Dynamics of the Price Distribution in a General Model of State-Dependent Pricing

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Abstract

This paper analyzes the effects of monetary policy shocks in a DSGE model that allows for a general form of smoothly state-dependent pricing by firms. As in Dotsey, King, and Wolman (1999) and Caballero and Engel (2007), our setup is based on one fundamental property: firms are more likely to adjust their prices when doing so is more valuable. The exogenous timing (Calvo 1983) and fixed menu cost (Golosov and Lucas 2007) models are nested as special cases of our setup.

Our model is calibrated to match the steady-state distribution of price adjustments in microdata; realism calls for firm-specific productivity shocks. Computing a dynamic general equilibrium requires us to calculate how the distribution of prices and productivities evolves over time. We solve the model using the method of Reiter (2006), which is well-suited to this type of problem because it combines a fully nonlinear characterization of the steady state value function and distribution with a linearization of the aggregate dynamics.

In our calibrated model, increased money growth causes a persistent rise in inflation and output. Uncorrelated money growth shocks have only a small effect on output in the menu cost model, so our calibrated model is closer to the Calvo specification in this case. Correlated money shocks, on the other hand, cause a large increase in consumption on impact in all three specifications, though the persistence of consumption is twice as large in the Calvo specification as it is in the other two.

Impulse responses differ depending on the distribution at the time the shock occurs. In particular, a surprise increase in money growth has different effects starting from the steady state distribution than it does if all firms have recently adjusted their prices.

Keywords: price stickiness, state-dependent pricing, stochastic menu costs, generalized (S,s), heterogeneous agents, distributional dynamics

JEL Codes: E31, E52, D81
1 Introduction

Sticky prices are an important ingredient in modern dynamic general equilibrium models, including those used by central banks for policy analysis. Yet the particular way of modelling price stickiness remains as controversial as ever. The Calvo (1983) model of fixed probability of adjustment has become extremely popular due to its analytical tractability, but it lacks the theoretical appeal of a microfounded model immune to the Lucas critique. In an influential article Golosov and Lucas (2007) propose a model of price-stickiness founded on the idea of a fixed “menu cost” of adjusting prices. They calibrate their model to match certain moments of the distribution of price changes found in US micro evidence. Since their model predicts only small and transitory real effects of monetary shocks, Golosov and Lucas claim that the Calvo setup exaggerates these effects and is therefore misleading for policy analysis.

We consider a general model of smoothly state-dependent pricing by firms that nests a variety of pricing models. Similar to Dotsey, King, and Wolman (1999) and Caballero and Engel (2007), our setup rests on one fundamental property: firms are more likely to adjust their prices when doing so is more valuable. The Calvo and fixed menu cost models are nested as (extreme) special cases of our setup. We calibrate our model to match the distribution of price adjustments found in recent US micro evidence (Klenow and Kryvstov (2005), Midrigan (2006), Nakamura and Steinsson (2007)); in particular, unlike Golosov and Lucas (2007), we are able to reproduce the fact that small price changes are very common in the data, alongside with frequent large price adjustments (Midrigan 2006).

We show that increased money growth causes a persistent rise in both inflation and output. The result of Golosov and Lucas (2007) that uncorrelated money growth shocks have only a small and transitory effect on output hinges critically on the inability of their model to generate small price changes. Hence, in the case of uncorrelated money shocks, our calibrated model behaves more closely to the Calvo specification. Correlated money shocks, on the other hand, cause a large increase in consumption on impact in all three specifications (including the fixed menu cost one), though the persistence of consumption is twice as large in the Calvo specification as it is in the other two.

Our state-dependent pricing mechanism is calibrated to match the data from micro studies of pricing behavior. As many authors have emphasized, matching these data requires us to allow for firm-specific shocks. But this leads to a heterogeneous agent problem: a state variable is the entire distribution of firms on prices and productivities. While the methods for calculating distributional dynamics are still new, the leading method of Krusell-Smith (1998) may not be well-suited to deal with the nonlinearities of the menu cost model.

Thus, our second innovation is to characterize the distributional dynamics using the computational method of Reiter (2006). This method is well suited for problems in which idiosyncratic shocks are more relevant than aggregate shocks for the individual decision maker, because it is fully nonlinear in idiosyncratic factors even though it imposes linearity in aggregate factors. Moreover, it is easy to implement because each step in the calculation is a familiar numerical procedure. First, it requires calculating the steady state equilibrium, which involves solving a backwards induction problem on a grid repeatedly until a fixed point for the aggregate price level is found. Second, the aggregate dynamics are solved linearly, which can be done with standard methods (e.g. Klein 2000; Sims 2001) in spite of the fact that this involves a very large system of equations representing values and densities at all points on the grid.

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\[ \text{Footnote 2: This holds even for narrowly defined product categories.} \]
1.1 Related literature

The literature on state-dependent pricing has rarely attempted to calculate the full dynamics, with idiosyncratic shocks, in general equilibrium.

- Some important intuition has been obtained from partial equilibrium studies, e.g. Caballero and Engel (1993, 2007)
- General equilibrium models that abstract from idiosyncratic shocks, e.g. Dotsey, King, and Wolman (1999)
- Strong restrictions on shock processes to obtain tractable distributional dynamics, e.g. Caplin and Spulber (1987), Gertler and Leahy (2005)

Recently a few studies have tried to go further.

- Golosov and Lucas (2007) have approximately characterized transition paths and some aspects of the dynamics by assuming that some general equilibrium variables are roughly constant.
- Midrigan (2006) has simulated distributional dynamics using an assumption like that of Krusell and Smith (1998), namely, that the mean price level gives sufficient information to characterize the effect of the price distribution on the value function. However, he focusses mostly on the aggregate steady state, providing only limited information on the dynamics, with no impulse response functions.
- Klenow and Kryvtsov (QJE 2008) simulate partial equilibrium versions of a variety of pricing models (including GL, DKW(1999), Calvo); demonstrate that neither fits the evidence well. Suggest that some “second generation” SDP models may fit better: Midrigan (2006), Gertler and Leahy (2006), DKW (2006) with idio shocks. They don’t state that by filling in the “missing middle” of small price changes, the GL result of near-neutrality of money shocks is gone. The question is not whether pricing is TD or SD, it is whether money matters or not.
- Our pricing model is robust to the details of the particular friction generating stickiness (we should still compare it with Woodford’s micro-founded hazard function). But the point is that ANY hazard function parametrized to yield small price changes will generate important real effects of money shocks. The GL result is not robust, since it does not survive a generalization that takes it closer to the micro evidence.

Our computational method, in contrast, allows the distribution to affect decisions in a fairly arbitrary way, so we need not make very strong assumptions to calculate equilibrium. The computational method is quick and tractable, allowing us to report general equilibrium transition paths and impulse responses of many variables, including moments of the distribution. Moreover, our model of price adjustment nests most of the common state-dependent pricing frameworks, so we can easily compare alternative models.

2 Partial equilibrium: the firm’s problem

Our paper will argue that bounded rationality motivates a simple and plausible generalization of rational equilibrium behavior that leads naturally to a price-setting framework which can be flexibly parameterized so as to nest a variety of popular models of pricing, including many which are usually considered full rationality models. Before we describe the whole structure of our DSGE
economy, it is helpful to study the partial equilibrium pricing decision of a monopolistic producer, to see how our pricing setup works.

Like Golosov and Lucas (2007), we assume price changes are driven primarily by idiosyncratic shocks. Thus, if firms are entirely rational, fully informed, and capable of frictionless adjustment, they will adjust their prices every time a new shock is realized. But we instead assume prices are “sticky”, in a well-defined sense: the probability of adjusting is less than one, but is greater when the benefit from adjusting is greater. What we mean by “the benefit from adjusting” becomes clear as soon as we write down the Bellman equations that describe the firm’s decision. There is a value associated with optimally choosing a new price today (while bearing in mind that prices will not always be adjusted in the future); likewise there is a value associated with leaving the current price unchanged today (likewise bearing in mind that prices will not always be adjusted in the future). The difference between these two values is the benefit from adjusting (or the loss from failing to adjust). The function \( \lambda(L) \) that gives the adjustment probability as a function of the loss \( L \) from failing to adjust is taken as a primitive of the model. We will choose a specification for \( \lambda \) that makes it easy to nest different models by appropriately setting a few parameters.

There are at least two ways of interpreting this framework. It could be seen as a model of stochastic menu costs, as in Dotsey, King, and Wolman (1999) or Caballero and Engel (1999). If rational, fully-informed firms draw an \( \text{iid} \) adjustment cost \( x \) every period, with cumulative distribution function \( F(x) \), then they will adjust their behavior whenever the adjustment cost \( x \) is less than or equal to the loss \( L \) from failing to adjust. Therefore, their probability of adjustment is \( \lambda(L) \) when the loss from nonadjustment is \( L \).

But perhaps this is an unnecessarily literal interpretation of the model. Alternatively, following Akerlof and Yellen (1985) for example, “stickiness” could be seen as a minimal deviation from rational expectations behavior. Under full rationality, full information, and zero adjustment costs, economic agents always adjust to the new optimal setting of their control variables; here instead we simply assume they sometimes fail to adjust. Perhaps failure to adjust occurs because information itself is “sticky” (as in Reis, 2006); or perhaps because managers face information processing constraints (as in Woodford 2008); rather than taking a stand on this, we just regard our assumption as an axiom to be imposed on near-rational, near-full-information behavior. Our framework is “close” to full rationality both because we can choose a \( \lambda \) function that is close to one for most \( L \), and more importantly because large mistakes are less likely than trivial ones. In this sense, our framework will permit us to deviate smoothly from standard rational decision making, to nest and compare other nearby forms of behavior.

2.1 The monopolistic competitor’s decision

Suppose then, following Golosov and Lucas (2007), that each firm \( i \) produces output \( Y_{it} \) under a constant returns technology, with labor \( N_{it} \) as the only input, and faces an idiosyncratic productivity process \( A_{it} \):

\[
Y_{it} = A_{it}N_{it}
\]

Firms are monopolistic competitors, facing the demand curve \( Y_{it} = \xi_t P_{it}^{-\epsilon} \), where \( \xi_t \) represents aggregate demand, and we assume they fulfill all demand at the price they set. They hire in competitive labor markets at wage rate \( W_t \), so per-period profits are

\[
\Pi_{it} = P_{it}Y_{it} - W_tN_{it} = \left( P_{it} - \frac{W_t}{A_{it}} \right) \xi_t P_{it}^{-\epsilon}
\]

We call the aggregate state of the economy \( \Omega_t \). There is no need to specify the structure of \( \Omega_t \) yet, except to say that it is a Markov process which determines all the aggregate endogenous variables:
\[
\xi_t = \xi(\Omega_t), \ W_t = W(\Omega_t).
\]
Idiosyncratic productivity \(A_{it}\) is driven by an unchanging Markov process, \(iid\) across firms and unrelated to \(\Omega_t\). Thus \(A_{it}\) is correlated with \(A_{i,t-1}\) but is uncorrelated with all other shock processes in the model. The assumption that demand shocks are related to aggregate conditions, while productivity shocks are purely idiosyncratic, is inessential for our methodology; we ignore more general cases only to keep notation simple. The case we focus on is equivalent to that considered in Golosov and Lucas (2007), and also resembles that in Reis (2006).

To implement our assumption that adjustment is more likely when it is more valuable, we must define the values of adjustment and nonadjustment. If a firm fails to adjust (so that \(P_{it} = P_{it-1}\)), then its current profits and its future prospects will both depend on its productivity \(A_{it}\) and on its price \(P_{it}\). Therefore these both enter as state variables in the value function of a nonadjusting firm, \(V(P_{it}, A_{it}, \Omega_t)\), which also depends on the aggregate state of the economy. When a firm adjusts, we assume it chooses the best price conditional on its current productivity shock and on the aggregate state. Therefore, the value function of an adjusting firm, after netting out any costs that may be required to make the adjustment, is just \(V^*(A_{it}, \Omega_t) \equiv \max_P V(P, A_{it}, \Omega_t)\). The value of adjusting to the optimal price, written in the same units as the value function, is then

\[
D(P_{it}, A_{it}, \Omega_t) \equiv \max_P V(P, A_{it}, \Omega_t) - V(P_{it}, A_{it}, \Omega_t)
\]

Of course, we don’t want the real probability of adjustment to differ when values are denominated in euros instead of pesetas. In order to take the function \(\lambda\) that maps the value of adjusting into the probability of adjusting as a primitive of the model, we must be sure to write it in the appropriate units. Under either interpretation of the model, the most natural units are those of labor time. Under the stochastic menu cost interpretation, the labor effort of changing price tags or rewriting the menu is likely to be a large component of the cost. Under the bounded rationality interpretation, even though we don’t explicitly model the computation process, we suppose the probability of adjustment is related to the labor effort associated with obtaining new information and/or recomputing the optimal price. Therefore, the function \(\lambda\) should depend on the loss from failing to adjust, converted into units of labor time by dividing by the wage rate. That is, the probability of adjustment is \(\lambda(L(P_{it}, A_{it}, \Omega_t))\), where \(L(P_{it}, A_{it}, \Omega_t) = \frac{D(P_{it}, A_{it}, \Omega_t)}{W(\Omega_t)}\) and \(\lambda\) is a given weakly increasing function which we take as a primitive of the model.

Conditional on adjustment, we have assumed that the firm sets the optimal price, \(P^*(A_{it}, \Omega_t) \equiv \arg \max_P V(P, A_{it}, \Omega_t)\). For clarity, we will distinguish between the firm’s beginning-of-period price, \(\tilde{P}_{it} = P_{it-1}\), and the price at which it produces and sells at time \(t, P_{it}\), which may or may not differ from \(\tilde{P}_{it}\). The adjustments are determined by the function \(\lambda\):

\[
\begin{aligned}
P_{it} &= \begin{cases} 
P^*(A_{it}, \Omega_t) & \text{with probability } \lambda \left( \frac{D(P_{it}, A_{it}, \Omega_t)}{W(\Omega_t)} \right) \\
\tilde{P}_{it} = P_{it-1} & \text{with probability } 1 - \lambda \left( \frac{D(P_{it}, A_{it}, \Omega_t)}{W(\Omega_t)} \right)
\end{cases}
\end{aligned}
\]

Function \(\lambda\) must satisfy \(\lambda' \geq 0\). In particular, we will consider the class

\[
\lambda(L) \equiv \frac{L^\varsigma}{\alpha^\varsigma + L^\varsigma}
\]

with \(\alpha\) and \(\varsigma\) positive. This function equals 0.5 when \(L = \alpha\), and is concave for \(\varsigma \leq 1\) and S-shaped for \(\varsigma > 1\). It has fatter tails than the normal \(cdf\), which may help it match the fat tails of the observed adjustment distribution emphasized by Midrigan (2006).

We can now state the Bellman equation that defines the value of producing at any given price. It differs somewhat depending on whether we impose the stochastic menu cost interpretation of our model or the bounded rationality interpretation; we begin with the latter because it is slightly simpler. Given the firm’s price \(P\) and its productivity shock \(A\), current profits are
\[
\left(P - \frac{W(\Omega)}{A}\right)\xi(\Omega)P^{-\epsilon}. \text{ The firm anticipates adjusting or not adjusting in the next period depending on the benefits of adjusting at that time. Therefore, using primes to denote next period’s values, the Bellman equation is:}
\]

\[
V(P, A, \Omega) = \left(P - \frac{W(\Omega)}{A}\right)\xi(\Omega)P^{-\epsilon} + \\
E \left\{ Q(\Omega, \Omega') \left[\left(1 - \lambda \left(\frac{D(P, A', \Omega')}{W(\Omega')}\right)\right) V(P', A', \Omega') + \lambda \left(\frac{D(P, A', \Omega')}{W(\Omega')}\right) D(P, A', \Omega')\right]\right\} | A, \Omega
\]

where \( Q(\Omega, \Omega') \) is the firm’s discount factor and the expectation refers to the distribution of \( A' \) and \( \Omega' \) conditional on \( A \) and \( \Omega \). Note that on the left-hand side of this equation, and in the term that represents current profits, \( P \) refers to a given firm \( i \)’s price \( P_{it} \) at the time of production. In the expectation on the right, \( P \) represents the price \( P_{i(t+1)} \) at the beginning of period \( t + 1 \), which may (probability \( \lambda \)) or may not (probability \( 1 - \lambda \)) be adjusted prior to time \( t + 1 \) production.

We can simplify substantially by noticing that the value on the right-hand side of the equation is just the value of continuing without adjustment, plus the expected gains due to adjustment:

\[
V(P, A, \Omega) = \left(P - \frac{W(\Omega)}{A}\right)\xi(\Omega)P^{-\epsilon} + \\
E \left\{ Q(\Omega, \Omega') \left[\left(V(P', A', \Omega') + \lambda \left(\frac{D(P, A', \Omega')}{W(\Omega')}\right) D(P, A', \Omega')\right]\right]\right\} | A, \Omega
\]
or equivalently, our most compact expression:

**Bellman equation in partial equilibrium, with aggregate shocks:**

\[
V(P, A, \Omega) = \left(P - \frac{W(\Omega)}{A}\right)\xi(\Omega)P^{-\epsilon} + E \{ Q(\Omega, \Omega') [V(P, A', \Omega') + G(P, A', \Omega')] \} | A, \Omega
\]  
(1)

where \( G(P, A', \Omega') \equiv \lambda \left(\frac{D(P, A', \Omega')}{W(\Omega')}\right) D(P, A', \Omega') \) represents the expected gains due to adjustment.

The difficulty of this model can be seen from the fact that the wage, the aggregate demand factor, the stochastic discount factor, and therefore also the value function all depend on the aggregate state \( \Omega \). In general equilibrium, there will be many firms \( i \) facing different idiosyncratic shocks \( A_{it} \) and stuck at different prices \( P_{it} \) at any time \( t \). The state of the economy will therefore include the entire distribution of prices and productivities. The reason for the popularity of the Calvo model is that even though firms have many different prices, up to a first-order approximation only the average price matters for equilibrium. Unfortunately, this property does not hold in general. In the current context, we need to recognize that all equilibrium quantities are explicitly functions of the distribution of prices and productivity across the economy. To calculate equilibrium, we therefore need an algorithm that takes account of the distributional dynamics.

To implement the algorithm of Reiter (2006), we must first solve the steady state general equilibrium before solving equilibrium with aggregate shocks. So consider an aggregate steady state, meaning that \( \Omega, W, \) and \( \xi \) are constant. We indicate the steady state value function by dropping \( \Omega \) as an argument, and the Bellman equation becomes

**Bellman equation in partial equilibrium steady state:**

\[
V(P, A) = \left(P - \frac{W}{A}\right)\xi P^{-\epsilon} + R^{-1} E \{ V(P, A') + G(P, A') | A\}
\]  
(2)

where \( R^{-1} \) is the steady state of the stochastic discount factor \( Q \), and

\[
G(P, A') \equiv \lambda \left(\frac{D(P, A')}{W}\right) D(P, A'), \quad D(P, A') \equiv \max_{P'} V(P', A') - V(P, A')
\]
This steady state Bellman equation is a standard dynamic programming problem, except for the timing of the max operator. A natural solution method is backwards induction on a two-dimensional grid $\Gamma \equiv \Gamma^P \times \Gamma^A$, where $\Gamma^P$ is a finite grid of possible values of $P_t$, and $\Gamma^A$ is a grid of possible values of $A_t$. However, before we define notation that confines the dynamics to a grid, it is useful describe the general equilibrium and detrend the model with respect to money growth, leaving all quantities in real terms.

2.2 Alternative sticky price frameworks

We will want to compare our simulation results to a number of alternative pricing frameworks. This is straightforward, because many models of sticky pricing can be nested in Bellman equation (1) by an appropriate choice of the capital gains function $G$.

1. **Calvo pricing**: Suppose prices adjust each period with probability $\tilde{\lambda}$, where $\tilde{\lambda}$ is an exogenous constant. Then the Bellman equation is the same as (1), if we set $\lambda(D/W) \equiv \lambda$.

2. **Fixed menu costs**: Suppose it costs $\kappa$ units of labor to adjust prices in any given period, where $\kappa$ is an exogenous constant called the "menu cost". Then the Bellman equation is given by (1), with $G = \lambda(D/W)D$ replaced by $G = 1\{D \geq \kappa W\}(D - \kappa W)$, where $1\{D \geq \kappa W\}$ is an indicator function taking value one when $D \geq \kappa W$ and zero otherwise.

3. **Stochastic menu costs**: Suppose it costs $\kappa$ units of labor to adjust prices in any given period, where $\kappa$ is an i.i.d. random variable with c.d.f. $\lambda(\kappa)$. Then the Bellman equation is given by (1), with $G = \lambda(D/W)D$ replaced by $G = \lambda(D/W)\Phi(D - W \kappa)$, where $\Phi(D > W \kappa)$ is an indicator function taking value one when $D > W \kappa$ and zero otherwise.

4. **Information-constrained pricing**: Woodford (2008) proposes a model in which managers decide on when to review a price based on imprecise awareness of current market conditions. His model implies the following adjustment probability function:

$$
\lambda(D/W) = \frac{\tilde{\lambda} \exp \left( \frac{D/W - \kappa}{\theta} \right)}{(1 - \tilde{\lambda}) + \tilde{\lambda} \exp \left( \frac{D/W - \kappa}{\theta} \right)}
$$

where $\kappa$ is a fixed cost of purchasing information, and $\theta$ represents the marginal cost of information.

3 General equilibrium

We next embed this partial equilibrium decision framework in an otherwise standard New Keynesian general equilibrium, following the setup of Golosov and Lucas (2007). In addition to the firms, there is a representative household and a monetary authority that chooses the money supply.

3.1 Households

The household’s period utility function is

$$
u(C_t) - x(N_t) + v(M_t/P_t)
$$

discounted by factor $\beta$ per period. Consumption $C_t$ is a Spence-Dixit-Stiglitz aggregate of differentiated products:

$$
C_t = \left\{ \int_0^1 C_{it}^* \, di \right\}^\frac{1}{\gamma}
$$

(3)
$N_t$ is labor supply, and $M_t/P_t$ is real money balances. The household’s period budget constraint is

$$\int_0^1 P_t C_{it} di + M_t + R_t^{-1} B_t = W_t N_t + M_{t-1} + T_t + B_{t-1} + \Pi_t$$

where $\int_0^1 P_t C_{it} di$ is total nominal spending on the differentiated goods. $B_t$ is nominal bond holdings, with interest rate $R_t - 1$; $T_t$ represents lump sum transfers received from the monetary authority, and $\Pi_t$ represents dividend payments received from the firms.

Assuming households choose $\{C_{it}, N_t, B_t, M_t\}_{t=0}^\infty$ so as to maximize expected discounted utility subject to the budget constraint, we obtain the following necessary conditions. Optimal allocation of consumption across the differentiated goods implies

$$C_{it} = (P_t/P_{it})^\chi C_{it}$$

(4)

where $P_t$ is the following price index:

$$P_t \equiv \left( \int_0^1 P_{it}^{1-\chi} di \right)^{\frac{1}{1-\chi}}$$

(5)

which also lets us rewrite nominal spending as $P_t C_t = \int_0^1 P_{it} C_{it} di$. Optimal labor supply and money holdings imply the first-order conditions

$$x'(N_t) = u'(C_t) W_t / P_t$$

(6)

$$v'(M_t/P_t) = u'(C_t)(1 - R_t^{-1})$$

(7)

and the Euler equation is

$$R_t^{-1} = \beta E_t \left( \frac{P_{t+1} u'(C_{t+1})}{P_{t+1} u'(C_t)} \right)$$

(8)

### 3.2 Monetary policy

As in Golosov and Lucas (2007), the money supply follows an exogenous, nonstationary stochastic process:

$$M_t \mu_t = M_{t-1}$$

(9)

where $\mu_t = \mu \exp(z_t)$, and $z_t$ is an AR(1) process:

$$z_t = \phi z_{t-1} + \epsilon_t^z$$

(10)

where $0 \leq \phi < 1$ and $\epsilon_t^z \sim i.i.d. N(0, \sigma_z^2)$ is a money growth shock. Thus the money supply trends upward by approximately factor $1/\mu > 1$ per period on average.

Seigniorage revenues are paid to the household as a lump sum transfer, and the government budget is balanced each period. Therefore the government’s budget constraint is

$$M_t = M_{t-1} + T_t$$

3One of the preceding equations is superfluous: (4) plus (3) implies (5), and likewise (4) plus (5) implies (3).
3.3 Aggregate consistency

Bond market clearing is simply \( B_t = 0 \). Market clearing for good \( i \) implies the following demand and supply relations for firm \( i \):

\[
Y_{it} = A_{it} N_{it} = C_{it} = P_t^e C_i P_{it}^{-e}
\]

Since \( C_t \) and \( P_t \) must both be functions of the aggregate state \( \Omega_t \), this takes the form assumed in our description of the firm’s problem if we set \( \xi_t = P_t^e C_i \). We can now calculate total labor demand too:

\[
N_t = \int_0^1 C_{it} \frac{di}{A_{it}} = P_t^e C_t \int_0^1 P_{it}^{-e} A_{it}^{-1} di
\]

At this point, we have spelled out all equilibrium conditions: household and monetary authority behavior has been described in this section, and the firms’ decision was stated in Section 2. Thus can now identify the aggregate state variable \( \Omega_t \). We have only included one aggregate shock in the model, namely, the money supply \( M_t \). Since the growth rate of \( M_t \) is AR(1) over time, the latest deviation in growth rates, \( z_t \), is a state variable too. There is also a continuum of idiosyncratic shocks, namely the productivity shocks \( A_{it} \), \( i \in [0,1] \). Finally, since firms cannot instantly adjust their prices, they are state variables too. More precisely, the state includes the joint distribution of prices and productivity shocks at the beginning of the period, prior to adjustment.

We will use the notation \( \hat{P}_t \) to refer to firm \( i \)'s price at the beginning of period \( t \), prior to adjustment; this may of course differ from the price \( P_t \) at which it produces, because the price may be adjusted before production. Therefore we will distinguish the distribution of production prices and productivity, which we write as \( \Phi_t(P_t, A_t) \), from the distribution of beginning-of-period prices and productivity, \( \hat{\Phi}_t(\hat{P}_t, A_t) \). Since beginning-of-period prices and productivities determine all equilibrium decisions at \( t \), we can define the state at time \( t \) as \( \Omega_t \equiv (M_t, z_t, \hat{\Phi}_t) \).

3.4 The firm’s problem in general equilibrium

The setup of sections 3.1-3.3 holds regardless of how firms set prices. In particular, regardless of the price-setting mechanism, \( C_t \), \( N_t \), \( P_t \), \( W_t \), \( R_t \), \( C_{it} \), \( P_{it} \), and \( M_t \) must obey equations (3) - (11). Next, we adapt our pricing setup to this general equilibrium environment. We write the model in the boundedly rational interpretation where the gains from adjustment in state \((P, A, \Omega)\) are \( G(P, A, \Omega) \equiv \lambda \left( \frac{D(P, A, \Omega)}{W(P, A)} \right) D(P, A, \Omega) \), but it is straightforward to rewrite it for other types of price stickiness.

We assume that the representative household owns the firms. Therefore the appropriate stochastic discount factor is

\[
Q(\Omega_t, \Omega_{t+1}) = \beta \frac{P(\Omega_t) u'(C(\Omega_{t+1}))}{P(\Omega_{t+1}) u''(C(\Omega_t))}
\]

In partial equilibrium, the firm’s demand function was \( \xi_t = C_t P_{it}^{-e} \). Using (4), in general equilibrium we have \( \xi_t = C_t P_{it}^{-e} \). Therefore, for a firm that sets sticky prices, the value of producing with price \( P_t \) and productivity \( A_t \) is

Bellman equation in general equilibrium:

\[
\begin{align*}
V(P_{it}, A_{it}, \Omega_t) &= \left( P_t - \frac{W(\Omega_t)}{A_{it}} \right) P(\Omega_t)^e C(\Omega_t) P_{it}^{-e} + \\
&+ \beta E_t \left\{ \frac{P(\Omega_t) u'(C(\Omega_{t+1}))}{P(\Omega_{t+1}) u''(C(\Omega_t))} \left[ V(P_{it}, A_{i,t+1}, \Omega_{t+1}) + G(P_{it}, A_{i,t+1}, \Omega_{t+1}) \right] \right\} A_{it}, \Omega_t
\end{align*}
\]
where
\[ G(P_{it}, A_{i,t+1}, \Omega_{t+1}) \equiv \lambda \left( \frac{D(P_{it}, A_{i,t+1}, \Omega_{t+1})}{W(\Omega_{t+1})} \right) D(P_{it}, A_{i,t+1}, \Omega_{t+1}) \]
and
\[ D(P_{it}, A_{i,t+1}, \Omega_{t+1}) \equiv \max_{P'} V(P', A_{i,t+1}, \Omega_{t+1}) - V(P_{it}, A_{i,t+1}, \Omega_{t+1}) \]

Notice that these equations involve only \( P, C, W, \) and \( P_i \). Therefore these equations give us the information needed to determine the idiosyncratic price process. Letting \( P^*(A_{i,t+1}, \Omega_{t+1}) \) denote the optimal choice in the maximization problem above, the price process is
\[ P_{i,t+1} = \begin{cases} 
  P^*(A_{i,t+1}, \Omega_{t+1}) & \text{with probability } \lambda \left( \frac{D(P_{it}, A_{i,t+1}, \Omega_{t+1})}{W(\Omega_{t+1})} \right) \\
  P_{it} & \text{with probability } 1 - \lambda \left( \frac{D(P_{it}, A_{i,t+1}, \Omega_{t+1})}{W(\Omega_{t+1})} \right) .
\end{cases} \]

3.5 Detrending

The value function and all prices have been written so far in nominal terms. It is natural to assume that the model can be rewritten in real terms. Thus suppose we deflate all prices by the nominal money stock, defining \( p_t \equiv P_t/M_t, p_{it} \equiv P_{it}/M_t, \) and \( w_t \equiv W_t/M_t \). Given the nominal distribution \( \Phi_t(P_t, A_t) \) and the money stock \( M_t \), let us denote by \( \Psi_t(p_t, A_t) \) the distribution over real production prices \( p_{it} \equiv P_{it}/M_t \). Likewise, let \( \Psi_t(p_t, A_t) \) be the distribution of real beginning-of-period prices \( \widetilde{p}_t \equiv \widetilde{P}_t/M_t \), in analogy to the beginning-of-period distribution of nominal prices \( \Phi_t(P_t, A_t) \). It is true that the model can be rewritten in real terms, then the level of the money supply, \( M_t \), is irrelevant for determining real quantities. Therefore, to describe the real equilibrium, it suffices to condition on the real state variable \( \Xi_t \equiv (z_t, \Psi_t) \), instead of the full nominal state \( \Omega_t \equiv (M_t, z_r, \Phi_t) \). The “real” value function \( \nu \) should likewise be the nominal value function, divided by the current money stock, and should be written as a function of real variables. That is,
\[ V(P_{it}, A_{it}, \Omega_{t}) = M_t \nu \left( \frac{P_{it}}{M_t}, A_{it}, \Xi_t \right) = M_t \nu (p_{it}, A_{it}, \Xi_t) \]

Deflating in this way, the system can be rewritten as follows (see the appendix for details).

**Detrended Bellman equation, general equilibrium:**

\[ v(p_{it}, A_{it}, \Xi_t) = \left( p_{it} - \frac{w(\Xi_t)}{A_{it}} \right) \left( \frac{p_{it}}{p(\Xi_t)} \right)^{-\xi} C(\Xi_t) + \beta E_t \left\{ \frac{p_{it+1}w'(C(\Xi_{t+1}))}{p(t+1)w'(C(\Xi_t))} \left[ v \left( \mu_{t+1}p_{it}, A_{i,t+1}, \Xi_{t+1} \right) + g \left( \mu_{t+1}p_{it}, A_{i,t+1}, \Xi_{t+1} \right) \right] | A_{it}, \Xi_t \right\} \]

where
\[ g \left( \mu_{t+1}p_{it}, A_{i,t+1}, \Xi_{t+1} \right) \equiv \lambda \left( \frac{d(\mu_{t+1}p_{it}A_{i,t+1}, \Xi_{t+1})}{w(\Xi_{t+1})} \right) \]
\[ d \left( \mu_{t+1}p_{it}, A_{i,t+1}, \Xi_{t+1} \right) \equiv \max_{P'} v(P', A_{i,t+1}, \Xi_{t+1}) - v \left( \mu_{t+1}p_{it}, A_{i,t+1}, \Xi_{t+1} \right) \]

Let \( p^*(A_{i,t+1}, \Xi_{t+1}) \) denote the optimal choice in the maximization problem above. Taking into account the fact that the firm starts period \( t + 1 \) with the eroded price \( \widetilde{p}_{i,t+1} \equiv \mu_{t+1}p_{it} \), the price process is
\[ p_{i,t+1} = \begin{cases} 
  p(A_{i,t+1}, \Xi_{t+1}) & \text{with probability } \lambda \left( \frac{d(\mu_{t+1}p_{it}A_{i,t+1}, \Xi_{t+1})}{w(\Xi_{t+1})} \right) \\
  \mu_{t+1}p_{it} & \text{with probability } 1 - \lambda \left( \frac{d(\mu_{t+1}p_{it}A_{i,t+1}, \Xi_{t+1})}{w(\Xi_{t+1})} \right) .
\end{cases} \]
In other words, when the firm’s nominal price is not adjusted at time $t+1$, its real price is deflated by factor $\mu_{t+1}$.

4 Computing general equilibrium: steady state

4.1 Discrete numerical model

In order to actually solve the model numerically, we approximate the economy by assuming that individual states, in real terms, always lie in a finite grid. For a discrete model of this type, the firm's decision can be calculated by backwards induction. This is an entirely standard solution method, but we will now spell it out in detail, both to see how it fits into our general equilibrium and in order to clarify our calculations later when we solve the more difficult case of aggregate shocks.

Thus, suppose we approximate our model on a two-dimensional grid $\Gamma \equiv \Gamma^p \times \Gamma^A$, where $\Gamma^p \equiv \{p^1, p^2, \ldots, p^{\#p}\}$ is a logarithmically-spaced grid of length $\#p$ of possible values of $p_i$, and $\Gamma^A \equiv \{a^1, a^2, \ldots, a^{\#A}\}$ is a logarithmically-spaced grid of length $\#A$ of possible values of $A_i$. Also, define $\Delta^p \equiv \log(p^{t+1}/p^t)$ as the logarithmic step size in grid $\Gamma^p$. From here on, we will frequently use superscripts to identify notation related to various grids. In this approximation, we can think of the distributions $\Psi$ and $\bar{\Psi}$ as matrices of size $\#p \times \#A$ in which the $(j, k)$ element represents the fraction of the population of firms in state $(p^i, a^k)$.

Likewise, we can now think of the value function as a $\#p \times \#A$ matrix $V$ of values $v_{jk} \equiv v(p^i, a^k)$ associated with the prices and productivities $(p^i, a^k) \in \Gamma$. We can then construct splines to evaluate the value function at points $p \notin \Gamma^p$ off the price grid, when necessary. In particular, we define the policy function

$$p^*(A) \equiv \arg \max_p v(p, A)$$

without requiring that it be chosen from the grid $\Gamma^p$, because (as we will see below) our solution method requires policies to vary continuously with their arguments. It is useful to collect the policies at the productivity grid points $a^k \in \Gamma^A$ as a row vector $p^* \equiv \{p^*(a^1), \ldots, p^*(a^{\#A})\} \equiv \{p^1(a^1), \ldots, p^{\#A}(a^{\#A})\}$. Next, we define the $\#p \times \#A$ matrix of adjustment values $D$ in which the $(j, k)$ element is

$$d_{jk} \equiv \max_p v(p, a^k) - v_{jk}$$

that is, the value of adjustment when the idiosyncratic state is $(p^i, a^k) \in \Gamma$. (Here again, the chosen $p$ is not constrained to lie in $\Gamma^p$.) Finally, we can define expected gains by a $\#p \times \#A$ matrix $G$ in which the $(j, k)$ element is

$$g_{jk} \equiv \lambda (d_{jk}/w) d_{jk}$$

We can now write down the discrete Bellman equation and the discrete distributional dynamics in a precise way. The dynamics involve three main steps. First, consider a firm with beginning-of-period price $\tilde{p}_t = p^i \in \Gamma^p$ and $A_t = a^k \in \Gamma^A$. This firm’s production price will be $p_{it} = p^{k_i}$ with probability $\lambda(d_{jk}/w)$, or will remain unchanged $(p_{it} = \tilde{p}_t = p^i)$ with probability $1 - \lambda(d_{jk}/w)$. If adjustment occurs, we maintain our grid-based approximation by rounding $p^{k_i}$ up or down stochastically to the nearest grid points. Therefore, we calculate the production distribution $\Psi_t$
from the beginning-of-period distribution \( \tilde{\Psi}_t \), as follows:

\[ \text{prob}(p_{it} = p^j | \tilde{p}_{it} = p^j, A_{it} = a^k) = \begin{cases} 
1 - \lambda(d^{ik} / w) & \text{if } p^j = p^l \\
\lambda(d^{ik} / w) \left( \frac{p^l - p^j}{p^l - p^*} \right) & \text{if } p^j = \min \{ p \in \Gamma^p : p \geq p^k \} \\
\lambda(d^{ik} / w) \left( \frac{p^* - p^j}{p^l - p^*} \right) & \text{if } p^j = \max \{ p \in \Gamma^p : p < p^k \} \\
0 & \text{otherwise}
\end{cases} \]

In these formulas, we assume \( \Gamma^p \) is chosen wide enough so that \( p^1 < p^* < p^{#^p} \) for all \( k \).

These equations can be summarized in matrix notation. Let \( E \) be a \( \#^p \times \#^p \) matrix of ones. Let \( \lambda(D/w) \) be a \( \#^p \times \#^A \) matrix with element \( (j, k) \) equal to \( \lambda(d^{jk} / w) \). Also, given \( a^k \in \Gamma^A \), define \( l(k) \) so that \( p^{(k)} = \min \{ p \in \Gamma^p : p \geq p^k \} \). Then let \( P \) be the \( \#^p \times \#^A \) matrix taking the value \( \frac{p^{(k)} - p^{(k)-1}}{p^{(k)} - p^*} \) in row \( l(k) \), column \( k \); and taking value \( \frac{p^j - p^{(k)}}{p^l - p^*} \) in row \( l(k) - 1 \), column \( k \); and taking value zero elsewhere. Then the relation between distributions \( \Psi_t \) and \( \tilde{\Psi}_t \) is:

\[ \Psi_t = (E - \lambda(D/w)) \ast \tilde{\Psi}_t + P \ast (E \ast (\lambda(D/w) \ast \tilde{\Psi}_t)) \]

(15)

where (as in MATLAB) the operator \( \ast \) represents element-by-element multiplication, and \( \ast \) represents ordinary matrix multiplication.

The second step in the distributional dynamics is to adjust real prices to take into account steady state money growth. Suppose for simplicity that deflating by the money growth rate causes real prices to decrease by an integer number of "steps" in the price grid; in other words, suppose \( \#^p \equiv \log \mu / \Delta \) is a nonnegative integer. Then, rounding up to \( p^l \) when prices fall off bottom of the grid, \( p_{it} = p^j \) implies \( \tilde{p}_{it, t+1} = \max \{ p^1, p^l, p^{#^p} \} \). \(^4\) We can think of this as defining a matrix \( R \) of size \( \#^p \times \#^p \) which equals one in row \( m(l) = \max \{1, l - \#^p\} \) of column \( l \), and is zero elsewhere.\(^5\) This allows us to interpret the row \( m \), column \( l \) element of \( R \) as:

\[ R_{m}^l = \text{prob}(\tilde{p}_{it, t+1} = p^m | p_{it} = p^l) \]

In other words, \( R \) is a Markov matrix that governs the transition from real prices in period \( t \) at the time of production, to real prices at the beginning of \( t + 1 \). Note that since the number of points in the policy grid is large, \( R \) is likely to be enormous, but the fact that it is also extremely sparse will make our calculations feasible.

The third and final step in the distributional dynamics is to take into account the Markov matrix \( S \) that governs the idiosyncratic productivity shocks \( A_i \). The row \( m \), column \( k \) element of \( S \) is

\[ S_{mk} = \text{prob}(A_{it} = a^m | A_{it} = a^k) \]

Combining the second and third steps, we can calculate the beginning-of-period distribution \( \tilde{\Psi}_{t+1} \) at \( t + 1 \) as a function of the time \( t \) distribution of production prices \( \Psi_t \):

\[ \tilde{\Psi}_{t+1} = R \ast \Psi_t \ast S' \]

(16)

The simplicity of this equation comes partly from the fact that the exogenous shocks to \( A_{i,t+1} \) are independent of the inflation adjustment that links \( \tilde{p}_{i,t+1} \) with \( p_{it} \). Also, exogenous shocks are represented from left to right in the matrix \( \Psi_t \), so that their transitions can be treated by right

\(^4\)In other words, we assume that any nominal price that would have a real value less than \( p^1 \) after inflation is automatically adjusted upwards so that its real value is \( p^1 \). This assumption is made for numerical simulation reasons only, and has a negligible impact on the equilibrium as long as we choose a sufficiently wide grid \( \Gamma^p \). If we were to compute examples with trend deflation, we would need to make an analogous adjustment to prevent real prices from surpassing the maximum grid point \( p^{#^p} \).

\(^5\)It is straightforward to generalize the definition of \( R \) to the case where \( \#^p \) is not an integer; see section 5.
multiplication, while policies are represented vertically, so that transitions related to policies can be treated by left multiplication.

The same transition matrices show up when we write the Bellman equation in matrix form. Let \( U \) be the \( \#^p \times \#^a \) matrix of current payoffs, with element \( u^{jk} = (p^j - \frac{\Psi^j}{\Psi}) C \left( \frac{p^j}{p} \right)^{-\epsilon} \) for \((p^j, a_k) \in \Gamma\). Then the Bellman equation is

**Steady state general equilibrium Bellman equation, matrix version:**

\[
V = U + \beta R' \ast (V + G) \ast S
\]  

(17)

Since the Bellman equation iterates backwards in time, it involves probability transitions represented by \( R \) and \( S \), whereas the distributional dynamics iterate forward in time and therefore contain \( R \) and \( S' \).

The functional equations (15), (16), and (17) can equivalently be seen as a system of \( 4 \#^p \#^a + \#^p + \#^a + 3 \) scalar equations. These equations involve the unknown matrices \( V, D, \Psi \), and \( \Psi' \); the vector \( p^* \); and the scalars \( w, p \), and \( C \): a total of \( 4 \#^p \#^a + \#^p + \#^a + 3 \) unknown scalars. The remaining \( 4 + \#^p \) equations needed to close the general equilibrium are

\[
u'(C) = \frac{px'(N)}{w}
\]  

(18)

\[
1 - \frac{v'(1/p)}{u'(C)} = \beta \mu
\]  

(19)

\[
N = \sum_{j=1}^{\#^p} \sum_{k=1}^{\#^a} \Psi^{jk} \left( \frac{p^j}{p} \right)^{-\epsilon} \frac{C}{A}
\]  

(20)

\[
p^{1-\epsilon} = \sum_{j=1}^{\#^p} \sum_{k=1}^{\#^a} \Psi^{jk} (p^j)^{1-\epsilon}
\]  

(21)

In these equations, \( j \in \{1, 2, ..., \#^p\} \) indicates possible prices, and \( k \in \{1, 2, ..., \#^a\} \) indicates possible productivities. Note that since the last equations are related to goods demand and supply, the relevant distribution is \( \Psi \), which is associated with the time of production. We have dropped the equation that defines the consumption aggregator \( C \) since it is implied by the others. Overall, then, we have \( 4 \#^p \#^a + \#^p + \#^a + 4 \) scalar equations to determine an equivalent number of unknowns: \( v^{jk}, d^{jk}, \Psi^{jk}, \Psi'^{jk}, p^{*k}, C^j, C, N, w, \) and \( p \).

### 4.2 Results: steady state

This steady state model can be calibrated by comparing its predictions to cross-sectional data on price changes, like those reported in Klenow and Kryvstov (2005), Midrigan (2006), and Nakamura and Steinsson (2007). In a companion paper, Costain and Nakov (2008), we report detailed results from a variety of specifications, and compare the model’s behavior under low and high steady-state inflation rates. Here we simply briefly discuss our preferred estimate from that paper. We will simulate our model at monthly frequency, for consistency with the results reported in the empirical literature. Also, since these papers all attempt to remove price changes attributable to temporary “sales”, our simulation results should be interpreted as a model of “regular” price changes unrelated to sales.

We take the steady state growth rate of money, 0.64% per quarter, and our utility parameterization, from Golosov and Lucas (2007). Therefore we set the discount factor to \( \beta = 1.04^{-1} \) per year. Consumption utility is CRRA, \( u(C) = \frac{1}{\gamma} C^{1-\gamma} \), with \( \gamma = 2 \). Labor disutility is linear, \( x(N) = \xi N \), with \( \xi = 6 \). The utility of real money holdings is logarithmic, \( v(m) = \nu \log(m) \),
with \( \nu = 0.0323 \) (this value is chosen so that the wage is one in steady state). The elasticity of substitution in the consumption aggregator is \( \epsilon = 7 \).

Given these utility parameters, we can next calibrate the idiosyncratic productivity shock process and the adjustment process to match data on the distribution of regular price changes. We assume productivity is AR(1) in logs:

\[
\log A_{it} = \rho \log A_{i,t-1} + \epsilon_{it}^a
\]

where \( \epsilon_{it}^a \) is a mean-zero, normal, iid shock. The only free parameters in the productivity process are \( \rho \) and \( \sigma^2_t \), the variance of \( \epsilon_{it}^a \). The adjustment process has two free parameters, \( \alpha \) and \( \zeta \), in the function class we have imposed, \( \lambda(d) = d^\zeta / (\alpha^\zeta + d^\zeta) \).

Our utility specification makes the steady state calculation especially simple, because for given parameters it can be reduced to a fixed point problem in the real price level \( p \). Guessing \( p \) permits us to invert the Euler equation (19) to calculate \( C = (1 - \beta p) \gamma \), and we can then calculate the wage from (18) as \( w = \xi p C \gamma \). This gives us enough information to construct the matrix \( U \), so we can solve the Bellman equation (17) and then find the steady state price distribution \( \Psi \) from (15) and (16). Knowing the price distribution \( \Psi \), we can calculate the price level \( p \) from (21). Finding a fixed point in \( p \) thus allows us to construct the steady state equilibrium.

Table 1 reports our preferred estimate from Costain and Nakov (2008), (labelled SDSP, for ‘state-dependent sticky prices’), together with data from several empirical papers.\(^7\) It also reports a Calvo (1983) version of the model and a fixed menu cost version (as in Golosov and Lucas 2007). These alternative versions use the same productivity process, while fixing the rate of price adjustment at its average in the data (the Calvo version) or calibrating the fixed menu cost to best fit the data. The SDSP calibration is chosen to match three moments: the median frequency of price adjustment, the median absolute price adjustment, and the fraction of price adjustments less than 5% in absolute value. As the table shows, the fixed menu cost version of the model fits the data poorly because it generates no small price adjustments. The Calvo model, instead, generates far too many small price adjustments. As our previous paper shows, this failure of the Calvo model can be corrected by choosing a more variable productivity process, but if so then the Calvo model leads to much larger welfare losses (the average welfare loss rises to over 3% of the median value of the firm.) By contrast, the average welfare loss in the SDSP calibration is only one half of one percent of the median value of the firm.

Figure 1 graphs a variety of objects that characterize the equilibrium. In the first plot we see the value function, as a function of prices and marginal cost (one over productivity); the lowest value occurs when the highest marginal cost is paired with the lowest price. The fourth and sixth plots show the distributions \( \Psi \) at the beginning of the period and \( \Psi \) at the time of production. The production distribution \( \Psi \) looks rather like a sail-backed dinosaur: the “sail” represents the mass of firms that have adjusted to the optimal price conditional on current productivity. At the beginning of the next period, this mass gets spread out by the productivity shock process, resulting in the smooth distribution \( \Psi \) seen in the fourth graph. Graphing the policy function in the eighth plot shows that the firm sets prices closer to the mean than would be the case under flexible prices, in anticipation of mean reversion of the technology process. The last graph shows the distribution of nominal price adjustments, which is mildly bimodal around zero, and resembles quite closely the distributions of supermarket price changes shown in Midrigan (2006), Figure 1.

\( ^{6}\)Our numerical method requires us to treat \( A \) as a discrete variable, so we use Tauchen’s method (Mertens, 2006) to approximate this AR(1) process on the discrete grid \( \Gamma^4 \). We use a grid of 101 points representing five standard deviations of the process \( A \). Our price grid of 501 logarithmically-spaced points extends 10% past the prices that would be chosen at the highest and lowest values of \( A \) if prices were fully flexible. This results in price steps of 0.28% in our baseline estimate.

\( ^{7}\)The column marked “target” is a simple average of the numbers from the three empirical papers. The simulations reported here are the same as those shown in Table 2 of Costain and Nakov (2008).
Finally, it is helpful to consider the computational implications of the relatively large but infrequent price adjustments seen in the data. With a median absolute price change of 8%, a typical price movement by firms in our baseline SDSP simulation is a jump of 30 steps in the price grid $p$. Clearly then, at any point in time most firms lie many steps away from their optimal prices; the table shows that the typical deviation from the optimal price ranges from 3.3% (in terms of the median) to 5.4% (on average), depending on the model. This suggests that constraining price adjustment to a finite grid is relatively unimportant both for price dynamics and for welfare analysis. We confirm this fact in Table 1 by recomputing the model (under the SDSP calibration) on a much coarser grid, with only 25 possible productivities (spanning $\pm 2.5$ standard deviations instead of $\pm 5$ standard deviations) and only 25 possible prices. Thus, in the coarser grid, each price step represents a 3.2% price change, instead of 0.32% in the previous calculation.

This dramatic coarsening of the grid has only minor consequences for the performance of the model. The statistic that changes most is the standard deviation of price adjustments, which rises from 10.4% to 11.2%. The other statistics are barely altered, including the welfare losses caused by price stickiness. Thus, computing the dynamics on a finite grid seems unimportant for the results, even when the grid is quite coarse. This is very helpful for our purposes, because it suggests that the more numerically challenging problem of characterizing the distributional dynamics can also be studied on a coarse grid.

5 Computing general equilibrium: dynamics

To characterize our model’s distributional dynamics in general equilibrium under aggregate shocks, we implement the algorithm of Reiter (2006). Reiter’s method recognizes that the large system of nonlinear equations we solved to calculate the general equilibrium steady state can also be interpreted as a system of nonlinear first-order autonomous difference equations that describe the dynamics of a grid-based approximation of our general equilibrium with aggregate shocks. In the absence of strong strategic complementarities or an inappropriate Taylor rule that might give rise to indeterminacy, such an equation system can be solved by perfectly standard linear simulation techniques. We will solve for the saddle-path stable solution of our linearized model using the QZ decomposition, following Klein (2000).

The crucial thing to notice about Reiter’s method is that it combines linearity and nonlinearity in a way appropriate for the model at hand. In our model, idiosyncratic shocks are likely to be larger and more economically important for individual firms’ decisions than aggregate shocks. This is true in many macroeconomic contexts (e.g. precautionary saving) and in particular Klenow and Kryvstov (2005), Golosov and Lucas (2007), and Midrigan (2006) argue that firms’ pricing decisions appear to be driven primarily by idiosyncratic shocks. Therefore, to deal with large idiosyncratic shocks, we treat functions of idiosyncratic states in a fully nonlinear way, by calculating them on a grid. As we emphasized above, this grid-based solution can be regarded as a large system of nonlinear equations: separate equations for all grid points. By linearizing each of these equations with respect to the aggregate dynamics, we recognize that aggregate changes are unlikely to affect individual value functions in a strongly nonlinear way. That is, we are implicitly assuming that both money supply shocks $\mu$ and changes in the distributions $\Psi$ and $\overline{\Psi}$ have sufficiently smooth effects on individual values that a linear treatment of these effects is sufficient.

Thus, we will write the general equilibrium dynamics as a system of difference equations. For

---

8 The reader might expect the median (or average) price adjustment to coincide with the median (or average) distance from the optimal price in the case of the Calvo model. The only reason they are not equal in the table is that the distribution of price adjustments is determined by the beginning-of-period distribution $\overline{\Psi}$, whereas we report the distance from the optimal price at the time of production. That is, we report the distance from the optimal price with respect to the distribution $\Psi$ that pertains after adjustments have taken place.
parsimonious notation in this context, we indicate dependence on the aggregate state by time subscripts, instead of by writing endogenous variables as functions of \( \Omega_t \). We will see that the difference equation system is a straightforward generalization of the steady state equations from the previous section. First, the time \( t \) money growth process is \( \mu_t = \mu \exp(\epsilon_t) \), where

\[
z_t = \phi_z z_{t-1} + \epsilon_t^z
\]  

where \( \epsilon_t^z \) is an iid normal shock with mean zero and standard deviation \( \sigma_z \).

Second, the firms’ Bellman equation can be written as a \( \#^p \times \#^A \) matrix system of equations for each \((p^j, a^k) \in \Gamma\). Let \( U_t \) be the matrix of current profits, so that the \((j, k)\) element of \( U_t \) is

\[
u_{t}^{jk} \equiv \left(p^j - \frac{w_{t}^{j}}{a^k}\right)C_t \left(\frac{p^j}{p_t}\right)^{-\epsilon} \equiv \left(p^j - \frac{w(\Xi_t)}{a^k}\right)C(\Xi_t) \left(\frac{p^j}{p(\Xi_t)}\right)^{-\epsilon}
\]  

Write the value function as a matrix \( V_t \), with \((j, k)\) element equal to \( v_t(p^j, a^k) \equiv v(p^j, a^k, \Xi_t) \) for \((p^j, a^k) \in \Gamma\). We can write the Bellman equation as

**Dynamic general equilibrium Bellman equation, matrix version:**

\[
V_t = U_t + \beta E_t \left\{ \rho t w(C_{t+1}) \frac{p_t^{j}}{p_{t+1}^{j}} R_{t+1}^{j} * (V_{t+1} + G_{t+1}^{j}) * S \right\} 
\]  

All quantities in the Bellman equation are analogous to corresponding quantities in the steady state equilibrium. The matrix \( G_{t+1}^{j} \) is defined by

\[
G_{t+1}^{j} \equiv \lambda(D_{t+1}/w_{t+1}) \ast D_{t+1}
\]  

where the \((l, m)\) element of \( D_{t+1} \) is

\[
d_{t+1}^{lm} \equiv d_{t+1}(p^l, a^m) \equiv \max_{p^l} v_{t+1}(p^l, a^m) - v_{t+1}(p^l, a^m)
\]  

The expectation \( E_t \) in the Bellman equation refers only to the effects of the time \( t + 1 \) money shock \( \mu_{t+1} \), because the shocks and dynamics of the idiosyncratic state \((p^j, a^k) \in \Gamma\) are completely described by the matrices \( R_{t+1}^{j} \) and \( S \). Note that \( S \) has no time subscript, and is exactly the same matrix described in the previous section. The Markov matrix \( R_{t+1}^{j} \) differs from the steady state matrix \( R \) only because in the fully dynamic equilibrium we must detrend by the realized money shock \( \mu_{t+1} \) instead of trend money growth \( \mu \). The row \( n \), column \( l \) element of \( R_{t+1}^{j} \), which we will call \( R_{t+1}^{nl} \), is

\[
R_{t+1}^{nl} = \rho \text{ob}(\bar{p}_{t+1}, p^{\mu|p_{t+1} = p^l, \mu_{t+1}}) = \begin{cases} 
1 & \text{if } \mu_{t+1} = p^l = p^{\mu} \\
\frac{\rho_{t+1}^{p^{\mu} - p^{\mu-1}}}{\rho_{t+1}^{p^{\mu} - p^{\mu-1}} + \mu_{t+1}^{p^{\mu} - p^{\mu}} - p^{\mu}} & \text{if } p^{\mu} < p^{\mu} = \min\{p \in \Gamma^p : p \geq \mu_{t+1}^{p^l}\} \\
0 & \text{if } p^{\mu} \leq p^{\mu} = \max\{p \in \Gamma^p : p < \mu_{t+1}^{p^l}\} \\
\end{cases}
\]

As for the dynamics of the distribution, the two steps are analogous to the steady state case:

\[
\Psi_{t} = (E - \lambda(D_{t}/w_{t})). * \Psi_{t} + P_{t} \ast (E * (\lambda(D_{t}/w_{t})). * \Psi_{t})
\]  

\[
\tilde{\Psi}_{t+1} = R_{t+1}^{j} \ast \Psi_{t} \ast S'
\]  

Matrix \( P_t \) is constructed from the policy function

\[
p_{t}^{jk} \equiv p_{t}(a^k) \equiv p^*(a^k, \Omega_t)
\]
in the same way as in the steady state. If \( p_l^{(k)} \) is the first price grid point greater than or equal to \( p_t^{k} \), then \( P_t \) takes value \( \left( \frac{p_l^{(k)} - p_t^{(k-1)}}{p_t^{(k)} - p_t^{(k-1)}} \right) \) in row \( l(k) \), column \( k \); and value \( \left( \frac{p_l^{(k)} - p_t^{k}}{p_t^{(k)} - p_t^{(k-1)}} \right) \) in row \( l(k) - 1 \), column \( k \); and is zero elsewhere.

Finally, the remaining equations that must be satisfied by the dynamic general equilibrium are

\[
x'(N_t) = \frac{w_t}{p_t} u'(C_t)
\]

\[
1 - \frac{v'(1/p_t)}{w'(C_t)} = \beta E_t \left( \frac{\mu_{t+1}}{p_{t+1}} \frac{p_t u'(C_{t+1})}{p_t u'(C_t)} \right)
\]

\[
N_t = \sum_{j=1}^{#p} \sum_{k=1}^{#a} \Psi_t^{jk} \left( \frac{p_j}{p_t} \right)^{1-\epsilon} \frac{C_t}{A^\epsilon}
\]

\[
p_t^{1-\epsilon} = \sum_{j=1}^{#p} \sum_{k=1}^{#a} \Psi_t^{jk} \left( \frac{p_j}{p_t} \right)^{1-\epsilon}
\]

### 5.1 Linearization

In Appendix B we further simplify this equation system for the case of linear labor disutility, \( x(N) = \xi N \), which means the first-order condition for labor reduces to \( \xi p_t = w_t u'(C_t) \), so we don’t actually need to solve for \( N_t \) to calculate the rest of the equilibrium.\(^9\) We show that in this case the system can be expressed in terms of the 'jump' variables \( V_t, C_t, \) and \( p_t \); the endogenous, predetermined state variable \( \Psi_t \equiv \Psi_{t-1} \) (the lagged distribution of idiosyncratic states); and the exogenous state \( \mu_t \). All other variables can easily be eliminated from the system, leaving us with \( \#TOT = 2\#p\#a + 3 \) nonlinear, first-order, autonomous difference equations governing the same number of processes. If we then collapse all the endogenous variables into a vector of the form

\[
\vec{X}_t \equiv \left( \text{vec}(V_t)', C_t, \ p_t, \ \text{vec}(\Psi_t)' \right)'
\]

then the entire system of expectational difference equations governing the equilibrium has the following form:

\[
E_t F \left( \vec{X}_{t+1}, \vec{X}_t, z_{t+1}, z_t \right) = 0
\]

where \( E_t \) is an expectation conditional on \( \mu_t \) and all previous shocks. If we linearize system \( F \) with respect to all arguments by constructing the Jacobian matrices \( A \equiv D_{\vec{X}_{t+1}} F, B \equiv D_{\vec{X}_t} F, C \equiv D_{z_{t+1}} F, \) and \( D \equiv D_{z_t} F, \) then we obtain the system

\[
E_t A \vec{X}_{t+1} + B \vec{X}_t + E_t C z_{t+1} + D z_t = 0
\]

This equation system has the form considered by Klein (2000), so we solve our model using his QZ decomposition method.\(^10\)

---

\(^9\)The assumption \( x(N) = \xi N \) is not essential; the more general case with nonlinear labor disutility simply requires us to simulate a larger equation system that includes \( N_t \).

\(^10\)Alternatively, the equation system can be rewritten in the form of Sims (2001). We chose to implement the Klein method because it is especially simple and transparent to program.
6 Results: dynamics

6.1 Effects of money growth shocks

We now study the impulse response functions implied by money supply shocks in several versions of our model, as illustrated in Figures 3 and 4 (5%). The figures show the response to a 1% increase in money supply growth in the SDSP calibration of our model, and compares it with the response in the Calvo and fixed menu cost specifications. Each specification is calculated starting from the ergodic distribution of prices and productivities associated with that specification (trend inflation is assumed to be 0.64% per quarter). Figure 3 shows impulse responses under the assumption that money supply growth is iid, while Figure 4 assumes money growth has monthly autocorrelation of 0.8 (0.51 at quarterly frequency). For numerical tractability we compute equilibrium on the coarse grid of 25 productivities and 25 price levels analyzed in Table 1 and Figure 2, which yields a distribution of price changes similar to that on a much finer grid.

The impulse responses show how an increase in money growth causes both inflation and consumption to rise. The response of our calibrated model (lines with disks) mostly lies between the responses seen in the Calvo (lines with squares) and fixed menu cost (lines with crosses) specifications. All three versions are simulated under the same parameters, and the same aggregate and idiosyncratic shock processes, changing only the specification of the adjustment probability function $\lambda$. As Golosov and Lucas (2007) have emphasized, the average price level adjusts more rapidly in the fixed menu cost specification, so there is a larger spike in the inflation rate and smaller, less persistent changes in real variables in that specification than in the others. This difference is especially pronounced under iid money growth, in which case our preferred SDSP calibration resembles more closely the Calvo model than the fixed menu cost model. Interestingly, under the more realistic assumption of autocorrelated money growth, the three models differ much less. While the total response of inflation over time is necessarily larger with autocorrelated money, there are also much larger real effects. Consumption and labor jump strongly on impact in all three specifications, though their response is substantially more persistent in the Calvo model than in the other two.

To better understand these differences in adjustment, we also show the impulse responses of some statistics related to the distributional dynamics. In Figure 3, we see that following an uncorrelated 1% rise in the money supply, the fraction of firms adjusting rises by 2.5 percentage points on impact in the menu cost specification (from 10% to 12.5% monthly), compared with a rise of just 0.3% in the SDSP specification. At the same time, the average price change in the menu cost model (+4.8%) increases much more than in either the SDSP (+1.7%) or the Calvo (+1%) specification. The reason for this can be seen in figure 4 showing the identity of adjusting firms following a money growth shock. In the menu cost case, adjusting firms are far out in the tails of the distribution (fig 4.1), making the average price change very sensitive to the aggregate shock (see fig.4.2). In the Calvo case, because adjusting firms are drawn randomly and many of them make small price changes, the average price change is much less sensitive to the shock. Our calibrated SDSP model is an intermediate case in which some selection takes place; nevertheless it is much closer to the Calvo than to the menu cost model.

Notice that, to a first-order approximation, the effect of the money shock on inflation can be decomposed into a part due to the fraction of adjusters $\phi^o$ (extensive margin) and a part due to the average price change $\sum p$ (intensive margin):

$$\frac{\partial \pi_t}{\partial \mu_t} = \frac{\partial (\sum p)}{\partial \mu_t} \phi^o + \frac{\partial \phi^o}{\partial \mu_t} \sum p^o.$$  

Recall that in our steady-state calibration $\phi^o = 0.10$ is much larger than $\sum p^o \approx 0.02$. In addition, variation in the total mass of adjusters in response to the shock ($\partial \phi^o/\partial \mu_t$) is likely to be muted, as
the larger fraction of price increases is offset by the smaller fraction of price decreases. This suggests that in a low-inflation environment in which price changes are driven mainly by idiosyncratic factors, the intensive margin is likely to dominate the variance of inflation. This is indeed the case in all specifications studied here and it is consistent with Klenow and Kryvtsov’s (2008) finding that, at least in US data, inflation reflects movements in the size of price changes rather than in the fraction of firms changing price.

By exaggerating the selection effect to the point of ruling out any small price changes, the fixed menu cost model makes the average price change extremely sensitive to aggregate shocks, which results in a strong inflation reaction and a muted output response. Our SDSP specification which matches much better the distribution of price changes from micro evidence implies much less responsive average price change and inflation, and a much stronger output response.

Unlike idiosyncratic technology shocks or uncorrelated money shocks, with autocorrelated money growth shocks (Figure 5.1) firms which fail to adjust expect to see their real price drifting further out of line over time. This is especially costly for Calvo firms for which the value loss from a lower-than-optimal price is much larger than the loss from a higher-than-optimal price (see Figure 5.2). The latter asymmetry affects to a much lesser extent firms subject to menu costs: their value function is much flatter and more symmetric around the optimal price, compared to Calvo firms. This is why firms subject to menu costs are less willing to anticipate future inflation by setting a higher price today, compared to Calvo firms which have a stronger incentive to do so.\(^\text{11}\) This has an opposite effect on the size of the average price change to the selection effect described earlier and illustrated in Figure 4. As a result, compared to iid money shocks, with autocorrelated money growth shocks the average price change under Calvo pricing gets closer to that under menu costs. This implies more similar responses of inflation and output too across the three specifications.

We also plot the response of price dispersion, defined as

\[
\Delta_t = \int A_{it}^{-1} \left( \frac{P_{it}}{P_t} \right)^{-\epsilon} \, di.
\]

Price dispersion in our setup is partly caused by the presence of idiosyncratic productivity shocks. But additional price dispersion, above that derived from the idiosyncratic shocks themselves, causes inefficient variation in demand across goods, and results in an inefficiency wedge in the transformation of labor input into output:

\[
\int L_{it} di = \Delta_t Y_t.
\]

In the case of uncorrelated shocks price dispersion moves in opposite directions in the Calvo and menu cost frameworks. In the Calvo model, price dispersion rises after a money growth shock, since those firms that are able to adjust their prices now choose a greater increase. In the menu cost model, instead, price dispersion may fall, because adjustment speeds up substantially.\(^\text{12}\) In the SDSP model, these two effects largely offset each other, so that price dispersion varies less; the overall effect is roughly a 0.5% rise in price dispersion in the case of autocorrelated shocks.

We have also repeated these impulse response calculations starting from a 10% annual inflation rate, as in the late 1970s in the US. There is little change in the results (so they are not shown). A 1% money supply shock has a slightly stronger effect on inflation, and thus a slightly weaker

\(^\text{11}\) Midrigan (2006) call this “front-loading” by Calvo firms. He stresses the different curvature of the value functions under Calvo and menu cost contracts, but does not point to the asymmetry as central to explaining the difference in “front-loading”.

\(^\text{12}\) This is similar to the effect of steady state inflation in the menu cost specification in Costain and Nakov (2008). In that paper, the standard deviation of price changes falls with inflation because it causes the probability of adjustment to increase. In the limit, as the probability of adjustment approaches one, differences in price adjustments reflect differences in idiosyncratic productivity only, with no remaining effect from price stickiness.
effect on consumption, when the baseline inflation rate is higher. Also, higher inflation makes the responses of the three models slightly more similar.

In their paper, Golosov and Lucas (2007) show that money supply shocks calibrated to be sufficiently large to match the standard deviation of inflation would explain a very small part of output fluctuations. But our impulse response calculations cast doubt on this claim, because the response of output to money is much smaller in the fixed menu cost model than it is in our preferred SDSP specification if shocks are uncorrelated, and moreover all the output responses are much larger when money shocks are correlated. Therefore, like Golosov and Lucas, we compare the output variability implied by all three versions of the model in Table 2. We set the autocorrelation $\phi_z = 0.5$, and choose the standard deviation of the money shock for each version of the model so that it matches 100% of the observed US inflation volatility. We then calculate the implied variability of output. Remarkably, under the SDSP specification, these money shocks would explain essentially all US output fluctuations. Under the Calvo specification, the calibrated money shocks would cause 181% of the output fluctuations observed in the US. With menu costs, the figure is much lower, slightly under 50%, but even this is much higher than Golosov and Lucas found, since they focused on the case of uncorrelated shocks. We also calculate a “Phillips curve” by regressing output on money growth; we obtain a coefficient of approximately one, much larger than that reported by Golosov and Lucas (see Table 2).

6.2 Discussion

To better understand how the impulse responses differ across models and depending on the autocorrelation of the shock, we decompose the change in the inflation rate into three main effects which have played a role in recent papers. Our decomposition is designed to clarify the relation between the effects identified in previous papers’ decompositions, which have sometimes been defined in mutually inconsistent ways. First, fixing the set of firms that adjust, inflation will rise if each one raises its price more than it otherwise would have done. Like Caballero and Engel (2007), we call this the intensive margin effect. Second, if the average adjustment is positive, inflation will rise if the fraction of firms adjusting increases (holding fixed the size distribution of adjustments); we call this the extensive margin effect. Third, fixing the fraction of firms that adjust, inflation will rise if adjustment opportunities are reallocated from firms desiring a small or negative price change to others desiring a large price increase; like Golosov and Lucas (2007) we call this the selection effect.

To see how we can cleanly distinguish these three effects, let us follow Caballero and Engel (2007) and consider the distribution of desired adjustments in log price space instead of the distribution of losses in value space. Call a firm’s desired adjustment $x \equiv P^*(A, \Omega) - P$. Suppose we start from the beginning-of-period steady state distribution of desired adjustments, which we will call $\hat{\varphi}(x)$. Assume, for simplicity only, that we can write the probability of adjustment as a function of the desired adjustment, $\lambda(x)$. Then in the steady state, the price increase at the beginning of the period is

$$\pi = \int x \hat{\varphi}(x) \lambda(x) dx$$

Now suppose (again, only for simplicity) that if the money supply increases unexpectedly by $\Delta \mu$ relative to its steady state growth rate, all desired prices rise by $\Delta P^*$. Then after the shock, fraction $\hat{\varphi}(x)$ of firms have desired adjustment $x + \Delta P^*$, and will adjust with probability

13These strong simplifying assumptions are made only for notational clarity and comparability with Caballero and Engel (2007). The numerical calculations reported in Table 3 do not impose these simplifications; instead, we use analogous formulas consistent with our model, explicitly taking into account the fact that $\lambda$ is a function of the value loss, not the price gap.
\[ \hat{\lambda}(x) + \Delta \hat{\lambda}(x) = \lambda(x) + \lambda'(x) \Delta p^*, \] so in this case the price increase will be

\[ \pi + \Delta \pi = \int (x + \Delta p^*) \hat{\varphi}(x) \left[ \lambda(x) + \Delta \hat{\lambda}(x) \right] dx. \]

Differentiating, the effect of money growth on the inflation rate is

\[ \frac{\partial \pi}{\partial \mu} = \frac{\partial p^*}{\partial \mu} \int \hat{\varphi}(x) \lambda(x) dx + \int x \hat{\varphi}(x) \frac{\partial \lambda(x)}{\partial \mu} dx \]

which is a generalization of equation (17) from Caballero and Engel (2007). We find it helpful to go one step more and divide up the effects again, as follows:

\[ \frac{\partial \pi}{\partial \mu} = \frac{\partial p^*}{\partial \mu} \int \hat{\varphi}(x) \lambda(x) dx + \int x \hat{\varphi}(x) \frac{\partial \lambda(x)}{\partial \mu} dx + \int x \hat{\varphi}(x) \left[ \frac{\partial \lambda(x)}{\partial \mu} - \frac{\partial \hat{\lambda}(x)}{\partial \mu} \right] dx \equiv I + E + S \]

where \( \frac{\partial \lambda(x)}{\partial \mu} \) measures how the fraction of firms adjusting varies with money growth. In this equation, the first term is what we call the intensive margin, so we label it \( I \), the second term \( E \) is what we call the extensive margin, and the third term \( S \) is our selection effect.

Previous papers have grouped these terms in different ways. Our definition of the intensive margin is the same as that in Caballero and Engel (2007), but they use "extensive margin" to refer to the sum of \( E \) and \( S \), thus mixing the effect of changing the fraction of firms adjusting with the effect of selecting large increases in place of small ones. Klenow and Kryvstov (2007), on the other hand, use "extensive margin" in the same sense we do, to mean variation in the fraction of firms adjusting, but their "intensive margin" is the sum of \( I \) and \( S \), so it mixes the effect of raising the desired price for all firms with the effect of reallocating adjustment opportunities across firms. Since only term \( I \) is present in the "time dependent" Calvo model, we feel it is useful to separate this from the selection effect \( S \) that can be important for state-dependent models, as discussed by Golosov and Lucas (2007).\(^{14}\)

Table XXX decomposes the initial impact of a money shock into its intensive, extensive, and selection components. In the Calvo model, by construction, only the intensive margin is active. The extensive margin is active in the other two models, but as Figure 3 suggested, it is never very important for inflation. This is true even for the menu cost model with uncorrelated shocks, where the frequency changes a lot. This is because in the steady state, the average price adjustment is small, so raising the frequency of adjustment while holding the average adjustment fixed has a quantitatively small effect on inflation.

**AFTER DISCUSSING DECOMPOSITION MUST DISCUSS FRONTLOADING**

### 6.3 The role of the distribution

Figures 6 and 7 illustrate the distributional dynamics of the model, and some of the implied nonlinearities. Figure 6 shows transitional dynamics of the SDSP calibration of the model, starting from a variety of different initial conditions; Figure 7 shows how the impulse response to a money supply shock changes when starting from different initial conditions.

Figure 6.1 shows the impact of a large monetary shock, but the calculation is carried out in a different way from the calculations above. Instead of starting at the steady state distribution

\(^{14}\)When Nakamura and Steinsson (2006) explain how their finding that inflation is strongly driven by variations in the frequency of price increases is compatible with Klenow and Kryvstov’s (2007) claim that most inflation is caused by variation in the average adjustment, they are effectively distinguishing between the extensive margin and selection effects.
and feeding in a money shock, we simply shift the distribution of real prices two grid points to the left. That is, we decrease real prices by 6.4%, which is equivalent to an uncorrelated increase in money supply growth by 6.4%. By calculating the effects of the shock in this way, we take nonlinearities into account, since our computational method allows full nonlinearity between one grid point and the next. Some of the impulse responses are proportional to those shown before; for example, the response of inflation in Fig. 5.1 is roughly six times larger than that shown for the SDSP calibration in the second panel of Fig. 2. But the fraction of firms adjusting increases in a more-than-linear way, rising more than tenfold, which makes sense since the value of adjustment should increase nonlinearly in the distance from the optimal price. Price dispersion also increases more than linearly, and labor, surprisingly, falls strongly on impact. Figure 6.2 performs a similar exercise, shifting the distribution of productivities by two grid points, which is equivalent to an uncorrelated 6.4% increase in productivity. Inflation falls, consumption rises, and labor falls, as expected in a sticky-price model.

Figures 6.3 and 6.4 instead show the transitional dynamics starting from degenerate distributions. In Figure 6.3, we graph the dynamics under the assumption that, for some reason, all prices are initially set to the mean price. Initially, therefore, the fraction of firms adjusting is 2.5 percentage points above its steady state, and price dispersion rapidly increases from far below its steady state. The degenerate initial price distribution is inefficient, so consumption also starts 1% below its steady state. Fig. 6.4 shows a related exercise: all firms start from the mean productivity and the mean price. Therefore this experiment can be seen as the effect of unexpectedly “turning on” the idiosyncratic shock process. In this case most firms initially feel no need to adjust their prices, until their productivity begins to drift away from its mean value. Therefore, the fraction of firms adjusting is initially below its steady state value, and price dispersion converges more slowly to steady state than it did in Fig. 6.3.

Figures 6.5 and 6.6 temporarily suppress the effects of frictions, in two ways. Fig. 6.5 studies the transition path starting from a distribution in which all firms have set their optimal flexible prices. So this experiment can be seen as the effect of unexpectedly “turning on” price stickiness. Unsurprisingly, as the second-to-last panel of Fig. 1 shows, firms choose more price dispersion under flexible prices; the pricing policy function is flatter under sticky prices. Therefore, when price stickiness is turned on, price dispersion gradually falls by 22%. The fraction of firms adjusting is initially below trend, since firms are nonetheless quite close to the prices they would prefer to choose under price stickiness. Also, the flexible prices are more efficient than the steady state distribution of sticky prices, so initially in this experiment consumption is above trend while labor is below. Fig. 6.6 is similar, but instead of starting all firms from their preferred flexible prices, we start them from their preferred sticky prices. This might be seen as the effect of “introducing the euro”: it is as if all firms are forced to change prices at one point in time, taking into account the fact that their prices will be sticky in the future. Conditional on this one-time change, we see that the fraction of firms adjusting is thereafter below steady state, since they start out at their preferred sticky price. The one-time adjustment also allows them to “catch up” with past inflation, so inflation is thereafter below steady state. Price dispersion is also temporarily above trend. That is, because of price stickiness, prices in the steady state distribution vary less strongly with productivity than the sticky-price policy function itself does.

Since the value of adjustment varies greatly with initial conditions, the effects of a monetary policy shock will also vary. Fig. 7 shows an example. For the SDSP calibration, it shows the effect of an uncorrelated 1% increase in money supply, either starting from the steady state distribution (as seen previously in Fig. 3), or with all firms starting from the optimal sticky price. In both cases, we aim to show only the effect of the money supply shock itself, so to graph the blue circled curve in Fig. 7 we first compute a path starting from optimal sticky prices plus a money shock; then we compute a path starting from optimal sticky prices without a money shock (as seen in Fig.
of price dispersion twice as much when starting from the optimal sticky price than it does starting from the steady state distribution, and labor must also increases substantially more in this case.

7 Conclusions

We have computed the impact of money supply shocks in a model of state-dependent pricing, characterizing the dynamics of the distribution of prices while allowing this distribution to have general equilibrium effects on firms’ and consumers’ decisions. State-dependent pricing somewhat weakens the real effects of monetary shocks, compared with the Calvo model. Nonetheless, these real effects remain important under the calibration that best fits microeconomic pricing data, especially when money supply shocks are autocorrelated (0.8 monthly), as they are in reality. With this autocorrelation, the initial effect of money supply on output in our state-dependent model is approximately the same as in the Calvo model, with half the persistence.

8 Appendix A: Detrending

Suppose the model can be rewritten in real terms by deflating all prices by the nominal money stock, defining $p_t \equiv P_t/M_t$, $p_{it} \equiv P_{it}/M_t$, and $w_t \equiv W_t/M_t$. Given the nominal distribution $\Phi_t(P_t, A_t)$ and the money stock $M_t$, we denote by $\Psi_t(p_t, A_t)$ the distribution over real production prices $p_{it} \equiv P_{it}/M_t$. Likewise, let $\Psi_t(p_t, A_t)$ be the distribution of real beginning-of-period prices $\tilde{p}_{it} \equiv \tilde{P}_{it}/M_t$, in analogy to the beginning-of-period distribution of nominal prices $\Phi_t(\tilde{P}_t, A_t)$. If it is true that the model can be rewritten in real terms, then the distribution $\Psi_t(\tilde{p}_t, A_t)$, together with the most recent money growth rate $z_t$, is a sufficient aggregate state variable to determine real quantities. That is, to describe the real equilibrium, it is not necessary to condition functions on $M_t$, only on $z_t \equiv (z_t, \Psi_t)$.

Therefore, if there exists a real equilibrium, the real aggregate functions can be written in terms of $z_t$ only, and must satisfy $C_t = C(z_t) = C(\Omega_t)$, $N_t = N(z_t) = N(\Omega_t)$, $p_t = p(z_t) = P(\Omega_t)/M_t$ and $w_t = w(z_t) = W(\Omega_t)/M_t$. Deflating from one period to the next will depend on the growth rate of money supply from one period to the next (but not on the level of the money supply). Thus the stochastic discount factor will be

$$q(\Xi_t, \Xi_{t+1}) = \beta M_t p(\Xi_t) u'(C(\Xi_{t+1})) M_{t+1} p(\Xi_{t+1}) u'(C(\Xi_t)) = \beta u(z_{t+1}) \frac{p(\Xi_t) u'(C(\Xi_{t+1}))}{p(\Xi_{t+1}) u'(C(\Xi_t))}$$

The “real” value function $v$ should likewise be the nominal value function, divided by the current money stock, and should be written as a function of real prices. Therefore we have

$$V(p_{it}, A_{it}, \Omega_t) = M_t v \left( \frac{P_{it}}{M_t}, A_{it}, \Xi_t \right) = M_t v (p_{it}, A_{it}, \Xi_t)$$

If a firm’s nominal price at time $t$ is $P_{it}$, then the value of maintaining this price fixed at time $t+1$ would be

$$v \left( \frac{P_{it}}{M_t}, A_{it}, \Xi_t \right) = M_t v (p_{it}, A_{it}, \Xi_t)$$

15If money growth is uncorrelated, then $z_t$ has no effect on the distribution of $\mu_{t+1}$, so the aggregate state can be summarized by $\Phi_t$ only. But when money growth is correlated we must keep track of $\Xi_t \equiv (\Phi_t, z_t)$ instead.
can be written in nominal or real terms as
\[
V(P_{it}, A_{i,t+1}, \Omega_{t+1}) = M_{t+1} v \left( \frac{P_{it}}{M_{t+1}}, A_{i,t+1}, \Xi_{t+1} \right)
\]
\[
= M_{t+1} v \left( \frac{M_{ip} M_{t+1}}{M_{t+1}}, A_{i,t+1}, \Xi_{t+1} \right) = M_{t+1} v \left( \mu_{t+1} p_{it}, A_{i,t+1}, \Xi_{t+1} \right)
\]

Likewise, if for any time \( t \) nominal price \( P_{it} \) we have the definitions
\[
D(P_{it}, A_{i,t+1}, \Omega_{t+1}) \equiv \max_{P'} V(P', A_{i,t+1}, \Omega_{t+1}) - V(P_{it}, A_{i,t+1}, \Omega_{t+1})
\]
\[
G(P_{it}, A_{i,t+1}, \Omega_{t+1}) \equiv \lambda \left( \frac{D(P_{it}, A_{i,t+1}, \Omega_{t+1})}{W(\Omega_{t+1})} \right) D(P_{it}, A_{i,t+1}, \Omega_{t+1})
\]

then we can define
\[
D(P, A_{i,t+1}, \Omega_{t+1}) \equiv M_{t+1} d \left( \frac{P}{M_{t+1}}, A_{i,t+1}, \Xi_{t+1} \right) = M_{t+1} d \left( \mu_{t+1} p_{it}, A_{i,t+1}, \Xi_{t+1} \right)
\]
\[
G(P, A_{i,t+1}, \Omega_{t+1}) \equiv M_{t+1} g \left( \frac{P}{M_{t+1}}, A_{i,t+1}, \Xi_{t+1} \right) = M_{t+1} g \left( \mu_{t+1} p_{it}, A_{i,t+1}, \Xi_{t+1} \right)
\]

Using this deflated notation, we can rewrite the Bellman equation as
\[
M_{t} v(p_{it}, A_{it}, \Xi_{t}) = M_{t} \left( p_{it} - \frac{w(\Xi_{t})}{A_{it}} \right) \left( \frac{p_{it}}{p(\Xi_{t})} \right)^{-\epsilon} C(\Xi_{t}) + \beta E_t \left\{ \frac{M_{t+1} p(\Xi_{t}) u'(C(\Xi_{t+1}))}{M_{t+1} p(\Xi_{t+1}) u'(C(\Xi_{t+1}))} M_{t+1} \left[ v \left( \mu_{t+1} p_{it}, A_{i,t+1}, \Xi_{t+1} \right) + g \left( \mu_{t+1} p_{it}, A_{i,t+1}, \Xi_{t+1} \right) \right] \bigg| A_{it}, \Xi_{t} \right\}
\]

Note that \( M_{t} \) cancels from both sides of the equation, and \( M_{t+1} \) cancels inside the expectation. Therefore we obtain
\[
v(p_{it}, A_{it}, \Xi_{t}) = \left( p_{it} - \frac{w(\Xi_{t})}{A_{it}} \right) \left( \frac{p_{it}}{p(\Xi_{t})} \right)^{-\epsilon} C(\Xi_{t}) + \beta E_t \left\{ \frac{p(\Xi_{t}) u'(C(\Xi_{t+1}))}{p(\Xi_{t+1}) u'(C(\Xi_{t+1}))} \left[ v \left( \mu_{t+1} p_{it}, A_{i,t+1}, \Xi_{t+1} \right) + g \left( \mu_{t+1} p_{it}, A_{i,t+1}, \Xi_{t+1} \right) \right] \bigg| A_{it}, \Xi_{t} \right\}
\]
where
\[
g \left( \mu_{t+1} p_{it}, A_{i,t+1}, \Xi_{t+1} \right) \equiv \lambda \left( d \left( \mu_{t+1} p_{it}, A_{i,t+1}, \Xi_{t+1} \right) \right) \left( \frac{w(\Xi_{t+1})}{w(\Xi_{t})} \right) d \left( \mu_{t+1} p_{it}, A_{i,t+1}, \Xi_{t+1} \right)
\]
\[
d \left( \mu_{t+1} p_{it}, A_{i,t+1}, \Xi_{t+1} \right) \equiv \max_{P'} v(P', A_{i,t+1}, \Xi_{t+1}) - v \left( \mu_{t+1} p_{it}, A_{i,t+1}, \Xi_{t+1} \right)
\]

Let \( p^*(A_{i,t+1}, \Xi_{t+1}) \) denote the optimal choice in the maximization problem above. Taking into account the fact that the firm starts period \( t + 1 \) with the eroded price \( \tilde{p}_{i,t+1} = \mu_{t+1} p_{it} \), the price process is
\[
p_{i,t+1} = \begin{cases} 
    p(A_{i,t+1}, \Xi_{t+1}) & \text{with prob } = \lambda \left( \frac{d(\mu_{t+1} p_{it}, A_{i,t+1}, \Xi_{t+1})}{w(\Xi_{t+1})} \right) \\
    \mu_{t+1} p_{it} & \text{with prob } = 1 - \lambda \left( \frac{d(\mu_{t+1} p_{it}, A_{i,t+1}, \Xi_{t+1})}{w(\Xi_{t+1})} \right)
\end{cases}
\]

In other words, when the firm’s nominal price is not adjusted at time \( t + 1 \), its real price is deflated by factor \( \mu_{t+1} = \mu e^{z_{t+1}} \).
9 Appendix B. Implementing the Klein (2000) method

Under linear labor disutility, $x(N) = \xi N$, general equilibrium dynamics are summarized by the following equations:

$$V_t = U_t + \beta E_t \left\{ \frac{p_t u'(C_{t+1})}{p_{t+1} u'(C_t)} R_{t+1} * (V_{t+1} + G_{t+1}) * S \right\}$$

$$1 - \frac{u'(1/p_t)}{u'(C_t)} = \beta E_t \left( \mu_{t+1} \frac{p_t u'(C_{t+1})}{p_{t+1} u'(C_t)} \right)$$

$$p_t^{1-\epsilon} = \sum_{j=1}^{#^B} \sum_{k=1}^{#^A} E_t \left( \tilde{\Psi}_{t+1}^j \right) (p^j)^{1-\epsilon}$$

$$E_t \left( \tilde{\Psi}_{t+1} \right) = (E - \lambda(D_t/w_t)) * \tilde{\Psi}_t + P_t * (E * (\lambda(D_t/w_t) * \tilde{\Psi}_t))$$

$$z_{t+1} = \phi_z z_t + C_{t+1}$$  \hspace{1cm} (36)

where $\tilde{\Psi}_t \equiv \Psi_{t-1}$ is the lagged distribution of idiosyncratic states from the time of production.

These equations form a system of $#^T \geq 2 #^P #^A + 3$ first-order expectational difference equations in the jump variables $V_t$, $C_t$, and $p_t$, the endogenous state variables $\Psi_t$, which are predetermined in the sense that $\tilde{\Psi}_{t+1} = E_t \tilde{\Psi}_{t+1}$, and the exogenous state process $z_t$. To see this, note that given $z_t$ and $z_{t+1}$ we can construct $\mu_t$, and thus $R_t$ and $R_{t+1}$. Given $R_t$, we can construct $\tilde{\Psi}_t = R_t * \tilde{\Psi}_t * S'$ from $\tilde{\Psi}_t$. Under linear labor disutility, we can construct $w_t = \xi p_t / u'(C_t)$, which gives us all the information needed to construct $U_t$. Finally, given $V_t$ and $V_{t+1}$ we can construct $D_t$ and $D_{t+1}$ and thus $\lambda(D_t/w_t)$ and $G_{t+1}$. Thus we have all the quantities that appear in the equation system, which can be summarized as a nonlinear system of the form

$$E_t \mathcal{F} \left( \overline{X}_{t+1}, \overline{X}_t, z_{t+1}, z_t \right) = 0$$  \hspace{1cm} (37)

using the notation

$$\overline{X}_t \equiv \left( vec(V_t)', C_t, p_t, vec(\tilde{\Psi}_t)' \right)'$$

By linearization we then obtain a system of the form (34), which Klein’s (2000) method is designed to solve.

10 Appendix C. Reduction of dimension

The linearization of (34) can be performed numerically by considering small deviations in all elements of $\overline{X}_t$, $\overline{X}_{t+1}$, $z_t$ and $z_{t+1}$, and the software of Klein (2000) or Sims (2001) can then be applied directly to solve the linearized system. The simulations reported in the paper apply this method to the model approximated on a coarse grid with 25 possible shocks $A$ and 25 possible prices $p$. As we argued in the text, using a coarse grid is unlikely to be economically important in the context of our model.

In future calculations we intend to check this assertion by combining a fine representation of the steady state with a coarse representation of the dynamics. There are two reasons why the dynamic calculation suffers from a severe curse of dimensionality. First, the Jacobians $A$ and $B$ have size $#^T \times #^T$ (so that if there are hundreds of grid points, the Jacobians will contain millions of elements) and are not sparse. The other major problem is that the computation time
for QZ decomposition is cubic in the number of equations $\#^{TOT}$. Both of these issues make the system too large for ordinary desktop computers when a fine grid is used in the second step of Reiter’s algorithm.

However, the fact that firms’ decisions are likely to depend less strongly on aggregate variations than on idiosyncratic shocks again helps us out, because it means that the grid used to characterize the steady state response to idiosyncratic shocks need not be the same as the grid used to characterize the deviation of the value function from the steady state value function. Thus, we will calculate the steady state value function on the fine policy grid $\Gamma^p \times \Gamma^A$, but perturb it on a coarser grid $\gamma^p \times \gamma^A$, where $\gamma^p \subset \Gamma^p$ and $\gamma^A \subset \Gamma^A$. We can then treat the value function as the sum of the steady state value plus a deviation which is piecewise linear over the coarser grid $\gamma^p \times \gamma^A$. So for example, if $\gamma^p$ and $\gamma^A$ consist of each tenth grid point of $\Gamma^p$ and $\Gamma^A$, then the number of equations and variables related to the value function in the linearized system (35) is reduced by a factor of one hundred. An alternative way of understanding this procedure is that it is a collocation method that imposes piecewise linear form on the deviations of the value function from the steady state value function, and treats each tenth point in $\Gamma^p$ as a collocation point.
References


[27] Midrigan, Virgiliu (2006), Menu Costs, Multi-Product Firms and Aggregate Fluctuations, Manuscript, New York University


Simulations from SDSP model, calculated on fine grid. First line: value \( V \), loss \( D \), and adjustment probability \( \lambda \), as functions of real price \( p \) and cost shock \( A^{-1} \). Second line: beginning of period distribution, adjustment distribution, and distribution at time of production. Third line: adjustment probability \( \lambda \) as a function of \( D \), policy function, and distribution of monthly non-zero price changes.
Table 1. Steady-state results

Menu cost: \((\sigma_{\epsilon}^2, \rho)\) same as SDSP; \(\kappa = 0.03\)
Calvo: \((\sigma_{\epsilon}^2, \rho)\) same as SDSP; \(\lambda = 0.10\)
SDSP: \((\sigma_{\epsilon}^2, \rho, \varsigma, \alpha) = (0.0021, 0.9351, 0.3675, 5.7347)\), computed with 501 or 25 grid points for \(p\)

<table>
<thead>
<tr>
<th>Model:</th>
<th>MC</th>
<th>Calvo</th>
<th>SDSP 501 pt</th>
<th>SDSP 25 pt</th>
<th>Target</th>
<th>Dataset</th>
</tr>
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<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>NS</td>
</tr>
<tr>
<td>Frequency of price changes</td>
<td>10.3</td>
<td>10</td>
<td>10.1</td>
<td>10.2</td>
<td>10.0</td>
<td>10.0</td>
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<tr>
<td>Mean absolute price change</td>
<td>12.3</td>
<td>5.64</td>
<td>8.9</td>
<td>9.1</td>
<td>8.8</td>
<td>8.5</td>
</tr>
<tr>
<td>Median absolute price change</td>
<td>11.9</td>
<td>4.25</td>
<td>7.9</td>
<td>8.1</td>
<td>8.5</td>
<td>8.5</td>
</tr>
<tr>
<td>Mean price increase</td>
<td>11.9</td>
<td>6.47</td>
<td>9.3</td>
<td>9.6</td>
<td>9.5</td>
<td>9.5</td>
</tr>
<tr>
<td>Median price increase</td>
<td>11.7</td>
<td>4.89</td>
<td>8.3</td>
<td>8.8</td>
<td>7.3</td>
<td>7.3</td>
</tr>
<tr>
<td>Std of price changes</td>
<td>12.4</td>
<td>7.28</td>
<td>10.4</td>
<td>11.2</td>
<td>11.8</td>
<td>11.8</td>
</tr>
<tr>
<td>Percent of price increases</td>
<td>60</td>
<td>60</td>
<td>59</td>
<td>58</td>
<td>62</td>
<td>67</td>
</tr>
<tr>
<td>Percent of price changes &lt;5% in abs value</td>
<td>0.03</td>
<td>56.7</td>
<td>29</td>
<td>28</td>
<td>36</td>
<td>40</td>
</tr>
<tr>
<td>Flow of menu cost as % of firms’ revenues</td>
<td>0.72</td>
<td>0.5</td>
<td>0.5</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Median distance from optimal price</td>
<td>3.29</td>
<td>3.65</td>
<td>3.90</td>
<td>3.86</td>
<td></td>
<td></td>
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<tr>
<td>Average distance from optimal price</td>
<td>3.80</td>
<td>5.09</td>
<td>5.29</td>
<td>5.42</td>
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<td></td>
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<tr>
<td>Median loss as % of median value</td>
<td>0.021</td>
<td>0.083</td>
<td>0.043</td>
<td>0.043</td>
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<td></td>
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<tr>
<td>Average loss as % of median value</td>
<td>0.040</td>
<td>0.45</td>
<td>0.16</td>
<td>0.17</td>
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<tr>
<td>Std of loss as % of median value</td>
<td>0.045</td>
<td>1.25</td>
<td>0.34</td>
<td>0.34</td>
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<td></td>
</tr>
</tbody>
</table>

Note: All statistics refer to regular consumer price changes excluding sales, and are stated in percent. “Target” is a simple average of the corresponding statistics reported by Nakamura and Steinsson (NS), Klenow and Kryvtsov (KK) and Virgiliu Midrigan’s (VM) averages for two supermarket chains.
Figure 2. Steady-state distribution of nonzero price adjustments: fine and coarse grids

Horizontal axis: log price change. Vertical axis: frequency

Table 2. Variance decomposition and Phillips curves of alternative models

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>SDSP II</th>
<th>Calvo</th>
<th>Menu cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Std of money shock (x100)</td>
<td>0.45</td>
<td>0.80</td>
<td>0.22</td>
<td></td>
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<tr>
<td>Std of quarterly inflation (x100)</td>
<td>0.246</td>
<td>0.246</td>
<td>0.246</td>
<td>0.246</td>
</tr>
<tr>
<td>Share explained by $\mu$ shock alone</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Std of quarterly output growth (x100)</td>
<td>0.51</td>
<td>0.52</td>
<td>0.92</td>
<td>0.25</td>
</tr>
<tr>
<td>Share explained by $\mu$ shock alone</td>
<td>102%</td>
<td>181%</td>
<td>49%</td>
<td></td>
</tr>
<tr>
<td>Slope coefficient in PC regression (standard error)</td>
<td>1.03 (0.015)</td>
<td>1.16 (0.034)</td>
<td>0.70 (0.012)</td>
<td></td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.74</td>
<td>0.44</td>
<td>0.97</td>
<td></td>
</tr>
</tbody>
</table>

Note: $\phi_z = 0.5$
Figure 3. Impulse-response functions: uncorrelated money shock.

Panels 1-5: difference from steady state in percentage points. Panels 6-9: percent deviation from steady state.
Figure 4.1: Identity of adjusters in Menu cost, Calvo, SDSP after a 1% uncorrelated money growth shock
Figure 5. Impulse-response functions: autocorrelated money shock.
Panels 1-5: difference from steady state in percentage points. Panels 6-9: percent deviation from steady state.
Table 3: Decomposing initial impact of money shocks

<table>
<thead>
<tr>
<th></th>
<th>$\frac{\sigma}{\eta}$</th>
<th>$\frac{\sigma \delta}{\eta}$</th>
<th>$\tau$</th>
<th>$E$</th>
<th>$S$</th>
</tr>
</thead>
</table>

**Uncorrelated shocks:**
- Calvo
- SDSP
- Menu cost

**Correlated shocks:**
- Calvo
- SDSP
- Menu cost
Figure 6A. Value functions with different curvature and asymmetry: Menu cost, Calvo, SDSP, inflation = 0.64% quarterly
FIG 6B: SAME GRAPH, INFLATION = 10% (like in the graph you sent me)
Figure 7.1 Transitional dynamics from shifted distribution: 6.4% increase in money.
Panels 1-5: difference from steady state in percentage points. Panels 6-9: percent deviation from steady state.
Figure 7.2 Transitional dynamics from shifted distribution: 6.4% increase in aggregate productivity.
Panels 1-5: difference from steady state in percentage points. Panels 6-9: percent deviation from steady state.
Figure 7.3 Transitional dynamics from shifted distribution: all firms from same price.

Panels 1-5: difference from steady state in percentage points. Panels 6-9: percent deviation from steady state.
Figure 7.4 Transitional dynamics from shifted distribution: all firms from same price and same cost.

Panels 1-5: difference from steady state in percentage points. Panels 6-9: percent deviation from steady state.
Figure 7.5 Transitional dynamics from shifted distribution: all firms from optimal flexible price.

Panels 1-5: difference from steady state in percentage points. Panels 6-9: percent deviation from steady state.
Figure 7.6 Transient dynamics from shifted distribution: all firms from optimal sticky price.

Panels 1-5: difference from steady state in percentage points. Panels 6-9: percent deviation from steady state.
Figure 8. Impulse-responses from different initial distributions.

Panels 1-5: difference from steady state in percentage points. Panels 6-9: percent deviation from steady state.