Deep Habits and the Cyclical Behaviour of Equilibrium Unemployment and Vacancies*

Federico Di Pace†
Birkbeck College

Renato Faccini‡
Bank of England

October 12, 2009

Abstract

We extend the standard textbook search and matching model by introducing deep habits in consumption. The cyclical fluctuations of vacancies and unemployment in our model can replicate those observed in the US data, with labour market tightness being twenty times more volatile than consumption. Vacancies display a hump-shaped response to technology shocks as well as autocorrelation coefficients that are in line with the empirical evidence. Our model preserves the cyclicality of the wages for the new hires and the calibration is consistent with the estimated elasticity of unemployment to unemployment benefits. The numerical simulations generate an artificial Beveridge curve which is in line with the data.

Keywords: Consumption; Business Cycles; Labour Market Fluctuations; Search and Matching; Wage Bargaining.

JEL Classification Numbers: E21;E24; E32; J41; J64

---

*The views expressed in this paper are those of the authors and do not necessarily represent those of the Bank of England. This is work in progress and should not be cited or quoted without the permission of the authors.

†The author greatly acknowledges the hospitality received from the Structural Economic Analysis Division at the Bank of England. Financial Support from the Economic and Social Research Council (award PTA-030-2005-00549) and from Experian PLC is greatly acknowledged. Address: Department of Economics, Mathematics and Statistics, Birkbeck, University of London, Malet Street, London WC1E 7HX. E-mail: fdipace@ems.bbk.ac.uk.

‡Corresponding Author. Address: Threadneedle Street, London EC2R 8AH. E-mail: Renato.Faccini@bankofengland.co.uk.

1 Introduction

In a seminal contribution, Shimer (2005) shows that the standard textbook matching model is unable to account for the high volatility of unemployment and vacancies as observed in the US data. Following his work, many studies have attempted to generate new sources of amplification in labour market variables. On the one hand, Gertler and Trigari (2009), Hall (2005a), Hertweck (2006), Kennan (2009), Menzio (2005), Moen and Rosen (2006) and Rudanko (2009) have shown that introducing wage rigidities for new hires into standard matching models can generate fluctuations in labour market variables that are in line with the data. But Pissarides (2009) and Haefke, Sonntag, and van Rens (2009) argue that this assumption does not appear to be supported by microeconometric evidence. These findings suggest that explanations of the unemployment volatility puzzle should preserve the cyclical volatility of wages. On the other hand, Hagedorn and Manovskii (2008) have shown that an alternative calibration of the standard search and matching model can generate sufficiently large fluctuations in unemployment and vacancies. Their results are driven by the value of the non-market activity being set close to the value of search to the worker. Costain and Reiter (2008) have however shown that this calibration implies an implausible elasticity of labour supply to unemployment insurance.

We develop a model that is immune to the above critiques. Following the work by Ravn, Schmitt-Grohe, and Uribe (2006), we introduce deep habits into a search and matching model to explain the cyclical behaviour of labour market variables. In their model habits are formed at the good-specific level rather than at the aggregate level. To facilitate the comparison with Shimer (2005), we assume that technology shocks are the only source of fluctuations, and employment is the only factor of production. Ravn, Schmitt-Grohe, and Uribe (2006) show that deep habits are key in matching the counter-cyclical behaviour of mark-ups in the US. In this study, we show that deep habits are also key in generating volatility in labour market variables.

In our model, artificial series for vacancies and unemployment match the empirical moments calculated using US data. As a result, the volatility of labour market tightness is about twenty times the volatility of aggregate consumption. The model overcomes the critique posed by Fujita and Ramey (2007), who show that the standard matching framework fails to generate hump-shaped vacancy responses to technology shocks. Our model is also successful at matching the persistence of vacancies with the data.

This work is also related to Faccini and Ortigueira (2008), where it is shown that investment-specific technology shocks can generate cyclical fluctuations in labour market variables that match the data. Our paper improves on search and matching models with time-varying job destruction rates, such as Faccini and Ortigueira (2008), Ramey (2008) and Shimer (2005), to the extent that we are able to generate a downward sloping Beveridge curve. The correlation between vacancies and unemployment in our model is close to the corresponding measure in the data.

Rotemberg (2006) shows that a search and matching model with exogenous shocks to mark-ups can match the volatilities in labour market variables. In his model, a mark-up shock leads to an expansion in employment and to a fall in the marginal product of labour which dampens the increase in the real wage paid to new workers.

This mechanism of amplification stands in sharp contrast with the mechanism of the standard model, where technology shocks are the only source of fluctuations. As pointed out by Shimer (2005), Nash bargaining implies that wages absorb most of the incentives for job creation, which are generated by changes in productivity.

Our main contribution is that we endogenise the dynamics of mark-ups into a search and matching model by introducing deep habits. In our economy, deep habits magnify the impact of technology shocks on vacancies by generating fluctuations in the real marginal cost, which can be interpreted as the shadow value of output. In the standard matching model, job creation depends on the current return of an additional worker, that is, on the difference between the productivity and the wage. In a model with monopolistic competition, this current period return is evaluated at the shadow value of output. As a result, fluctuations in the real marginal cost affect the incentive to post vacancies.

As in Ravn, Schmitt-Grohe, and Uribe (2006), fluctuations in the real marginal cost are driven by the *inter-temporal effect* and the *price-elasticity effect* effect. A technology shock increases the present value of per unit profits and generates an incentive for firms to invest in consumer base. Firms do so by building up the stock of habits. In turn, this increase in habits is achieved by higher current sales, which entails a decrease in the mark-up and an increase in the real marginal cost. This effect is known as the intertemporal effect. In addition, an expansion in aggregate demand increases the price elasticity, which in turn decreases the mark-up and raises the marginal cost. Ravn, Schmitt-Grohe, and Uribe (2006) refer to this effect as the price elasticity effect. The main difference between our mechanism of amplification and Rotemberg (2006) is the following: In Rotemberg (2006) there is no intertemporal effect, but only a price elasticity effect, which is generated by exogenous shocks to the elasticity of demand.

The paper is organized as follows. The first section presents the model. The second section discusses the main mechanism of amplification and propagation. The third section compares the results of our simulations with the data. The last section concludes.

2 The Model

2.1 The Labour market

The labour market is frictional in that firms fill jobs by posting vacancies. The technology that matches jobs with workers is given by

$$M_t = M v_t^{\xi} u_t^{1-\xi}, \tag{1}$$

where M_t denotes the aggregate flow of hires at time t, u_t denotes aggregate unemployment, v_t aggregate vacancies. The parameter $\xi \in (0, 1)$ is the elasticity of the matching function with respect to unemployment and the parameter M is the efficiency of matching. At time t, vacancies are filled with probability $q(\theta_t) \equiv M_t/v_t = \theta_t^{\xi-1}$, where $\theta_t = v_t/u_t$ denotes labour market tightness. The assumption of constant returns to scale in the matching function implies that workers find jobs with probability $M_t/u_t = \theta_t q(\theta_t)$.

Following Blanchard and Gali (2008), we assume that workers matched at the beginning of time t become immediately productive. The law of motion for aggregate employment, denoted by n_t, can be written as

$$n_t = M_t + (1 - \rho)\, n_{t-1}, \tag{2}$$

where ρ is the exogenous job destruction rate. Unemployment evolves according to the following law of motion

$$u_t = 1 - (1 - \rho)\, n_{t-1}. \tag{3}$$

This condition states that the stock of workers searching for a job at time t is given by the measure of workers who did not have a job at $t-1$, $1 - n_{t-1}$, plus the measure of workers who lost their job at the end of $t-1$, ρn_{t-1}.

2.2 The Problem of the Household

The economy is populated by a unit measure of identical households, indexed by $i \in [0,1]$. Households have preferences over different consumption varieties, indexed by $j \in [0,1]$. Workers can be either employed or unemployed. Members of the household employed at firm j earn a wage rate w_{jt} and suffer disutility from working, while the unemployed members receive unemployment subsidies b. We assume that the workers can perfectly insure against idiosyncratic shocks. This assumption implies that all members of the households enjoy the same level of consumption. We denote by $r_{t,t+1}$ the price of the state contingent asset at time t that pays one unit of consumption at time $t+1$. Following Ravn, Schmitt-Grohe, and Uribe (2006), we assume that household preferences exhibit "external habit formation" in consumption at the good-specific level rather than at the aggregate level. This consumption externality has been coined as deep habits or, alternatively, as "catching up with the Joneses good by good".

Household i solves two problems: an intratemporal and an inter-temporal problem. The former problem is to minimise total consumption expenditure, $\int_0^1 p_{jt} c^i_{jt} dj$, subject to the following consumption object

$$x^i_t = \left[\int_0^1 \left(c^i_{jt} - \zeta s_{jt-1} \right)^{1-1/\epsilon} dj \right]^{1/(1-1/\epsilon)}, \tag{4}$$

where s_{jt} denotes the stock of external habit in the consumption of good j at time $t-1$, $\zeta \in [0,1]$ the degree of external habit formation of each variety and ϵ the intratemporal elasticity of substitution of the habit adjusted consumption across varieties. The stock of habits is assumed to evolve over time according to the following law of motion

$$s_{jt} = \vartheta s_{jt-1} + (1 - \vartheta)\, c_{jt}, \tag{5}$$

where the parameter $\vartheta \in (0,1)$ measures the speed of adjustment of the stock of external habits to changes in the average level of consumption of variety j. A value of ϑ equal to 0 implies that the stock of habits exhibits no persistence.

By minimising expenditure with respect to c_t^j, we derive the individual consumption demands of variety j by household i

$$c_{jt}^i = \left(\frac{p_{jt}}{p_t}\right)^{-\epsilon} x_t^i + \zeta s_{jt-1}, \tag{6}$$

where $p_t \equiv \left[\int_0^1 p_{jt}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$ is the nominal price index and p_{jt} is the price of good j. The consumption demand for each variety j is decreasing in the relative price of good j, p_{jt}/p_t, and increasing in both the level of habit adjusted consumption, x_t^i, and, for positive values of ζ, in the aggregate stock of habits s_{jt-1}. Aggregating across households implies that $P_t x_t^i = \int_0^1 P_{jt}\left(c_{jt}^i - \zeta s_{jt-1}\right) dj$.

The second problem of the household i is to maximise their lifetime utility by choosing the consumption object, x_t^i, and state contingent bonds, d_{t+1}^i. The period utility is defined as

$$\mathcal{U}\left(x_t^i, n_{it}\right) = \frac{\left(x_t^i\right)^{1-\sigma} - 1}{1-\sigma} - \chi \frac{\left(n_t^i\right)^{1+\varphi}}{1+\varphi},$$

where the first term of this expression denotes utility from habit adjusted consumption and the second term disutility of work. The term $n_t^i = \int_0^1 n_{jt}^i dj$ represents aggregate employment at the household i and σ denotes the inter-temporal elasticity of substitution between the consumption object at time t and $t+1$. The parameter φ is the inverse of the Frisch elasticity of labour supply and χ is a constant.

The life-time utility of household i is given by

$$\max_{x_t^i} \quad E_t \sum_{s=0}^{\infty} \beta^s \mathcal{U}\left(x_{t+s}^i, n_{t+s}^i\right), \tag{7}$$

where E_t is the mathematical expectation operator conditional on the information available at time t and $\beta \in (0,1)$ is the subjective discount factor. The household i's budget constraint expressed in terms of the aggregate consumption good is given by

$$x_t^i + \omega_t + E_t r_{t,t+1} d_{t+1}^i = d_t^i + \int_0^1 w_{jt} n_{jt}^i dj + \left(1 - n_t^i\right) b + \pi_t^i, \tag{8}$$

where ω_t is equal to $\zeta \int_0^1 \left(\frac{p_{jt}}{p_t}\right) s_{jt-1} dj$, and π_t^i are the real profits of the firms distributed to each household i at time t. We assume that households face an additional constraint that prevents them from engaging in Ponzi games. Finally, the law of motion of employment is given by the following equation

$$n_t^i = (1-\rho)\, n_{t-1}^i + \theta_t q(\theta_t) u_t^i. \tag{9}$$

Household i maximises (7) by choosing the processes x_t^j and d_{t+1}^i subject to condition (8) and a no Ponzi-game constraint. The firm takes π_t, ω_t, w_t and d_0 as given. The first order conditions are

$$\lambda_t^i = \mathcal{U}_x\left(x_t^i, n_t^i\right), \tag{10}$$

and

$$\lambda_t^i r_{t,t+1} = \beta E_t \lambda_{t+1}^i. \tag{11}$$

The first equation is simply the marginal utility of the consumption object. The second equation is the standard Euler condition that sets the marginal cost of consumption at time t equal to the marginal benefit of consumption at time $t+1$. In addition, the non-arbitrage condition with respect to employment is given by

$$\mathcal{W}_{jt}^i = w_{jt}^i - \left[b - \frac{\mathcal{U}_n\left(x_t^i, n_t^i\right)}{\lambda_t^i} \right] + E_t \beta \frac{\lambda_{t+1}^i}{\lambda_t^i} (1-\rho) \mathcal{W}_{jt+1}^i \left[1 - \theta_{t+1} q(\theta_{t+1})\right], \tag{12}$$

where $\mathcal{W}_{jt}^i$ denotes the value to the household of having an additional worker employed at firm j.

2.3 The Problem of the Firm

There is a unit mass of monopolistically competitive firms, each of which produces a particular variety of the final good. Each variety j is produced using labour as the sole factor of production. The production process exhibits decreasing returns to scale, and it is given by $y_{jt} = A_t n_{jt}^{\alpha}$. The variable y_{jt} denotes the output of firm j and A_t is an aggregate technology shock that follows a stochastic process of the form

$$\ln A_t = \varrho_a \ln A_{t-1} + \varepsilon_{at} \quad \text{with} \quad \varepsilon_{at} \sim N\left(0, \varsigma_a\right), \tag{13}$$

where ϱ_a is the persistence of the technology shock and ς_a is the standard deviation of the innovation ε_{at}. Firms open vacancies at the beginning of each period in order to control the level of employment. When posting vacancies, firm j takes the matching probabilities as given.

The assumptions of decreasing returns in production and Nash bargaining imply that both the marginal product of labour and wages depend on the firm's size. Following Stole and Zwiebel (1996), Cahuc, Marque, and Wasmer (2008) argue that if vacancies are opened *before* wages are bargained, firms should anticipate how vacancies affect the negotiated wage. This bargaining problem is known as intrafirm bargaining. In contrast, we assume that the opening of vacancies, the hiring of workers and the bargaining of wages occur *simultaneously*. In principle, intrafirm bargaining has the potential to drive a wedge between large firm models and the standard one-worker-one-firm characterization as in Shimer (2005). We abstract from intrafirm bargaining to isolate the effect of deep habits on labour market variables[1].

Opening vacancies is costly in that the resources that could be otherwise devoted to producing the consumption good are diverted to hiring. We therefore assume that each firm faces a series of resource constraints of the form:

$$A_t n_{jt}^{\alpha} = c_{jt} + C\left(v_{jt}\right), \tag{14}$$

[1] Faccini and Ortigueira (2008) show that, when neutral technology shocks are the only source of fluctuation, intra-firm bargaining has negligible implications for the cyclical behaviour of labour market variables. Another study by Krause and Lubik (2007) finds similar results.

where $C(v_{jt}) = \kappa v_{jt}$ is the vacancy cost function.

By adding up the cross-sectional individual demands for good j, we can recover aggregate demand for good j. Analytically, we simply integrate expression (6) over i to obtain

$$c_{jt}^{i} = \left(\frac{p_{jt}}{p_t}\right)^{-\epsilon} x_t + \zeta s_{jt-1}, \tag{15}$$

where $x_t = \int_0^1 x_{it} di$ is a habit adjusted measure of aggregate consumption across varieties. The demand for good j depends on the sum of a price elastic term, $(p_{jt}/p_t)^{-\epsilon} x_t$, and a price inelastic term ζs_{jt-1}. An expansion in aggregate demand increases the weight of the price elastic term in the demand function, which implies that the price elasticity of demand for good j is positively related to aggregate demand. Since mark-ups are inversely related to the price elasticity of demand, the deep habits model predicts that mark-ups are time-varying and counter-cyclical.

Monopolistic competition implies that each firm sets their price by taking the prices of all other firms as given. At the announced price, each firm stands ready to meet the demand for its own variety. We define the profit of firm j at time t by $\phi_{jt} = p_{jt} c_{jt}/p_t - w_t n_{jt}$. The problem of the representative firm is to choose the processes c_{jt}, p_{jt}, s_{jt} and v_{jt} so as to maximise the present discounted value of expected profits,

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} \left[\frac{p_{jt+s}}{p_{t+s}} c_{jt+s} - w_{jt+s} n_{jt+s}\right],$$

subject to the resource constraint, (14), the law of motion of employment,

$$n_{jt} = (1-\rho) n_{jt-1} + v_{jt} q(\theta_t), \tag{16}$$

the law of motion of the habit stock, (5), and demand for good j, (15). The first order conditions with respect to vacancies, consumption, the stock of habits and prices are:

$$C'(v_{jt}) mc_{jt} = \daleth_{jt} q(\theta_t), \tag{17}$$

$$mc_{jt} = \frac{p_{jt}}{p_t} - \nu_{jt} + (1-\vartheta)\psi_{jt}, \tag{18}$$

$$\psi_{jt} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left[\vartheta \psi_{jt+1} + \zeta \nu_{jt+1}\right], \tag{19}$$

and

$$c_{jt} = \epsilon \left(\frac{p_{jt}}{p_t}\right)^{-\epsilon-1} x_t \nu_{jt}. \tag{20}$$

where mc_{jt}, $\daleth_{jt}$, ψ_{jt} and ν_t are the Lagrange multipliers associated with the constraints in equations (5), (14), (15) and (16), respectively. The shadow value of output, which is denoted in our model by mc_{jt}, is the contribution of an additional unit of output to the profits of the firm, being also equal to the firm's real marginal costs. In addition, the non-arbitrage condition with respect to employment is

$$\daleth_{jt} = \alpha A_t mc_{jt} n_{jt}^{\alpha-1} - w_{jt} + E_t \beta \frac{\lambda_{t+1}}{\lambda}(1-\rho)\daleth_{jt+1}, \tag{21}$$

where $\mathcal{J}_{jt}$ denotes the Lagrange multiplier associated with the employment constraint, which is equal to the current period marginal value of employment to the firm. By combining (17) with (21), we can find an expression for the job creation condition

$$\frac{C'(v_{jt})}{q(\theta_t)} mc_{jt} = \alpha A_t mc_{jt} n_{jt}^{\alpha-1} - w_{jt} + \beta(1-\rho) E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{C'(v_{jt+1})}{q(\theta_{t+1})} mc_{jt+1}. \quad (22)$$

This condition states that firms will expand employment up to the point where the marginal cost equals the marginal benefit of employing an additional worker. The LHS of expression (22) measures the expected cost of increasing employment at the margin. Since the adjustment costs in our set-up are interpreted as forgone output, the expected cost of employment must be equal to the additional cost of posting a vacancy, denoted by $C'(v_{jt})$, times the average duration of a vacancy, $1/q(\theta_t)$, evaluated at the shadow value of output.

The first order condition with respect to consumption, equation (18), expresses the shadow value of output as the sum of three components. The first two terms represent the current period revenues associated with a marginal increase in output. This is equal to the revenue p_{jt}/p_t, obtained on the marginal unit of production net of the forgone revenue on inframarginal quantities, ν_{jt}. The third component of equation (18) is the shadow value of the habit stock and is equal to the present value of the expected revenues associated with an additional sale of good j.

2.4 Wage Bargaining

Wages are renegotiated every period and solve a standard Nash bargaining problem. This bargaining protocol implies that wages maximise the weighted product of the worker's and the firm's value of employment. Formally, the wage solves the following condition

$$w_{jt} = \arg\max \mathcal{W}_{jt}^{\gamma} \mathcal{J}_{jt}^{1-\gamma},$$

where γ denotes the bargaining power of the worker. The first order condition yields the standard sharing rule:

$$\gamma \mathcal{J}_{jt} = (1-\gamma) \mathcal{W}_{jt}. \quad (23)$$

By plugging (12) and (21) into (23), and using $(1-\gamma) E_t \mathcal{W}_{jt+1} = \gamma E_t \mathcal{J}_{jt+1}$, it follows that:

$$w_{jt} = \gamma \alpha A_t mc_{jt} n_{jt}^{\alpha-1} + (1-\gamma)\left[b - \frac{U_n(x_t, n_t)}{\lambda_t}\right] + \gamma\beta(1-\rho) E_t \frac{\lambda_{t+1}}{\lambda_t} \mathcal{J}_{jt+1} \theta_{t+1} q(\theta_{t+1}).$$

Replacing the first order condition for vacancies, (17), into the above equation we obtain

$$\begin{aligned} w_{jt} &= \gamma \alpha A_t mc_{jt} n_{jt}^{\alpha-1} + (1-\gamma)\left[b - \frac{U_n(x_t, n_t)}{\lambda_t}\right] \\ &\quad + \gamma\beta(1-\rho) E_t \frac{\lambda_{t+1}}{\lambda_t} C'(v_j) mc_{jt+1} \theta_{t+1}. \end{aligned} \quad (24)$$

This condition states that the real wage is a weighted average of the marginal product of the worker plus the cost of replacing the worker and the opportunity cost of working. This wage equation differs from the standard equation because it depends on both current and expected future marginal costs. As we show later in the text, the real marginal cost plays a key role in generating amplification in our model. An implication of this equation is that the dynamics of the real wage will track the real marginal cost quite closely.

2.5 Aggregation and the Competitive Equilibrium

Using the assumption of symmetry, aggregation in our model economy is simple and can be obtained by integrating over households and firms. We simply remove the i and j subscripts from the equations in the main text to obtain the aggregate economic behaviour in our model economy. A stationary competitive equilibrium is a set of stationary processes ν_t, λ_t, μ_t, ϕ_t, n_t, w_t, v_t, u_t, M_t, c_t, x_{ct}, mc_t, θ_t, $r_{t,t+1}$ satisfying equations (1)-(3), (5), (10)-(11), (13), (15), (17)-(21) and (24), given the exogenous stochastic process $\{A_t\}_{t=0}^{\infty}$ and the initial conditions s_{t-1} and n_{t-1}.

3 The source of amplification

In this section we discuss the mechanism of amplification. To gain intuition, we take a closer look at the job creation condition

$$\frac{C'(v_{jt})}{q(\theta_t)} mc_{jt} = \alpha A_t mc_{jt} n_{jt}^{\alpha-1} - w_{jt} + (1-\rho)\beta E_t \frac{\lambda_{t+1}}{\lambda_t} \frac{C'(v_{jt+1})}{q(\theta_{t+1})} mc_{jt+1}.$$

Employment fluctuations in our model are driven by the shadow value of output, which is strongly procyclical and volatile. The real marginal cost enters the job creation condition mainly by affecting the marginal revenue product and the real wage. The sum of these two terms is the current period net return of a marginal worker, which is increasing in mc_{jt}. As a result, an increase in the real marginal cost will create a higher incentive to post vacancies.

To understand the dynamics of the real marginal cost, we copy the first order condition with respect to consumption:

$$mc_{jt} = \frac{p_{jt}}{p_t} - \nu_{jt} + (1-\vartheta)\psi_{jt}. \tag{25}$$

As we have previously discussed, the first two terms on the RHS of equation (25) are the current period revenues associated with a marginal increase in output. The third component is the present value of the expected revenues associated with an additional sale of the good. We start by discussing the role of this last term.

By iterating forward equation (15), we can derive an expression for ψ_{jt} as a sum of the discounted shadow values of expected sales

$$\psi_{jt} = \zeta E_t \sum_{s=0}^{\infty} \vartheta^s \beta^{s+1} \frac{\lambda_{t+s+1}}{\lambda_t} \nu_{jt+s+1}.$$

Therefore, the introduction of deep habits magnifies the impact of a positive technology shock on the present value of per unit profits via higher expected future sales. This effect is what Ravn, Schmitt-Grohe, and Uribe (2006) refer to as the *inter-temporal effect* of deep habits. Higher future sales in our model translate into higher vacancies through the job creation condition. It is worth noticing that this expression is increasing in the degree of habit formation ζ.

The *inter-temporal effect* of deep habits is re-enforced by a *price-elasticity* effect. By combining (15) and (20), we can derive an expression for ν_{jt} that depends on the relative price of good j, $\nu_{jt} = -c_j \partial \left(p_{jt}/p_t\right)/\partial c_j$. The variable ν_{jt} is positive because the partial derivative $\partial \left(p_{jt}/p_t\right)/\partial c_j$ is negative. Factoring p_{jt}/p_t on the first two terms on the RHS of (25), we can write $p_{jt}/p_t - \nu_{jt} = p_{jt}/p_t \left(1 + \varepsilon_{p,c}\right)$, where $\varepsilon_{p,c}$ denotes the price elasticity of consumption demand for good j. Since the price elasticity is pro-cyclical with deep habits, it follows that a positive technology shock increases the real marginal cost. An alternative way to see this effect is to substitute for ν_{jt} using equation (20). In the symmetric equilibrium, where $p_{jt} = p_t$, it follows that $\nu_t = c_t/\left(\epsilon x_t\right)$. Substituting for $x_t = c_t - \zeta s_{t-1}$, we can re-write ν_t as

$$\nu_t = \frac{c_t}{\epsilon \left(c_t - \zeta s_{t-1}\right)}.$$

This expression is decreasing with c_t, and hence the price elasticity is pro-cyclical. In the following section we disentangle the impact of the *price-elasticity effect* and the *inter-temporal effect* on the marginal cost by analysing the impulse responses.

Since the dynamics of the wage is driven by the real marginal cost, both variables will exhibit about the same volatility. However, we are not concerned with this result. Pissarides (2009) and Haefke, Sonntag, and van Rens (2009) show that what matters for job creation is only the volatility of wages for the new hires. The volatility of this subset of wages in the data appears to be compatible with the assumption of full flexibility in the standard model. Therefore, these studies call for solutions of the unemployment volatility puzzle that preserve the cyclicality of wages.

4 Calibration

In this section we assign numerical values to the parameters of the model following a standard calibration exercise. We rely on two sources of information. Some parameters are set using *a priori* information. The remaining ones are set such that the stationary equilibrium of the model matches a number of stylised facts observed in the post WWII US economy. One period of time in our model equals one quarter in the data.

We discuss first the parameters that are set using *a priori* information. The subjective discount factor, β, is set to 0.99, implying a quarterly real interest rate of about 1 percent. The degree of external habit formation ζ, the persistence of the habit stock, ϑ, and the intertemporal elasticity of substitution, σ, are set to 0.86, 0.85 and 2, respectively, as estimated by Ravn, Schmitt-Grohe, and Uribe (2004). The rate of job destruction ρ is set to 0.08 as estimated by Davis, Haltiwanger, and Schuh (1996). The elasticity of the matching function ξ is set to 0.5, the standard value used in the literature. This choice lies within the range of plausible values, between 0.5 and 0.7, as reported by Petrongolo and Pissarides (2001) in a survey of the literature. The inverse of the Frisch elasticity of labour supply ϕ is set to 1. The value of this parameter is in line with the business cycle literature. The coefficient χ in the utility function is normalized to 1 and only the bargaining power parameter γ is set arbitrarily. We choose the standard value of 0.5 in order to facilitate comparability with the existing literature, which typically focuses on the case of symmetric Nash bargaining.

We set the remaining five parameters, namely the elasticity of substitution ϵ, the elasticity of the production function with respect to employment, α, the constant of the vacancy cost function, κ, the constant of the matching function, M, the bargaining power of the worker, b, to match: i) a 20% equilibrium mark-up as in Schmitt-Grohe and Uribe (2007); ii) a labour share of income of 66%; iii) an unemployment rate of 10% as in Hall (2005b). This rate of unemployment, which is somewhat higher than in the data can be justified by interpreting workers who are unmatched in the model as being both actively and passively searching for a job. These passive job seekers are sometimes defined as workers marginally attached to the labour force, meaning that they would be willing to work if they received an offer. As such, they are not captured by the standard ILO definition of unemployment. As pointed out by Trigari (2006), this interpretation is consistent with the abstraction in the model from participation decisions; iv) a vacancy filling rate of 70% as in Trigari (2006) [2]; v) a replacement ratio - computed as the sum of unemployment benefits and the disutility of work over the wage - equal to 70%, as in Gertler, Sala, and Trigari (2008).

The choices of γ and b are very important for the analysis of cyclical fluctuations in labour market variables. Hagedorn and Manovskii (2008) argue that, in the standard textbook matching model with small firms, the combination of a replacement ratio of 95% and a bargaining power of 0.05 generates labour market fluctuations that are in line with the empirical evidence. In their study, the replacement ratio is equal to the value of non-market activity that includes both unemployment subsidies and the value of leisure. Their calibration has been criticised on the grounds that it generates an excessive sensitivity of unemployment to unemployment benefits. In our calibration, the replacement ratio generates a semi-elasticity of unemployment to unemployment benefits around 2, in line with that estimated by Costain and Reiter (2008). Our choice for the bargaining power $\gamma = 0.5$ is standard, and together with our choice of b, it ensures that our results are not driven by the Hagedorn and Manovskii effect.

Our calibration implies a quarterly job finding rate of about 80%. This result is not directly implied by our calibration exercise and it corresponds to the empirical estimates calculated by Shimer (2005)[3]. We set the persistence of the technology shock ϱ_a to 0.95. And, finally, we set the standard deviation of the shock such that the implied volatility of consumption is matched to the data. Table 1 provides a summary of the parameters used in the baseline calibration of our hypothetical model economy.

[2]The same value is used by Cooley and Quadrini (1999) and den Haan, Ramey, and Watson (2000).

[3]Shimer estimates a monthly job finding rate of 0.45. This would imply a quarterly finding rate of 0.83 following the formula $f_q = 1 - (1 - f_m)\hat{}3$, where f_q and f_m denote the quarterly and the monthly finding rate, respectively.

Table 1: Calibrated Parameter Values

Description	Parameter	Value
Intertemporal Elasticity of Substitution	σ	2
Discount Factor	β	0.99
Frisch Elasticity	φ	1
Degree of External Habit Formation	ζ	0.86
Persitence of the Habit Stock	ψ	0.85
Efficiency of the Matching Technology	M	0.75
Elasticity of the Matching Function	ξ	0.5
Cost of posting a vacancy	κ	0.16
Bargaining Power	γ	0.5
Unemployment Benefit	b	0.41
Labour Share	α	2/3
Elasticity of Substitution between varieties	ϵ	8.6
Persistence of Productivity Shock	ρ	0.95
Standard Deviation	ς	0.0018

5 Model Evaluation

We start this section by documenting the stylised facts of the US labour market, and we then assess the ability of our model to replicate these facts. We close the section by discussing the responses of the labour market variables to a productivity shock.

5.1 US data

Table 2 reports the standard deviations, the autocorrelations and the cross-correlations of both consumption and labour market variables using US data. All data series are quarterly and cover the period ranging from 1951Q1 to 2006Q2. The summary statistics refer to the cyclical component of the logged variables, which are detrended using a Hodrick-Prescott filter with a smoothing parameter of 1600.

Notes. Standard Deviations and correlations in this table correspond to quarterly series, detrended using a Hodrick-Prescott filter with smoothing parameter of 1600.

A key stylised fact is that vacancies (v) and unemployment (u) are about 11 times and 10 times more volatile than consumption. The combination of these two facts implies that labour market tightness, $\theta = v/u$, is about 20 times more volatile than consumption. The inability of the standard search and matching model to replicate the data in this dimension has triggered a significant amount of research. Another important stylised fact is that vacancies are highly autocorrelated, with a first order autocorrelation coefficient of 0.91. The data also displays a strongly negative contemporaneous correlation between vacancies and unemployment. This correlation coefficient, which is equal to -0.92, is also known as the slope of the Beveridge curve.

Table 2: Summary Statistics, Quarterly U.S. Data 1951-2006

	c	n	v	u	θ
σ_x/σ_c	1	1.04	11.03	9.77	20.37
Autocorrelation	0.82	0.92	0.91	0.87	0.90
	1	0.6	0.77	-0.65	0.73
	-	1	0.87	-0.91	0.91
Correlation Matrix	-	-	1	-0.92	0.98
	-	-	-	1	-0.98
	-	-	-	-	1

5.2 Simulation results

The model is solved using first order perturbation methods following the study by Schmitt-Grohe and Uribe (2004). We simulate the model by generating 100 artificial series of 100,000 observations each for c, n, v, u, and θ. We then compute the summary statistics for each of series and average across repetitions. The results are reported in Table 3 below.

Table 3: Labour Productivity Shocks

		c	n	v	u	θ
σ_x/σ_c		1	1.03	12.67	9.09	20.58
Autocorrelation		0.96	0.95	0.9	0.95	0.94
	c	1	0.89	0.80	-0.88	0.88
	n	-	1	0.94	-0.95	0.99
Correlation Matrix	v	-	-	1	-0.78	0.96
	u	-	-	-	1	-0.92
	θ	-	-	-	-	1

The simulated statistics reveal that the model is able to replicate the data surprisingly well. Vacancies and unemployment are about 12.7 and 9 times more volatile than aggregate consumption. This result implies that labour market tightness is 20 times more volatile than consumption, which corresponds to the value that we observe in the data. In our model deep habits generate a mechanism of amplification that is sufficiently strong to solve the Shimer's puzzle.

Table 4: Autocorrelations of the Vacancy series

Lags	1	2	3	4
US Data	0.91	0.69	0.43	0.15
Model with Deep Habits	0.9	0.84	0.57	0.39

Notes. Correlations in this table correspond to quarterly series, detrended using a Hodery-Prescott filter with smoothing parameter of 1600.

The model performs well also on other dimensions. Table 4 shows that the dynamic autocorrelation of the artificial vacancy series is in line with its empirical counterpart. Moreover, as we show in the following section, vacancy series are hump-shaped in response to a technology shock. This is another important result since Fujita and Ramey (2007) have emphasised the inability of the standard model to generate persistent vacancy series and hump-shaped responses. All artificial series of the labour market variables exhibit first order autocorrelation coefficients that are in line with the values observed in the data. Similarly, the simulated cross-correlations have about the same order of magnitude as the actual correlations.

The contemporaneous correlation between vacancies and unemployment in our model is -0.77, and this result compares with a value of -0.92 in the data. Our model improves upon models with time-varying stochastic job destruction rates, which exhibit a counterfactual upward-sloping Beveridge curve.

5.3 Impulse Responses

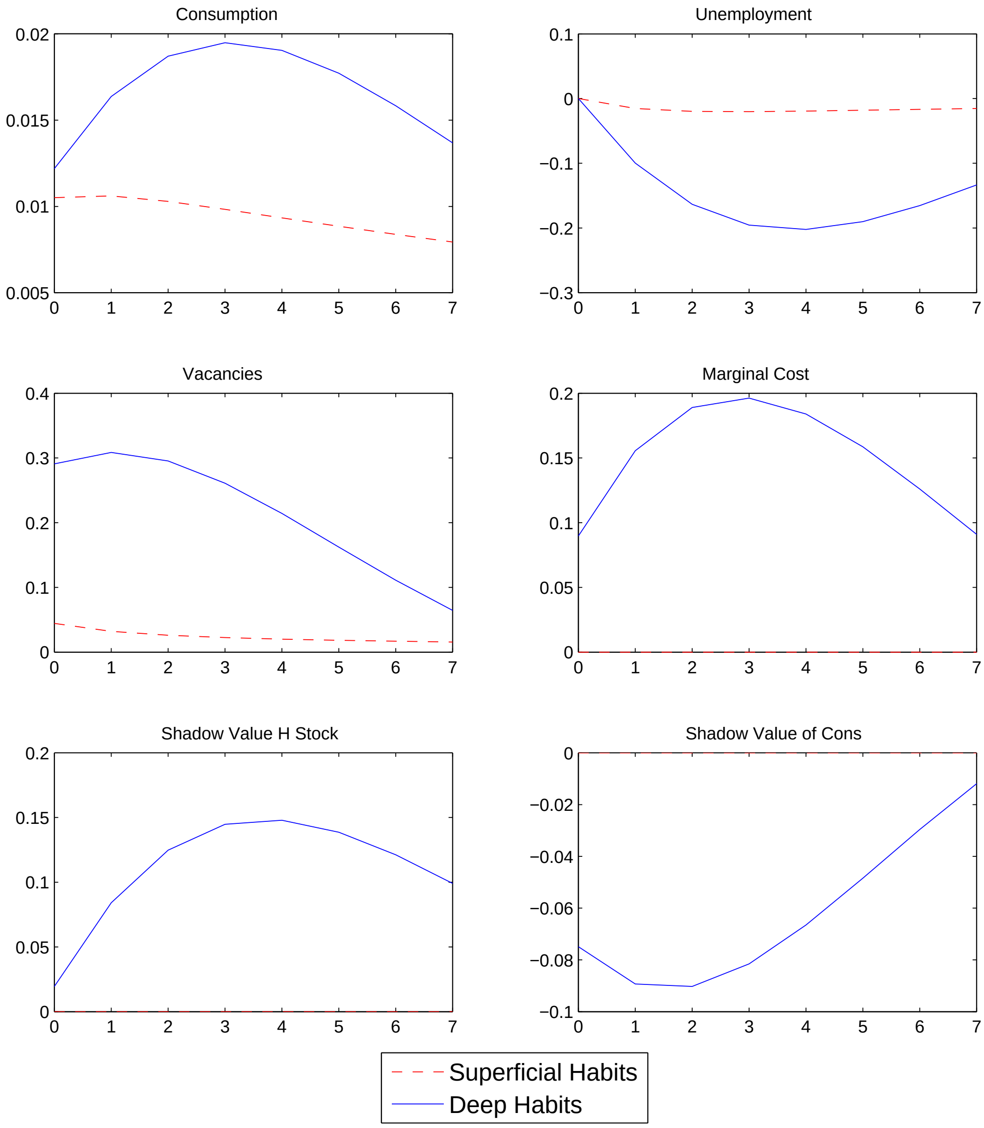
In this section we study how the model responds to a 1% increase in the technology shock. In order to disentangle the effect of deep habits, we compare the dynamic properties of our model with an otherwise identical model featuring superficial habits. The real marginal cost is constant with superficial habits and procyclical with deep habits. As discussed in Section 3, the real marginal cost plays a key role in generating amplification of a technology shock to labour market variables.

Figure 5.3 displays the impulse responses for consumption, unemployment, vacancies and the real marginal cost in both models with deep habits and superficial habits. In addition we show the responses of the shadow value of the habit stock, ψ_t, and the shadow value of consumption, ν_t. As explained in section 3, these two variables explain the fluctuations in the real marginal cost. Equation (25) shows that the real marginal cost is increasing in ψ_t and decreasing in ν_t. The shadow value of the habit stock captures the *inter-temporal effect* of deep habits and the shadow value of consumption the *price elasticity effect*.

Figure 5.3 shows that, following a technology shock, both the inter-temporal effect and the price-elasticity effect increase the real marginal costs. Both sources are quantitatively important. The figure shows that in the deep habit model an increase in the real marginal cost generates strong responses in vacancies and unemployment. When we shut down the real marginal cost channel by replacing deep habits with superficial habits, the model exhibits no amplification to a technology shock.

In the standard search and matching model a technology shocks renders employment more productive, and firms respond by increasing vacancies. However, with Nash bargaining, wages soak up most of the incentives for job creation, and the model exhibits little amplification properties. Under deep habits a technology shock generates an additional incentive for firms to post vacancies. Firms exploit the higher productivity of labour to build up the stock of habits. In addition, the price elasticity of demand increases with aggregate consumption, reinforcing the expansionary effect on vacancies. and employment.

As a result, a technology shock increases consumption, the shadow value of the habit stock, the real marginal cost and vacancies on impact. While consumption and



Notes. Impulse Responses are measured in percent deviation from the steady state. Horizontal axes display the number of quarters after the shock.

Figure 1: Impulse responses to a technology shock under deep and superficial habits

the real marginal cost peak after three quarters, the shadow value of the habit stock and vacancies peak after the fourth quarter and the first quarter respectively. Unemployment and the shadow value of consumption fall in the wake of a productivity shock. The through in unemployment occurs after four periods and that of the shadow value of consumption after two periods. All series are hump-shaped.

6 Conclusions

Search and matching models featuring nominal wage rigidities for new hires have been successful at generating amplification and propagation of technology shocks to labour market variables. These models provide a successful alternative to Hagedorn and Manovskii (2008), who show that a high opportunity cost of work generates sufficient volatility in unemployment and vacancies. However, recent evidence has cast doubts on these assumptions. Pissarides (2009) calls for alternative explanations of the unemployment volatility puzzle that are consistent with the estimated elasticity of labour supply to unemployment insurance and preserve the cyclical volatility of wages.

This paper takes up on this challenge. To this end, we extend the standard search and matching model by introducing deep habits in consumption. To facilitate the comparison with Shimer (2005), we assume that technology shocks are the only source of fluctuations and employment is the only factor of production. We show that the introduction of deep habits into a search and matching model creates a powerful mechanism of amplification whereby technology shocks are transmitted onto labour market variables. Deep habits generate a procyclical real marginal cost, which is in line with the data. This procyclicality in the real marginal cost is at the source of the amplification. Our model is capable of matching the data in several dimensions. Our model solves the unemployment volatility puzzle and generates persistent, hump-shaped vacancy series as well as plausible correlations of labour market variables.

Our paper reinforces the conclusions of Ravn, Schmitt-Grohe, and Uribe (2006) that deep habits have a wide range of macroeconomic implications. Previous work by Ravn, Schmitt-Grohe, and Uribe (2006), Ravn, Schmitt-Grohe, Uribe, and Uuskula (2008) and Ravn, Schmitt-Grohe, and Uribe (2008) shows that deep habits can account for the countercyclicality of mark-ups, the positive response of consumption to a government expenditure shock, the price puzzle and the incomplete pass-through. Our work uncovers an important implication of deep habits for the labour market.

References

BLANCHARD, O., AND J. GALI (2008): "Labor Markets and Monetary Policy: A New-Keynesian Model with Unemployment," NBER Working Papers 13897, National Bureau of Economic Research, Inc.

CAHUC, P., F. MARQUE, AND E. WASMER (2008): "A Theory Of Wages And Labor Demand With Intra-Firm Bargaining And Matching Frictions," *International Economic Review*, 49(3), 943–972.

COOLEY, T. F., AND V. QUADRINI (1999): "A neoclassical model of the Phillips curve relation," *Journal of Monetary Economics*, 44(2), 165–193.

COSTAIN, J. S., AND M. REITER (2008): "Business cycles, unemployment insurance, and the calibration of matching models," *Journal of Economic Dynamics and Control*, 32(4), 1120–1155.

DAVIS, S., J. HALTIWANGER, AND S. SCHUH (1996): *Job Creation and Destruction*. MIT Press.

DEN HAAN, W. J., G. RAMEY, AND J. WATSON (2000): "Job destruction and the experiences of displaced workers," *Carnegie-Rochester Conference Series on Public Policy*, 52(1), 87–128.

FACCINI, R., AND S. ORTIGUEIRA (2008): "Labor-Market Volatility in the Search-and-Matching Model: The Role of Investment-Specific Technology Shocks," Discussion paper.

FUJITA, S., AND G. RAMEY (2007): "Job matching and propagation," *Journal of Economic Dynamics and Control*, 31(11), 3671–3698.

GERTLER, M., L. SALA, AND A. TRIGARI (2008): "An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining," *Journal of Money, Credit and Banking*, 40(8), 1713–1764.

GERTLER, M., AND A. TRIGARI (2009): "Unemployment Fluctuations with Staggered Nash Wage Bargaining," *Journal of Political Economy*, 117(1), 38–86.

HAEFKE, C., M. SONNTAG, AND T. VAN RENS (2009): "Wage Rigidity and Job Creation," Kiel Working Papers 1504, Kiel Institute for the World Economy.

HAGEDORN, M., AND I. MANOVSKII (2008): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited," *American Economic Review*, 98(4), 1692–1706.

HALL, R. E. (2005a): "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, 95(1), 50–65.

——— (2005b): "Job loss, job finding, and unemployment in the us economy over the past fifty years," NBER Working Papers 11678, NBER.

HERTWECK, M. S. (2006): "Strategic Wage Bargaining, Labor Market Volatility, and Persistence," Discussion paper.

KENNAN, J. (2009): "Private Information, Wage Bargaining and Employment Fluctuations," *Review of Economic Studies*.

KRAUSE, M. U., AND T. A. LUBIK (2007): "The (ir)relevance of real wage rigidity in the New Keynesian model with search frictions," *Journal of Monetary Economics*, 54(3), 706–727.

MENZIO, G. (2005): "High Frequency Wage Rigidity," Discussion paper, mimeo.

MOEN, E. R., AND A. ROSEN (2006): "Incentives in Competitive Search Equilibrium and Wage Rigidity," CEPR Discussion Papers 5554, C.E.P.R. Discussion Papers.

PETRONGOLO, B., AND C. A. PISSARIDES (2001): "Looking into the Black Box: A Survey of the Matching Function," *Journal of Economic Literature*, 39(2), 390–431.

PISSARIDES, C. A. (2009): "The Unemployment Volatility Puzzle: Is Wage Stickiness the Answer?," *Econometrica*.

RAVN, M. O., S. SCHMITT-GROHE, AND M. URIBE (2004): "Deep Habits," NBER Working Papers 10261, National Bureau of Economic Research, Inc.

——— (2006): "Deep Habits," *Review of Economic Studies*, 73(1), 195–218.

——— (2008): "Incomplete cost pass-through under deep habits," *Proceedings*.

RAVN, M. O., S. SCHMITT-GROHE, M. URIBE, AND L. UUSKULA (2008): "Deep Habits and the Dynamic Effects of Monetary Policy Shocks," Discussion paper.

ROTEMBERG, J. J. (2006): "Cyclical wages in a search-and-bargaining model with large firms," *Proceedings*.

RUDANKO, L. (2009): "Labor market dynamics under long-term wage contracting," *Journal of Monetary Economics*, 56(2), 170–183.

SCHMITT-GROHE, S., AND M. URIBE (2004): "Solving dynamic general equilibrium models using a second-order approximation to the policy function," *Journal of Economic Dynamics and Control*, 28(4), 755–775.

SCHMITT-GROHE, S., AND M. URIBE (2007): "Optimal Simple And Implementable Monetary and Fiscal Rules," *Journal of Monetary Economics*, 54, 17021725.

SHIMER, R. (2005): "The Cyclical Behavior of Equilibrium Unemployment and Vacancies," *American Economic Review*, 95(1), 25–49.

STOLE, L. A., AND J. ZWIEBEL (1996): "Intra-firm Bargaining under Non-binding Contracts," *Review of Economic Studies*, 63(3), 375–410.

TRIGARI, A. (2006): “The Role of Search Frictions and Bargaining for Inflation Dynamics,” Working Papers 304, IGIER (Innocenzo Gasparini Institute for Economic Research), Bocconi University.