GMM based inference for panel data models

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Abstract

We apply a range of GMM based inference procedures to the first-order dynamic panel data model. We use moment conditions from either the first differenced or levels model or both. In addition to standard Wald and LM procedures we consider some recently developed weak instrument robust GMM statistics. By Monte Carlo simulation we address size and power in finite samples of hypothesis tests on the autoregressive coefficient. Our results indicate that conventional tests are subject to considerable size distortions in a significant part of the parameter space. Weak instrument robust statistics, however, have good size properties while maintaining sufficient power in especially the system model.

1. Introduction

The Generalized Method of Moments (GMM) is a commonly employed technique to estimate the parameters in panel data models with endogenous regressors and

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unobserved individual specific heterogeneity. In such models least-squares estimators, i.e. fixed effects or random effects estimators, are inconsistent for a finite number of time periods and a large number of cross-section observations. Since in micro-economics the typical dimension of a panel data set is a short time span for a large cross-section alternative consistent GMM estimators have been proposed (Arellano and Bond, 1991; Arellano and Bover, 1995; Blundell and Bond, 1998). Arellano and Bond (1991) transform the model into first differences (DIF) to wipe out the individual specific effects. Sequential moment conditions are then used where lagged levels of the variables are instruments for the endogenous differences. Arellano and Bover (1995) and Blundell and Bond (1998) propose the use of extra moment conditions arising from the model in levels (LEV) when certain stationarity conditions of the initial observation are satisfied. The resulting system (SYS) GMM estimator combines moment conditions for the model in first differences with moment conditions for the model in levels.

Especially the system GMM estimator proposed by Blundell and Bond (1998) has become increasingly popular in applied economic research using panel data. We do not intend to give an exhaustive overview of specific applications, but in labor economics (labor supply and demand), development economics (effectiveness of foreign aid), health economics (health expenditures, organization of health care, aging, addiction), industrial organization (mergers & acquisitions, evaluation of competition policy), international economics (effects of trade policy and economic integration), macroeconomics (economic growth, optimal currency areas) and finance (banking regulation) this method has been applied extensively.

Main reason for the popularity of GMM in applied economic research using panel data is that GMM provides asymptotically efficient inference assuming a minimal set of statistical assumptions. Despite these optimal asymptotic properties of GMM statistics, however, their behaviour in finite samples can be rather peculiar due to weakness and/or abundance of moment conditions and dependence on crucial nuisance parameters. It has been well documented (see e.g. Blundell and Bond, 1998) that the DIF GMM estimator can have very poor finite sample properties in terms of bias and precision when the series are persistent, as the instruments are then weak predictors of the endogenous changes. The SYS GMM estimator has been shown in Monte Carlo studies by e.g. Blundell and Bond (1998) and Blundell, Bond and Windmeijer (2000) to have much better finite sample properties in terms of bias and root mean squared error than that of the DIF GMM estimator. However, Bun and Windmeijer (2010) show that there is a weak instrument problem also for the equation in levels. They show that the weak instrument problem in the LEV model manifests itself not so much in terms of absolute bias of coeficient estimators, but merely in estimation bias relative to that of inconsistent least squares estimators and size distortions of Wald type tests. In a Monte Carlo study they show that these anomalous properties extend to the SYS GMM estimator and corresponding Wald test.

In addition to time series persistence, in panel data models estimated by GMM also the variance ratio is relevant for the weak instrument problem. The variance ratio is defined as the variance of the individual specific effects divided by the variance of the idiosyncratic errors. Especially the finite sample performance of LEV and SYS estimators deteriorate with increasing variance ratio, even when the time series persistence of the panel data is moderate only. Regarding coefficient estimators, the vulnerability of the LEV and SYS moment conditions for this nuisance parameter has been documented already in the bias approximations of Bun and Kiviet (2006) and Hayakawa (2007), and in the simulation findings of Kiviet (2007). Bun and Windmeijer (2010) show that Wald statistics based on LEV and SYS coefficient estimators show similar vulnerability to the variance ratio.

These results suggest that accurate inference in finite samples is troublesome when we use the available linear moment conditions for panel data models in conventional GMM estimation and testing procedures. Coefficient estimators can be badly biased and standard t or Wald tests can be heavily size distorted for a significant part of the parameter space. Instead, what we need are test procedures based on statistics with finite sample distributions largely invariant to both parameters of interest and important nuisance parameters.

In this paper we analyze by Monte Carlo simulation the size and power of such alternative testing procedures in finite samples. We will use the first-order dynamic panel data model without any additional explanatory variables. In this very simple set-up the autoregressive coefficient is the parameter of interest and the variance ratio an important nuisance parameter.

We analyze a wide range of statistics to test null hypotheses concerning the autoregressive parameter. First, we consider the use of Wald testing procedures based on (refinements of) the two-step GMM estimator. We exploit a corrected estimate of the variance of the two-step GMM estimator developed by Windmeijer (2005) and the continuous updated GMM estimator (Hansen et al., 1996). Second, we consider also the LM test, which in this case reduces the number of estimated parameters to zero. Third, we apply some recently developed (Stock and Wright, 2000; Kleibergen, 2005) weak instrument robust GMM statistics. Fourth, we will compare GMM statistics making use of either DIF moment conditions or LEV or both (SYS).

The remaining of the paper is as follows. In Section 2 we describe in more detail the various GMM based statistics and their application to the dynamic panel data model. In Section 3 we report some selected results from an extensive Monte Carlo study. Section 4 concludes.

2. Generalized Method of Moments

2.1. GMM statistics

We consider estimation of a scalar parameter θ using the k-dimensional moment equation

$$E\left[f_i(\theta)\right] = 0, \qquad 1, ..., N, \tag{2.1}$$

where we have suppressed the dependence of the moment equation on the actual panel data for N individuals. We denote the unique value of θ for which these moment conditions hold with θ_0. The variance matrix of the moment conditions (evaluated at θ_0) is defined by

$$V_{ff}(\theta_0) = E\left[f_i(\theta_0) f_i(\theta_0)'\right].$$

Furthermore, the derivative of the moment conditons is defined by

$$q_i(\theta) = \frac{\partial f_i(\theta)}{\partial \theta}, \qquad q_N(\theta) = \frac{1}{N}\sum_{i=1}^{N} q_i(\theta).$$

The GMM estimator of θ_0 minimizes the following objective function with respect to θ:

$$f_N(\theta)' W_N f_N(\theta)$$

where $f_N(\theta) = \frac{1}{N}\sum_{i=1}^{N} f_i(\theta)$ and W_N a weight matrix. Under standard regularity conditions the GMM estimator has a limiting normal distribution with asymptotic variance depending on the particular choice of the weight matrix W_N. Using a known weight matrix, e.g. the identity matrix, results in a consistent one-step GMM estimator denoted by $\hat{\theta}_1$. When we use a one-step estimator in combination with the inverse of the estimated variance matrix of the moment conditons as weight matrix, i.e.

$$\begin{aligned} W_N(\hat{\theta}_1) &= \hat{V}_{ff}(\hat{\theta}_1)^{-1} \\ &= \frac{1}{N}\sum_{i=1}^{N} f_i(\hat{\theta}_1)f_i(\hat{\theta}_1)', \end{aligned}$$

the optimal two-step GMM estimator results denoted by $\hat{\theta}$. A consistent estimator of its asymptotic variance is

$$\widehat{var}(\hat{\theta}) = \frac{1}{N}\left(q_N(\hat{\theta})'W_N(\hat{\theta}_1)q_N(\hat{\theta})\right)^{-1}.$$

The standard Wald test (based on the two-step GMM estimator) for testing $H_0 : \theta = \theta_0$ can simply be expressed as

$$W(\theta_0) = \frac{\left(\hat{\theta} - \theta_0\right)^2}{\widehat{var}(\hat{\theta})}. \tag{2.2}$$

Windmeijer (2005) proposed a finite sample correction to the estimated asymptotic variance of the two-step GMM estimator. Replacing $\widehat{var}(\hat{\theta})$ in (2.2) by this alternative estimator (labeled $\widehat{var}_c(\hat{\theta})$) we get the corrected Wald statistic

$$W_c(\theta_0) = \frac{\left(\hat{\theta} - \theta_0\right)^2}{\widehat{var}_c(\hat{\theta})}. \tag{2.3}$$

We also consider the Wald statistic based on the continuously updating GMM estimator (Hansen et al., 1996) of θ (labeled $\hat{\theta}_{cue}$), i.e.

$$W_{cue}(\theta_0) = \frac{\left(\hat{\theta}_{cue} - \theta_0\right)^2}{\widehat{var}(\hat{\theta}_{cue})}. \tag{2.4}$$

The standard GMM-LM statistic (Newey and West, 1987) for testing $H_0 : \theta = \theta_0$ can be written as

$$\begin{aligned} LM(\theta_0) \;=\; & N f_N(\theta_0)'\hat{V}_{ff}(\theta_0)^{-1}q_N(\theta_0)\left(q_N(\theta_0)'\hat{V}_{ff}(\theta_0)^{-1}q_N(\theta_0)\right)^{-1} \\ & q_N(\theta_0)'\hat{V}_{ff}(\theta_0)^{-1}f_N(\theta_0). \end{aligned} \tag{2.5}$$

Furthermore, we consider two weak instrument robust GMM statistics due to Stock and Wright (2000) and Kleibergen (2005). The Stock and Wright (2000) statistic can be written as

$$S(\theta_0) = Nf_N(\theta_0)'\hat{V}_{ff}(\theta_0)^{-1}f_N(\theta_0). \tag{2.6}$$

The Kleibergen (2005) statistic is equal to:

$$\begin{aligned} KLM(\theta_0) &= Nf_N(\theta_0)'\hat{V}_{ff}(\theta_0)^{-1}\hat{D}_N(\theta_0)\left(\hat{D}_N(\theta_0)'\hat{V}_{ff}(\theta_0)^{-1}\hat{D}_N(\theta_0)\right)^{-1} \\ &\quad \hat{D}_N(\theta_0)'\hat{V}_{ff}(\theta_0)^{-1}f_N(\theta_0), \end{aligned} \tag{2.7}$$

where (see Kleibergen (2005, 2007, 2008) for more details):

$$\begin{aligned} \hat{D}_N(\theta) &= q_N(\theta) - \hat{V}_{qf}(\theta)\hat{V}_{ff}(\theta)^{-1}f_N(\theta), \\ \hat{V}_{qf}(\theta) &= \frac{1}{N}\sum_{i=1}^{N} q_i(\theta)f_i(\theta)' - q_N(\theta)f_N(\theta)'. \end{aligned}$$

The S and KLM statistics have limiting $\chi^2(k)$ and $\chi^2(1)$ distributions respectively regardless of strong or weak identification. Under strong identification all other statistics have a limiting $\chi^2(1)$ distribution, but under weak identification their null distribution may be radically different.

2.2. moment conditions for the AR(1) panel data model

We now continue with the discussion of the particular moment conditions for the first-order autoregressive panel data model:

$$\begin{aligned} y_{it} &= \theta y_{i,t-1} + u_{it}, \qquad i = 1, ..., N;\ t = 2, ..., T, \\ u_{it} &= \eta_i + v_{it} \end{aligned} \tag{2.8}$$

where it is assumed that η_i and v_{it} have an error components structure with

$$E(\eta_i) = 0,\ E(v_{it}) = 0,\ E(v_{it}\eta_i) = 0, \qquad i = 1, ..., N;\ t = 1, ..., T \tag{2.9}$$

$$E(v_{it}v_{is}) = 0, \qquad i = 1, ..., N \text{ and } t \neq s, \tag{2.10}$$

and the initial condition satisfies

$$E(y_{i1}v_{it}) = 0, \qquad i = 1, ..., N;\ t = 2, ..., T. \tag{2.11}$$

Under these assumptions the following $(T-1)(T-2)/2$ linear moment conditions are valid for the model in first differences

$$E\left(y_i^{t-2}\Delta u_{it}\right)=0, \qquad t=3,...,T, \tag{2.12}$$

where $y_i^{t-2}=(y_{i1},y_{i2},...,y_{it-2})'$ and $\Delta u_{it}=u_{it}-u_{i,t-1}=\Delta y_{it}-\theta\Delta y_{i,t-1}$.

Defining

$$Z_{di}=\left[\begin{array}{ccccccc} y_{i1} & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & y_{i1} & y_{i2} & \cdots & 0 & \cdots & 0 \\ . & . & . & \cdots & . & \cdots & . \\ 0 & 0 & 0 & \cdots & y_{i1} & \cdots & y_{iT-2} \end{array}\right]; \ \Delta u_i=\left[\begin{array}{c} \Delta u_{i3} \\ \Delta u_{i4} \\ \vdots \\ \Delta u_{iT} \end{array}\right],$$

the DIF moment conditions (2.12) can be more compactly written as

$$E\left(Z_{di}'\Delta u_i\right)=0. \tag{2.13}$$

Blundell and Bond (1998) exploit additional moment conditions from the assumption on the initial condition (see Arellano and Bover (1995)) that

$$E\left(\eta_i\Delta y_{i2}\right)=0, \tag{2.14}$$

which holds when the process is mean stationary:

$$y_{i1}=\frac{\eta_i}{1-\theta}+v_{i1}, \tag{2.15}$$

with $E\left(v_{i1}\right)=E\left(v_{i1}\eta_i\right)=0$. If (2.9), (2.10), (2.11) and (2.14) hold then the following $(T-1)(T-1)/2$ moment conditions are valid for the model in levels

$$E\left(u_{it}\Delta y_i^{t-1}\right)=0, \qquad t=3,...,T, \tag{2.16}$$

where $\Delta y_i^{t-1}=(\Delta y_{i2},\Delta y_{i2},...,\Delta y_{it-1})'$. Defining

$$Z_{li}=\left[\begin{array}{ccccccc} \Delta y_{i2} & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \Delta y_{i2} & \Delta y_{i3} & \cdots & 0 & \cdots & 0 \\ . & . & . & \cdots & . & \cdots & . \\ 0 & 0 & 0 & \cdots & \Delta y_{i2} & \cdots & \Delta y_{iT-1} \end{array}\right]; \ u_i=\left[\begin{array}{c} u_{i3} \\ u_{i4} \\ \vdots \\ u_{iT} \end{array}\right],$$

the LEV moment conditions (2.16) can be written as

$$E\left(Z_{li}'u_i\right)=0. \tag{2.17}$$

The full set of linear SYS moment conditions under assumptions (2.9), (2.10), (2.11) and (2.14) is given by

$$\begin{aligned} E\left(y_i^{t-2}\Delta u_{it}\right) &= 0 \qquad t = 3, ..., T; \\ E\left(u_{it}\Delta y_{i,t-1}\right) &= 0 \qquad t = 3, ..., T, \end{aligned} \tag{2.18}$$

or

$$E\left(Z_{si}'p_i\right) = 0, \tag{2.19}$$

where

$$Z_{si} = \begin{bmatrix} Z_{di} & 0 & \cdots & 0 \\ 0 & \Delta y_{i2} & & 0 \\ . & . & \ddots & . \\ 0 & 0 & \cdots & \Delta y_{iT-1} \end{bmatrix}; \; p_i = \begin{bmatrix} \Delta u_i \\ u_i \end{bmatrix}.$$

Summarizing, using the general notation from (2.1) we exploit for the DIF, LEV and SYS models the following sets of moment conditions

$$\begin{aligned} f_{di}(\theta) &= Z_{di}'(\Delta y_i - \theta\Delta y_{i,-1}), \\ f_{li}(\theta) &= Z_{li}'(y_i - \theta y_{i,-1}), \\ f_{si}(\theta) &= Z_{si}'\begin{bmatrix} \Delta y_i - \theta\Delta y_{i,-1} \\ y_i - \theta y_{i,-1} \end{bmatrix}, \end{aligned}$$

where the number of moment conditions $k = (T-1)(T-2)/2$ for $f_{di}(\theta)$ and $f_{li}(\theta)$, while $k = (T-1)(T-2)/2 + T - 2$ for $f_{si}(\theta)$.

2.3. discussion

In the simulations we will analyze the in Section 2.1 described GMM statistics for testing hypotheses on the autoregressive parameter in the dynamic panel data model (2.8). We will now discuss their relative merits in more detial using related literature on GMM and dynamic panel data models.

The Wald statistic in (2.2) depends primarily on GMM coefficient and asymptotic variance estimators. Hence, the finite sample distribution of the Wald statistic in (2.2) can deviate substantially from its limiting chi-squared distribution when coefficient estimators and/or asymptotic variance estimators are inaccurate. One important aspect of this inaccuracy is that estimators can be biased.

In moderately large samples especially asymptotically most efficient GMM coefficient estimators may show substantial bias by the use of an abundance of moment conditions, see Ziliak (1997) and Koenker and Machado (1999). In dynamic panel data models the number of available moment conditions increases rapidly with the number of time observations. By using asymptotic expansion techniques Bun and Kiviet (2006) show for this model that the order of finite sample bias of GMM estimators depends indeed on the number of moment conditions used in estimation. Furthermore, their bias approximations indicate that bias of GMM coefficient estimators for the DIF model depend primarily on the autoregressive dynamics. But more importantly, bias of LEV and SYS GMM estimators depends heavily on the aforementioned variance ratio. This dependence of estimation bias on both parameters of interest and crucial nuisance parameters suggests weak identification problems for a significant part of the parameter space.

Indeed, Blundell and Bond (1998) show that the DIF GMM estimator can have very poor finite sample properties in terms of bias and precision when the series are persistent, as the instruments are then only weakly correlated with the endogenous regressors. However, for the dynamic panel data model Bun and Windmeijer (2010) show that there is a weak instrument problem for the LEV and SYS GMM estimators too. They show that the weak instrument problem manifests itself not so much in terms of absolute bias of coefficient estimators, but merely in estimation bias relative to that of inconsistent least squares estimators and size distortions of Wald type tests.

The continuously updating GMM estimator (Hansen et al., 1996) has been shown in Monte Carlo studies to have less finite sample bias than the two-step GMM estimator. Also test statistics based on this estimator are more reliable. Using asymptotic expansion techniques Newey and Smith (2004) derive some desirable theoretical properties about the higher order bias of the generalized empirical likelihood (GEL) estimator. The class of GEL estimators includes the continuous updating estimator of Hansen et al. (1996). Hence, we expect the CUE-GMM Wald statistic (2.4) to be more reliable compared with the two-step Wald statistic.

Even if GMM coefficient estimators have negligible bias, still estimators of the asymptotic variance matrix can be severely biased in finite samples. Reason is that the estimated asymptotic variance of the two-step GMM estimator depends on a preliminary consistent one-step coefficient estimator. For panel data models Arellano and Bond (1991) already showed by simulation that asymptotic standard errors of two-step GMM estimators can be severely downward biased. Using asymptotic expansion techniques Windmeijer (2005) proposes a finite sample correction to the estimated asymptotic variance of the two-step GMM estimator. Bond and Windmeijer (2005) conclude that in terms of size properties the corrected two-step Wald statistic (2.3) performed equally well as the one-step Wald statistic. However, in case of weak identification they are still size distorted because coefficient estimators are biased as discussed above.

The LM statistic (2.5) has the advantage over Wald-type statistics that it is evaluated under the restrictions imposed by the null hypothesis. In the case of only a scalar parameter this reduces the number of estimated parameters to zero. For the dynamic panel data model Bond and Windmeijer (2005) find more favorable performance of the LM test compared with Wald type tests. However, in case of weak identification still size distortions can be seen. In other words, the finite sample distribution of the GMM-LM statistic still depends on crucial nuisance parameters. This can be explained by the fact that in case of linear moment conditions the LM and one-step Wald statistics are numerically equivalent when exploiting the same covariance matrix estimate (Newey and West, 1987). In practice they differ somewhat because they are based on constrained and unconstrained estimates of the covariance matrix, but we expect them to behave very similar at least under the null hypothesis. Now one-step Wald statistics depend on one-step GMM estimators which in dynamic panel data models have been shown (Bun and Kiviet, 2006) to depend heavily on either the autoregressive dynamics or variance ratio or both. Hence, the LM statistic faces the same lack of invariance problem regarding nuisance parameters as Wald statistics.

Summarizing, conventional GMM Wald and LM statistics are subject to significant size distortions especially in case of weak instruments. Main reason is that null distributions of these statistics in finite samples depend on crucial nuisance parameters and this dependence becomes more pronounced in case of weak identification. Regarding the dynamic panel data model weak identification occurs when the autoregressive dynamics are persistent and/or the variance of the unobserved time invariant heterogeneity is large.

To overcome size distortions of GMM with weak instruments, test statistics have been proposed with limiting null distributions robust to instrument quality. Hence, these limiting distributions provide a better approximation to finite sample distributions especially in case of weak identification. Stock and Wright (2000) propose the S statistic (2.6), while Kleibergen (2005) develops the KLM statistic (2.7).

Regarding the dynamic panel data model it is expected that in finite samples the null distributions of these weak instrument robust statistics are largely invariant to both autoregressive dynamics and variance ratio. Regarding its application to panel data models with sequential moment conditons, a possible disadvantage of the S statistic is its dependence on the number of moment conditions. As noted before the number of available moment conditions grows rapidly with the number of time observations. Hence, especially the power of the S statistic might be negatively affected even when the number of time observations in the panel data is moderate. The KLM statistic provides a solution to this lack of robustness to many moment conditions. The simulations of Kleibergen (2005) show that the KLM statistic provides a more powerful test than the S statistic.

3. simulation results

Our simulation study contributes in various ways to existing simulation results on the finite sample behaviour of GMM statistics for the simple AR(1) panel data model (2.8). First, contrary to previous simulation studies we analyze the behaviour of GMM test statistics under both null and alternative hypotheses. Second, we vary both the autoregressive coefficient and variance ratio in order to get a more comprehensive overview of the finite sample properties in case of weak identification. Third, we also analyze weak instrument robust GMM statistics. To the best of our knowledge these statistics have not been analyzed before for the dynamic panel data model.

We generate data voor y_{it} according to

$$\begin{aligned} y_{i1} &= \frac{\eta_i}{1-\theta} + v_{i1}, \\ y_{it} &= \theta y_{i,t-1} + \eta_i + v_{it}, \qquad t = 2, ..., T, \end{aligned}$$

$$\begin{aligned} v_{i1} &\sim IIN\left(0, \frac{\sigma_v^2}{1-\theta^2}\right),\ \eta_i \sim IIN\left(0, \sigma_\eta^2\right), \\ v_{it} &\sim IIN\left(0, \sigma_v^2\right), \qquad t = 2, ..., T. \end{aligned}$$

Without loss of generality we set $\sigma_v^2 = 1$ and we use $\sigma_\eta^2 = \{0, 1, 3, 6, 10\}$. Regarding sample size we use $T = \{3, 4, 5, 6, 7\}$ and $N = \{250, 500, 1000\}$. For each combination of T, N and σ_η^2 we vary θ from 0.1 to 0.95 with step length 0.025.

We consider tests of $H_0 : \theta = \theta_0$ with $\theta_0 = \{0.5, 0.6, 0.7, 0.8, 0.9, 0.95\}$. We test H_0 always with a nominal size of 5%. We analyze actual size and power of tests based on the following GMM statistics: Wald (W), Wald with corrected variance (W_c), Wald based on continuously updated GMM estimator (W_{cue}), LM (LM), Kleibergen (2005) statistic (KLM) and Stock and Wright (2000) statistic (S). We analyze the preformce of these 6 statistics for the DIF, LEV and SYS model. The resulting power curves are based on 1000 Monte Carlo replications.

We first show selected simulation results for $T = 7$ and $N = 250$. Figure 1 shows power curves for $\theta_0 = 0.5$ and $vr = 1$, hence under H_0 the weak instrument problem is relatively minor in all three (DIF, LEV, SYS) models. In this case we observe for all 6 GMM statistics that actual size is quite close to nominal size in especially the DIF and LEV models. In general LEV and SYS statistics have steeper power curves than DIF statistics. Also there is for many DIF and LEV statistics a sudden decline in power when θ gets closer to 1. Perhaps most striking is the fact that for the KLM statistic this sudden decline has disappeared in the SYS model. In the SYS model the KLM statistic seems to perform best, i.e. it shows no size distortion, while maintaining good power properties.

In Figure 2 we continue our analysis changing θ_0 from $\theta_0 = 0.5$ to $\theta_0 = 0.8$. We expect now a weak instrument problem especially in the DIF and, hence, SYS models. Increasing the persistence of the panel data we indeed observe considerable size distortions for the 3 Wald statistics, especially in the DIF model. In the SYS model these size distortions are even aggrevated. Again SYS KLM seems to perform best in this case.

In Figure 3 we go back to $\theta_0 = 0.5$, but increase the variance ratio from $vr = 1$ to $vr = 3$. We now expect a weak instrument problem especially in the LEV model. Increasing the variance ratio we hardly see any differences in power curves for the DIF model. In the LEV model we observe some size distortions and power curves are flatter for higher values of θ. But differences are most pronounced for the SYS model where we see considerable size distortions. Only the power curves of the two weak instrument robust stastistics (KLM and S) seem largely invariant to the change in the variance ratio with again SYS KLM having most favorable performance.

In Figure 4 we analyze the impact of a simultaneous change of both θ_0 and vr resulting in the design with $\theta_0 = 0.8$ and $vr = 3$. We now expect a weak instrument problem in both DIF and LEV models. Wald-type tests now perform poorly in all 3 models. Especially in the SYS model size distortions are huge. For example, for the Wald statistic the actual rejection probability under H_0 is around 85%.

Summarizing the results from Figures 1-4, null distributions of Wald type tests depend heavily on crucial nuisance parameters. In case of weak instruments Wald tests can have large size distortions. Null distributions of especially weak instrument robust statistics are, as expected, largely invariant to these crucial model parameters. Most striking feature is the favorable performance of SYS KLM. Where DIF and LEV KLM statistics show a sudden power decline for larger values of θ and/or vr this feature disappears in the SYS model.

Finally, we analyze the effects of changing sample size. We concentrate our analysis on the SYS model only. As expected increasing N from 250 to 1000 results in more favorable results in case of strong identification as can be seen from the upper left panel of Figure 5. However, when θ_0 and vr become large the weak instrument problem again arises as can be concluded from the upper right panel of Figure 5. Wald tests are heavily size distorted showing that the weak instrument problem is relevant even in large samples. Decreasing the number of time periods from $T = 7$ to $T = 4$ lowers this size distortion, hence apparently the number of moment conditions used in estimation matters a lot for Wald tests. Overall SYS KLM shows the best performance.

4. concluding remarks

We have analyzed by simulation the finite sample properties of a wide range of GMM statistics for the first-order dynamic panel data model without any additional regressors. For three versions of this model, i.e. first-differences or levels or both, we have analyzed size and power of standard Wald and LM statistics as well as some recently developed weak instrument robust GMM statistics. Our simulation design contains a wide range of parametrizations covering both strongly and weakly identified cases. The latter occurs when there is high persistence or a relatively large degree of time invariant unobserved heterogeneity or both.

Our results confirm existing evidence on the poor finite sample properties of the Wald-type tests even in case of reasonable strong identification. Although the LM test is an improvement in this case, it still shows considerable size distortions in case of weak instruments. In this case especially the weak instrument robust statistics show favorable performance. Overall it seems that the KLM statistic applied to the system model performs best in terms of both size and power.

Having established the favorable performance of weak instrument robust statistics for the relatively simple panel AR(1) model we intend to generalize the analysis to models with more than one endogenous regressor. This is necessary because in many applications there are apart from the autoregressive dynamics additional endogenous or weakly exogenous regressors. It is expected that in such models standard Wald and LM statistics will behave poorly again. However, the presence of more than one regressor subject to feedback mechanisms from the dependent variable may cause additional complications regarding the use of weak instrument robust GMM statistics.

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Figure 1: power curves, $\theta_0 = 0.5, vr = 1, T = 7, N = 250$

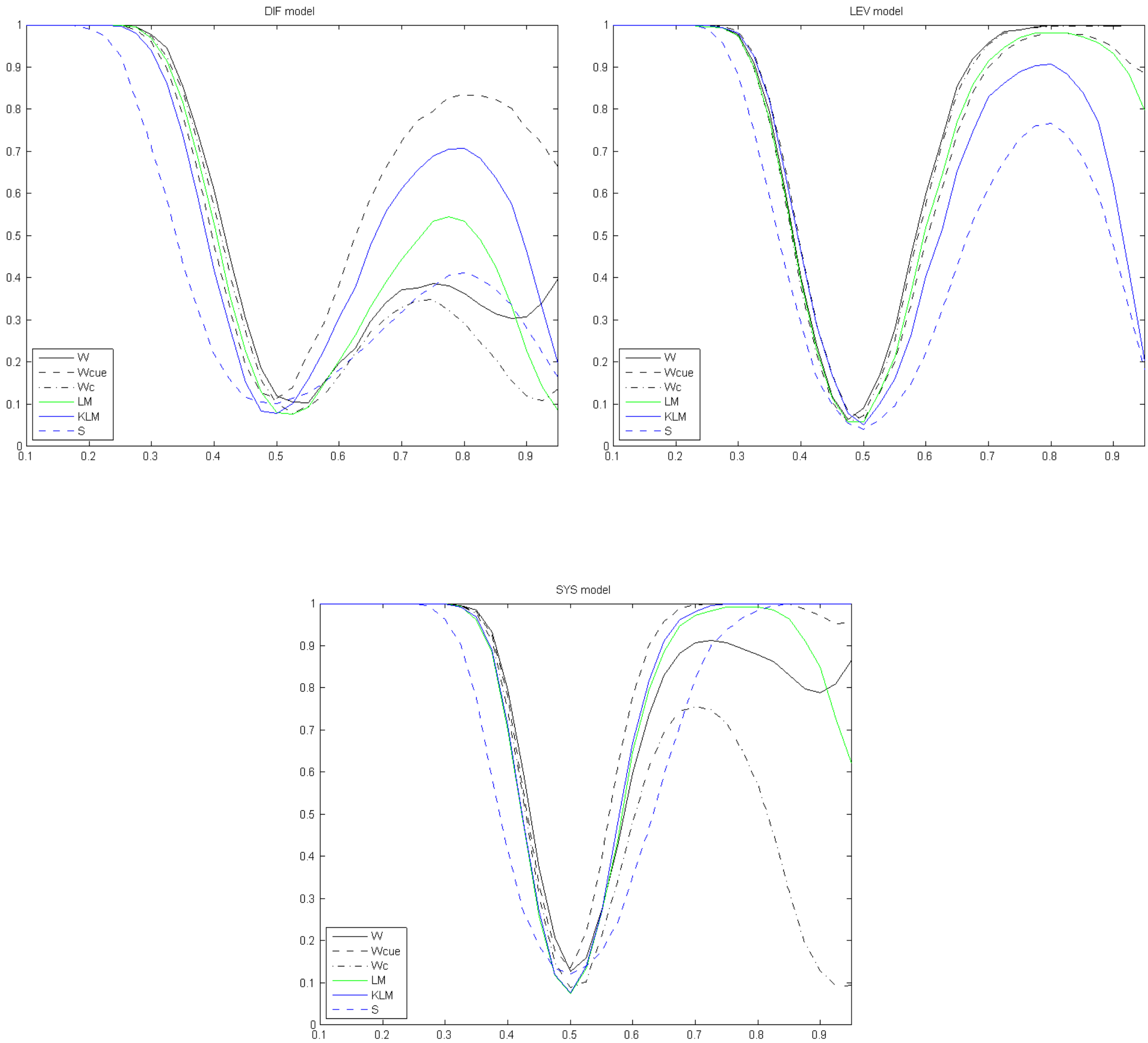


Figure 2: power curves, $\theta_0 = 0.8, vr = 1, T = 7, N = 250$

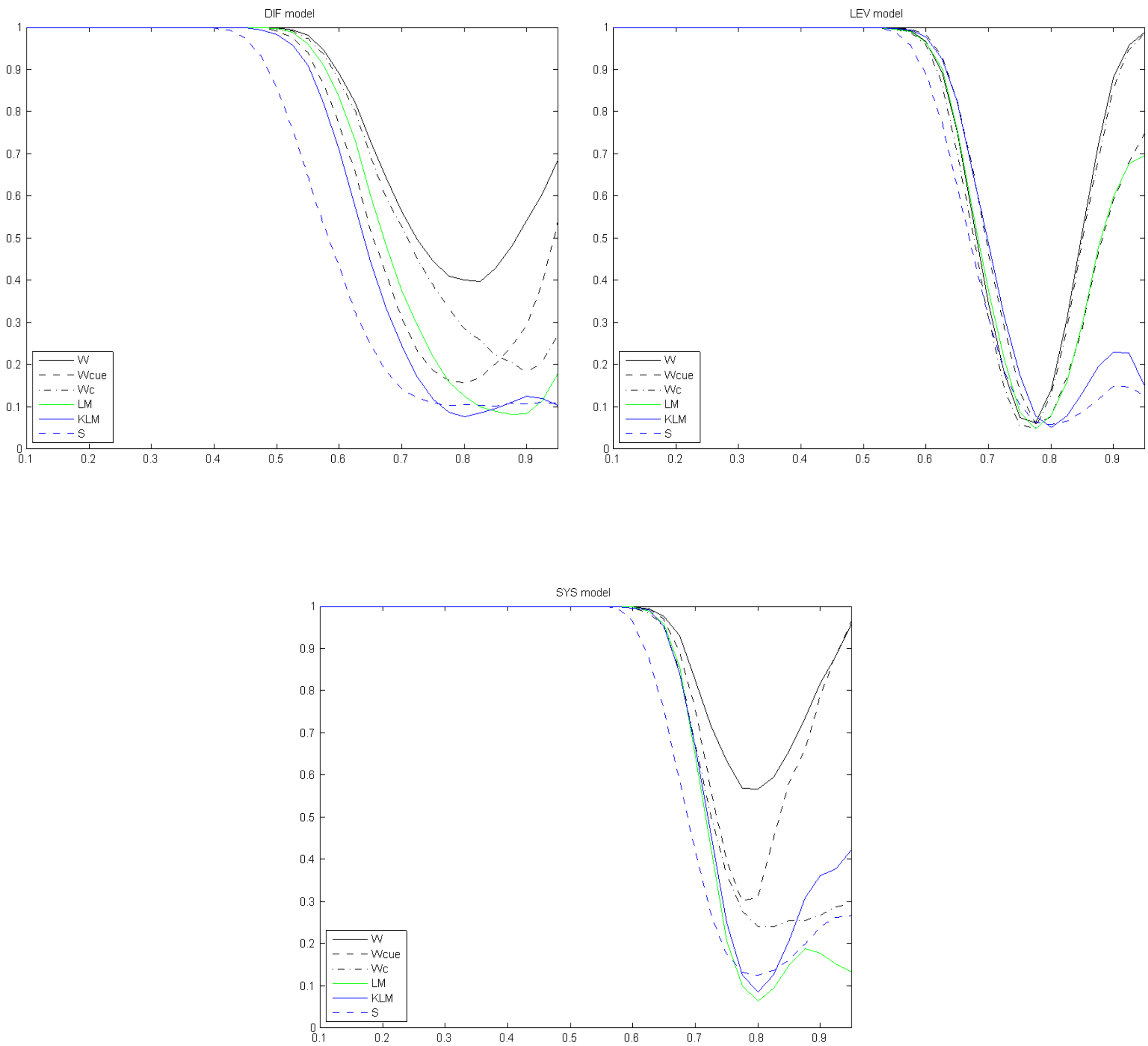


Figure 3: power curves, $\theta_0 = 0.5, vr = 3, T = 7, N = 250$

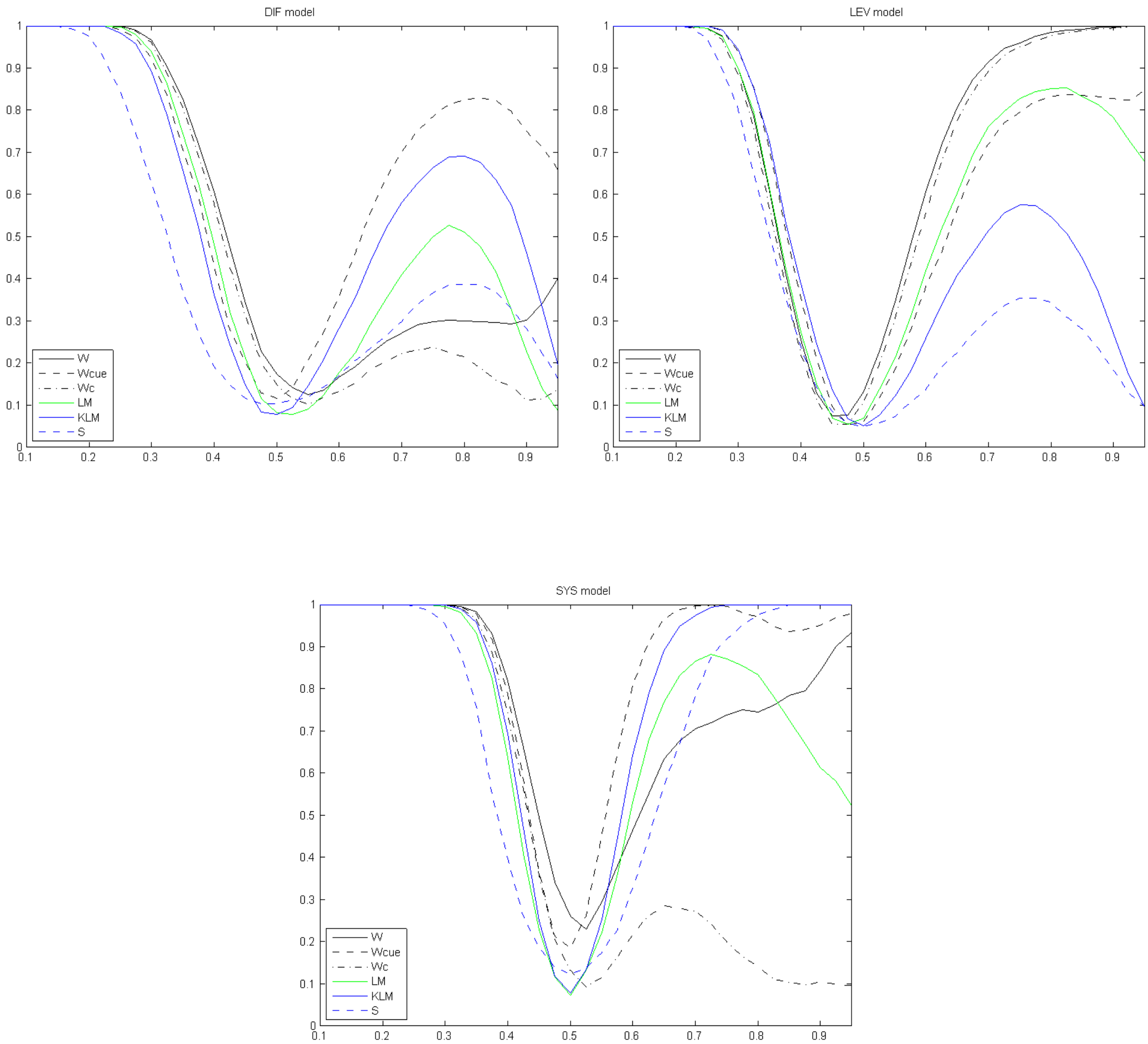


Figure 4: power curves, $\theta_0 = 0.8, vr = 3, T = 7, N = 250$

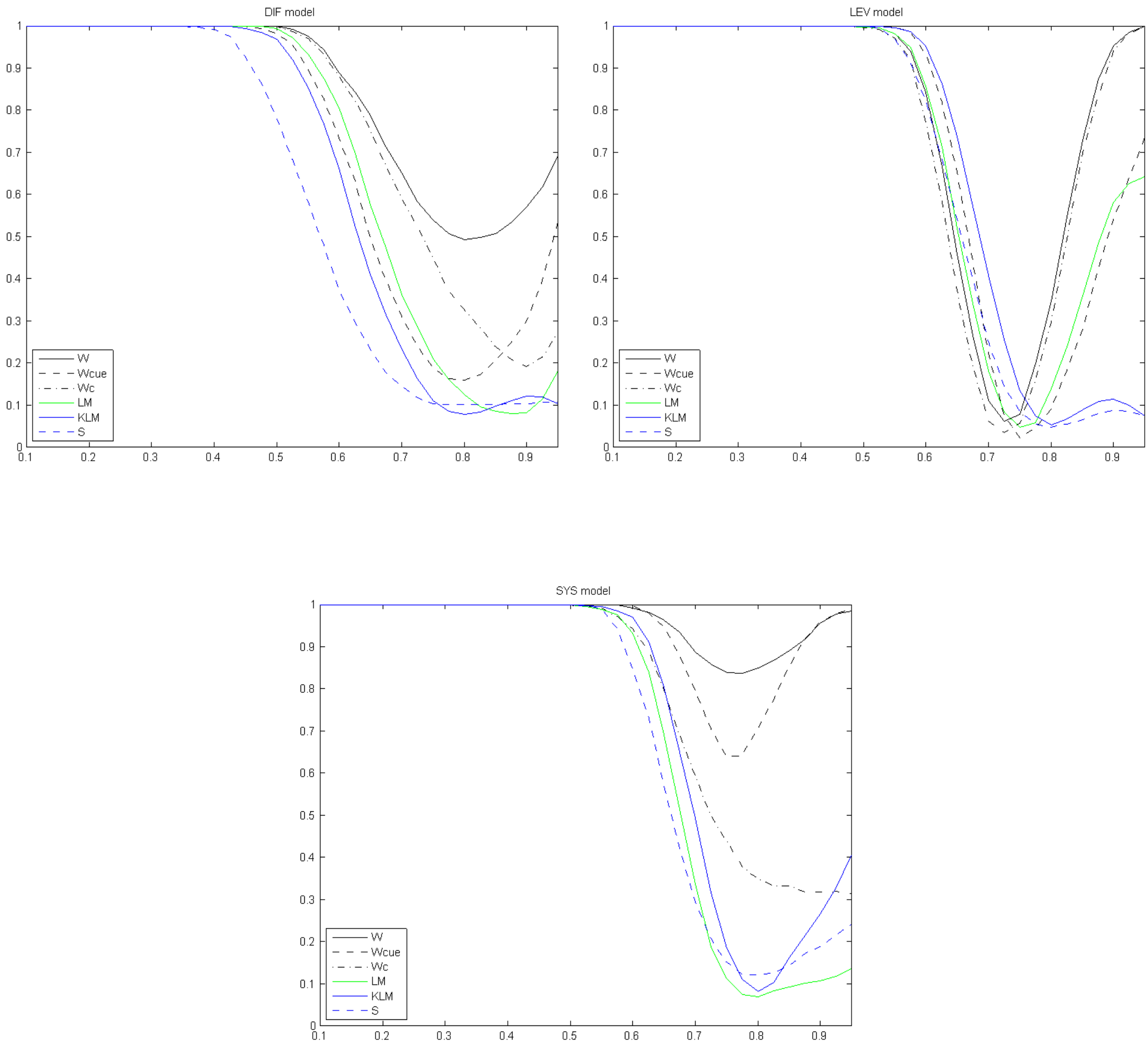
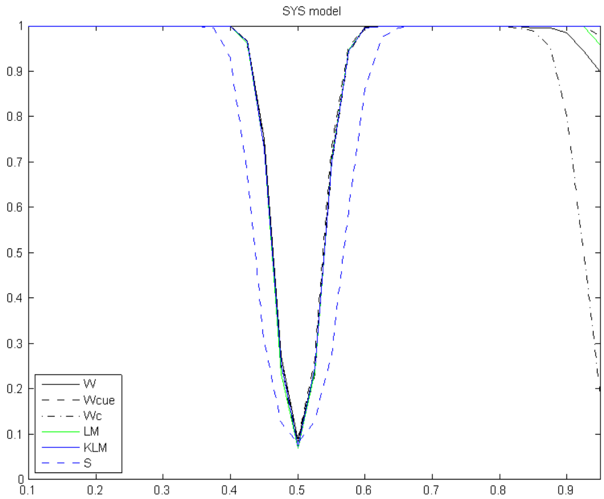
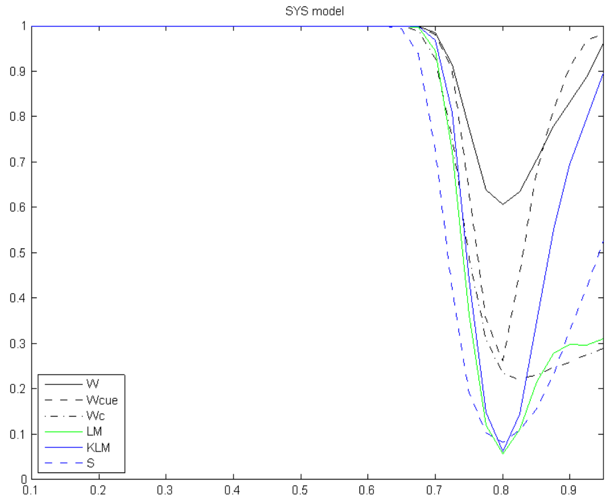


Figure 5: power curves, SYS model



θ_0= 0.5, $vr = 1, T = 7, N = 1000$



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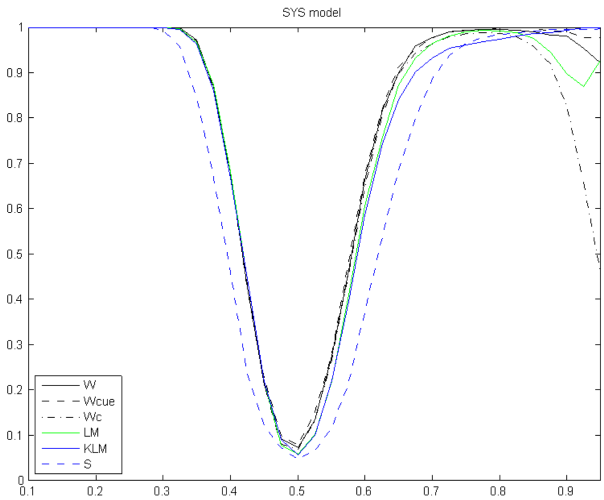
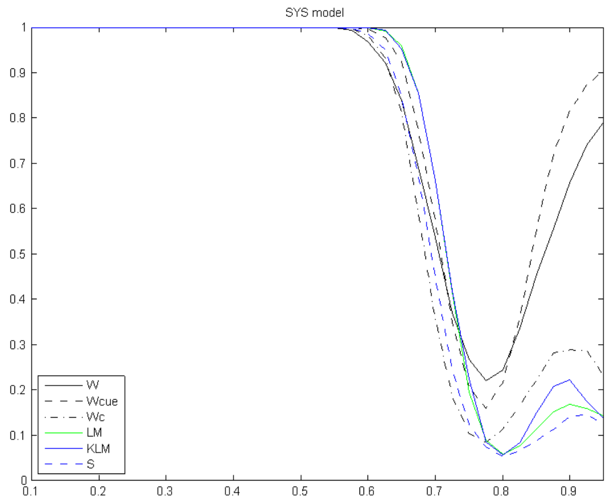


θ_0= 0.5, $vr = 1, T = 4, N = 1000$



θ_0= 0.8, $vr = 3, T = 4, N = 1000$