Endogenous Institution Formation under a Catching-up Strategy in Developing Countries\textsuperscript{1}

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(This Version: November, 2008)

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Abstract

This paper explores endogenous institution formation under a catching-up strategy in developing countries. Since the catching-up strategy is normally against the comparative advantages of the developing countries, it can not be implemented through *laissez-faire* market mechanisms, and a government needs to establish non-market institutions to implement the strategy. In a simple two-sector model, the authors show that an institutional complex of price distortion, output control, and a directive allocation system is sufficient to implement the best allocation for the catching-up strategy. Furthermore, removing any of the three components will make it no longer implementable. The analysis also compares the best allocation and prices under the catching-up strategy with their counterparts under no distortions. The results of this paper provide important implications for understanding the institution formation in the developing countries that were pursuing a catching-up strategy after World War II.

**Key Words:** development strategy; institution; price distortion; output control; directive allocation system

**JEL Classification:** D02, O17, O20, P41
1 Introduction

After World War II, many former colonial or semi-colonial countries in Asia, Africa and Latin America achieved their political independence. After that, how to realize rapid economic development and also achieve economic independence became new challenges for political leaders in those countries. Compared with industrialized countries, such as the United States and countries in Western Europe, these newly independent and less developed countries had extremely low income levels, living standards and economic growth rates. A common feature of the developing countries was that they heavily specialized in the production and export of primary commodities, and imported most industrial goods from the wealthy industrialized countries. Historically, the lack of modern industrial sectors, especially heavy industries, which were the basis of military strength and economic power, had forced many countries in the developing world to yield to the colonial powers. After achieving independence, the political leaders and elites in those countries naturally experienced an impetus towards the rapid development of large heavy industries, and commonly adopted an ideology of economic nationalism (Lal and Myint, 1996, chapter 7).

In the 1940s and 1950s, with still fresh memories of the Great Depression in capitalist countries, economists and policy makers began to cast doubt on the sound functioning of a laissez-faire market mechanism, even in fledgling market economies. The rise of Keynesism as a new economic doctrine in the Western countries also reflected the belief that the market encompassed insurmountable defects, and government needed to provide supplementary policy measures to sustain a stable and well-functioning economy. In sharp contrast to the deep crisis in the Western countries in the 1930s, the socialist Soviet Union, having adopted a planned economic system and prioritized the development of heavy industries, was experiencing rapid economic growth and successfully transformed itself from a traditional agrarian economy into an industrial economy\(^1\). That seemingly great success of the Soviet Union, at least at that time, influenced the political leaders of the developing coun-

\(^1\)After succeeding Lenin and consolidating his power, in 1929, Stalin started to pursue in earnest the prioritized development of heavy industries through a series of five-year plans (Gregory and Harrison, 2005; Gregory and Stuart, 2001). The share of heavy industry in Soviet industrial output rose rapidly (Moravcik, 1965; Allen, 2003) and the Soviet Union quickly became a global military power before World War II.
tries, whether they were socialist countries or not, and encouraged them to adopt similar strategies to achieve rapid industrialization\(^2\).

With the desire for rapid industrialization, the governments in the developing countries took various measures to facilitate the growth of the industrial sector. And it was commonly believed that a high proportion of economic surplus should be directed to investment in industrial sectors, just as had been done in the period of "socialistic primary accumulation" in the Soviet Union.

One important finding is that, though the developing countries were quite different from each other in political regime, economic size, geographic features and cultural traditions, very similar institutions were established in those countries after World War II, to implement the catching-up strategy of rapid industrialization. In this paper, we will investigate the rationale behind those institutional arrangements, and use a simple model to show that such institution formation is endogenous to a government’s adoption of catching-up strategies.

Institution here is defined as a set of non-technologically determined constraints that govern and shape economic agents’ interactions, in part helping them to form expectations of other agents’ actions and the results of their own actions (Lin and Nugent, 1995; Greif, 1998). It is clear from this definition that institutions consist of both formal entities, like laws, state-mandated rules, and commercial contracts, and informal ones, like social norms, customs and ideologies.

Among the similar institutional arrangements across the developing countries implementing catching-up strategies, the first and most important was price distortion, such as the "price scissor" adopted mainly in socialist countries and the "import substitution" policy adopted in many other developing countries. Price scissor was a kind of price discrimination against non-industrial sectors, such as agriculture. For example, in China and the Soviet Union, agricultural products were procured by the government at

\(^2\)In Krueger’s (1995) article in the *Handbook of Development Economics*, she summarized five dominant thoughts in development economics, as below: 1) the desire and drive for "modernization"; 2) the interpretation of "industrialization" as the route to modernization; 3) the belief in "import substitution" as a necessary policy to provide protection for new "infant" industries; 4) the distrust of the private sector and the market and the belief that government should take the leading role in development; and 5) a distrust of the international economy and pessimism that exports from developing countries could grow.
prices lower than market equilibrium prices, and sold at those low prices to residents in urban areas where the industries were located. "Import substitution" implies that governments export domestic primary goods, while prohibiting the import of industrial products, so as to protect the development of domestic industrial sectors. It was adopted by many Latin American countries from the 1930s to the late 1980s. Under import substitution, the prices of domestic industrial products were normally higher than corresponding international prices. In essence, import substitution is also, at least partially, a kind of price distortion.

The second kind of institutional arrangement is directive allocation systems for output and production factors. In many developing countries, the procurement and sale of commodities at regulated prices were typically operated and controlled by the governments, and the allocation of production factors was commonly through directive allocation, rather than market mechanisms. As a result the public sector played an important role in non-socialist developing countries implementing catching-up strategies. For example, in India, 62.1% of total productive capital and 26.7% of total labor in industry was in the public sector by 1978-1979 (Krueger, 1995), and in Brazil, in 1984, 81 of the 200 largest enterprises were state-owned, which accounted for 74.2% of the total capital and 56.3% of the total net income (Lin, et. al, 1999, Chapter 2).

Third and finally, the governments adopting a catching-up strategy heavily intervened in the production decision of enterprises, through such measures as government ownership, direct government operation, investment licensing, etc. It is understandable that, in socialist countries, nearly all the economic activities were directly or indirectly under the control of government. However, in non-socialist developing countries, there was also extensive government intervention in the production activities of enterprises. For example, in post war India, government permission was required for new investment by any of the 20 largest industrial houses, and it was permissible only when that investment could not be carried out by other industrial houses (Krueger, 1995). In fact, in those developing countries, even the private sectors were subject to substantial control by the state political and

\[ \text{It was documented that, in India, the effective rate of protection (ERP) were well above 100% for 39 industries (of a 76 industry classification) in 1968-69 (Bhagwati and Srinivasan,1975), and in Turkey, the ERPs of the infant industries were normally over 200% even after twenty years of their establishment (Krueger and Tuncer, 1982).} \]
economic apparatus, among which the most important is output control.

In this paper, we will argue that the institutional complex of price distortion, output control and directive allocation is indeed endogenous to the government choice of catching-up strategies in the developing countries (Lin, et. al, 1994, 1996, 2007). The endowment structures of the developing countries commonly feature scarcity in capital and abundance in labor supply, while the development of industries, especially heavy industries, requires huge amounts of capital investment. Given the endowment structures of the developing countries, prioritizing the development of industrial sectors is obviously against the comparative advantages of the economies, and enterprises in heavy industry sectors are not viable in free competitive markets (Lin, 2003). Therefore, that kind of catching-up strategy can not be implemented through *laissez-faire* market mechanisms. To implement that strategy, the government needs to exclude market functioning and establish certain institutional arrangements that can facilitate the development of the industrial sectors.

First of all, a government pursuing catching-up strategies needs to reduce the costs of industrial production. Capital is expensive in the developing countries and market interest rates are high, which implies that capital-intensive production should be quite costly and non-profitable in the developing countries. To reduce production costs, the government needs to fix the interest rates at low levels, and reduce the prices of consumption goods, such as agriculture products, because cheaper consumption goods will reduce the labor costs of the industrial sectors. Through this kind of price distortion, the government can reduce the costs of industrial production. As was mentioned before, "price scissor" in both socialist and non-socialist economies and the policy of "import substitution" adopted by many developing country governments can be grouped into the institutional arrangement of price distortion.

Second, the government needs to set up a directive allocation system. Price distortion implies that the prices of many products are artificially set

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4The institution in this paper refers to institution arrangement when is a set of behavioral rules that govern agents' behaviors in specified domains, not institutional structure which is totality of institutional arrangements (Lin and Nugent, 1995).

at below market-clearing levels, and as a result, there exists excess demand for those products. To sufficiently allocate the under-priced products and factors to target industries, the government needs to establish a directive allocation system, and secure its position as monopolistic purchaser of those goods. Otherwise, either non-industrial sectors will compete with industrial sectors for those under-priced goods, or they can be sold in other markets at higher prices. This also explains the agriculture collectivization movements in China and the Soviet Union, and why public ownership and direct government control were also prevalent in many non-socialist developing countries.

Finally, the government also needs to intervene in the enterprises’ production activities. When workers in enterprises can freely decide how much to produce or how high to set their wages, they will take action for their own benefits. For example, if wages are fixed by the government at low levels, the workers will have low incentives to exert effort. In this case, an output requirement set by the government may guarantee the lowest level of effort input by the workers.

There has been a large literature studying the endogenous formation of economic or political institutions and the government’s role in economic development. The early literature commonly assumed that the government acts as a benevolent social guardian of the economy, and its role is to establish certain institutions and take relevant measures just to compensate for the defects of the market mechanism. But later observation of inefficient government intervention and government corruption has led to new consideration of the role of the government and the formation of economic and political institutions. A new strand of literature began to explore this problem from the perspective of political economy, by clearly introducing the incentives of politicians in the government and interest groups (Grossman and Helpman, 1994; Shleifer and Vishny, 1994, 1998; Acemoglu, 2006, 2007). According to this theory, the government intervenes in the economy and establishes certain institutional arrangements to maximize the payoffs of special interest groups, or merely for the purpose of rent-seeking. Shleifer and Vishny (1994, 1998) emphasize the grabbing-hand nature of governments and suggest that government controls over enterprises come about when politicians can use public resources to buy off enterprise managers, for their cooperation in redistributing economic or political rents. Acemoglu (2006) shows in a simple model that political elites choose policies to in-
crease their income and to directly or indirectly transfer resources from the rest of society to themselves, which leads to distortions and thus inefficient allocation in the economy.

However, in the developing countries that were pursuing the catching-up strategies, the establishment of various institutional arrangements by the governments was obviously not for the purpose of acquiring rents by the political elites, and neither were there particular beneficiary groups of those institutional arrangements, at least in the short run (Lin, et al., 2006). In such economic environments where economic activities were highly regulated and the role of the market was relatively limited, the political elites did not have much room for rent-seeking either. On the contrary, during the transition period of many socialist countries in the 1990s, with the collapse of traditional plan systems, there were much more serious corruption problems in the governments than before. In fact, the political leaders of those developing countries adopted the catching-up strategies under the good will that national prosperity can soon be achieved through rapid industrialization.

In this paper, in a simple two-sector model, we will prove that, in the developing countries, the formation of that special institutional complex of price distortion, directive allocation and output control is endogenous to the governments’ adoption of a catching-up strategy. Furthermore, that institution complex is both necessary and sufficient for implementing the best allocation for catching-up strategy. In our two-sector model, effort is introduced as an inherent control variable of economic agents, which is not observable and cannot be directly controlled by the government. But the behaviors of the agents can be indirectly regulated by the external economic environments, which is designed purposely by the government. So the problem of the government is how to design optimal institutional arrangements to implement the best allocation for catching-up strategy, given the interest conflict and incentive problem of the economic agents.

We prove in our model that the best allocation is implementable by a combination of price distortion, directive allocation system and output control, and none of these three components is dispensable for that implementation. The intuition behind this result is that, without output control, the agents will reduce their effort inputs and less outputs in both sectors will be produced, and as a result, less surplus can be mobilized for capital accumulation. Without price distortion, the equilibrium price of consumption
goods will be higher, and therefore raise the labor cost of production. And without directive allocation of capital, the agents can adjust their capital demands to effort inputs, which will finally result in more consumption and less capital accumulation.

The remaining parts of this paper are organized as follows: Section 2 presents a basic two-sector model, and characterizes the optimal allocation and shadow prices under no distortion. Section 3 introduces the catching-up strategy into the model. By solving a central planner’s problem, we characterize the best allocation for implementing that catching-up strategy. Section 4 turns to study the implementation of that best allocation by the government, and proves that that best allocation is implementable by an institutional complex of price distortion, output control and directive allocation system. Section 5 is a counter-factual analysis, where we alternatively remove one part of that institutional complex, while keeping the other two parts unchanged, and explore its impact on final allocations. Section 6 is a short conclusion.

2 The Model and Optimal Allocation

2.1 Model Setup

There are two sectors in the economy: capital goods sector and consumption goods sector. The production of either goods requires both capital and labor inputs. Total capital $K = K_1 + K_2$, where $K_1$ is capital input in the capital goods sector and $K_2$ in the consumption goods sector. Similarly, we have total labor $L = L_1 + L_2$. It is assumed that labor is not intersectoral transferrable in our economy\(^6\). For simplicity and without loss of generality, we normalize $L = 1$ and let $L_1 = \lambda$ and $L_2 = 1 - \lambda$, $\lambda \in (0, 1)$. The endowment structure of the economy is measured by per capita capital, $k = K/L$, which is normally low in the developing countries.

The production of capital goods (goods 1) is more capital intensive than that of consumption goods 2 (goods 2), and their production functions are

$$Y_j = F_j(K_j, e_j L_j) \quad j = 1, 2$$

\(^6\)For example, in China, peasants in rural areas are not allowed to work in manufactories in urban areas, under the regulation of hukou system. Also, the production of different goods requires different skills, and it is difficult for workers in one sector to acquire the required skills for production in another sector in short time.
where $e_j$ is the per capita effort input of a worker in sector $j$, and $e_j L_j$ may be interpreted as effective labor input. We assume workers in the same sector are homogenous. The production function $F_j(\cdot, \cdot)$ is homogenous of degree one in $K_j$ and $e_j L_j$. Per capita output and per capita capital in sector $j$ are respectively $y_j = \frac{Y_j}{L_j}$ and $k_j = \frac{K_j}{L_j}$. Total capital balance constraint implies that $\lambda k_1 + (1 - \lambda) k_2 = k$. For per capital output, we have

$$y_j = F_j(\frac{K_j}{L_j}, e_j) = f_j(k_j, e_j) \quad j = 1, 2$$

It is obvious that $f_j(k_j, e_j)$ is homogenous of degree one in $k_j$ and $e_j$. Consumption goods 2 can be used only for consumption and are not preservable, and capital goods 1 can be used for either consumption or investment.

Though economic agents in both sectors are assumed to have the same preference, they are heterogenous in essence since they are in different sectors, having different skills, and labor is not intersectoral transferrable. The utility function of agents in sector $j$ is

$$U_j(x_{1j}, x_{2j}, e_j) = u(x_{1j}, x_{2j}) - C(e_j)$$

where $x_{ij}$ is the amount of goods $i$ consumed by an agent in sector $j$, and $e_j$ is his effort input. $u(\cdot, \cdot)$ is strictly concave in $(x_{1j}, x_{2j})$ and satisfies the Inada conditions, and the disutility of effort, $C(e_j)$, is strictly convex satisfying the conditions that $C'(0) = 0$ and $C'(\infty) = \infty$.

### 2.2 Optimal Allocation

For later comparison with distorted allocation under catching-up strategy, we first study the optimal allocation in the benchmark case of no distortion. The social welfare function is standard and defined as

$$W(x_{ij}, e_j) = \lambda [u(x_{11}, x_{21}) - C(e_1)] + (1 - \lambda) [u(x_{12}, x_{22}) - C(e_2)]$$
and the optimal allocation is to maximize social welfare under resource and technology constraints, that is

\[
\begin{align*}
\mathcal{P}^O & \quad \max_{\mathcal{P}^O} \ W(x_{11}, x_{12}, x_{21}, x_{22}; e_1, e_2) \\
\text{s.t.} & \quad \lambda x_{11} + (1 - \lambda) x_{12} = \lambda f_1(k_1, e_1) \\
& \quad \lambda x_{21} + (1 - \lambda) x_{22} = (1 - \lambda) f_2(k_2, e_2) \\
& \quad \lambda k_1 + (1 - \lambda) k_2 = k
\end{align*}
\]

The Lagrangian of problem \(\mathcal{P}^O\) is

\[
\mathcal{L} = \lambda [u(x_{11}, x_{21}) - C(e_1)] + (1 - \lambda) [u(x_{12}, x_{22}) - C(e_2)] \\
- \mu_1 [\lambda x_{11} + (1 - \lambda) x_{12} - \lambda f_1(k_1, e_1)] \\
- \mu_2 [\lambda x_{21} + (1 - \lambda) x_{22} - (1 - \lambda) f_2(k_2, e_2)] \\
- \mu_3 [\lambda k_1 + (1 - \lambda) k_2 - k]
\]

The solution to this problem is given by the following optimal conditions

\[
\begin{align*}
u'_{11} &= \mu_1 \\
u'_{12} &= \mu_1 \\
u'_{21} &= \mu_2 \\
u'_{22} &= \mu_2 \\
C'(e_1) &= \mu_1 f'_1(e_1) \\
C'(e_2) &= \mu_2 f'_2(e_2) \\
\mu_1 f'_{1,k_1} &= \mu_2 f'_{2,k_2} = \mu_3
\end{align*}
\]

and the technology and resource constraints

\[
\begin{align*}
\lambda x_{11} + (1 - \lambda) x_{12} &= \lambda f_1(k_1, e_1) \\
\lambda x_{21} + (1 - \lambda) x_{22} &= (1 - \lambda) f_2(k_2, e_2) \\
\lambda k_1 + (1 - \lambda) k_2 &= k
\end{align*}
\]

These conditions give the optimal allocation \(\{x_{ij}^*, e_j^*, k_j^*\}\) and the shadow prices of goods 1,2 and capital in optimal allocation \(\{\mu_1, \mu_2, \mu_3\}\). According to the second theorem of welfare economics, under our assumptions of technology, preference and labor heterogeneity, the optimal allocation is supportable by a Walrasian equilibrium. If we take capital goods 1 as
numeraire, then the support prices \( \{ p^*, r^*, w_j^* \} \) are such that

\[
p^* = \frac{\mu_2}{\mu_1} \quad r^* = \frac{\mu_3}{\mu_1} = p^* f_{2,k_2}^* \quad w_1^* = f_{1,e_1}^* \quad w_2^* = p^* f_{2,e_2}^*
\]

Therefore, the optimal allocation can be implemented by a market mechanism \(^7\). We next characterize the optimal allocation and shadow prices in a specific example.

**Example 1** \( C(e) = \frac{1}{2} e^2 \quad u(x_{1j}, x_{2j}) = \gamma \ln x_{1j} + (1-\gamma) \ln x_{2j} \quad f_1(k_1, e_1) = A_1 k_1^{\alpha} e_1^{1-\alpha} \quad f_2(k_2, e_2) = A_2 k_2^\beta e_2^{1-\beta} \) with \( 1 > \alpha > \beta > 0 \).

Solving the optimal conditions and technology and resource constraints, we get that

\[
\begin{align*}
x_{11} &= \frac{\gamma}{\mu_1} \\
x_{12} &= \frac{1 - \gamma}{\mu_2} \\
x_{21} &= \frac{1^+}{\mu_1} \\
x_{22} &= \frac{1^-}{\mu_2} \\
e_1^{1+\alpha} &= \mu_1 (1 - \alpha) A_1 k_1^\alpha \\
e_2^{1+\beta} &= \mu_2 (1 - \beta) A_2 k_2^\beta \\
\mu_3 &= \mu_1 A_1 \left( \frac{k_1}{e_1} \right)^{\alpha-1} \\
\mu_3 &= \mu_2 A_2 \left( \frac{k_2}{e_2} \right)^{\beta-1} \\
x_{11} &= \lambda A_1 k_1^{\alpha} e_1^{-\alpha} \\
x_{21} &= (1 - \lambda) A_2 k_2^\beta e_2^{1-\beta} \\
k &= \lambda k_1 + (1 - \lambda) k_2
\end{align*}
\]

There are nine equations and nine unknowns, and it is easy to get the close-form solutions. We summarize the main results of optimal allocation in the example below:

**Proposition 2** In the two-sector model, the optimal allocations \( \{ x_{1j}^*, e_j^*, k_j^* \} \)

\(^7\)For the implementation problem, an implicit assumption is that it is a private market economy, and producers face perfect competition.
The above example are given by

\[
x_{11}^* = x_{12}^* = \frac{A_1\alpha^\alpha(1 - \alpha)}{[\alpha\gamma + \beta(1 - \gamma)]^\alpha} k^\alpha
\]

\[
x_{21}^* = x_{22}^* = \frac{A_2\beta^\beta(1 - \beta)}{[\alpha\gamma + \beta(1 - \gamma)]^\beta} k^\beta
\]

\[
e_1^* = \left[ \frac{\gamma(1 - \alpha)}{\lambda} \right]^{\frac{1}{2}}
\]

\[
e_2^* = \left[ \frac{(1 - \gamma)(1 - \beta)}{1 - \lambda} \right]^{\frac{1}{2}}
\]

\[
k_1^* = \frac{\alpha\gamma}{\alpha\gamma + \beta(1 - \gamma)} k
\]

\[
k_2^* = \frac{\beta(1 - \gamma)}{\alpha\gamma + \beta(1 - \gamma)} k
\]

There is a linear relationship between \( k_j^* \) in optimal allocation, that is,

\[
k_1^* = \frac{\alpha\gamma(1 - \lambda)}{\beta\lambda(1 - \gamma)} k_2^*
\]

and the per capita capital of the capital goods sector is higher than that of the consumption goods sector if and only if \( \frac{\alpha\gamma(1 - \lambda)}{\beta\lambda(1 - \gamma)} > 1 \), which is equivalent to

\[
\lambda < \frac{\alpha\gamma}{\beta(1 - \gamma) + \alpha\gamma}
\]

In the developing countries, most of the labor lies in the agriculture sector and the ratio of labor in the industry sector, \( \lambda \), is low. We may assume the above condition is satisfied in the developing countries. The optimal outputs of goods 1 and 2 are respectively

\[
q_1^* = \frac{x_{11}^*}{\lambda} = \frac{A_1\alpha^\alpha(1 - \alpha)}{\lambda^{\frac{1-\alpha}{2}} [\alpha\gamma + \beta(1 - \gamma)]^\alpha} k^\alpha
\]

\[
q_2^* = \frac{x_{21}^*}{1 - \lambda} = \frac{A_2\beta^\beta(1 - \beta)}{(1 - \lambda)^{\frac{1-\beta}{2}} [\alpha\gamma + \beta(1 - \gamma)]^\beta} k^\beta
\]
and the solutions to the Lagrangian multipliers are

\[ \mu_1^* = \frac{\gamma}{x_{11}} = \frac{\gamma^{1-\alpha}}{A_1 \alpha (1-\alpha) \lambda^{\frac{1-\alpha}{2}}} k^{-\alpha} \]

\[ \mu_2^* = \frac{1-\gamma}{x_{21}} = \frac{(1-\gamma)^{1-\beta}}{A_2 \beta \gamma^{\frac{1-\beta}{2}}} k^{-\beta} \]

\[ \mu_3^* = \frac{\alpha \gamma + \beta (1-\gamma)}{k} \]

\( \mu_j^* \)'s are respectively the nominal shadow prices of capital goods, consumption goods and capital. Given our assumption of preference and production functions, we know from the second theorem of welfare economics that the above optimal allocations are supportable by a Walrasian equilibrium, for some initial allocation of capital endowments \( (\tilde{k}_1, \tilde{k}_2) \). When applying the second theorem of welfare economics, we have implicitly assumed that the government is able to enact lump-sum redistribution of the initial capital endowments in these two sectors. If we take the capital goods as numeraire, we get the following results about the supporting prices.

**Corollary 3** In a Walrasian equilibrium that supports the optimal allocations, the equilibrium relative price of good 2, the interest rate and wage levels, \( \{p^*, r^*, w_1^*, w_2^*\} \), are given by

\[ p^* = \frac{\mu_2}{\mu_1} = \frac{A_1 \alpha}{A_2 \beta} \left( \frac{(1-\alpha) \lambda}{\gamma} \right)^{\frac{1-\alpha}{2}} \frac{k^{\alpha-\beta}}{[\alpha \gamma + \beta (1-\gamma)]^{\alpha-\beta}} \quad (17) \]

\[ r^* = \frac{\mu_3}{\mu_1} = A_1 \alpha \left( \frac{1-\alpha}{\gamma} \right)^{\frac{1-\alpha}{2}} \frac{k^{\alpha-1}}{[\alpha \gamma + \beta (1-\gamma)]^{\alpha-1}} \quad (18) \]

\[ w_1^* = \frac{A_1 \alpha^2 \gamma}{\lambda^2} \left( \frac{1-\alpha}{\gamma} \right)^{\frac{1-\alpha}{2}} k^\alpha \quad (19) \]

\[ w_2^* = \left( \frac{(1-\gamma)(1-\beta)}{1-\lambda} \right)^{\frac{1}{2}} A_1 \alpha \left( \frac{(1-\alpha) \lambda}{\gamma} \right)^{\frac{1-\alpha}{2}} k^\alpha \quad (20) \]
3 Catching-Up Strategy and a Central Planner Problem

In this section, we introduce the catching-up strategy into the model, and by solving a central planner’s problem, we characterize the best allocation for implementing that strategy. In the next section, we will study how to implement that best allocation, given the interest conflicts between the agents and the government. Similar to Sah and Stiglitz (1984, 1987), we study this problem in a static model, and thus abstract the dynamic effects of industrial accumulation. However, it will be clear that a static model is sufficient to capture the main trade-offs that determine the endogeneity of institutional formation under a catching-up strategy.

As was stated above, when implementing a catching-up strategy, the government in a developing country tries to mobilize enough economic surplus to invest in industry. In our model, since consumption goods can only be used for consumption, the total surplus that is available for investment in industrial sectors is the net output of capital goods, $I$, that is,

$$I = \lambda f_1(k_1, e_1) - \lambda x_{11} - (1 - \lambda)x_{12}$$  \hspace{1cm} (21)

The total amount of industrial accumulation is equal to the difference between total output and consumption of capital goods. We assume the government’s objective function is a weighted average of social welfare and industrial accumulation $I$ in the following form

$$U^G(x_{ij}, e_j) = W(x_{ij}, e_j) + \phi I$$  \hspace{1cm} (22)

where $\phi$ is a weight proxy of $I$ in government’s objective function. When $\phi = 0$, $U^G$ returns to the benchmark case of the social welfare function. And the central planner problem for the government is thus

$$\max_{P^C} \lambda [u(x_{11}, x_{21}) - C(e_1)] + (1 - \lambda) [u(x_{12}, x_{22}) - C(e_2)] + \phi I$$

s.t. \hspace{0.5cm} \begin{align*}
\lambda x_{11} + (1 - \lambda)x_{12} + I &= \lambda f_1(k_1, e_1) \\
\lambda x_{12} + (1 - \lambda)x_{22} &= (1 - \lambda)f_2(k_2, e_2) \\
\lambda k_1 + (1 - \lambda)k_2 &= k
\end{align*}$$
The Lagrangian of problem $\mathcal{P}^C$ is

$$
\mathcal{L} = \lambda [u(x_{11}, x_{21}) - C(e_1)] + (1 - \lambda) [u(x_{12}, x_{22}) - C(e_2)] + \phi I \\
- \bar{\mu}_1 [\lambda x_{11} + (1 - \lambda)x_{12} + I - \lambda f_1(k_1, e_1)] \\
- \bar{\mu}_2[\lambda x_{12} + (1 - \lambda)x_{22} - (1 - \lambda)f_2(k_2, e_2)] \\
- \bar{\mu}_3[\lambda k_1 + (1 - \lambda)k_2 - k]
$$

Solving the optimal conditions, we have

$$
\begin{align*}
    u'_{11} &= u'_{12} = \phi = \bar{\mu}_1 \\
    u'_{21} &= u'_{22} = \bar{\mu}_2 \\
    C'(e_1) &= \bar{\mu}_1 f'_{1,e_1} \\
    C'(e_2) &= \bar{\mu}_2 f'_{2,e_2} \\
    \bar{\mu}_1 f'_{1,k_1} &= \bar{\mu}_2 f'_{2,k_2} = \bar{\mu}_3
\end{align*}
$$

and the technology and technology constraints (21), (5) and (6). These conditions characterize the best allocations $\{\hat{x}_{ij}, \hat{e}_j, \hat{k}_j\}$ and shadow prices $\{\bar{\mu}_1, \bar{\mu}_2, \bar{\mu}_3\}$ that maximize the objective functions of the government pursuing a catching-up strategy. Using the same example as before, it is easy to get the following optimal conditions:

$$
\begin{align*}
x_{11} &= x_{12} = \frac{\gamma}{\mu_1} = \frac{\gamma}{\phi} \\
x_{21} &= x_{22} = \frac{1 - \gamma}{\mu_2} \\
e_1^{1+\alpha} &= \phi (1 - \alpha) A_1 k_1^\alpha \\
e_2^{1+\beta} &= \bar{\mu}_2 (1 - \beta) A_2 k_2^\beta \\
\bar{\mu}_3 &= \phi \alpha A_1 (\frac{k_1}{e_1})^{\alpha -1} \\
\bar{\mu}_3 &= \bar{\mu}_2 \beta A_2 (\frac{k_2}{e_2})^{\beta -1} \\
x_{11} &= \lambda A_1 k_1^\alpha e_1^{1-\alpha} - I \\
x_{21} &= (1 - \lambda) A_2 k_2^\beta e_2^{1-\beta} \\
k &= \lambda k_1 + (1 - \lambda)k_2
\end{align*}
$$

It is difficult to get the close-form solutions to problem $\mathcal{P}^C$, but the results
summarized as below are enough for comparison with the results in the optimal allocations.

**Proposition 4** In a static two-sector economy, the best allocation \(\{\hat{x}_{ij}, \hat{\epsilon}_j, \hat{k}_j\}\) for implementing the catching-up strategy are

\[
\hat{x}_{11} = \hat{x}_{12} = \frac{\gamma}{\phi} = \frac{A_1\alpha^\alpha(1 - \alpha)^{\frac{1-\alpha}{2}}\lambda^{\frac{1-\alpha}{2}}\left[\frac{\gamma + I\phi}{\alpha(\gamma + I\phi) + \beta(1 - \gamma)^{\alpha}}\right]}{\frac{1}{2}}k^\alpha
\]

(23)

\[
\hat{x}_{21} = \hat{x}_{22} = \frac{A_2\beta^\beta(1 - \beta)^{\frac{1-\beta}{2}}(1 - \lambda)^{\frac{1-\beta}{2}}(1 - \gamma)^{\frac{1+\beta}{2}}k^\beta}{\frac{1}{2}}k
\]

(24)

\[
\hat{\epsilon}_1 = \left[\frac{(1 - \alpha)(\gamma + I\phi)}{\lambda}\right]^{\frac{1}{2}}
\]

(25)

\[
\hat{\epsilon}_2 = \left[\frac{(1 - \gamma)(1 - \beta)}{1 - \lambda}\right]^{\frac{1}{2}}
\]

(26)

\[
\hat{k}_1 = \frac{\alpha(\gamma + I\phi)k}{\alpha(\gamma + I\phi) + \beta(1 - \gamma)^{\lambda}}
\]

(27)

\[
\hat{k}_2 = \frac{\beta(1 - \gamma)k}{\alpha(\gamma + I\phi) + \beta(1 - \gamma)^{1 - \lambda}}
\]

(28)

The best allocations for the catching-up strategy are given in quasi-reduced forms, and for given \(\hat{I}\), there is also a linear relationship between capital allocation in both sectors

\[
\hat{k}_1 = \frac{\alpha(\gamma + I\phi)(1 - \lambda)}{\beta\lambda(1 - \gamma)}\hat{k}_2
\]

It is clear that whenever \(\hat{I} > 0\), more capital needs to be allocated to the capital goods sector in the *best* allocation. A quite intuitive result is that the more bias toward the development of the capital goods sector, the more output and accumulation of capital goods in the best allocation, which is summarized in the following corollary.

**Corollary 5** In the central planner problem \(P^C\), the nominal GDP of the capital goods sector increases with \(\phi\), that is,

\[Y'_1(\phi) > 0\]
where \( Y_1(\phi) = p_1 \cdot \lambda q_1 = \phi \left( I + \frac{1}{\phi} \right) \), and

\[
I'(\phi) > 0
\]

for \( I \geq 0 \).

**Proof.** From the condition of

\[
I = \frac{A_1 \alpha^\alpha (1-\alpha)^{1-\alpha} \lambda^{1-\alpha} (\gamma + I\phi)^{1-\alpha} k^{\alpha} - \gamma}{\alpha \left( \gamma + I\phi + \beta(1-\gamma) \right)^\alpha}
\]

we have

\[
A\phi Y_1(\phi)^{\frac{\alpha-1}{2}} = [\alpha Y_1(\phi) + \beta(1-\gamma)]^\alpha
\]

where \( A = A_1 \alpha^\alpha (1-\alpha)^{1-\alpha} \lambda^{1-\alpha} \). It is easy to get that

\[
Y_1'(\phi) \left[ \frac{\alpha^2}{\alpha \left( \gamma + I\phi + \beta(1-\gamma) \right)^\alpha} + \frac{1-\alpha}{2(\gamma + I\phi)} \right] = \frac{1}{\phi} > 0
\]

so \( Y_1'(\phi) > 0 \). Also from this condition, we have

\[
I'(\phi) = \frac{1}{H} \{ (\gamma + \phi I) [\alpha(1-\alpha)\phi I + 2\alpha \gamma + \beta(1-\gamma)(1+\alpha)] + (1-\alpha)(1-\gamma)\gamma \beta \}
\]

where \( H = \phi^2 \{ \alpha(\alpha+1)(\gamma + \phi I)^2 + 2\beta(1-\gamma)(\gamma + \phi I) \} > 0 \). A sufficient condition for \( I'(\phi) \) is \( I \geq 0 \). \( \blacksquare \)

The next result gives a necessary and sufficient condition under which the best accumulation of capital goods is zero.

**Corollary 6** \( \hat{I} = 0 \) iff

\[
\phi = \frac{\gamma^{\frac{1-\alpha}{2}} [\alpha \gamma + \beta(1-\gamma)]^\alpha}{A_1 \alpha^\alpha (1-\alpha)^{1-\alpha} \lambda^{1-\alpha} k^{-\alpha}} = p_1^* \tag{29}
\]

When \( \phi > p_1^* \), \( \hat{I} \) is strictly positive and increasing in \( \phi \).

Interestingly, the right hand side of the equation is just the nominal price of capital goods in the case of optimal allocations when there is no distortion, that is \( \mu_1^* \). So the result says that when the weight of industrial accumulation is equal to the nominal price of capital goods in the optimal allocation case, the government does not need to sacrifice capital goods consumption for industrial accumulation. And when \( \phi > p_1^* \), the industrial
accumulation $\hat{I}$ is not only strictly positive but increasing in $\phi$. We also have that the outputs of both sectors in the best allocation are

\[
\hat{q}_1 = \frac{A_1 \alpha^\alpha (1 - \alpha)^{\frac{1 - \alpha}{2}} \left[ \gamma (\gamma + I \phi) \right]^{\frac{\alpha-1}{2}}}{\lambda^{\frac{1 - \alpha}{2}}} k^\alpha + \frac{I}{\lambda} \quad (30)
\]
\[
\hat{q}_2 = \frac{A_2 \beta^\beta (1 - \beta)^{\frac{1 - \beta}{2}} \left[ \gamma (\gamma + I \phi) + \beta (1 - \gamma) \right]^{\frac{\beta+1}{2}}}{(1 - \lambda)^{\frac{1 - \beta}{2}}} k^\beta \quad (31)
\]

**Corollary 7** When $\phi > p_1^*$, we have

\[
\hat{k}_1 > k_1^* \quad \hat{k}_2 < k_2^*
\]
\[
\hat{e}_1 > e_1^* \quad \hat{e}_2 = e_2^*
\]
\[
\hat{q}_1 > q_1^* \quad \hat{q}_2 < q_2^*
\]

The result is quite intuitive. In a central planner problem, when the government (the central planner) is pursuing a catching-up strategy, more capital will be allocated to the capital goods sector, workers in the capital goods sector will work harder and capital goods output will increase. The solutions to the Lagrangian multipliers are

\[
\bar{\mu}_1 = \frac{\phi}{A_1 \alpha^\alpha (1 - \alpha)^{\frac{1 - \alpha}{2}} \lambda^{\frac{1 - \alpha}{2}}} \left[ \gamma (\gamma + I \phi) \right]^{\frac{\alpha-1}{2}} k^{-\alpha}
\]
\[
\bar{\mu}_2 = \frac{1 - \gamma}{x_{21}} = \frac{1 - \gamma}{A_2 \beta^\beta (1 - \beta)^{\frac{1 - \beta}{2}} (1 - \lambda)^{\frac{1 - \beta}{2}}} \left[ \gamma (\gamma + I \phi) + \beta (1 - \gamma) \right]^{\frac{\beta+1}{2}} k^{-\beta}
\]
\[
\bar{\mu}_3 = \bar{\mu}_2 k^\beta = \frac{\alpha (\gamma + I \phi) + \beta (1 - \gamma)}{k}
\]

**4 Implementation of a Catching-up Strategy**

In this section we study the government problem of how to implement the best allocation in the real world when there are interest conflicts between the government and economic agents. We assume capital is owned by agents with an initial capital endowment of $k_j$ for an agent in sector $j$. An agent in sector $j$ has two income sources: one is capital income $r_k$, the other is labor income $w_j e_j$\(^8\). An agent decides his demand and effort input to maximize his utility under various policy constraints imposed by the government,

\(^8\)Here we normalize the hours of work for each agent as 1.
with an objective function (2) different from that of the government (22). For example, unlike the government, the economic agents may not care much about the development of heavy industries or accumulation of capital goods. To implement the best allocation for the catching-up strategy, the government needs to establish relevant institutions and impose various constraints on agents’ economic activities. Since that catching-up strategy cannot be implemented through market mechanisms, restricting the functioning of market mechanism is a natural objective of the government in its design and establishment of relevant institutional arrangements. Those institutional arrangements can be divided into three categories, and each corresponds to special policy variables in our model:

1. **Distorted price system**, commonly known as "price scissor" in the literature, but the policy of "import substitution" is also a kind of price distortion arrangement in essence. By distorted price system, we mean in our model that product and factor prices are control variables of the government, including the relative price of consumption goods $p$, interest rate $r$ and wage levels $w_j$.

2. **Directive allocation system**, which implies that the allocation of products and factors are through government directives rather than through market mechanism. In our model we study only the case in which the allocation of capital is under the control of government, while both consumption and capital goods are still allocated through the market mechanism. This setting weakens the role of government in allocating resources and products, especially for the governments in socialist countries. However, this weakened setting is enough to demonstrate the function of a directive allocation system under catching-up strategies. In our model, it implies that $k_j$ is a control variable of the government.

3. **Deprivation of autonomy of producers**, which means that production activities in the economy are under close control of the government, such as quantity control, investment licensing, export licensing, etc. In our model, deprivation of production autonomy implies that producers in both sectors face the output obligations, $q_j$, imposed by the government.
Therefore, the control variables of the government are \( \{p, r, w_j; q_j; k_j\} \), and the choice variables of an agent in sector \( j \) are his demand for products \( x_{ij} \) and his effort input \( e_j \). We investigate the interaction between the government and agents in a two-stage game of complete information. First the government determines the economic environment by specifying the choice of policy variables \( \{p, w_j, r; q_j; k_j\} \). Second, facing the economic constraints, the agents in both sectors make their own utility-maximization decisions, and then the final allocation is realized as an equilibrium outcome. As the first step, we solve the best responses of the economic agents, given the exogenous parameters \( \{p, w_j, r; q_j; k_j\} \). We normalize the reservation utility for agents in both sectors as 0.

### 4.1 Best Responses of Agents

Facing the exogenously given economic institutions or constraints \( \{p, w_j, r; q_j; k_j\} \), the problem of an agent in sector \( j \) is as follows:

\[
\begin{align*}
\mathcal{P}_j &= \max_{\{x_{ij}, e_j\}} u(x_{1j}, x_{2j}) - C(e_j) \\
& \text{s.t.} \quad x_{1j} + px_{2j} \leq w_j e_j + r k_j \\
& \quad f_j(k_j, e_j) \geq q_j
\end{align*}
\]

The price restrictions \( \{p, r, w_j\} \) affect the income level and consumption decision of the agent, the output requirement \( q_j \) jointly with the capital allocation \( k_j \) determine the minimal effort input of the agent. The Lagrangian of the problem is

\[
\mathcal{L}_j = u(x_{1j}, x_{2j}) - C(e_j) - \eta_1 (x_{1j} + px_{2j} - w_j e_j - r k_j) - \eta_2 [q_j - f_j(k_j, e_j)]
\]

The budget constraint is binding at optimality, and the optimal conditions are

\[
\begin{align*}
\frac{u'_{1j}}{u'_{2j}} &= \eta_1 \\
\frac{C'(e_j)}{w_j e_j + r k_j} &= \eta_2 f_j(k_j, e_j) + \eta_1 w_j \\
\eta_2 \geq 0 \quad \text{and} \quad \eta_2 [q_j - f_j(k_j, e_j)] &= 0
\end{align*}
\]
Our interest is in the case that the quantity constraint is also binding at optimality, otherwise the output obligation imposed by the government does not affect the optimal effort inputs of the agents. It corresponds to the case that \( \eta_2 > 0 \) and \( q_j - f_j(k_j, e_j) = 0 \). And the Lagrangian multipliers satisfy

\[
\eta_1 = u_{1j}' \quad \eta_2 = \frac{C'(e_j) - w_ju_{1j}'}{f_{j,e_j}'} > 0
\]

The condition of \( \eta_2 \) implies that the marginal cost of effort is higher than the marginal revenue/adjusted-utility of effort input when the output constraint is binding. The value function of problem \( \mathcal{P}^A \) is the indirect utility function of agents in sector \( j \), denoted as \( v_j(p, r, w_j, q_j, k_j) \), and the optimal effort input is \( e_j(q_j, k_j) \).

**Lemma 8** Given the economic environment \( \{p, r, w_j, q_j, k_j\} \), the indirect utility function of agents in sector \( j \) satisfies

\[
\begin{aligned}
\frac{\partial v_j}{\partial p} &= -u_{1j}' x_{2j} < 0 \\
\frac{\partial v_j}{\partial r} &= u_{1j}' k_j > 0 \\
\frac{\partial v_j}{\partial w_j} &= u_{1j}' e_j > 0 \\
\frac{\partial v_j}{\partial q_j} &= \frac{C'(e_j) - w_ju_{1j}'}{f_{j,e_j}'} < 0 \\
\frac{\partial v_j}{\partial k_j} &= \left[ C'(e_j) - w_ju_{1j}' \right] f_{j,k_j} > 0 \\
\frac{\partial v_j}{\partial k_j} &= u_{1j}' r > 0
\end{aligned}
\]  

(32)

and for optimal effort supply \( e_j(q_j, k_j) \), we have

\[
\begin{aligned}
\frac{\partial e_j}{\partial q_j} &= \frac{1}{f_{j,e_j}} > 0 \\
\frac{\partial e_j}{\partial k_j} &= -\frac{f_{j,k_j}}{f_{j,e_j}} < 0
\end{aligned}
\]  

(33)

The results of comparative statics analysis of \( v_j \) and \( e_j \) will be extensively used in solving the optimal policy problem of the government in the next section. The Appendix also provides the results of comparative statics analysis on the demand functions \( x_{1j} \) and \( x_{2j} \). Using the same example as before, we derive the close-form demand functions, effort inputs and the indirect utility function of the agent as below.

**Lemma 9** Given the economic environment \( \{p, r, w_j, q_j, k_j\} \), in the example
above, the effort inputs of the agents are

\[ e_2(q_2, k_2) = \left( \frac{q_2}{A_2k_2^\alpha} \right)^{\frac{1}{1-n}} \]  
(34)

\[ e_1(q_1, k_1) = \left( \frac{q_1}{A_1k_1^\alpha} \right)^{\frac{1}{1-n}} \]  
(35)

and the demand functions of agents in sector \( j \) are

\[ x_{1j}(p, r, w_j, q_j, k_j) = \gamma(w_je_j + r\tilde{k}_j) \]  
(36)

\[ x_{2j}(p, r, w_j, q_j, k_j) = \frac{(1-\gamma)(w_je_j + r\tilde{k}_j)}{p} \]  
(37)

The indirect utility function of agents in sector \( j \) are

\[ v_j(p, r, w_j, q_j, k_j) = \ln B + \ln \left[ \gamma^\gamma(1-\gamma)^{1-\gamma} \right] + \ln(w_je_j + r\tilde{k}_j) - (1-\gamma) \ln p - \frac{1}{2}e_j^2 \]  
(38)

### 4.2 Optimal Institutions for Implementing CAD Strategy

Having solved the best responses of the agents in both sectors, we now study the optimal policy choices of the government that adopts the catching-up strategy of rapid industrialization. As before, the objective function of the government is a weighted average of social welfare and industrial accumulation, and the problem of the government is

\[
\begin{align*}
\text{max} & \quad W(p, r, w_j, q_j, k_j) + \phi I \\
\text{s.t.} & \quad v_1(p, w_1, r, q_1, k_1) \geq 0 \\
& \quad v_2(p, w_2, r, q_2, k_2) \geq 0 \\
& \quad \lambda x_{11} + (1-\lambda)x_{12} + I = \lambda q_1 \\
& \quad \lambda x_{21} + (1-\lambda)x_{22} = (1-\lambda)q_2 \\
& \quad \lambda k_1 + (1-\lambda)k_2 = k
\end{align*}
\]

By substituting the budget constraints of both sectors and the balance condition for consumption goods sector into the technology constraint of sector 1, we get a new expression for industrial accumulation, that is,

\[ I(p, r, w_j, q_j, k_j) = \lambda[q_1 - w_1e_1(q_1, k_1)] + (1-\lambda)[pq_2 - w_2e_2(q_2, k_2)] - rk \]  
(39)
It is obvious that the total industrial accumulation is equal to the sum of total surplus of both production sectors. From (39), we get the following results of comparative statics that, given other things equal, the total industrial accumulation $I$ satisfies

$$\frac{\partial I}{\partial q_j} > 0 \quad \frac{\partial I}{\partial p} > 0 \quad \frac{\partial I}{\partial r} < 0 \quad \frac{\partial I}{\partial w_j} < 0 \quad \frac{\partial I}{\partial k} > 0$$

And the government’s problem is reformulated as:

$$\mathcal{P}(M) \quad \max_{\mathcal{M}} \quad W(p, r, w_j; q_j; k_j) + \phi I(p, r, w_j, q_j, k_j)$$

$$s.t. \quad v_1(p, w_1, r, q_1, k_1) \geq 0$$

$$v_2(p, w_2, r, q_2, k_2) \geq 0$$

$$\lambda k_1 + (1 - \lambda)k_2 = k$$

where $\phi > 0$ measures the weight of industrial accumulation in the government’s objective function. Here we have normalized the reservation utility of the agents in both sectors to be 0. The restriction of $v_j \geq 0$ also implies that the government has limited implementation power, and extreme exploitation of a certain sector is not possible. $M = \{p, r, w_j; q_j, k_j\}$ represents the set of available policy instruments of the government, in which $k_1$ and $k_2$ are interdependent through $\lambda k_1 + (1 - \lambda)k_2 = k$. Let $v_G(M)$ be the value function of problem $\mathcal{P}(M)$, and $M' \subset M$ be a subset of $M$, that is, $M'$ represents a smaller set of policy instruments. It is obvious that

**Proposition 10** For $M' \subset M$, we have $v_G(M') \leq v_G(M)$.

**Proof.** The proof is self-evident. For $\forall M' \subset M$, suppose control variables $z_j \in M'$, and $y_j \in M \setminus M'$. Suppose the optimal solution of $\mathcal{P}(M')$ are $\{z_j^*, y_j^*\}$ where $z_j^*$’s are chosen by the government and $y_j^*$’s are equilibrium levels of $y_j$. In problem $\mathcal{P}(M)$, the government can simply choose $z_j = z_j^*$ and $y_j = y_j^*$, then achieve the same value of $v_G(M')$. ■

However, if we relax this constraint or introduce discrimination against certain group of agents, i.e. the peasants, this framework can be used to study inequality under the catching-up strategy, or famine in former socialist economies.
The Lagrangian of problem $\mathcal{P}(M)$ is

$$L = \left[ \lambda v_1(p, w_1, r, q_1, k_1; \bar{k}_1) + (1 - \lambda)v_2(p, w_2, r, q_2, k_2; \bar{k}_2) \right]$$

$$+ \phi \left[ \lambda (q_1 - w_1 e_1(q_1, k_1)) + (1 - \lambda)(pq_2 - w_2 e_2(q_2, k_2)) - rk \right]$$

$$+ \mu_1 v_1(p, w, q_1, k_1; \bar{k}_1) + \mu_2 v_2(p, r, q_2, k_2; \bar{k}_2) - \mu_3 [\lambda k_1 + (1 - \lambda)k_2 - k]$$

$\mu_1 \geq 0$ and $\mu_2 \geq 0$ are Kuhn-Tucker multipliers. The first-order conditions are

$$p : \quad (\lambda + \mu_1) \frac{\partial}{\partial p} + [(1 - \lambda) + \mu_2] \frac{\partial}{\partial p} + \phi(1 - \lambda)q_2 = 0$$

$$r : \quad (\lambda + \mu_1) \frac{\partial}{\partial r} + [(1 - \lambda) + \mu_2] \frac{\partial}{\partial r} - \phi k = 0$$

$$w_1 : \quad (\lambda + \mu_1) \frac{\partial}{\partial w_1} - \phi \lambda e_1 = 0$$

$$w_2 : \quad [(1 - \lambda) + \mu_2] \frac{\partial}{\partial w_2} - \phi(1 - \lambda)e_2 = 0$$

$$q_1 : \quad (\lambda + \mu_1) \frac{\partial}{\partial q_1} + \phi \lambda (1 - w_1 e_1 q_1) = 0$$

$$q_2 : \quad [(1 - \lambda) + \mu_2] \frac{\partial}{\partial q_2} + \phi(1 - \lambda)(p - w_2 e_2 q_2) = 0$$

$$k_1 : \quad (\lambda + \mu_1) \frac{\partial}{\partial k_1} - \phi \lambda w_1 e_1 \k_1 - \mu_3 \lambda = 0$$

$$k_2 : \quad [(1 - \lambda) + \mu_2] \frac{\partial}{\partial k_2} - \phi(1 - \lambda)w_2 e_2 k_2 - \mu_3 (1 - \lambda) = 0$$

$$\mu_3 : \quad \lambda k_1 + (1 - \lambda)k_2 - k = 0$$

$$\mu_1 : \quad \mu_1 \geq 0 \quad \mu_1 v_1(p, w, q_1, k_1) = 0$$

$$\mu_2 : \quad \mu_2 \geq 0 \quad \mu_2 v_2(p, r, q_2, k_2) = 0$$

We restrict our attention to the cases that both $v_1$ and $v_2$ are strictly positive at optimality, which implies that $\mu_1 = \mu_2 = 0$. Substituting the results of the agent’s indirect utility function, (32), and effort supply, (33), we get the optimal conditions as below:
\[ p : \quad -\lambda u'_{11}x_{21} - (1 - \lambda)u'_{12}x_{22} + \phi (1 - \lambda)q_2 = 0 \]
\[ r : \quad \lambda u'_{11}k_1 + (1 - \lambda)u'_{12}k_2 - \phi k = 0 \]
\[ w_1 : \quad u'_{11}e_1 - \phi e_1 = 0 \]
\[ w_2 : \quad u'_{12}e_2 - \phi e_2 = 0 \]
\[ q_1 : \quad -\frac{C'(e_1) - w_1u'_{11}}{f'_{1,e_1}} + \phi \left( 1 - \frac{w_1}{f'_{1,e_1}} \right) = 0 \]
\[ q_2 : \quad -\frac{C'(e_2) - w_2u'_{12}}{f'_{2,e_2}} + \phi \left( p - \frac{w_2}{f'_{2,e_2}} \right) = 0 \]
\[ k_1 : \quad [C'(e_1) - w_1u'_{11}] \frac{f'_{1,k_1}}{f'_{1,e_1}} + \phi w_1 \frac{f'_{1,k_1}}{f'_{1,e_1}} - \mu_3 = 0 \]
\[ k_2 : \quad [C'(e_2) - w_2u'_{12}] \frac{f'_{2,k_2}}{f'_{2,e_2}} + \phi w_2 \frac{f'_{2,k_2}}{f'_{2,e_2}} - \mu_3 = 0 \]
\[ \mu_3 : \quad \lambda k_1 + (1 - \lambda)k_2 - k = 0 \]

From the conditions of \( w_j \), we have at optimality
\[ u'_{11} = u'_{12} = \phi \]

Given this, the condition of \( p \) is implied by the balance conditions of consumption goods, and the condition of \( r \) is implied by the identity of \( \lambda k_1 + (1 - \lambda)k_2 = k \). Substituting into the conditions of \( q_j \), we get
\[ C'(e_1) = \phi f'_{1,e_1} \]
\[ C'(e_2) = p\phi f'_{2,e_2} \]

Substituting into the conditions of \( k_j \), we get
\[ f'_{1,k_1} = p f'_{2,k_2} \]

Together with the balance conditions
\[ k = \lambda k_1 + (1 - \lambda)k_2 \]
\[ (1 - \lambda)q_2 = \lambda x_{21} + (1 - \lambda)x_{22} \]

There are seven equations with eight unknowns, so in this case, there is some leeway for the government to implement certain allocation \( \{x_{ij}, k_j, e\} \). This comes from the result that the optimal condition of interest rate, \( r \), becomes redundant given the conditions of \( w_j \)’s are satisfied. The reason for this result is that in the agent problem, effort input is uniquely determined.
by output regulation and capital allocation, and given this, \( r \), and \( w_j \)’s jointly determine the agent’s income level. Though total income level and relative price, \( p \), are uniquely determined by the optimal conditions, the composition of interest and labor income is not uniquely determined when both \( r \) and \( w_j \)’s are control variables of the government. We introduce a natural behavioral restriction on government’s selection of the interest rate, that is, the government sets the interest rate at the level of marginal capital output of both sectors 

\[
  r = f'_{1,k_1} = pf'_{2,k_2}
\]

**Proposition 11** For a government that implements CAD strategy using three groups of policy instruments, the optimal choice of the policy variables \( \{p, r, w_j; q_j, k_j\}, j = 1, 2 \), satisfies

\[
  u'_{11} = u'_{12} = \phi \\
  C'(e_1) = \phi f'_{1,e_1} \\
  C'(e_2) = p\phi f'_{2,e_2} \\
  r = f'_{1,k_1} = pf'_{2,k_2} \\
  k = \lambda k_1 + (1 - \lambda)k_2 \\
  (1 - \lambda)q_2 = \lambda x_{21} + (1 - \lambda)x_{22}
\]

where \( x_{ij} \) and \( e_j \) are given by the solutions to problem \( P^A \).

Using the same example as above, we will prove that the best allocation for catching-up strategy is implementable, and will characterize the optimal institutional arrangements for implementing that strategy. In that example, we know the demand functions are

\[
  x_{1j}(p, w_j) = \gamma(w_j e_j + r\bar{k}_j) \\
  x_{2j}(p, w_j) = \frac{(1 - \gamma)(w_j e_j + r\bar{k}_j)}{p}
\]

and the effort inputs are

\[
  e_1(k_1, q_1) = \left( \frac{q_1}{A_1 k_1^\alpha} \right)^{\frac{1}{1-\alpha}} \\
  e_2(k_2, q_2) = \left( \frac{q_2}{A_2 k_2^\beta} \right)^{\frac{1}{1-\beta}}
\]

25
We consider the case where both \( v_1 \) and \( v_2 \) are strictly positive at optimality, which implies that \( \mu_1 = \mu_2 = 0 \). The optimal conditions are thus:

\[
\begin{align*}
\frac{\gamma}{x_{11}} &= \frac{\gamma}{x_{12}} = \phi \\
e_1 &= \phi(1 - \alpha)A_1k_1^\alpha e_1^{-\alpha} \\
e_2 &= \phi p(1 - \beta)A_2k_2^\beta e_2^{-\beta} \\
r &= \alpha A_1k_1^{\alpha-1}e_1^{1-\alpha} = p\beta A_2k_2^{\beta-1}e_2^{1-\beta} \\
k &= \lambda k_1 + (1 - \lambda)k_2 \\
(1 - \lambda)q_2 &= \lambda x_{21} + (1 - \lambda)x_{22}
\end{align*}
\]

Substituting the expressions of \( x_{ij} \) and \( e_j \), we have

\[
\begin{align*}
\phi(w_1e_1 + r\hat{k}_1) &= \phi(w_2e_2 + r\hat{k}_2) = 1 \\
\phi(1 - \alpha)A_1k_1^\alpha &= e_1^{1+\alpha} \\
\phi p(1 - \beta)A_2k_2^\beta &= e_2^{1+\beta} \\
\alpha A_1k_1^{\alpha-1}e_1^{1-\alpha} &= p\beta A_2k_2^{\beta-1}e_2^{1-\beta} = r \\
\lambda k_1 + (1 - \lambda)k_2 &= k \\
\left(1 - \gamma\right)(w_1e_1 + r\hat{k}) &= (1 - \lambda)q_2 \\
p &= \left(1 - \gamma\right)(w_1e_1 + r\hat{k})
\end{align*}
\]

To implement the best allocation \( \{\hat{x}_{ij}, \hat{k}_j, \hat{e}_j\} \), the government can simply selects \( k_j = \hat{k}_j \), which are

\[
\begin{align*}
\hat{k}_1 &= \frac{\alpha(\gamma + \hat{I}\phi)}{\alpha(\gamma + \hat{I}\phi) + \beta(1 - \gamma)} k \\
\hat{k}_2 &= \frac{\beta(1 - \gamma)}{\alpha(\gamma + \hat{I}\phi) + \beta(1 - \gamma)} \frac{k}{1 - \lambda}
\end{align*}
\]

where \( \hat{I} \) is given by

\[
\frac{\gamma}{\phi} = \frac{A_1\alpha(1 - \alpha)^{1-\alpha} \lambda^{1-\frac{\alpha}{2}} \left[\gamma(\gamma + \hat{I}\phi)\frac{\alpha-1}{2}\right]}{\left[\alpha(\gamma + \hat{I}\phi) + \beta(1 - \gamma)\right]^{\alpha}} k^{\alpha}
\] (40)
and selects \( \hat{q}_j \) such that
\[
\hat{q}_1 = A_1 \hat{k}_1 \hat{e}_1^{1-\alpha} \\
\hat{q}_2 = A_2 \hat{k}_2 \hat{e}_2^{1-\beta}
\]
These \( \{ \hat{q}_j, \hat{k}_j \} \) satisfy the optimal conditions (2-5). The interest rate is equal to the marginal output of capital, that is
\[
\hat{r} = \alpha A_1 \hat{k}_1^{\alpha-1} \hat{e}_1^{1-\alpha} \\
\hat{r} = A_1 \alpha \left( \frac{1 - \alpha}{\gamma + \hat{I} \phi} \right)^{\frac{1-\alpha}{\gamma}} \left[ \alpha (\gamma + \hat{I} \phi) + \beta (1 - \gamma) \right]^{1-\alpha} k^{\alpha-1} \tag{41}
\]
Since \( \phi(w_j e_j + r \hat{k}_j) = 1 \), we have
\[
\hat{p} = \frac{(1 - \gamma)}{\phi(1 - \lambda) \hat{q}_2} \\
\hat{p} = A_1 \alpha \left( \frac{(1-\alpha)\lambda}{\gamma + \hat{I} \phi} \right)^{\frac{1-\alpha}{\gamma}} \cdot \frac{k^{\alpha-\beta}}{A_2 \beta \left( \frac{(1-\beta)(1-\lambda)}{1-\gamma} \right)^{\frac{1-\beta}{\gamma}}} \left[ \alpha (\gamma + \hat{I} \phi) + \beta (1 - \gamma) \right]^{\alpha-\beta} \tag{42}
\]
and the government-set wage levels
\[
\hat{w}_j = \frac{1}{\hat{e}_j} \left( \frac{1}{\phi} - \hat{r} \hat{k}_j \right) \quad j = 1, 2
\]
It can be checked that \( \{ \hat{p}, \hat{r}, \hat{w}_j; \hat{q}_j, \hat{k}_j \} \) satisfy the above optimal conditions, and
\[
\hat{x}_{ij} \left( \hat{p}, \hat{r}, \hat{w}_j; \hat{q}_j, \hat{k}_j \right) = \hat{x}_{ij}
\]
We summarize the main result of implementation as follows:

**Proposition 12** The best allocation for the catching-up strategy, \( \{ \hat{x}_{ij}, \hat{e}_j, \hat{k}_j \} \), is implementable by an institutional complex of price distortion, output control and directive allocation system. Specifically, \( \{ \hat{p}, \hat{r}, \hat{w}_j; \hat{q}_j, \hat{k}_j \} \) implements that best allocation.

This is the main result of this paper. It demonstrates that the best allocation for a catching-up strategy, which is not implementable in a free market mechanism, can be implemented by an institutional complex of price distortion, output control and directive allocation. In the best allocation of
\{\hat{x}_{ij}, \hat{e}_j, \hat{k}_j\}, \hat{k}_j \] itself is a policy variable, and the government can directly select capital allocation \(k_j = \hat{k}_j\). Though effort input is not observable, it is ex post verifiable given the capital allocation \(k_j\) and production function \(f_j(\cdot, \cdot)\). So through further output control \(\hat{q}_j\), the government can guarantee the minimal effort input of \(\hat{e}_j\). Finally, by setting the relative prices, \(\{\hat{p}, \hat{r}, \hat{w}_j\}\), the government is able to determine the income and consumption levels of the agents. For the comparison between \(\hat{p}\) and the optimal price \(p^*\), we have

**Corollary 13** When \(\phi > p^*_1\), in implementing the catching-up strategy, the government will distort downward the consumption good price, that is, \(\hat{p} < p^*\).

**Proof.** Divide (17) by (42), we get

\[
\frac{p^*}{\hat{p}} = \left[ \frac{\gamma + \hat{I}\phi}{\gamma} \right]^{1+\frac{1-\alpha}{\gamma}} \left[ \frac{\alpha(\gamma + \hat{I}\phi) + \beta(1-\gamma)}{\alpha\gamma + \beta(1-\gamma)} \right]^{\alpha-\beta} > 1
\]

since \(\alpha > \beta\) and \(\hat{I} > 0\) given \(\phi > p^*_1\). ■

The above corollary tells us that when a government wants to successfully implement the catching-up strategy, it needs to reduce the relative price of consumption goods. From this result, we understand the rationale behind the selection of "price scissor" or "import substitution" policies by many developing countries when they were pursuing a development strategy of rapid industrialization. To compare the distorted interest \(\hat{r}\) to optimal interest \(r^*\), we have

\[
\frac{\hat{r}}{r^*} = \left[ \frac{(1-\alpha)\lambda}{\gamma + \hat{I}\phi} \right]^{1-\alpha} \left[ \frac{\alpha(\gamma + \hat{I}\phi) + \beta(1-\gamma)}{\alpha\gamma + \beta(1-\gamma)} \right]^{1-\alpha}
\]

\[
= \left( \frac{\gamma}{\gamma + \hat{I}\phi} \right)^{1-\alpha} \left[ \frac{\alpha(\gamma + \hat{I}\phi) + \beta(1-\gamma)}{\alpha\gamma + \beta(1-\gamma)} \right]^{1-\alpha}
\]

and therefore for \(\hat{r} < r^*\), we need

\[
\left[ \frac{\alpha(\gamma + \hat{I}\phi) + \beta(1-\gamma)}{\alpha\gamma + \beta(1-\gamma)} \right]^2 < \frac{\gamma + \hat{I}\phi}{\gamma} \iff \left[ 1 + \frac{\alpha\hat{I}\phi}{\alpha\gamma + \beta(1-\gamma)} \right]^2 < 1 + \frac{\hat{I}\phi}{\gamma}
\]
Let $z = \bar{I} \phi$ and $b = \alpha \gamma + \beta (1 - \gamma)$, the above inequality can be reformulated as
\[ \left[ 1 + \frac{\alpha z}{b} \right]^2 < 1 + \frac{z}{\gamma} \iff z \left[ \alpha^2 \frac{z}{b} - \left( \frac{b}{\gamma} - 2 \alpha \right) \right] < 0 \]
which is equivalent to
\[ \bar{I} < \frac{1}{\alpha^2 \gamma \phi} \left[ \beta^2 (1 - \gamma)^2 - \alpha^2 \gamma^2 \right] \]

**Corollary 14** When $\phi > p_{1}^*$, (1) If $\beta (1 - \gamma) > \alpha \gamma$, then
\[
\begin{cases} 
\dot{r} < r^* & \text{if } \bar{I} < \frac{1}{\alpha^2 \gamma \phi} \left[ \beta^2 (1 - \gamma)^2 - \alpha^2 \gamma^2 \right] \\
\dot{r} \geq r^* & \text{otherwise}
\end{cases}
\]
(2) If $\beta (1 - \gamma) \leq \alpha \gamma$, then $\dot{r} \geq r^*$ always.

The condition $\beta (1 - \gamma) > \alpha \gamma$ is equivalent to $\gamma < \beta / (\alpha + \beta)$, which means that agents spend only a small portion of their incomes on capital goods consumption. It is quite realistic for and generally satisfied by agents in the developing countries, who spend most of their money on food and nondurable goods consumption. Under this realistic condition, when the accumulation level is relatively low, implementing a catching-up strategy requires reducing interest rates. However, when the accumulation level is too high, it will finally drive up interest rates. Extreme accumulation implies that much capital is allocated to the capital goods sector (see (27)), and given that the effort input of the consumption goods sector unchanged, it will eventually increase the marginal output of the capital of sector and the equilibrium interest rate. Or intuitively, given the objective function of the government, there exists a substitution relationship between consumption and accumulation and extreme accumulation will squeeze out consumption and drive up the marginal utility of consumption.

## 5 Necessity of the Three Institutional Components

In this part we explore in detail the effects of alternatively removing one of the three components of the endogenous institutional arrangements on final allocations and prices. Alternative removal implies that each time only one
of the three components of the institutional complex is removed\textsuperscript{10}. Each removal changes one facet of the economic institutional arrangement, and endows the agents with more freedom than before in their economic decisions. For a policy variable that is removed from the pool of the government’s policy instruments, its equilibrium value will be determined through a market mechanism. We will compare the equilibrium values of those variables with their counterparts in best allocation.

5.1 Necessity of Output Control

What is the impact of removing output controls, when the other two components of the institutional complex are kept unchanged? In this case, the agents face only the institutional constraints of price distortion and directive allocation of capital, but they can freely decide how much to produce. Under the less restrictive institutional constraints of \( \{p, r, w_j; k_j\} \), the problem for the agent in sector \( j \) is now

\[
\max_{(x_{1j}, x_{2j})} \quad u(x_{1j}, x_{2j}) - C(e_j) \\
\text{s.t.} \quad x_{1j} + px_{2j} \leq w_j e_j + r k_j
\]

We easily get the optimal conditions that

\[
\frac{\partial u_{2j}(x_{1j}, x_{2j})}{\partial (x_{1j}, x_{2j})} = p u_{1j}'(x_{1j}, x_{2j}) \\
C'(e_j) = w_j u_{1j}'(x_{1j}, x_{2j}) \\
x_{1j} + px_{2j} = w_j e_j + r k_j
\]

We denote the solution to this problem as \( \{x_{1j}^Q, x_{2j}^Q, e_j^Q\} \), where the superscript \( Q \) means that there is no quantity control over the producers. By intuition, without output control, the agents will invest less effort and the result will be less output of the corresponding product, which is proved in the following proposition.

**Proposition 15** Given the institutional constraints of price distortion and directive capital allocation, \( \{\hat{p}, \hat{r}, \hat{w}_j; \hat{k}_j\} \), the effort input of an agent facing

\textsuperscript{10}Removing all the components implies a situation of no government intervention at all (free market economy).
no output control is lower than that in the best allocation, that is,

$$e_j^Q \leq \hat{e}_j$$  \hspace{1cm} (43)

**Proof.** Suppose not, then $\hat{e}_j < e_j^Q$ and the output of product $f_j(k_j, e_j^Q) > \hat{q}_j$, which implies that the quantity constraint is not binding in $(P^A)$. So $\hat{e}_j$ cannot be the best effort input level in the best allocation for implementing the catching-up strategy. ■

**Corollary 16** $x_{ij}^Q \leq \hat{x}_{ij}$ and $\hat{q}_j \geq f_j(\hat{k}_j, e_j^Q)$.

**Proof.** It is self-evident, given $\hat{e}_j \geq e_j^Q$ and the formula of demand functions. ■

These results demonstrate that there is an "over" supply of effort under the full constraints of the institutional complex, in the sense that the marginal cost of effort input is higher than its marginal revenue (remember the optimal conditions of problem $(P^A)$). So when a government implements the catching-up strategy, it will compel the agents to exert more effort than they would do in free market situations. So to guarantee a certain level of effort inputs, the government needs to impose the output requirements on the production of the agents, and output control is indispensable for implementing the best allocation for the catching-up strategy.

### 5.2 Necessity of Price Distortion

When the agents face only quantity control and directive allocation of capital and there is no price regulation, agent $j$’s problem is to choose optimal consumption and effort inputs under the constraints of $\{q_j; k_j\}$. In this case, the demand for products in both sectors is determined only by consumer demand, not by government demand. The problem for agent $j$ is thus

$$\max_{\{x_{ij}, e_j\}} \quad u(x_{1j}, x_{2j}) - C(e_j)$$

s.t. \quad $x_{1j} + px_{2j} \leq w_j e_j + r \tilde{k}_j$

$$q_j = f_j(k_j, e_j)$$

The key difference between this problem and problem $P^A$ lies in the fact that when price is liberalized, equilibrium price levels are determined only by market equilibrium. It is obvious that the distorted prices in the best
allocation, \{\hat{p}, \hat{r}, \hat{w}_j\}, are not sustainable as market equilibrium prices, since there is at least excess supply of capital goods. So removing the component of price distortion implies that the final allocation is (strictly) inferior to that under the full constraints of the institutional complex.

In the problem above, \(e_j\) is uniquely determined by \(q_j = f_j(k_j, e_j)\). In our specific example, the demand function is given by

\[
x_{1j}(p, w_j) = \gamma(w_j e_j + r k) \quad x_{2j}(p, w_j) = \frac{(1 - \gamma)(w_j e_j + r k)}{p}
\]

The market equilibrium requires that

\[
\lambda\gamma(w_1 e_1 + r k) + (1 - \lambda)\gamma(w_2 e_2 + r k) = \lambda f_1(k_1, e_1)
\]

\[
\lambda \frac{(1 - \gamma)(w_1 e_1 + r k)}{p} + (1 - \lambda)\frac{(1 - \gamma)(w_2 e_2 + r k)}{p} = (1 - \lambda)f_2(k_2, e_2)
\]

The constraint of output controls also requires that

\[
f_1(k_1, e_1) = q_1
\]

\[
f_2(k_2, e_2) = q_2
\]

and we get

\[
p^P = \frac{(1 - \gamma) \lambda}{(1 - \lambda) \gamma} \frac{q_1}{q_2}
\]

and again, the superscript \(P\) means that there is no price regulation by the government.

**Proposition 17** Given the same optimal policy controls \{\hat{k}_j; \hat{q}_j\}, the relative price of consumption goods under additional price control is lower than that without price control, that is

\[
\hat{p} < p^* < p^P
\]

**Proof.** Remember that

\[
\hat{p} = \frac{\lambda(1 - \gamma)}{(1 - \lambda)(\gamma + I\phi)} \cdot \frac{\hat{q}_1}{\hat{q}_2}
\]

\[
p^* = \frac{\lambda(1 - \gamma)}{(1 - \lambda) \gamma} \cdot \frac{q_1^*}{q_2^*}
\]

and \(\hat{q}_1 > q_1^*\) and \(\hat{q}_2 < q_2^*\). \(\blacksquare\)
It is intuitive that when removing the price regulation, the equilibrium price of consumption goods becomes higher, but it is even higher than $p^*$, the price that supports the optimal allocation in a Walrasian equilibrium. This is because the production of both sectors is still distorted under output control and directive allocation of capital.

5.2.1 Necessity of a Directive Allocation System

The government’s objective of establishing the directive allocation system is to allocate those under-priced products and production factors to target sectors, i.e. the capital goods sector. Specifically, in our model, removing the directive allocation system implies that the allocation of capital $k_j$ is through market allocation schemes, rather than through government directive allocations. In this subsection, we investigate the effects of removing the directive allocation system.

First, it is natural that, in a free market, if the interest rate is lower than the market-clearing level (i.e. $\hat{r} < r^*$), there will exist excess demand for it, and if $\hat{r} > r^*$, there then exists excess supply of capital. So it is self-evident that the government needs to establish a directive allocation system to direct the allocation of products and production factors at controlled prices. In this subsection, we study a somewhat different environment in which the agents still face output controls imposed by the government. And we will see why the best allocation is not achievable when there are only price and output controls $\{p, r, w_j; q_j\}$. In this case, the agents can decide their capital demand or supply so that their marginal rate of transformation between capital and effort is equal to the corresponding level of the marginal rate of substitution. Specifically, we are interested in the question: When capital is tradable at the regulated price $\hat{r}$, do the agents have incentives to sell or purchase additional capital at the initial allocation of $\{\hat{x}_{ij}, \hat{e}_j, \hat{k}_j\}$? We define the additional demand/supply of capital by an agent in sector $j$ as

$$\Delta k_j = k_j - \hat{k}_j$$

(46)

where $k_j$ is the final per capita capital used for production in sector $j$. And
the agent's problem is

$$\max_{\{x_{1j}, x_{2j}, e_j, k_j\}} u(x_{1j}, x_{2j}) - C(e_j)$$

subject to

$$x_{1j} + px_{2j} + r\Delta k_j \leq w_j e_j + r\bar{k}_j$$

$$f_j(k_j, e_j) \geq q_j$$

Given the fixed output requirement $q_j$, an agent can reduce capital demand by increasing effort input, or vice versa. And the optimal combination of capital and effort inputs is determined by relative prices and output requirements $\{p, r, w_j; q_j\}$. The Lagrangian of the problem is

$$\mathcal{L} = u(x_{1j}, x_{2j}) - C(e_j) - \eta_1 [x_{1j} + px_{2j} + r\Delta k_j - w_j e_j - r\bar{k}_j] - \eta_2 [q_j - f_j(k_j, e_j)]$$

and the optimal conditions are

$$x_{1j} : \quad u'_{1j} = \eta_1$$

$$x_{2j} : \quad u'_{2j} = \eta_1 p$$

$$e_j : \quad C'(e_j) = \eta_1 w_j + \eta_2 f'_{j,e_j}$$

$$k_j : \quad \eta_1 r = \eta_2 f'_{j,k_j}$$

$$\eta_1 : \quad x_{1j} + px_{2j} + r\Delta k_j = w_j e_j + r\bar{k}_j$$

$$\eta_2 : \quad q_j = f_j(k_j, e_j)$$

By rearrangement, the optimal conditions are now

$$pu'_{1j} = u'_{2j}$$

$$\frac{f'_{j,e_j}}{f'_{j,k_j}} = \frac{C'(e_j) - u'_{1j}w_j}{u'_{1j}r}$$

$$x_{1j} + px_{2j} = w_j e_j - r\Delta k_j + r\bar{k}_j$$

$$q_j = f_j(k_j, e_j)$$

Remember in trinity institutional arrangements, the conditions for marginal rate of transformation is

$$\frac{f'_{j,e_j}}{f'_{j,k_j}} = \frac{C'(e_j)}{u'_{1j}r}$$ \quad (47)$$

while in this case, it is

$$\frac{f'_{j,e_j}}{f'_{j,k_j}} = \frac{C'(e_j)}{u'_{1j}r} - \frac{w_j}{r}$$ \quad (48)$$

Using our specific example, the condition for the marginal rate of trans-
formation under only price and output controls, \( \{\hat{p}, \hat{r}, \hat{w}_j; \hat{q}_j\} \), is

\[
\frac{(1 - \alpha_j)k_j}{\alpha_j \hat{e}_j} = \frac{\hat{w}_j \hat{e}_j + \hat{r}k}{\hat{r}} e_j - e_j \Delta k_j - \frac{\hat{w}_j}{\hat{r}}
\]

and the corresponding MRT conditions under trinity institutional arrangements is

\[
\frac{(1 - \alpha_j)\hat{k}_j}{\alpha_j \hat{e}_j} = \frac{\hat{w}_j \hat{e}_j + \hat{r}k}{\hat{r}} \hat{e}_j
\]

where \( \alpha_1 = \alpha \) and \( \alpha_2 = \beta \). We want to compare the value of \( k_j \) to \( \hat{k}_j \).

As a starting point for analysis, we first assume \( k_j = \hat{k}_j \), which implies that \( \Delta k_1 = 0 \) and \( e_j = \hat{e}_j \) since the output requirements remain unchanged. For sector \( j \) agent, it is obvious that

\[
\frac{(1 - \alpha_j)\hat{k}_j}{\alpha_j \hat{e}_j} > \frac{\hat{w}_j \hat{e}_j + \hat{r}k}{\hat{r}} \hat{e}_j - \frac{\hat{w}_j}{\hat{r}}
\]

To get the optimal condition to hold, we need to reduce the capital input below \( \hat{k}_j \) and increase the effort input over \( \hat{e}_j \), and we get the following result.

**Proposition 18** When there are only regulations on prices and outputs, \( \{\hat{p}, \hat{r}, \hat{w}_j; \hat{q}_j\} \), and capital is tradable, the demand for capital in either sector is strictly lower than that in three-in-one environments, that is, \( k_j < \hat{k}_j \), for \( j = 1, 2 \).

Given the opportunity of trading capital and if that trade can really happen, we will have that the incomes of agents in both sectors will be higher than before, that is, \( w_j e_j - r \Delta k_j + rk > \hat{w}_j \hat{e}_j + \hat{r}k \), which will result in greater demand for both products than under trinity institutional arrangements.

**Corollary 19** When there are only regulations on prices and outputs, \( \{\hat{p}, \hat{r}, \hat{w}_j; \hat{q}_j\} \), and capital trade is feasible, there will be more consumption of capital goods and less industrial accumulation than in the best allocation, specifically,

\[
x^K_{1j} > \hat{x}_{1j} \quad I^K < \hat{I}
\]

It is worth mentioning that the above results are based on the condition that capital trade is feasible. Only under this condition could the agents
purchase or sell capital at the regulated price \( \hat{r} \). And with that opportunity, the agents are able to adjust their effort inputs and consumption decisions, and therefore affect the final allocation in this economy.

6 Conclusion

After the Second World War, many newly independent countries adopted the catching-up strategy of rapid industrialization. However, for those developing countries, prioritizing the development of industrial sectors, especially the heavy industry sectors, was against the comparative advantages of their economies. And the catching-up strategy was not implementable by laissez-faire market mechanisms. To implement that strategy, the governments need to establish a series of institutions that exclude the functioning of markets and are able to mobilize enough economic surplus for industrial accumulation. Under the catching-up strategy, similar institutional arrangements were established in those developing countries, no matter whether they were socialist countries.

In this paper, we develop a simple two-sector model to explain the rationale behind that institution formation in those developing countries. We prove that the best allocation for a catching-up strategy can be implemented by an institutional complex that consists of price distortion, output control and a directive allocation system. We further explore the counterfactual effects of alternatively removing one part of the institutional complex on final economic allocations. We show the following results: (1) removing price regulation will result in higher price of consumption goods, (2) removing output control will induce less effort inputs by the agents, and (3) removing the directive capital allocation system will result in more consumption and less industrial accumulation. In short, none of these three components of this institutional complex is dispensable for a successful implementation of the catching-up strategy. In a simple two-sector model, this paper provides a framework for understanding the endogenous institution formation under the catching-up strategy in developing countries after World War II, and shows that how a government can establish certain institutional arrangements to implement allocations that are not implementable by laissez-faire market mechanisms.

The formation and evolution of economic institutions in developing coun-
tries under various development strategies is an important topic for research. This paper represents a first step in this line of research, and more work certainly needs to be done in the future. First, this paper studies just the formation of special economic institutions under the catching-up strategy, and does not explore the dynamic effects of that institutional arrangements on the long run economic growth, which requires the development of dynamic models. Second, the framework developed in this paper can be extended to investigate various issues in economic development, such as famines in the developing countries and socialist countries, the relationship between inequality and development strategy, and so on. And finally, for the testable hypotheses that arise from the theoretical model, we also need to verify those hypotheses through rigorous empirical research.

A Comparative Statics Analysis of $x_{ij}$’s

For demand functions, we have

$$\frac{\partial x_{1j}}{\partial p} < 0 \quad \frac{\partial x_{1j}}{\partial w_j} > 0 \quad \frac{\partial x_{1j}}{\partial r} > 0 \quad \frac{\partial x_{1j}}{\partial k_j} < 0 \quad \frac{\partial x_{1j}}{\partial q_j} > 0 \quad \frac{\partial x_{1j}}{\partial k} > 0$$

$$\frac{\partial x_{2j}}{\partial p} < 0 \quad \frac{\partial x_{2j}}{\partial w_j} > 0 \quad \frac{\partial x_{2j}}{\partial r} > 0 \quad \frac{\partial x_{2j}}{\partial k_j} < 0 \quad \frac{\partial x_{2j}}{\partial q_j} > 0 \quad \frac{\partial x_{2j}}{\partial k} > 0$$

Proof. The optimal conditions for the problem is

$$u'_{1j} = \eta_1$$
$$u'_{2j} = \eta_1 p$$
$$C'(e_j) = \eta_2 f_j e_j + \eta_1 w_j$$
$$w_j e_j + r k = x_{1j} + px_{2j}$$
$$q_j = f_j(k_j, e_j)$$
by total differentiation, we get the following results\(^1\)

\[
\begin{pmatrix}
pu''_{11} - u''_{21} & pu''_{12} - u''_{22} & 0 \\
1 & p & -w_j \\
0 & 0 & f'_{j,e_j}
\end{pmatrix}
\begin{pmatrix}
dx_{1j} \\
dx_{2j} \\
d\epsilon_j
\end{pmatrix}
= 
\begin{pmatrix}
-u'_1 dp \\
-x_{2j} dp + kdr + e_j dw_j + rdk \\
-f'_{j,k_j} dk_j + dq_j
\end{pmatrix}
\begin{pmatrix}
|H| < 0
\end{pmatrix}
\]

from which we have

\[
dx_{1j} = \frac{1}{|H|} \begin{vmatrix}
-u'_1 dp & pu''_{12} - u''_{22} & 0 \\
-x_{2j} dp + kdr + e_j dw_j + rdk & p & -w_j \\
-f'_{j,k_j} dk_j + dq_j & 0 & f'_{j,e_j}
\end{vmatrix}
\]

Therefore,

\[
\frac{\partial x_{1j}}{\partial p} = \frac{f'_{j,e_j}}{|H|} \begin{vmatrix}
-u'_1 & pu''_{12} - u''_{22} \\
-x_{2j} & p \\
\end{vmatrix}
\]

\[
\frac{\partial x_{1j}}{\partial w_2} = \frac{f'_{j,e_j}}{|H|} \begin{vmatrix}
0 & pu''_{12} - u''_{22} \\
e_j & p \\
\end{vmatrix}
> 0
\]

\[
\frac{\partial x_{1j}}{\partial r} = \frac{f'_{j,k_j}}{|H|} \begin{vmatrix}
pu''_{12} - u''_{22} & 0 \\
p & -w_j
\end{vmatrix}
< 0
\]

\[
\frac{\partial x_{1j}}{\partial q_j} = \frac{q_j}{|H|} \begin{vmatrix}
pu''_{12} - u''_{22} & 0 \\
p & -w_j
\end{vmatrix}
> 0
\]

\[
\frac{\partial x_{1j}}{\partial k} = \frac{f'_{j,e_j}}{|H|} \begin{vmatrix}
0 & pu''_{12} - u''_{22} \\
r & p \\
\end{vmatrix}
> 0
\]

And

\[
dx_{2j} = \frac{1}{|H|} \begin{vmatrix}
pu''_{11} - u''_{21} & -u'_1 dp \\
1 & -x_{2j} dp + kdr + e_j dw_j + rdk & -w_j \\
0 & -f'_{j,k_j} dk_j + dq_j & f'_{j,e_j}
\end{vmatrix}
\]

\(^1\) About the notation here,

\[
u''_{21} = \frac{\partial^2 u}{\partial x_{2j} \partial x_{1j}}
\]
Therefore,

\[
\frac{\partial x_{2j}}{\partial p} = f_{j,e_j} \begin{vmatrix} pu_{11}^j - u_{21}^j & -u_1^j \\ 1 & -x_{2j} \end{vmatrix} < 0
\]
\[
\frac{\partial x_{2j}}{\partial w_j} = f_{j,e_j} \begin{vmatrix} pu_{11}^j - u_{21}^j & 0 \\ 1 & e_2 \end{vmatrix} > 0
\]
\[
\frac{\partial x_{2j}}{\partial r} = f_{j,e_j} \begin{vmatrix} pu_{11}^j - u_{21}^j & 0 \\ 1 & k \end{vmatrix} > 0
\]
\[
\frac{\partial x_{2j}}{\partial k_j} = f_{j,e_j} \begin{vmatrix} pu_{11}^j - u_{21}^j & 0 \\ 1 & -w_2 \end{vmatrix} < 0
\]
\[
\frac{\partial x_{2j}}{\partial q_j} = -q_j \begin{vmatrix} pu_{11}^j - u_{21}^j & 0 \\ 1 & -w_2 \end{vmatrix} > 0
\]
\[
\frac{\partial x_{2j}}{\partial k} = f_{j,k_j} \begin{vmatrix} pu_{11}^j - u_{21}^j & 0 \\ 1 & r \end{vmatrix} > 0
\]

References


[14] Lin, J. Yifu, Marshall Lecture at Cambridge University, mimeo


