The Impact of Cost-Reducing R&D Spillovers on the Ergodic Distribution of Market Structures

Christopher A. Laincz* and Ana Rodrigues**

Abstract
We study a dynamic duopoly model of R&D to analyze the impact of imperfect appropriability on market structure and welfare. We pursue this analysis by extending the Markov-Perfect dynamic industry model proposed by Ericson and Pakes (1995), through the introduction of a non-proprietary productivity component to R&D as part of a dynamic, stochastic process. We find that when spillovers are costlessly obtained increases in the extent of spillovers leave industry concentration largely unchanged but rates of innovation fall leading to losses in welfare through both reduced consumer surplus and firm values. In contrast, when spillovers require absorptive capacity investment in own R&D, larger spillovers lead to declines in concentration level. However, rates of innovation increase and welfare rises. The difference lies in the degree of substitutability between own and external R&D sources. When own and external R&D are perfect substitutes, firms’ R&D investment declines. When spillovers can only be obtained through absorptive R&D, the degree of substitutability falls, generating higher rates of innovation, particularly by smaller firms, leading to a less concentrated market structure.

* Drexel University, LeBow College of Business, Department of Economics, Philadelphia, PA 19104 (claincz@drexel.edu) (Corresponding Author).
** University of York, Department of Economics, York, UK (asdr101@york.ac.uk).

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1 Introduction

Imperfect appropriability of the returns to R&D generates a negative incentive for firms to innovate creating underinvestment in R&D relative to the optimum. At the same time, imperfect appropriability may mitigate socially wasteful R&D to the extent that R&D is duplicative across firms. The initial literature on the problems of imperfect appropriability and free riding in R&D analyzes the welfare implications in the presence of the trade-off between private incentives to innovate and efficient use of knowledge. Michael Spence (1984) develops a deterministic model where symmetric firms invest in cost-reducing R&D with spillovers to rivals. He finds that the absence of appropriability leads unambiguously to suboptimal underinvestment in R&D. Policies that increase appropriability still leave the market inefficient because the R&D costs required to achieve a given rate of industry cost reduction are higher with full appropriability. He concludes that the best market performance is achieved in an industry with low appropriability coupled with subsidies to restore incentives to invest and improve allocative efficiency.

However, Levin and Reiss (1988) and Cohen and Levinthal (1989) show that treating external knowledge as a pure public good, acquired at zero cost by all firms in the industry, is in the genesis of the results pointing to subinvestment in the presence of R&D spillovers. They argue that spillovers are not acquired costlessly, but firms need to establish absorptive capacity to take advantage of spillovers. This point is in line with Rosenberg (1974) and Nelsons' (1982) claim that, in order to be able to take advantage of externalities, the firm has to undertake some R&D investment of its own. For example, a firm may need laboratories with research personnel that can interpret and make use of the industry pool of knowledge.

Later work by D’Aspremont and Jacquemin (1988) with further refinements by Henriques (1990), Suzumura (1992), Simpson and Vonortas (1994) and Ziss (1994) analyzes these issues with regards to joint research ventures. These models, as those above, are typically two-stage oligopolies wherein firms first choose R&D then compete in the product market and acquire spillovers costlessly from rival firms. They generally find that large spillovers are beneficial to welfare despite the negative incentives for R&D investment and, in some cases, the joint monopoly (or joint venture) solution outperforms the competitive solution in terms of welfare because it reduces duplication of research effort.\footnote{Song (2005), in an applied dynamic analysis of research joint ventures, reaches similar conclusions in his study of SEMATECH.}
All of these analyses start from the assumption of a *given market structure* (usually symmetric for tractability reasons) and then proceed to evaluate the R&D incentives and welfare results. By market structure we mean the degree of asymmetry (or lack thereof in the symmetric case) in market shares between firms. But as Dasgupta and Stiglitz (1981) and Louy (1979) make clear, the market structure itself is endogenous to the incentives to conduct R&D with or without spillovers. In addition, Kamien and Schwartz (1971) and Reinganum (1983) argue for the relevance of uncertainty in R&D versus determinism when modelling rivalry. Reinganum shows that uncertainty, generally assumed away in the literature, is not innocuous for the case of preemptive patenting, and its presence reverses the strength of incentives between an incumbent and potential rival in conducting R&D. Moreover, Flaherty (1980) carefully examines the stability properties of market structures in a dynamic game of perfect foresight with cost reducing R&D. She finds that the symmetric case is unstable but the asymmetric case is stable to an unanticipated shock under reasonable conditions including the linear demand model commonly used in this literature. Empirically, Mansfield (1984) argues that “…an industry’s concentration level tends to be low if its members’ products and processes can be imitated easily and cheaply… Apparently, differences among industries in the technology transfer process may be able to explain much more of the interindustry variation in concentration levels than is generally assumed.”

Thus, characterizations of the appropriability issue based on the analysis of two-stage, perfect foresight, symmetric market structure models may very well not be the best representation of such industries. Ultimately, it calls into question the welfare results which often hinge on the degree of appropriability. That leads to the following questions that we address: 1) How do knowledge spillovers affect the incentives to conduct R&D and rates of innovation in a dynamic context? 2) In the presence of R&D with spillovers what market structures are most likely to emerge? and 3) What are the welfare consequences with spillovers?

We study a dynamic duopoly model of R&D to analyze the implications of imperfect appropriability of R&D on welfare and market structure. We pursue this analysis by extending the Markov-Perfect dynamic industry model proposed by Ericson and Pakes (1995) (hereafter, "E-P"), through the introduction of a non-proprietary productivity component to R&D as part of a dynamic, stochastic process. This model allows for the *joint determination* of both market structure and R&D as endogenous variables whereas two-stage models must assume a starting market structure. The feedback effect between the degree of concentration and the rate of innovation in
the presence of imperfect appropriability is the novel feature of this model.

We examine the equilibrium market structure and welfare under two different appropriability scenarios: 1) firms obtain R&D spillovers \textit{costlessly}; and 2) R&D spillovers require investment in an absorptive capacity. In Spence’s (1984) costless characterization R&D falls with spillovers, but in Levin and Reiss (1988) and Cohen and Levinthal (1989) R&D can increase with spillovers due to need for absorptive capacity. We extend these results to a dynamic stochastic framework, but we also go further by addressing the impact of spillovers on the equilibrium level of concentration. We find that when spillovers require absorptive capacity as a by-product of R&D investment, concentration levels decline with increases in the extent of spillovers and welfare rises through increased consumer surplus. However, when firms obtain spillovers costlessly, increases in the extent of spillovers have ambiguous effects on concentration levels while consumer surplus declines because the rate of innovation falls leading to higher prices.

The remainder of the paper proceeds as follows: Section 2 begins by laying out the two cases for how firms acquire spillovers from R&D and then specifies a fully dynamic model. Section 3 presents comparative statics on the policy functions in the costless case, while Section 4 examines the absorptive capacity case. The paper then proceeds to illustrate the effects highlighted through numerical simulation in Section 5. Section 6 concludes.

\section{The Model}

We model a duopolistic industry where firms produce homogeneous goods and engage in Cournot competition in the spot market. Firms can engage in R&D to lower their marginal costs in the future and thus (potentially) increase their market share \textit{vis-a-vis} their rivals. The Cournot-Nash outcome in the spot market yields higher profits for the firm with the relatively lower marginal costs and hence drives the incentives for investing in R&D and taking advantage of R&D spillovers. The equilibrium of this class of models is Markov-Perfect in the sense of Maskin and Tirole (1988), and a rational expectations equilibrium, i.e., equilibrium is reached when the expectations of the firms about future states are the future states that are actually a consequence of those expectations (See E-P, 1995).
2.1 R&D and Appropriability

Following Levin and Reiss (1988) and Cohen and Levinthal (1989), we express the total amount of innovative R&D that firm $i$ can utilize to pursue its innovative purposes, $m_i$, as:

$$m_i = x_i + \gamma(x_i)bx_j$$  \hspace{1cm} (1)

where $x_i$ represents firm $i$’s own R&D investment while the rival’s R&D is denoted $x_j$. The function $\gamma(x_i)$ determines the absorptive capacity of the firm and is assumed strictly concave in $x_i$ and satisfies $0 \leq \gamma(x_i) \leq 1 \forall x_i$ and $\gamma(0) = 0$. The parameter $b$ lies in the range of $0 \leq b \leq 1$ and captures the extent of the intra-industry spillovers, i.e. the fraction of rivals’ R&D that is useful and available to firm $i$. The extent depends on the ease of imitation, patent policies, worker mobility, the amount of knowledge embodied in the output of the innovation process, the degree of tacit knowledge required, etc. Larger $b$ implies that a higher proportion of firms’ R&D investment enters the industry pool of knowledge. If $b$ is set to zero, then R&D is perfectly appropriable and we obtain the specification in E-P. If $b$ equals unity, then all R&D in the industry is publicly accessible, but not necessarily used because absorptive capacity itself may be costly.\footnote{We model spillovers here based on the flow of R&D in the manner of the majority of the previously discussed literature such that our results can be contrasted. In our concluding section, we briefly discuss the potential effects and differences if we considered spillovers to arise from stocks of knowledge instead of flows of R&D. However, empirical evidence suggests that within a firm R&D is highly persistent and concentrated among large firms such that flows may be a reasonable proxy for stocks (See Malerba, Orsenigo, and Peretto, 1997).}

Here we specify the same functional form for absorptive capacity as the R&D function used in E-P, but the specifics depend on the case being considered.

$$\gamma(x_i) = \begin{cases} 
1 & \text{Costless Case} \\
\frac{\gamma x_i}{1 + \gamma x_i} & \text{Absorptive Capacity Case} 
\end{cases}$$  \hspace{1cm} (2)

We consider two cases of limited appropriability as shown in (2).\footnote{In addition, we considered a third case wherein R&D investment in own knowledge and investment in attempting to obtain spillovers were separable functions. Here, one can think of a separable function for spillovers in two ways: either the need for a basic R&D lab to keep abreast of research developments or as a mechanism for pure imitation of rivals. It turns out the analysis and results of this case are very similar to the costless case we analyze here because the separable case allows for similar substitution patterns by the firms. Since the separable case added little to the results, we chose to focus on the two main approaches found in the literature.} First, following the early literature on appropriability of R&D, we consider the case that R&D spillovers are costless to obtain.\footnote{For example, see Spence (1984) and D’Aspremont and Jacquemin (1988).} The absorptive capacity function becomes $\gamma(x_i) = 1$, $\forall x_i$ (including $x_i = 0$) and the
amount of usable R&D becomes:

\[ m_i = x_i + bx_j. \]  

(3)

This specification implies that a firm need not do any research of its own in order to increase efficiency. It also entails a strong negative incentive for own R&D because innovative activity creates external benefits for rivals. That negative incentive increases with the extent of spillovers (or lack of appropriability), i.e. as \( b \) increases.

In the second case, the absorptive capacity of firm \( i \) is a monotonically increasing concave function of own R&D investment, i.e. \( \gamma_x > 0 \) and \( \gamma_{xx} < 0 \). The parameter \( \gamma \) in the equation above captures the productivity of this type of investment in increasing firm \( i \)'s absorptive capacity and captures the ease of learning the specificities of the technological knowledge, e.g. highly sophisticated and complex in contrast to easily recognizable knowledge. The specification thus distinguishes between the extent (captured by \( b \)) and the productivity (captured by \( \gamma \)) of spillovers as in Levin and Reiss (1988). They note that extent reflects the degree of patent protection and secrecy, for example, but the productivity reflects the applicability of acquired R&D. Crucially, in the absorptive capacity case, R&D encompasses both the role of innovation and absorption of external knowledge. Thus, the marginal contribution to usable knowledge will exceed unity whenever an external pool of R&D activity exists, \( x_j > 0 \), in the presence of spillovers, \( b > 0 \).

This case converges to the costless case for an arbitrarily small level of \( x_i > 0 \) as

\[ \lim_{\gamma \to \infty} \left[ \frac{\gamma x_i}{1 + \gamma x_i} \right] = 1. \]  

(4)

In the costless case, the contribution of spillovers to knowledge depends only on the extent. Thus, a firm can choose to eschew own R&D entirely and still innovate, but under absorptive capacity some own R&D is required for innovation.

The outcome of firm \( i \)'s innovative activity, captured by the random variable \( \nu_i \), is given by:

\[
\nu_i \begin{cases} 
1 & \text{with probability } p(\nu_i) \\
0 & \text{with probability } 1 - p(\nu_i)
\end{cases}
\]  

(5)

where

\[ p(\nu_i) = \frac{am_i}{1 + am_i} = \frac{a(x_i + \gamma (x_i) bx_j)}{1 + a(x_i + \gamma (x_i) bx_j)}. \]  

(6)
Thus $\nu_i$ is a binary variable indicating successful innovation when $\nu_i = 1$ and failure otherwise. $a$ denotes the productivity of usable knowledge which represents the technological opportunity in the industry, i.e., the difficulty of innovating in the industry as it relates to the stage of development of scientific knowledge and other knowledge specific characteristics. The knowledge production function in (6) determines the probability that a firm is successful in R&D, hence the outcome of R&D is uncertain in our model.

The link from R&D to profitability is through process innovations and we track the efficiency levels of firms as the sum of innovations or number of times it was successful. The marginal costs at any point in time depend on the efficiency level of the firm at time $t$:

$$mc_i = mc_0 e^{-\eta w_i}$$ (7)

where $w_i \in \mathbb{Z}^+$ stands for the efficiency level of firm $i$. $\eta > 0$ captures the rate at which marginal costs decrease with a unit increase in the efficiency level and $mc_0$ represents the marginal cost of a firm with $w_i = 0$. Higher efficiency levels imply lower marginal costs. The industry is characterized, at time $t$, by the efficiency level of the firms represented by the vector $s_t = [w_{it}, w_{jt}]$ where $w_{it}$ represents firm $i$’s efficiency level at time $t$ (though we drop the time subscripts throughout the remainder of the paper).

The evolution of the efficiency levels is given by:

$$w_i' = w_i + \nu_i - \varepsilon$$ (8)

where prime indicates the next period. $\nu_i$ is the firm specific random variable that takes on the value of 1 if the firm successfully innovates and 0 otherwise as given above in (5). The second random variable, $\varepsilon$, is also binary taking on the value of 1 with exogeneous probability $\delta > 0$ and zero otherwise. We think of an outcome of $\varepsilon = 1$ as representing an exogenous increase in the factor price index for the industry, as in E-P (1995). This shock to the efficiency level induces some degree of correlation in firms’ fates, as it changes all firms’ costs simultaneously. The role of $\varepsilon$ in the dynamic model (specified below) bounds the efficiency levels from above in equilibrium and generates pressure on all firms to continue engaging in R&D to maintain profits.\footnote{One comment relating our work to the previous literature is in order. Because we characterize the R&D process as one of uncertainty, our mapping from R&D effort to future marginal costs differs from most previous work (e.g. Spence, 1984, Levin and Reiss, 1988, and Cohen and Levinthal, 1989). However, consider a deterministic}
The probability of an increase in the efficiency level of firm $i$ is then:

$$\text{Probability}(w_i' = w_i + 1) = (1 - \delta) \frac{a(x_i + \gamma(x_i) b x_j)}{1 + a(x_i + \gamma(x_i) b x_j)}$$  \hspace{1cm} (9)$$

For illustration purposes, Figure 1 shows a parameterized example of the R&D success function in the costless case. The x-axis displays own R&D efforts and the y-axis shows the total probability of success, the probability that follows directly from own R&D (which is equivalent to the zero spillovers case) and the incremental probability that follows from spillovers. Because knowledge is perfectly and costlessly acquired, own R&D has no impact on the effect of the spillovers and the probability that the firm is successful rises merely because they are able to observe rivals’ activities. The figure shows the extreme impact of this assumption, in that a firm conducting absolutely no R&D has a positive (and potentially large) probability of innovating simply because they benefit directly from their rivals’ expenditures. However, the marginal contribution of external knowledge falls with greater levels of own R&D.

Figure 2 displays the absorptive capacity case. The highest curve is the total probability of success, while at low levels of own R&D the major benefits derive from absorptive capacity and not own innovative activity. These added benefits initially rise with absorptive capacity, but the additional benefit of the spillovers diminishes at higher levels of own R&D.

2.2 The Spot Market:

The demand for the industry product is assumed linear and is given by the following inverse demand function:

$$P(Q) = A - B(Q) \quad \text{with } A, B > 0$$  \hspace{1cm} (10)$$

where $P(Q)$ is the price of the good produced, and $Q$ the industry output. At the beginning of each period firms compete in quantities and solve the following standard profit maximization problem:

$$\max \pi_i = (P(Q) - mc_i)q_i - f$$  \hspace{1cm} (11)$$

function mapping R&D effort directly into marginal costs where marginal costs are a decreasing, convex function of R&D effort. If one appends a bounded error term, you would obtain a similar structure to the one discussed above in expectation. The reason the error term needs to be bounded here is that our structure imposes lower and upper limits to how much marginal costs can change over a single period. Over a longer time horizon the bounds expand generating greater potential changes in the long-run through the dynamic set up of the model.
where $q_i$ is the quantity produced by firm $i$ and $f$ is the fixed cost of production.

The Cournot-Nash equilibrium will determine the following optimal quantity choices:

$$q_i^* = \frac{A + mc_j - 2mc_i}{3B} \tag{12}$$

which will jointly determine the following equilibrium price:

$$P^* = \frac{A + mc_i + mc_j}{3} \tag{13}$$

Given equilibrium price and quantities, equilibrium firms’ profits are:

$$\pi_i^* = \max \left\{ -f, \frac{[A + mc_j - 2mc_i]^2}{9B} - f \right\} \tag{14}$$

The firm will choose whether to produce or not based on how its marginal costs compare with that of their rival as seen in the numerator of (14). If its marginal costs are too large relative to its competitor, the firm will choose not to produce and just pay fixed costs.

### 2.3 Optimization

In each period of time, firms choose the level of R&D expenditures that maximizes the expected present discounted value of their future stream of profits. The state of the industry is summarized by the vector of the efficiency levels of the firms. Denote $pr(w_i', s' \mid w_i, s)$ as a firm’s perceptions of the joint probability that its efficiency will evolve to $w_i'$ in the next period, and that the market structure it faces will be $s'$, conditional the firm’s current state and the current market structure. The optimal R&D choice solves the following Bellman equation subject to a non-negativity constraint on R&D:

$$V_i(w_i, s) = \max_{x_i \geq 0} \{ \pi(w_i, s) - c_x x_i + \beta E_i V_i(w_i', s') \} \tag{15}$$

where $\beta = 1/(1 + r)$, with $r$ standing for the interest rate, is the discount factor common to all firms, and $\pi(w_i, s)$ denotes the profits to firm $i$ with efficiency level $\omega_i$ in the spot market with market structure $s$. $c_x$ represents the unit cost of R&D.

To understand the impact of the extent of spillovers, we need to understand the basic shape
of the value function and policy functions. Figure 3 shows the solution to the value function for firm 1 using the parameters listed in Table 1 with \( b = 0 \) and Figure 4 shows the policy function. The shapes shown here are not particular to the parameters used, but apply generally (See E-P, 1995, and Pakes and McGuire 1994). The x-axis represents firm 1’s efficiency level, the y-axis is the rival firm’s (firm 2) efficiency level, and the z-axis is firm 1’s value in Figure 3 and R&D effort in Figure 4. Higher efficiency levels mean lower marginal costs. Notice that holding the rival firm’s efficiency level fixed, the value function takes a distinctly convex shape at low levels of efficiency, hits an inflection point then becomes concave. At the extreme ends, the value function is relatively flat. In these regions, firms enter coasting states as discussed in E-P (1995). Firms too small to compete cease R&D because the marginal gains are too small relative to the costs given that its rival has such a large advantage. However, as the gap in efficiency levels shrinks, the value increases at an increasing rate creating larger marginal increments in value. Eventually, with higher relative efficiency levels, the firm enters the concave portion of the value function. This region represents a firm with most or all of the market that achieves little benefit from reducing costs because it captures little, if any, additional market share from its rival and it faces diminishing returns in the product market itself for additional cost reductions.

Looking at the policy function we see that as firm 1’s efficiency increases R&D investment increases holding the rival firm’s efficiency fixed. Eventually this R&D investment reaches a maximum (around 5 where the marginal increase in value is at its greatest) and begins declining as the marginal gains from further investment fall. When firm 1 is the more efficient firm, as the rival firm’s efficiency, \( w_2 \), increases there is initially a decline in own R&D but little difference as the rival firm becomes significantly more efficient.

In the optimization problem for firm \( i \) it is convenient to let \( C_S(w_i + 1, s'_m) \) denote the expected value of the firm conditional on success in innovating, and let \( C_F(w_i, s'_m) \) denote the expect value in the case it fails to develop an innovation. Then one can rewrite the general value function (15) above as:

\[
V_i(w_i, s) = \max_{x_i > 0, s > 0} \left\{ \frac{\pi(w_i, s) - c_x x_i +}{1 + \frac{\alpha(x_i + \gamma(x_i) x_{02})}{1 + \alpha(x_i + \gamma(x_i) x_{02})}} C_S(w_i + 1, s'_m) + \frac{1}{1 + \frac{\alpha(x_i + \gamma(x_i) x_{02})}{1 + \alpha(x_i + \gamma(x_i) x_{02})}} C_F(w_i, s') \right\} \tag{16}
\]

Both \( C_S \) and \( C_F \) can be further broken down into components that hinge on whether the rival is
successful or not. Thus (16) expands to become:

\[
V_i(w_i, s) = \max_{x_i > 0} \left\{ \beta \left[ \frac{\partial V_i}{\partial x_i} = -c_x + \frac{a \frac{\partial m_i}{\partial x_i}}{(1 + a m_i)^2} [C_S - C_F] + \frac{a \frac{\partial m_j}{\partial x_i}}{(1 + a m_j)^2} (C_{S}^* - C_{F}^*) \right] \right\}
\]

and the values \(C_{ij}\) represent success or failure for the firm and the rival firm, respectively, with \(S\) indicating success for the firm and \(F\) indicating failure. Thus, \(C_{SS}\) indicates both firms innovate, \(C_{SF}\) indicates the firm is successful but the rival fails, \(C_{FS}\) is the reverse, and \(C_{FF}\) indicates both fail to innovate. Note that due to the monotonic increase in value with own efficiency and decrease with rival efficiency, the following relationships hold: \(C_{SF} > C_{SS}, C_{FF} > C_{FS}\). A firm is best off when it succeeds and its rival fails, while the lowest value obtains when the firm fails but its rival succeeds in innovating.

The expansion here demonstrates where in the optimization problem the firm internalizes the effect of increasing its own R&D expenditure on its rival’s probability of success. Increasing own R&D obviously increases the probability of achieving the value \(C_{S}\) in (16) but in the presence of spillovers raises the probability of outcome values of \(C_{SS}\) and \(C_{FS}\) where the rival also succeeds.

Taking the derivative with respect to \(x_i\) and simplifying:

\[
\frac{\partial V_i}{\partial x_i} = -c_x + \frac{a \frac{\partial m_i}{\partial x_i}}{(1 + a m_i)^2} [C_S - C_F] + \frac{a \frac{\partial m_j}{\partial x_i}}{(1 + a m_j)^2} (C_{S}^* - C_{F}^*)
\]

where \(C_{S}^*\) and \(C_{F}^*\) refer to the values if the rival succeeds or fails, respectively. \(C_{F}^* > C_{S}^*\) since the firm’s value is higher when its rival is a less efficient producer. Thus, the term in the second line of (18) is negative and reduces the incentives to engage in R&D while the preceding term represents the gains from innovation and provides the incentive for raising R&D. The term \(-\frac{c_x}{\beta}\) represents the cost and reduces R&D.

**Proposition 1** The second-order condition for a maximization is everywhere negative for an interior solution.

**Proof.** See the appendix. ■
The proof of Proposition 1 and all of the propositions that follow have been placed in Appendix I. We now turn to analyzing the costless and absorptive capacity cases.

3 Costless Spillovers

3.1 Baseline Case

The dynamic nature and complexity of the optimization precludes closed form solutions. However, in order to understand how the incentives and the distribution of market structures changes under costless and absorptive capacity functions, we will begin with a baseline case. The baseline case is a restricted version of the costless model wherein each firm takes the rival’s probability of success as exogenous. That is, the firm does not internalize the effect of its own R&D program on its rival’s pool of knowledge. The reason for using this case, as we show below, is that the degree of spillovers, b, will have essentially no impact on the rate of innovation under this restriction. With no impact on the rate of innovation, then the distribution of market structures, i.e. the degree of asymmetry between the firms, will remain largely unchanged. Then, we show below that relative to this baseline case, the effects of spillovers having diverging effects in the two full models of interest, the costless and absorptive capacity cases. That is, the baseline case described here serves as a point of reference for comparing the full models under the two methods of obtaining spillovers. We compare the differences in the optimization problem and show explicitly how and why the level of R&D, rate of innovation, and market structures differ depending on whether spillovers are acquired costlessly or through absorptive capacity.

In the presence of costless spillovers if the firm does not internalize the effect of own R&D on the rival’s probability of success, the firm treats the probabilities inside the brackets of (17) above as given, as in the E-P model.6 Thus, the firm chooses the value maximizing level of R&D, \( x_i \), and we obtain the first-order condition from the following value function:

\[
V_i(w_i, s) = \max_{x_i \geq 0} \left\{ \pi(w_i, s) - c_x x_i + \beta \left[ \frac{a(x_i + \gamma(x_i) b x_j)}{1 + a(x_i + \gamma(x_i) b x_j)} C_S(w_i' + 1, s_m') + \frac{1}{1 + a(x_i + \gamma(x_i) b x_j)} C_F(w_i, s') \right] \right\},
\]

(19)

6 This baseline case still generates a rational expectations equilibrium in the sense that firms’ forecasts of values are the values attained and can be solved for using the Pakes-McGuire algorithm. We show this equilibrium in section 5.
The first order condition is:

\[ 0 = \frac{-c_x}{\beta} + \frac{a}{(1 + a(x_i + bx_j))^2} (C_S - C_F) \]  

which yields a solution for \( x_i^* \) and \( m_i^* \) dependent on the parameters and the value of success/failure for the firm as follows:

\[ x_i^* = \frac{1}{a} \left( \frac{a\beta (C_S - C_F)}{c_x} \right)^{1/2} - \frac{1}{a} - bx_j, \text{ and} \]  
\[ m_i^* = x_i + bx_j = \frac{1}{a} \left( \frac{a\beta (C_S - C_F)}{c_x} \right)^{1/2} - \frac{1}{a}. \]  

In (21), \( x_i^* \) is the optimal level of R&D given the productivity of R&D, \( a \), the discount factor, \( \beta \), the unit cost of R&D, \( c_x \), the incremental value for successful R&D, \( C_S - C_F \), the extent of spillovers, \( b \), and the level of R&D undertaken by the rival firm, \( x_j \). From this equation it is easy to see the direct negative effect of the extent of spillovers on optimal R&D levels. An increase in \( b \) lowers the optimal level of R&D since the firm directly (and costlessly) benefits from rival R&D, provided \( x_j > 0 \). This effect has been widely discussed in the literature (Spence, 1984) and we refer to it as the substitution effect. Moreover, except for the final term, \( bx_j \), the solution is identical to that in E-P (1995) without spillovers and the total knowledge, shown in (22), matches their solution with the only difference being that \( m_i \) includes knowledge obtained through spillovers.

When we look at the impact of spillovers on the optimal level of R&D, note that there are two endogenous variables in (21), namely the incremental value of successful R&D, \( C_S - C_F \), and the level of rival R&D, \( x_j \). For notational convenience, let \( \Delta C = C_S - C_F \). Rewriting the expression with \( \Delta C \) and \( x_j \) as functions of \( b \) we have:

\[ x_i^* = \frac{1}{a} \left( \frac{a\beta \Delta C (b)}{c_x} \right)^{1/2} - \frac{1}{a} - bx_j (b). \]  

Taking the first derivative with respect to the extent parameter \( b \) yields:

\[ \frac{\partial x_i^*}{\partial b} = \frac{1}{2} \left( \frac{a\beta \Delta C (b)}{c_x \Delta C (b)} \right)^{1/2} \frac{\partial \Delta C (b)}{\partial b} - x_j (b) - b \frac{\partial x_j (b)}{\partial b}. \]  

The first term reveals that R&D could rise or fall with an increase in the extent of spillovers.
depending on how the incremental value of successful R&D changes. All the parameters and the term in parentheses are positive leaving the sign of the effect dependent on the sign of \( \frac{\partial \Delta C(b)}{\partial b} \). Essentially, the effect depends on how the slope of the value function changes with an increase in the extent of the spillovers. This effect can take on either positive or negative values. In fact, the term contains two effects. First, there is a positive cost effect. Because firm value rises with the level of efficiency, a firm can maintain the same position relative to its rival at a lower cost in terms of R&D expenditure. However, this R&D cost effect while increasing values, can have positive or negative effects on the marginal values. The effect will be increasing whenever R&D is increasing in efficiency and decreasing otherwise because the effect depends on the relative reduction in R&D expenditures.

Second, the marginal value may rise or fall depending upon how the transition probabilities change. Another way to think about this is to ask which firm, the less or more efficient, is willing to work harder for an improvement in relative efficiency when spillovers are greater?\(^7\) If being successful is more likely to lead to a positive change in the level of efficiency relative to its rival, the marginal value increases and so does the incentive for R&D. On the other hand, if the spillovers lead to a more active rival, sustaining an advantage becomes more difficult and costly which flattens the value function. The slope will increase for some firms in some market structures, but decrease in others. We cannot analytically say more about \( \frac{\partial \Delta C(b)}{\partial b} \), but our simulations show this effect to be ambiguous as discussed and, more importantly, quite small relative to the other effects. Therefore, as we proceed in our analytical exposition, we treat these effects are negligible in order to focus on the crucial differences in incentives for engaging in R&D that arise due to the costless and absorptive capacity specifications.

The substitution terms in (24), \(-x_j(b) - b \frac{\partial x_j(b)}{\partial b}\), display the direct effect of the change in spillovers on the amount of available knowledge. The secondary effect, captured by \(-b \frac{\partial x_j(b)}{\partial b}\), shows that when spillovers increase a rival’s R&D expenditures, it leads to a decrease in own R&D and essentially the firm is more willing to free ride on its rival’s activity. In contrast, a decrease in rival R&D, leads to an increase in own R&D and less dependence on spillovers. The degree of substitutability in strategies is directly proportional to the extent parameter. The overall direction of R&D depends on the elasticity of R&D expenditures of the rival with respect

\(^7\) The question is similar to the approach taken in Budd, Harris, and Vickers (1993) who look at whether asymmetry between two firms tends to increase or decrease in a dynamic model of duopoly without spillovers.
to the extent of spillovers.

**Proposition 2** In the costless, baseline case holding values constant with respect to marginal changes in \( b \), \( \frac{\partial C(b)}{\partial b} = 0 \), the optimal level of R&D declines with \( b \) if the rival’s elasticity of R&D with respect to \( b \) is greater than -1, \( \varepsilon_{x,b} > -1 \).

**Proof.** See the appendix. ■

The condition \( \varepsilon_{x,b} > -1 \) implies that although an increase in spillovers may lower a rival’s level of R&D it does not lower it sufficiently far to actually reduce the spillover pool on the margin.

To see how the elasticity affects the available pool of knowledge in the costless case we look at the elasticity of knowledge w.r.t. \( b \):

\[
\frac{\partial m_i}{\partial b} = x_j + b \frac{\partial x_j}{\partial b}
\]

\[
\frac{\partial m_i}{\partial b} \times \frac{1}{x_j} = 1 + \varepsilon_{x,b}
\]

Intuitively, if knowledge were negatively elastic, i.e. \( \varepsilon_{x,b} < -1 \), then increases in \( b \) would increase firm \( i \)’s R&D expenditure as seen in (24). Under symmetry this situation cannot hold since it would imply that R&D expenditures react in opposite directions for identical firms.

**Proposition 3** In the costless baseline duopoly, if firm \( i \) and firm \( j \) are symmetric, then \( -1 \leq \varepsilon_{x,b}^i, \varepsilon_{x,b}^j \leq 0 \).

**Proof.** See the appendix. ■

This condition may not hold when the firms are asymmetric. However, it does indicate that when the industry has a low concentration, R&D will decline for both firms.

While the overall incentives driven by the substitution effect reduce R&D expenditure, the linear costless specification for knowledge spillovers generates an extreme result when we look at the rate of innovation. Recall that firms combine the optimal amount of own and rival R&D to form usable knowledge which then enters the R&D production function given in (6). Thus,
changes in these probabilities show how the rate of innovation in the industry varies with spillovers.

\[ \Pr (u(x^*_i)) = \frac{am_i}{1 + am_i} \]

\[ = a \left[ \left( \frac{1}{\alpha} \left( \frac{a\beta \Delta C(b)}{c_x} \right)^{1/2} - \frac{1}{\alpha} - bX_{-i} (b) \right) + bX_{-i} \right] \]

\[ = 1 - \left( \frac{a\beta \Delta C(b)}{c_x} \right)^{-1/2} \]

(25)

The result above is identical to the case when \( b = 0 \) (the standard result in E-P), which leads to the following proposition.

**Proposition 4** In the baseline case when knowledge spillovers are costlessly obtained, neither the direct substitution effect nor the indirect substitution effect alter the optimal rate of innovation. Only alterations in the marginal values of innovation will change the rate of innovation and hence alter the degree of concentration. I.e., holding the values constant, \( \frac{\partial \Pr (u(x^*_i))}{\partial b} = 0 \).

**Proof.** See the appendix.

The rate of innovation in the baseline case is the same as the no spillovers case, \( b = 0 \) (and the no spillovers results do not depend on how spillovers are obtained, obviously). Proposition 4 will be instrumental in understanding how the costless and absorptive capacity cases differ under the full models of interest. To preview the results we obtain, when firms do internalize the effect of their own R&D on the rival’s chances of innovation, we show that in the costless case the rate of innovation declines. In contrast, under absorptive capacity, the rate of innovation rises. That difference alone is interesting, but also has implications for the distribution of the market structures and welfare which we characterize in Section 5.

The key aspect of Proposition 4 resides in the fact that substitution effect neutralizes the impact of spillovers on own R&D, leaving the rate of innovation identical to the rate of innovation without spillovers. The reason is the linear specification, standard in the literature, which directly adds outside knowledge to own R&D. The optimal choice of own R&D directly accounts for the spillovers, and without any change in the value function, would lead to the identical rate of innovation without spillovers. As a result, the only remaining effect on the probability of success or rate of innovation, for any given market structure, resides in how the spillovers alter the shape of the value function. If there are no effects on the shape of the value function, then spillovers
are irrelevant. Section 5, which simulates the model, demonstrates the impact of the changes in the value function which, again, are small.

Given that the rates of innovation are unchanged (and do not depend on the elasticity of R&D w.r.t $b$) then total R&D efforts must be declining as the extent of the spillovers rises. This result is intuitively straightforward. Firms utilize the outside knowledge as a substitute for their own efforts and reduce costs which leads to lower R&D efforts, but the use of outside knowledge allows the rates of innovation to remain the same. The only changes to the market structure then, in the baseline case will come through in changes in the slope of the value function itself. Therefore, we expect to see very little change in the market structure with increased spillovers when we simulate the model.

When the firms are asymmetric, we can further show that at least one firm decreases R&D as $b$ rises and in that case it is the firm with the lower level of R&D that decreases R&D further.

**Proposition 5** In the costless baseline duopoly, the elasticities of R&D with respect to $b$, must satisfy either $0 > \epsilon_{x,b}^i, \epsilon_{x,b}^j > -1$ or $\epsilon_{x,b}^i < -1$ and $\epsilon_{x,b}^j > 0$ (or $\epsilon_{x,b}^j < -1$ and $\epsilon_{x,b}^i > 0$). Furthermore, when $\epsilon_{x,b}^i < -1$ and $\epsilon_{x,b}^j > 0$, then $x_j > x_i$.

**Proof.** See the appendix. ■

In our simulations, when we obtain highly asymmetric market structures, the more efficient, higher market share firm conducts more R&D than its rival. Thus, the follower tends to free ride when the market structure is asymmetric. However, at some point the firm with lower R&D levels hits the non-negativity constraint on R&D. The inelastic case appears the more reasonable situation since it does imply that holding R&D of one firm fixed, an increase in the extent of spillovers implies an overall increase in usable knowledge. Thus, while we can show that at least one firm decreases R&D as spillovers increase, and that overall knowledge available remains constant, it is possible that one firm will actually increase R&D investment with higher spillovers because the rival reduces R&D elastically. In the symmetric case, both firms lower R&D.
3.2 The Full Model, Costless Case

Now we return to the costless case when firms internalize the effect of their own R&D on rivals’ chances of success. From the value function:

\[ V_i(w_i, s) = \max_{x_i \geq 0} \left\{ \beta \left[ \frac{\pi(w_i, s) - c_x x_i + a(x_i + \gamma(x_i) b x_j)}{1 + a(x_i + \gamma(x_i) b x_j)} \right] + \frac{1}{1 + a(x_i + \gamma(x_i) b x_j)} \left[ \frac{a(x_i + \gamma(x_i) b x_j)}{1 + a(x_i + \gamma(x_i) b x_j)} \right] \right\} \]

we obtain the following first order condition:

\[ FOC : 0 = -\frac{c_x}{\beta} + \frac{a}{(1 + am_i)^2} (C_S - C_F) + \frac{ab}{(1 + am_j)^2} [C_S^* - C_F^*]. \]

There are two differences between this first order condition and that of the baseline case. First, the crucial difference between the baseline case and the full model lies with the final term. This term is unambiguously negative, but not present in the baseline case. The term reflects the negative incentive to engage in R&D because it raises the rival’s ability to innovate. Therefore, the optimal level of \( x_i \) falls with this term. Intuitively, firms choose lower levels of R&D when they account for the fact that the rival becomes more likely to lower costs through the spillovers. That means the rate of innovation falls with spillovers unlike the baseline case where they remained constant. That statement is conditional on the second difference. The difference in values for success and failure, represented by \( C_S - C_F \), or \( \Delta C \) from before, undoubtedly changes. However, analytically we cannot say much about them without solving a parameterized version of the model.

Our simulations show these effects to be small relative to the other effects.

Solving implicitly for \( m_i \) we have:

\[ m_i^* = \frac{1}{a} \left[ \left( \frac{a \beta \Delta C}{c_x} \right) + \frac{ab \beta}{c_x} \left( \frac{1 + am_i}{1 + am_j} \right)^2 [C_S^* - C_F^*] \right]^{1/2} - \frac{1}{a} \]

which differs from the baseline case in the appearance of the entire second term in brackets, \( \frac{ab \beta}{c_x} \left( \frac{1 + am_i}{1 + am_j} \right)^2 [C_S^* - C_F^*] \). That term does not appear in the baseline case and it is unambiguously
negative because $C_S^* < C_F^*$. Thus, the level of available knowledge is lower than the no spillovers case (identical to the baseline case) holding the difference in values constant. Moreover, the term is increasing in magnitude with the degree of spillovers, $b$.

**Proposition 6** When spillovers are costless, holding changes in values constant, the level of available knowledge, the rate of innovation, and the overall level of R&D investment in the full model is less than in the baseline case.

**Proof.** See the appendix. ■

Proposition 5 establishes that, when spillovers are costless, the rate of innovation falls with increased spillovers. This effect will produce opposite results on concentration. First, with falling rates of innovation, we expect the market to become less concentrated as firms spend less effort in extending and maintaining their cost advantages. However, above we saw that in the case where one firm increases R&D as the extent of spillovers increases, it is the firm with the larger level of R&D to start with. In our simulations in the next section, the firm with the larger R&D program is also typically the firm with the large cost advantage. Thus, this effect would serve to increase concentration. The total level of R&D investment by both firms declines, although we cannot explicitly prove whether one firm’s R&D level is higher or lower compared with the baseline case.

We turn to the absorptive capacity case next, where we find largely the opposite effects.

## 4 Absorptive Capacity

### 4.1 Baseline Case

We now consider the case when firms acquire spillovers by establishing absorptive capacity through their own R&D program. Starting with the original value function, under absorptive capacity, but assuming, for the case of comparison to the baseline case, that firms do not account for the impact of own R&D on rivals, the first-order condition is:

$$0 = \frac{-c_x}{\beta} + a \left( \frac{1 + \frac{\gamma}{(1 + x_j)^2} b x_j}{(1 + a m_i)^2} \right) (C_S - C_F).$$

Solving first for the implicit optimal total level of knowledge, $m_i^*$ as:

$$m_i^* = \left( \frac{\beta \Delta C}{ac_x} \right)^{1/2} \left( 1 + \frac{\gamma}{(1 + x_i)^2} b x_j \right)^{1/2} - \frac{1}{a}$$  \hspace{1cm} (28)$$
we find the optimal level of own R&D:

\[ x_{i,AC}^* = \left( \frac{\beta \Delta C}{ac_x} \right)^{1/2} \left( \frac{\partial m_i^*}{\partial x_i^*} \right)^{1/2} - \frac{1}{a} - \frac{\gamma x_i^*}{(1 + \gamma x_i^*)} bx_j(b). \]  

(29)

The expression in (29), while not an explicit solution for \( x_i^* \), is readily compared with the baseline costless case in (21). There are two differences in these solutions. First, in (29) the second term in parentheses does not appear in the costless case, expressed as \( \frac{\partial m_i}{\partial x_i} \), is:

\[ \frac{\partial m_i}{\partial x_i} = 1 + \frac{bx_j}{(1 + \gamma x_i)} \geq 1 \]

and must be greater than or equal to unity. In the costless case, because external knowledge is added linearly, \( m_i = x_i + bx_j \), then \( \frac{\partial m_i}{\partial x_i} = 1 \). Under absorptive capacity, the marginal increase in knowledge from own R&D exceeds unity whenever \( b \) and \( x_j \) are non-zero, therefore generating a greater incentive to engage in R&D.

The second difference lies with the direct negative spillover effect, the last term in (29), which is now modified by the absorptive capacity expression. Because the effect of absorptive capacity, \( \frac{\gamma x_i^*}{(1 + \gamma x_i)} \), lies between 0 and 1, the direct negative spillover effect plays less of a role when absorptive capacity is required to utilize external knowledge. The greater own R&D, the greater the effect of spillovers through absorptive capacity. Thus, we see a shifting of incentives towards increased R&D under absorptive capacity.

**Proposition 7** In the baseline case, holding changes in values constant, when R&D spillovers are obtained through investing in absorptive capacity, R&D is higher than when spillovers are obtained costlessly whenever \( b > 0 \).

**Proof.** See the appendix. ■

In effect, the presence of absorptive capacity changes the margins on which the R&D decision is made. With increased productivity of own R&D through spillovers, absent in the costless case, optimal R&D expenditures rise. Solving for the rate of innovation:

\[ Pr(\upsilon(x_i^*)) = \frac{am_i}{1 + am_i} \]
we can derive:

\[
\Pr (v (x_i^*)) = 1 - \left( \frac{c_x}{a\beta \Delta} \right)^{1/2} \left( \frac{(1 + \gamma x_i^*)^2}{(1 + \gamma x_i^*)^2 + \gamma b x_j} \right)^{1/2}.
\]

(30)

Note that the final term in parentheses is \(\left( \frac{\partial m_i}{\partial x_i} \right)^{-1/2}\). That expression must lie between 0 and 1 since \(\frac{\partial m_i}{\partial x_i} \geq 1\). Therefore, for an interior solution, the rate of innovation is greater than in the costless baseline case. Since the marginal value of own R&D rises due to absorptive capacity in (30) for the same parameters, the rate of innovation is higher than when costlessly obtained. In the costless baseline case, the rate of innovation did not change with spillovers. Here, the increase in the extent of spillovers raises the incentive to engage in R&D and acquire spillovers.

**Proposition 8** In the baseline case, holding changes in values constant, when knowledge spillovers are obtained through absorptive capacity, the level of available knowledge and the rate of innovation is greater than when knowledge spillovers are obtained costlessly whenever \(b > 0\) and \(x_j > 0\).

**Proof.** See the appendix. ■

We cannot say, however, whether R&D rises or falls relative to the no spillovers case because of the competing effects. On the one hand, R&D will fall for the same reasons we observed previously wherein the greater availability of outside knowledge provides a substitute for expenditures on R&D. However, in order to make greater use of external R&D, the firm will want to engage in more of its own R&D to enhance its absorptive capacity.

The higher rates of innovation will promote greater competition and reduce the level of concentration. Once absorptive capacity is introduced, the firm with the smaller R&D program also enjoys the larger marginal gain from the spillovers in terms of its probability of successful innovation.

### 4.2 The Full Model, Absorptive Capacity Case

Now, to complete the comparison, we study the first-order condition in the full model of the absorptive capacity case. The optimality condition is:

\[
0 = \frac{-c_x}{\beta} + \frac{a \left( 1 + \frac{\gamma}{(1 + \gamma x_i^*)^2 + \gamma b x_j} \right) b x_j}{(1 + \gamma x_i^*)^2} (C_S - C_F) + \frac{ab x_j}{(1 + \gamma x_i^*)^2} (C^*_S - C^*_F).
\]

(31)
which is similar to the first-order condition of the full model version of the costless case repeated here:

\[ FOC = 0 = \frac{-c_x}{\beta} + \frac{a}{(1 + am_i)^2} (C_S - C_F) \]

\[ + \frac{ab}{(1 + am_j)^2} [C^*_{S} - C^*_{F}] . \]

Note the effect of absorptive capacity which enters the final two terms in the numerators. The first of these two terms reflects the positive incentives to engage in R&D and contains \( 1 + \gamma (1 + \gamma x_i) b x_j \). This component reflects the same increase in the margin of success of the firm due to taking advantage of spillovers. This component takes a value of 1 if the rival does no R&D and increases with higher levels of \( x_j \). Thus, the magnitude of the second term increases with spillovers generating higher levels of R&D at the optimum. The second, and negative term, shows the marginal increase in the probability of rival success, as in the costless case, but is reduced by the derivative of the absorptive capacity function which is less than one. Overall then, the positive incentives for R&D rise and the negative incentives decline in magnitude.

Solving implicitly for \( m_i^* \):

\[ m_i^* = \frac{1}{a} \left[ \left( \frac{a \beta}{c_x} \right) (1 + \gamma' (x_i) b x_j) \Delta C + \frac{ab \beta x_j}{c_x} \left( \frac{1 + am_i}{1 + am_j} \right)^2 \gamma' (x_i) [C^*_{S} - C^*_{F}] \right]^{1/2} - \frac{1}{a} \]  

(32)

The negative component (second term inside the brackets) enters as we saw in the full costless case. Thus, overall knowledge is reduced, as expected, by the negative incentive stemming from spillovers helping the rival. However, when compared with the full costless case, we see that knowledge, innovation, and R&D are higher. The first derivative of the absorptive capacity function, \( \gamma' (x_i) \), enters both the positive and negative terms. Since \( 0 \leq \gamma' (x_i) < 1 \), the positive term increases in absolute value while the negative term falls. Thus, knowledge is higher under the strategic absorptive capacity case than under strategic costless case.

**Proposition 9** In the full model, holding changes in values constant, and knowledge spillovers are obtained through absorptive capacity, the level of knowledge is higher than when spillovers are obtained costlessly.

**Proof.** See the appendix. ■

Intuitively, these results make sense. When R&D investment is required to take advantage of
spillovers, it increases the incentives for firms to engage in R&D because it raises marginal benefits from external knowledge. In addition, the marginal external benefit to rivals is mitigated because their ability to succeed depends on their own engagement in R&D.

With the opposing effects in the fully strategic case, we cannot make a general statement about whether knowledge is higher or lower than the costless baseline case where the rate of innovation was unaffected by $b$. However, in the case of symmetry we can show that knowledge (and therefore the rate of innovation) is higher.

**Proposition 10** In the full model, holding changes in values constant, when knowledge spillovers are obtained through absorptive capacity, and the firms are symmetric, the level of knowledge, and the rate of innovation, is higher than when spillovers are obtained costlessly in the baseline case. Therefore the rate of innovation rises with the extent spillovers under absorptive capacity.

**Proof.** See the appendix. ■

With the increase in knowledge over the costless baseline case, the rate of innovation rises with spillovers provided the firms are symmetric. We can say more though when the firms are not symmetric. (32) reveals a push towards symmetry. The effect is easiest to observe from solving for the rate of innovation:

$$
\Pr (v(x_i^*)) = 1 - \left[ \left( \frac{a \beta \Delta C}{c_x} + \frac{a \beta}{c_x} \gamma' (x_i) bx_j \left( \Delta C + \left( \frac{1 + am_i}{1 + am_j} \right)^2 \Delta C^* \right) \right]^{-1/2}.
$$

If there are no spillovers, $b = 0$, we obtain exactly the same solution as in (25), because the entire second term in brackets vanishes. With spillovers, the terms in front, $\left( \frac{a \beta}{c_x} \gamma' (x_i) bx_j \right)$, are all positive leaving the sign dependent on the relative magnitudes of $\Delta C$ and $\left( \frac{1 + am_i}{1 + am_j} \right)^2 \Delta C^*$, where $\Delta C$ is the marginal gain in value for success and $\Delta C^*$ is the marginal loss in value from the rival’s success. Under asymmetry, whether the rate of innovation rises depends on $\left( \frac{1 + am_i}{1 + am_j} \right)^2$ and here we observe a push towards symmetry. When firm $i$ conducts relatively more R&D than its rival $j$, the ratio increases (exceeds 1) meaning the firm places more weight on the negative consequence of R&D, namely creating spillovers for the rival. Thus, the larger firm with the higher R&D level will reduce R&D and rate of innovation when this incentive dominates. On the other hand, whenever firm $i$ conducts relatively less R&D the ratio is less than one and the rate of innovation rises. As shown in Proposition 10, when the firms are symmetric, $\Delta C > \Delta C^*$, and thus the entire term is overall positive leading to higher rates of innovation. Therefore, the
smaller firm with the lower R&D program will have greater incentive to increase R&D and its probability of innovation will rise.

Note how these results contrast with the baseline costless case where we began. There, spillovers did not affect the rate of innovation. Now, spillovers will raise the rate of innovation when the firms are symmetric, and when they are asymmetric, the incentives shift towards the smaller firm raising its rate of innovation. Contrasted with the costless case, where the rates of innovation unambiguously fell, we see a very different effect on the market structure.

To summarize the analysis, Table 2 shows our results for comparative purposes and our expectations for the simulations that follow. Compared to the no spillovers situation, \( b = 0 \), R&D unambiguously declines in the costless spillovers cases, both in the baseline and full model, though it falls more in the full model where the rate of innovation declines. In the absorptive capacity cases, however, the level of knowledge and the rate of innovation increase. R&D may increase or decrease, but if R&D declines, it will decline less than in the costless case. Whether the rate of innovation rises more in the baseline case of absorptive capacity or the full model is unclear because of the endogeneity of the market structure. On the one hand, firms internalize the negative effect on their own values of increasing the knowledge available to the rival firm. That effect clearly lowers the rate of innovation. However, the additional push towards symmetry that this effect entails would raise the overall rate of innovation because competition and the marginal gains from success are largest when the firms are symmetric.

For welfare, we anticipate the following. With no change in the rate of innovation, we expect little change in consumer surplus in the costless baseline case. However, with lower expenditures on R&D, firm values should rise depending on how the value function itself changes (where we have been assuming no change above). In the full model, costless case, the fall in the rate of innovation will raise prices and lower welfare, although this may be offset to some extent by lower concentration. On the other hand, in both absorptive capacity cases, we expect that consumer surplus will increase through both higher rates of innovation which lead to lower costs and the general push towards more symmetry which increases competition and further lowers prices.
5 Simulation

5.1 Methodology

The preceding analysis captures the impact of spillovers on optimal R&D decisions, and hence the rates of innovation. It does not however illustrate the effect of spillovers on the level of concentration and welfare though we could infer the direction from the analysis. Thus, we use this section to illustrate the effects discussed above and to show how the market structure changes with the extent of spillovers by simulating the model.

The numerical algorithm solves for the Markov-perfect Nash equilibrium policy functions for investment and values associated with each possible market structure. The algorithm delivers the optimal strategies \( \{x^*_i(w_i,s)\} \) for all \( w_i \in W \), and all \( s \in S \). The simulation program then uses these equilibrium policies to stochastically generate the evolution of the market structure of the industry, which is an ergodic process. We simulated the model 100,000 times under each scenario to obtain the numerical results for the ergodic distributions of market structures and the expected discounted value of the welfare measures. We also vary the extent of spillovers, \( b \), from zero to unity, i.e. from fully appropriable R&D to complete spillovers, with jumps of 10\% in the extent of knowledge spillovers. The parameter values used for the simulations of the model are given in Table 1. In a few cases, firms began to exceed the state space when spillovers reached \( b = 0.9 \) and 1.0. Thus, we omit those results here since the range from 0 to 0.8 demonstrates the results of the preceding section.

5.2 Costless Spillovers

We begin with the representation of spillovers as they have been primarily used in the literature preceding the work of Levin and Reiss (1988), but continuing in the analysis of the welfare implications of research joint ventures (e.g. D’Aspremont and Jacquemin, 1988, Suzumura, 1992). The graphs in Figure 5 show how R&D, the rate of innovation, firm values, and concentration change as the extent of spillovers, \( b \), increases. In all graphs we distinguish between the leader and the follower. The lead firm is the firm with the highest efficiency level (largest market share).\(^8\) In doing so it allows us to see the changes incentives for the large and small firms as spillovers

\(^8\) When the firms have identical efficiency levels we assign one to being the leader and the other the follower. Since the firms are facing identical problems, the identities do not matter or alter the results in any way.
increase.

For purposes of comparison and relating the findings of the previous section to the simulations, we also simulated the baseline models. As shown in Proposition 4, the rate of innovation will only change if there is variation in the difference in values, $\Delta C$, but these changes are quite small. The probability of success hardly varies even though R&D expenditures decline substantially and in percentage terms more so for the follower firm. There is also a small increase in values as the firms spend less on R&D. Since the market consists of only two firms, the market share of the largest firm, the C1 index, is a sufficient statistic. While some variation exists, the changes show that there is less than a one percentage point shift in the market shares on average. In the other cases below we see more substantial changes.

Figure 6 shows the ergodic distribution for the market structures when $b = 0.0$, $b = 0.2$, $b = 0.5$, and $b = 0.8$. The x-axes show the efficiency level for the leader firm and the y-axes show the efficiency level for the follower firm. Thus, the x-y plane represents all possible market structures which is the cross-product of the two efficiency levels. The z-axis then shows the distribution of the market structures that occur in equilibrium. The upper left diagram shows the distribution of market structures without spillovers, $b = 0.0$. Two modes emerge where the taller peak represents an asymmetric, concentrated market structure. Here, the lead firm possesses a significant efficiency advantage over its rival and thus captures much of the market share. The second mode, in contrast, lies along the diagonal where firms’ hold equal, or nearly equal, efficiency levels. Thus, the parameterization chosen provides a good baseline for comparison as we will be able to detect changes in the mass under these two modes that represent quite different market structures.

As $b$ increases, the distribution does change but not dramatically so as we would expect when the rates of innovation remain largely the same. The changes that do occur follow from small adjustments in the slope of the value function.\footnote{Using the simulations, we calculate that the changes to the value function, assumed away for analytical tractability reasons in the preceding section, account for only about 17% of the change in R&D. Moreover, the direction of the effects varies, sometimes positive, sometimes negative. In contrast, the component we do analyze, the substitution effect, is consistently negative (as expected) and accounts for the remainder of the effects and dominates.} Compare these results with Figures 7 and 8 which show the same set of graphs for the full model, i.e. accounting for the impact of their R&D on the rival. The difference is striking. As in Proposition 6, R&D falls further, while the rate of innovation for the lead firm declines and rises slightly for the follower. There is a small decrease in
concentration for the middle levels of the extent of spillovers, but this change still only represents a two percentage point change, at most, in the C1 concentration ratio.

To understand the non-monotonic movement in the concentration ratio, the distribution graphs in Figure 8 show that as both firms reduce R&D expenditures, the symmetric mode, where both firms are relatively efficient, disappears. The mass of the distribution moves towards the asymmetric mode at the same time the frequency of low efficiency levels for both firms increases. Thus, we see some shift towards lower concentration as the two firms compete less fiercely in R&D, but a greater likelihood of a more asymmetric market since it is the follower’s incentives that decline the most.

5.3 Absorptive Capacity

Under the baseline absorptive capacity specification, Figure 9 displays the effects of increasing spillovers on the mean equilibrium levels of R&D, probability of success, firm values and C1. For both the leader and the follower, R&D investment falls on average due to the negative incentive associated with less appropriability, but also with spillovers less R&D is required to achieve any given level of success. The change in total R&D efforts is smaller than under the costless case above expected from Proposition 7. Firm values also rise, but with a more pronounced increase for the smaller firm. The concentration in industry also declines.

The key difference shown in Figure 9 is the reversal of efforts by the leader and the follower. Under no spillovers, or costless spillovers, the lead firm maintained higher R&D efforts and, even in the presence of spillovers, a higher rate of innovation as in Proposition 8. However, with costly absorptive capacity required to obtain external knowledge, as the extent of the spillovers increases, we see the smaller, follower firm engaging in more R&D and reaching a higher rate of innovation. These changes, in turn do affect the market structure in a fairly dramatic, but intuitive way.

Figure 10 shows the substantial effects of the change in incentives. As the extent of the spillovers increases, the market become more symmetric. Both of the same modal market structures are present. However, the probability mass shifts from the highly unequal market structure to the more symmetric market structure as the extent of spillovers rises. Given the shift towards symmetry, not surprisingly the C1 concentration ratio falls with spillovers. The market share shifts approximately seven percentage points, which moves the mean market share distribution to about 55-45 from 62.5-37.5.
We show the results of the full model under absorptive capacity in Figures 11 and 12. The outcome is very similar to the baseline case, but the additional push towards symmetry in (32) shows that the follower’s average R&D expenditure exceeds the leader’s for lower levels of spillovers. The level of knowledge and rates of innovation are clearly higher than in the costless case (Proposition 9). As a result, the lower right panel shows an even larger decrease in the degree of concentration. This stronger push towards symmetry appears in Figure 12 where at $b = 0.8$ the symmetric mode is clearly dominant indicating higher R&D competition between the firms.

Overall, the simulations reveal how the costless and absorptive capacity cases differ. When investment in R&D is required to take advantage of spillovers, the marginal benefits of R&D increase. That leads to more intense competition and greater benefits from R&D fall on the follower firm. In turn that leads to a less concentrated market structure with higher rates of innovation. The higher rates of innovation raise consumer surplus because competition intensifies. In contrast, costlessly obtained spillovers reduce the rate of innovation as firms substitute rival R&D for their own. The impact on the market structure is non-monotonic with a tendency towards decreased concentration as overall rates of innovation decline, but towards increased concentration as the incentives of the less efficient firm lead it to reduce R&D even more than the leader for high levels of spillovers.

Finally, Table 3 compares firm values and consumer surplus under high spillovers ($b = 0.8$) versus the no spillovers case ($b = 0$). We continue to distinguish between the lead and follower firms because the impact differs. In the costless baseline case, where the rate of innovation was largely unchanged, both firms benefit but the majority of the benefits fall on the weaker of the two firms because spillovers allow the firm to free ride on its rival and reduce R&D expenditures. There is a small positive change in consumer surplus, but it is an order of magnitude smaller than changes in the other cases. Under the full costless model, the follower firm continues to benefit, but the lead firm’s value declines while consumer surplus falls with higher prices due to lower rates of innovation.

In contrast, under absorptive capacity, consumer surplus increases with the higher rates of innovation and the less concentrated market structure. The effect is more pronounced in the full model where the market pushes towards greater symmetry. The increased symmetry is also reflected in the mean changes in the values of the lead and follower firms. The follower firm’s value rises in both cases, but by nearly 50% in the full model. In contrast, the lead firm’s value rises
slightly when firms ignore the externality but falls when the market becomes more competitive.

The opposing welfare effects in the two key models (when firms do not ignore the impact on rivals) are of interest because the outcomes show that the net change in welfare (primarily driven by consumer surplus) depends on how spillovers are obtained. Costless acquisition of external knowledge reduces welfare primarily through lowering the rate of innovation. This result is in line with the early work by Spence (1984) and others who found that spillovers decreased the incentive to engage in R&D too much relative to the optimum.

On the other hand, we find that welfare rises if external knowledge is obtained through absorptive capacity. In their static, deterministic models with symmetry in firms’ research policies, Levin and Reiss (1988) and Cohen and Levinthal (1990) find that externalities in R&D might increase the amount of R&D investment undertaken in an industry leading to welfare gains. The predictions implied by the model presented here suggest their results cannot be extended to a dynamic, stochastic framework with firm heterogeneity. However, it is important to emphasize, that the fall in R&D investment associated with increased spillovers does not imply that the efficiency level of firms will decrease nor that the rate of innovation will decline. While firms reduce their R&D levels due to spillovers, the probabilities of successful innovation rise. That implies that the overall rate of innovation for the industry, the joint rate of success, increases which raises consumer surplus. Moreover, our analysis also adds the effect of a decrease in concentration which further improves consumer surplus.

While the preceding was meant to be illustrative of the results of the analytical section, we also conducted a sensitivity analysis of our parameterization. Specifically, we varied the key parameters governing R&D productivity, $a$, the rate of cost reduction, $\eta$, the level of fixed costs $f$, the rate of factor price increases, $\delta$, and the productivity of absorptive capacity, $\gamma$. We found no change in the qualitative results presented here.10

6 Conclusions

Our model of an infinite horizon duopoly engaged in R&D competition and Cournot-Nash competition in the spot market shows strong effects on the ergodic distribution of the market structures except when spillovers are costlessly obtained and firms behave non-strategically. In light

10 These results are available on request.
of Mansfield’s (1984) stylized fact stating that industries exhibit greater degrees of competition when imitation is easier, we find that result in the model under absorptive capacity, but not in the costless case. However, whether spillovers are welfare enhancing or not depends on whether they are obtained costlessly or through absorptive capacity. Costless spillovers reduce the level of R&D, the rate of innovation, and lower welfare despite some decreases in concentration. The latter follows from the higher prices in equilibrium because firms are innovating less. In contrast, when firms need to establish absorptive capacity to acquire external knowledge the rate of innovation rises for two reasons. First, the positive impact on the incentives to increase R&D raise the marginal productivity of R&D from the firm’s point of view. Second, the effect is strongest for the firm that would, in the absence of spillovers conduct the least amount of R&D. The result is a push towards a more symmetric market structure where the overall level of R&D is highest and the rates of innovation increase. Higher rates of innovation lead to lower long-run prices for consumers which raises welfare.

There are two key issues worth addressing but beyond the scope of this paper. The first is that we abstract from entry and exit. We do not consider the added incentives for a firm to engage in R&D to capture more market share (the whole market in effect) by driving its rival out of the market completely. In addition, entry may offset the welfare gains as the appropriability problems becomes more severe for any one firm when multiple rivals exist. This model can incorporate entry and exit as well as increasing the number of firms, but again that is beyond the scope of this paper.

Secondly, we have followed, for the most part, the specifications of imperfect appropriability used for the past 25 years in the literature on R&D, spillovers, and competition. However, these papers often discuss, but rarely model or measure explicitly, external knowledge as a stock, rather than a flow (which is how we, and many of the paper cited, treat it in the model). Consider a large dominant firm that owes its position to having a vastly superior production process to all others, but which, for whatever reason, ceases to engage in further R&D. Under the specifications discussed here that firm can no longer be a source of spillovers. Yet it seems more than plausible that other firms will still try to emulate that most efficient firm and seek information on that production process. Thus, we believe another avenue for future work is to treat the efficiency levels as a stock of innovations which enter the knowledge production function rather than the flow of R&D. This alteration may well change incentives and the resulting equilibrium market.
structures, and thus we feel it is a worthy avenue for future work.

References


1 Appendix: Proofs of Propositions

Proposition 1 The second-order condition for a maximization is everywhere negative for an interior solution.

Proof. Starting from:

\[
\frac{\partial V_i}{\partial x_i} = -\frac{c_x}{\beta} + \frac{a \frac{\partial m_i}{\partial x_i}}{(1 + am_i)^2} \left[ \left( \frac{am_j}{1 + am_j} \right) C_{SS} + \left( \frac{1}{1 + am_j} \right) C_{SF} \right] \\
- \frac{a \frac{\partial m_i}{\partial x_i}}{(1 + am_i)^2} \left[ \left( \frac{am_j}{1 + am_j} \right) C_{FS} + \left( \frac{1}{1 + am_j} \right) C_{FF} \right] \\
\left( \frac{am_i}{1 + am_i} \right) \frac{a \frac{\partial m_j}{\partial x_i}}{(1 + am_j)^2} (C_{SS} - C_{SF}) + \left( \frac{1}{1 + am_i} \right) \frac{a \frac{\partial m_j}{\partial x_i}}{(1 + am_j)^2} (C_{FS} - C_{FF})
\]

\[
\frac{\partial V_i^2}{\partial x_i^2} = -\frac{2a^2}{(1 + am_i)^3} \left( \frac{\partial m_i}{\partial x_i} \right)^2 [C_S - C_F] + \frac{a}{(1 + am_i)^2} \left( \frac{\partial^2 m_i}{\partial x_i^2} \right) [C_S - C_F] \\
+ \frac{a^2}{(1 + am_i)^2 (1 + am_j)} \left( \frac{\partial m_i}{\partial x_i} \right) \left( \frac{\partial m_j}{\partial x_i} \right) [C_{SS} + C_{FF} - C_{SF} - C_{FS}] \\
- \frac{2a^2}{(1 + am_i)^3} \left( \frac{\partial^2 m_i}{\partial x_i^2} \right) [C_S - C_F^*] + \frac{a^2}{(1 + am_j)^2} \left( \frac{\partial^2 m_j}{\partial x_i^2} \right) [C_{S}^{*} - C_{F}] \\
+ \frac{2a^2}{(1 + am_i)^3} \left( \frac{\partial^2 m_j}{\partial x_i^2} \right) [C_{SS} + C_{FF} - C_{SF} - C_{FS}]
\]

From the definitions of \( m_i \) and \( m_j \):

\[
\frac{\partial m_i}{\partial x_i} = 1 + \gamma'(x_i) bx_j \geq 1, \quad \frac{\partial^2 m_i}{\partial x_i^2} = \gamma''(x_i) bx_j < 0 \\
0 < \frac{\partial m_j}{\partial x_i} = \gamma(x_j) b \leq 1, \quad \frac{\partial^2 m_j}{\partial x_i^2} = 0
\]

The second-order condition can be slightly written by combining the 2nd and 4th lines and since the last term goes to zero:

\[
\frac{\partial V_i^2}{\partial x_i^2} = -\frac{2a^2}{(1 + am_i)^3} \left( \frac{\partial m_i}{\partial x_i} \right)^2 [C_S - C_F] + \frac{a}{(1 + am_i)^2} \left( \frac{\partial^2 m_i}{\partial x_i^2} \right) [C_S - C_F] \\
+ \frac{2a^2}{(1 + am_i)^2 (1 + am_j)} \left( \frac{\partial m_i}{\partial x_i} \right) \left( \frac{\partial m_j}{\partial x_i} \right) [C_{SS} + C_{FF} - C_{SF} - C_{FS}] \\
- \frac{2a^2}{(1 + am_j)^3} \left( \frac{\partial^2 m_j}{\partial x_i^2} \right) [C_{SS} - C_{SF} - C_{FS} - C_{SF}]
\]

The terms in the first line above are unambiguously negative. The sign of the
second two lines needs to be established. Focusing on the final, unambiguously positive term first where \( C_S^* - C_F^* < 0 \), we can derive an equivalent expression from the FOC. The first-order condition is:

\[
- \frac{c_x}{\beta} + \frac{a}{(1 + am_j)^2} \left( \frac{\partial m_i}{\partial x_i} \right) [C_S - C_F] + \frac{a}{(1 + am_j)} \mu \frac{\partial m_j}{\partial x_i} [C_S^* - C_F^*] = 0.
\]

After some algebra we can show that:

\[
- \frac{2a^2}{(1 + am_j)^3} \left( \frac{\partial m_j}{\partial x_i} \right)^2 [C_S^* - C_F^*] = \frac{2a^2(1 + am_j)}{(1 + am_i)^2(1 + am_j)^2} \left( \frac{\partial m_i}{\partial x_i} \right) \left( \frac{\partial m_j}{\partial x_i} \right) [C_S - C_F]
\]

\[
- \frac{2acx}{\beta(1 + am_j)} \left( \frac{\partial m_j}{\partial x_i} \right)
\]

The final equality shows the last term in the second-order condition equals the right-hand side, where the first term is positive and the second-term, owing to the minus sign in front, is negative. Thus the positive term in the second-order condition is something smaller in absolute value than

\[
\frac{2a^2(1 + am_j)}{(1 + am_i)^2(1 + am_j)^2} \left( \frac{\partial m_i}{\partial x_i} \right) \left( \frac{\partial m_j}{\partial x_i} \right) [C_S - C_F].
\]

We can compare this term with the second line of the second order condition above:

\[
\frac{2a^2(1 + am_j)}{(1 + am_i)^2(1 + am_j)^2} \left( \frac{\partial m_i}{\partial x_i} \right) \left( \frac{\partial m_j}{\partial x_i} \right) [C_S + C_{FF} - C_{SF} - C_{FS}].
\]

The second-order condition will hold if the combined value is negative. Adding the terms and factoring we have:

\[
\left[ \frac{2a^2}{(1 + am_i)^2(1 + am_j)^2} \left( \frac{\partial m_i}{\partial x_i} \right) \left( \frac{\partial m_j}{\partial x_i} \right) \right] \left( \frac{C_{SS}}{C_{SS}} + C_{FF} - C_{SF} - C_{FS} + am_jC_{SS} + C_{FS} - am_jC_{SF} - C_{FF} \right)
\]

The term in brackets is positive, while the term in parentheses reduces to:

\[
(1 + am_j)(C_{SS} - C_{SF}) < 0
\]

which is negative since \( C_{SF} > C_{SS} \). Thus, the second-order condition holds. 

Q.E.D. ■

**Proposition 2** In the costless, baseline case holding values constant with respect to marginal changes in \( b \), \( \frac{\partial \Delta C(b)}{\partial b} = 0 \), the optimal level of R&D declines with \( b \) if the rival’s elasticity of R&D with respect to \( b \) is greater than -1, \( \varepsilon_{x,b} > -1 \).
Proof. From the optimal solution for R&D in the costless non-strategic case:

\[
\begin{align*}
\frac{\partial x_i^*}{\partial b} \bigg|_{\Delta C} &= -x_j (b) - b \frac{\partial x_j (b)}{\partial b} \\
\frac{\partial x_j^*}{\partial b} \bigg|_{\Delta C} &= \frac{1}{x_j (b)} & \text{and} & \frac{\partial x_j^*}{\partial b} \bigg|_{\Delta C} = -1 - \varepsilon_{x,b}.
\end{align*}
\]

Q.E.D. \[\blacksquare\]

Note that proposition 3 in the paper corresponds to the second part of the proposition presented here.

**Proposition 3** In a duopoly, the elasticities of R&D with respect to b, \( \varepsilon_{x,b} \), in the costless baseline case must satisfy either \( 0 > \varepsilon_{x,b}^i, \varepsilon_{x,b}^j > -1 \) or \( \varepsilon_{x,b}^i < -1 \) and \( \varepsilon_{x,b}^j > 0 \). If firm \( i \) and firm \( j \) are symmetric, then \( -1 \leq \varepsilon_{x,b}^i, \varepsilon_{x,b}^j \leq 0 \). If \( \varepsilon_{x,b}^i < -1 \) and \( \varepsilon_{x,b}^j > 0 \), then \( x_j > x_i \).

Proof. First, consider the case where for both firm \( i \) and \( j \), \( \varepsilon_{x,b}^i, \varepsilon_{x,b}^j < -1 \). Then \( \frac{\partial m_j}{\partial b} \frac{1}{x_i} = 1 + \varepsilon_{x,b}^i < 0 \). By Proposition 4, \( \frac{d \Pr(v_j)}{d b} \bigg|_{\Delta C} = 0 \Rightarrow \frac{\partial x_j}{\partial b} > 0 \).

But then \( \varepsilon_{x,b}^j > 0 \) which contradicts the assumption that \( \varepsilon_{x,b}^j < -1 \). Therefore both \( \varepsilon_{x,b}^i \) and \( \varepsilon_{x,b}^j \) cannot be less than \(-1\). Second, consider the case where for both firm \( i \) and \( j \), \( \varepsilon_{x,b}^i, \varepsilon_{x,b}^j > 0 \). Then \( \frac{\partial m_j}{\partial b} \frac{1}{x_i} = 1 + \varepsilon_{x,b}^i > 0 \). By Proposition 4, \( \frac{d \Pr(v_j)}{d b} \bigg|_{\Delta C} = 0 \Rightarrow \frac{\partial x_j}{\partial b} < 0 \). But then \( \varepsilon_{x,b}^j < 0 \) which contradicts the assumption that \( \varepsilon_{x,b}^j > 0 \). Therefore both \( \varepsilon_{x,b}^i \) and \( \varepsilon_{x,b}^j \) cannot be positive.

Third, consider the case where \( \varepsilon_{x,b}^i < -1 \) and \( -1 < \varepsilon_{x,b}^j < 0 \). Then \( \frac{\partial m_i}{\partial b} \frac{1}{x_i} = 1 + \varepsilon_{x,b}^i < 0 \). By Proposition 4, \( \frac{d \Pr(v_i)}{d b} \bigg|_{\Delta C} = 0 \Rightarrow \frac{\partial x_i}{\partial b} > 0 \). But then \( \varepsilon_{x,b}^j > 0 \) which contradicts the assumption that \( \varepsilon_{x,b}^j < 0 \). Therefore \( \varepsilon_{x,b}^i < -1 \) and \( -1 < \varepsilon_{x,b}^j < 0 \) cannot hold and by symmetry \( \varepsilon_{x,b}^j < -1 \) and \(-1 < \varepsilon_{x,b}^i < 0 \) cannot hold. By steps 1 and 3 above, the only remaining possibility under symmetry where \( \varepsilon_{x,b}^i = \varepsilon_{x,b}^j \) is \(-1 \leq \varepsilon_{x,b}^i, \varepsilon_{x,b}^j \leq 0 \). From Proposition 4, since \( \frac{d \Pr(v_i)}{d b} \bigg|_{\Delta C} = 0 \), then \( \frac{dm_i}{db} = \frac{dm_j}{db} = 0 \), or

\[
\frac{dm_i}{db} = \frac{\partial x_i}{db} + x_j + b \frac{\partial x_j}{db} = \frac{dm_j}{db} = \frac{\partial x_j}{db} + x_i + b \frac{\partial x_i}{db} = 0
\]

\( x_i - x_j = (1 - b) \left( \frac{\partial x_i}{\partial b} - \frac{\partial x_j}{\partial b} \right) \)

Thus, if \( \varepsilon_{x,b}^i < -1 \) and \( \varepsilon_{x,b}^j > 0 \), then \( x_j > x_i \). Q.E.D. \[\blacksquare\]
Proposition 4  In the baseline case when knowledge spillovers are costlessly obtained, neither the direct substitution effect nor the indirect substitution effect alter the optimal rate of innovation. Only alterations in the marginal values of innovation will change the rate of innovation and hence alter the degree of concentration. I.e., holding the values constant, $\frac{\partial \Pr(v(x^*_i))}{\partial b} = 0$.

Proof. Substituting the optimal R&D level (21) into the knowledge production function:

$$
\Pr(v(x^*_i)) = \frac{am_i}{1 + am_i} = \frac{a(x^*_i + bX_{-i})}{1 + a(x^*_i + bX_{-i})} = \frac{a \left( \frac{1}{a} \left( \frac{a\beta \Delta C(b)}{c_x} \right)^{1/2} - \frac{1}{a} - bX_{-i}(b) \right) + bX_{-i}(b)}{1 + a \left( \frac{1}{a} \left( \frac{a\beta \Delta C(b)}{c_x} \right)^{1/2} - \frac{1}{a} - bX_{-i}(b) \right) + bX_{-i}} = 1 - \left( \frac{a\beta \Delta C(b)}{c_x} \right)^{-1/2}
$$

Q.E.D.

Proposition 5  In the costless baseline duopoly, the elasticities of R&D with respect to $b$, must satisfy either $0 > \varepsilon_{x,b}^i > -1$ or $\varepsilon_{x,b}^j < -1$ and $\varepsilon_{x,b}^i > 0$ (or $\varepsilon_{x,b}^j < -1$ and $\varepsilon_{x,b}^i > 0$). Furthermore, when $\varepsilon_{x,b}^i < -1$ and $\varepsilon_{x,b}^j > 0$, then $x_j > x_i$.

Proof. See Proposition 3 above.

Proposition 6  When spillovers are costless, holding changes in values constant, the level of available knowledge, the rate of innovation, and the overall level of R&D investment in the full model is less than in the baseline case.

Proof. In the baseline case:

$$
m^*_{BASELINE} = \left[ \left( \frac{a\beta \Delta C}{c_x} \right) \right]^{1/2}
$$

whereas in the fully model $m^*_{FULL}$ is given by

$$
m^*_{FULL} = \frac{1}{a} \left[ \left( \frac{a\beta \Delta C}{c_x} \right) + \frac{ab\beta}{c_x} \left( \frac{1 + am_i}{1 + am_j} \right)^2 \left[ C^*_S - C^*_F \right] \right]^{1/2} - \frac{1}{a}.
$$
Since $C^*_1 < C^*_2$, then $m^{\text{FULL}}_i < m^{\text{COSTLESS}}_i$. Since the rate of innovation is determined by the probability of successful which is a strictly monotonically increasing function of $m$, the rate of innovation declines. Finally a lower level of $m$ immediately implies that the total levels of R&D must decline. $Q.E.D.$

**Proposition 7** In the baseline case, holding changes in values constant, when R&D spillovers are obtained through investing in absorptive capacity, R&D is higher than when spillovers are obtained costlessly whenever $b > 0$.

**Proof.** Compare the R&D solutions for the baseline and full costless cases. Two terms appear in the latter but not in the former. First, we have

$$\frac{\partial m_i}{\partial x_i} = 1 + \frac{bx_j}{(1 + \gamma x_i^*)^2} \geq 1$$

which raises the optimal R&D level. Second, the absorptive capacity function modifies the direct spillover effect

$$-\frac{\gamma x_i^*}{(1 + \gamma x_i^*)} bx_j (b)$$

but $\frac{\gamma x_i^*}{(1 + \gamma x_i^*)}$ is bounded between 0 and 1, while in the costless case it takes on the value of 1. Therefore the overall negative effect is reduced in absolute value leading to higher levels of $x^*_i$. $Q.E.D.$

**Proposition 8** In the baseline case, holding changes in values constant, when knowledge spillovers are obtained through absorptive capacity, the level of available knowledge and the rate of innovation is greater than when knowledge spillovers are obtained costlessly whenever $b > 0$ and $x_j > 0$.

**Proof.** Starting from (25):

$$\Pr (v (x^*_i)) = 1 - \left(\frac{c_x}{a \beta \Delta C (b)}\right)^{1/2} \left(\frac{\partial m_i^* (b)}{\partial x_i^*}\right)^{-1/2}$$

and noting that:

$$\frac{\partial m_i}{\partial x_i} = 1 + \frac{bx_j}{(1 + \gamma x_i^*)^2} \geq 1,$$

immediately implies that the entire term in (25) is larger relative to the costless case where $\frac{\partial m_\text{costless}}{\partial x_i} = 1$. By Proposition 4, then

$$\frac{d \Pr (v_i)}{db} > 0.$$

Thus,

$$\frac{d \Pr (v_i)}{db} = \frac{a}{(1 + am_i)^2} \frac{dm_i}{db} > 0.$$
which only holds provided \( \frac{d m_i}{dt} > 0 \). \( Q.E.D. \)

**Proposition 9** In the full model, holding changes in values constant, and knowledge spillovers are obtained through absorptive capacity, the level of knowledge is higher than when spillovers are obtained costlessly.

**Proof.** Comparing the optimal level of knowledge in the costless and absorptive capacity cases respectively we have:

\[
m_i^{\text{COSTLESS}} = \frac{1}{a} \left[ \frac{a \beta \Delta C}{c_x} + \frac{ab \beta}{c_x} \left( \frac{1 + am_i}{1 + am_j} \right)^2 [C^*_S - C^*_F] \right]^{1/2} - \frac{1}{a}
\]

\[
m_i^{\text{AC}} = \frac{1}{a} \left[ \left( \frac{a \beta \Delta C}{c_x} \right) (1 + \gamma' (x_i) bx_j) + \frac{ab \beta}{c_x} \left( \frac{1 + am_i}{1 + am_j} \right)^2 \gamma' (x_i) x_j [C^*_S - C^*_F] \right]^{1/2} - \frac{1}{a}
\]

In both solutions, inside the brackets the first term is positive since \( \Delta C > 0 \) while the second term is negative since \( C^*_S - C^*_F < 0 \). The first term is overall larger since \( \gamma' (x_i) bx_j \geq 0 \) while the negative term is reduced by \( 0 \leq \gamma' (x_i) \leq 1 \), which make the absorptive capacity level of \( m_i^* \) larger than in the costless case. \( Q.E.D. \)

**Proposition 10** In the full model, holding changes in values constant, when knowledge spillovers are obtained through absorptive capacity, and the firms are symmetric, the level of knowledge, and the rate of innovation, is higher than when spillovers are obtained costlessly in the baseline case. Therefore the rate of innovation rises with the extent spillovers under absorptive capacity.

**Proof.** Using (23) and (29), the difference in available knowledge between the two cases is given by the following terms:

\[
\frac{ab \beta x_j}{c_x} \gamma' (x_i) \Delta C + \frac{ab \beta}{c_x} \left( \frac{1 + am_i}{1 + am_j} \right)^2 \gamma' (x_i) x_j [C^*_S - C^*_F]
\]

\[
= \frac{ab \beta x_j}{c_x} \gamma' (x_i) \left[ (C_S - C_F) + \left( \frac{1 + am_i}{1 + am_j} \right)^2 (C^*_S - C^*_F) \right]
\]

Under symmetry then the expression becomes:

\[
\frac{ab \beta x_j}{c_x} \gamma' (x_i) [C_S - C_F + C^*_S - C^*_F]
\]

Since \( C_S > C^*_F > C^*_S > C_F \), then

\[
C_S - C^*_F > C^*_S - C^*_F, \text{ and using } C^*_F > C_F, \text{ then} \n\]

\[
C_S - C_F > C_S - C^*_F > C^*_S - C^*_F
\]

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Thus, the entire term is positive indicating an increase in knowledge relative to the baseline, costless case under symmetry. \textit{Q.E.D.} ■
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Table 2: Summary of Analytical Results for Increases in $b$

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<th>R&amp;D</th>
<th>Rate of Innovation</th>
<th>Concentration</th>
<th>Welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costless Baseline</td>
<td>↓</td>
<td>No change</td>
<td>No change</td>
<td>↑ slightly</td>
</tr>
<tr>
<td>Abs Cap. Baseline</td>
<td>↑</td>
<td>↑</td>
<td>↓</td>
<td>↑</td>
</tr>
<tr>
<td>Costless Full Model</td>
<td>↓</td>
<td>↓</td>
<td>?</td>
<td>?/↓</td>
</tr>
<tr>
<td>Abs Cap. Full Model</td>
<td>↑</td>
<td>↑</td>
<td>↓</td>
<td>↑</td>
</tr>
</tbody>
</table>
Table 3: Percentage Changes in Welfare with Spillovers
(Percentage Represents Change in Surplus from No Spillovers, $b = 0.0$, to High Spillovers, $b = 0.8$)

<table>
<thead>
<tr>
<th>Case</th>
<th>Lead Firm Surplus</th>
<th>Follower Firm Surplus</th>
<th>Consumer Surplus</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Costless Baseline</td>
<td>7.7%</td>
<td>26.7%</td>
<td>0.8%</td>
<td>6.5%</td>
</tr>
<tr>
<td>Abs Cap. Baseline</td>
<td>2.3%</td>
<td>35.0%</td>
<td>6.8%</td>
<td>8.8%</td>
</tr>
<tr>
<td>Costless Full Model</td>
<td>-15.2%</td>
<td>8.9%</td>
<td>-16.8%</td>
<td>-13.2%</td>
</tr>
<tr>
<td>Abs Cap. Full Model</td>
<td>-5.7%</td>
<td>46.4%</td>
<td>7.9%</td>
<td>8.1%</td>
</tr>
</tbody>
</table>
Figure 1: Costless Case  
(b=0.5, a=0.5, Xj=4.0)

Figure 2: Absorptive Capacity Case  
(b=0.5, gamma=5, a=0.5, Xj=4.0)
Figure 3: Value Function in Duopoly
Figure 4: Firm 1's R&D Investment
Figure 5: Baseline Costless Case
Changes in Mean Values of R&D, Rate of Innovation, Firm Values, and C1 Concentration Ratio
(Solid Line is Leader Firm and Dashed Line is Follower Firm)
Figure 6: Costless, Baseline Case
Impact of Spillovers on Distribution of the Market Structure
Figure 7: Costless Case, Full Model
Changes in Mean Values of R&D, Rate of Innovation, Firm Values, and C1 Concentration Ratio
(Solid Line is Leader Firm and Dashed Line is Follower Firm)
Figure 8: Costless, Full Model
Impact of Spillovers on Distribution of the Market Structure

b=0.0

b=0.2

b=0.5

b=0.8
Figure 9: Baseline Absorptive Capacity Case
Changes in Mean Values of R&D, Rate of Innovation, Firm Values, and C1 Concentration Ratio
(Solid Line is Leader Firm and Dashed Line is Follower Firm)
Figure 10: Absorptive Capacity, Baseline Case
Impact of Spillovers on Distribution of the Market Structure
Figure 11: Absorptive Capacity Case, Full Model
Changes in Mean Values of R&D, Rate of Innovation, Firm Values, and C1 Concentration Ratio
(Solid Line is Leader Firm and Dashed Line is Follower Firm)
Figure 12: Absorptive Capacity, Full Model
Impact of Spillovers on Distribution of the Market Structure