Fear and Market Failure: Global Imbalances and "Self-insurance"[1]

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Abstract

Standard models of general equilibrium would predict modest current account surpluses in the emerging markets if they face higher risk than the US itself. But, with pronounced Loss Aversion in Emerging Markets, precautionary saving can generate substantial 'global imbalances', especially if there is an inefficient supply of global 'insurance'. A combination of fear and market failure generates imbalances as a general equilibrium outcome. In principle, lower real interest rates will ensure aggregate demand equals supply at a global level: but disequilibrium may result if the required real interest rate is negative.

A precautionary savings glut appears to us to be a temporary phenomenon, however, destined for correction as and when adequate reserve levels are achieved. But if the process of correction is triggered by 'Sudden Stop' on capital flows to the US, might this not lead to "hard landing" that is forecast by several leading macroeconomists?

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Introduction

Are the present current account imbalances – including notably a US external deficit running at 7% of its national output – part of the normal ebb and flow of trade and finance; or are there special factors at work? Is laissez faire the appropriate policy or is there a case for policy coordination? It depends who you ask; and when.

According to Nouriel Roubini, looking for factors to account for current global imbalances is rather like solving a 'whodunnit': and the two most plausible culprits turn out to be the US and Asia. He notes, for example, that from 2000 to 2004 US fiscal policy showed a large swing into deficit of almost 6% of GDP. But matters have, he argues, changed since 2005: the US fiscal deficit has shrunk somewhat but the external deficit continues to widen. The 'global savings glut' identified by Mr Bernanke, allied with weak investment in East Asia following the crisis experience of 1997/8 are named as key factors.

A very different perspective is offered by David Backus *et al.* (2006) who bridle at the use of the term global imbalances; and by the same token they reject the idea of looking for special factors or 'culprits' associated with them. What we observe, they suggest, is business-as-usual: for this no special explanation (nor any policy initiatives) are required. This seems, broadly speaking, to be the perspective of Alan Greenspan (2007, p.360) who remarks: "It is tempting to conclude that the US current account deficit is essentially a by-product of long-term forces, and is thus largely benign. After all, we do seem to have been able to finance it with relative ease in recent years". The long-term factors referred to include a major decline in portfolio "home bias"; and an acceleration in US productivity growth.

Deficits that can be financed smoothly in good times may prove to be difficult to finance when credit markets tighten, however. So the answer may also depend on the state of financial confidence. As the global turmoil of summer 2007 has shown, when trust evaporates in credit markets, CDOs and other asset backed commercial paper can become unsaleable in New York and London, and many of the biggest names in global investment banking can be left nursing large losses.

To investigate how plausible it is that standard optimising behaviour explains what we observe – and, if not, what special factors might be introduced - we employ a simple global model of trade and finance which incorporates elements of higher macroeconomic risk in emerging markets. (It is a variety of dynamic, stochastic general equilibrium: but for analytical tractability, there are only two blocs, referred to as US and EM, two states, and two periods.) We find that it can encompass a broad range of outcomes, depending on the completeness of markets and the behaviour of consumers. On the assumption that markets are complete and preferences are well behaved, for example, risk may be spread globally without significant imbalances. Some conclude that the explanation of imbalances must reflect differences between US and the emerging markets in terms of economic developments: Mendoza et al. (2007) and Dollar and Kraay (2006) emphasise lack of financial development in emerging markets as a source of precautionary savings. In this paper, however, we take a different route by including less orthodox, but more realistic, features into a model of macroeconomic volatility.

What are these factors? We refer to them in summary fashion as fear and market failure. The former refers to the scarring effects that the East Asia crisis has had on countries in the region; for which the concept of Loss Aversion is introduced in modelling aggregate demand by emerging markets. The latter refers to the absence of efficient means to spread the risk.

But even when countries are profoundly loss averse - determined to avoid the downside consumption shocks of the recent past – efficient asset markets can, in principle, provide the necessary cover without a savings glut. It is when appropriate insurance is not available that fearful consumers self-insure through saving, and global imbalances begin to emerge. See Aizenman and Lee (2005) for a test of precautionary behaviour versus the "mercantilist" views of economists such as Dooley et al (2004).

Joseph Stiglitz has noted that "The East Asian countries that constitute the class of '97 – the countries that learned the lessons of instability the hard way in the crises that began in that year – have boosted their reserves in part because they wanted to make sure that they won't need to borrow from the IMF again. Others, who saw their neighbours suffer, came to the same conclusion – it is imperative to have enough reserves to withstand the worst of the world's economic vicissitudes." A combination of fear and market failure generates this scenario as a general equilibrium outcome. Our approach is one which effectively integrates the precautionary motive for accumulating reserves into a model of global imbalances. The effect of precautionary behaviour in depressing real interest rates (and possibly employment) is checked by the US acting as 'consumer-of-last-resort'.

But this global equilibrium is no sure thing. Were the collateral backing for US consumers to prove inadequate, as recent turbulence in financial markets may have presaged - they might be unable to continue acting as global long-stop. [Many believe that house prices in the US (and elsewhere) have risen above sustainable levels over the last decade – the ex-Chairman of the US Fed included[4]: the collapse of a house price bubble in the US would probably be enough to stop the US acting as 'consumer of last resort'.]

A check to US demand would surely lead to a fall in real interest rates. But could they fall far enough to balance global demand? The Japanese experience after the stock market collapsed was a decade of very low interest rates and some price deflation: effectively real interest rates were bounded above zero. If the US is faced with a 'Sudden Stop' in its deficit financing, real interest rates might likewise hit a floor – a Liquidity Trap - leading to global recession. If real interest cannot adjust to match demand and supply in the good markets, then like Keynes in the 1930s (and Krugman in the 1990s), one may need to consider non-market-clearing outcomes where income is endogenous.

The paper is structured as follows. Section 1 develops the benchmark model of general equilibrium where macroeconomic volatility in Emerging Market economies (henceforth referred to as 'EM') is shared with the US without substantial surpluses or deficits, even when markets are incomplete. Section 2 introduces Loss Aversion in Emerging Markets, with and without insurance. In the absence of complete markets,

[4] On September 17, Financial Times reported that Mr Greenspan to have acknowledged the existence of a bubble in US house prices.

we find that macroeconomic volatility can lead to substantial precautionary savings and negative real interest rates. Interestingly, the absence of insurance allows us to use Fisher's approach to characterise a global equilibrium of fear and market failure. Section 3 considers the possibility of introducing strategic elements into asset markets; and compares our results with recent work by Jeanne and Ranciere (2006). Section 4 considers the possible emergence of Keynesian equilibrium due to a Liquidity Trap and/ or a 'Sudden Stop' in capital flows. The paper concludes that a savings glut could lead to global recession if it is combined with financial panic that prevents the US from acting as "consumer of last resort".

1. A 2x2x2 Global Model

To analyse implications of asymmetric macroeconomic shocks, we use a simplified dynamic stochastic general equilibrium (DSGE) model in the tradition of Mas-Colell *et al* (1995) and Obstfeld and Rogoff (1996). This stylised one good model has two time periods, two states of nature and two countries – the US and EM (referred to as 1 and 2 respectively, with the asterisk suffix '*' used to denote EM where appropriate). The framework is similar to that used in Miller *et al* (2005, 2006), to study the global implications of irrational exuberance in the US stock market; but here the US invests in risky assets and supplies safety and security in exchange (Hausmann and Sturzenegger, 2005).

Rather than postulating growth differentials, with high productivity in the US accounting large US deficits as Mr Greenspan (2007, p. 360) seems to suggest, we assume identical expected growth but differential risk. Specifically growth prospects in EM have greater volatility than for the US, modelled by adding a mean-preserving spread. Though this does not have a great impact in a standard general equilibrium framework, this is no longer true when downside risk is aggravated by a form of Loss Aversion (where the utility of consumption in period 2 which lies below that reached in the previous period is sharply discounted.) In a stochastic environment, the resulting risk sensitivity can lead the EM to acquire substantial insurance; and to act 'as if' it underestimates the mathematical expectation of growth.

When the relevant insurance is not available (or its provision is not credible), EM can always 'self-insure' – saving instead of swapping financial promises. Combining inadequate insurance with Loss Aversion provides a ready explanation for low interest rates, high EM savings and a US deficit.

To put this in context, consider the case of China. After what happened to many East Asian countries in 1997/8[5], it is clear that interruptions to trend growth are perfectly possible: and the rampant Chinese Dragon may be no more immune to shocks than were the Asian Tigers. In the words of Peter Nolan (2004, pp48-49):

> Today, the Chinese economy is growing fast, but the lesson from the past, especially the Asian Financial Crisis, is that perceptions can change overnight. China is today the last remaining large 'Growth story' in the world; it already has a huge 'bubble' of FDI, with the largest FDI inflows of any economy in the world… It is easy to imagine how the bubble might burst, and the flow of capital be reversed, with huge potential de-stabilizing consequences for the economy and society. There would then be a full-blown 'Chinese Financial Crisis'. A central goal of policy must be to avoid such an outcome.

If there is concern that consumption on the downside should not fall relative to past levels, China can of course seek insurance by selling FDI and buying US government bonds: and it can also seek to self-insure by acquiring US bonds via the current account. If, for any reason, the first option is limited, then self-insurance will be seen as the only way to avoid an unappealing prospect – the prospect, perhaps, of humiliation like that suffered by its near neighbour South Korea in 1997/1998 when it had to go cap in hand to the IMF and G7 and sacrifice sovereignty to get the financial support it needed in the crisis.[6]

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[5] When, in the crisis, trend growth rates effectively changed sign.

[6] Stiglitz (2006, p248) comments "The East Asian countries that constitute the class of '97 – the countries that learned the lessons of instability the hard way in the crises that began in that year – have boosted their reserves in part because they wanted to make sure that they won't need to borrow from the IMF again. Others, who saw their neighbours suffer, came to the same conclusion – it is imperative to have enough reserves to withstand the worst of the world's economic vicissitudes."

These considerations may suggest that strategic factors play a role that is not captured in the competitive framework we use here[7]– that some sort of insurance market game may be in process.

The basic model - and some extensions

The pattern of endowments assumed is indicated in Table 1. Both blocs are endowed with one unit at time one, and in expected terms each bloc grows at the rate g . In the absence of uncertainty each bloc would consume its endowment and, with log utility, real interest rates would equal growth rate plus the pure rate of time preference. (Thus expected growth of 3% and time preference of half that would imply a global real interest rate of 4.5%.)

With uncertainty, consider the case where future endowments for EM can take one of two values: high and low, with a standard deviation of σ around the mean rate of growth. (For convenience, each of the two outcomes is treated equi-probable; but the assumed symmetry for the shocks to EM growth is made for expositional reasons: the key results are independent of the probability weights, as noted below.) Assumed endowments are shown in the table.

	USA (country 1)	**EM(country 2)**	
		State 1 (with probability π)	**State 2** (with probability $1-\pi$)
Period 1	$Y_1 = 1$	$Y_1^* = 1$	$Y_1^* = 1$
Period 2	$Y_2(1) = Y_2(2) = 1 + g$	$Y_2^*(1) = 1 + g + \sigma$	$Y_2^*(2) = 1 + g - \sigma$

Table 1. The pattern of endowments

What we find is that for a benchmark case with standard log preferences is that macroeconomic volatility in EM is not sufficient to generate global imbalances. Dropping the assumption of complete markets does not in itself have much effect on the benchmark results, as indicated in the top right of the table. But volatility has substantial effects for EM with Loss Aversion. As shown in the second row, the Loss

[7] We are grateful to Sayantan Ghosal for this observation.

Aversion constraint begins to bind for σ>2g even if markets are complete, but it binds for σ>g when there is no insurance.

Macroeconomic Volatility, Markets and Preferences		**Financial Markets**	
		Complete	Incomplete
EM Consumer preferences	No Loss Aversion	No Global Imbalances	Small Global Imbalances
	Loss Aversion	Global Imbalances if σ>2g	Global Imbalances if σ>g

Table 2. The emergence of Global Imbalances

(a) The benchmark case

To study the pattern of savings and world real interest rates, assume for a benchmark case that markets are complete and also that representative consumers in each block countries share identical preferences. With log preferences, for example, lifetime utility in country 1 (US) will be

$$U(C_1, C_2(\cdot)) = \ln(C_1) + \beta[\pi \ln(C_2(1)) + (1-\pi)\ln(C_2(2))] \qquad (1)$$

where β is time preference, C_1 and $C_2(\cdot)$ are period 1 and period 2 consumption respectively; and the budget constraint is

$$C_1 + q(1)C_2(1) + q(2)C_2(2) = Y_1 + q(1)Y_2(1) + q(2)Y_2(2) \equiv W \qquad (2)$$

where $q(s) > 0$ ($s = 1,2$) are Arrow prices measured in period 1 sure-consumption, and W is the present value of US's total wealth.

Given Arrow prices, US's optimal consumption implied by its first order conditions are simply

$$C_1 = \frac{W}{1+\beta}. \quad (3)$$

$$C_2(1) = \frac{\beta\pi}{q(1)} C_1 \quad (4)$$

$$C_2(2) = \frac{\beta(1-\pi)}{q(2)} C_1 \quad (5)$$

Those for EM follow the same forms.

Market clearing, with global endowments consumed in each period and state, determines the equilibrium Arrow prices and real interest rates as follows:

$$q(1) = \pi\beta Y_1^W / Y_2^W(1) \quad (6)$$

$$q(2) = (1-\pi)\beta Y_1^W / Y_2^W(2) \quad (7)$$

$$\sum_s q(s) = 1/(1+r) \quad (8)$$

Using the superscript W to indicate world endowment. The pattern of consumption is obtained by substituting (6) and (7) into (3), (4) and (5).

With the endowments specified in Table 1, EM has an incentive to save in period 1, as is evident from a comparison of EM wealth relative to US wealth:

$W^* = (W - (q(2) - q(1))\sigma) < W$

where σ is the standard deviation of the EM endowment and $q(2) > q(1)$.

Because EM wealth is relatively lower, so is consumption, i.e.

$C_1^* = W^* / (1+\beta) < W / (1+\beta) = C_1$.

So EM saves, matched by a US current account deficit; and the more volatile is EM's endowment in period 2 (i.e. the greater is σ), the higher will be its period 1 savings. But with log utility and efficient provision of 'insurance', the savings effects are distinctly modest, as will be seen in Table 3.

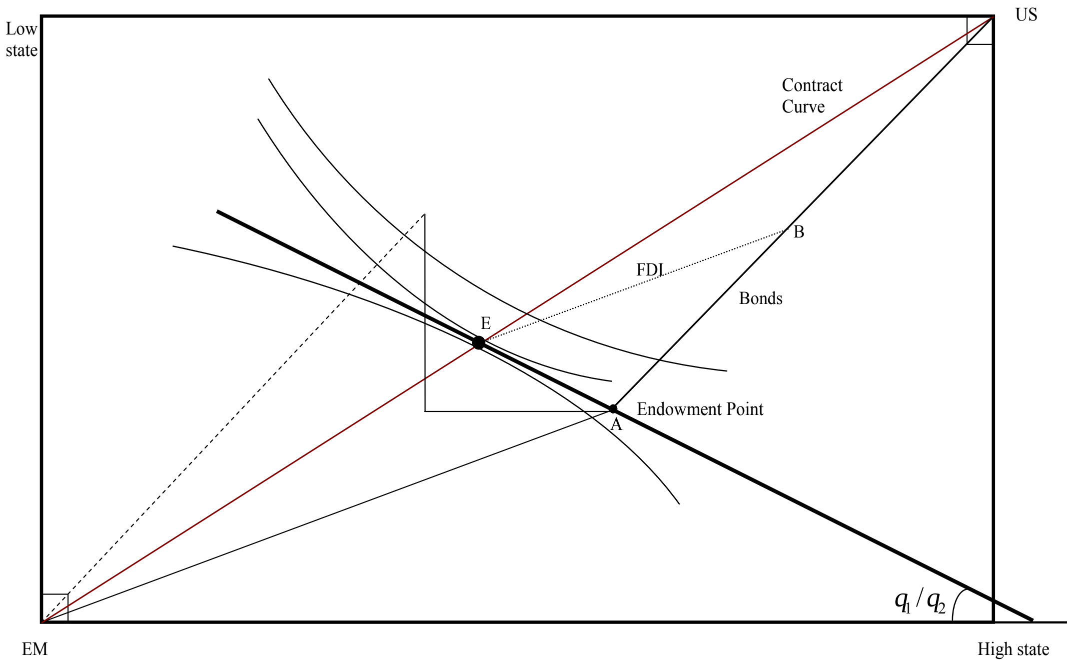

Figure 1: Buying insurance: a swap of GDP-Bonds.

Given the asymmetry of global endowments, consumption risk in the EM is not diversified away: but it is shared as shown in Figure 1, an Edgeworth box diagram describing allocations in period 2, as in Mas-Colell *et al* (p.593, 1995). [Outcomes for the high payoff state are on the horizontal and for the low payoff state on the vertical, and utility for EM is measured from the lower left corner and that for US from the upper right: identical probability assessments and utility functions imply that the contract curve is the diagonal in the figure.]

The autarky endowment point is at *A*, where for the US – with identical endowments in both states – this lies on the 45-degree line measured from the upper right corner. For the EM, however, disparity in the endowment between the two states means that it lies to the right of the 45-degree line drawn from the bottom left corner. Ignoring the effect of the first period savings on reallocating entitlements (as they are so small, see Table 3), general equilibrium consumption is shown at point E (on the contract curve). This involves the EM exchanging claims on output in state 1 for claims in state 2 at the relative price indicated by the slope of AE. The slope of this vector in absolute terms is $\left| q(1)/q(2) \right| < 1$, reflecting the relative abundance of goods in high state.

Note that consumption at E may be achieved by the sale of GDP-bonds from the US (vector AB) in exchange for GDP-bonds of the EM (labelled FDI in the Figure, see vector BE)[8]:sales of US government securities in exchange for FDI in China may provide an approximation.

As indicated above, with log preferences the quantitative effects of macroeconomic volatility in EM are virtually zero. There is almost no effect on the global interest rate and a tiny US deficit of only one 'basis point' of GDP for parameter values of $\beta = 0.985$, $\pi = 1/2$, g = 0.03 and σ = 0.03; and increasing volatility by a factor of four (to σ = 0.12), only increases the US deficit by one fifth of a percentage point of GDP and shaves a mere 30 basis points off the world interest rate.

Relaxing the strong assumptions made in this benchmark can change this as is indicated in the next table, where volatility is well above the mean growth rate.

σ = 0.12, g = 0.3		**Financial Markets**	
		Complete	Incomplete
EM Consumer preferences	No Loss Aversion	Deficit 0.2 Real Rate 4.2	Deficit 0.34 Real Rate 3.9
	Loss Aversion	Deficit 1.2 Real Rate 3.5	Deficit 4.6 Real Rate -4.3

Table 3. Macroeconomic Volatility, the US Deficit and the Real Interest Rate

Note: US deficits are measured as percentage of period 1 GDP. Equi-probable states are assumed; but, for the case of incomplete markets and Loss Aversion and bonds only, the results are independent of state probabilities.

(b) Incomplete markets and some precautionary savings

[8] For further discussion of GDP bonds, see Griffith-Jones and Sharma (2006), Griffith-Jones and Shiller (2006).

Ruling out insurance does generate a small amount of precautionary savings: EM will save more in period 1 to avoid potential utility losses were it to consume its unequal endowments in period 2, and the extra savings will bring down the global rate of interest. Instead of being driven purely by wealth effects, savings in emerging markets will now incorporate precautionary motive.

But with log utility, eliminating insurance does not predict a savings glut in EM. EM saving as percentage of GDP (and the US deficit) is twice as large as in the Benchmark case, but it still remains very small even when standard deviation of the shock to its endowment rises to 12%. (The effect of increasing risk on the world interest rate is more pronounced: it falls by 60 basis points, to less than 4%, when the standard deviation increases from 3 to 12%.)

Durdu et al (2007) find substantial precautionary savings emerges from macroeconomic volatility when sudden stops are triggered by macro shocks: i.e., countries are reduced to financial autarky at times of crisis. Here we explore a different route, not financial autarky but limited insurance with "exotic preferences" (Backus et al., 2004).

2. Global Equilibrium with Loss Aversion

(a) Loss aversion with insurance

In this section, we modify the preferences of the EM by incorporating two elements from Prospect Theory (Kahneman and Tversky, 1979): namely, *reference dependence* and *loss aversion*. We assume that consumption achieved in the previous period acts as a reference in the current period, so the measurement of utility depends on whether there is a "loss" or a "gain" in current consumption relative to this reference. To capture loss aversion, we assume that, close to the reference point, the increase in utility of a unit "gain" in current consumption (relative to the reference) is much smaller than the decrease in utility of a unit "loss" in current consumption.

Specifically, let the utility of state *i* consumption be defined as

$$u(C_2^*(i)) = \begin{cases} \ln(C_2^*(i)/C_1^*) & if \quad C_2^*(i) \geq C_1^* \\ \lambda \ln(C_2^*(i)/C_1^*) & if \quad C_2^*(i) < C_1^* \end{cases} \tag{11}$$

where $\lambda > 1$ indicates the degree of loss aversion. (Note that the utility measure becomes negative for consumption below reference level.)

To make the following treatment tractable, we consider a limiting case of loss aversion, namely, $\lambda \to +\infty$. Under this simplification which implies extreme disutility of any contraction of consumption, (11) is equivalent to imposing the constraint that

$$C_2^*(i) \geq C_1^* \tag{12}$$

The procedure used here, of imposing the constraint that next period's consumption in any state of the world should not fall below consumption in the current period, could also be viewed as an extreme form of *habit formation* as widely used in macroeconomic models. Chari, Kehoe and McGrattan (2002), in their attempts to determine whether sticky prices can lead to volatile and persistent real exchange rate movements, for example, assume in one experiment that the utility from consumption depends not on current consumption but its level relative to a fraction of last period's aggregate consumption. A similar formulation has also been used by Campbell and Cochrane (1999), Carroll, Overland, and Weil (2000), Ravn, Schmitt-Grohe, and Uribe (2004). As Carroll *et al* show, with this form of habit-persistence in consumption, higher growth may lead to higher saving.

In what follows, we show that loss aversion can also increase savings, but only if consumption would otherwise have fallen below the reference trigger. With complete contingent securities, US optimal consumption is derived in the same way as in Section 2.1. But EM's optimal consumptions are solutions to the following problem:

$$Max_{C_1^*, C_2^*(i)} \{\ln(C_1^*) + \beta[\pi \ln(C_2^*(1)) + (1-\pi)\ln(C_2^*(2))]\} \tag{13}$$

subject to the budget constraint

$$C_1^* + q^{LA}(1)C_2^*(1) + q^{LA}(2)C_2^*(2) = Y_1^* + q^{LA}(1)Y_2^*(1) + q^{LA}(2)Y_2^*(2) \equiv W^* \tag{14}$$

and (12).

How does loss aversion in EM change the equilibrium prices and allocation? We summarise these results in the following propositions.

Proposition 1. If $\sigma \leq 2g$, equilibrium prices and allocation are the same as those in Section 1(a).

Proof: See Appendix A.

Note that with complete Arrow securities, both countries can share risks. This risk-sharing means that both countries consume more or less equal proportions of the aggregate state endowment. So if the standard deviation of EM endowment in period 2 is small, EM is effectively insured against low consumption in the bad state. Therefore, no additional saving is required.

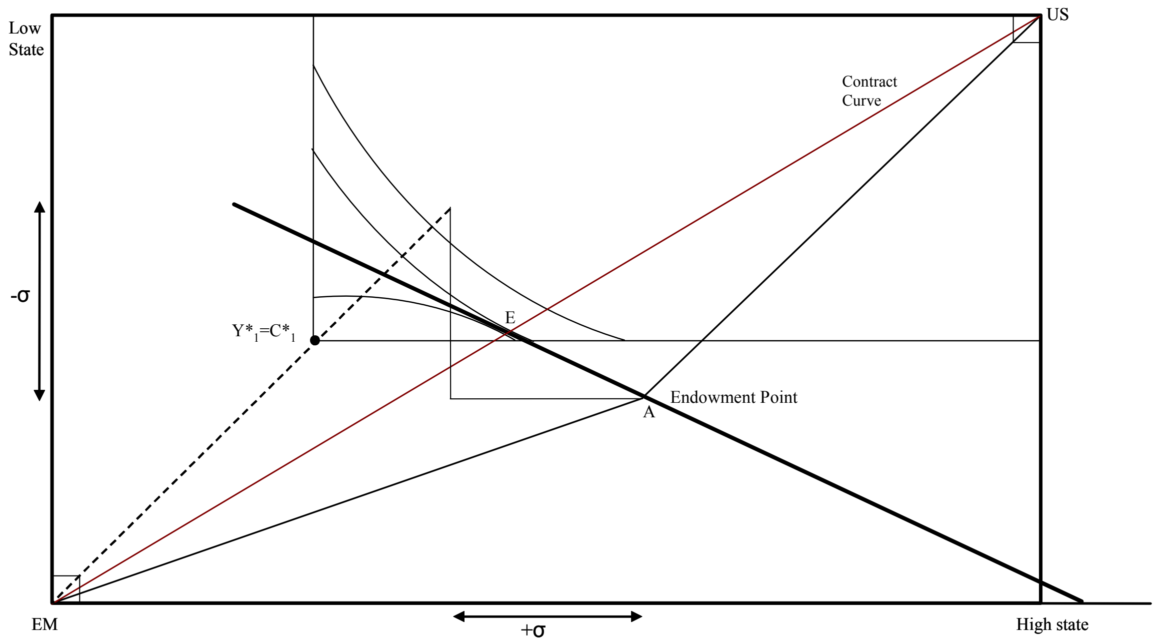

Figure 2. Unchanged equilibrium when the loss aversion constraint fails to bind

Proposition 1 is illustrated in Figure 2, where point E represents optimal consumption allocation with loss aversion. The loss aversion constraint is represented the L-shaped lined emanating from the point $Y_1^* = C_1^*$, and all EM consumption allocations to the north-east of this point satisfy the constraint. As point E lies on the contract curve north-east of point $Y_1^* = C_1^*$, the loss aversion constraint is not binding; and equilibrium in Figure 2 is identical to that in Figure 1. When risk increases, however, equilibrium can change as indicated in the following proposition.

Proposition 2. For the endowment structure given in Table 2, if $\sigma > 2g$, then

(1) $q^{LA}(1) > q(1)$ and $q^{LA}(2) > q(2)$;

(2) $q^{LA}(2)/q^{LA}(1) > q(2)/q(1)$;

(3) $r^{LA} < r$;

(4) $C_1^*(LA) < C_1^*$.

(5) $C_2^*(2, LA)/C_2^*(1, LA) > C_2^*(2)/C_2^*(1)$

Proof: See Appendix B.

Results in Proposition 2 are quite intuitive. If the standard deviation of period 2 EM endowment is large, simple risk sharing is not sufficient to ensure that the consumption in the bad state remains above the reference level for EM. So loss aversion increases EM's demand for insurance in period 2. As this raises the relative price $q^{LA}(2)/q^{LA}(1)$, EM also increases savings as a substitute for high cost insurance. (Note that period 1 savings for EM not only act as a substitute for insurance but also reduce the reference consumption in period 2, making the constraint less likely to bind.) Proposition 2(5) implies that consumption allocation in period 2 when loss aversion constraint is binding lies above the contract curve associated with no loss aversion.

Suppose we allow EM to have a different parameters for time preference, β', and the subjective probability parameter, π', while keeping those of US as before, can we replicate the outcomes in Proposition 2 without evoking the assumption of loss aversion? The results for this "as if" exercise are given in the following proposition.

Proposition 3. For a set of parameters $\{\beta, \pi; \beta', \pi'\}$ (and given restriction on endowments as in Proposition 2) to replicate the equilibrium results in Proposition 2, it is sufficient that

(1) $\pi' = \dfrac{\beta\pi(1 + q^{LA}(2))}{(1+\beta)q^{LA}(2) + \beta\pi} < \pi$

(2) $\beta' = \dfrac{(1+\beta)q^{LA}(2) + \beta\pi}{1+\beta(1-\pi)} > \beta$

Proof: See Appendix C.

Proposition 3 indicates that the effects of introducing loss aversion on the part of the EM will (when the constraint is binding) be to increase its perceived pessimism ($\pi' < \pi$) and to make it more forward-looking ($\beta' > \beta$).

How loss aversion can impact on global equilibrium is illustrated using Figure 4b, the Edgeworth box used earlier. As before, points A and E represent EM's second period endowment and consumption allocation in the absence of loss aversion. With large enough σ, however, the loss aversion constraint becomes binding and EM will increase its first period savings (Proposition 2(4)), moving its second period effective endowment from A (along the 45 degree line) to B. The binding of the loss aversion constraint will also change relative Arrow prices (Proposition 2(2)), making EM's period 2 budget constraint flatter (see line BE'). The intersection of the budget line BE' with the loss aversion constraint $C_2^*(L) = C_1^*$ defines the new equilibrium E' which lies above the contract curve due to Proposition 2(5). From A, a combination of savings and an asset swap of US bonds for FDI allows for consumption at point E' satisfying the loss aversion constraint.

As indicated in Proposition 3, this new equilibrium may also be replicated without loss aversion if EM has lower time preference (higher β) and greater pessimism (lower π). As can be seen in Figure 3, the increase in pessimism in the EM, calibrated by a fall in π'/π, has two effects: first it makes the contract curve concave, and second it changes the relative Arrow prices which makes EM's budget constraint flatter. The decrease in the time preference, calibrated by the increase in β'/β, has the effect of increasing EM's savings and so shifting its second period effective endowments from A to B. The intersection of the budget constraint BE' with the modified contract curve defines the equilibrium.

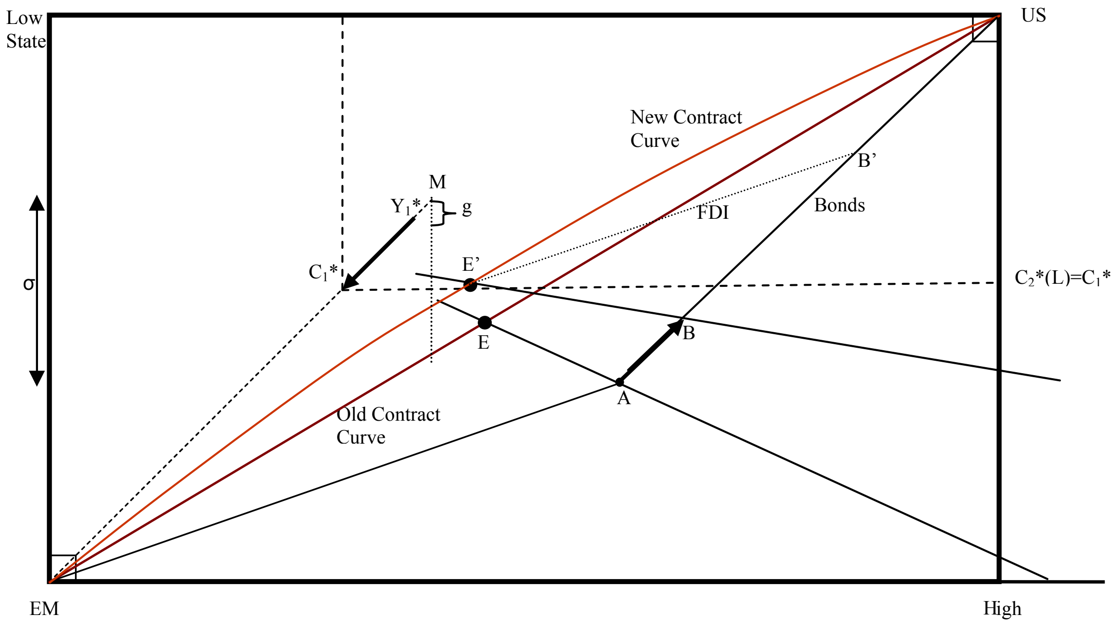

Figure 3: Savings and insurance with Loss Aversion; it's 'as if' time preference has fallen and pessimism has increased in EM.

The quantitative significance of loss aversion on real interest rates and savings is indicated in lines 6 and 7 for the third case considered in Table 3. With the standard deviation of up to 6%, the constraint is not binding, so the real interest rates and savings are the same as in the benchmark case. But the effect of loss aversion becomes apparent when the standard deviation increases to 12%: this generates a substantial increase in the EM savings and a marked fall in the global interest rates. As a consequence, the US deficit rises by 1% of GDP as a 0.7% fall of the real interest rates encourages US consumption.

Table 4 indicates the parameter values necessary to replicate the equilibrium with loss aversion by changing time preferences and state probability assessment on the part of EM. When $\sigma = 12\%$, for example, $\beta' = 1.03\beta$ and $\pi' = 0.98\pi$ will generate the same equilibrium allocation as under loss aversion.

σ	Loss aversion constraint	β' / β	π' / π
3%	Not binding	1	1
6%	Not binding	1	1
12%	Binding	1.03	0.98
18%	Binding	1.07	0.97

Table 4. Loss aversion, less time preferences and more pessimism.

(b) Loss aversion with no insurance

Since the East Asia crisis EMEs seem no longer inclined to trust the IMF as a provider of emergency finance. But, as Griffith-Jones and Shiller (2006) and Wolf (2005) point out, global financial markets are notoriously incomplete. What happens if EM countries with Loss Aversion face substantial volatility without insurance? The answer is substantial precautionary saving, so long as volatility exceeds the growth rate.

Proposition 4. For $\sigma \leq g$, the equilibrium real interest rates and consumption allocations are the same for bonds only without loss aversion, see (c) above[9].

But for $\sigma > g$, the constraint $C_2^*(2) \geq C_1^*$ binds, and the equilibrium real interest rate is

$$1+r = \frac{-\psi + \sqrt{\psi^2 + 4\beta Y_1 Y_2 /(1+\beta)^2}}{2\beta Y_1 /(1+\beta)} \qquad (14)$$

where $\psi = \beta Y_1 /(1+\beta) + Y_1 - Y_2^*(2) - Y_2 /(1+\beta)$.

The consumption allocation for EM is given by

$$C_2^*(1) = Y_2^*(1) + (1+r)(Y_1 - Y_2^*(2))/(2+r) \qquad (15)$$

$$C_2^*(2) = C_1^* = [(1+r)Y_1 + Y_2^*(2)]/(2+r) \qquad (16)$$

and the consumption allocation for US can be obtained simply by using the market clearing conditions.

Proof: For $\sigma \leq g$, one can show that $C_2^*(2) \geq C_1^*$, so real interest rates and consumption allocation in Section 1(b) still constitute the equilibrium solution. For $\sigma > g$, however, solutions in Section 1(b) violate the constraint $C_2^*(2) \geq C_1^*$. Imposing binding constraint yields the optimal consumption for EM as in (15) and (16). The optimal consumption for US, derived in the same way as in Section 1(b), gives

$$C_1 = \frac{1}{1+\beta}\left(Y_1 + \frac{Y_2}{1+r}\right) \equiv \frac{W}{1+\beta} \qquad (17)$$

$$C_2(1) = C_2(2) = \frac{\beta(1+r)W}{1+\beta} \qquad (18)$$

---

[9] Note that the Loss Aversion constraint binds for a smaller σ than for when insurance is available: as risk cannot be shared at a global level, EM has to self-insure by increasing savings even for a moderate σ .

Using market clearing condition $C_1 + C_1^* = 2Y_1$ one arrives at the equilibrium real interest rates represented by (14). Using (14), one can back out the equilibrium consumption for both US and EM.

As we discuss below, when the loss aversion constraint does bind, *the global real interest rate is determined independently of state probabilities*. (For the simple reason that the focus of all eyes in EM is the bad outcome.) This implies that the implications of the global model can now be shown using a familiar Fisher diagram of intertemporal choice, see Figure 4 where the horizontal axis measures endowments and consumption in period 1, and the vertical the outcomes in period 2.

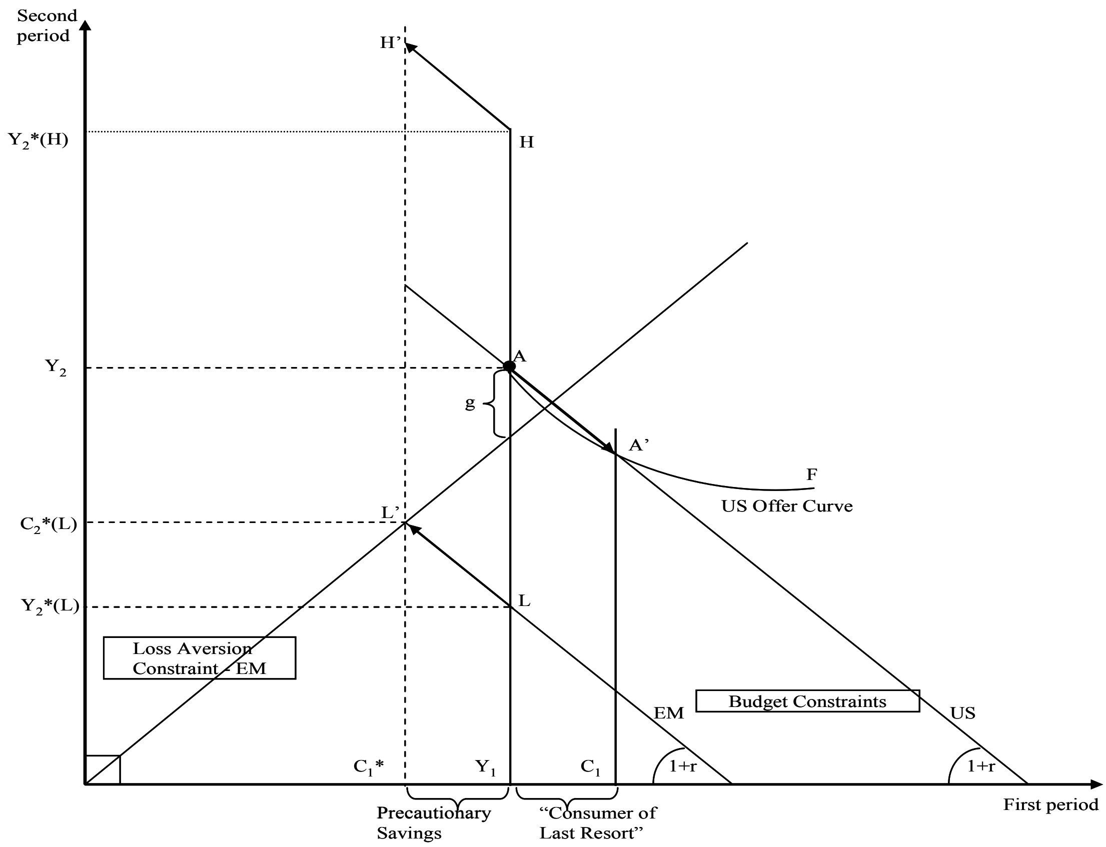

Figure 4: Global Equilibrium: Precautionary Saving and "the consumer of last resort"

Point A describes the income in both periods for US and the hyperbola AF represents US offer curve[10]. For any given interest rate, the intersection of the appropriate US

[10] The parametric representation of the US offer curve is given by the US inter-temporal budget constraint and the proportionality condition, $C_2 / C_1 = (1+r)\beta$, implied by its first order conditions. Replacing the real interest rates in one of the equations using the other gives the US offer curve.

budget constraint AA' and the offer curve AF determines the optimal inter-temporal consumption allocation of the US (at point A') and the US current account deficit.

Turning to the EM, point A also indicates first period income and *average* second period endowments for EM: with volatility is shown by the spread between H and L. Whenthe constraint is binding its inter-temporal budget constraint, $C_1^* + C_2^*(L)/(1+r) = Y_1 + Y_2^*(L)/(1+r)$, is represented by the downward sloping line passing through low state endowment L. To satisfy the constraint, consumption in the first period and for the low state in the second period must lie on the 45-degree line OC. Thus the intersection of the budget line LL' and the 45-degree line determines the EM precautionary savings, indicated by the horizontal distance $C_1^* Y_1$. As σ increases, and point L moves downwards, precautionary savings will go up.

How is the world interest rate to be determined? For markets to clear in period 1, the real interest rate has to fall sufficiently so that extra consumption by the US balances precautionary savings by the emerging markets. Faced with a savings glut, the US has to act as the global 'consumer of last resort' (Wolf, 2006). [Diagrammatically, the real interest rate must be chosen such that vector of excess consumption AA' is equal and opposite to the precautionary savings vector LL'.] Increased volatility will clearly cut real interest rates: an increase in σ will increase in EM's savings, so -- to ensure that this is matched by the US deficit -- the budget line AA' has to rotate anti-clock-wise, reducing real interest rates. The Fisher diagram reveals three points of interest.

(i) EM consumes below its weight

As equilibrium L' is on a budget line lying below the usual Fisherian inter-temporal budget constraint, it is 'as if' income is only partially capitalised. Loss aversion can, it seems, generate outcomes observationally equivalent to those in a non-stochastic model of Caballero *et al* (2006), where income is also imperfectly capitalised. (See Cooper, 2005 for a similar perspective).

(ii) State probabilities do not matter

As note above, the outcomes for savings and global interest rates in period 1 do not depend on the state probabilities. Figure 6 illustrates the global equilibrium for the case where high and low states are equi-probable: so the US trading vector AA' is balanced by the equally weighted EM's trading vectors LL' and HH'. Consequently, when high state is realised in period 2, this model predicts a massive increase in the EM's state consumption (given by point H'). But this unrealistic prediction can easily be modified without changing savings behaviour. [Consider for example, the case of asymmetric shocks where there is low probability of a large negative shock and a high probability of a small positive shock, see Jeanne and Ranciere (2006).

To keep the mean-preserving feature of the EM's second period state endowments, requires $\pi(Y_2+\sigma_H)+(1-\pi)(Y_2-\sigma_L)=Y_2$ or $\sigma_H=\frac{1-\pi}{\pi}\sigma_L$, where σ_H is the shock in the high state and σ_L that in the low state. By fixing σ_L at the same level as that in Figure 6, one can choose π close to 1 to make σ_H arbitrarily small. This would yield the same equilibrium savings as drawn in Figure 6, but reduce EM's high state consumption substantially. This is because savings and the interest rate do not depend on how likely the low state is but on how bad it is.]

(iii) Equilibrium real interest rates can be negative.
Volatility can drive interest rates to zero and below. In terms of the figure, negative rates will occur when the point A' on the US offer curve required to match these savings is sufficiently far to the right that the budget line has an absolute slope less than unity.

The algebraic condition for volatility to push real rates into negative territory is as follows:

Proposition 5. Given the endowment structure specified in Table 2, the real interest rate $r \le 0$ for $\sigma \ge (2-2\beta)/(1+\beta)+[2/(1+\beta)+1]g$.

Proof: From (14), imposing the condition $r \le 0$, one obtains the parameter restriction given in the above proposition.

This is illustrated numerically in the last two lines of Table 3, where for σ=12% savings reaches 4.6% and the equilibrium real interest rate fall to -4.3%[11].

The relationship between real interest rates and risk for parameters of our benchmark model is illustrated in more detail in Figure 7 where the horizontal axis measures the negative shock to EM's period 2 endowment and the equilibrium real interest rates is plotted on the vertical axis. When the loss aversion constraint is not binding real interest rates decrease very slowly with increasing σ; but when the loss aversion constraint is binding the real interest rates fall sharply as risk increases. From Proposition 5, the critical level of σ beyond which the real interest turns negative turns out to be about 7.5% for the parameters used here.

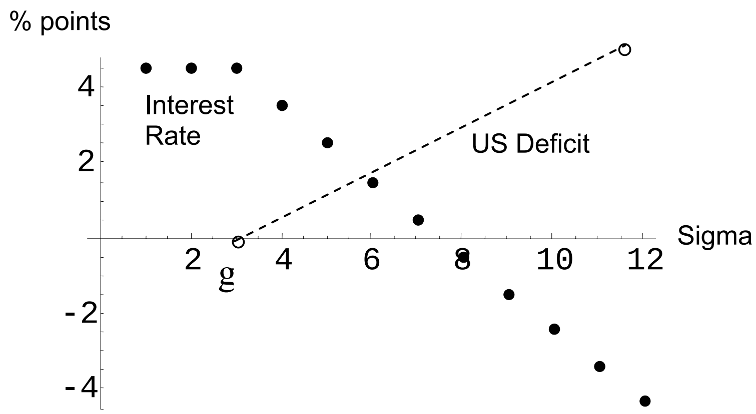

Figure 5. Loss Aversion and Macroeconomic Volatility

Note: For Jeanne and Ranciere (2006) σ of 10% might lead to imbalances of about 5%.

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3. Strategic considerations; and a comparison

All the calculations reported above assume competitive equilibrium even when the set of assets is incomplete. But, as Dooley and Garber (2005) point out, the big players in asset markets are governments who can manipulate supply. Furthermore, Meissner and Taylor have shown how Britain in the years 1870–1913 and US in years 1981 –

[11] Note that in their paper on the optimal level of international reserves for emerging market countries, Jeanne and Ranciere (2005) assume a crisis output cost of 10% in their benchmark calibration.

2003 have been able to enjoy a "privilege" in the form of higher yields earned on external assets than paid on external liabilities – worth about 0.5% of GDP per annum in both cases. Maybe so, but Meissner and Taylor warn that such monopoly power is a fading asset: the privilege is much higher in earlier years than later.[12]

Could one modify the competitive equilibrium by allowing for monopoly power on the part of the US? Instead of supplying safe asset on a competitive basis, US could, for example, select the utility maximising point on the demand for safe asset from the EM: or could it act as a dynamic monopolist?[13] As indicated by Table below this might generate outcomes between limit cases (of complete markets and no insurance) reported in the paper.

	Complete markets	**Market power**	**No insurance**
Standard preferences	No savings; fair Insurance	More saving; Overpriced insurance	Some precautionary savings
Loss aversion in EM	Precautionary savings if $\sigma>2g$; fair Insurance	High Precautionary Savings; overpriced insurance	Very High Precautionary Savings if $\sigma>g$;

Table 5. General Equilibrium solutions: a place for strategic analysis?

A comparison

It may be interesting to compare what we get from a general equilibrium approach with results reported in a recent IMF study of the optimal reserves by Jeanne and Ranciere (2005). For an emerging market economy facing a low spread in capital markets, the risk of a 10% fall in output should lead to reserve holdings of 9.37% of GDP, see discussion of Table 3 in their paper. Note that, as all these reserves will be used to maintain consumption when there is a shock and they are all reconstituted one period later, it is as if such a shock is associated with a corresponding *savings rate* of nine and a half percent of GDP over the post crisis period of reserve build up. As

[12]The gradual disappearance of the privilege is examined in Thamotheram, 2006.

[13] Supplying dollars at high prices as the RoW accumulates reserves, with a dollar devaluation when reserve stock reaches equilibrium, see Section 7.2 below.

there is no insurance in their model, this is to be compared with our no insurance case, where the build-up of reserve assets *precedes* the crisis. For a shock with a downside of 12% our figure for savings is about four and a half percent. While this is only about half as much as for Jeanne and Ranciere (2006), this may be because we allow for consumption smoothing across the two periods while their static simplification rules this out.[14]

Two observations may be made – over the period of time that reserves are built up, and over the implications for sustainability. As a preliminary, note that the actual reserve holdings by China greatly exceed the savings figures just discussed: from around 16% of GDP in 2000 they almost doubled to reach 29% in 2003, Jeanne and Ranciere (2006, Table 1). This suggests that treating the issue in a two period context (as the IMF study and we do) is too restrictive. The level of reserves may be built up over a period of two or three years – and it can be expanded by assets swaps as well as external surpluses, as the case with insurance has shown.

The second observation is that the reserve build-up is essentially a *transitional* phenomenon: once reserves have reached their desired level, there is no need for high precautionary savings[15]. The high savings, low interest rate outcomes we have studied are not to be thought of as steady-state equilibria, but as temporary phenomena. Putting it more bluntly, the precautionary approach implies that the current pattern of imbalances is not sustainable. Vulnerable aspects of the current equilibrium are considered next.

4. Risk of global recession

(a) A 'Sudden Stop' for the US consumer?

In the Arrow-Debreu framework we are using, contracts made in the initial period are enforceable without cost. In practice, however, default risk for loan contracts commonly leads to the requirement of collateral. How might imposing collateral requirements alter the predictions of a simple GE model? A key issue will be whether

[14] For countries facing high interest rates, however, the optimal reserve holding is calculated to be only about 1½% of GDP – with a correspondingly lower saving rate, Jeanne and Ranciere (2006, Table 3).
[15] We can show this in the GE context by changing the initial holding of bonds by the RoW, which play the same role as reserves as in the analysis of Jeanne and Ranciere (2006).

borrowers have a sufficient amount of pledgeable assets that can be seized by the lender in case of default.[16]

In reality housing can play that role, with consumers spending in excess of current income on the security of their property. In these circumstances, a house price bubble could substantially ease the collateral constraint: and there is some evidence that there has been a house price bubble in the US and in Europe[17]. The sub-prime mortgage crisis in America has, moreover, signalled the fear that borrowers in that category may not have the income flows to service their loans. Attempts to seize collateral on default could lead to a fall in the value of collateral and according to an Economist Briefing (September 22, p.35) 'The prospect of a coordinated global housing slump … remains a plausible risk.'

If over-inflated house prices have been sustaining high consumption in the US, falling property prices could effectively lead to a 'sudden stop' for US consumers. This could have important affects on the predictions of our simple model, as a limit on borrowing could prevent the US from acting as consumer of last resort. In terms of figure x for example it will be as if the US offer curve becomes vertical at some point. If the limit was binding, this would lead to a fall in global aggregate demand. A cut in interest rates might be expected to induce more consumption, but if demand is sufficiently interest inelastic, there is there is no guarantee positive real interest rates are consistent with equilibrium.

(b) The Liquidity Trap

It might appear that markets will clear so long as the real interest rate is free to adjust. But even with no collateral limits on US consumption, it was found that market-clearing interest rates have to be *negative* for substantial risk ($\sigma > 7.5\%$). What if there is a constraint on US consumption and a zero lower bound on the real interest rate? This will imply that the US deficit is less than high savings in the EM in these

[16] See Kiyotaki, N and J. H. Moore (1997) for model of moral hazard and collateralised borrowing.

[17] Economist Magazine, September 15, 2007, p104.

circumstances: in other words, global demand will fall short of global supply at full employment levels of income and zero (or lower) real interest rates.

When might such a bound be relevant? In a non-inflationary world, real interest rates can be lowered by cutting nominal rates, but since nominal interest rates cannot go below zero, the real interest rate too has a zero bound. (Nor would adding price flexibility help, unless prices are expected to rise.) The case of Japan, where the collapse of the Nikkei in the early 1990s was followed by a decade of near zero inflation and interest rates may serve to illustrate. But if one was to impose an exogenous zero bound on the real rates, how is the model to be solved? One would presumably have to make assumptions of what happens when markets do not clear: that supply contracts until global demand and supply balance, for example[18]. What this involves, of course, is switching from the 'full employment' world of Arrow Debreu to some Keynesian alternative.

Faced with an excess of savings over investment, Classical economists in the 1930s had argued that interest rates would fall as needed to equate savings and investment (and preserve full employment): But Keynes objected that interest rates would be subject to a lower bound set by the Liquidity Trap and, for this reason, income would become endogenous, falling until savings matched investment. The Japanese experience has led to a resurgence of interest in Keynesian equilibria, most notably in the 1998 Brookings Paper by Paul Krugman subtitled "Japan's Slump and the Return of the Liquidity Trap".

5. Conclusion

A model of global equilibrium where countries outside the US face higher risk than the US itself can lead to current account surpluses in the EM. If it is driven by Loss Aversion, such precautionary savings can cause substantial 'global imbalances', particularly if there is an inefficient supply of global insurance. In principle, this simply requires lower real interest rates to ensure that aggregate demand equals

[18] This is, in fact, something like what happened after the East Asian crisis when countries in the region went into sharp recession and the US acted as the 'consumer of last resort'.

supply at the global level (though the required real interest may turn out to be negative). A situation with low interest rates and high savings outside the US thus appears to be an efficient global equilibrium: but is it sustainable?

A precautionary savings glut appears to us to be a temporary phenomenon, destined for correction as and when adequate reserve levels are achieved. If the process of correction is triggered by panic, could it not lead to the inefficient outcomes that concern macroeconomists such as Eichengreen and Park, Roubini and Setser, and Martin Wolf? The unprecedented savings levels recorded in East Asia since 1997/8 financial crises and the prolonged failure of Japan to escape from a Liquidity Trap would then appear as early warning signals: and the failure to effect a smooth transfer after the first World War, leading as it did to a Liquidity Trap and the emergence of Keynesian under-employment economics, as a precedent that should not be ignored. When precautionary savings is combined with financial panic, history offers no guarantee of full employment.

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Appendices

A. Equilibrium with precautionary savings.

Denote S the first period saving by US (the amount of bonds purchased), its optimal level is determined by the solution to the following problem:

$$Max_S\{\ln(C_1)+\beta\ln(C_2)\} \quad \text{(Z1)}$$

subject to

$$C_1 = Y_1 + S \quad \text{(Z2a)}$$

$$C_2 = Y_2 - (1+r)S \quad \text{(Z2b)}$$

where $(1+r)$ is the gross real interest rates.

The optimal saving implies the period 1 consumption

$$C_1 = \frac{1}{1+\beta}\left(Y_1 + \frac{Y_2}{1+r}\right) \quad \text{(Z2c)}$$

Let C_2^* be EM's period 2 medium level, i.e., $C_2^* = [C_2^*(1)+C_2^*(2)]/2$, then EM's optimal level of consumption is the solution to the following problem:

$$Max_S\{\ln(C_1^*)+\beta[\pi\ln(C_2^*+\sigma)+(1-\pi)\ln(C_2^*-\sigma)]\} \quad \text{(Z3)}$$

subject to

$$C_1^* = Y_1^* - S \quad \text{(Z4a)}$$

$$C_2^* = Y_2^* + (1+r)S \quad \text{(Z4b)}$$

where $Y_2^* = [Y_2^*(1)+Y_2^*(2)]/2$ indicates EM's period 2 medium endowment.

One can solve for a similar problem for EM to yield its period 1 consumption

$$C_1^* = Y_1 - \frac{-\xi+\sqrt{\xi^2-4(1+\beta)\varsigma}}{2(1+\beta)} \quad \text{(Z5)}$$

where

$$\xi = Y_2^*(1)+Y_2^*(2)+\beta[\pi Y_2^*(2)+(1-\pi)Y_2^*(1)]-\beta(1+r)Y_1$$

$$\varsigma = Y_2^*(1)Y_2^*(2)-\beta(1+r)Y_1[\pi Y_2^*(2)+(1-\pi)Y_2^*(1)]$$

Imposing equilibrium condition

$$C_1 + C_1^* = 2Y_1$$

yields the following fixed point condition for real interest rates

$$\left(\frac{Y_2}{1+r} - \beta Y_1\right)^2 + \xi\left(\frac{Y_2}{1+r} - \beta Y_1\right) + (1+\beta)\varsigma = 0 \qquad \text{(Z6)}$$

Equations (Z5) and (Z6) are used to generate numerical results for the Bonds-only case in Table 3.

Appendix B. Proof of Proposition 1

Note that the modification of Foreign preferences only affects the partial equilibrium allocation for the Foreign country. To solve for the optimal consumptions for the Foreign country, we first replace C_1^* in (13) and (12) using budget constraint (14) to form the following Lagrangean:

$$L = \ln(W^* - q^{LA}(1)C_2^*(1) - q^{LA}(2)C_2^*(2)) + \beta[\pi \ln(C_2^*(1)) + (1-\pi)\ln(C_2^*(2))]$$
$$+ \lambda_1[C_2^*(1) - W^* + q^{LA}(1)C_2^*(1) + q^{LA}(2)C_2^*(2)] + \lambda_2[C_2^*(1) - W^* + q^{LA}(1)C_2^*(1) + q^{LA}(2)C_2^*(2)]$$

The first order conditions become

$$-\frac{q^{LA}(1)}{C_1^*} + \frac{\beta\pi}{C_2^*(1)} + \lambda_1[1 + q^{LA}(1)] + \lambda_2 q^{LA}(1) = 0 \qquad \text{(A1)}$$

$$-\frac{q^{LA}(2)}{C_1^*} + \frac{\beta(1-\pi)}{C_2^*(2)} + \lambda_1 q^{LA}(2) + \lambda_2[1 + q^{LA}(2)] = 0 \qquad \text{(A2)}$$

$$\lambda_1(C_2^*(1) - C_1^*) = 0,\ \lambda_1 \geq 0,\ C_2^*(1) \geq C_1^* \qquad \text{(A3)}$$

$$\lambda_2(C_2^*(2) - C_1^*) = 0,\ \lambda_2 \geq 0,\ C_2^*(2) \geq C_1^* \qquad \text{(A4)}$$

(A3) and (A4) are complementary slackness conditions.

Given $Y_2^*(1) > Y_2^*(2)$, there are only three possible cases: (i) $\lambda_1 = \lambda_2 = 0$, (ii) $\lambda_1 = 0$ and $\lambda_2 > 0$, and (iii) $\lambda_1 > 0$ and $\lambda_2 > 0$.

For $\lambda_1 = \lambda_2 = 0$, (A1), (A2) and budget constraint (14) imply

$$C_1^* = W^*/(1+\beta). \qquad \text{(A5)}$$

$$C_2^*(1) = \beta\pi C_1^* / q^{LA}(1) \qquad \text{(A6)}$$

$$C_2^*(2) = \beta(1-\pi)C_1^* / q^{LA}(2) \qquad \text{(A7)}$$

The equilibrium conditions ensure that $q^{LA}(1) = q(1)$ and $q^{LA}(2) = q(2)$ (where $q(1)$ and $q(2)$ are given by (6) and (7)).

Using constraints $C_2^*(1) \geq C_1^*$ and $C_2^*(2) \geq C_1^*$, one arrives at $Y_2^W(1) \geq Y_1^W$ and $Y_2^W(1) \geq Y_1^W$, or $2g - \sigma \geq 0$ for endowments given in Table 1. As $q^{LA}(1) = q(1)$ and $q^{LA}(2) = q(2)$, the general equilibrium allocation will be the same as in Section 1(a).

Appendix C. Proof of Proposition 2

Consider the second case outlined above, namely, $\lambda_1 = 0$ and $\lambda_2 > 0$. The first order conditions become

$$-\frac{q^{LA}(1)}{C_1^*} + \frac{\beta\pi}{C_2^*(1)} + \lambda_2 q^{LA}(1) = 0 \tag{B1}$$

$$-\frac{q^{LA}(2)}{C_1^*} + \frac{\beta(1-\pi)}{C_2^*(2)} + \lambda_2[1 + q^{LA}(2)] = 0 \tag{B2}$$

$$\lambda_1 = 0,\ C_2^*(1) \geq C_1^* \tag{B3}$$

$$C_2^*(2) - C_1^* = 0, \lambda_2 > 0, \tag{B4}$$

Solving them yields

$$C_1^* = C_2^*(2) = \frac{1+\beta(1-\pi)}{(1+\beta)(1+q^{LA}(2))} W^* \tag{B5}$$

$$C_2^*(1) = \frac{\beta}{1+\beta} \frac{\pi}{q^{LA}(1)} W^* \tag{B6}$$

$$\lambda_2 = \frac{1+\beta}{W^*} \frac{q^{LA}(2) - \beta(1-\pi)}{1+\beta(1-\pi)} \tag{B7}$$

To ensure (B5) – (B7) constitute optimal solutions for the Foreign country, we need to impose the restrictions on the Lagrange multipliers as those given at the beginning. Condition $\lambda_1 = 0$ implies $C_2^*(1) \geq C_1^*$. From (B5) and (B6), this requires

$$\beta\pi(1 + q^{LA}(2)) \geq [1 + \beta(1-\pi)] q^{LA}(1) \tag{B8}$$

From (B7), condition $\lambda_2 > 0$ requires

$$q^{LA}(2) > \beta(1-\pi) \tag{B9}$$

To solve for the equilibrium prices, we impose the following market clearing conditions:

$$C_2(1) + C_2^*(1) = Y_2^W(1) \tag{B10}$$

$$C_2(2) + C_2^*(2) = Y_2^W(2) \tag{B11}$$

Condition (B10) implies

$$q^{LA}(1) = \frac{\beta\pi}{1+\beta(1-\pi)} \frac{Y_1^W + q^{LA}(2) Y_2^W(2)}{Y_2^W(1)} \tag{B12}$$

This Arrow price relationship is exactly the same as the one in complete markets without loss aversion.

Replacing $q^{LA}(1)$ in the state price relationship implied by (B11) yields the following quadratic equation for $q^{LA}(2)$

$$(d/q(2) - b)[q^{LA}(2)]^2 + \Delta q^{LA}(2) + a\beta(1-\pi) - d = 0 \tag{B13}$$

where

$$a = \left(\frac{1}{2} + \frac{\beta\pi}{1+\beta(1-\pi)} \frac{Y_2^*(1)}{Y_2^W(1)} \right) Y_1^W$$

$$b=\left(\frac{Y_2^*(2)}{Y_2^W(2)}+\frac{\beta\pi}{1+\beta(1-\pi)}\frac{Y_2^*(1)}{Y_2^W(1)}\right)Y_2^W(2)$$

$$d=(1+\beta)\beta(1-\pi)Y_1^W$$

$$\Delta=d/q(2)+b\beta(1-\pi)-a-d$$

Since $d/q(2)-b>0$ and $a\beta(1-\pi)-d<0$, (B13) has a positive and a negative roots. Choosing the positive solution gives

$$q^{LA}(2)=\frac{-\Delta+\sqrt{\Delta^2-4(d/q(2)-b)(a\beta(1-\pi)-d)}}{2(d/q(2)-b)} \quad \text{(B14)}$$

Applying (B9) to (B14) yields

$Y_2^W(2)<Y_1^W$

or

$$2g-\sigma<0 \quad \text{(B15)}$$

With (B15) and assumptions made in Table 1 ($Y_2^W(1)>Y_1^W$), (B8) is satisfied. So (B15) is the parameter restriction used in Proposition 2.

Rearranging (B13), one can show

$$(a+bq^{LA}(2))(q^{LA}(2)-\beta(1-\pi))=d(1+q^{LA}(2))(q^{LA}(2)/q(2)-1) \quad \text{(B16)}$$

As a, b, d and $q^{LA}(2)$ are all positive, with $q^{LA}(2)-\beta(1-\pi)>0$ implied by (B15), (B16) can hold if and only if

$q^{LA}(2)/q(2)-1>0$

So $q^{LA}(2)>q(2)$.

Since (B12) is the Arrow price relationship in complete markets without loss aversion, so if $q^{LA}(2)=q(2)$, (B12) must imply $q^{LA}(1)=q(1)$. As $q^{LA}(1)$ varies positively with $q^{LA}(2)$ in (B12), $q^{LA}(2)>q(2)$ implies $q^{LA}(1)>q(1)$. To see how relative prices $q^{LA}(2)/q^{LA}(1)$ must increase in the presence of loss aversion, we rearrange (B12) to yield

$$\frac{1+\beta(1-\pi)}{\beta\pi}Y_2^W(1)=\frac{Y_1^W}{q^{LA}(1)}+\frac{q^{LA}(2)}{q^{LA}(1)}Y_2^W(2)$$

As $q^{LA}(1)>q(1)$, the above equation implies $q^{LA}(2)/q^{LA}(1)>q(2)/q(1)$.

The effect of loss aversion on the equilibrium real interest rates is straightforward to gauge. Because $q^{LA}(2)>q(2)$ and $q^{LA}(1)>q(1)$, so

$$1+r^{LA}=\frac{1}{q^{LA}(1)+q^{LA}(2)}<1+r=\frac{1}{q(1)+q(2)}.$$

To show (5) in Proposition 2, note that from (B5), (B6), (4)—(7), one has

$$\frac{C_2^*(2,LA)}{C_2^*(1,LA)}\Big/\frac{C_2^*(2)}{C_2^*(1)}=\frac{1+\beta(1-\pi)}{\beta\pi}\frac{q^{LA}(1)}{1+q^{LA}(2)}\frac{q(2)}{q(1)}$$

Replacing the relative prices $q^{LA}(2)/q^{LA}(1)$ using (B12), one can show that the right hand side of the above equation is strictly greater than 1.

Appendix D. Proof of Proposition 3.

With parameters $\{\beta', \pi'\}$ for the Foreign country, the optimal consumption without loss aversion gives arise the following set of first order conditions:

$$C_1^* = W^*/(1+\beta') . \tag{C1}$$

$$C_2^*(1) = \beta'\pi'W^*/[(1+\beta')q^{LA}(1)] \tag{C2}$$

$$C_2^*(2) = \beta'(1-\pi')W^*/[(1+\beta')q^{LA}(2)] \tag{C3}$$

Equating (C1) and (B5) gives

$$\beta' = \frac{(1+\beta)q^{LA}(2)+\beta\pi}{1+\beta(1-\pi)} \tag{C4}$$

Equating (C3) and (B7) yields

$$\pi' = \frac{\beta\pi(1+q^{LA}(2))}{(1+\beta)q^{LA}(2)+\beta\pi} \tag{C5}$$

As $q^{LA}(2) > \beta(1-\pi)$, one can easily show that $\beta' > \beta$ and $\pi' < \pi$.