Delay and Haircuts in Sovereign Debt: Recovery and Sustainability

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Abstract

A striking aspect of recent sovereign debt swaps is that, conditional on default, long delay is positively correlated with size of the haircut. In this paper, we develop a model of debt restructuring to account for this – highlighting economic recovery and sustainability as complementary reasons for delay. We show how growth (recovery) and sustainability together can explain multi-period delay, due initially to wait for recovery and then to permit debtor to signal about the sustainability concern. We argue that it is the extension of delay that leads to the larger haircut.

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Key Words: Debt restructuring, delay, growth, sustainability, asymmetric information, signalling

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I. INTRODUCTION

Over the past few decades, a large number of developing and less developed countries have rescheduled their payments on loans owed to the foreign creditors. Bulow and Rogoff (1989) investigate the bargaining process that governs the rescheduling agreements or negotiated partial defaults on these countries’ debts. They analyse debt restructuring by applying the alternating-offers bargaining approach of Rubinstein (1982), assuming that the sovereign debtor and creditor bargain over welfare gains from trade. This application has its attractions, and such bargaining framework has been extended by Bhattacharya and Detragiache (1994) to include the role of the Multilateral Institutions in their study of debt reduction operations. A key prediction from this type of bargaining framework is that Pareto efficient settlement will be reached without significant delay.

However, a careful attention to several sovereign bond restructurings reveals that negotiations to restructure sovereign debt are usually time-consuming and involve some delays, which could be costly for different parties involved. On the part of the sovereign debtor, the country could experience disruptions in its access to international capital market or international trades, while the creditors could suffer large losses in the value of their investments (Pitchford and Wright, 2008). Why is it so difficult to restructure sovereign debts in a timely manner? The answer to this question indeed varies, depending on the viewpoint and the approach undertaken in looking at this issue.

Some literature highlights the problem associated with multiple creditors in the sovereign debt restructuring process. Kletzer (2002), Haldane et al. (2004) and Weinschelbaum and Wynne (2005) study how the problem of creditor coordination could pose important problem in the restructuring
process; however, delay does not occur. In the theoretical analysis of Pitchford and Wright (2008), delay in the bargaining between sovereign debtor and international creditors could result from an imperfect creditor coordination. They argue that when the enforcement of the sovereign’s contractual commitments is weak, the strategic holdup by some creditors could be the main cause of delay in the restructuring of sovereign debt.

Other strand of literature provides an alternative explanation for delay in the sovereign debt restructuring. Merlo and Wilson (1998) have shown that, in the absence of state contingent contracts, an efficient delay can exist when both the sovereign debtor and the creditor understand that the size of the bargaining surplus follows a stochastic process, i.e. the economy could be in a boom or a slump. In such framework, settlement may well be delayed until the economy recovers.

This paper differs from the aforementioned literature in that creditor coordination problem is absent from our analysis as our model focuses on the bargaining between a sovereign debtor and a creditor. As a consequence, delay in sovereign debt negotiation cannot be caused by the strategic holdup between the creditors as in Pitchford and Wright (2008). With stochastic bargaining surplus, the result from our analysis is consistent with Merlo and Wilson (1998) in that a one-period delay could occur to permit for economic recovery. However, since the “efficient delay” that results from Merlo and Wilson framework benefits both debtor and creditor, this is not consistent with the observation that several of the sovereign bond restructurings during 1998 and 2005 have been characterised by a positive correlation between protracted negotiation (delay) and haircuts (write-down of face value).1

1 See Section 3 for further discussion about the empirical evidence on delay and haircuts and the restructuring of Argentine debts as the ‘Case in Point’.

Why are haircuts relevant in sovereign debt restructuring process? Sturzeneg-
ger and Zettelmeyer (2007) discuss how defaults and subsequent settlement affect the creditors. The losses that defaults have inflicted on creditors are largely based on the comparison between the (remaining) payment stream that was originally promised to investors and the payment stream associated with the restructured instruments, both discounted at a common interest. Sturzenegger and Zettelmeyer (2005) calculate the haircuts associated with bond exchanges and restructurings of 1998 to 2005. They find that there are very large variations in the average level of haircuts across debt restructuring episodes. Present value “haircuts” ranged from around 5-20 percent for Uruguay (2003) to over 50 percent for Russia (2000) and over 70 percent for Argentina (2005), with the remaining exchanges falling mostly in the 20 to 40 percent range.

Is long delay in restructuring process positively correlated with size of the haircut? This paper aims to develop a simple model, which helps us finding answer to this question. To investigate whether a positive correlation between delay and haircuts exists, we study an infinite-horizon bargaining model with stochastic bargaining surplus and when there is asymmetric information about the debtor’s sustainability constraint. Our results show that there will be a positive probability of a two-period delay in bargaining: initially reflecting recovery, followed by signalling by debtor about sustainability constraint.

The remainder of the paper is structured as follows. Section 2 presents the model and the derivation of Perfect Bayesian Equilibrium under different scenarios. In Section 3, we discuss the empirical evidence on delay and haircuts in sovereign debt, while Section 4 concludes. The technical materials and the derivation of Perfect Bayesian Equilibrium in pure strategy can be found in the appendix.
II. THE MODEL

The Environment

We model a sovereign debtor who is in default. Conditional on default, there is bargaining between debtor and creditor in term of restructuring the sovereign debts. The resource that is available for bargaining is the tax revenue generated by accessing the international capital market. We suppose that the debtor uses the tax revenue for two main purposes, namely spending on public activities and debt repayment. For the debtor to repay the debts, he has to divert some money away from other public activities. The debtor’s “offer” to the creditor then corresponds to the amount of tax revenue diverted from other public activities. For expositional purpose, we will refer to the tax revenue in our discussion in the rest of the paper as \( \pi \), where \( \pi > 0 \). Since we look at the world in which the debtor is in default, we suppose that the value of \( \pi \) at the initial period is low, denoted by \( \pi_L \), to capture the economic recession. However, in the subsequent periods, we allow \( \pi \) to be stochastic, \( \pi \) can continue to be low at \( \pi_L \) with probability \( p \) or grows to a higher level, \( \pi_H \), with probability \( 1 - p \), where a growing \( \pi \) corresponds to an increase in the tax revenue.

Sustainability constraint, \( s \), represents the minimum fraction of the tax revenue the debtor has to retain for himself. Alternatively, it denotes a minimum acceptable share of the bargaining surplus for the sovereign debtor. Such constraint denotes a maximum amount of tax revenue available to service debt in default consistent with economic and political stability of the debtor country. The sustainability constraint determines the type of the debtor. When \( s \) is close to 0, the debtor is Optimistic, while when \( s \) is close to 0.

\(^2\)In general, the bargaining surplus refers to the gain for sovereign debtor from a successful debt restructuring.
a particular level $\bar{s} > 0$, the debtor is Cautious. The Optimistic and Cautious debtors differ in the utility they obtain from consuming their part of the bargaining surplus. Let $CR$, $C$ and $O$ denote creditor, Cautious debtor and Optimistic debtor, respectively. While the utility for the Optimistic debtor is linear in his own payoff, there is a discontinuity in the Cautious debtor’s utility to reflect his concern about the sustainability of any settlement. Let $u_i$ represents the utility of type-$i$ debtor and $\Delta_i$ denotes the type-$i$’s debtor’s share of the available bargaining surplus, where $i = C, O$. Formally, for the Optimistic debtor, $u_O = \Delta_O$ for all values $\Delta_O$, while for the Cautious debtor, $u_C = 0$ for $\Delta_C < \bar{s}$, and $u_C = \Delta_C$ for $\Delta_C \geq \bar{s}$.

We suppose that, if the game continues to $t = 2$, the nature chooses $s \in \{0, \bar{s}\}$ with probability $\{q, 1 - q\}$. We assume that the uncertainty with respect to $\pi$ and with respect to debtor’s type are resolved at $t = 2$. However, there is an asymmetric information over the debtor’s type: the sovereign debtor knows his own type, while the creditor does not know the type of debtor. The creditor believes that the debtor is Optimistic with probability $q_0$, where $q_0$ denotes the initial prior of the creditor.

We consider an infinite-horizon bargaining model. The bargaining game is specified as follows. We assume that the sovereign debtor makes the offer at $t = 1$, but, in the subsequent periods, each party has an equal probability of making an offer (being the proposer). Offers are made at discrete points in time, namely, at times $1, 2, 3, \ldots$. An offer is a number greater than or equal to zero and less than or equal to $\pi$. If the offer is accepted, the game ends; otherwise, the game continues to the next period. This process continues until the offer is accepted. Suppose that a common discount factor is $\delta < 1$. If there is disagreement, the country is excluded
from access to the international capital market. As a consequence, the sovereign debtor cannot tax the domestic economic activity that rely on the continuous access to the international capital market. Formally, this is translated into an assumption that the disagreement payoffs are zero for both players. The timing of events is summarised in Table 1 below.

<table>
<thead>
<tr>
<th>Size of $\pi$</th>
<th>Debtor’s type</th>
<th>Proposer</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 1$</td>
<td>$\pi_L$</td>
<td>Debtor makes offer</td>
<td>If offer accepted, game ends. If offer rejected, game continues.</td>
</tr>
<tr>
<td>$t = 2$</td>
<td>$\pi_L$ or $\pi_H$ with ${p, 1-p}$; Uncertainty over $\pi$ is resolved.</td>
<td>Debtor makes offer with prob $\frac{1}{2}$ Creditor makes offer with prob $\frac{1}{2}$</td>
<td>If offer accepted, game ends. If offer rejected, game continues.</td>
</tr>
<tr>
<td>$t = 3, ...$</td>
<td>$s \in {0, \tilde{s}}$ with ${q, 1-q}$; Uncertainty over debtor’s type is resolved</td>
<td>Debtor makes offer with prob $\frac{1}{2}$ Creditor makes offer with prob $\frac{1}{2}$</td>
<td>If offer accepted, game ends. If offer rejected, game continues.</td>
</tr>
</tbody>
</table>

| Debtor makes offer with prob $\frac{1}{2}$ Creditor makes offer with prob $\frac{1}{2}$ | If offer accepted, game ends. If offer rejected, game continues. |

**Table 1:** Timing of events

In general, we solve for the *Perfect Bayesian Equilibrium* of the model. We look at three cases: (1) the size of $\pi$ is fixed at $\pi_L$ for all periods and the type of debtor is known, (2) the bargaining surplus, $\pi$, can change over

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3In Eaton and Gersovitz (1981), when there is default, the debtor cannot access the international capital market to borrow for consumption smoothing.
time with some probability but the debtor’s type is known, and (3) \( \pi \) can change over time and there is an asymmetric information about the debtor’s type. We consider each of these cases below.

**Case 1: The Benchmark Case**

In this case, we assume that the size of bargaining surplus is fixed at \( \pi_L \) for all periods. Moreover, there is a complete information about the type of the debtor: the debtor could either be Optimistic or Cautious. These assumptions imply that this game is, in fact, reduced to a standard Rubinstein bargaining game (Rubinstein, 1982). We solve for the Perfect Bayesian equilibrium. Since this is an infinite-horizon game, one cannot use the backwards induction method. We arbitrarily select one period and derive the equilibrium offers and payoffs for the creditor and the debtor when the debtor is Optimistic and Cautious, respectively. It is important to note that, in the case of a Cautious debtor, his concern about the sustainability of any settlement, i.e. the presence of a sustainability constraint, reduces the amount of surplus that is available for bargaining from \( \pi_L \) to \( \pi_L - \delta \).

The following proposition summarises our results under this benchmark case.

**Proposition 1** *In the Perfect Bayesian equilibrium, each player makes the same offer in each period whenever he is chosen to make an offer. If the debtor is Optimistic, the offer made by the debtor and the creditor whenever they are chosen to make an offer will always be \( \frac{\delta \pi_L}{2} \) and the payoff for each party will always be \( \frac{\pi_L}{2} \). If the debtor is Cautious the offer made by the debtor and the creditor whenever they are chosen to make an offer will always be \( \frac{\delta (\pi_L - \delta)}{2} \). The payoffs for the Cautious debtor and the creditor will always be \( \left( \frac{\pi_L + \delta}{2} \right) \) and \( \left( \frac{\pi_L - \delta}{2} \right) \), respectively. Moreover, both parties always reach an agreement without delay.*
Proof. See Appendix A. ■

Case 2: Stochastic Bargaining Surplus

Under this case, the debtor’s type is known: the debtor is either Optimistic or Cautious. Although we suppose that the bargaining surplus at $t = 1$ is $\pi_L$, in the subsequent periods, $\pi$ can be $\pi_L$ with probability $p$ or increase to a higher level, $\pi_H$, with probability $1 - p$, where $\pi_L$ and $\pi_H$ capture the prospects of recession and recovery, respectively. We assume that the uncertainty with respect to $\pi$ is resolved at $t = 2$. We solve for the Perfect Bayesian equilibrium.

We begin with period $t = 2$. First, let us consider the case where the debtor is Optimistic. Following the arguments presented in the benchmark case, in the Perfect Bayesian equilibrium, the payoffs for the Optimistic debtor and the creditor at $t = 2$ are $\left(\frac{\pi + \delta}{2}, \frac{\pi}{2}\right)$, respectively. Next, in the case of a Cautious debtor, in the Perfect Bayesian equilibrium, again following the arguments presented in the benchmark case, the payoffs for the Cautious debtor and the creditor at $t = 2$ are $\left(\frac{\pi + \delta}{2}, \frac{\pi - \delta}{2}\right)$, respectively.

Moving to the first period, we calculate the continuation values for each player. Let the expected size of the tax revenue, $\pi$, be denoted by $E\pi$, where $E\pi = p\pi_L + (1 - p)\pi_H$, and let $\delta$ denote a common discount factor. In the case of an Optimistic debtor, the continuation value is the same for the debtor and the creditor, namely $\frac{\delta E\pi}{2}$. In the case where the debtor is Cautious, the continuation values for the debtor and the creditor are $\frac{\delta (E\pi + \delta)}{2}$ and $\frac{\delta (E\pi - \delta)}{2}$, respectively.

At $t = 1$, the sovereign debtor is a proposer. The continuation values computed earlier would limit the offers that can be made by the debtor\textsuperscript{4}.

\textsuperscript{4}The current bargaining surplus, $\pi_L$, can be shared between debtor or creditor, but doing so means giving up on the prospect of economic growth. Thus, it is a primitive endogenous growth model like that of Merlo and Wilson (1998).
With relatively low growth prospects, the bargaining model predicts an immediate settlement. Let $E_g$ denote the expected growth of the economy and let $r$ denote the time rate of discount, where $E_g = \frac{(E \pi - \pi \lambda)}{\pi \lambda}$ and $r = \frac{1-\delta}{\delta}$. Delay occurs at $t = 1$ when the debtor’s best offer falls below the creditor’s continuation value, rendering it to be rejected by the creditor.

Proposition 2 summarises the condition which make delay becomes attractive for both types of debtor.

**Proposition 2** When the debtor’s type is known but there is uncertainty with respect to $\pi$, there will be a positive probability for a one-period delay in the bargaining to restructure sovereign debts provided that the expected growth of the economy exceeds the rate of discount, i.e. $r < E_g$. Such delay is to permit for economic recovery.

**Proof.** See Appendix A.

This condition for one-period delay is essentially the same as that in Merlo and Wilson (1998).

**Case 3: Stochastic Bargaining Surplus and Asymmetric Information about Debtor’s Type**

Now, we study the case in which $\pi$ can change over time in a stochastic manner and there is an asymmetric information about the debtor’s type. Our goal is to obtain a two-period delay in bargaining, initially to permit for economic recovery followed by signalling about debtor’s type. We solve for the Perfect Bayesian equilibrium with signalling in mixed strategy.$^5$

**Proposition 3** There exists a mixed-strategy Perfect Bayesian equilibrium with two-period delay, initially to permit for recovery followed by signalling

$^5$ The derivation of Perfect Bayesian equilibrium with signalling in pure strategy requires the presence of inside option and is contained in Appendix B.
about sustainability constraint. The existence of a mixed-strategy Perfect Bayesian equilibrium with delay in the second period to permit the debtor to signal for sustainability constraint requires that $\pi (1 - q_1) < \bar{s}$ be satisfied.

To ensure that the Optimistic and the Cautious debtor want to delay in the first period, it requires that $\pi_L - \delta \left[ \frac{\delta (E \pi - \bar{s})}{2} \right] < \hat{a}$ and $\pi_L - \delta \left[ \frac{\delta (E \pi + \bar{s})}{2} \right] < \hat{a}$, respectively. The minimum payoff for the creditor to accept the debtor’s offer, $\hat{a}$, is given by:

\[
\hat{a} = \delta \left\{ \frac{\delta (E \pi - \bar{s})}{2} - (1 - q_0) \left[ \frac{\delta (E \pi - \bar{s})}{2} - E \pi + E x'_2 \right] \right\},
\]

where $\delta$ denotes the common discount factor, $q_0$ denotes prior belief of the the creditor and $E x'_2 > E \pi - \frac{\delta (E \pi - \bar{s})}{2}$.

**Proof.** See Appendix A. ■

To show the correlation between delay and size of the haircut in sovereign debt, it is necessary that we compare the creditor’s payoff in the case with one-period delay (Case 2) and the case with two-period delay (Case 3). The creditor’s continuation payoffs from rejecting the debtor’s offer at $t = 1$ under each case are summarised in the table below.

<table>
<thead>
<tr>
<th>Debtor’s type</th>
<th>One-period Delay</th>
<th>Two-period Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimistic debtor</td>
<td>$\frac{\delta E \pi}{2}$</td>
<td>$\hat{a}$</td>
</tr>
<tr>
<td>Cautious debtor</td>
<td>$\frac{\delta (E \pi - \bar{s})}{2}$</td>
<td>$\hat{a}$</td>
</tr>
</tbody>
</table>

**Table 2: Delay and haircuts in debt restructuring (Mixed Strategy)**

We can see from Table 2 that, when there is a two-period delay, the creditor’s payoff is lower relative to the case with one-period delay. It follows that prolonged delay is positively correlated with a larger haircut in our
model of sovereign debt swap. The key result in this model is that it is the extension of delay that leads to a large haircut.

We summarise the above discussion with the following proposition:

**Proposition 4** Relative to the case with one-period delay, the second-period delay due to signalling concerns will lower creditor’s payoffs and lead to larger haircuts.

III. EVIDENCE ON DELAY AND HAIRCUT

We begin this section by examining the key aspects of the recent debt swaps. According to Roubini and Setser (2004b), several of the sovereign debt restructurings involving international investors from 1998 to 2005 have been largely characterised by protracted negotiations. Supplemented by the calculations of haircuts associated with the recent bond restructurings by Sturzenegger and Zettlemeyer (2005), we are able to investigate the relationship between delay and haircuts, as indicated in Table 3.
Table 3: Sovereign Debt Restructurings 1998-2005

Sources: Table 14 and 15 in Sturzenegger and Zettlemeyer (2005); Table A.3 in Roubini and Setser (2004b)

Table 3 enables us to understand one of the striking aspects of recent sovereign debt swaps, namely, conditional on default, long delay in debt negotiation is positively correlated with size of the haircuts. For instance, default by Ecuador took a year to resolve and led to a 60 percent haircut. Others include the defaults by Russia and Argentina, which involved the largest amount of debt and resulted in the largest haircuts. Each of these two defaults took more than a year and a half of negotiation. The model of sovereign debt swaps developed in this paper does account for this. The main result in this model is that it is the extension of delay that leads to
the large haircuts. Moreover, this model highlights economic recovery and sustainability considerations as complementary reasons for delay.

By examining the recent accounts of sovereign debt restructurings, we observe both economic recovery and sustainability concerns. Some governments have interest of pulling their countries out of the recession in order to build their electoral popularity with their citizens. In this case, the concerns that early settlement might endanger recovery results in a delay in restructuring of sovereign debt. This is consistent with the analysis of Merlo and Wilson (1998). Other governments, for example, pursue a strategy of imposing a tight constraint on the resources available to the private creditors because they have the determination of avoiding a repeat of default on sovereign debt which could lead to social and economic disruption (Dhillon et al., 2006). Such strategy indeed reflects the commitment of the debtor’s government to sustainability.

Could Argentina be a case in point? The causes for delay in Argentina could be both political and economic. Though sovereign default was declared at the end of 2001, political factors militated against early restructuring. There was a problem of legitimacy as the country was being governed by an interim administration led by President Duhalde. At that time, the Argentine economy was in a severe recession. The priority of the President was to engineer recovery and “not to pursue outstanding structural reforms, among which debt resolution was the most important” (Bruno, 2004, p.1620).

Serious efforts to restructure Argentine debt did not begin until the interim administration was replaced in the elections of 2003; but the striking rate of recovery of GDP during President Duhalde’s administration suggests that “debt restructuring would have been postponed even if there had been no problem of legitimacy” (Dhillon et al., 2006). It was in September 2003,
at the meetings of the IMF and the World Bank in Dubai, that the Argentine government led by President Kirchner finally revealed its negotiating stance. It was determined to reach a durable settlement with its creditors: the consequences of default had proved so disastrous that a prompt but unsustainable settlement was unacceptable.

The specific strategy for reducing the debt exposure of the economy involved three principal commitments by the Argentine government: (i) to run a primary surplus of 3 percent of GDP, (ii) to limit the cost of debt service, and (iii) to exempt the preferred creditors from restructuring. The first two commitments effectively determined the overall size of the write-down, while the third commitment – to pay full compensation to the preferred creditors\(^6\) – meant there was little left for other private creditors. Without taking account of past-due interest, these constraints left an annual flow of only about a billion dollars on GDP valued at $137 billion – a ‘Dubai residual’ of less than one percentage point of GDP for private creditors holding debt with a face value of around $80 billion\(^7\).

The Dubai proposals articulated by the Argentine government were promptly rejected by creditor groups. However, improvements offered in the course of 2004 together with a decline in the global interest rates meant that a debt swap was finally accepted by 76 percent of the creditors in 2005. There were clearly two separate phases leading up to the debt swap. In the first phase, from end-2001 to mid-2003, the Argentine economy was recovering strongly from a deep recession and there appeared to be a consensus between both parties to await recovery – a consensus reinforced by the political difficulties faced by the Duhalde regime. As for the second phase, one could interpret

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\(^6\) These included both International Financial Institutions (such as the IMF, the World Bank and the IADB) and domestic bondholders who had lent into arrears.

\(^7\) See Dhillon et al. (2006) for more detail.
the meagre Dubai offer made by Argentina in September 2003 as driven by sustainability concerns. In the context of our analysis, this low offer was designed to be rejected, leading to delay and a reappraisal of the debtor’s type, and finally to a settlement that respected these sustainability concerns.

Anecdotal support for this interpretation is provided in Liascovich (2005, pp. 226-227) in his biography of Mr. Lavagna, the Argentine Finance Minister at the time\footnote{An interesting discussion of the negotiating strategy – centred on the need to ensure sustainability – is available in Levy Yeyati and Valenzuela (2007), Chapter 11.}. His writing on the low Dubai offer and the sustainability concerns lying behind it is as follows:

Some time before the offer [at Dubai], Lavagna was already preparing the field: he realised that after the offer “there are going to be sad faces everywhere”. And indeed the first reaction of the creditors was of rejection...But the Argentine offensive was not restricted to Dubai. [President] Kirchner in New York, one day after the offer, had an interview with President George Bush, who said, “Keep on negotiating firmly with the creditors”. And the Argentine President used the auditorium of UN General Assembly to criticise the international financial organisations for support in reduction debt and [promoting] growth. “It’s never been known to recover debts from the dead”, he said in his speech.

As for improving the offer to achieve a final settlement, Liascovich (2005, p.247) continues:

The strategy was to maintain the posture of Dubai as long as possible, so that the effect of a new offer would come as a
relief...On the 1st of June [2004] in Buenos Aires Lavagna presented an improved offer which would be the definitive version of the swap. After eight months of insistence, from President Kirchner downwards, that the offer at Dubai would not change, this new offer was a better deal for the creditors. There was no change in the instruments involved in the swap, but there was in the recognition of unpaid interest.

The close coincidence of the final swap with the sustainability requirement calculated by the Argentine government is consistent with our model of bargaining with sustainability, as long as creditors played an important role in determining the final offer. Sgard (2004), for example, argues that the swap was made on terms designed to appeal to investment banks who had apparently bought up much of the outstanding debt.

IV. CONCLUSION

A striking aspect of recent sovereign debt restructurings is that, conditional on default, long delay is positively correlated with size of the haircut. Most existing studies on sovereign debt swaps do not address this issue. In this paper, we develop a bargaining model to account for this, highlighting economic recovery and sustainability considerations as complementary reasons for delay.

With stochastic bargaining surplus and asymmetric information about the debtor’s sustainability concern, we show that two-period delay can occur, initially reflecting recovery followed by signalling about sustainability. The main result of the model shows that prolonged delay is positively correlated

\[ s = \text{ratio of } \frac{s}{\pi} \text{ ratio of 55 percent, while the swap itself is estimated to represent a payoff about 53 percent.} \]
with a large haircut. We offer this sequential game of recovery and signalling about sustainability as an interpretation of events in Argentina.

In their assessment of the reasons for delay, Roubini and Setser (2004a) stress the roles played by heterogeneity of creditors and contracts. They argue that the IMF should play a key role in coordinating creditors\textsuperscript{10}. The model we propose abstracts from issues of creditor coordination and focuses on the role of economic recovery and of signalling about sustainability. Avoiding efficient delay due to recovery prospects can in principle be achieved by the contingent contracts, and Argentina did finally issue GDP-linked bonds as a part of the swap. These were initially greatly undervalued by the market at the time of issue but have subsequently been priced much more favourably. Could this be a good augury for future debt restructuring?

In the model we propose in this paper, the IMF could help by providing information. When the growth prospects are not common knowledge, for example, the IMF could resolve uncertainty about future growth. Likewise, when sustainability concerns are in dispute, the IMF could publish its own best estimates, which should remove the need for a Cautious debtor to use delay as a signal for sustainability considerations\textsuperscript{11}.

A potentially serious challenge to carrying out this informational role is that the IMF faces a conflict of interest: as a senior creditor, it presumably has an incentive to exaggerate sustainability requirements in favour of the debtor so as to minimise other claims on the debtor’s resources. Would such induced compassion for debtors not be checked by its creditor-dominated Executive Board? If not, this informational task could be delegated else-

\textsuperscript{10}Roubini and Setser (2004b) argue that the IMF could orchestrate ‘debtor in possession finance’. See Miller and Thomas (2006) for an account of the role played by the New York Courts in coping with creditor heterogeneity.

\textsuperscript{11}Blustein (2005) provides discussion on the official sustainability assessments of Argentina made by the IMF but kept confidential.
where, perhaps to the Inter-American Development Bank for cases of Latin American debt restructuring, for example.

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References


Appendix

Appendix A - Proofs of Propositions
Proof of Proposition 1

We derive the equilibrium payoffs for the creditor and the debtor when the debtor is Optimistic and Cautious, respectively. We begin with the case of the Optimistic debtor, followed by the case of the Cautious debtor.

**Optimistic Debtor.** Let $x_{CR}$ and $x_O$ denote the payoffs for the Optimistic debtor when the creditor and the debtor are the proposers, respectively, and let $\pi_L - x_{CR}$ and $\pi_L - x_O$ denote the share of the available bargaining surplus for the creditor. Using the standard arguments\footnote{Such arguments are as follows. In the equilibrium, either party will agree to a debt restructuring proposal if the proposal offers the party at least as much in discounted present value as it can expect to attain by waiting until the next period, given the strategies of both parties.}, to compute the payoffs for the creditor and the Optimistic debtor along the Perfect Bayesian equilibrium, the following two equations need to be solved:

\[
  x_{CR} = \frac{\delta}{2} (x_{CR} + x_O), \quad (A1)
\]

\[
  \pi_L - x_O = \frac{\delta}{2} (\pi_L - x_{CR} + \pi_L - x_O). \quad (A2)
\]

Equations (A1) and (A2) have a unique solution, given by $(x^*_O, x^*_{CR})$:

\[
  x^*_O = \frac{2 - \delta}{2} \pi_L \quad \text{and} \quad x^*_{CR} = \frac{\delta \pi_L}{2}. \quad (A3)
\]

In the Perfect Bayesian equilibrium, if the debtor is known to be Optimistic and he is chosen to make an offer, he always offers $\pi_L - x^*_O = \frac{\delta \pi_L}{2}$ to the creditor and if the creditor is chosen to make an offer, his offer to the debtor will always be $x^*_{CR} = \frac{\delta \pi_L}{2}$. Given that each party has an equal probability of making an offer (being the proposer), it follows that the payoffs for the Optimistic debtor and the creditor are given by $(\frac{\pi_L}{2}, \frac{\pi_L}{2})$, respectively.
Cautious Debtor. Next, we consider a situation in which the debtor is known to be Cautious. The Cautious debtor has concern about the sustainability of any settlement. The presence of a sustainability constraint reduces the amount of surplus that is available for bargaining from $\pi_L$ to $\pi_L - \bar{s}$. Let $\tilde{x}_{CR}$ and $\tilde{x}_C$ denote the Cautious debtor’s payoff when the creditor and the Cautious debtor are the proposer, respectively. Again, using the standard arguments, to compute the payoffs for the creditor and the Cautious debtor along the Perfect Bayesian equilibrium, the following two equations need to be solved:

$$\tilde{x}_{CR} = \frac{\delta}{2} (\tilde{x}_{CR} + \tilde{x}_C), \quad (A4)$$

$$(\pi_L - \bar{s}) - \tilde{x}_C = \frac{\delta}{2} \left( \left[ (\pi_L - \bar{s}) - \tilde{x}_{CR} \right] + \left[ (\pi_L - \bar{s}) - \tilde{x}_C \right] \right). \quad (A5)$$

Equations (A4) and (A5) have a unique solution, given by $(\tilde{x}_C^*, \tilde{x}_{CR}^*)$:

$$\tilde{x}_C^* = \frac{(2 - \delta)(\pi_L - \bar{s})}{2} \quad \text{and} \quad \tilde{x}_{CR}^* = \frac{\delta(\pi_L - \bar{s})}{2} \quad (A6)$$

In the Perfect Bayesian equilibrium, if the debtor is known to be Cautious and he is chosen to make an offer, he always offers $(\pi_L - \bar{s}) - \tilde{x}_C^* = \frac{\delta(\pi_L - \bar{s})}{2}$ to the creditor and if the creditor is chosen to make an offer, his offer to the debtor will always be $\tilde{x}_{CR}^* = \frac{\delta(\pi_L - \bar{s})}{2}$. An agreement will be reached. Given that each party has an equal probability of making an offer (being the proposer), it follows that the payoffs for the debtor and the creditor are given by: $\left( (\frac{\pi_L + \bar{s}}{2}), (\frac{\pi_L - \bar{s}}{2}) \right)$, respectively. Similarly, we can solve all periods prior to this arbitrary period recursively. Q.E.D.

Proof of Proposition 2

The condition for delay can be derived as follows. Let us begin with the case of an Optimistic debtor. The best offer that the Optimistic debtor
can make is the excess of the available bargaining surplus over his own contin-
uation value, given by \( \pi_L - \frac{\delta E\pi}{2} \). If this offer falls below the creditor’s continuation value, \( \frac{\delta E\pi}{2} \), this offer will not be accepted. Formally, the condition for the first-period delay for the Optimistic debtor is given by:

\[
\pi_L - \frac{\delta E\pi}{2} < \frac{\delta E\pi}{2},
\]

which can be written as

\[
\frac{1 - \delta}{\delta} < \frac{(E\pi - \pi_L)}{\pi_L}.
\]

This is equivalent to the condition for delay in terms of the expected growth rate and the discount rate:

\[
\frac{r}{\delta} < \frac{Eg}{\delta},
\]

which requires that the expected growth of the economy exceeds the rate of discount. Given that this condition is satisfied, the Optimistic debtor will delay in the first period. According to this condition, delay becomes more attractive with bullish growth prospects.

Next, we consider the case of a Cautious debtor. The best offer that the Cautious debtor can make is the excess of the available bargaining surplus over his own continuation value, given by \( \pi_L - \frac{\delta (E\pi + \bar{s})}{2} \). If this offer falls below the creditor’s continuation value, \( \frac{\delta (E\pi - \bar{s})}{2} \), this offer will not be accepted. Formally, the condition for the first-period delay for the Cautious debtor is given by:

\[
\pi_L - \frac{\delta (E\pi + \bar{s})}{2} < \frac{\delta (E\pi - \bar{s})}{2},
\]

which requires that the expected growth of the economy exceeds the rate of discount. Given that this condition is satisfied, the Optimistic debtor will delay in the first period. According to this condition, delay becomes more attractive with bullish growth prospects.

Next, we consider the case of a Cautious debtor. The best offer that the Cautious debtor can make is the excess of the available bargaining surplus over his own continuation value, given by \( \pi_L - \frac{\delta (E\pi + \bar{s})}{2} \). If this offer falls below the creditor’s continuation value, \( \frac{\delta (E\pi - \bar{s})}{2} \), this offer will not be accepted. Formally, the condition for the first-period delay for the Cautious debtor is given by:

\[
\pi_L - \frac{\delta (E\pi + \bar{s})}{2} < \frac{\delta (E\pi - \bar{s})}{2},
\]

which requires that the expected growth of the economy exceeds the rate of discount. Given that this condition is satisfied, the Optimistic debtor will delay in the first period. According to this condition, delay becomes more attractive with bullish growth prospects.
which can be re-written as:

\[ r < E_g. \]

This condition also requires that the expected growth of the economy exceeds the rate of discount. Given that this condition is satisfied, the Cautious debtor will delay in period one to permit for economic recovery. \textit{Q.E.D.}

\textbf{Proof of Proposition 3}

We begin with period \( t = 3 \), when there is a complete information about the debtor’s type. First, let us take the case where the debtor is Optimistic. Using arguments identical to those in the benchmark case, in the Perfect Bayesian equilibrium, if the Optimistic debtor is chosen to make an offer, his offer to the creditor is \( \frac{\delta \pi}{2} \) and if the creditor is chosen to make an offer, the creditor’s offer is \( \frac{\delta \pi}{2} \). Given that the choice of proposer depends on the toss of a coin, the payoffs for the Optimistic debtor and the creditor at \( t = 3 \) are \( \left( \frac{\pi}{2}, \frac{\pi}{2} \right) \), respectively.

Next, we look at the case in which the debtor is Cautious. In the Perfect Bayesian equilibrium, if the Cautious debtor is chosen to make an offer, his offer to the creditor is \( \frac{\delta(\pi - \delta)}{2} \); and, if the creditor is chosen to make an offer, his offer to the debtor is \( \frac{\delta(\pi - \delta)}{2} \). The payoffs for the Cautious debtor and the creditor at \( t = 3 \) are \( \left( \frac{\pi + \delta^2}{2}, \frac{\pi + \delta^2}{2} \right) \), respectively.

Let the posterior beliefs of the creditor over the two types of the debtor be denoted by \( q_2 \). In Table A1, we consider the creditor’s offers at extreme values of \( q_2 \).
Creditor’s belief as to debtor’s type | Creditor’s offer to debtor | Payoff for creditor |
--- | --- | ---
$q_2 = 1$ (It’s an Optimist) | $\frac{\delta \pi}{2}$ | $\pi - \frac{\delta \pi}{2}$ |
$q_2 = 0$ (It’s Cautious) | $\frac{\delta (\pi - s)}{2}$ | $(\pi - \bar{s}) - \frac{\delta (\pi - \bar{s})}{2}$ |

Table A1: Debtor’s and Creditor’s offers with extreme beliefs

For less extreme beliefs, i.e. $0 < q_2 < 1$, the creditor’s expected payoff from a high offer of $\frac{\delta (\pi - s)}{2}$, which is acceptable to both types of debtor, will be $(\frac{2-\delta}{2})(\pi - \bar{s})$, while the creditor’s expected payoff from a low offer of $\frac{\delta s}{2}$, which is only acceptable to the Optimistic debtor, will be $q_2 (\pi - \frac{\delta s}{2})$. If $q_2 (\pi - \frac{\delta s}{2}) > (\frac{2-\delta}{2})(\pi - \bar{s})$, the creditor will do better by making a low offer but otherwise for $q_2 (\pi - \frac{\delta s}{2}) < (\frac{2-\delta}{2})(\pi - \bar{s})$. When $q_2 (\pi - \frac{\delta s}{2}) = (\frac{2-\delta}{2})(\pi - \bar{s})$, the two offers give the creditor the same expected payoff. This condition implies that $q_2 = \frac{\pi - \bar{s}}{\pi}$. Since $0 < \bar{s} < \pi$, it follows that $q_2 < 1$. We assume that, if the creditor is indifferent between a low and a high offer, he will choose a high offer.

The creditor’s offers as a function of his belief are shown in the table below:

<table>
<thead>
<tr>
<th>Creditor’s belief as to debtor’s type</th>
<th>Creditor’s offer to debtor</th>
<th>Expected payoff for creditor</th>
</tr>
</thead>
</table>
$q_2 > \frac{\pi - \bar{s}}{\pi}$ (Probably an Optimist) | $\frac{\delta \pi}{2}$ | $q_2 (\pi - \frac{\delta \pi}{2})$ |
$q_2 \leq \frac{\pi - \bar{s}}{\pi}$ (Probably Cautious) | $\frac{\delta (\pi - s)}{2}$ | $(\frac{2-\delta}{2})(\pi - \bar{s})$ |

Table A2: Debtor’s and Creditor’s offers at $t = 3$ for all values of belief
As for the debtor, the Optimistic debtor’s offer to the creditor is \( \frac{\delta \pi}{2} \), while the Cautious debtor’s offer to the creditor is \( \frac{\delta (\pi - s)}{2} \).

From the perspective of the second period, given the common discount factor \( \delta \) and bearing in mind that each player has a 50% probability of being a proposer, the continuation values for the debtor and the creditor are given by:

<table>
<thead>
<tr>
<th>Creditor’s belief as to debtor’s type</th>
<th>Continuation values for debtor</th>
<th>Continuation values for creditor</th>
</tr>
</thead>
</table>
| \( q_2 > \frac{\pi - s}{\pi} \)  
(Probably an Optimist)               | \( \frac{\delta \pi}{2} \)    | \( \frac{\delta}{2} \left( q_2 \left( \frac{2 - \delta}{2} \right) \pi + \frac{\delta \pi}{2} \right) \) |
| \( q_2 \leq \frac{\pi - s}{\pi} \)  
(Probably Cautious)                  | \( \frac{\delta (\pi + s)}{2} \) | \( \frac{\delta (\pi - s)}{2} \) |

**Table A3:** Continuation values for both parties at \( t = 2 \)

One observation that arises from Table A3 is that, when \( q_2 \to 1 \), the continuation value for the creditor approaches \( \frac{\delta \pi}{2} \) as in the case with complete information about debtor’s type.

Moving backwards, we now consider the bargaining at \( t = 2 \). There is an asymmetric information about the debtor’s type. Let us fix \( q_2 \) for which the continuation values for the creditors are as in Table A3 and denote such continuation belief by \( q'_2 \), where \( q'_2 \leq \frac{\pi - s}{\pi} \). The continuation values for the creditor and the debtor corresponding to such continuation belief of the creditor as to the debtor’s type are given by \( \frac{\delta (\pi - s)}{2} \) and \( \frac{\delta (\pi + s)}{2} \), respectively.

Suppose that the debtor is chosen to make an offer at \( t = 2 \). Let \((x_2, \pi - x_2)\) denote the offer made by the debtor. Suppose that \( \tilde{x}_2 \) solves: \( \pi - \tilde{x}_2 = \frac{\delta (\pi - s)}{2} \). By computation, we find that \( \tilde{x}_2 = \left( \frac{2 - \delta}{2} \right) \pi + \frac{\delta s}{2} \). Let \( x'_2 \) be
any positive number such that $\pi - x'_2 < \frac{\delta(\pi - \bar{s})}{2}$, where $\bar{x}_2 < x'_2 < \pi$. Thus, $x'_2 > \left(\frac{2-\delta}{2}\right)\pi + \frac{\delta\bar{s}}{2}$.

The Optimistic debtor offers $(\bar{x}_2, \pi - \bar{x}_2)$ with a probability $(1 - \beta)$ and offers $(x'_2, \pi - x'_2)$ with a probability $\beta$, while the Cautious debtor offers $(x'_2, \pi - x'_2)$ with a probability 1. The posterior belief of the creditor at $t = 3$, which is $q_2 = \frac{\pi - \bar{s}}{\pi}$, can be obtained using the Bayesian updating rule: $q_2 = \frac{\beta q_1}{\beta q_1 + (1 - q_1)}$, where $q_1$ is the creditor’s belief at $t = 2$. It follows that

$$\frac{\beta q_1}{\beta q_1 + (1 - q_1)} = \frac{\pi - \bar{s}}{\pi}.$$  \hspace{1cm} (A9)

Solving equation (A9) for $\beta$ yields:

$$\beta^* = \left(\frac{\pi - \bar{s}}{\bar{s}}\right)\left(\frac{1 - q_1}{q_1}\right).$$

Since we need $\beta < 1$, it requires that the following condition is satisfied:

$$\left(\frac{\pi - \bar{s}}{\bar{s}}\right)\left(\frac{1 - q_1}{q_1}\right) < 1,$$

or

$$\pi (1 - q_1) < \bar{s}.$$  \hspace{1cm} (A10)

As long as $\bar{s} > \pi (1 - q_1)$, there exists $\beta \in (0, 1)$ which solves equation (A9). It follows that, for the creditor, by observing $x'_2$, $q_2 = \frac{\pi - \bar{s}}{\bar{s}}$. Given $q_2 = \frac{\pi - \bar{s}}{\pi}$, creditor is indifferent between accepting $(\bar{x}_2, \pi - \bar{x}_2)$ and rejecting it and obtains his continuation value. From condition (A10), we have $q_1 > \frac{\pi - \bar{s}}{\bar{s}}$. Let us denote the lower bound for $q_1$, i.e. $\frac{\pi - \bar{s}}{\bar{s}}$, by $q$. It is clear that $q^*_2 < q$.

Creditor accepts $(\bar{x}_2, \pi - \bar{x}_2)$ with a probability $\gamma$ and rejects it with a
probability \((1 - \gamma)\). For any \(\theta \in [0, 1]\), let

\[
d(\theta) = \theta \left( \frac{\delta(\pi + \bar{s})}{2} \right) + (1 - \theta) \left( \frac{\delta \pi}{2} \right).
\]

After observing \((\tilde{x}_2, \pi - \tilde{x}_2)\), the creditor chooses \(\theta = 1\), while after observing \((x'_2, \pi - x'_2)\), the creditor chooses \(\theta = 0\). The debtor’s payoff from \(\tilde{x}\) is given by \(\gamma \tilde{x}_2 + (1 - \gamma) \left( \frac{\delta \pi}{2} \right)\), and the debtor’s payoff from \(x'_2\) is given by \(\frac{\delta(\pi + \bar{s})}{2}\). Note that \(\tilde{x}_2 > \frac{\delta(\pi + \bar{s})}{2} > \frac{\delta \pi}{2}\). It follows that \(\exists \gamma \in (0, 1)\), which solves:

\[
\gamma \tilde{x}_2 + (1 - \gamma) \left( \frac{\delta \pi}{2} \right) = \frac{\delta(\pi + \bar{s})}{2}. \tag{A11}
\]

By solving equation (A11) for \(\gamma\) yields:

\[
\gamma^* = \frac{\delta \bar{s}}{2(\tilde{x}_2 - \frac{\delta \pi}{2})}. \tag{A12}
\]

Thus, there exists a mixed-strategy Perfect Bayesian equilibrium with a delay in the second period to permit the debtor to signal for sustainability constraint if condition (A10) is satisfied. The details of the signalling equilibrium in mixed strategy are as follows:

At \(t = 2\), the Optimistic debtor offers \((\tilde{x}_2, \pi - \tilde{x}_2)\) with a probability \((1 - \beta^*)\) and offers \((x'_2, \pi - x'_2)\) with a probability \(\beta^*\). The Cautious debtor offers \((x'_2, \pi - x'_2)\) with a probability 1. The creditor rejects the offer \((\tilde{x}_2, \pi - \tilde{x}_2)\) with a probability \(\gamma^*\) and rejects the offer \((x'_2, \pi - x'_2)\) with a probability 1. Then, at \(t = 3\), the creditor’s belief as to the debtor’s type is \(q_2 = \frac{\pi - \bar{s}}{\pi}\). The creditor’s payoff is \(\frac{\delta(\pi - \bar{s})}{2}\). The debtor’s payoff depends on \(x_2\). If \(x_2 = \tilde{x}_2\), his payoff is \(\frac{\delta \pi}{2}\), while if \(x_2 = x'_2\), his payoff is \(\frac{\delta(\pi + \bar{s})}{2}\).

Moving to \(t = 1\), we calculate the continuation values for the creditor and both type of debtor. There are two scenarios to be considered: when
the debtor knows his own type and when he does not. If the debtor knows his own type, the continuation payoff for the Optimistic debtor is $$\delta \left[ \frac{\delta (\pi + \bar{s})}{4} \right]$$; the continuation payoff for the Cautious debtor is $$\delta \left[ \frac{\delta (\pi + \bar{s})}{2} \right]$$; and the continuation payoff for the creditor is

$$\delta \left[ \frac{\delta (\pi - \bar{s})}{2} \right] \left( 1 - \left( \frac{\pi - \bar{s}}{\bar{s}} \right) (1 - q_0) \right) + \left( \frac{\pi - \bar{s}}{\bar{s}} \right) (1 - q_0) (\pi - x'_2)$$,

where $$q_0$$ denotes the prior belief of the creditor as to the debtor’s type.

However, if the debtor does not know his own type, the creditor’s continuation payoff is the same as before but the debtor’s payoff is

$$\frac{\delta^2 (\pi + \bar{s})}{2} \left\{ \frac{2 - q_0}{2} \right\}$$.

In the first period, the sovereign debtor is a proposer. It follows from the above two scenarios that the expected payoff for the creditor (from rejecting the debtor’s offer at $$t = 1$$) is given by $$\hat{a}$$, where

$$\hat{a} = \delta \left\{ \frac{\delta (E \pi - \bar{s})}{2} - (1 - q_0) \left( \frac{E \pi - \bar{s}}{\bar{s}} \right) \left[ \frac{\delta (E \pi - \bar{s})}{2} - E \pi + E x'_2 \right] \right\}$$,

(A13)

where $$E x'_2 > E \pi - \frac{\delta (E \pi - \bar{s})}{2}$$ since $$x'_2 > \pi - \frac{\delta (\pi - \bar{s})}{2}$$. Alternatively, $$\hat{a}$$ can be interpreted as the minimum payoff for the creditor to accept the debtor’s offer.

Next, we derive the conditions which make delay becomes attractive for each type of debtor. For the Optimistic debtor, the best offer that he can make is the excess of the available bargaining surplus over his own continuation value, i.e. $$\pi_L - \delta \left[ \frac{\delta (E \pi + \bar{s})}{4} \right]$$. If such offer falls below the expected payoff for the creditor, $$\hat{a}$$, the debtor’s offer will not be accepted, resulting in a delay in the first period. Formally, the condition for delay for the Optimistic debtor is given by:

$$\pi_L - \delta \left[ \frac{\delta (E \pi + \bar{s})}{4} \right] < \hat{a}$$.

(A14)
Similarly, for the Cautious debtor, his best offer is $\pi_L - \delta \left[ \frac{\delta(E_\pi + \bar{s})}{2} \right]$. If such offer falls below $\hat{a}$, the offer will not be accepted. Formally, the condition for delay for the Cautious debtor is

$$\pi_L - \delta \left[ \frac{\delta(E_\pi + \bar{s})}{2} \right] < \hat{a}. \quad (A15)$$

Then, provided that the derived condition for delay in the second period (i.e. condition (A10)) is also satisfied, there will be a positive probability of a two-period delay in bargaining: initially reflecting recovery, followed by signalling by debtor about sustainability.

Consider the RHS of equation (A13). Since $0 < q_0 < 1$, $(\frac{E_\pi - \bar{s}}{4}) > 0$ and $E_\pi > E_\pi - \frac{\delta(E_\pi - \bar{s})}{2}$, it straightforward to show that $\hat{a} < \frac{\delta E_\pi}{2}$ and $\hat{a} < \frac{\delta(E_\pi - \bar{s})}{2}$; therefore, both conditions (A14) and (A15) are more stringent than $r < E_g$. It follows that the conditions for first-period delay in the model with asymmetric information about the debtor’s type are more stringent than in the full information case discussed in Case 2. The reason is that there is a positive risk of the creditor facing a Cautious debtor, which lowers the creditor’s continuation payoff. \textit{Q.E.D.}

\textit{Appendix B - Inside Option and Perfect Bayesian Equilibrium with Signalling in Pure Strategy}

In this Appendix B, we study whether, in the case in which $\pi$ can change over time in a stochastic manner and there is an asymmetric information about the debtor’s type, the Perfect Bayesian signalling equilibrium in pure strategy exists. In the main text, we have shown that, in the absence of the inside options, the model is solvable but the equilibrium is in mixed strategy. To get the pure-strategy equilibrium, the inside option is necessary. We interpret the inside option as follows: in the event that there is sustainability
constraint, the debtor government receives political payoff from delaying. The inside option is denoted by $\omega_i$, where $i = C, O$. The inside options for the Optimistic debtor and the Cautious debtor are $\omega_O = 0$ and $\omega_C > 0$, respectively.

We begin our analysis from period $t = 3$, where there is a complete information about the debtor’s type. Let $q_2$ denote the posterior belief of the creditor as to the debtor’s type. First, let us take the case where the debtor is Optimistic. Using arguments identical to those in the benchmark case in the main text, in the Perfect Bayesian equilibrium, if the Optimistic debtor is chosen to make an offer, his offer to the creditor is $\frac{\delta \pi}{2}$ and if the creditor is chosen to make an offer, the creditor’s offer is $\frac{\delta \pi}{2}$. Given that the choice of proposer depends on the toss of a coin, the payoffs for the Optimistic debtor and the creditor at $t = 3$ are $\frac{\pi}{2}$ and $\frac{\pi}{2}$, respectively.

Next, we look at the case in which the debtor is Cautious. In the Perfect Bayesian equilibrium, if the Cautious debtor is chosen to make an offer, his offer to the creditor is $\frac{\delta (\pi - s)}{2}$; however, if the creditor is chosen to make an offer, his offer to the debtor is $\frac{\delta (\pi - s)}{2}$. Again, given that the choice of proposer depends on the toss of a coin, the payoffs for the Cautious debtor and the creditor at $t = 3$ are $\left(\frac{\pi + s}{2}\right)$ and $\left(\frac{\pi - s}{2}\right)$, respectively.

From the perspective of the second period, given the common discount factor $\delta$, in the case of an Optimistic debtor, the continuation values for the debtor and the creditor are $\frac{\delta \pi}{2}$ and $\frac{\delta \pi}{2}$, respectively. In the case of a Cautious debtor, the continuation values for the debtor and the creditor are $\frac{\delta (\pi + s)}{2}$ and $\frac{\delta (\pi - s)}{2}$, respectively. At $t = 2$, there is an asymmetric information about the debtor’s type: while the debtor knows his own type, the debtor’s type is unknown to the creditor.

We begin by considering the situation in which the debtor is chosen
to make the offer. At the separating equilibrium, the offer made by the Optimistic debtor is \((y_2, \pi - y_2)\), while the offer made by the Cautious debtor is \((\tilde{y}_2, \pi - \tilde{y}_2)\).

First, after observing the Optimistic debtor’s offer, the induced posterior beliefs for the creditor given the debtor’s offer is \(q_2 = 1\), and after observing the Cautious debtor’s offer, the induced posterior beliefs for the creditor given the debtor’s offer is \(q_2 = 0\). When \(q_2 = 1\), the continuation value for the creditor at \(t = 2\) is \(\frac{\delta \pi}{2}\). The Optimistic debtor’s offer should sufficiently high to induce the creditor to accept; thus, it requires that \(\pi - y_2 = \frac{\pi \delta}{2}\) or \(y_2 = \pi \left(\frac{2 - \delta}{2}\right)\). When \(q_2 = 0\), the continuation value for the creditor at \(t = 2\) is \(\frac{\delta(\pi - \tilde{\pi})}{2}\). At the separating equilibrium, the offer by the Cautious debtor should fall below the creditor’s continuation value, i.e. \(\pi - \tilde{y}_2 < \frac{(\pi - \tilde{\pi}) \delta}{2}\) or \(\tilde{y}_2 > \frac{(2 - \delta) \pi}{2} + \frac{\delta \tilde{\pi}}{2}\), which will induce the creditor to reject. Thus, the payoff for the Optimistic debtor is \(\left(\frac{2 - \delta}{2}\right) \pi\), while the payoff for the Cautious debtor is \(\frac{(\pi + \tilde{\pi}) \delta}{2} + \omega_C\), which is his continuation value plus his inside option.

Next, we need to derive the conditions required to ensure that the two types of debtor reveal themselves in a separating equilibrium. These conditions would ensure that (i) the Optimist does not imitate the Cautious debtor; (ii) the Cautious debtor does not imitate the Optimist and (iii) the Cautious debtor wants to delay.

**Will the Optimist imitate the Cautious debtor?** Let us fix \(q_2\). After observing the Cautious offer, it follows that \(q_2 = 0\). To ensure that the Optimist reveals himself, it requires that he will not do better by imitating the Cautious debtor, i.e. that the benefit of settling quickly exceeds the present discounted value of continuing to \(t = 3\) and pretend to be a Cautious debtor. The condition, which ensures that the Optimistic debtor reveals
himself, requires that:

\[
\frac{(\pi + \bar{s}) \delta}{2} < \left( \frac{2 - \delta}{2} \right) \pi,
\]

which can be re-written as

\[
\delta < \frac{2\pi}{(\bar{s} + 2\pi)}.
\]  \tag{B1}

Will the Cautious debtor imitate the Optimist? If the Cautious debtor makes the Optimistic offer, then \(q_2 = 1\). For the Cautious debtor to want to delay, the present discounted value of continuing and being identified as Cautious plus the inside option, \(\omega_C\), should exceed the benefits from settling quickly, i.e.

\[
\frac{(\pi + \bar{s}) \delta}{2} + \omega_C > \left( \frac{2 - \delta}{2} \right) \pi.
\]

Will the Cautious debtor want to delay? If the Cautious debtor makes an offer of \(\frac{\delta(\pi - \bar{s})}{2}\), this will signal to the creditor the debtor’s type \((q_2 = 0)\). Given that \(q_2 = 0\), the creditor will choose to accept the debtor’s offer. For the Cautious debtor to want to delay, it requires that:

\[
\frac{(\pi + \bar{s}) \delta}{2} + \omega_C > \pi - \frac{(\pi - \bar{s}) \delta}{2},
\]

or

\[
\delta > \frac{(\pi - \omega_C)}{\pi}. \tag{B2}
\]

By combining conditions (B1) and (B2), we obtain the sufficient condition for a separating equilibrium and a delay due to signalling about debtor’s type:

\[
\bar{s} < \frac{(\bar{s} + 2\pi) \omega_C}{\pi} \quad \text{or} \quad \omega_C > \frac{\bar{s}\pi}{(\bar{s} + 2\pi)}. \tag{B3}
\]
To summarise, when the debtor is chosen to make the offer at \( t = 2 \), in a separating equilibrium, his payoffs and offers will be as summarised in Table B1.

<table>
<thead>
<tr>
<th>Induced posterior beliefs for the creditor given the debtor’s offer</th>
<th>Debtor’s payoff</th>
<th>Offer to creditor</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q_2 = 1 ) (Debtor is an Optimist)</td>
<td>( y_2 = \left( \frac{2 - \delta}{2} \right) \pi )</td>
<td>( \pi - y_2 = \frac{\delta \pi}{2} )</td>
<td>High offer will induce creditor to accept</td>
</tr>
<tr>
<td>( q_2 = 0 ) (Debtor is Cautious)</td>
<td>( \hat{y}_2 &gt; \left( \frac{2 - \delta}{2} \right) \pi + \frac{\delta s}{2} ), ( \pi - \hat{y}_2 &lt; \frac{(\pi - s) \delta}{2} )</td>
<td></td>
<td>Offer too low to be accepted by creditor</td>
</tr>
</tbody>
</table>

**Table B1:** Debtor’s offers and payoffs in a separating equilibrium

Next, we consider the situation in which the creditor is chosen to make an offer at \( t = 2 \). Let the creditor’s offer be represented by \( (\hat{y}_2, \pi - \hat{y}_2) \). At a separating equilibrium, Optimistic debtor would accept the creditor’s offer, while Cautious debtor would reject. After observing that his offer is being accepted, the induced posterior beliefs for the creditor is \( q_2 = 1 \). Since the continuation value for the debtor is \( \frac{\delta \pi}{2} \), this implies that \( \hat{y}_2 > \frac{\delta \pi}{2} \) or \( \hat{y}_2 = \frac{\delta \pi}{2} \). In this case, the payoff for the creditor is \( \pi - \frac{\delta \pi}{2} \). After observing that his offer is being rejected, the induced posterior beliefs for the creditor is \( q_2 = 0 \). With the debtor’s continuation payoff of \( \frac{(\pi + s) \delta}{2} \), this implies that \( \hat{y}_2 < \frac{(\pi + s) \delta}{2} \). In this case, the creditor receives his continuation value, \( \frac{(\pi - s) \delta}{2} \). The expected payoff for the creditor from making an offer of \( \hat{y}_2 = \frac{\delta \pi}{2} \) is

\[
q_0 \left( \pi - \frac{\delta \pi}{2} \right) + (1 - q_0) \left( \frac{(\pi - s) \delta}{2} \right).
\]
To summarise, when the creditor is chosen to make the offer at $t = 2$, in the separating equilibrium, given that the Optimistic debtor accepts and the Cautious debtor rejects the creditor’s offer, the creditor’s offer and expected payoff are as presented in Table B2:

<table>
<thead>
<tr>
<th>Induced posterior beliefs for the creditor in a separating equilibrium</th>
<th>Expected payoff for creditor</th>
<th>Offer to debtor</th>
<th>Outcome</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_2 = 1$ if debtor accepts the offer; $q_2 = 0$ if debtor rejects the offer</td>
<td>$q_0 \left( \pi - \frac{\delta \pi}{2} \right) + (1 - q_0) \left( \frac{(\pi - \bar{s}) \delta}{2} \right)$</td>
<td>$\frac{\delta \pi}{2}$</td>
<td>Optimist accepts the creditor’s offer but the Cautious debtor rejects</td>
</tr>
</tbody>
</table>

Table B2: Creditor’s offer and expected payoff in a separating equilibrium

It is straightforward to see that the Optimist will accept the creditor’s offer as he is indifferent between accepting and rejecting it. The Cautious debtor will reject the creditor’s offer as $\frac{\delta \pi}{2} < \frac{(\pi + \bar{s}) \delta}{2} + \omega_C$. It remains to be checked that the creditor has no incentive to deviate by making an offer that either (i) both types of debtor reject and/or (ii) both types of debtor accept.

(i) The creditor makes an offer which will be rejected by both types of debtor: $\hat{y}_2 < \frac{\delta \pi}{2}$. If both types of debtor reject the offer, $q_2 = q_0$. Since the expected payoff for the creditor if $q_0 > \frac{(\pi - \bar{s})}{\pi}$ is $\frac{\delta \pi}{2}$, it requires that:

$$\frac{\delta \pi}{2} < q_0 \left( \pi - \frac{\delta \pi}{2} \right) + (1 - q_0) \left( \frac{(\pi - \bar{s}) \delta}{2} \right).$$
As \( \pi - \frac{\delta \pi}{2} > \frac{\delta x}{2} > \frac{(\pi - s)x}{2}, \exists 0 < \bar{q}_0 < 1 \) such that, if \( q_0 > \bar{q}_0 \), the above inequality is satisfied. Therefore, to ensure that the creditor has no incentive to deviate by making an offer that will be rejected by both types of debtor, it requires that \( q_0 > \max\left\{ \bar{q}_0, \frac{(\pi - s)}{\pi} \right\} \).

(ii) The creditor makes an offer which will be accepted by both types of debtor: \( \hat{y}_2 = \frac{(\pi + s)x}{2} \). Since the expected payoff for the creditor is \( \pi - \frac{(\pi + s)x}{2} \), it requires that
\[
\pi - \frac{(\pi + s)x}{2} < \frac{\delta \pi}{2},
\]
which, after rewriting, we obtain the following condition:
\[
2\pi \left( 1 - \delta \right) < \bar{s},
\]
i.e. the sustainability concerns have to be sufficiently important to outweigh the cost of delay. Rewriting this inequality as:
\[
\frac{2(1 - \delta)}{\delta} < \frac{\bar{s}}{\pi}, \tag{B4}
\]

For the purpose of illustration, we see that for \( \delta = 0.8 \) for example, condition (B4) requires that the perceived needs of sustainability would need to be at least 50% of the available bargaining surplus, \( \pi \), in order for delay to be an attractive strategy for the Cautious debtor. Moreover, the required sustainability ratio, \( \frac{s}{\pi} \), rises sharply as higher rates of discount (lower \( \delta \)) increase the cost of this signalling strategy. Since this condition (B4) is always true for \( \delta \) sufficiently close to 1, it follows that the equilibrium offer by the creditor is \( \hat{y}_2 = \frac{\delta \pi}{2} \) with payoff of \( \frac{\delta \pi}{2} \) for the Optimistic debtor and payoff of \( \frac{\delta(\pi + s)}{2} \) for the Cautious debtor. The expected payoff for the creditor is \( q_0 \left( \pi - \frac{\delta \pi}{2} \right) + (1 - q_0) \left( \frac{(\pi - s)x}{2} \right) \).
Next, we compute the continuation value for the creditor and both types of debtor at $t = 1$. There are two scenarios to be considered: (i) the debtor knows his own type, and (ii) the debtor does not know his own type. We consider each of these two scenarios below.

In the first scenario, at $t = 1$, the debtor knows his own type. In this case, the continuation payoff for the Optimistic debtor at $t = 1$ is

$$\delta \left[ \frac{1}{2} \left( \frac{2 - \delta}{2} \right) \pi \right] + \frac{1}{2} \left( \frac{\delta \pi}{2} \right) = \frac{\delta \pi}{2};$$

the continuation payoff for the Cautious debtor at $t = 1$ is

$$\delta \left[ \omega_C + \frac{(\pi + s) \delta}{2} \right];$$

and the continuation payoff for the creditor at $t = 1$ is

$$\delta \left[ q_0 \left( \frac{\delta \pi}{2} \right) + (1 - q_0) \left( \frac{\pi - s \delta}{2} \right) \right].$$

In the second scenario, at $t = 1$, debtor does not know his own type. In this case, the creditor’s payoff is the same as in the first scenario. The debtor’s payoff is

$$q_0 \delta \left[ \frac{\pi (2 - \delta)}{2} + \frac{\delta \pi}{2} \right] + (1 - q_0) \frac{\delta}{2} \left[ 2 \omega_C + (\pi + s \delta) \right].$$

In what follows, we consider the bargaining game at $t = 1$. In the first period, the sovereign debtor is a proposer. By computation, it follows from the first and the second scenarios that the expected payoff for the creditor
from rejecting the debtor’s offer at $t = 1$, denoted by $\tilde{a}$, is defined as:

$$
\tilde{a} = \delta \left[ p \left\{ q_0 \left( \frac{\delta\pi_L}{2} \right) + (1 - q_0) \left( \frac{(\pi_L - \bar{s})\delta}{2} \right) \right\} + (1 - p) \left\{ q_0 \left( \frac{\delta\pi_H}{2} \right) + (1 - q_0) \left( \frac{(\pi_H - \bar{s})\delta}{2} \right) \right\} \right],
$$

or

$$
\tilde{a} = \delta \left[ \frac{\delta E\pi}{2} - \frac{(1 - q_0)\delta \bar{s}}{2} \right].
$$

Alternatively, $\tilde{a}$ can be interpreted as the minimum payoff for the creditor to accept the debtor’s offer.

The continuation values computed earlier would limit the offers that can be made by the debtor. Let us derive the conditions which make delay becomes attractive. We begin with the case of an Optimistic debtor. The best offer that the Optimistic debtor can make is the excess of the available bargaining surplus over his own continuation value, given by $\pi_L - \frac{\delta E\pi}{2}$. If this offer falls below the expected payoff for the creditor, $\tilde{a}$, this offer will not be accepted. Formally, the condition for the first-period delay for the Optimistic debtor is given by:

$$
\pi_L - \frac{\delta E\pi}{2} < \tilde{a}, \quad (B5)
$$

where $E\pi = p\pi_L + (1 - p)\pi_H$ denotes the expected size of $\pi$. Given that Condition (B5) is satisfied, the Optimistic debtor will delay in period 1. Since $\tilde{a} = \delta \left[ \frac{\delta E\pi}{2} - \frac{(1-q)\delta \bar{s}}{2} \right]$, condition (B5) becomes

$$
\frac{2}{\delta^2} - \frac{E\pi}{\delta \pi_L} + \frac{\bar{s}}{\pi_L} - \frac{q\bar{s}}{\pi_L} - 1 < E_g,
$$

where $E_g = \frac{(E\pi - \pi_L)}{\pi_L}$ denotes the expected growth of the economy. This seems to suggest that the first-period delay becomes attractive for the Op-
timistic debtor with bullish growth prospects but otherwise with relatively low growth prospects.

Next, we consider the case of a Cautious debtor. The best offer that the Cautious debtor can make is the excess of the available bargaining surplus over his continuation value, given by $\pi_L - \delta \left[ \omega_C + \frac{\delta(E \pi + \bar{s})}{2} \right]$. If this offer falls below the creditor’s continuation value, $\tilde{a}$, this offer will not be accepted. Formally, the condition for the first-period delay for the Cautious debtor is given by:

$$\pi_L - \delta \left[ \omega_C + \frac{\delta(E \pi + \bar{s})}{2} \right] < \tilde{a}, \quad \text{(B6)}$$

and, given that the following condition (B6) is satisfied, the Cautious debtor will delay in period one. Since $\tilde{a} = \delta \left[ \frac{\delta E \pi}{2} - \frac{(1-q) \delta \bar{s}}{2} \right]$, condition (B6) can be re-written as follows:

$$\frac{2}{\delta^2} - \frac{2 \omega_C}{\delta \pi_L} - \frac{E \pi}{\pi_L} - \frac{q \bar{s}}{\pi_L} - 1 < Eg,$$

where, again, this condition seems to suggest that the first-period delay becomes attractive for the Cautious debtor with bullish growth prospects.

Then, provided that the derived conditions for delay in the second period are also satisfied, there will be a positive probability for a two-period delay in bargaining: initially reflecting recovery (growth), followed by signalling by debtor about sustainability.

Note that, because $\tilde{a} < \frac{\delta E \pi}{2}$, condition (B5) is more stringent than condition (A7). Similarly, since $\tilde{a} < \frac{\delta(E \pi - \bar{s})}{2}$ and $\delta \left[ \omega_C + \frac{\delta(E \pi + \bar{s})}{2} \right] < \frac{\delta(E \pi + \bar{s})}{2}$, condition (B6) is more stringent than condition (A8). Therefore, the conditions for first-period delay in the model with asymmetric information about the debtor’s type are more stringent than in the full information case discussed in Case 2. The reason is that there is a positive risk of the creditor
facing a Cautious debtor, which lowers the creditor’s continuation payoff.

To study the correlation between delay in the sovereign debt restructuring and the size of the haircut, it is necessary that we compare the creditor’s payoff in the case with one-period delay (Case 2) and the case with two-period delay (Case 3). The creditor’s continuation payoffs from rejecting the debtor’s offer at $t = 1$ under each case are summarised in the table below.

<table>
<thead>
<tr>
<th>Debtor’s type</th>
<th>One-period Delay</th>
<th>Two-period Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimistic debtor</td>
<td>$\frac{\delta E^c}{2}$</td>
<td>$\hat{a}$</td>
</tr>
<tr>
<td>Cautious debtor</td>
<td>$\frac{\delta (E^c - \delta)}{2}$</td>
<td>$\hat{a}$</td>
</tr>
</tbody>
</table>

**Table B3**: Delay and haircuts in debt restructuring (Pure Strategy)

We see from Table B3 that, when there is two-period delay, the creditor’s payoff is lower relative to the case with one-period delay. It follows that prolonged delay is positively correlated with a larger haircut in our model of sovereign debt swap. The key result is that it is the extension of delay that leads to a larger haircut.