aspect, entrepreneurial demand for labour service is determined by equalizing the real wage with marginal product of labour:

$$w_t = X_t(1-\alpha) \frac{Y_t}{N^G_t}; \quad (8)$$

Let’s leave the detailed exposition of financial frictions to the bank’s problem discussed below. To finish the entrepreneur’s problem, it is necessary to analyse the transition of their net worth. The existence of credit market implies that entrepreneurs are not allowed to fully self finance. In other words, they cannot accumulate their net worth forever. We can achieve this by assuming the exit and entry of entrepreneurs out and into the entrepreneurial sector. The probability that an entrepreneur will survive until the next period is $\nu$ (i.e. there is a probability $1-\nu$ that he dies in between periods), so entrepreneurs only have finite expected horizon $1/(1-\nu)$ for operation. This assumption is vital, as it ensures that entrepreneurs never accumulate enough net wealth to finance new capital expenditure entirely and have to go to
the credit market for external funds. The size of the entrepreneurial sector is constant, with new arrivals replacing departed entrepreneurs. The newly entered entrepreneurs receive some transferred seed money, $S_t$, for operation. We can derive the evolution of entrepreneurs’ net worth as follows:

$$NW_{t+1} = x_t \nu [R_t^k Q_{t-1} K_t - \frac{R_{t-1}^n}{E_{t-1} \pi_t} EFP_t (Q_{t-1} K_t - NW_t)] + (1-\nu)S_t;$$  \hspace{1cm} (9)$$

where the first term in the square bracket represents the ex post return of holding capital in $t$ and the second is the cost of borrowing, which is the real interest rate implied by the loan contract signed in $t-1$. As borrowers sign a debt contract that specifies a nominal interest rate, the loan repayment in real terms depends on the ex post real interest rate. Thus an increase (decrease) in inflation will reduce (increase) the real cost of debt repayment and push up (down) the entrepreneurial net worth. The stochastic nature of net worth evolution is introduced by a random disturbance term $x_t$, which follows the process

$$\ln x_t = \rho_x \ln x_{t-1} + \varepsilon_{xt};$$  \hspace{1cm} (10)$$

where $\rho_x \in (0,1), \varepsilon_{xt} \sim iid(0, \sigma_x^2)$. This random term shifts entrepreneurial net worth up and down independently of movements in fundamentals. Christiano et al. (2010) interpret this shift factor as a reduced form way to capture what Alan Greenspan has called ‘irrational exuberance’, or simply asset price bubbles. NT follows Gilchrist and Leahy (2002) to treat this as a shock to the efficiency of contractual relations between borrowers and lenders so as to influence the degree of asymmetric information and costly state verification problem. I interpret this disturbance as a credit demand shock as it perturbs the financial condition of entrepreneurs and their demand for external finance. As shown below, this is justified by looking at the impulse response that $x_t$ drives aggregate level of loan and EFP into the same direction, a distinguished characteristic of demand shock.  

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4 Without this seed money, entering entrepreneurs would have no net worth, and so they would not be able to buy any capital. Also, among the entrepreneurs who survive there are some who are bankrupt and have no net worth. Without a transfer they would not be able to buy capital either.

5 Note this explanation is not contradicted with either Christiano et al. (2010) or NT. Specifically, a positive shock to entrepreneurs’ net worth (asset bubble) can be thought of as a negative shock to credit demand since more investment can be financed internally; it can also be treated as a shock to the contractual efficiency that pushes down the EFP.
2.3 Banks

The function of external finance channel in the model economy is determined by financial intermediaries/banks. They issue deposits to collect funds from households and then convert those funds into lending as corporate loans to entrepreneurs. To simplify the analysis, I omit any regulation of reserve or the existence of inter-bank markets. The latter justifies the existence of a representative bank in the model economy. The absence of reserve requirement and positive loan rate imply that the bank will lend out whatever is deposited: \( L_t = D_t \).

Based on the assumption in Goodfriend and McCallum (2007), the volume of loan supply (equivalent as the demand for deposit funds) is designed to be determined by a model of loan production, or more accurate, loan management, which is involved with collateral assessment and labour monitoring. This setup is motivated to capture the supply side of the credit market since in BGG the financial intermediaries exist passively to satisfy the demand of external funds by entrepreneurs. In what follows, the loan management is assumed to be conducted by combining the collateral for evaluation and labour effort for monitoring. The specification is in Cobb-Douglas fashion as follows:

\[
L_{t+1} = F_t(Q_t K_{t+1})^\gamma N_t^{1-\gamma};
\]

where \( L_{t+1} \) is the amount of loan lending in period \( t+1 \) determined at the end of period \( t \). \( Q_t \) is the price of capital at the end of period \( t \), thus \( Q_t K_{t+1} \) is the value of collateral at the beginning of time \( t+1 \). \( N_t^{F} \) is the labour effort involved in loan monitoring, and \( \gamma \) denotes for the share of collateral in loan production. \( F_t \) is an exogenous technology measure capturing total factor productivity in banking sector (loan supply shock), following

\[
\ln F_t = (1 - \rho_f) \ln F + \rho_f \ln F_{t-1} + \varepsilon_{ft};
\]

with \( \rho_f \in (0,1) \), \( \varepsilon_{ft} \sim iid (0, \sigma_f^2) \). It is noteworthy that Eq. (10) distinguishes itself to the original setting in Goodfriend and McCallum (2007) that only economy-wide capital is used as collateral for loan production. The reasons are twofold. First, government bond is not necessary here since the model refrains from the analysis of it; the omission is a simpli-

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6 This can be partly justified that the reserve requirement is mostly for demand deposit, not time deposit considered here. Moreover, the bank in the model is in broader sense to capture the economy-wide credit.
fication. Moreover, to resemble BGG’s expression of financial friction (shown below), it is more appropriate to exclude bond from the loan production function.

On the other hand, as described in section 2.2, entrepreneurs obtain the loan to finance the purchase of next period capital in excess of their net wealth $NW_{t+1}$:

$$L_{t+1} = Q_t K_{t+1} - NW_{t+1};$$  \hfill (13)

Eq. (11) and Eq. (13) together characterize the equilibrium in credit markets.

The instantaneous profit of the bank at the end of period $t$ is the new arriving deposit funds and gross interest payment on existing loans, less the labour cost for monitoring, cost of collateral service, the new issuing loans and the gross interest payment on existing deposits. The bank chooses the collateral $Q_t K_{t+1}$, labour service $N^F_t$ and newly issued loan $L_{t+1}$ to maximize the expected life-time profit in favour of the bank owners, households. The profit maximization problem of the bank is given by

$$\text{Max}_{Q_t, K_{t+1}, N^F_t, L_{t+1}} E_t \sum_{h=0}^{\infty} \beta^h U_{C_t+h} \left\{ D_{t+h} + R^l_t L_{t+h} - L_{t+h} - R^d_t D_{t+h} - 
\quad W_{t+h} N^F_{t+h} - r^q_{t+h} Q_{t+h} - K_{t+h} \right\};$$

subject to the bank balance sheet constraint and loan production function, where $\beta^h U_{C_t+h}/U_{C_t}$ is the household’s stochastic discount factor. The F.O.Cs for this optimization problem are:

$$E_t \{ R^{ul}_{t+1} - R^{dl}_{t+1} \} = \frac{\gamma \beta_t \gamma_t}{\gamma_t L_{t+1} / Q_t K_{t+1}};$$  \hfill (14)

$$E_t \{ R^{ul}_{t+1} - R^{dl}_{t+1} \} = \frac{W_t}{(1-\gamma) L_{t+1} / N^F_t};$$  \hfill (15)

Eq. (14) and Eq. (15) apply the usual Baumal (1952) conditions, which equalize the marginal cost of intermediation to the factor prices of inputs divided by the marginal product of the inputs. As highlighted in Goodfriend and McCallum (2007), this marginal cost captures the idealized net uncollateralized external finance premium (UEFP) in the model, under the condition that entrepreneurs come to borrow without any collateral. Thus entrepreneurs have to pay full cost of intermediation: labour monitoring plus collateral service. In the other
extreme, if entrepreneurs possess the full amount of collateral to borrow, they pay the full cost at the same time get back the return of collateral services. Therefore, the net EFP for entrepreneurs is only the labour monitoring cost, which is the fraction $1 - \gamma$ of the total cost. We call this the fully collateralized external finance premium (CEFP) in the model economy, represented as:

$$CEFP_{t+1} - 1 = \frac{W_i}{L_{t+1}/N_F} ;$$

In reality, the actual amount of EFP lies between UEF$P_{t+1} - 1 = (UEFP_{t+1} - 1)[1 - \gamma \frac{NW_{t+1}}{Q_i K_{t+1}}] ;$ (16)

Combine Eq. (2.16) and Eq. (2.15), after some rearrangements, to get

$$EFP_{t+1} - 1 = \frac{1}{1 - \gamma} F_i \left[ \left( 1 - \frac{NW_{t+1}}{Q_i K_{t+1}} \right)^{\frac{1}{1 - \gamma}} \left[ 1 - \gamma \frac{NW_{t+1}}{Q_i K_{t+1}} \right] \right] ;$$ (17)

Eq. (17) highlights the key relationship between $EFP$ and the ratio of internal funds to purchased capital value, $NW_{t+1}/Q_i K_{t+1}$, from the bank’s optimization behaviour. Given that $F_i$ and $w_i$ are exogenous, and capital price is at steady state value of unity, we can derive the following proposition:

**Proposition 1**: In equilibrium, assume $F_i$ and $w_i$ are exogenously given, and capital price is in steady state value. External finance premium is a decreasing and convex function of the ratio of net worth to purchased capital value.

See Appendix B for the proof of proposition 2.1. This proposition implies a very important inference comparable to BGG: The external finance premium is higher the more entrepreneurs rely on external funds. Figure 2.1 plots the gross EFP against the ratio of net worth to the value of purchased capital with baseline calibration ($\gamma = 0.77 , F_i = 2.69 , w_i = 2.12$):
Figure 1 shows that the EFP decreases as less external funds is needed with given value of purchased capital (less leverage) in an diminishing rate. This implies that EFP in steady state will increase dramatically even after you reduce the internal funds relative to capital value by only a small amount. This shows the mechanism of the accelerator effect embedded in the banking sector of the baseline model. To see the dynamic relationship, I derive the log-linear form of Eq. (17) around the non-stochastic steady state:

\[
\epsilon f \hat{\rho}_t = - \frac{\text{EFP} - 1}{\text{EFP}} \left( \gamma \frac{\text{NW}}{K} \left( \frac{\text{NW}}{K} \right) + \frac{\gamma}{1 - \gamma} \frac{\text{NW}}{K} \right) \left( n \hat{w}_{t+1} - \hat{k}_{t+1} - \hat{q}_t \right) \\
\frac{\text{EFP} - 1}{\text{EFP}} \left( \hat{w}_t - \frac{1}{1 - \gamma} \hat{f}_t \right) \quad ; \quad (17L)
\]

Eq. (17L) elaborates the behind the scene accelerator effect from the banking sector in the baseline model. The short run dynamics of EFP depends on the dynamics of the net worth to capital value ratio, real wage for labour monitoring and exogenous loan production technology. Thus Eq. (17L) is highly comparable with the counterpart reduced form equation in BGG framework of the form:

\[
\epsilon f \hat{\rho}_{t+1} = -\psi(n \hat{w}_{t+1} - \hat{k}_{t+1} - \hat{q}_t) ;
\]
BGG claims that the elasticity of external finance premium with respect to the ratio of internal funds to total value of capital is derived from an optimal contracting problem between entrepreneurs and financial intermediaries. Higher net worth relative to value of purchased capital makes more funds of entrepreneurial sector sink into the project. Thus the incentives are more aligned between entrepreneurs and banks so as to reduce the asymmetric information problem and EFP. The baseline model with loan management also predicts a similar aggregate relationship as in BGG, despite the fact that the corresponding elasticity is shown differently by an expression nesting steady state value of EFP and internal funds to total value of capital ratio, and the parameter value of collateral share in loan production\(^7\). Besides this, the baseline model also highlights the importance of the real wage to influence the dynamics of EFP before subjecting to the exogenous shock in the banking sector\(^8\). All these promising increments are not considered in BGG and many other studies of financial accelerator.

### 2.4 Capital producers

Capital producers are included to rationalize the fluctuations in the real capital price \(Q\), since the volatile asset prices contribute to the fluctuations of entrepreneurial wealth. Consider there are perfectly competitive capital producers in the economy to control the supply of capital. They combine the purchased capital and investment funds to produce new capital, \(\tilde{K}\), according to

\[
\tilde{K} = \Phi \left( \frac{I}{K} \right) K;
\]

with \(\Phi(0) = 0, \Phi'(.) > 0, \Phi''(.) < 0\). This increasing and concave function captures the presence of adjustment costs in the production of capital goods. Capital producers choose the investment expenditure in order to maximize their profit, \(Q, \tilde{K} - I\), taking the relative price of capital as given. The first-order condition is

\(^7\)The elasticity in BGG equals to \(\psi\) only after figuring out the optimal loan contract between entrepreneurs and financial intermediaries; it also depends on micro structure of the contract environment (e.g., average fraction of monitoring cost after the entrepreneurs default). See the appendix of BGG for details.

\(^8\)Real wage becomes relevant because it is the factor price in loan management and affects the marginal cost of intermediation activity.
\[ Q_t = \left[ \Phi' \left( \frac{I_t}{K_t} \right) \right]^{-1} ; \quad (18) \]

Here I restrict the capital production function so that the relative price of capital is unity in steady state. Capital producers’ decision is linked with the entrepreneurs’ capital-purchasing decision via the variation in the price of capital.

The aggregate capital stock evolves according to

\[ K_{t+1} = \Phi \left( \frac{I_t}{K_t} \right) K_t + (1 - \delta) K_t \quad (19) \]

Note that capital is homogeneous, so there is no difference between newly-produced and old capital. Old capital used by entrepreneurs is rented out for the production of new capital, and then returned at the same price as the newly-produced capital.

### 2.5 Retailers

The retail sector is applied to introduce nominal rigidity into this economy. Here I assume that entrepreneurs sell all of their output goods to retailers. Retailers purchase the homogenous wholesale goods from entrepreneurs, differentiate them using a linear technology at no resource cost and sell as final goods to households, capital producers and the government sector. In this way, the retailers have the monopolistic power to set the prices of these final goods. The reason why retailers are incorporated together with entrepreneurs is to avoid the complication of aggregating individual entrepreneur’s demand for capital and his net worth when entrepreneurs themselves are imperfect competitors. Ultimately retailers’ monopolistic profits belong to the households who own them, in contrast to entrepreneurs who are independent agents possessing their own wealth. Before exploring the retailers’ problem in details, I firstly derive the aggregation of final goods. The final goods \( Y_t \) are bundles of differentiated goods \( Y_t(j) \), \( j \in [0,1] \), provided by the continuum of monopolistically competitive retailers. The aggregation follows the framework of Dixit and Stiglitz (1977) as

\[ \text{Recall that the assumption of linear technology for retailers ensures that the amount of final goods varies one-for-one with the amount of wholesale goods in the economy.} \]
\[ Y_t = \left[ \int_0^{\varepsilon_p} \left( \int_0^{\varepsilon_p-1} Y_t(j)^{\varepsilon_p-1} dj \right)^{\varepsilon_p} \right]^{\varepsilon_p-1}; \]

where \( \varepsilon_p \) is the elasticity of substitution between different goods. The optimal allocation of expenditure across differentiated goods implies a downward sloping demand function for goods \( j \):

\[ Y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon_p} Y_t; \]

where \( P_t(j) \) denotes the price of good \( Y_t(j) \), \( Y_t \) denotes the aggregate demand, and \( \varepsilon \) also measures the price elasticity of demand among differentiated goods. \( P_t \) denotes the price index of final goods given by

\[ P_t = \left[ \int_0^{1} P_t(j)^{1-\varepsilon_p} dj \right]^{1-\varepsilon_p}; \]

Following Calvo (1983), and a discrete version as in Yun (1996), I assume that each retailer cannot reoptimize its selling price unless it receives a random signal. The probability that each retailer can reoptimize his price in a given period is \( 1 - \theta_p \), independently of other firms and of the time elapsed since the last adjustment. Thus the average length of time a price remains unchanged is \( 1/(1 - \theta_p) \). Retailer \( j \) who has the opportunity to reset its price in a given period \( t \) choose the price, \( P_t^*(j) \), that maximizes its expected discounted profits until the period when they are next able to change their price. On the other hand, the retailer who doesn’t have the opportunity to reset price must charge the price that was in effect in the preceding period indexed by the steady state gross rate of inflation, \( \pi \). Retailer \( j \)’s optimization problem is:

\[ \text{Max} E_t \sum_{h=0}^{\infty} \left[ (\beta \theta_p)^h \frac{U_{C_{t+h}}}{U_{C_t}} \frac{P_t^*(j)\pi^h - X_{t+h}P_{t+h}}{P_{t+h}} Y_{t+h}(j) \right]; \]
subject to the demand function of $Y_t(j)$. Note that the stochastic discount factor for expected profits consists of the probability that retailers can change their price and the households’ intertemporal marginal rate of substitution. The F.O.C for the optimal problem is

$$P^*(j) = \frac{E_p}{\varepsilon_p - 1} \frac{E_t \sum_{h=0}^{\infty} \{(\beta \theta_p)^h U_{Ct+h+1}/U_{Ct+h} Y_{t+h}(j) X_{t+h}\}}{E_t \sum_{h=0}^{\infty} \{(\beta \theta_p)^h U_{Ct+h+1}/U_{Ct+h} Y_{t+h}(j) \pi^h / P_{t+h}\}}; \quad (20)$$

The aggregate price index is given by

$$P_t^{1-\varepsilon_p} = \theta \left( \pi P_{t-1} \right)^{1-\varepsilon_p} + (1 - \theta) P_t^{1-\varepsilon_p};$$

Log-linear approximations of the F.O.C and aggregate price index imply the following New Keynesian Phillips curve:

$$\hat{\pi} = \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \beta \theta_p)(1 - \theta_p)}{\theta_p} \hat{\lambda}_t; \quad (20L)$$

where $\hat{\lambda}_t$ is the log deviation of real marginal cost from steady state.

### 2.6 Government and monetary policy

Finally I set the budget constraint for the government and the policy rule of the monetary authority to close the whole model. The aggregate final output goods consist of households’ consumption, capital producers’ investment expenditure and the government expenditure, $G_t$. Every period, the market for final goods clears as

$$Y_t = C_t + I_t + G_t; \quad (21)$$

where government expenditure is financed by lump-sum taxes and money creation

$$G_t = \frac{M_t - M_{t-1}}{P_t} + T_t;$$
For monetary policy, I assume the monetary authority exogenously sets the gross growth rate of money, \( \mu \), such that the supply of real money balance evolves according to\(^\text{10}\)

\[
m_t = \mu m_{t-1} \frac{P_{t-1}}{P_t};
\]

(22)

The money growth rate is assumed to follow a stochastic AR(1) process as

\[
\ln \mu_t = \rho_m \ln \mu_{t-1} + \varepsilon_{mt};
\]

(23)

where \( \rho_m \in (0,1) \), \( \varepsilon_{mt} \sim iid(0, \sigma_m^2) \).

### 2.7 Equilibrium

In the baseline model economy, the equilibrium is defined as a set of endogenous variables \( \{Y, C, I, N^G, N^F, K, NW, L, m, Q, w, R^k, \pi, EFP, X\} \) that satisfies households’ decision rules (1) and (2), wage inflation curve (3), entrepreneurs’ optimal conditions (6), (7) and (8), banks’ decision rule (17), capital producers’ optimal condition (18), New Keynesian Phillips curve derived from retailers’ problem, (20), resource constraints (4), (9), (11), (13), (19), (21), and the money growth rule (22). Thus the log-linear version of the system around the non-stochastic steady state can be derived as:

\[
\hat{\lambda}_t = \hat{\lambda}_{t+1} + \hat{\eta}^n_t - \hat{\lambda}_t; \\
\eta_m \hat{m}_t = \frac{1}{1 - R^k} \hat{\eta}^n_t - \hat{\lambda}_t;
\]

(1L)

\[
\hat{\omega}_t = \beta \hat{\omega}_{t+1} + \frac{(1 - \beta \theta_w)(1 - \theta_w)}{\theta_w(1 + \eta_w \varepsilon_w)} [\eta_n (\frac{N^G}{1 - N} \hat{n}^G_t + \frac{N^F}{1 - N} \hat{n}^F_t) - \hat{\lambda}_t - \hat{\omega}_t]; \\
\hat{y}_t = \hat{a}_t + \alpha \hat{k}_t + (1 - \alpha) \hat{n}^G_t;
\]

(3L)

\[
\hat{r}^k_{t+1} = \frac{X \alpha Y/K}{R^k} (\hat{x}_{t+1} + \hat{y}_{t+1} - \hat{k}_{t+1}) + \frac{(1 - \delta)}{R^k} \hat{q}_{t+1} - \hat{q}_t;
\]

(6L)

\[
\hat{r}_t^{LE} = \hat{r}_t^{n} - \hat{\pi}_t + eff \hat{p}_t; \\
\hat{\pi}_t = \hat{y}_t + \hat{\omega}_t - \hat{n}^G_t;
\]

(7L)

(8L)

\(^{10}\) The choice of money supply rule instead of interest rate rule is because of the large sample span from 1964 to 2009, following NT.
\[
\frac{1}{vR^2} (n\hat{\nu}_{t+1} - \hat{x}_t) = \frac{K}{NW} \hat{r}_t^e - \left( \frac{K}{NW} - 1 \right) (\hat{r}_t^a - \pi_t) - \left( \frac{K}{NW} - 1 \right) \text{eff} \hat{p}_t + n\hat{\nu}_t; 
\]

(9L)

\[
\hat{\nu}_{t+1} = \hat{f}_t + \gamma \hat{\nu}_{t+1} + \gamma \hat{q}_t + (1 - \gamma)n\hat{\nu}_{t+1} + (1 - \frac{NW}{K})l_{t+1};
\]

(11L)

\[
\hat{\nu}_{t+1} = -\hat{q}_t + \frac{NW}{K} n\hat{\nu}_{t+1} + (1 - \frac{NW}{K})l_{t+1};
\]

(13L)

\[
eff \hat{p}_t = -\frac{EFP - 1}{EFP} \left[ \frac{\gamma NW/K}{1 - \gamma} + \frac{NW/K}{1 - \gamma} \right] \left( n\hat{\nu}_{t+1} - \hat{\nu}_{t+1} - \hat{q}_t \right); \]

(17L)

\[
\hat{q}_t = \varphi (\hat{r}_t - \hat{k}_t);
\]

(18L)

\[
\hat{k}_t = \hat{\alpha} + (1 - \delta) \hat{k}_t;
\]

(19L)

\[
\hat{\pi}_t = \beta \hat{\pi}_{t+1} + \left( 1 - \beta \theta \right) \left( 1 - \theta \right) \hat{\pi}_t;
\]

(20L)

\[
\hat{y}_t = \frac{C}{Y} \hat{c}_t + \frac{1}{Y} \hat{i}_t;
\]

(21L)

\[
\hat{\mu}_t = \hat{m}_t - \hat{m}_{t-1} + \hat{\pi}_t;
\]

(22L)

Given the log-linear version of the stochastic processes

\[
\hat{\alpha}_t = \rho_a \hat{\alpha}_{t-1} + \epsilon_{\alpha_t};
\]

(5L)

\[
\hat{x}_t = \rho_x \hat{x}_{t-1} + \epsilon_{x_t};
\]

(10L)

\[
\hat{f}_t = \rho_f \hat{f}_{t-1} + \epsilon_{f_t};
\]

(12L)

\[
\hat{\mu}_t = \rho_m \hat{\mu}_{t-1} + \epsilon_{\mu_t};
\]

(23L)

Eq. (1L) to (13L) and (17L) to (23L) are the log-linear version corresponding to Eq. (1) to (13) and (17) to (23). Following the convention, all the variables with hat on top denote percentage deviations from non-stochastic steady state, where I omit the conditional expectations operator on the assumption of ‘Certainty Equivalence’. Using Uhlig’s undetermined coefficients procedure yields a state space solution of the form\(^\text{11}\):

\(^\text{11}\) The detailed methodological exposition is given in Uhlig (1999), whereby the Matlab programme is available at his homepage (http://www2.wiwi.hu-berlin.de/institute/wpol/html/toolkit.htm).
\[
\hat{s}_{t+1} = \Omega_1 \hat{s}_t + \Omega_2 \varepsilon_{t+1};
\]
\[
\hat{d}_t = \Omega_3 \hat{s}_t;
\]
where the state variable vector, \( \hat{s}_t \), includes predetermined and exogenous variables; \( \hat{d}_t \) is the vector of control variables; and the vector \( \varepsilon_t \) contains the random innovations. The coefficient matrices, \( \Omega_1 \), \( \Omega_2 \), and \( \Omega_3 \), have elements that depend on the structural parameters of the model. Therefore, the state space solution, (24) and (25) is used later to construct underlying shocks and simulate the model.

### 3 Calibration

Before using the log-linear system above to construct the shocks and simulate the model, it is necessary to set values to all the structural parameters. In what follows, I set parameter values to calibrate the baseline model to quarterly data of the post war US economy. Since those parameter values are central to my shock extraction process, I try to keep them as close as possible to standard choice in the literature generally, and to NT specifically. There are in total 25 parameters, including those characterizing the shock processes. The discount rate \( \beta \) is set equal to 0.99 to match the average annual steady state real interest rate of 4%. The elasticity of substitution for consumption \( \eta^c \), real balances \( \eta^m \) and leisure \( \eta^x \) are all set equal to conventional value 1.5, implying a nearly logarithmic utility function. Habit persistence parameter \( \xi \) is 0.6. The weight on real balances \( \psi^m \) equals to 0.0019 to match the average M1 velocity of consumption. To reconcile the average working time of around 30%, the weight on leisure \( \psi^x \) is set to 2.47. The share of capital in goods production function \( \alpha \) and the capital depreciation rate \( \delta \) are fairly standard in real business cycle (RBC) literature, to which we set value of 0.36 and 0.025. For entrepreneurs’ surviving rate in the end of each period \( \nu \), I will use the value of 0.978, implying entrepreneurial average life of 45 quarters. The next two parameters, \( \varphi \) and \( \psi \), are key to BGG’s financial accelerator mechanism, since \( \varphi \) measures the level of capital adjustment cost and so the response of investment to shocks and \( \psi \) directly captures the degree of financial accelerator effect. Despite the dispute over
Table 1 Parameters in the model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>Household’s discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>( \eta', \eta'', \eta' )</td>
<td>Intertemporal elasticity of substitution</td>
<td>1.5</td>
</tr>
<tr>
<td>( \psi'' )</td>
<td>Weight on real balances in utility</td>
<td>0.0019</td>
</tr>
<tr>
<td>( \psi' )</td>
<td>Weight on leisure in utility</td>
<td>2.47</td>
</tr>
<tr>
<td>( \xi )</td>
<td>Habit persistence</td>
<td>0.6</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Share of capital in goods production</td>
<td>0.36</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Capital depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>( \theta_p )</td>
<td>Retailers’ probability of not able to reset price</td>
<td>0.5</td>
</tr>
<tr>
<td>( \theta_w )</td>
<td>Households’ probability of not able to reset wage</td>
<td>0.75</td>
</tr>
<tr>
<td>( \varepsilon_p )</td>
<td>Goods elasticity of demand</td>
<td>11</td>
</tr>
<tr>
<td>( \varepsilon_w )</td>
<td>Labour elasticity of demand</td>
<td>4</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Entrepreneurs’ surviving rate</td>
<td>9.78</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>Curvature of capital adjustment cost function</td>
<td>1</td>
</tr>
<tr>
<td>( \psi )</td>
<td>EFP elasticity of net worth to collateral value ratio</td>
<td>0.037</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Share of collateral in loan production</td>
<td>0.803</td>
</tr>
<tr>
<td>( \rho_a )</td>
<td>Autocorrelation of goods productivity shock</td>
<td>0.95</td>
</tr>
<tr>
<td>( \rho_m )</td>
<td>Autocorrelation of money growth rate</td>
<td>0.65</td>
</tr>
<tr>
<td>( \rho_x )</td>
<td>Autocorrelation of loan demand shock</td>
<td>0.9</td>
</tr>
<tr>
<td>( \rho_f )</td>
<td>Autocorrelation of loan supply shock</td>
<td>0.9</td>
</tr>
<tr>
<td>( \varepsilon_a, \varepsilon_m, \varepsilon_x, \varepsilon_f )</td>
<td>Standard deviations of the four shocks</td>
<td>0.0075</td>
</tr>
</tbody>
</table>

these two parameter values, I follow NT to set them to be 1 and 0.037 respectively. For the share of collateral in loan production function \( \gamma \), I refer to all relevant studies considering loan production (e.g., Goodfriend and McCallum (2007), Benk, Gillman and Kejak (2005), (2008) and (2010), and Leao (2003)). Since their chosen values for \( \gamma \) lie between 0.65 (lower bound) and 0.89 (upper bound), any value between the two bounds are plausible. I pick up aloan production (e.g., Goodfriend and McCallum (2007), Benk, Gillman and Kejak (2005),

---

12 Originally, BGG set \( \varphi \) equal to 0.25 and \( \psi \) equal to 0.05. These define a relatively low level of capital adjustment cost and high degree of financial accelerator effect. Christenson and Dib (2008) used maximum likelihood method to estimate the value of \( \varphi \) be 0.59 and \( \psi \) be 0.042 for US economy in the post Volcker era. Meier and Muller (2006) found an even higher value of \( \varphi \), 0.65, but insignificant \( \psi \).
Table 2  Steady state in the model economy

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Steady state values</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A$</td>
<td>Goods sector productivity level</td>
<td>1</td>
</tr>
<tr>
<td>$F$</td>
<td>Loan productivity level</td>
<td>2.15</td>
</tr>
<tr>
<td>$R_d$</td>
<td>Risk-free rate</td>
<td>1.01</td>
</tr>
<tr>
<td>$R^c$</td>
<td>Gross return on capital</td>
<td>1.0176</td>
</tr>
<tr>
<td>$R^n$</td>
<td>Nominal interest rate</td>
<td>1.0194</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Inflation rate</td>
<td>1.0092</td>
</tr>
<tr>
<td>$EFP$</td>
<td>External financing premium</td>
<td>1.0075</td>
</tr>
<tr>
<td>$N^G$</td>
<td>Labour service in goods sector</td>
<td>0.3</td>
</tr>
<tr>
<td>$N^F$</td>
<td>Labour service in banking sector</td>
<td>0.005</td>
</tr>
</tbody>
</table>

II. Steady-state ratios

<table>
<thead>
<tr>
<th>Ratio</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Y/K$</td>
<td>Output to capital</td>
<td>0.13</td>
</tr>
<tr>
<td>$I/Y$</td>
<td>Investment to output</td>
<td>0.19</td>
</tr>
<tr>
<td>$C/Y$</td>
<td>Consumption to output</td>
<td>0.61</td>
</tr>
<tr>
<td>$G/Y$</td>
<td>Govt expenditure to output</td>
<td>0.2</td>
</tr>
<tr>
<td>$L/K$</td>
<td>Leverage ratio</td>
<td>0.489</td>
</tr>
<tr>
<td>$N^F/N$</td>
<td>Financial hour to total hour</td>
<td>1.7%</td>
</tr>
</tbody>
</table>

aloan production (e.g., Goodfriend and McCallum (2007), Benk, Gillman and Kejak (2005), (2008) and (2010), and Leao (2003)). Since their chosen values for $\gamma$ lie between 0.65 (lower bound) and 0.89 (upper bound), any value between the two bounds are plausible. I pick up a value of 0.803 to make the elasticity of EFP with respect to the net worth to capital ratio in the banking model match that set in NT. The parameters associated with price and wage rigidity also follow that in NT, where the elasticity of demand for goods $\varepsilon_p$, and labour $\varepsilon_w$, are 11 and 4 such that the steady state markups are 10% in the goods market and 33% in labour market, and the probability of not reoptimizing for price setters $\theta_p$, is 0.5 while that for wage setters $\theta_w$, is 0.75.

Now we have calibrated values for 17 out of the 25 parameters, the last 8 are the parameters capturing the process of the underlying 4 shocks. Since the shock processes are
constructed in the next section, I give initial values for the 8 parameters in advance so that the model can be solved numerically. Some key steady state values in the model are also highlighted as follows. The external finance premium is generally unobservable in reality, hence we can only refer to some close indicators to pin down the steady state value. It is set to 1.0075 for baseline, corresponding to an annual risk spread of 300 basis points, approximating the post war average spread between the corporate bond rate and the three-month treasury bill rate. This is consistent with the estimates in Queijo (2009) and lies within the range reported in De Fiore and Uhlig (2005)\(^ {13}\). The steady state quarterly gross inflation is set to 1.0092, implying the nominal interest rate of 1.0194. Following NT, the steady state leverage ratio of entrepreneurs is set to 48.9%, which means the ratio of net worth to value of purchased capital is 0.511. The steady state consumption, investment and government expenditure share of GDP are given by 0.603, 0.192 and 0.205, respectively to match the historical average. In labour market, the steady state ratio of monitoring hour relative to goods produce hour is 1.7%. All the parameters and their calibrated values are described in table 1 while steady states are summarized in table 2.

4 Empirical Results

Based on the calibration discussed above, I carry on the evaluation of the empirical performance of the model. First of all, the four underlying shocks are constructed using the method proposed in Benk et al. (2005), (2008) and (2010) and NT. To check the robustness, the DSGE extracted TFP and monetary shock processes are compared to their counterparts derived from traditional estimation. Moreover, the two financial shocks are plotted against the post war recessions indicated by NBER on the one hand, and against the proxies of EFP on the other. The empirical performance of the model with financial shocks, both or either one, is evaluated by calculating second moments, historical decomposition and variance decomposition.

\(^{13}\) In De Fiore and Uhlig (2005), the reported range for annual risk premiums on bonds and loans in U.S. is between 160 and 340 basis points.
4.1 Construction of shocks

The assumed processes of the underlying four shocks are not appropriate for simulation until they are specified to be consistent with the baseline model. Two main reasons lie behind this. First of all, while goods sector productivity shock and monetary shock have non-controversy origins and can be easily backed up by conventional approach\textsuperscript{14}, there are no well agreed counterparts for financial shocks, especially when we are considering the shocks from both supply and demand sides. Assuming different financial structures in the model economy might imply different shock processes. For instance, in Benk et al. (2008) exchange credit model, the autocorrelation for credit shock is 0.93, and the standard deviation of innovation is 0.019. While in Atta-Mensah and Dib (2008) of credit creation model, the two corresponding parameters are 0.78 and 0.047 respectively. Christiano et al. (2010) report 0.53 and 0.025 for the financial wealth shock in their model. Based on these, I have to estimate the financial shock processes independently to capture the model consistent ones. Moreover, as argued in Ingram et al. (1994) and studies following up, any model that is in accord with the several time series that make up US macroeconomic data must feature multiple shocks that are correlated at all leads and lags. At least we cannot avoid the possibility of the correlations between the innovations driving the shock process. Therefore constructing consistent shocks nested in the model is not only desirable, but also necessary.

To construct the four underlying shocks, the procedure in NT is adopted\textsuperscript{15}, which is briefly described as follows. As assumed in section 2, the four shocks follow AR(1) processes. By giving initial values for the autocorrelation parameters, we can solve the model and recover the Markov decision rules numerically, which are written in state-space form as shown in Eq. (24) and (25). The model’s endogenous control variables are stacked in vector $\hat{d}$, and the endogenous and exogenous state variables are contained in vector $\hat{s}$. The sequence of the variables in $\hat{s}$ is ordered in such a way that the endogenous predetermined state variables appear first and the exogenous states follow up. Eq. (25) can now be written more explicitly as

\textsuperscript{14} Goods sector productivity shock can be estimated from constructed Solow Residuals. Monetary shock can be estimated by using data on money supply.

\textsuperscript{15} Benk et al. (2005), (2008) and (2010) apply the same procedure extensively in a series of papers. A similar application can also be found in Chari et al. (2007) where they are trying to realize all the underlying wedges.
\[
\hat{d}_t = \Omega_{31} \hat{s}_{1t} + \Omega_{32} \hat{s}_{2t} \\
= \Omega_{31} [k, n \hat{w}_{t-1}, \hat{q}_{t-1}, \hat{\mu}_{t-1}, \hat{\epsilon}_{t-1}, \hat{f}_{t-1}'] + \Omega_{32} [\hat{a}_t, \hat{\mu}_t, \hat{x}_t, \hat{f}_t']; \quad (25')
\]

By solving the model, we recover the two coefficients matrices, \( \Omega_{31} \) and \( \Omega_{32} \). In this case, we can estimate the processes of the four shocks if we assign values to \( \hat{d}_t \) and \( \hat{s}_{1t} \) from the data. This is straightforward from the ordinary least squares estimators for \( [\hat{a}_t, \hat{\mu}_t, \hat{x}_t, \hat{f}_t'] \) via the following transformation:

\[
[\hat{a}_t, \hat{\mu}_t, \hat{x}_t, \hat{f}_t'] = (\Omega_{32}' \Omega_{32})^{-1} \Omega_{32}' [\hat{d}_t - \Omega_{31} \hat{s}_{1t}]; \quad (26)
\]

The identification of the four underlying shocks requires the data for at least four variables contained in \( \hat{d}_t \). More than four variables simply give an over identification estimation for the shocks. The choice of the preferred combination of variables is discussed below.

Given the estimated series of the four shocks, what should be focused on next is to estimate each autocorrelation coefficient of the four processes. To account for the possible correlations between disturbances (heteroskedasticity), I apply the following seemingly unrelated regressions estimator (SURE)\(^{16}\):

\[
\begin{bmatrix}
\hat{a}_t \\
\hat{\mu}_t \\
\hat{x}_t \\
\hat{f}_t
\end{bmatrix} = 
\begin{bmatrix}
\rho_a & 0 & 0 & 0 \\
0 & \rho_m & 0 & 0 \\
0 & 0 & \rho_x & 0 \\
0 & 0 & 0 & \rho_f
\end{bmatrix}
\begin{bmatrix}
\hat{a}_{t-1} \\
\hat{\mu}_{t-1} \\
\hat{x}_{t-1} \\
\hat{f}_{t-1}
\end{bmatrix} + 
\begin{bmatrix}
\varepsilon_{at} \\
\varepsilon_{mt} \\
\varepsilon_{xt} \\
\varepsilon_{ft}
\end{bmatrix}; \quad (27)
\]

After obtaining the estimates of the first order autocorrelation coefficients for \( \hat{a}_t, \hat{\mu}_t, \hat{x}_t, \) and \( \hat{f}_t \), I substitute them back to the solution algorithm to get a new matrix \( \Omega_{32} \), then estimate the shock process again and proceed in an iterative fashion. Successive versions of \( \Omega_{32} \) are calculated until \( \rho_a, \rho_m, \rho_x \) and \( \rho_f \) converge. Then the ultimate estimated autocorrelations and variance-covariance matrix (VCM) are used in the solution algorithm to simulate the model.

As noted earlier, it should be borne in mind that the choice of the preferred combination of variables contained in \( \hat{d}_t \) is crucial to generate robust time series of the underlying shocks.

\(^{16}\) The reason why the off-diagonal elements in the autocorrelation matrix are zero will be discussed below.
Both Benk et al. (2005) and NT argue that different combinations of variables in \( \hat{d} \) yield different shock processes so that it is not easy to identify how to pick up the correct bunch of variables. To solve this potential problem, NT proposed a rule of thumb criteria which states that the sensible combination should produce estimated processes for productivity shock and monetary shock that are highly correlated with their conventionally constructed counterparts from single equation estimation\(^{17}\). Based on this, they generate a time series for the shock to entrepreneurial net worth (loan demand shock) on the condition that the constructed TFP and monetary shock have high correlation with their conventional counterparts: 0.76 and 0.94. I follow the same strategy here to estimate both the loan demand and supply shocks. During the estimation, I tried different combinations of variables in \( \hat{d} \) and distinguish the most plausible one that gives TFP and monetary shock highly correlated with their counterparts\(^{18}\). In particular, I picked up six variables that are suitable from \( \hat{d} \): \([\hat{y}_t, \hat{n}_t^G, \hat{n}_t^F, \hat{w}_t, \hat{\sigma}_t]\). Note that all of them are logged and linearly detrended, refer to Appendix B for data description. To rationalize the choice, \( \hat{y}_t, \hat{n}_t^G \) and \( \hat{m}_t \) are chosen to make plausible TFP and monetary shock while \( \hat{n}_t^F \) is used to capture the dynamics of credit supply shock, \( \hat{f}_t\). \( \hat{w}_t \) and \( \hat{\sigma}_t \) are also included in the estimation so that the credit demand shock is recovered close to that in NT\(^{19}\).

Figure 2.2 plots the DSGE constructed and traditionally estimated TFP processes covering the sample period between 1964Q2 and 2009Q4. It is clear to see that the DSGE derived shock (solid line) mimics the traditionally estimated shock (dashed line) very well, with a very high correlation coefficient of 0.97 between them. On the other hand, the comparison of monetary shock between the two derivations (shown in figure 3) is less satisfied with the corresponding correlation of 0.76. The main discrepancy stems from the first half of the sample period, where the DSGE derived shock always underpredicts that from traditional estimation. This feature also presents in NT’s estimation of monetary shock, despite the fact that they have a considerably higher correlation of 0.94 between the two. Since there are no extant conventional counterparts of financial shock, it is currently impossible to assess the

\(^{17}\) Productivity shock is easily constructed via the detrended Solow residuals, given the data on per capita GDP, capital stock and labour. Monetary shock is more straightforward to recover by using the data on M1.

\(^{18}\) Only the most plausible shock processes are plotted here while those from other combination are available upon request.

\(^{19}\) Actually, \( \hat{m}_t \) and \( \hat{\omega}_t \) are endogenous state variables and belong to \( \hat{\sigma}_{t+1} \). Picking them up is justified since their values are also determined in the Markov decision rules and can be treated equally as variables in \( \hat{d}_t \).
robustness of the estimation for financial shocks as we did for the previous two shocks. This also rationalizes the use of SURE as a plausible way to get autocorrelation coefficients and VCM discussed above. Nevertheless, the high correlation between the previous two and their corresponding counterparts justifies the validity of the two financial shocks from state-space derivation, plotted together in figure 4. The estimation of entrepreneurial net worth shock (left axis) is closely linked with that in NT\textsuperscript{20}, implying the shock construction process is not sensitive to the number of shocks. This shows a one step further validation of the loan productivity shock (right axis) generated here.

The time series properties of the four shocks can be summarized as follows. First of all, the first order autocorrelation coefficients for the four shocks are $\rho_a = 0.9433$, $\rho_m = 0.4189$, $\rho_x = 0.9796$, and $\rho_f = 0.8216$. The two financial shocks (demand and supply sides) are both more persistent than the growth rate of M1, but straddle the TFP shock. The net worth shock is more persistent than TFP while the loan productivity shock is less persistent. This ordering is consistent with that described in NT for the three shocks (excluding loan productivity) on the one hand and here on the other for the two shocks (TFP and money) derived from conventional estimation\textsuperscript{21}. Turning to the VCM of the disturbances, both of the two VCM from DSGE construction and from traditional estimation are shown below:

\textsuperscript{20} Refer to figure 3 in Nolan and Thoenisson (2009).
\textsuperscript{21} The conventional derived shocks show that TFP is more persistent than the growth rate of M1, with $\rho_a = 0.9556$, $\rho_m = 0.6097$. 
The comparison between DSGE constructed and traditionally estimated VCM shows that both of the DSGE TFP and money growth are slightly more volatile than their traditional counterparts. Moreover, the two are positively correlated in the two cases while the correlation between the DSGE derived ones is a little higher. The positive correlation between TFP and money growth is indicative of an historical accommodation of supply-side shocks by the Fed. Now focus on the VCM of DSGE constructed disturbances per se. The loan productivity (supply shock) is negatively correlated with TFP, but positively correlated with money growth, implying the loan supply side is more accommodative to monetary condition 22. On the other hand, the net worth shock is negatively correlated with money growth (consistent with NT), but positively correlated with TFP (by contrast with NT). It seems more favourable to positive correlation between TFP and asset bubble since asset prices always burst during recessions. The negative correlation to money growth implies that the Fed goes against asset price bubbles 23. It is noteworthy that the correlation between loan productivity and net worth

\[
VCM_{DSGE} = 10^{-4} \times \begin{bmatrix} 0.4123 & 0.1823 & 0.1718 & -0.9288 \\ 0.1823 & 2.4582 & -1.0387 & 1.8035 \\ 0.1718 & -1.0387 & 1.0586 & -0.0648 \\ -0.9288 & 1.8035 & -0.0648 & 23.1235 \end{bmatrix}; \quad VCM_{Tra} = 10^{-4} \times \begin{bmatrix} 0.3965 & 0.1145 \\ 0.1145 & 2.3861 \end{bmatrix};
\]

22 Benk et al. (2008) constructed an exchange credit shock process that also possesses positive correlation to money growth and negative correlation to TFP as the loan productivity extracted here. This shows a way of consistency despite that their shock is to consumption credit and my shock is for investment.

23 There is a literature focusing on whether the central bank should respond to asset price when conducting monetary policy. See Bernanke and Gertler (1999).
shock is slightly negative ($-0.0648 \times 10^{-4}$), for which it shows the identification of supply side from demand side shock in credit markets. The volatilities of the four shocks are ordered consistently to relevant studies in the literature. The net worth shock is more volatile than TFP but less volatile than money growth; in line with NT. The loan productivity is the most volatile one; also appears in Benk et al. (2005). This can be possibly understood by the fact that shock in a specific sector is much more volatile than TFP which is an aggregate shock that results in the smoothing of all the idiosyncratic shocks from different sectors.

It would be interesting to relate the DSGE extracted shocks with the post war NBER business cycle reference dates, which track recessions starting at the peak of a business cycle and ending at the trough. Figures 2 to 4 also highlight the 7 main recession episodes in the sample between 1964Q2 and 2009Q4 (including the most recent recession triggered by the sub-prime mortgage crisis in 2007): 1969Q4-1970Q4, 1973Q4-1975Q1, 1980Q1-1980Q3, 1981Q3-1982Q4, 1990Q3-1991Q1, 2001Q1-2001Q4 and 2008Q4-2009Q4 (end of sample). Figure 2.4 shows an apparent picture that every recession happened when both of the entrepreneurial net worth and loan production are in contraction. This is a fairly striking result, implying only one of the two financial shocks, either demand or supply side, is not strong enough to cause an economy-wide recession. For instance, the non recession era such as (1964-1965), (1985-1987) and (1992-1994) witness a contraction of entrepreneurial net worth while the loan productivity is in expansion. The boom in the supply of credit offsets the contraction in entrepreneurial sector and avoids economy-wide recessions. The reverse is
also true as shown in the recession episodes of 1973Q4-1975Q1 and 1981Q3-1982Q4 that the economic down-turn ceases earlier because the entrepreneurial sector recovers sooner than the credit supply. It is noteworthy that the recent recession could have recovered earlier since the loan supply started to expand during 2008 before the breakdown of Lehman Brothers, which reverse the credit expansion into a deeper contraction, as shown in the red line of figure 4. This implication is also apparent in Hirakata et al. (2010). On the other hand, the ink between the peaks and troughs of the business cycle and the realized TFP and money growth shocks is less obvious than the financial shocks, of which is also present in NT.

To assess the validity of the DSGE constructed financial shock, NT also plot it against the proxy of external finance premium, the spread between AAA rated corporate bonds and the 3-month Treasury bill rate, and find a strong negative correlation between the two. Here the same assessment procedure is followed, where both the loan productivity and net worth shock are plotted against the proxies of EFP. The spread of AAA rated corporate bonds as well as that of BAA and high yield bonds are used as the proxies. Figure 5 and 6 plot the HP-filtered net worth and loan productivity shock respectively against the three HP-filtered proxies.

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24 Refer to the figure 4 of their constructed shock processes with Bayesian approach.

25 Gertler and Lown (1999) argue that the high-yield bond spread emerges as a particularly useful indicator of the external finance premium and financial conditions more generally in the last two or three decades.
The two figures indicate that both of the two shocks are significantly negatively correlated with the three proxies of EFP. With stronger correlation, it seems that larger fraction of the cyclical EFP is accommodating the demand side of credit market. Nevertheless, the supply side effect shouldn’t be ignored completely. It is also found that the loan productivity leads those spreads for two or three quarters since the correlation between contemporaneous spreads and lagged loan productivity is higher in absolute value (not shown here), though the increment is fairly small (to about -0.37). This feature is not appearing for net worth shock. The corresponding correlation between TFP and spreads, or money growth and spreads is fairly weak: (-0.03 to -0.13) and (0.14 to 0.21).

### 4.2 Impulse responses to financial shocks

This section briefly examines the impulse responses of the model economy to the two financial shocks. Figure 7 and 8 plot the responses of six variables that are attractive: output, investment, loan, capital price (Tobin’s q), net worth and external finance premium. It is apparent that the two negative shocks both drive down the economy as expected. The effect of the net worth shock is very strong and persistent; the economy goes to downturn for a very long period, as shown in NT. The responses to loan supply contraction is relatively weak, but still significant. The effect is also less persistent as the economy reverts back to steady state quickly. One interesting thing to notice is that the responses of loan volume are in opposite
Fig. 7 Impulse responses to financial shock (net worth)

Direction subject to the two shocks which are in the same direction. This rationalizes the earlier claim that the entrepreneurial net worth shock behaves more like a credit demand shock, while the loan productivity resembles a supply shock. Subject to a negative shock to net worth, the wealth of entrepreneurs contracts hugely because of the persistent effect; drops much deeper than that of capital demand. This makes the demand for external funds larger than before, pushing up the EFP. Thus a negative shock to net worth behaves like a positive shock to loan demand, driving up loan (quantity) and EFP (price) simultaneously. On the other hand, a negative shock to loan productivity resembles a credit supply contraction, accompanied with declining loan volume (quantity) and increasing EFP (price).

4.3 Second moments

Comparing the second moments of the model simulated series with the moments of the empirical series from the data is the traditional way to evaluate the performance of the business cycle models. Here I follow this strategy to assess how the business cycle performance of the model is altered after we add in the two financial shocks individually on
the one hand and together on the other. Table 3 summarizes the second moments of the key variables from data (1964:1-2009:4) and compares them with the data generated by four models (estimates from 100 repeated stochastic simulation) that are identical except that: Model 1 has both financial shocks; Model 2 wipes out the supply sided financial shock; Model 3 gets rid of the net worth shock; Model 4 has no financial shock.

For the volatility part, as shown in panel A, the models with financial shocks, either one or both, come closer to data for output, investment, loan, hours, M1, inflation and EFP. Among the models with financial shocks, Model 3 comes closer to data for consumption, investment, real wage while Model 2 better matches for loan, hours and M1. For EFP, Model 3 performs as well as Model 2. The latter reconciles with the result highlighted in Nolan and Thoenissen (2008). There are two variables that all the models fail to predict the moments completely. One is the net worth for which all the models overpredict by 4 to 8 times; the other one is the nominal interest rate where all types predict less than 10% to the empirical counterpart.

For the correlation with output, as shown in panel B, all the models (Model 1 to Model 4) correctly predicts the sign of the correlation with GDP for consumption, investment, hours,

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26 The statistics for the model without financial frictions are not compared since Nolan and Thoenissen (2008) have done that extensively.

27 The shocks in Model 2, Model 3 and Model 4 are constructed separately by the same methodology described above.
Table 3 Second Moments (Data 1964Q2 to 2009Q4)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Data</th>
<th>Model 1 (NW&amp;LP shock)</th>
<th>Model 2 (NW shock only)</th>
<th>Model 3 (LP shock only)</th>
<th>Model 4 (No F_shocks)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% S.D.</td>
<td>Relative to output</td>
<td>% S.D.</td>
<td>Relative to output</td>
<td>% S.D.</td>
</tr>
<tr>
<td>Output</td>
<td>1.58</td>
<td>1.00</td>
<td>1.60</td>
<td>1.00</td>
<td>1.61</td>
</tr>
<tr>
<td>Consumption</td>
<td>1.29</td>
<td>0.82</td>
<td>1.56</td>
<td>0.97</td>
<td>1.60</td>
</tr>
<tr>
<td>Investment</td>
<td>5.26</td>
<td>3.33</td>
<td>8.12</td>
<td>5.07</td>
<td>8.19</td>
</tr>
<tr>
<td>Loan</td>
<td>2.29</td>
<td>1.45</td>
<td>1.74</td>
<td>1.09</td>
<td>1.84</td>
</tr>
<tr>
<td>Hours</td>
<td>1.88</td>
<td>1.19</td>
<td>2.07</td>
<td>1.29</td>
<td>2.03</td>
</tr>
<tr>
<td>Real wage</td>
<td>0.97</td>
<td>0.61</td>
<td>0.76</td>
<td>0.48</td>
<td>0.75</td>
</tr>
<tr>
<td>Real M1</td>
<td>3.21</td>
<td>2.03</td>
<td>2.57</td>
<td>1.61</td>
<td>2.63</td>
</tr>
<tr>
<td>Nominal rate</td>
<td>0.41</td>
<td>0.26</td>
<td>0.03</td>
<td>0.02</td>
<td>0.03</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.29</td>
<td>0.18</td>
<td>0.53</td>
<td>0.33</td>
<td>0.52</td>
</tr>
<tr>
<td>EFP</td>
<td>0.35</td>
<td>0.22</td>
<td>0.43</td>
<td>0.27</td>
<td>0.33</td>
</tr>
<tr>
<td>Net worth</td>
<td>2.25</td>
<td>1.42</td>
<td>17.04</td>
<td>10.65</td>
<td>17.23</td>
</tr>
</tbody>
</table>

B. Contemporaneous correlation with output (S.E.)

<table>
<thead>
<tr>
<th>Variables</th>
<th>Consumption</th>
<th>Investment</th>
<th>Loan</th>
<th>Hours</th>
<th>Real wage</th>
<th>Real M1</th>
<th>Nominal rate</th>
<th>Inflation</th>
<th>EFP</th>
<th>Net worth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.87</td>
<td>0.90</td>
<td>0.26</td>
<td>0.86</td>
<td>0.14</td>
<td>0.14</td>
<td>0.38</td>
<td>0.15</td>
<td>-0.65</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>0.34 (0.14)</td>
<td>0.82 (0.05)</td>
<td>-0.24 (0.16)</td>
<td>0.86 (0.03)</td>
<td>-0.05 (0.15)</td>
<td>0.19 (0.14)</td>
<td>0.19 (0.15)</td>
<td>0.40 (0.09)</td>
<td>-0.72 (0.07)</td>
<td>0.81 (0.06)</td>
</tr>
<tr>
<td></td>
<td>0.34 (0.12)</td>
<td>0.81 (0.05)</td>
<td>-0.24 (0.13)</td>
<td>0.86 (0.03)</td>
<td>-0.03 (0.13)</td>
<td>0.19 (0.13)</td>
<td>0.34 (0.12)</td>
<td>0.37 (0.09)</td>
<td>-0.77 (0.06)</td>
<td>0.80 (0.06)</td>
</tr>
<tr>
<td></td>
<td>0.96 (0.01)</td>
<td>0.95 (0.01)</td>
<td>-0.17 (0.15)</td>
<td>0.86 (0.04)</td>
<td>-0.28 (0.15)</td>
<td>0.95 (0.01)</td>
<td>-0.10 (0.09)</td>
<td>0.69 (0.05)</td>
<td>-0.64 (0.08)</td>
<td>0.94 (0.02)</td>
</tr>
<tr>
<td></td>
<td>0.97 (0.01)</td>
<td>0.96 (0.01)</td>
<td>-0.17 (0.14)</td>
<td>0.85 (0.04)</td>
<td>-0.25 (0.16)</td>
<td>0.96 (0.01)</td>
<td>-0.12 (0.09)</td>
<td>0.67 (0.05)</td>
<td>-0.90 (0.03)</td>
<td>0.94 (0.02)</td>
</tr>
</tbody>
</table>

M1, inflation, EFP and net worth. For nominal interest rate, Model 1 and 2 can predict the correct sign while Model 3 and 4 fail. None of the four models predict the sign for loan and real wage correctly despite that both variables have nearly acyclical behaviour. On the quantitative perspective, Model 1 and 2 underpredict the correlation with output of consumption, investment while Model 3 and 4 overpredict. All the four versions of the model overpredict the correlation for M1, inflation and net worth. The only variable that all the four models capture perfectly simultaneously is hours. EFP, the most important variable in this study, is captured better by models with financial frictions and Model 3 performs best.
Comparing across models with and without financial shocks reveals mixed results in the performance assessment. The contribution of financial shocks is not straightforward to identify. This complements some recent findings that financial accelerator plays limited role in the model’s transmission mechanism\textsuperscript{28}. The main contribution of financial shocks in terms of matching the data’s second moments over the sample period is the ability to match the second moments of the EFP. For both the volatility and correlation with output, Model 3 with loan productivity shock on top of TFP and monetary shock performs the best. Although Model 1 and 2 are not far away from Model 3, the importance of supply side financial shock to determine the dynamics of EFP is revealed clearly.

### 4.4 Historical decomposition of EFP

To further confirm the conjecture from previous section that the loan productivity shock plays an important role in determining the dynamics of EFP, this part decomposes the variation of cyclical EFP into the four underlying shocks. Figure 9 displays the time path of AAA spread (EFP proxy) and the contribution of each structural shock. The solid black line is the data of cyclical AAA spread from 1964Q2 to 2009Q4. Red and blue bars refer to the contribution of monetary and TFP shock respectively while green and purple bars represent the contribution of the two financial shocks; net worth and loan productivity. The effect of TFP to the cyclical AAA spread is minor. The contribution of monetary shock seems

\textsuperscript{28} See Meier and Muller (2006), Christensen and Dib (2008) who use strict econometric testing.
Table 4  Variance decomposition

<table>
<thead>
<tr>
<th>Variables</th>
<th>Percentage owing to:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TFP</td>
</tr>
<tr>
<td>Output</td>
<td>8.72%</td>
</tr>
<tr>
<td>Consumption</td>
<td>19.84%</td>
</tr>
<tr>
<td>Investment</td>
<td>1.96%</td>
</tr>
<tr>
<td>Loan</td>
<td>18.86%</td>
</tr>
<tr>
<td>Hours</td>
<td>13.69%</td>
</tr>
<tr>
<td>Nominal rate</td>
<td>12.63%</td>
</tr>
<tr>
<td>Inflation</td>
<td>27.01%</td>
</tr>
<tr>
<td>EFP</td>
<td>3.63%</td>
</tr>
<tr>
<td>Net worth</td>
<td>2.16%</td>
</tr>
</tbody>
</table>

larger, but still moderate. This can explain why the second moments (both the volatility and correlation with output) generated by model excluding financial shocks are far away from their empirical counterparts. The figure clearly shows that the seven notable economic recessions within the sample period, 1969Q4-1970Q4, 1973Q4-1975Q1, 1980Q1-1980Q3, 1981Q3-1982Q4, 1990Q3-1991Q1, 2001Q1-2001Q4 and 2008Q4-2009Q4, correspond well with the episodes when the AAA spreads are around cyclical peaks. Meanwhile, the cyclical peaks are contributed mostly by the two financial shocks. Each recession episode is accompanied by the situation when both of the two financial shocks predict a high EFP. The recent financial crisis is a notable case such that the EFPs driven by net worth as well as loan productivity are both unprecedentedly high. On the other hand, if the two shocks predict different sign for EFP, the AAA spread is only moderate and the economy is not in recession (e.g., mid 1980s and mid 1990s). After combining figure 4 and 9, it is fair to claim that both of the two shocks from financial sector are significant and important to determine the cyclical behaviour of EFP.

4.5 Variance decompositions

In this section, I measure the contribution of each of the four shock processes to EFP as well as other key macroeconomic time series. Here I follow the procedure applied in Hirakata et al. (2010). Table 4 reports the variance decompositions for output, consumption, investment,
loan, hours, federal funds rate, inflation, EFP and net worth. It is apparent that the net worth shock accounts for the largest fraction of variation for output, investment, loan, hours, federal funds rate, EFP and net worth itself. These results are in line with that in NT, predicting the net worth shock is the dominant force for the variation of above variables. Not surprisingly to see the fluctuations of inflation is mainly determined by the monetary shock, which is the genuine natural cause of inflation. For EFP, we can still find the significant influence from the loan productivity shock, despite the main dominant force is still net wealth shock. About one third of the total effect on EFP from financial sector comes from the supply side of the credit market.

2.5 Conclusion

In this chapter, I make one step further based on the previous chapter to quantitatively assess the role played by the shocks originated from the banking sector in the U.S. business cycle. Specifically, I build a model that generally follows the setup of BGG and NT except for the banking sector, where I replace the optimal contracting problem of BGG with an explicit profit maximization problem in the banking sector subject to a loan production function. In consequence, both the demand side (entrepreneurial sector) and supply side (banking sector) of the credit market contribute to the financial frictions. In the model context, four exogenous shocks, including two conventional structural shocks (TFP and monetary) and the two financial sector shocks (entrepreneurial net wealth and loan productivity), are constructed by combining the data and the model’s linear state-space solution. The shock to the technology in loan production resembles the disturbance originating in the banking sector. Together with the shock to entrepreneurial net wealth, we can analyse the contribution of the disturbance from both the supply and demand side of the financial market.

The main results can be summarized as follows. First of all, the DSGE extracted TFP and monetary shock are observationally similar to their counterparts constructed with traditional estimation while entrepreneurial net worth shock is close to the one constructed in NT. Secondly, every post war recession is crashed with the situation that both the entrepreneurial

29 In calculating the variance decompositions, I first calculate the volatility of the endogenous variable conditional on each of the shocks, and then sum these volatilities to calculate the share of each shock.
net worth and the loan productivity shock are in contraction; either one of them is not strong enough to cause an economy-wide recession. Thirdly, both of the extracted loan productivity and net worth shock are negatively correlated with proxies of EFP. Fourth, the variance decomposition indicates that loan productivity shock is an important source of EFP variation, though the dominant driving force is still net worth shock. Finally, net worth shock is still a dominant factor even after we include the loan productivity shock.

This study rationalizes one interesting point raised in NT that the entrepreneurial net worth shock remains contractionary after recessions have ended. This is because the shock from the other side of the credit market, the loan productivity, starts the expansion. Another notable point from the analysis is subject to the recent economic recession. The terrible economic downturn could stop earlier since the loan supply is found to expand during 2008. However, the breakdown of Lehman Brothers reverses the credit expansion into a deeper contraction. This point is still searching for the empirical support.

For future research, it is always attractive to incorporate the banks’ balance sheet condition into the analysis. The importance of bank capital, either for the transmission mechanism or as an independent source of aggregate fluctuations, is still under exploration.
Appendix

A.1  Proof of proposition 1

Given the equation of external finance premium expressed in steady state

$$EFP - 1 = \frac{1}{1 - \gamma} \frac{1}{1 - w} F \left(1 - \frac{\text{NW}}{\text{QK}}\right)^{\frac{\gamma}{1 - \gamma}} \left[1 - \frac{\text{NW}}{\text{QK}}\right];$$  \hspace{1cm} (A1)

The first and second derivatives of EFP with respect to net worth capital ratio are

$$\frac{\partial EFP}{\partial (\text{NW}/\text{QK})} = -\frac{1}{1 - \gamma} \frac{1}{1 - w} F^{\frac{1}{1 - \gamma}} \frac{\gamma}{1 - \gamma} \left(1 - \frac{\text{NW}}{\text{QK}}\right)^{\frac{2\gamma - 1}{1 - \gamma}} \left[1 - \frac{\text{NW}}{\text{QK}}\right];$$ \hspace{1cm} (A2)

$$\frac{\partial^2 EFP}{\partial (\text{NW}/\text{QK})^2} = \frac{1}{1 - \gamma} \frac{1}{1 - w} F^{\frac{1}{1 - \gamma}} \frac{\gamma}{1 - \gamma} \left[2\gamma - 1\right] \left(1 - \frac{\text{NW}}{\text{QK}}\right)^{\frac{3\gamma - 2}{1 - \gamma}} \left[1 - \frac{\text{NW}}{\text{QK}}\right];$$  \hspace{1cm} (A3)

It is easy to verify that the first order derivative comes out with negative sign while the second order derivative is positive, so prove the proposition.

A.2  Data description

Data are expressed in per-capita terms using population over 16 (expressed in billions) in quarterly base.

GDP, $Y_t$: Gross Domestic Product, in billions of dollars, deflated by the Implicit Price Deflator of GDP. Source: Bureau of Economic Analysis.

Consumption, $C_t$: Personal Consumption Expenditures (non-durables plus services), in billions of dollars, deflated by the Implicit Price Deflator of GDP. Source: Bureau of Economic Analysis.

Investment, $I_t$: Private Fixed Investment, in billions of dollars, deflated by the Implicit Price Deflator of GDP. Source: Bureau of Economic Analysis.
Inflation, $\pi_t$: first difference of the log of the Implicit Price Deflator of GDP. Source: Bureau of Economic Analysis.

Real wage, $w_t$: Real Average Hourly Compensation for nonfarm business sector. Source: Bureau of Economic Analysis.

Wage inflation, $\omega_t$: first difference of the log of the Average Hourly Compensation for nonfarm business sector. Source: Bureau of Economic Analysis.

Capital, $K_t$: quarterly series is constructed using annual capital stock data and quarterly data on investment expenditure. Source: Bureau of Economic Analysis.

Tobin’s q, $Q_t$: constructed using Eq. (2.18L) and the data on investment and capital.

Net worth, $NW_t$: nonfarm nonfinancial corporate business net worth (market value) taken from the flow of funds account.

Money, $M_t/P$: Real per capita M1.

Nominal interest rate, $R^n_t$: 3-month average of the daily effective federal funds rate. Source: Federal Reserve System.

Hours worked, $N^{G}_t$: nonfarm business sector index, hours of all persons. Source: Bureau of Labor Statistics.

References


