A panel unit root test in the presence of cross-section dependence

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Summary A panel unit root test is derived based on a Lagrangian Multiplier for panels with a cross-section dependence modeled using factor models. The test statistic is shown to be different from square of test statistic of Pesaran (2007) even for a case of no constant and no trend, hence the test statistic is not simply a different calculation of the suggestion made in Pesaran (2007). Implementation of the test is very simple and it is easy to understand its mechanics. Furthermore, the limiting distribution of of individual cross-sectionally augmented test statistics which appear to be functions of standard wiener processes are shown to be free of nuisance parameters. The critical values of the proposed test statistics are tabulated. The small sample size properties of the proposed test statistics are shown to outperform the suggestion of Pesaran (2007) in terms of power, in many cases, i.e. the proposed test statistic is more powerful, in particular, for alternatives involving some individuals with unit root and some with stationary behaviors, which makes it quite appealing for practical purposes. The asymptotics are established under $N/T \to \delta > 0$ with N being the number of cross-sections and T the length of time series.

Keywords: *Panel Data, Unit Root, Cross-section Dependence, Factor Models, Lagrangian Multiplier.*

1. INTRODUCTION

It is a well established fact that leaving the cross-sectional dependence out would lead to serious size distortions and power loss. Chang (2002), Choi (2002), Phillips and Sul (2003), Bai and Ng (2004), Moon and Perron (2004), Smith *et al.* (2004), Breitung and Das (2005), Choi and Chue (2007), Pesaran (2007), Demetrescu *et al.* (2009) and Pesaran *et al.* (2009) propose different panel unit root tests allowing for cross-section dependence.

Chang (2002) uses a nonlinear instrumental approach to deal with the cross-sectional dependence. Choi (2002) proposes a special form of covariance matrix to model the cross-sectional dependence. This is while Bai and Ng (2004), Moon and Perron (2004), and Phillips and Sul (2003) turn to residual factor models to capture the cross-sectional dependence. Phillips and Sul (2003) propose an orthogonalization procedure that asymptotically eliminates the common factors. Bai and Ng (2004) and Moon and Perron (2004) use similar approaches in the sense that they estimate the factors and work with de-factored time series. Pesaran (2006) proposed that cross-sectional means of differenced data, and cross-section mean of lagged data are good proxies for unknown factors. This fact is used in Pesaran (2007) and Pesaran *et al.* (2009) to proxy for unobserved factors instead of using factor estimation which involves estimating the number of factors then the factors themselves.

One important distinguishing feature of these tests is the asymptotic relationship between dimensions of panel under consideration. For example Phillips and Sul (2003), Moon and Perron (2004), and Bai and Ng (2004) require a fairly big time series dimension (T) compared to cross-section dimension (N), i.e. they need $N/T \to 0$. Chang (2002) has introduced a test which is valid for fixed N as T gets big (cf. Pesaran (2007)) and the test of Smith *et al.* (2004) has the same restriction. Pesaran (2007) introduces a framework in which he allows for $N/T \to \delta$ for a positive real valued δ. The small sample size properties of existing tests have been examined extensively.[1] Furthermore, test of Pesaran (2007) is extremely simple to implement. Following the approach to Pesaran (2007), this paper proposes a single factor structure for the error term to tackle the cross-sectional dependence, while the test statistic proposed here is base on Lagrangian Multiplier. The proposed test has the same asymptotic behavior for N and T as the proposed test of Pesaran (2007) but in the contrary to time series literature on testing for unit roots, it is not equivalent to a t-statistic based test.

In testing for the unit root in panels, cross-sectional units are all nonstationary under the null while the alternative could take two possible forms.[2] First, all of the cross-sectional units are assumed to be stationary. Secondly some of the cross-sectional units are allowed to be nonstationary. As mentioned in Pesaran (2007) a panel unit root test can only be consistent whose alternative is of the second form.[3] This note seems, however, to have remained just as an important hint to econometricians, while they tend to address on the power features of their proposed tests for simulated panels whose all of cross-sections are stationary. In the current work we address this issue and observe that the test of Pesaran (2007) is not always desirable when the fraction of stationary cross-sectional units is away from one. Nevertheless, our proposed test outperforms a t-statistic based test as the alternative gets closer to the null, i.e. as the number of nonstationary units in the alternative grows.

The structure of the paper is as follows. The panel data model considered in this paper is discussed in the next section and the proposed test statistic is also presented. Section 3 states the asymptotic results and provides some discussion on how the proposed test differs from the test proposed by Pesaran (2007). Simulation studies and size and power features of the proposed test are contrasted with the test of Pesaran (2007) in section 4. Section 5 concludes. Appendix A and B contain the mathematical proofs and tables, respectively.

2. MODEL

Consider a simple linear heterogeneous panel data model of the following form

$$y_{it} = (1 - \phi_i)\,\mu_i + \phi_i y_{i,t-1} + u_{it}, i = 1, 2, ..., N; t = 1, 2, ..., T$$

The error term has a single-factor structure, i.e. $u_{it} = \gamma_i f_t + \varepsilon_{it}$, for which ε_{it} is the idiosyncratic error term and f_t is the unobserved common factor associated with the factor loading γ_i. With $\alpha_i = (1 - \phi_i)\,\mu_i$ and $\beta_i = -(1 - \phi_i)$ rewrite the model as

$$\Delta y_{it} = \alpha_i + \beta_i y_{i,t-1} + \gamma_i f_t + \varepsilon_{it} \tag{2.1}$$

for which we are interested to introduce a test for the following hypothesis

$$H_0 : \beta_i = 0 \ \vee i \tag{2.2}$$

[1]cf. Breitung and Pesran (2007) and Choi (2006).

[2]Except Demetrescu *et al.* (2009) that use a KPSS approach.

[3]This is a note mentioned in an unpublished work by Im K. S. and M. H. Pesaran (2003): On the Panel Unit Root Tests Using Nonlinear Instrumental Variable.

against an alternative of the form

$$H_a : \beta_i < 0,\ i = 1, 2, ..., N_1,\ \beta_i = 0,\ i = N_1 + 1, N_1 + 2, ..., N$$

Where N_1/N tends to a nonzero fraction $0 < \delta \leq 1$ as $N \to \infty$. Under the null we have that

$$\Delta y_{it} = \gamma_i f_t + \varepsilon_{it} \tag{2.3}$$

Using Pesaran (2007) one can write this equation in a matrix format as

$$\Delta \mathbf{y}_i = \delta_i \Delta \overline{\mathbf{y}} + \xi_{it}$$

for which $\Delta \mathbf{y}_i = (\Delta y_{i1}, \Delta y_{i1}, ..., \Delta y_{iT})'$, $\Delta \overline{\mathbf{y}} = (\Delta \overline{y}_1, \Delta \overline{y}_2, ..., \Delta \overline{y}_T)'$, $\delta_i = \frac{\gamma_i}{\overline{\gamma}}$, and $\xi_{it} = \varepsilon_{it} - \delta_i \overline{\varepsilon}_t$.

Let $\overline{\mathbf{M}}_w = \mathbf{I}_T - \overline{\mathbf{W}}\left(\overline{\mathbf{W}}'\overline{\mathbf{W}}\right)^{-1}\overline{\mathbf{W}}'$ with $\overline{\mathbf{W}} = \left(\tau, \Delta\overline{\mathbf{y}}, \overline{\mathbf{y}}_{-1}, \mathbf{y}_{i,-1}\right)$ where $\overline{\mathbf{y}}_{-1} = \left(\overline{y}_0, \overline{y}_1, ..., \overline{y}_{T-1}\right)'$ and $\mathbf{y}_{i,-1} = (y_{i,0}, y_{i,1}, ..., y_{i,T-1})'$ and $\overline{\mathbf{M}}_\delta = \mathbf{I}_T - \Delta\overline{\mathbf{y}}\left(\Delta\overline{\mathbf{y}}'\Delta\overline{\mathbf{y}}\right)^{-1}\Delta\overline{\mathbf{y}}'$. Then

$$\overline{\mathbf{M}}_\delta \Delta \mathbf{y}_i = \overline{\mathbf{M}}_\delta \xi_i$$

and

$$\overline{\mathbf{M}}_w \Delta \mathbf{y}_i = \overline{\mathbf{M}}_w \xi_i$$

where $\xi_i = (\xi_{i1}, \xi_{i2}, ..., \xi_{iT})' \sim \left(\mathbf{0}, \omega_i^2 \mathbf{I}_T\right)$

$$\omega_i^2 = \sigma_i^2 \left(1 - \frac{2\delta_i}{N}\right) + \frac{\delta_i^2}{N}\overline{\sigma}^2 = \sigma_i^2 + O\left(\frac{1}{N}\right)$$

and $\overline{\sigma}^2$ is the mean of σ_i^2 over i.

The proposed test statistics is based on the following which is clearly and LM type test.

$$LM_i = T\left(1 - \frac{\Delta\mathbf{y}_i \overline{\mathbf{M}}_\delta \overline{\mathbf{M}}_w \overline{\mathbf{M}}_\delta \Delta\mathbf{y}_i}{\Delta\mathbf{y}_i \overline{\mathbf{M}}_\delta \Delta\mathbf{y}_i}\right) = T\left(1 - \frac{\Delta\mathbf{y}_i \overline{\mathbf{M}}_w \Delta\mathbf{y}_i}{\Delta\mathbf{y}_i \overline{\mathbf{M}}_\delta \Delta\mathbf{y}_i}\right) \tag{2.4}$$

which using (7) and (8) can be written as

$$LM_i = T\left(1 - \frac{\xi_i' \overline{\mathbf{M}}_w \xi_i}{\xi_i' \overline{\mathbf{M}}_\delta \xi_i}\right)$$

and with $\nu_i = \xi_i / \omega_i \sim (\mathbf{0}, \mathbf{I}_T)$ we have

$$LM_i = T\left(1 - \frac{\nu_i' \overline{\mathbf{M}}_w \nu_i}{\nu_i' \overline{\mathbf{M}}_\delta \nu_i}\right) \tag{2.5}$$

The following assumption are considered all through this paper.

Assumption 1. Idiosyncratic error term, ε_{it}, is independently distributed across time and cross sections, and has finite fourth moment, and is a mean zero random variable with variance σ_i^2.

Assumption 2. unobserved common factor , f_t, does not exhibit serial correlation has mean zero and variance σ_f^2 with finite fourth moment. Without loss of generality we can assume the variance of the common factor to be unity.

Assumption 3. ε_{it}, f_t, and γ_i are independently distributed for all i.

3. LM BASED UNIT ROOT TEST

THEOREM 3.1. *Given assumptions 1 and 2, for the process defined in (2.1) and generated under (2.2), for a fixed T, we have*

$$\frac{1}{T}LM_i \xrightarrow{N} 1-\frac{\frac{\varepsilon_i'\varepsilon_i}{\sigma_i^2}-\left(\Gamma_1^*\mathbf{q}_{iT}^{(1)}+\mathbf{q}_{iT}^{(2)}\right)'\left(\Gamma_1^*\Psi_{fT}\Gamma_1^*+I_r\Gamma_2^*\Psi_{fiT}^{(1)}\Gamma_2^*+\Gamma_3^*\Psi_{fiT}^{(2)}\right)^{-1}\left(\Gamma_1^*\mathbf{q}_{iT}^{(1)}+\mathbf{q}_{iT}^{(2)}\right)}{\frac{\varepsilon_i'\varepsilon_i}{\sigma_i^2}-\frac{\varepsilon_i'\mathbf{f}(\mathbf{f}'\mathbf{f})^{-1}\mathbf{f}'\varepsilon_i}{\sigma_i^2}} \tag{3.1}$$

where LM_i is defined in (2.5), and all the matrices in the limit expression can be found in the Appendix A.

THEOREM 3.2. ***Theorem 2.*** *Given assumptions 1-3, and $\lim_{N\to\infty}\overline{\gamma}=\gamma_*\neq 0$, if $N\to\infty$ is followed by $T\to\infty$, then*

$$LM_i \underset{T}{\overset{N}{\to}} \ell_{if} \tag{3.2}$$

where ℓ_{if} is defined in Appendix A and is free from nuisance parameters.

To test for a unit root in a time series setting it is quite straightforward to see that for a no intercept no trend case an LM test statistic is the same as the square of a DF test statistic. However, this is not the case for the panel model considered here. Theorem 2 establishes asymptotic results for an intercept case. In (2.1) considering $\alpha_i = 0$ for all i, the limiting distribution of cross-sectionally augmented Lagrangian multiplier test can be derived and shown to be

$$LM_i = \frac{1}{D^2 - BE}\left(2HID - BI^2 - H^2E\right)$$

with the B, D, E, H, and I defined in Appendix A. On the other hand a cross-sectionally augmented DF test statistics, t_{DF_i}, proposed in Pesaran (2007) has the following limiting distribution for a no intercept and no trend case.

$$t_{DF_i} = \frac{I - \frac{HD}{B}}{\left[E - \frac{D^2}{B}\right]^{0.5}}$$

For these two limiting distributions one might observe that

$$LM_i = \left[t_{DF_i}\right]^2 + \frac{H^2}{B} \tag{3.3}$$

for which the additional term is in fact

$$\frac{H^2}{B} = \frac{\left[\int_0^1 W_f(r)\,dW_i(r)\right]^2}{\int_0^1 W_f^2(r)\,dr}$$

which originates from the factors present in the model.[4] In this respect one would not expect the same relationship between t_{DF_i} and LM_i as established for the simple time-series case for a no intercept and no trend case. In other words, factor augmentation step makes the two test statistics not to be equivalent in their traditional way. $\frac{H^2}{B}$ has a $\chi^2_{(1)}$

[4] W_f is a wiener process driving the common factor and W_i is a wiener process driving the idiosyncratic error term for cross-section i.

distribution which means that the cross-section dependence makes an LM type test to be more variable than a case where the cross-sectional units are independent.

Equation (2.5) sketches a Lagrangian Multiplier test which in time series analysis is known to be asymptotically equivalent to other types of tests. In a pure time series analysis framework and under some conventional assumptions, one would expect that an LM test statistic would not be different from testing procedures based on other possible definitions of the distance between the null and the alternative. But as we see from (3.3) this is not asymptotically true for the panel model considered here.

The literature on panel econometrics distinguishes between 2 types of cross-sectional dependence: strong and weak. A strong cross-sectional dependence is when the covariance of Δy_{it} possesses at least one eigen value that explodes as the cross-section dimension goes to infinity. One way to allow for a strong dependence is to consider a common factor in the error term. Theorem 1 and 2, do not allow for serial correlation in the error term which is important in practice. Serial correlation could come through the factor, f_t, or through the remainder of the error term after removing the common factor. Considering an AR representation for the factor would only require to correct the underlying Wiener process for f_t. Considering the serial correlation to come through the idiosyncratic error term would however involve a different asymptotic result which in fact could be written in the same fashion as Pesaran (2007) while augmenting more lags in the regression would help removing such an effect.

4. SIMULATION AND COMPARISON STUDIES

The computation of the critical values were based on 10000 regressions and calculating the value of the proposed test statistic in (2.4), while a correction for small sample sizes was always considered. In generating the individual series for generating the all of the critical values, time is considered to start from -50 with $y_{i,-50} = 0$. The panel dimensions are also spanning a wide range of possibilities i.e. $N,\ T \in \{10, 15, 20, 30, 50, 70, 100, 200\}$. Tables (1)-(3) and (13)-(15) tabulated the critical values of the proposed test for the average of cross-sectionally augmented LM statistic and for the individual cross-sectionally augmented cases, where 3 common model specifications are considered.

The Date Generating Process (DGP) considered to examine the small sample size of the proposed test takes into account this possibility that the alternative, as sketched in the introduction, can include cases in which only a subset of the cross-sections are stationary. This point has been set off in simulation studies done by Pesaran (2007) where cross-sectional augmentation in conducting a panel unit root test is discussed. As we will see test of Pesaran (2007) looses its power the more one deviates from a panel with purely stationary cross-sections. The simulation study for analyzing small sample size properties of the proposed test is contrasted with the tests proposed by Pesaran (2007).

The proposed test is called MH, the test of Pesaran (2007) is tagged CIPS and CIPS* represents the truncated version of CIPS. The simulation studies for sketching on the power and size comparison of MH and CIPS are carried out for $N,\ T \in \{10, 20, 30, 50, 100\}$.

Tables (4)-(6) tabulate the size and power of the proposed test in contrast with the Test of Pesaran (2007). For these tables the power is calculated for a simulated panel data set with one third of its individuals being I(1) and the rest being I(0). Table (4) pertains to a model where there is no intercept and no trend, and Tables (5) and (6) correspond respectively to cases of including an intercept and an intercept and a time trend, respectively.

In table (4) both tests are always correctly sized, while both almost have no power for N and T equal to 10. For the no intercept and no trend case, CIPS has relatively better power for $N = 10$, while as N increases, MH tends to slightly gain power. For the cases of N and T equal to 50, and $N = 100$ and $T = 50$ MH exhibits a substantially higher power. The results shown in table (5) are always in favor of CIPS as it always exhibits a slightly higher power, this is while the size is correct for both tests. Table (6) shows a substantially higher power for the proposed test compared to the power of CIPS.

Tables (7)-(9), tabulate power and size results for in the same fashion as tables (4)-(6) except that the power has been calculated for simulated panel data sets with half of its individuals being I(1) and the rest being I(0). Table (7) and (8) show results pretty much the same as the results of tables (4)-(5), while MH has a substantially higher power for the intercept and trend case tabulated in table (9).

In tables (10)-(12), power has been calculated for the case where two third of the unit in the simulated panel data are considered I(1) and the rest to be I(0). As this table shows, CIPS exhibits a lower relative power compared to MH in almost all the cases. This is while both tests exhibit lower power profile for an intercept case compared to the case of no intercept and the case of an intercept and a trend. For the intercept and trend case, shown in table (12), MH likewise tables (6) and (9) exhibits a substantially higher power compared to CIPS.

5. CONCLUSION

A Lagrangian Multiplier test is proposed for unit root testing in panels with cross-sectionally dependent units modeled through considering a simple factor structure. The test is applicable to test the existence of a unit root in each individual separately while averaging the test statistics across the units provides a tool for testing the existence of a unit root in the panel data. The asymptotics of the proposed test are derived and show to be independent of the nuisance parameters. For the no intercept case it is also shown that the proposed test is not the square of the test proposed by Pesaran (2007). The power and size of the proposed test is contrasted to those of CIPS. Power of the test is calculated or cases in which the panel data set is simulated with a one third, half, and two third of the units being I(1) and the rest being I(0). As the results show, in the intercept and trend case the proposed test has a drastically higher power compared to that CIPS. Furthermore, as the fraction of I(1) individuals increase to two third, the result show that MH has a relatively higher power for the no intercept case and the intercept case compared to that of CIPS. The size of both tests is always correct for all the cases. As the simulation results show, the proposed test deserves more theoretical investigations.

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REFERENCES

Bai, J. and S. Ng (2004). A PANIC attack on unit roots and cointegration. *Econometrica* *72*, 1127–1177.

Baltagi, B., G. Bresson and A. Pirotte (2007). Panel Unit Root Tests and Spatial Dependence. *Journal of Applied Econometrics 22*, 339–360.

Breitung, J., and S. Das (2005). Panel unit root tests under cross sectional dependence *Statistica Neerlandica 59*, 414–433.

Breitung, J. and M. H. Pesaran (2007). Unit roots and cointegration in panels. In Ma'tya's L. and Sevestre P. (Eds), *The Econometrics of Panel Data: Fundamentals and Recent Developments in Theory and Practice*, 279–322. Kluwer: Dordrecht.

Choi, I. (2002). Combination unit root tests for cross-sectionally correlated panels. *Mimeo*, Hong Kong University of Science and Technology.

Choi, I. (2006). Nonstationary panels. In Mills T. C. and K. Patterson (Eds)*Palgrave Handbooks of Econometrics, Volume 1*, 511–539. Palgrave Macmillan: Basingstoke.

Choi I. and T. K. Chue (2007). Subsampling hypothesis tests for nonstationary panels with applications to exchange rates and stock prices. *Journal of Applied Econometrics 22*, 233–264.

Chang Y. (2002). Nonlinear IV unit root tests in panels with cross-sectional dependency. *Journal of Econometrics 100*, 261–292.

Demetrescu M., U. Hassler and A. I. Tarcolea (2009). Testing for Stationarity in Large Panels with Cross-Dependence, and US Evidence on Unit Labor Cost Forthcoming in *Journal of Applied Statistics* .

Hamilton J. D. (1994). *Time Series Analysis* New Jersey: Princeton.

Moon, H. R. and B. Perron (2004). Testing for a unit root in panels with dynamic factors. *Journal of Econometrics 122*, 81–126.

Pesaran, M. H. (2006). Estimation and inference in large heterogeneous panels with cross section dependence. *Econometrica 74*, 967–1012.

Pesaran, M. H. (2007). A simple panel unit root test in the presence of cross-section dependence. *Journal of Applied Econometrics 22*, 265–312.

Pesaran M. H., L. V. Smith and T. Yamagata (2009) A Panel Unit Root Test in the Presence of a Multifactor Error Structure. Working paper, Cambridge University.

Phillips P. C. B. and D. Sul (2003) Dynamic panel estimation and homogeneity testing under cross section dependence. *Econometrics Journal 6*, 217–259.

Smith L. V., S. Leybourne S, T. K. Kim and P. Newbold (2004) More powerful panel data unit root tests with an application to mean reversion in real exchange rates. *Journal of Applied Econometrics 19*, 147–170.

APPENDIX A: PROOFS OF RESULTS

Proof of Theorem 1. In (2.4) first we consider $\nu_i'\overline{\mathbf{M}}_w\nu_i$.

$$\nu_i'\overline{\mathbf{M}}_w\nu_i = \nu_i'\nu_i - \left(\nu_i'\overline{\mathbf{W}}D\right)\left(D\overline{\mathbf{W}}'\overline{\mathbf{W}}D\right)^{-1}\left(D\overline{\mathbf{W}}'\nu_i\right) \tag{A.1}$$

where

$$D = \begin{bmatrix} T^{-1/2} & 0 & 0 & 0 \\ 0 & T^{-1/2} & 0 & 0 \\ 0 & 0 & T^{-1} & 0 \\ 0 & 0 & 0 & T^{-1} \end{bmatrix}$$

and

$$D\overline{\mathbf{W}}'\overline{\mathbf{W}}D = \begin{bmatrix} 1 & \frac{1}{T}\Delta\overline{\mathbf{y}}'\tau & \frac{1}{T^{\frac{3}{2}}}\overline{\mathbf{y}}'_{-1}\tau & \frac{1}{T^{\frac{3}{2}}}\tau'\mathbf{y}_{i,-1} \\ \frac{1}{T}\Delta\overline{\mathbf{y}}'\tau & \frac{1}{T}\Delta\overline{\mathbf{y}}'\Delta\overline{\mathbf{y}} & \frac{1}{T^{\frac{3}{2}}}\Delta\overline{\mathbf{y}}'\overline{\mathbf{y}}_{-1} & \frac{1}{T^{\frac{3}{2}}}\Delta\overline{\mathbf{y}}'\mathbf{y}_{i,-1} \\ \frac{1}{T^{\frac{3}{2}}}\overline{\mathbf{y}}'_{-1}\tau & \frac{1}{T^{\frac{3}{2}}}\Delta\overline{\mathbf{y}}'\overline{\mathbf{y}}_{-1} & \frac{1}{T^2}\overline{\mathbf{y}}'_{-1}\overline{\mathbf{y}}_{-1} & \frac{1}{T^2}\overline{\mathbf{y}}'_{-1}\mathbf{y}_{i,-1} \\ \frac{1}{T^{\frac{3}{2}}}\tau'\mathbf{y}_{i,-1} & \frac{1}{T^{\frac{3}{2}}}\Delta\overline{\mathbf{y}}'\mathbf{y}_{i,-1} & \frac{1}{T^2}\overline{\mathbf{y}}'_{-1}\mathbf{y}_{i,-1} & \frac{1}{T^2}\mathbf{y}'_{i,-1}\mathbf{y}_{i,-1} \end{bmatrix} \tag{A.2}$$

$$D\overline{\mathbf{W}}'\nu_i = \begin{bmatrix} \frac{1}{\sqrt{T}}\tau'\nu_i \\ \frac{1}{\sqrt{T}}\Delta\overline{\mathbf{y}}'\nu_i \\ \frac{1}{T}\overline{\mathbf{y}}'_{-1}\nu_i \\ \frac{1}{T}\mathbf{y}'_{i,-1}\nu_i \end{bmatrix} \tag{A.3}$$

Using (2.3) the following equations follow.

$$\Delta\overline{\mathbf{y}} = \overline{\gamma}\mathbf{f}+\overline{\varepsilon} \tag{A.4}$$

$$\mathbf{y}_{i,-1} = \mathbf{y}_{i0}\tau + \gamma_i\mathbf{s}_{f,-1} + \mathbf{s}_{i,-1} \tag{A.5}$$

$$\overline{\mathbf{y}}_{-1} = \overline{\mathbf{y}}_0\tau + \overline{\gamma}\mathbf{s}_{f,-1} + \overline{\mathbf{s}}_{-1} \tag{A.6}$$

with $\tau = (1, 1, ...1)$, $\mathbf{s}_{i,-1} = (0, s_{i,1}, s_{i,2}, ..., s_{i,T-1})'$, $\overline{\mathbf{s}}_{-1} = (0, \overline{s}_1, \overline{s}_2, ..., \overline{s}_{T-1})$, $s_{i,t} = \sum_{j=1}^{t}\varepsilon_{ij}$, $\overline{s}_t = N_{-1}\sum_{j=1}^{N} s_{it}$, $\mathbf{s}_{f,-1} = (s_{f0}, s_{f1}, ..., s_{f,T-1})'$, and $s_{ft} = \sum_{j=1}^{t} f_j$. Equation (A.6) can be used to eliminate the partial sum of unobserved factors from equation (A.5), namely

$$\mathbf{y}_{i,-1} = (y_{i0} - \delta_i\overline{y}_0)\,\tau - \delta_i\overline{\mathbf{y}}_{-1} + \mathbf{s}_{i,-1} - \delta_i\overline{\mathbf{s}}_{-1}$$

Let $\mathfrak{s}_{i,-1} = (\mathbf{s}_{i,-1} - \delta_i\overline{\mathbf{s}}_{-1})\,/\omega_i$ and rewrite as

$$\mathbf{y}_{i,-1} = (y_{i0} - \delta_i\overline{y}_0)\,\tau - \delta_i\overline{\mathbf{y}}_{-1} + \omega_i\mathfrak{s}_{i,-1} \tag{A.7}$$

$\mathfrak{s}_{i,-1}$ is a random walk associated with $\nu_i = \xi_i/\omega_i \sim (\mathbf{0}, \mathbf{I}_T)$. Now using (A.4)-(A.7), along with some results from Pesaran (2006) we can simplify the elements of (A.2) and (A.3) as follows.

$$\frac{1}{T}\Delta\overline{\mathbf{y}}'\tau = \overline{\gamma}\left(\frac{\mathbf{f}'\tau}{T}\right) + \left(\frac{\mathbf{f}'\overline{\varepsilon}}{T}\right)$$

$$\frac{1}{T^{\frac{3}{2}}}\overline{\mathbf{y}}'_{-1}\tau = \overline{y}_0\left(\frac{1}{\sqrt{T}}\right) + \overline{\gamma}\left(\frac{\mathbf{s}'_{f,-1}\tau}{T^{\frac{3}{2}}}\right) + \left(\frac{\overline{\mathbf{s}}'_{-1}\tau}{T^{\frac{3}{2}}}\right)$$

$$\frac{1}{T^{\frac{3}{2}}}\tau'\mathbf{y}_{i,-1} = y_{i0}\left(\frac{1}{\sqrt{T}}\right) - \gamma_i\left(\frac{\mathbf{s}'_{f,-1}\tau}{T^{\frac{3}{2}}}\right) + \left(\frac{\mathbf{s}'_{i,-1}\tau}{T^{\frac{3}{2}}}\right)$$

$$\frac{1}{T}\Delta\overline{\mathbf{y}}'\Delta\overline{\mathbf{y}} = \overline{\gamma}^2\left(\frac{\mathbf{f}'\mathbf{f}}{T}\right) + 2\overline{\gamma}\left(\frac{\mathbf{f}'\overline{\varepsilon}}{T}\right) + \left(\frac{\overline{\varepsilon}'\overline{\varepsilon}}{T}\right)$$

$$\begin{aligned}\frac{1}{T^{\frac{3}{2}}}\Delta\overline{\mathbf{y}}'\overline{\mathbf{y}}_{-1} &= \overline{\gamma y}_0\left(\frac{\mathbf{f}'\tau}{T^{\frac{3}{2}}}\right) + \overline{y}_0\left(\frac{\overline{\varepsilon}'\tau}{T^{\frac{3}{2}}}\right) + \overline{\gamma}^2\left(\frac{\mathbf{f}'\mathbf{s}_{f,-1}}{T^{\frac{3}{2}}}\right)\\ &+\overline{\gamma}\left(\frac{\mathbf{f}'\overline{\mathbf{s}}_{-1}}{T^{\frac{3}{2}}}\right) + \overline{\gamma}\left(\frac{\overline{\varepsilon}'\mathbf{s}_{f,-1}}{T^{\frac{3}{2}}}\right) + \left(\frac{\overline{\varepsilon}'\overline{\mathbf{s}}_{-1}}{T^{\frac{3}{2}}}\right)\end{aligned}$$

$$\begin{aligned}\frac{1}{T^{\frac{3}{2}}}\Delta\overline{\mathbf{y}}'\mathbf{y}_{i,-1} &= \overline{\gamma}y_{i0}\left(\frac{\mathbf{f}'\tau}{T^{\frac{3}{2}}}\right) + \overline{\gamma}\gamma_i\left(\frac{\mathbf{f}'\mathbf{s}_{f,-1}}{T^{\frac{3}{2}}}\right) + \overline{\gamma}\left(\frac{\mathbf{f}'\mathbf{s}_{i,-1}}{T^{\frac{3}{2}}}\right)\\ &+y_{i0}\left(\frac{\overline{\varepsilon}'\tau}{T^{\frac{3}{2}}}\right) + \gamma_i\left(\frac{\overline{\varepsilon}'\mathbf{s}_{f,-1}}{T^{\frac{3}{2}}}\right) + \left(\frac{\overline{\varepsilon}'\mathbf{s}_{i,-1}}{T^{\frac{3}{2}}}\right)\end{aligned}$$

$$\begin{aligned}\frac{1}{T^2}\overline{\mathbf{y}}'_{-1}\overline{\mathbf{y}}_{-1} &= \frac{\overline{y}_0^2}{T} + \overline{\gamma}^2\left(\frac{\mathbf{s}'_{f,-1}\mathbf{s}_{f,-1}}{T^2}\right) + \left(\frac{\overline{\mathbf{s}}'_{-1}\overline{\mathbf{s}}_{-1}}{T^2}\right)\\ &+2\overline{\gamma y}_0\left(\frac{\mathbf{s}'_{f,-1}\tau}{T^2}\right) + 2\overline{y}_0\left(\frac{\overline{\mathbf{s}}'_{-1}\tau}{T^2}\right) + 2\overline{\gamma}\left(\frac{\mathbf{s}'_{f,-1}\overline{\mathbf{s}}_{-1}}{T^2}\right)\end{aligned}$$

$$\begin{aligned}\frac{1}{T^2}\overline{\mathbf{y}}'_{-1}\mathbf{y}_{i,-1} &= \overline{y}_0 y_{i0}\left(\frac{1}{T}\right) + \overline{y}_0\gamma_i\left(\frac{\tau'\mathbf{s}_{f,-1}}{T^2}\right) + \overline{y}_0\gamma_i\left(\frac{\tau'\mathbf{s}_{i,-1}}{T^2}\right)\\ &+\overline{\gamma}y_{i0}\left(\frac{\mathbf{s}'_{f,-1}\tau}{T^2}\right) + \overline{\gamma}\gamma_i\left(\frac{\mathbf{s}'_{f,-1}\mathbf{s}_{f,-1}}{T^2}\right) + \overline{\gamma}\left(\frac{\mathbf{s}'_{f,-1}\mathbf{s}_{i,-1}}{T^2}\right)\\ &+y_{i0}\left(\frac{\overline{\mathbf{s}}'_{-1}\tau}{T^2}\right) + \gamma_i\left(\frac{\overline{\mathbf{s}}'_{-1}\mathbf{s}_{f,-1}}{T^2}\right) + \left(\frac{\overline{\mathbf{s}}'_{-1}\mathbf{s}_{i,-1}}{T^2}\right)\end{aligned}$$

$$\begin{aligned}\frac{1}{T^2}\mathbf{y}'_{i,-1}\mathbf{y}_{i,-1} &= y_{i0}\left(\frac{1}{T}\right) + \gamma_i^2\left(\frac{\mathbf{s}'_{f,-1}\mathbf{s}_{f,-1}}{T^2}\right) + \left(\frac{\mathbf{s}'_{i,-1}\mathbf{s}_{i,-1}}{T^2}\right)\\ &+2y_{i0}\gamma_i\left(\frac{\tau'\mathbf{s}_{f,-1}}{T^2}\right) + 2y_{i0}\left(\frac{\tau'\mathbf{s}_{i,-1}}{T^2}\right) + 2\gamma_i\left(\frac{\mathbf{s}'_{f,-1}\mathbf{s}_{i,-1}}{T^2}\right)\end{aligned}$$

Now, noting that all the cross sectional means go to zero as N goes to infinity we can write

$$D\overline{\mathbf{W}}'\overline{\mathbf{W}}D \xrightarrow{N} \Gamma_1^*\Psi_{fT}\Gamma_1^* + I_r\Gamma_2^*\Psi_{fiT}^{(1)}\Gamma_2^* + \Gamma_3^*\Psi_{fiT}^{(2)} \tag{A.8}$$

where

$$\Gamma_1^* = \begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & \gamma^* & 0 & 0\\ 0 & 0 & \gamma^* & 0\\ 0 & 0 & 0 & \gamma_i \end{bmatrix},\ \Psi_{fT} = \begin{bmatrix} 1 & \frac{\mathbf{f}'\tau}{T} & \frac{\mathbf{s}'_{f,-1}\tau}{T^{\frac{3}{2}}} & \frac{\mathbf{s}'_{f,-1}\tau}{T^{\frac{3}{2}}}\\ \frac{\mathbf{f}'\tau}{T} & \frac{\mathbf{f}'\mathbf{f}}{T} & \frac{\mathbf{f}'\mathbf{s}_{f,-1}}{T^{\frac{3}{2}}} & \frac{\mathbf{f}'\mathbf{s}_{f,-1}}{T^{\frac{3}{2}}}\\ \frac{\mathbf{s}'_{f,-1}\tau}{T^{\frac{3}{2}}} & \frac{\mathbf{f}'\mathbf{s}_{f,-1}}{T^{\frac{3}{2}}} & \frac{\mathbf{s}'_{f,-1}\mathbf{s}_{f,-1}}{T^2} & \frac{\mathbf{s}'_{f,-1}\mathbf{s}_{f,-1}}{T^2}\\ \frac{\mathbf{s}'_{f,-1}\tau}{T^{\frac{3}{2}}} & \frac{\mathbf{f}'\mathbf{s}_{f,-1}}{T^{\frac{3}{2}}} & \frac{\mathbf{s}'_{f,-1}\mathbf{s}_{f,-1}}{T^2} & \frac{\mathbf{s}'_{f,-1}\mathbf{s}_{f,-1}}{T^2} \end{bmatrix}$$

$$I_r = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, \Gamma_2^* = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \gamma^* & 0 & 0 \\ 0 & 0 & \gamma^* & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \Psi_{fiT}^{(1)} = \begin{bmatrix} \frac{\tau s'_{i,-1}}{T^{\frac{3}{2}}} & \frac{f' s_{i,-1}}{T^{\frac{3}{2}}} & \frac{s_{i,-1} s'_{f,-1}}{T^2} & \frac{s_{i,-1} s'_{i,-1}}{T^2} \\ 0 & 0 & 0 & \frac{f' s_{i,-1}}{T^{\frac{3}{2}}} \\ 0 & 0 & 0 & \frac{s_{i,-1} s'_{f,-1}}{T^2} \\ 0 & 0 & 0 & \frac{\tau s'_{i,-1}}{T^{\frac{3}{2}}} \end{bmatrix}$$

$$\Gamma_3^* = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2\gamma_i \end{bmatrix}, \Psi_{fiT}^{(2)} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\mathbf{s}'_{f,-1}\mathbf{s}_{i,-1}}{T^2} \end{bmatrix}$$

Now we consider the elements of equation (A.3). First recall that $\delta_i = \frac{\gamma_i}{\overline{\gamma}}$, and $\xi_{it} = \varepsilon_{it} - \delta_i \overline{\varepsilon}_t$ with $\omega_i^2 = \sigma_i^2 + O\left(\frac{1}{N}\right)$ and $\mathfrak{s}_{i,-1} = \left(\mathbf{s}_{i,-1} - \delta_i \overline{\mathbf{s}}_{-1}\right)/\omega_i$ with $\mathfrak{s}_i = \mathfrak{s}_{i,-1} + \nu_i$. Then we can simplify the elements of (A.3) as follows.

$$\frac{1}{\sqrt{T}}\tau'\nu_i = \left(\frac{\tau'\varepsilon_i}{\omega_i\sqrt{T}}\right) - \delta_i\left(\frac{\tau'\overline{\varepsilon}}{\omega_i\sqrt{T}}\right)$$

$$\begin{aligned} \frac{1}{\sqrt{T}}\Delta\overline{\mathbf{y}}'\nu_i &= \overline{\gamma}\left(\frac{\mathbf{f}'\nu_i}{\sqrt{T}}\right) + \left(\frac{\overline{\varepsilon}'\nu_i}{\sqrt{T}}\right) \\ &= \overline{\gamma}\left(\frac{\mathbf{f}'\varepsilon_i}{\omega_i\sqrt{T}}\right) - \overline{\gamma}\gamma_i\left(\frac{\mathbf{f}'\overline{\varepsilon}}{\omega_i\sqrt{T}}\right) \\ &\quad + \left(\frac{\overline{\varepsilon}'\varepsilon_i}{\omega_i\sqrt{T}}\right) - \delta_i\left(\frac{\overline{\varepsilon}'\overline{\varepsilon}}{\omega_i\sqrt{T}}\right) \end{aligned}$$

$$\begin{aligned} \frac{1}{T}\overline{\mathbf{y}}'_{-1}\nu_i &= \overline{y}_0\left(\frac{\tau'\nu_i}{T}\right) + \overline{\gamma}\left(\frac{\mathbf{s}'_{f,-1}\nu_i}{T}\right) + \left(\frac{\overline{\mathbf{s}}'_{-1}\nu_i}{T}\right) \\ &= \overline{y}_0\left(\frac{\tau'\varepsilon_i}{\omega_i T}\right) - \overline{y}_0\delta_i\left(\frac{\tau'\overline{\varepsilon}}{\omega_i\sqrt{T}}\right) \\ &\quad + \overline{\gamma}\left(\frac{\mathbf{s}'_{f,-1}\varepsilon_i}{\omega_i T}\right) - \overline{\gamma}\delta_i\left(\frac{\mathbf{s}'_{f,-1}\overline{\varepsilon}}{\omega_i T}\right) \\ &\quad + \left(\frac{\overline{\mathbf{s}}'_{-1}\varepsilon_i}{\omega_i T}\right) - \delta_i\left(\frac{\overline{\mathbf{s}}'_{-1}\overline{\varepsilon}_i}{\omega_i T}\right) \end{aligned}$$

$$\begin{aligned} \frac{1}{T}\mathbf{y}'_{i,-1}\nu_i &= y_{i0}\left(\frac{\tau'\nu_i}{T}\right) + \gamma_i\left(\frac{\mathbf{s}'_{f,-1}\nu_i}{T}\right) + \left(\frac{\mathbf{s}'_{i,-1}\nu_i}{T}\right) \\ &= y_{i0}\left(\frac{\tau'\varepsilon_i}{\omega_i T}\right) - y_{i0}\delta_i\left(\frac{\tau'\overline{\varepsilon}}{\omega_i\sqrt{T}}\right) \\ &\quad + \gamma_i\left(\frac{\mathbf{s}'_{f,-1}\varepsilon_i}{\omega_i T}\right) - \gamma_i\delta_i\left(\frac{\mathbf{s}'_{f,-1}\overline{\varepsilon}}{\omega_i T}\right) \\ &\quad + \left(\frac{\mathbf{s}'_{i,-1}\varepsilon_i}{\omega_i T}\right) - \delta_i\left(\frac{\mathbf{s}'_{i,-1}\overline{\varepsilon}_i}{\omega_i T}\right) \end{aligned}$$

Hence equation (A.3) has the following limit as N goes to infinity:

$$D\overline{\mathbf{W}}'\nu_i \xrightarrow{N} \Gamma_1^*\mathbf{q}_{iT}^{(1)} + \mathbf{q}_{iT}^{(2)} \tag{A.9}$$

where

$$\mathbf{q}_{iT}^{(1)} = \begin{bmatrix} \frac{\tau'\varepsilon_i}{\sigma_i\sqrt{T}} \\ \frac{\mathbf{f}'\varepsilon_i}{\sigma_i\sqrt{T}} \\ \frac{\mathbf{s}'_{f,-1}\varepsilon_i}{\sigma_i T} \\ \frac{\mathbf{s}'_{f,-1}\varepsilon_i}{\sigma_i T} \end{bmatrix}, \mathbf{q}_{iT}^{(2)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{\mathbf{s}'_{i,-1}\varepsilon_i}{\sigma_i T} \end{bmatrix}$$

As N goes to infinity the limit of first term in equation (A.1), $\nu_i'\nu_i$ can simply be writen as

$$\nu_i'\nu_i = \frac{\varepsilon_i'\varepsilon_i + \delta_i^2\overline{\varepsilon}_i'\overline{\varepsilon}_i - 2\delta_i\varepsilon_i'\overline{\varepsilon}_i}{\omega_i^2} \xrightarrow{N} \frac{\varepsilon_i'\varepsilon_i}{\sigma_i^2} \tag{A.10}$$

For the denominator, $\nu_i'\overline{\mathbf{M}}_\delta\nu_i$ with $\overline{\mathbf{M}}_\delta = \mathbf{I}_T - \Delta\overline{\mathbf{y}}\left(\Delta\overline{\mathbf{y}}'\Delta\overline{\mathbf{y}}\right)^{-1}\Delta\overline{\mathbf{y}}'$, we have

$$\nu_i'\overline{\mathbf{M}}_\delta\nu_i = \nu_i'\nu_i - \nu_i'\Delta\overline{\mathbf{y}}\left(\Delta\overline{\mathbf{y}}'\Delta\overline{\mathbf{y}}\right)^{-1}\Delta\overline{\mathbf{y}}'\nu_i$$

for which it can easily be shown that

$$\nu_i'\overline{\mathbf{M}}_\delta\nu_i \xrightarrow{N} \frac{\varepsilon_i'\varepsilon_i}{\sigma_i^2} - \frac{\varepsilon_i'\mathbf{f}\left(\mathbf{f}'\mathbf{f}\right)^{-1}\mathbf{f}'\varepsilon_i}{\sigma_i^2} \tag{A.11}$$

Putting (A.8), (A.9), (A.10) and (A.11) together proves the result. □

Proof of Theorem 2. The only thing we should do is to let T to go to infinity for the results derived in Theorem 1. Using Hamilton (1994) and doing similar derivation the following preliminary results would be straightforward.

$\frac{\mathbf{f}'\tau}{T} \xrightarrow{T} 0$, $\frac{\mathbf{s}'_{f,-1}\tau}{T^{\frac{3}{2}}} \xrightarrow{T} \int_0^1 W_f(r)\,dr (= A)$, $\frac{\mathbf{f}'\mathbf{f}}{T} \xrightarrow{T} 1$, $\frac{\mathbf{f}'\mathbf{s}_{f,-1}}{T^{\frac{3}{2}}} \xrightarrow{T} 0$, $\frac{\mathbf{s}'_{f,-1}\mathbf{s}_{f,-1}}{T^2} \xrightarrow{T} \int_0^1 W_f^2(r)\,dr (= B)$, $\frac{\tau\mathbf{s}'_{i,-1}}{\sigma_i T^{\frac{3}{2}}} \xrightarrow{T} \int_0^1 W_i(r)\,dr (= C)$, $\frac{f'\mathbf{s}_{i,-1}}{T^{\frac{3}{2}}} \xrightarrow{T} 0$, $\frac{\mathbf{s}'_{i,-1}\mathbf{s}_{f,-1}}{\sigma_i T^2} \xrightarrow{T} \int_0^1 W_f(r)\,W_i(r)\,dr (= D)$, $\frac{\mathbf{s}'_{i,-1}\mathbf{s}_{i,-1}}{\sigma_i^2 T^2} \xrightarrow{T} \int_0^1 W_i^2(r)\,dr (= E)$, $\frac{\tau'\varepsilon_i}{\sigma_i\sqrt{T}} \xrightarrow{T} W_i(1) (= F)$, $\frac{\mathbf{s}'_{f,-1}\varepsilon_i}{\sigma_i T} \xrightarrow{T} \int_0^1 W_f(r)\,dW_i(r) (= H)$, $\frac{\mathbf{s}'_{i,-1}\varepsilon_i}{\sigma_i^2 T} \xrightarrow{T} \int_0^1 W_i(r)\,dW_i(r) = 1/2\left[W_i^2(1) - 1\right] (= I)$ and $\frac{\mathbf{f}'\varepsilon_i}{\sigma_i\sqrt{T}} \xrightarrow{T} W_{if}(1)$ which is a brownian motion.

Using the preliminary results, it would be obvious that for (A.8) we have the following,

$$\Psi_{fT} \xrightarrow{T} \begin{bmatrix} 1 & 0 & \int_0^1 W_f(r)\,dr & \int_0^1 W_f(r)\,dr \\ 0 & 1 & 0 & 0 \\ \int_0^1 W_f(r)\,dr & 0 & \int_0^1 W_f^2(r)\,dr & \int_0^1 W_f^2(r)\,dr \\ \int_0^1 W_f(r)\,dr & 0 & \int_0^1 W_f^2(r)\,dr & \int_0^1 W_f^2(r)\,dr \end{bmatrix}$$

$$\Psi_{fiT}^{(1)} \xrightarrow{T} \begin{bmatrix} \int_0^1 W_i(r)\,dr & 0 & \sigma_i\int_0^1 W_f(r)\,W_i(r)\,dr & \sigma_i^2\int_0^1 W_i^2(r)\,dr \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_i\int_0^1 W_f(r)\,W_i(r)\,dr \\ 0 & 0 & 0 & \int_0^1 W_i(r)\,dr \end{bmatrix}$$

$$\Psi_{fiT}^{(2)} \xrightarrow{T} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_i\int_0^1 W_f(r)\,W_i(r)\,dr \end{bmatrix}$$

Now consider (A.9) and observe that,

$$\mathbf{q}_{iT}^{(1)} \overset{T}{\rightarrow} \begin{bmatrix} W_i\left(1\right) \\ W_{if}\left(1\right) \\ \int_0^1 W_f\left(r\right) dW_i\left(r\right) \\ \int_0^1 W_f\left(r\right) dW_i\left(r\right) \end{bmatrix}, \ \mathbf{q}_{iT}^{(2)} \overset{T}{\rightarrow} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/2\left[W_i^2(1)-1\right] \end{bmatrix}$$

Hence the limit of (A.2) and (A.3) are known when $N \rightarrow \infty$ is followed by $T \rightarrow \infty$. Further consider that $\frac{1}{T}\frac{\varepsilon_i'\varepsilon_i}{\sigma_i^2} - \frac{1}{T}\frac{\varepsilon_i'\mathbf{f}\left(\mathbf{f}'\mathbf{f}\right)^{-1}\mathbf{f}'\varepsilon_i}{\sigma_i^2} \overset{T}{\rightarrow} 1$ as $\frac{\varepsilon_i'\mathbf{f}\left(\mathbf{f}'\mathbf{f}\right)^{-1}\mathbf{f}'\varepsilon_i}{\sigma_i^2} \overset{T}{\rightarrow} W_{fi}^2(1)$. Using these results, we see that

$$\ell_{if} = \frac{\begin{array}{c} 2ACHI - 2BCFI - 2HID - 2AFHE + 2CFHD + 2AFID \\ +BI^2 + H^2E + BF^2E - C^2H^2 - A^2I^2 - F^2D^2 \end{array}}{BE + 2ACD - D^2 - BC^2 - A^2E}$$

for which the letters are defined in the preliminary results mentioned above. □

APPENDIX B: TABLES

Table 1. Critical Values.

N	10	15	20	30	50	70	100	200
T				1 Percent				
10	1.5508	1.1205	0.9646	0.6680	0.4642	0.3609	0.2550	0.1743
15	1.9612	1.5260	1.2839	1.0788	0.8338	0.7234	0.6525	0.4999
20	2.0807	1.6669	1.4287	1.2199	0.9985	0.8946	0.8081	0.6637
30	2.3157	1.8198	1.6562	1.3156	1.1304	1.0308	0.9339	0.8235
50	2.3673	1.9890	1.7617	1.5203	1.2844	1.1691	1.0891	0.9682
70	2.5284	2.0768	1.8134	1.5634	1.3428	1.2362	1.1392	1.0316
100	2.4601	2.1049	1.8163	1.6924	1.3746	1.2713	1.1982	1.0871
200	2.5865	2.1175	1.9210	1.6680	1.4621	1.3550	1.3043	1.1930
				5 Percent				
10	0.8534	0.5692	0.4031	0.2461	0.0800	0.0081	-0.0380	-0.1099
15	1.1856	0.9154	0.7921	0.6198	0.4867	0.4125	0.3706	0.2952
20	1.3382	1.0768	0.8992	0.7914	0.6444	0.5809	0.5292	0.4629
30	1.4870	1.2022	1.0742	0.9185	0.7956	0.7325	0.6821	0.6185
50	1.5904	1.3379	1.2314	1.0486	0.9270	0.8709	0.8195	0.7488
70	1.6130	1.4022	1.2437	1.1090	0.9751	0.9033	0.8562	0.8042
100	1.6497	1.4061	1.2850	1.1687	1.0146	0.9433	0.9091	0.8612
200	1.7225	1.4512	1.3264	1.2098	1.0997	1.0143	0.9942	0.9255
				10 Percent				
10	0.4701	0.2468	0.1184	-0.0051	-0.1217	-0.1707	-0.2058	-0.2553
15	0.7928	0.6206	0.5145	0.3978	0.3042	0.2417	0.2101	0.1675
20	0.9542	0.7570	0.6261	0.5457	0.4604	0.4033	0.3772	0.3411
30	1.0968	0.8705	0.7921	0.6855	0.5933	0.5675	0.5284	0.4921
50	1.1982	1.0158	0.9259	0.7944	0.7285	0.6927	0.6553	0.6108
70	1.2203	1.0363	0.9665	0.8492	0.7529	0.7247	0.6961	0.6578
100	1.2726	1.0856	0.9786	0.9116	0.7941	0.7499	0.7333	0.7125
200	1.2996	1.1109	1.0180	0.9555	0.8804	0.8260	0.8030	0.7539

Note: Critical Values for average of individual cross-sectionally augmented Dickey-Fuller χ^2 distribution (no intercept no trend case).

Table 2. Critical Values.

N	10	15	20	30	50	70	100	200
T				1 Percent				
10	3.2079	3.0116	2.5106	2.4615	2.4012	2.0120	2.0084	2.0133
15	3.6851	3.2155	3.1460	3.0393	2.7282	2.5324	2.5180	2.3713
20	4.0785	3.5122	3.2087	3.1254	2.8773	2.8105	2.6803	2.6338
30	4.3525	3.8717	3.6303	3.4629	3.0840	2.9527	2.9442	2.8874
50	4.6332	4.1429	3.9989	3.7950	3.2978	3.2468	3.0737	3.0299
70	4.8808	4.3261	3.8731	3.7737	3.5778	3.3892	3.2023	3.1058
100	4.5044	4.4440	4.0831	3.7981	3.4603	3.3966	3.2519	3.0809
200	5.0958	4.4424	4.1469	3.7328	3.4780	3.5481	3.2640	3.1979
				5 Percent				
10	2.4452	2.2411	1.9154	1.9225	1.8850	1.6192	1.6165	1.5965
15	3.1227	2.7638	2.5969	2.5153	2.3283	2.1601	2.1596	2.0478
20	3.2090	2.9010	2.7362	2.6355	2.4686	2.3805	2.3377	2.3049
30	3.5238	3.2762	3.1308	2.9006	2.6727	2.5723	2.5632	2.5476
50	3.7299	3.5433	3.2582	3.1485	2.9087	2.8390	2.7713	2.6926
70	3.9583	3.4975	3.3046	3.1829	3.0504	2.9132	2.8431	2.7798
100	3.6629	3.5717	3.4625	3.2622	3.0246	2.9692	2.9233	2.8037
200	3.9532	3.8521	3.5240	3.2505	3.0803	3.0275	2.9165	2.8821
				10 Percent				
10	2.0735	1.8911	1.6538	1.5712	1.5500	1.4150	1.3655	1.3751
15	2.6268	2.4594	2.3017	2.1810	2.0661	1.9352	1.9459	1.8574
20	2.8575	2.5470	2.4318	2.4044	2.2360	2.2084	2.1580	2.1747
30	3.0436	2.9063	2.8272	2.5918	2.4448	2.3807	2.3831	2.3601
50	3.2345	3.0778	2.9059	2.8334	2.6763	2.5935	2.5587	2.4898
70	3.3957	3.1153	2.9165	2.8830	2.7600	2.7291	2.6205	2.6203
100	3.2567	3.1875	3.0216	2.9543	2.7911	2.7511	2.6893	2.6479
200	3.5017	3.2990	3.1174	2.9923	2.8459	2.8066	2.7435	2.6952

Note: Critical Values for average of individual cross-sectionally augmented Dickey-Fuller χ^2 distribution (intercept case).

Table 3. Critical Values.

N	10	15	20	30	50	70	100	200
T				1 Percent				
10	4.7419	4.3436	4.1490	3.9263	3.7680	3.6986	3.6632	3.5648
15	5.5272	5.0978	4.8821	4.6646	4.4486	4.3100	4.2336	4.1612
20	6.1133	5.5602	5.2137	5.0239	4.7548	4.5916	4.5511	4.4053
30	6.5916	5.9811	5.7251	5.3698	5.1528	4.9812	4.8698	4.7393
50	6.9154	6.4181	6.1566	5.6967	5.4473	5.2461	5.1767	4.9905
70	7.1913	6.6907	6.2952	5.9234	5.5989	5.4229	5.3233	5.1219
100	7.4516	6.7697	6.4669	6.0510	5.6677	5.5070	5.4191	5.2191
200	7.6008	6.8859	6.5853	6.1991	5.8446	5.6908	5.5214	5.3667
				5 Percent				
10	3.9900	3.6655	3.5300	3.3710	3.2208	3.1699	3.1279	3.0510
15	4.7948	4.4469	4.2867	4.1159	3.9563	3.8812	3.8208	3.7877
20	5.1743	4.8544	4.6880	4.4772	4.2946	4.2203	4.1703	4.0792
30	5.6280	5.2564	5.0952	4.8375	4.6708	4.5693	4.4865	4.4034
50	6.0076	5.6158	5.4257	5.1393	4.9741	4.8545	4.7755	4.6770
70	6.2238	5.7914	5.5818	5.3293	5.1323	4.9977	4.9231	4.7900
100	6.3497	5.9464	5.6597	5.4189	5.2063	5.0988	5.0073	4.8946
200	6.4383	6.0674	5.8433	5.5833	5.3294	5.1985	5.1251	5.0003
				10 Percent				
10	3.5551	3.2844	3.1686	3.0371	2.9287	2.8692	2.8298	2.7721
15	4.3717	4.0918	3.9509	3.8402	3.7170	3.6661	3.6230	3.5741
20	4.7248	4.4846	4.3707	4.1860	4.0537	3.9897	3.9524	3.8971
30	5.1670	4.8741	4.7334	4.5451	4.4316	4.3438	4.2795	4.2262
50	5.4901	5.1953	5.0634	4.8463	4.7121	4.6320	4.5598	4.4885
70	5.6833	5.3669	5.2029	5.0145	4.8572	4.7584	4.7019	4.6093
100	5.7921	5.5022	5.2782	5.1011	4.9509	4.8594	4.7889	4.6974
200	5.8746	5.6412	5.4600	5.2592	5.0587	4.9586	4.9113	4.8120

Note: Critical Values for average of individual cross-sectionally augmented Dickey-Fuller χ^2 distribution (intercept and trend case).

Table 4. Size and power.

N	Test	Size T					Power T				
		10	20	30	50	100	10	20	30	50	100
10	CIPS	0.0514	0.0514	0.0492	0.0466	0.0529	0.0546	0.1052	0.1738	0.4007	0.9336
	CIPS*	0.0495	0.0514	0.0492	0.0466	0.0529	0.0518	0.1051	0.1738	0.4007	0.9336
	MH	0.0487	0.0494	0.0499	0.0482	0.0541	0.0442	0.0756	0.1285	0.3386	0.9184
20	CIPS	0.0535	0.0549	0.0498	0.0475	0.0534	0.0540	0.1062	0.1929	0.5292	0.9913
	CIPS*	0.0515	0.0549	0.0498	0.0475	0.0534	0.0513	0.1061	0.1929	0.5292	0.9913
	MH	0.0516	0.0520	0.0538	0.0456	0.0504	0.0469	0.0927	0.1727	0.4974	0.9929
30	CIPS	0.0481	0.0493	0.0562	0.0460	0.0455	0.0485	0.0969	0.2013	0.5514	0.9909
	CIPS*	0.0451	0.0493	0.0562	0.0460	0.0455	0.0455	0.0968	0.2013	0.5514	0.9909
	MH	0.0469	0.0448	0.0531	0.0493	0.0485	0.0455	0.0897	0.2037	0.6061	0.9985
50	CIPS	0.0550	0.0454	0.0551	0.0466	0.0532	0.0526	0.1014	0.2344	0.6521	0.9996
	CIPS*	0.0524	0.0454	0.0551	0.0466	0.0532	0.0495	0.1014	0.2344	0.6521	0.9996
	MH	0.0540	0.0502	0.0529	0.0488	0.0507	0.0505	0.1050	0.2319	0.7226	0.9999
100	CIPS	0.0456	0.0518	0.0515	0.0460	0.0504	0.0391	0.0988	0.2508	0.7043	1.0000
	CIPS*	0.0437	0.0516	0.0515	0.0460	0.0504	0.0367	0.0988	0.2508	0.7043	1.0000
	MH	0.0448	0.0483	0.0479	0.0477	0.0508	0.0458	0.1089	0.2716	0.8315	1.0000

Note: Size and power for average of individual cross-sectionally augmented Dickey-Fuller χ^2 distribution (no intercept and no trend). Power is calculated for the case when one-third of the units are $I(1)$ and the rest $I(0)$.

Table 5. Size and power.

N	Test	Size T					Power T				
		10	20	30	50	100	10	20	30	50	100
10	CIPS	0.0634	0.0598	0.0531	0.0480	0.0711	0.0732	0.0851	0.1058	0.2094	0.7724
	CIPS*	0.0528	0.0595	0.0531	0.0480	0.0711	0.0619	0.0847	0.1057	0.2094	0.7724
	MH	0.0543	0.0553	0.0467	0.0458	0.0734	0.0469	0.0463	0.0541	0.1320	0.6992
20	CIPS	0.0607	0.0606	0.0483	0.0569	0.0521	0.0727	0.0833	0.0952	0.2708	0.8886
	CIPS*	0.0484	0.0604	0.0482	0.0569	0.0521	0.0613	0.0831	0.0952	0.2708	0.8886
	MH	0.0713	0.0619	0.0422	0.0508	0.0451	0.0576	0.0512	0.0476	0.1704	0.8306
30	CIPS	0.0394	0.0450	0.0431	0.0401	0.0503	0.0539	0.0612	0.0771	0.2083	0.9348
	CIPS*	0.0305	0.0450	0.0431	0.0401	0.0503	0.0433	0.0607	0.0771	0.2083	0.9348
	MH	0.0429	0.0510	0.0438	0.0453	0.0399	0.0356	0.0427	0.0532	0.1675	0.9127
50	CIPS	0.0413	0.0494	0.0481	0.0581	0.0541	0.0499	0.0574	0.0945	0.3472	0.9892
	CIPS*	0.0331	0.0492	0.0481	0.0581	0.0541	0.0400	0.0571	0.0945	0.3472	0.9892
	MH	0.0373	0.0581	0.0568	0.0498	0.0500	0.0268	0.0399	0.0706	0.2265	0.9819
100	CIPS	0.0579	0.0456	0.0471	0.0589	0.0555	0.0735	0.0554	0.0863	0.3853	0.9995
	CIPS*	0.0487	0.0450	0.0471	0.0589	0.0555	0.0617	0.0549	0.0859	0.3853	0.9995
	MH	0.0552	0.0545	0.0561	0.0493	0.0454	0.0440	0.0431	0.0694	0.2591	0.9979

Note: Size and power for average of individual cross-sectionally augmented Dickey-Fuller χ^2 distribution (intercept). Power is calculated for the case when one-third of the units are $I(1)$ and the rest $I(0)$.

Table 6. Size and power.

N	Test	Size T					Power T				
		10	20	30	50	100	10	20	30	50	100
10	CIPS	0.0495	0.0508	0.0452	0.0567	0.0500	0.0754	0.0570	0.0634	0.1343	0.4502
	CIPS*	0.0241	0.0498	0.0451	0.0567	0.0500	0.0478	0.0564	0.0632	0.1343	0.4501
	MH	0.0475	0.0470	0.0541	0.0570	0.0496	0.0555	0.0974	0.1687	0.3991	0.9312
20	CIPS	0.0458	0.0470	0.0514	0.0488	0.0544	0.0760	0.0530	0.0702	0.1351	0.6584
	CIPS*	0.0247	0.0462	0.0513	0.0488	0.0544	0.0462	0.0520	0.0700	0.1351	0.6584
	MH	0.0485	0.0469	0.0476	0.0504	0.0536	0.0651	0.1482	0.2932	0.6495	0.9946
30	CIPS	0.0464	0.0447	0.0491	0.0543	0.0574	0.0737	0.0474	0.0681	0.1529	0.7280
	CIPS*	0.0232	0.0438	0.0490	0.0543	0.0574	0.0445	0.0466	0.0676	0.1529	0.7280
	MH	0.0454	0.0484	0.0507	0.0527	0.0565	0.0745	0.2205	0.4218	0.8059	0.9992
50	CIPS	0.0512	0.0550	0.0485	0.0455	0.0447	0.0822	0.0594	0.0628	0.1530	0.8547
	CIPS*	0.0268	0.0545	0.0485	0.0455	0.0447	0.0477	0.0588	0.0624	0.1528	0.8547
	MH	0.0511	0.0504	0.0463	0.0447	0.0507	0.0858	0.2641	0.4872	0.8584	0.9999
100	CIPS	0.0443	0.0469	0.0536	0.0462	0.0519	0.0775	0.0496	0.0658	0.1692	0.9653
	CIPS*	0.0229	0.0451	0.0535	0.0462	0.0519	0.0476	0.0485	0.0656	0.1692	0.9653
	MH	0.0486	0.0475	0.0516	0.0492	0.0504	0.0906	0.3111	0.5908	0.9179	1.0000

Note: Size and power for average of individual cross-sectionally augmented Dickey-Fuller χ^2 distribution (intercept and trend). Power is calculated for the case when one-third of the units are $I(1)$ and the rest $I(0)$.

Table 7. Size and power.

N	Test	Size T					Power T				
		10	20	30	50	100	10	20	30	50	100
10	CIPS	0.0484	0.0493	0.0481	0.0428	0.0550	0.0576	0.1008	0.1700	0.4216	0.9311
	CIPS*	0.0461	0.0493	0.0481	0.0428	0.0550	0.0551	0.1008	0.1700	0.4216	0.9311
	MH	0.0472	0.0480	0.0462	0.0456	0.0521	0.0465	0.0714	0.1234	0.3587	0.9168
20	CIPS	0.0507	0.0518	0.0487	0.0458	0.0524	0.0519	0.1088	0.1921	0.5291	0.9894
	CIPS*	0.0472	0.0517	0.0487	0.0458	0.0523	0.0488	0.1087	0.1921	0.5291	0.9894
	MH	0.0554	0.0526	0.0520	0.0465	0.0521	0.0461	0.0927	0.1749	0.4956	0.9915
30	CIPS	0.0506	0.0493	0.0498	0.0466	0.0417	0.0510	0.0930	0.1990	0.5533	0.9917
	CIPS*	0.0479	0.0493	0.0497	0.0466	0.0417	0.0484	0.0928	0.1988	0.5533	0.9917
	MH	0.0484	0.0469	0.0483	0.0496	0.0433	0.0486	0.0884	0.1977	0.5941	0.9984
50	CIPS	0.0559	0.0517	0.0489	0.0522	0.0498	0.0543	0.1014	0.2291	0.6499	0.9998
	CIPS*	0.0534	0.0516	0.0489	0.0522	0.0498	0.0516	0.1013	0.2290	0.6499	0.9998
	MH	0.0547	0.0543	0.0464	0.0505	0.0523	0.0502	0.1086	0.2272	0.7192	0.9999
100	CIPS	0.0453	0.0518	0.0486	0.0490	0.0471	0.0384	0.1037	0.2454	0.6946	1.0000
	CIPS*	0.0433	0.0518	0.0486	0.0490	0.0471	0.0364	0.1037	0.2454	0.6946	1.0000
	MH	0.0524	0.0490	0.0518	0.0506	0.0496	0.0483	0.1131	0.2588	0.8292	1.0000

Note: Size and power for average of individual cross-sectionally augmented Dickey-Fuller χ^2 distribution (no intercept no trend). Power is calculated for the case when half of the units are $I(1)$ and the rest $I(0)$.

Table 8. Size and power.

N	Test	Size T					Power T				
		10	20	30	50	100	10	20	30	50	100
10	CIPS	0.0584	0.0564	0.0564	0.0505	0.0671	0.0703	0.0812	0.1123	0.2082	0.7705
	CIPS*	0.0494	0.0561	0.0563	0.0505	0.0671	0.0587	0.0810	0.1123	0.2082	0.7705
	MH	0.0558	0.0537	0.0485	0.0517	0.0723	0.0453	0.0474	0.0551	0.1298	0.7066
20	CIPS	0.0596	0.0615	0.0488	0.0588	0.0512	0.0729	0.0850	0.0951	0.2744	0.8951
	CIPS*	0.0482	0.0610	0.0488	0.0588	0.0512	0.0622	0.0845	0.0950	0.2744	0.8951
	MH	0.0701	0.0597	0.0416	0.0526	0.0414	0.0545	0.0492	0.0467	0.1627	0.8364
30	CIPS	0.0426	0.0445	0.0452	0.0420	0.0509	0.0552	0.0560	0.0827	0.2165	0.9340
	CIPS*	0.0329	0.0443	0.0451	0.0420	0.0509	0.0442	0.0558	0.0826	0.2165	0.9340
	MH	0.0475	0.0510	0.0495	0.0408	0.0425	0.0374	0.0397	0.0605	0.1706	0.9111
50	CIPS	0.0365	0.0497	0.0494	0.0606	0.0516	0.0514	0.0621	0.1006	0.3415	0.9883
	CIPS*	0.0292	0.0497	0.0494	0.0606	0.0516	0.0404	0.0616	0.1005	0.3415	0.9883
	MH	0.0289	0.0546	0.0612	0.0503	0.0535	0.0227	0.0458	0.0748	0.2290	0.9831
100	CIPS	0.0607	0.0462	0.0485	0.0615	0.0584	0.0746	0.0564	0.0893	0.3902	0.9995
	CIPS*	0.0484	0.0459	0.0485	0.0615	0.0584	0.0613	0.0563	0.0893	0.3902	0.9995
	MH	0.0546	0.0560	0.0589	0.0487	0.0474	0.0430	0.0451	0.0741	0.2678	0.9976

Note: Size and power for average of individual cross-sectionally augmented Dickey-Fuller χ^2 distribution (intercept). Power is calculated for the case when half of the units are $I(1)$ and the rest $I(0)$.

Table 9. Size and power.

N	Test	Size T					Power T				
		10	20	30	50	100	10	20	30	50	100
10	CIPS	0.0467	0.0531	0.0483	0.0574	0.0495	0.0723	0.0580	0.0637	0.1282	0.4598
	CIPS*	0.0257	0.0522	0.0482	0.0574	0.0494	0.0478	0.0572	0.0635	0.1281	0.4597
	MH	0.0461	0.0571	0.0495	0.0534	0.0522	0.0541	0.0978	0.1662	0.3968	0.9290
20	CIPS	0.0506	0.0482	0.0443	0.0518	0.0636	0.0790	0.0544	0.0648	0.1364	0.6700
	CIPS*	0.0269	0.0471	0.0443	0.0518	0.0636	0.0487	0.0538	0.0643	0.1364	0.6700
	MH	0.0497	0.0457	0.0466	0.0491	0.0581	0.0720	0.1501	0.2849	0.6511	0.9943
30	CIPS	0.0491	0.0522	0.0532	0.0570	0.0551	0.0822	0.0529	0.0687	0.1549	0.7210
	CIPS*	0.0253	0.0519	0.0531	0.0570	0.0551	0.0498	0.0516	0.0685	0.1548	0.7210
	MH	0.0487	0.0498	0.0509	0.0549	0.0559	0.0834	0.2215	0.4313	0.7968	0.9994
50	CIPS	0.0462	0.0578	0.0483	0.0464	0.0479	0.0774	0.0610	0.0640	0.1580	0.8542
	CIPS*	0.0237	0.0571	0.0481	0.0462	0.0478	0.0474	0.0605	0.0639	0.1580	0.8542
	MH	0.0514	0.0539	0.0501	0.0507	0.0503	0.0820	0.2598	0.4846	0.8581	0.9999
100	CIPS	0.0401	0.0477	0.0514	0.0459	0.0481	0.0711	0.0516	0.0657	0.1632	0.9661
	CIPS*	0.0209	0.0470	0.0512	0.0459	0.0480	0.0429	0.0499	0.0656	0.1631	0.9661
	MH	0.0442	0.0486	0.0518	0.0481	0.0447	0.0835	0.3062	0.5865	0.9222	1.0000

Note: Size and power for average of individual cross-sectionally augmented Dickey-Fuller χ^2 distribution (intercept and trend). Power is calculated for the case when half of the units are $I(1)$ and the rest $I(0)$.

Table 10. Size and power.

N	Test	Size T					Power T				
		10	20	30	50	100	10	20	30	50	100
10	CIPS	0.0505	0.0471	0.0506	0.0467	0.0518	0.0468	0.0636	0.1090	0.2246	0.6227
	CIPS*	0.0480	0.0470	0.0505	0.0467	0.0518	0.0441	0.0636	0.1090	0.2246	0.6227
	MH	0.0488	0.0463	0.0480	0.0500	0.0490	0.0537	0.0747	0.1308	0.2948	0.7986
20	CIPS	0.0526	0.0546	0.0471	0.0496	0.0501	0.0425	0.0683	0.1096	0.2441	0.6282
	CIPS*	0.0491	0.0546	0.0471	0.0496	0.0501	0.0391	0.0683	0.1096	0.2441	0.6282
	MH	0.0554	0.0533	0.0490	0.0461	0.0458	0.0491	0.0974	0.1563	0.3458	0.9003
30	CIPS	0.0490	0.0537	0.0525	0.0491	0.0430	0.0418	0.0632	0.1189	0.2591	0.5898
	CIPS*	0.0458	0.0537	0.0524	0.0491	0.0430	0.0393	0.0631	0.1189	0.2591	0.5898
	MH	0.0487	0.0460	0.0549	0.0472	0.0443	0.0465	0.0889	0.1720	0.3967	0.9306
50	CIPS	0.0514	0.0496	0.0490	0.0486	0.0496	0.0413	0.0654	0.1334	0.2931	0.6382
	CIPS*	0.0493	0.0494	0.0489	0.0486	0.0496	0.0386	0.0654	0.1334	0.2931	0.6382
	MH	0.0485	0.0498	0.0503	0.0457	0.0491	0.0483	0.0961	0.1861	0.4581	0.9760
100	CIPS	0.0476	0.0547	0.0493	0.0483	0.0521	0.0337	0.0732	0.1387	0.3235	0.6548
	CIPS*	0.0450	0.0547	0.0493	0.0483	0.0521	0.0319	0.0732	0.1387	0.3235	0.6548
	MH	0.0474	0.0543	0.0520	0.0501	0.0510	0.0480	0.1085	0.2096	0.5196	0.9953

Note: Size and power for average of individual cross-sectionally augmented Dickey-Fuller χ^2 distribution (no intercept no trend). Power is calculated for the case when two-third of the units are $I(1)$ and the rest $I(0)$.

Table 11. Size and power.

N	Test	Size T					Power T				
		10	20	30	50	100	10	20	30	50	100
10	CIPS	0.0551	0.0609	0.0569	0.0480	0.0675	0.0639	0.0613	0.0716	0.1172	0.4848
	CIPS*	0.0454	0.0603	0.0569	0.0480	0.0675	0.0535	0.0610	0.0715	0.1172	0.4848
	MH	0.0569	0.0544	0.0488	0.0505	0.0749	0.0493	0.0611	0.0668	0.1441	0.5919
20	CIPS	0.0590	0.0602	0.0499	0.0551	0.0521	0.0643	0.0641	0.0586	0.1286	0.4980
	CIPS*	0.0473	0.0601	0.0499	0.0551	0.0521	0.0543	0.0637	0.0584	0.1285	0.4979
	MH	0.0695	0.0609	0.0426	0.0509	0.0430	0.0641	0.0657	0.0737	0.1735	0.6167
30	CIPS	0.0497	0.0471	0.0462	0.0391	0.0486	0.0527	0.0457	0.0483	0.0989	0.5284
	CIPS*	0.0379	0.0468	0.0461	0.0391	0.0486	0.0426	0.0456	0.0482	0.0989	0.5284
	MH	0.0525	0.0534	0.0482	0.0396	0.0421	0.0475	0.0632	0.0815	0.1680	0.6695
50	CIPS	0.0421	0.0500	0.0533	0.0599	0.0487	0.0453	0.0448	0.0578	0.1511	0.6268
	CIPS*	0.0339	0.0495	0.0533	0.0599	0.0487	0.0362	0.0443	0.0578	0.1510	0.6267
	MH	0.0368	0.0560	0.0601	0.0491	0.0509	0.0335	0.0701	0.1053	0.2153	0.8059
100	CIPS	0.0589	0.0493	0.0465	0.0621	0.0578	0.0669	0.0389	0.0489	0.1634	0.7000
	CIPS*	0.0486	0.0491	0.0464	0.0621	0.0578	0.0562	0.0387	0.0489	0.1634	0.7000
	MH	0.0534	0.0561	0.0514	0.0492	0.0468	0.0516	0.0687	0.1067	0.2401	0.8704

Note: Size and power for average of individual cross-sectionally augmented Dickey-Fuller χ^2 distribution (intercept). Power is calculated for the case when two-third of the units are $I(1)$ and the rest $I(0)$.

Table 12. Size and power.

N	Test	Size T					Power T				
		10	20	30	50	100	10	20	30	50	100
10	CIPS	0.0477	0.0506	0.0460	0.0563	0.0443	0.0757	0.0537	0.0516	0.0857	0.2296
	CIPS*	0.0244	0.0499	0.0459	0.0563	0.0443	0.0488	0.0525	0.0516	0.0857	0.2296
	MH	0.0483	0.0528	0.0510	0.0512	0.0496	0.0917	0.2504	0.4219	0.7254	0.9865
20	CIPS	0.0500	0.0453	0.0502	0.0472	0.0539	0.0772	0.0463	0.0538	0.0711	0.3171
	CIPS*	0.0225	0.0447	0.0500	0.0472	0.0539	0.0459	0.0458	0.0536	0.0711	0.3171
	MH	0.0455	0.0480	0.0504	0.0507	0.0539	0.1013	0.3181	0.5341	0.8341	0.9989
30	CIPS	0.0504	0.0483	0.0547	0.0574	0.0488	0.0779	0.0445	0.0543	0.0900	0.3353
	CIPS*	0.0268	0.0480	0.0545	0.0574	0.0488	0.0491	0.0440	0.0543	0.0900	0.3353
	MH	0.0483	0.0481	0.0543	0.0519	0.0504	0.1074	0.3510	0.6033	0.8859	0.9983
50	CIPS	0.0503	0.0563	0.0486	0.0462	0.0491	0.0794	0.0525	0.0470	0.0771	0.4000
	CIPS*	0.0266	0.0555	0.0486	0.0462	0.0491	0.0487	0.0518	0.0469	0.0771	0.4000
	MH	0.0521	0.0541	0.0511	0.0483	0.0516	0.1218	0.4085	0.6593	0.9220	1.0000
100	CIPS	0.0482	0.0470	0.0516	0.0459	0.0535	0.0794	0.0421	0.0477	0.0706	0.5187
	CIPS*	0.0239	0.0458	0.0515	0.0459	0.0534	0.0485	0.0415	0.0476	0.0706	0.5187
	MH	0.0515	0.0471	0.0497	0.0480	0.0520	0.1266	0.4476	0.7266	0.9535	1.0000

Note: Size and power for average of individual cross-sectionally augmented Dickey-Fuller χ^2 distribution (intercept and trend). Power is calculated for the case when two-third of the units are $I(0)$ and the rest $I(1)$.

Table 13. Critical values.

N	10	15	20	30	50	70	100	200
T				1 Percent				
10	6.3314	6.4113	6.4425	6.3997	6.3605	6.3792	6.3970	6.3406
15	7.0610	7.1202	7.0357	7.0166	7.0790	7.1550	7.0720	7.0618
20	7.4977	7.5304	7.5036	7.4395	7.5067	7.4596	7.5198	7.4475
30	7.8030	7.8091	7.9920	8.0658	7.8163	7.8187	7.9168	7.8738
50	8.1428	8.2582	8.2682	8.0489	8.2872	8.2935	8.1717	8.1267
70	8.2113	8.3837	8.4066	8.2989	8.4178	8.3581	8.4575	8.3216
100	8.3703	8.3340	8.4505	8.4852	8.4346	8.5233	8.3881	8.5466
200	8.6712	8.5634	8.5353	8.5756	8.6022	8.4295	8.5445	8.4949
				5 Percent				
10	4.4608	4.4772	4.4806	4.5213	4.4717	4.4713	4.4656	4.4680
15	4.6093	4.6342	4.6169	4.6430	4.6820	4.6605	4.6427	4.6592
20	4.7327	4.7653	4.7774	4.6970	4.7935	4.7554	4.7794	4.7157
30	4.9329	4.7991	4.7979	4.9146	4.8892	4.8616	4.9222	4.8798
50	4.9341	4.9126	5.0154	4.9224	4.9354	4.9323	4.9522	4.8910
70	4.9803	5.0123	5.0280	4.9995	4.9497	4.9372	5.0293	4.9988
100	5.0341	4.9861	5.0112	5.0208	4.9914	5.0805	5.0512	5.0201
200	5.0563	5.0793	5.1270	5.1001	5.0737	5.0239	5.0650	5.1120
				10 Percent				
10	3.2516	3.2643	3.2843	3.2802	3.2504	3.2721	3.2391	3.2849
15	3.3169	3.2810	3.3095	3.3390	3.3503	3.3096	3.2910	3.2851
20	3.3131	3.3571	3.3527	3.2914	3.3902	3.3417	3.3788	3.3228
30	3.4678	3.3666	3.3398	3.4388	3.4162	3.3781	3.4211	3.3867
50	3.4910	3.4302	3.4747	3.4143	3.4436	3.4590	3.4231	3.4148
70	3.4253	3.4429	3.4793	3.4857	3.3946	3.3998	3.4637	3.4225
100	3.5084	3.4334	3.4802	3.4644	3.4488	3.4419	3.4769	3.4334
200	3.5022	3.4749	3.5112	3.4730	3.4691	3.4542	3.4667	3.5285

Note: Critical Values for individual cross-sectionally augmented Dickey-Fuller χ^2 distribution (no intercept no trend).

Table 14. Critical values.

N	10	15	20	30	50	70	100	200
T				1 Percent				
10	7.3890	7.4113	7.4256	7.4146	7.5176	7.4700	7.5023	7.4856
15	8.7821	8.7385	8.6751	8.7238	8.8653	8.8960	8.8172	8.7726
20	9.6097	9.5886	9.6214	9.5071	9.5516	9.5436	9.5900	9.5073
30	10.3446	10.3707	10.5496	10.3566	10.3879	10.4616	10.3089	10.4806
50	11.0592	11.1060	11.0017	11.2374	11.1048	11.1417	11.2320	11.3634
70	11.5673	11.4488	11.4865	11.4739	11.5045	11.5858	11.4390	11.5638
100	11.5992	11.6467	11.7292	11.9126	11.6055	11.8886	11.7749	11.7674
200	11.8330	12.1208	12.1025	12.0964	12.1818	12.2068	12.1155	12.0732
				5 Percent				
10	5.9787	5.9488	5.9790	5.9870	6.0309	5.9932	6.0090	6.0088
15	6.6065	6.5992	6.6432	6.5976	6.6455	6.6440	6.6629	6.6169
20	7.0976	7.0559	6.9486	7.0210	7.0295	6.9993	7.0629	6.9744
30	7.3981	7.4200	7.4846	7.3955	7.4619	7.4315	7.3706	7.3846
50	7.7709	7.7612	7.6794	7.8228	7.7704	7.7931	7.7762	7.8101
70	7.9382	7.8734	7.8592	7.9270	7.9380	7.9560	7.9563	7.8876
100	8.0233	7.9686	8.0020	8.0765	8.0610	8.0285	8.1017	8.1326
200	8.1345	8.1569	8.1847	8.2428	8.1991	8.1919	8.2574	8.1522
				10 Percent				
10	4.9554	4.9457	4.9663	4.9783	5.0165	4.9687	4.9590	5.0230
15	5.3974	5.3495	5.3857	5.3684	5.4071	5.3601	5.4429	5.3647
20	5.6589	5.6617	5.5923	5.6306	5.6449	5.5868	5.6737	5.6040
30	5.8554	5.8747	5.8740	5.8648	5.8829	5.8784	5.8586	5.8361
50	6.1224	6.1165	6.0260	6.1638	6.0777	6.0887	6.0981	6.1004
70	6.1863	6.1998	6.1799	6.2437	6.1975	6.1797	6.2109	6.1622
100	6.2804	6.2428	6.2683	6.2830	6.2566	6.2590	6.3112	6.3263
200	6.3146	6.3690	6.3884	6.4038	6.3446	6.3470	6.3586	6.3460

Note: Critical Values for individual cross-sectionally augmented Dickey-Fuller χ^2 distribution (intercept).

Table 15. Critical values.

N	10	15	20	30	50	70	100	200
T				1 Percent				
10	8.2368	8.2118	8.1826	8.2448	8.1754	8.2219	8.1574	8.1889
15	10.2180	10.1818	10.2969	10.4453	10.4607	10.2383	10.2599	10.2024
20	11.3849	11.5063	11.2904	11.2622	11.4143	11.5993	11.6644	11.3026
30	12.8379	12.9048	12.8237	12.7159	12.9204	12.7837	12.8349	12.9798
50	14.5571	14.1877	14.5868	14.4139	14.1526	14.5405	14.3143	14.4480
70	14.7590	14.6820	14.9989	14.5914	14.5014	14.9230	14.3269	14.5527
100	15.1148	15.4429	14.9199	15.2892	15.5925	15.6649	15.5674	15.0304
200	15.8268	15.4929	15.6251	16.1510	15.5023	15.8461	16.3602	15.8296
				5 Percent				
10	7.1533	7.1935	7.1813	7.1731	7.1301	7.2366	7.1834	7.1489
15	8.3051	8.3615	8.4036	8.4960	8.4631	8.3963	8.3543	8.3573
20	9.0718	9.0147	9.2101	9.0011	9.0866	9.1467	9.1738	9.0800
30	9.8689	9.9199	9.9591	9.8700	9.9179	9.8923	9.7927	9.9472
50	10.8157	10.7650	10.8156	10.7971	10.5490	10.7935	10.8206	10.7661
70	10.9098	10.8888	11.1158	10.7909	10.9580	11.1004	10.8922	10.9111
100	11.0859	11.3939	11.2052	11.4558	11.1709	11.3025	11.3504	11.2977
200	11.5493	11.4292	11.4132	11.6924	11.3978	11.4430	11.7467	11.6173
				10 Percent				
10	6.3898	6.3621	6.3689	6.3738	6.3857	6.4556	6.4036	6.3690
15	7.2122	7.2194	7.2738	7.2978	7.3349	7.2019	7.2490	7.1645
20	7.8287	7.6997	7.8092	7.6999	7.7838	7.8320	7.8578	7.7001
30	8.3677	8.3450	8.3981	8.2880	8.3525	8.3913	8.3477	8.3822
50	8.8385	8.9130	8.8884	8.9709	8.8197	8.9123	8.9507	8.9397
70	9.0213	9.0682	9.1705	8.9984	9.0375	9.2174	8.9744	9.0385
100	9.2334	9.3110	9.2334	9.3941	9.1080	9.2874	9.2543	9.3730
200	9.5010	9.3993	9.4579	9.5068	9.5149	9.5021	9.7164	9.6152

Note: Critical Values for individual cross-sectionally augmented Dickey-Fuller χ^2 distribution (intercept and trend).