A merger between firms with overlapping markets may lessen the degree of competition in such markets to the detriment of customers, even though it may also create the potential for efficiencies. In order to prevent possible abuse of market power due to such a merger a competition agency can condition its approval on the merging firms undertaking either to divest some of their assets or to accept a package of behavioural remedies. A fundamental problem for the agency is that it inevitably has less information than the merging firms do with respect to the conditions of competition in the market. Even if it has sufficient evidence to deduce that a significant lessening of competition would result from the initial merger proposal, it may be difficult to design appropriate remedies. Investigations can help to discover relevant information, but inquiries are costly. There are also significant incentive problems in that the merging parties do not have the incentive to reveal the truth. However, the agency does have one advantage, in that it can usually design the framework for negotiating with the firms. In this paper, we explore how bargaining over remedies can reveal relevant information. Conversely, we also explore how such negotiation can create inefficient outcomes.

The essential structure of our model is as follows. The agency would like to learn a 'true remedy' that would resolve competition concerns. The firm knows the 'true remedy', while the agency does not. The agency can conduct an independent investigation of a merger to obtain estimates of the true parameters of the merger. However, the agency can make a mistake. The mistake depends on the amount of resources that the agency invests into the investigation (time, number of people, money, etc.).

As a result of a merger investigation, the agency can allow the merger, allow the merger with some remedies, or block the merger. Both the EU and US merger regulations allow two phases of the investigation, though this structure is more formal in the EU. In the first round the agency conducts a preliminary investigation (one month in the EU). Then the agency compares the results obtained in this way, with the remedies that were offered by the merging firms. If the
offer is supported by the results obtained from its own investigation, then the agency approves
the merger with the agreed remedies, otherwise they proceed into the second round (i.e. Phase II
in the EU). In the second round, the agency conducts a more detailed investigation with more
resources invested into it (four months in the EU). However, going into the second round is
costly both for the merging firms and the agency. In this paper, we construct a model of a two-
phase negotiation over remedies between the agency and merging firms. In this way, remedies
may be agreed either in the first or second rounds (i.e. Phase I or Phase II).

There have been few attempts in the literature to address the issue of bargaining over remedies in
merger regulation. Farrell (2003) was one of the first to discuss a bargaining model of remedy
negotiation between an antitrust agency and merging parties. He applied a Nash bargaining
solution concept in determining the appropriate welfare standard of the competition agency while
considering efficiencies due to a merger. Glazer and Rubinstein (2004) develop a persuasion
game model in which one party (a speaker) wishes to persuade another party (a listener) to
accept a certain request. The value of the request depends on two parameters. These are known
only to the speaker, while the listener can check the value of at most one. The authors derived an
optimal rule that determines the listener's response: which aspect to check and whether to accept
or reject the speaker's request. Although they do not apply their model to remedy negotiation, it
is close in structure to the single phase version of our model.

In our model merging firms offer a particular remedy to fix competition concerns in the
knowledge that the competition agency might reject it and move into a costly but more informed
Phase II. Although we are particularly concerned with merger remedies, the bargaining process
we describe could be applied to any situation with asymmetric information between two parties,
and where the party with more information wants to retain the maximum possible amount of
assets but the party with less information wants to learn and enforce a ‘true’ or ideal distribution
of assets. For example, this may be relevant for certain divorce arrangements where out-of-court
settlement would be cheaper but less informed than a fully litigated outcome.

The questions that we try to answer are:

- How does the 2-phase structure of investigation and remedy negotiation affect outcomes?
- How efficient is the remedy negotiation process in revealing the truth?
- What types of error are likely and how harmful are they to welfare?
- How much resource should the agency put into its investigations?
The next section sets out our characterisation of merger remedies, and other main modelling assumptions. Our main propositions are in section 2, and section 3 concludes.

1. The Model

a) Characterisation of Remedies

We characterise a remedy by the parameter $\alpha$, where $\alpha=1$ if the merger proceeds without remedy, and $\alpha=0$ if the merger is completely prohibited or abandoned. For example, $\alpha$ may be thought of as the share of assets retained by the merging firms, which is equivalent to the amount of these assets if we normalise the assets of the target firm at 1. We assume that there is an $\alpha^{TR}$ in $(0, 1)$ that would leave the degree of competition in the industry effectively unchanged (e.g. the merger may lower costs such that customer price and service would be unchanged despite the merged firm’s larger market share). If $\alpha \leq \alpha^{TR}$, then the merger would not significantly lessen competition, but if $\alpha > \alpha^{TR}$, then it would. $\alpha^{TR}$ is known by the firms but not by the agency.

For simplicity, we assume that the expected profit from the merger is proportional to the fraction of assets that the firm can retain after remedies are approved by the antitrust agency, with gross profits of $\pi$ if no remedy is required (i.e. if $\alpha=1$). Clearly, the firms want to keep as many assets as possible. $\alpha^{O}$ is the remedy offered by the merging firms. To be interesting in the present context, we consider a merger proposal that would lessen competition if it were to proceed without remedy, but for which there exists a remedy short of prohibition that would restore competition while preserving at least some potential efficiencies (i.e. $\alpha^{TR} \in (0,1)$).

This simple characterisation allows us to focus on the negotiation process without having to make specific assumptions about underlying market structures or models of competition. While it is only an approximation to any particular set of circumstances, there are several cases for which it may be a reasonable approximation. For example, negotiation over remedies in the Cournot model with capacity constraints. The agency and the merging firms decide on the amount of divested assets ($\alpha =$ capacity) that would resolve competition concerns due to the
merger.\textsuperscript{1} Alternatively, negotiations may be over a threshold value of market concentration that would make collusion possible (Motta, 2004). In a differentiated products model of a merger between firms with multiple brands, a divestiture of different portfolios of brands would lead to different market outcomes. The competition agency wants to identify a true remedy, which is a marginal divested portfolio that does not lead to the abuse of market power ($\alpha = \text{size of portfolio}$).

It is possible that there may be more than one true $\alpha$ that would satisfy this, in which case we should consider the Pareto efficient set determined by the consequences for potential merger efficiencies.

\textit{b) Sequence of Decision Making}

The timing of the game is shown in Figure 1:

1) \textit{Merger decision}: firms decide to go IN or OUT of the merger approval process;  
2) \textit{Phase I}: agency conducts a limited investigation of the merger; the firms can, if they wish, propose a remedy package before the agency decides whether to approve the merger (with or without remedy, e.g. partial divestiture) or not; if the agency and the firms do not agree on the remedy, then the agency refers the case to Phase II;  
3) \textit{Quit option}: firms can proceed to Phase II or opt OUT of the merger;  
4) \textit{Phase II}: agency further investigates the merger, with more resources and precision; the firms can propose the same or a different remedy package before the agency decides whether to approve the merger (with or without remedy) or not; if the agency still considers the remedy proposal to be inadequate, then the agency rejects the merger.

For empirical context, in the European Union between 1990 (when specific merger control at this level was first introduced; ECMR, 1989) and early June 2006, 3044 mergers were large enough and sufficiently pan-European to fall within the scope for control.\textsuperscript{2} Of these, 5\% were remedied in Phase I, 5\% were referred to Phase II, and 2\% were withdrawn.\textsuperscript{3} Of the 129 cases entering Phase II, 60\% were remedied, 15\% prohibited and 17\% withdrawn.

\textsuperscript{1} Note that merging firms and the buyer have an anticompetitive incentive to cripple divested assets in order to reduce capacity in an industry (Farrell, 2003).

\textsuperscript{2} These percentages are calculated from the numbers available on the DG Competition website: http://ec.europa.eu/comm/competition/mergers/cases/stats.html. Note: it appears that these statistics may have been revised over the summer!

\textsuperscript{3} A further 2\% were referred to member states for review at the national level.
Our model leaves the design of the remedy entirely in the hands of the merging firms, with the agency holding veto power. This seems to be a reasonable characterisation of the EU system. However, other negotiation structures would be appropriate for other institutions. For example, remedy proposals may be left in the hands of the agency, with the firms choosing only to accept or to reject the agency’s proposal in stage 2 and/or 4. In such institutions, the agency could restrict the information it receives. At one extreme, it could ignore any remedy proposal coming from the merging firms, and rely entirely on its own investigation. Alternatively, it might condition its final decision on both the firms’ offer and its own investigation. If such an agency chose credibly to restrict itself to a choice between either the firms’ final offer or a prohibition, then this would be effectively the same as in our model.

c) Information

As a result of its Phase I investigation, the agency obtains its own estimate of the required remedy $x_1$. This signal $x_1$ is assumed to be unbiased in the sense that it is drawn out of the distribution of $x$ with mean $\alpha^{TR}$. For simplicity, we assume the distribution to be uniform such that $x \sim U[\alpha^{TR} - \sigma, \alpha^{TR} + \sigma]$ is the range. Both the agency and the firms know the interval of the distribution is $2\sigma$, but only the firms know the true mean. In other words, the agency knows the width of the interval out of which it draws $x_1$ but not the position of this interval on $[0, 1]$. However, we do assume the agency knows that the merger is remediable, so $0 < \alpha^{TR} - \sigma$ and $\alpha^{TR} + \sigma < 1$. For most of this paper we assume that $\sigma$ is exogenous. However, in a variant of the model, we assume the variance of the distribution of $x$ is determined by the amount of resources, $e$, invested in Phase I. The more resources invested, the smaller the range: $\sigma'(e) < 0$, with $\sigma''(e) > 0$. Finally, if there is a second round (Phase II), then the agency and the firm bear costs $K_A$ and $K_F$ respectively, and the agency receives a more accurate signal, $x_2$. We further assume for simplicity that this signal is perfectly accurate, so the agency would learn the true remedy following a Phase II inquiry.

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4 For example, the UK Competition Commission makes final stage 4 proposals in this way, with the firms having veto power in the sense that they can walk away from the merger proposal. Note that under either system, there can
d) Firms’ Objective

The objective of the merging firms is to choose a remedy offer so as to maximize expected profit (as compared with the pre-merger situation). Define $\pi$ as the expected profit increment due to the unremedied merger (i.e. post-merger profit less the sum of pre-merger profits). The merging firms are risk neutral so they choose an optimal remedy offer in Phase I, $\alpha^*$, to maximise:

$$\text{Max}_{\alpha} \left[ \Pr(\text{Approval in Phase I}) \alpha^O \pi + \Pr(\text{Approval in Phase II}) \left[ \alpha^\text{TR} \pi - K_F \right] \right]$$

(1)

e) Agency Objective and Errors

Compared with the pre-merger situation, there is no harm to consumers due to a merger if $\alpha \leq \alpha^\text{TR}$. However, if $\alpha > \alpha^\text{TR}$ we assume that excessive market power will harm consumers (relative to pre-merger) to the extent of $[\alpha - \alpha^\text{TR}]\beta$. We assume $\beta > \pi$, so profits do not outweigh consumer losses at the margin. Continuity in the profit and consumer welfare effects in the choice of remedy, combined with the discrete possibility that the merger may be abandoned, mean that there are three qualitatively different types of error in remedy selection. As usual, there can be Type I errors associated with the agency mistakenly preventing profitable opportunities that would not harm consumers, and Type II errors associated with allowing mergers that are harmful to consumers. In our model, we must add that these may be either ‘incremental’ in the sense that they arise from a choice of remedy that is somewhat too strong ($0 < \alpha < \alpha^\text{TR}$) or somewhat too limited ($0 < \alpha^\text{TR} < \alpha$), or they may be ‘drastic’ in the sense that an otherwise remediable merger fails to go ahead because the agency and firms fail to agree. We call the latter Type ID errors.\(^5\)

We assume that the objective of the agency is to minimise welfare losses due to the merger and the costs of the agency’s investigation. Ideally, it would induce an offer of the true remedy, but this may not be possible. The following welfare effects arise compared with the ideal remedy of $\alpha = \alpha^\text{TR}$ (see Table 1 below for a partial summary):

1. If the accepted remedy is smaller than the true remedy, $\alpha < \alpha^\text{TR}$, then the harm to consumers is zero but the merged firm foregoes some of its profit $[\alpha^\text{TR} - \alpha] \pi$.

---

\(^5\) There is no equivalently drastic Type II error because even if the agency wrongly accepted an offer of $\alpha = 1$, this would be qualitatively the same as what we have called an ‘incremental’ error. This serves to illustrate that the size of an ‘incremental’ error may be larger than a ‘drastic’ error, depending on $\alpha^\text{TR}$ and the relative size of $\beta$ and $\pi$. 

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2. If the remedy is insufficient $\alpha > \alpha^{\text{TR}}$, then the harm to consumers is $[\alpha - \alpha^{\text{TR}}]\beta$. However, if the agency equally values the benefits to shareholders, this must be set against a partially offsetting profit increment of $[\alpha - \alpha^{\text{TR}}] \pi$. More generally, we assume that the agency places a weight of $\varphi$ on profits relative to consumer surplus, where $\varphi \in [0,1]$. $\varphi$ represents the agency’s attitude to profits when they come at the expense of consumer welfare. If $\varphi = 0$, only consumers matter in this case; and if $\varphi = 1$, consumers and shareholders continue to be considered as equally worthy.\footnote{Thus, our earlier assumption that $\pi < \beta$ could be relaxed to $\varphi \pi < \beta$.}

3. If the agency rejects the merger, then the welfare loss is equal to foregone profits due to the merger, $\alpha^{\text{TR}} \pi$.

These welfare effects are summarised in Figure 3.

FIG. 3 [Welfare Effects of Inappropriate Remedy] HERE

Table 1: Errors and Welfare Losses Other than Investigation Costs

<table>
<thead>
<tr>
<th>Firms’ offer</th>
<th>$\alpha^O &lt; \alpha^{\text{TR}}$</th>
<th>$\alpha^O &gt; \alpha^{\text{TR}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approve</td>
<td>$[\alpha^{\text{TR}} - \alpha] \pi$</td>
<td>$[\alpha - \alpha^{\text{TR}}] [\beta - \varphi \pi]$</td>
</tr>
<tr>
<td></td>
<td>Type I</td>
<td>Type II</td>
</tr>
<tr>
<td>Prohibit</td>
<td>$\alpha^{\text{TR}} \pi$</td>
<td>$\alpha^{\text{TR}} \pi$</td>
</tr>
<tr>
<td></td>
<td>Type ID</td>
<td>Type ID</td>
</tr>
</tbody>
</table>

In addition to these market-based costs and benefits, there are investigation costs to consider in both the ‘inevitable’ Phase I and the ‘avoidable’ Phase II. By ‘inevitable’, we mean that once a merger control system is in place, Phase I costs are unavoidable if a merger satisfying the jurisdictional conditions is to take place; while ‘avoidable’ refers to the fact that a Phase II investigation is a case-specific choice by the agency.

4. The agency faces a loss due to the avoidable cost of its investigations, $K_A$.

5. The agency may also wish to take into account the avoidable compliance costs of the firms, $K_F$.

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f) Agency Approval Rules

After obtaining its own estimate of the true (ideal) remedy, the agency compares its preliminary finding, \( x_1 \), with the offer made by the firm, \( \alpha^O \), and applies an approval rule. In this paper, we consider the following approval rule: \( \text{if } x_1 > \alpha^O \), then approve the remedy, otherwise proceed to the next phase (if applicable). This is a special case of the linear approval rule: approve \( \text{iff } x_1 > \theta \alpha^O + \epsilon \). If there is a second round, the process is repeated; i.e. the firm sends a signal \( \alpha^O_2 \), and simultaneously the agency picks up its own signal \( x_2 \) out of the distribution of \( x \), where in Phase II, \( x \sim \text{U}[\alpha^{TR}, \sigma^2/3] \) and \( \sigma < \sigma^2 \). As already stated, in this paper we assume that the agency learns the value of the true remedies for sure in the second phase, \( \sigma^2=0 \).

2. Analysis and Results

It is convenient to define the following terms:

- Size of the remedy = \( 1 - \alpha^O \)
- Potential harm of the merger = \( 1 - \alpha^{TR} \)
- Excessive (deficient) remedy offer by the firms if \( \alpha^{TR} - \alpha^O > 0 \) (< 0)
- Accuracy of the agency’s investigation = \( 1 / \sigma \)
- Probability of prohibition = probability of failure to agree on a remedy
- Relative inaccuracy of the agency’s Phase I investigation = \( \frac{\sigma}{\alpha^{TR}} \)
  - The investigation is relatively inaccurate if \( 3 \sigma > \alpha^{TR} \)
  - The investigation is relatively accurate if \( 3 \sigma \leq \alpha^{TR} \)
- Relative cost of Phase II to the firms = \( \frac{K_F}{\alpha^{TR} \pi} \)
- ‘The firms’ Phase II cost relative to agency’s Phase I inaccuracy’ is defined as the ratio of the relative cost to the firms in Phase II to the relative inaccuracy of the agency’s Phase I investigation. This is:
  - High if \( \frac{K_F}{\pi} \geq 3 \sigma \)
  - Low if \( \frac{K_F}{\pi} < 3 \sigma \)
In order to identify the underlying mechanisms, we proceed by considering cases of increasing complexity. The first two cases have a passive agency in the sense that its investigation costs are exogenous and it follows a simple rule in accepting or rejecting remedy offers.

**Case 1: Single Phase Investigation With Exogenous Agency Effort**

If there is only one round (i.e. phase of investigation), so there is no opportunity for approval in round two, and if the agency does not choose an effort level, it has a simple choice: either approve the offered remedy or prohibit the merger.

**Proposition 1:** If agency investigation costs are exogenous and there is only a single phase of merger control:

a) The size of the remedy offer by the firms (e.g. assets offered for divestiture) is increasing in the potential harm of the merger. If the investigation is relatively inaccurate, the size of the offer is increasing in the accuracy of the investigation; but if it is already relatively accurate, then the size of offer declines as the agency’s accuracy improves.

b) The optimal remedy offer is excessive (i.e. creates a Type I error). This error is decreasing in the potential harm of the merger unless the investigation is relatively accurate (in which case the error is independent of $a^{TR}$. The error is increasing in the accuracy of the agency’s investigation if the investigation is relatively inaccurate, but decreasing if it is relatively accurate.

c) The probability of prohibition is strictly positive (i.e. Type ID error) unless the investigation is relatively accurate, in which case it is zero. For relatively inaccurate investigations, this probability is increasing in the potential harm of the merger and decreasing in the accuracy of the agency’s investigation.

**Proof**

The interval from which $x$ is drawn is $[\alpha^{TR} - \sigma, \alpha^{TR} + \sigma]$. If $\alpha^O$ were to fall outside this range, then the probability of approval would be either 1 (in which case $\alpha^O$ could be profitably reduced) or zero (in which case $\alpha^O$ could be profitably increased). See Fig. 2. Thus, $\alpha^O$ must be chosen within this interval, and the probability of approval of the offered remedies is the following:

$$\Pr(\alpha^O) = \Pr(Approval \ in \ Phase \ I) = \Pr(\alpha^O \leq x) = 1 - \Pr(x < \alpha^O) = \frac{1}{\sqrt{\frac{\alpha^O - \alpha^{TR}}{\sigma}}} \quad (2)$$
From (1) and (2), and given the absence of an opportunity to agree in Phase II, the objective of
the merging firms is $\max_o \left\{ \Pr(o) \alpha^O \right\}$. Suppose there is an internal solution, then the first
order condition can be rearranged to give the optimal offer (Proposition 1a):

$$\alpha^* = \frac{\alpha^{TR} + \sigma}{2} \quad (3)$$

The condition that this offer lies within the feasible range (i.e. $\alpha^O > \alpha^{TR} - \sigma$) is:

$$\alpha^{TR} < 3\sigma \quad (4)$$

If this condition does not hold, there is a corner solution with

$$\alpha^* = \alpha^{TR} - \sigma \quad (3a)$$

From (3) and (3a), the difference between the true offer and the optimal offer (Proposition 1b) is:

$$\alpha^{TR} - \alpha^* = \frac{1}{2} \left[ \alpha^{TR} - \sigma \right] > 0 \quad \text{if} \quad \alpha^{TR} < 3\sigma$$

$$\alpha^{TR} - \alpha^* = \sigma \quad \text{if} \quad \alpha^{TR} \geq 3\sigma \quad (5a)$$

(5) and (5a) are always positive by our assumption that the agency knows the merger is
remediable so $\alpha^{TR} - \sigma > 0$.

From (2) and (3), the probability of failure to agree (Proposition 1c) is:

$$1 - \Pr(\alpha^*) = 1 - \left[ \frac{1}{2} - \frac{1}{2} \frac{\alpha^{TR} + \sigma - \alpha^{TR}}{2\sigma} \right] = \frac{1}{4}\left[ 3 - \frac{\alpha^{TR}}{\sigma} \right] > 0 \quad \text{if} \quad \alpha^{TR} < 3\sigma$$

$$1 - \Pr(\alpha^*) = 0 \quad \text{if} \quad \alpha^{TR} \geq 3\sigma \quad (6a)$$

Thus, this probability lies in the range $[0, 1)$ and is strictly positive when condition (4) holds.

QED

Observe that Proposition 1b does not arise from risk aversion – we have explicitly assumed that
firms are risk neutral. The excessive remedy offer is due to the drastic downside of a failure to
agree, which creates an asymmetry of payoffs in balancing an offer that is slightly too big
compared with one that is slightly too small.

One apparently surprising result is the non-monotonic relationship between the Type I error and
the accuracy of the agency’s investigation. Intuitively, when the agency is very inaccurate, the
merging parties focus on the average of the maximum gain ($\alpha^{TR} + \sigma$) and failure to agree;
whereas once the agency is sufficiently accurate, it pays to offer whatever is necessary to ensure
that there is definite agreement ($\alpha^{TR} - \sigma$).
**Case 2: Two Phase Investigation With Exogenous Agency Effort**

We now allow firms a second chance at agreeing remedies, but at the cost of having to take part in a Phase II investigation.

**Proposition 2:** If agency investigation costs are exogenous, there are two phases of merger control, there is complete revelation of the optimal remedy \((\alpha^{TR})\) in Phase II if the investigation reaches that far, and if the firms do not face prohibitive costs if referred to Phase II:

a) The size of the Phase I remedy offer by the firms is increasing in the potential harm of the merger, If the investigation is relatively inaccurate, the size of the offer is increasing in the accuracy of the investigation; but if it is already relatively accurate, then the size of offer declines as the agency’s accuracy improves (i.e. as in case 1). If the investigation is relatively inaccurate, the size of Phase I offer is increasing in the cost to firms of a Phase II investigation relative to the potential profitability of an unremedied merger.

b) The optimal Phase I remedy offer may be either excessive or deficient. If the Phase II cost to the firms is low relative to the agency’s Phase I inaccuracy, a Type I error is more likely (and so a Type II error is less likely) the greater are the firms’ costs of Phase II investigation relative to the potential profits of an unremedied merger, and the greater the accuracy of the agency’s investigation. If the Phase II cost to the firms is high relative to the agency’s Phase I inaccuracy, then the firms’ costs and expected profits do not affect the error, but the remedy is always excessive and increasing in the accuracy of the agency’s investigation;

c) The probability of a Phase II investigation is strictly positive unless the Phase II cost to the firms is high relative to the agency’s Phase I inaccuracy, in which case it is zero. In the former case, the referral probability is decreasing in both the accuracy of the agency’s investigation, and the firms’ costs of Phase II investigation relative to the potential profits of an unremedied merger.

**Proof**

The firms know that the true remedy would be agreed if they go into Phase II, so their objective is:

\[
\text{Max}_{\alpha^o} \{\text{Pr}(\alpha^o)\alpha^o \pi + \left[1 - \text{Pr}(\alpha^o)\right] \alpha^{TR} \pi - K_F \} 
\]

Suppose there is an interior solution. From (2) and (7), and after simplification, the first order condition can be rearranged to give (Proposition 2a):
\[ \alpha^* = \alpha^{TR} + \frac{1}{2} \left[ \sigma - \frac{K_F}{\pi} \right] \]  

(8)

The condition that this is interior to the interval from which the agency draws its information (i.e. \( \alpha^0 > \alpha^{TR} - \sigma \)) is:

\[ \frac{K_F}{\pi} < 3\sigma \]  

(9)

If condition (9) does not hold, then

\[ \alpha^* = \alpha^{TR} - \sigma \]  

(8a)

Thus, from (8), (9), (8a) and \( K_F \geq 0 \), \( \alpha^* \in [\alpha^{TR} - \sigma, \alpha^{TR} + \sigma/2] \).

From (8), the difference between the true offer and the optimal offer (Proposition 2b) is:

\[ \alpha^{TR} - \alpha^* = \frac{1}{2} \left[ \sigma - \frac{K_F}{\pi} \right] \] if \( \frac{K_F}{\pi} < 3\sigma \) and

\[ \alpha^{TR} - \alpha^* = \sigma \] if \( \frac{K_F}{\pi} \geq 3\sigma \)  

(10)

(10a)

There is no restriction for (10) to be either positive or negative, so the offer may be deficient. On the other hand, if the offer is excessive it cannot be as excessive as in single phase merger appraisal (case 1) because the merger would be abandoned before going into Phase II if that would not be profitable unless \( \alpha^{TR} \pi > K_F \). Comparing (5) and (10), if there is a Type I error in case 2, it must be less than in case 1.

The probability of failure to agree in Phase I, and so of referral into Phase II, (Proposition 2c) is:

\[ 1 - \Pr(\alpha^*) = 1 - \frac{\alpha^{TR} + \frac{1}{2} \left( \sigma - \frac{K_F}{\pi} \right) - \alpha^{TR}}{2\sigma} = \frac{1}{4} \left[ 3 - \frac{K_F}{\sigma \pi} \right] > 0 \] if \( \frac{K_F}{\pi} < 3\sigma \) and

(11)

\[ 1 - \Pr(\alpha^*) = 0 \] if \( \frac{K_F}{\pi} \geq 3\sigma \)  

(11a)

The probability in (11) lies in the range \((0, \frac{1}{4})\). Also, because \( \alpha^{TR} \pi > K_F \), the probability of referral to Phase II in case 2 must exceed the probability of referral in case 1 (holding all parameter values except the bargaining procedure constant). \( QED \)
Proposition 2 identifies another bias in Phase I remedy offers, which works in the opposite direction to the asymmetry identified in case 1. Consider one extreme as $K_F/\pi \to 0$, so firms can ignore the costs they would incur in Phase II. This removes the downside of a rejected offer, so the merging firms will try to keep a larger share of assets. They still have an incentive to try to win approval of an insufficient offer, because this would be better than being left with $\alpha^{TR}$. If the agency’s market testing turns out to be lucky for the firms, they will win Phase I approval and the agency will incur a Type II error. At the other extreme, as $K_F \to \alpha^{TR}\pi$ so it would hardly be worth proceeding to Phase II, the bias from case 1 dominates (i.e. the large downside of a rejected offer) so the agency incurs a Type I error if it accepts what will be an over-generous offer.

Another way to interpret Proposition 2 is if we define the ‘size of merger’ as $\pi/K_F$. This defines size by the potential to increase profit, relative to compliance costs which are likely to exhibit economies of scale. Proposition 2 then shows that firms involved in bigger mergers will offer less generous remedies and are more likely to go into Phase II. Note that the latter prediction is due to the bargaining stance taken by the merging firms, and is not due to a judgement by the agency that it is ‘not worth’ pursuing small mergers.

**Case 3: Endogenous Agency Effort**

This case is allows the agency to decide how deeply to investigate the merger in Phase I, conditional on the information it has available at the time. This active role requires an explicit treatment of the agency’s objective function. Following the discussion in section 1d), as summarised in Table 1, the agency minimizes the following loss function:

$$\Pr(\alpha)\{ I(\alpha)\[(\alpha - \alpha^{TR})(\beta - \varphi\pi)] + [1 - I(\alpha)]\[(\alpha^{TR} - \alpha)\pi ] \} + \Pr(Phase II) (K_A + \varphi K_F) + e$$

where $I(\alpha) = \{1, \text{ if } \alpha \to \alpha^{TR}; 0, \text{ if } \alpha < \alpha^{TR}\}$

In the case of a single phase investigation, $\Pr(Phase II) = 0$, so the loss function specialises to:

$$\Pr(\alpha)\{ I(\alpha)\[(\alpha - \alpha^{TR})(\beta - \varphi\pi)] + [1 - I(\alpha)]\[(\alpha^{TR} - \alpha)\pi ] \} + e$$

However, the optimal effort level will be dependent on such an exogenous parameter as a true remedy, which by definition is not known to the agency.
Further Alternatives PROVISIONAL

So far in the paper we described a simultaneous move game (Case 1). At the same time the situation when the agency moves first is identical to the Case 1 as well, the agency chooses the effort level e, which determines the accuracy of the investigation, which in its turn would determine the optimal offer by the firm. (e → σ → αO). Therefore the equilibrium offer is equivalent to the Case 1

Another situation is when the Firm moves first. This implies a signalling game with the firms recognising how their offer will be interpreted, and trying credibly to persuade the agency to accept the minimum remedy. Given the setting of our model, the agency fixes the amount of resources it dedicates to every merger investigation. Knowing the accuracy of the investigation, the merging firms choose the optimal offer given equations (3) or (8). However, the agency might try to infer the value of αTR from the firms’ offer, αO using the same equations but without any hard proof of it. Unfortunately the agency is sometimes tempted to go ahead with the remedies without numerical support from its own investigations, but the remedies are dismissed by a judge at the appeal stage.

3. Conclusions

This paper provides a first attempt to understand how outcomes are determined by the 2-phase structure of most merger investigation institutions and related remedy negotiations. We find that there are inherent biases, and identifiable circumstances where we would expect remedy offers to be excessive, and where we would expect them to be deficient. In principle, these predictions could be confirmed or rejected empirically. We also find the circumstances in which a Phase II investigation is most likely. This is driven not by the potential harm of an unremedied merger – plenty of such mergers are remedied in Phase I – but by factors affecting the incentive of the merging parties to offer more or less generous remedies early in the investigation. Our model also opens the way for extensions that might increase our understanding of how an agency should best allocate resources across investigations.

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References


Figure 1: Characterisation of Remedy
Figure 2: Sequence of Decisions in Merger Regulation

1. Merger Proposal
   - M
   - No merger proposal

2. Phase I
   - CA
   - Firms propose merger (with remedy offer)
   - Agency approves merger (subject to remedy)

3. Quit Option
   - M
   - Proposal withdrawn

4. Phase II
   - CA
   - Firms make Phase II remedy offer
   - Agency approves merger (subject to remedy)
   - Prohibition

Figure 3: Welfare Effects of Inappropriate Remedy

Welfare (exclusive of investigation costs) Relative to No Merger

0 1

Prohibition

\[ \alpha^{TR} \]

No Remedy