Testing for cross-sectional correlation in panels with various cross-sectional dimensions (N) and various time-series dimensions (T)

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Abstract

The past decade has witnessed a rapid development in models with cross-sectional correlation. Apart from the traditional approach in spatial statistics, which uses the so-called spatial weights matrix (see, for instance, Kelejian and Prucha, 1999), a number of approaches are suggested in the literature of panel data econometrics. Notable examples include (a) Economic distance (Conley, 1999), (b) Factor model (Bai, 2003), and (c) Using proxies (Pesaran, 2006). Re-visiting the test for error components first suggested in Breusch and Pagan (1980) (see also the extensions in Honda, 1985), we interpret this test as a test for cross-sectional correlation and derive its asymptotic distribution under the following cases: (1) both time-series dimension (T) and cross-sectional dimension (N) go to ∞ (*jointly*), (2) $T \to \infty$ while N is fixed, and (3) $N \to \infty$ while T is fixed. Case (1) is in contrast with that in Honda (1985) (see also Quah, 1994) who considers $T = \rho N$ while $N \to \infty$ (the *diagonal path* limit named by Phillips and Moon, 1999). The results under Cases (2) and (3), to the best of our knowledge, are new. Interestingly, while the distributions under (1) and (2) are normal, that under (3) is not and it is not symmetric either. The critical values under (3) can be easily approximated by Monte Carlo simulations though. Finite $(2+\delta)th$ ($\delta > 0$) moments are required for independently but non-identically distributed (i.n.i.d) data, while only finite $2nd$ moments are required for i.i.d data. A Monte Carlo experiment is performed and it aims to throw light on the choice among the critical values suggested in the three cases, given a T and an N.

Key Words: cross-sectional correlation; cross-sectional dimension; Diagonal path limit; Joint limit; Sequential limit; time-series dimension

JEL Classification C12; C21; C23

1 Introduction

Throughout this paper, we consider the following *linear* panel regression model:

$$y_{it} = \beta' x_{it} + \nu_{it}, \tag{1.1}$$

where $i = 1, \ldots, N$ and $t = 1, \ldots, T$, x_{it} is an $m \times 1$-vector while both y_{it} and ν_{it} are scalars.

The past decade has witnessed a rapid development in models with cross-sectional correlation. More precisely, in one form or the other, the error term ν_{it} is assumed to be correlated across i, that is,

$$E[\nu_{it}\nu_{jt}] = 0, \text{ for some } i, j, i \neq j.$$

Apart from the traditional approach in spatial statistics, which uses the so-called spatial weights matrix (see, for instance, Kelejian and Prucha (1999)), a number of approaches for cross-sectional correlation are suggested in the literature of panel data econometrics. Notable examples include (a) modelling the *economic distance* suggested in Conley (1999), (b) *estimating the factors* suggested in Bai (2003,2006) and Bai and Ng (2002,2007), and (c) *proxying the factors* suggested in Pesaran (2006).

Suppose we are not sure which approach of *cross-sectional correlation* should be adopted, we may start with a very simple model, namely, the Anderson and Hsiao (1981) model which assumes the error term ν_{it} to be uncorrelated across i and t and test for the *cross-sectional correlation*. This is the approach taken by Pesaran (2004), Hsiao, Pesaran and Pick (2007) and Pesaran, Ullah and Yamagata (2008), who essentially improves upon one particular example of the test suggested in Breusch and Pagan (1980). More precisely, in sub-section 3.3 of Breusch and Pagan (1980), they consider the error-component model:

$$\nu_{it} = \alpha_i + \lambda_t + \epsilon_{it}, \tag{1.2}$$

where ϵ_{it}'s are uncorrelated across i and across t. One of the test statistics they consider is:

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left[\left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\hat{\nu}_{it}\right)^2-\hat{\sigma}^2_{NT}\right]/\sqrt{2}\hat{\sigma}^2_{NT}=\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left[\frac{1}{N}\sum_{i=1}^{N}\hat{\nu}_{it}\sum_{\substack{j=1\\j\neq i}}^{N}\hat{\nu}_{jt}\right]/\sqrt{2}\hat{\sigma}^2_{NT}, \qquad (1.3)$$

where $\hat{\sigma}^2_{NT}=(NT)^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{\nu}^2_{it}$, the residual $\hat{\nu}_{it}=y_{it}-\hat{\beta}'_{NT}x_{it}$, and the OLS estimator, $\hat{\beta}_{NT}=\left(\sum_{t=1}^{T}\sum_{i=1}^{N}x_{it}x'_{it}\right)^{-1}\left(\sum_{t=1}^{T}\sum_{i=1}^{N}x_{it}y_{it}\right)$. It is quite clear from (1.2) that the test statistic in (1.3) has some power against the alternative $H_a: var(\lambda_t)>0$.

As one can see from the next section (Theorem 2.1(1)), in one place or the other, we need to consider the asymptotic distribution (when $N,T\to\infty$ *jointly*):

$$\sum_{t=1}^{T}\xi_{NT,t}, \text{ where } \xi_{NT,t}=\frac{1}{\sqrt{T}N}\sum_{i=1}^{N}\nu_{it}\sum_{\substack{j=1\\j\neq i}}^{N}\nu_{jt}.$$

To this end, we consider the following central limit theorem (CLT) for the *double-indexed process* $\xi_{NT,t}$.

Theorem 1.1. *(Gnedenko and Kolmogorov, 1954 and Hall and Heyde, 1980) Let* $\xi_{NT,1},\ldots,\xi_{NT,T}$, *for* $N=1,2,\ldots$ *and* $T=1,2,\ldots$, *be a double-indexed random variables, independent within rows, such that as* $N,T\to\infty$,

$$(i) \quad \sum_{t=1}^{T}E\xi_{NT,t}\longrightarrow 0,$$

$$(ii) \quad \sum_{t=1}^{T}var\xi_{NT,t}\longrightarrow s^2<\infty.$$

The followings are equivalent (with $N,T\to\infty$*):*

$$(a) \quad \sum_{t=1}^{T}E\left(\xi_{NT,t}-E\xi_{NT,t}\right)^2 I_{\{|\xi_{NT,t}-E\xi_{NT,t}|>\varepsilon\}}\longrightarrow 0, \text{ for every } \varepsilon>0.$$

$$(b) \quad \sum_{t=1}^{T}\left(\xi_{NT,t}-E\xi_{NT,t}\right)^2\longrightarrow_p s^2.$$

$$(c) \quad \sup_{1\le t\le T}P\{|\xi_{NT,t}-E\xi_{NT,t}|>\varepsilon\}\longrightarrow 0, \text{ for every } \varepsilon>0,$$

and $\sum_{t=1}^{T}\xi_{NT,t}\longrightarrow_{\mathcal{L}}\mathcal{N}(0,s^2)$. □

On the one hand, Theorem 1.1 is the double-index version of Theorem 3.5 of Hall and Heyde (1980) (see also Gnedenko and Kolmogorov, 1954), and on the other hand, it extends Phillips and Moon (1999)'s double-indexed CLT. In their Theorem 2, Phillips and Moon (1999) show that the *Lindeberg condition* (Condition (a) in Theorem 1.1) suffices for a doubled-indexed CLT. By showing that the so-called *relative stability* condition (Condition (b) in Theorem 1.1) is necessary for asymptotic normality, we are able to construct a consistent estimator for the asymptotic variance of the normal distribution. See Remark (7) after Theorem 2.1 below. The proof is exactly the same as that in Gnedenko and Kolmogorov (1954) and Phillips and Moon (1999) and thus it is skipped.

Further, following the lines in Hall and Heyde (1980), it is tempting to weaken the independence assumption to martingale difference sequence. The weakening seems to be natural if one considers the partial sum $\sum_{t=1}^{T} \xi_{NT,t}$ in testing for cross-sectional correlation (see (2.1) below). It is unclear how the process is *ordered* if one considers the partial sum $\sum_{i=1}^{N} \varepsilon_{NT,i}$ in testing for time-series corelation (see (2.2) below for the definition of $\varepsilon_{NT,i}$). We leave this interesting extension to future research.

As expected, the Lindeberg condition in Theorem 1.1 can be replaced by a *third absolute moment condition*. The result is stated in the next theorem.

Theorem 1.2. *(Pollard, 2002) Let $\xi_{NT,1}, \ldots, \xi_{NT,T}$, for $N = 1, 2, \ldots$ and $T = 1, 2, \ldots$, be a double-indexed random variables, independent within rows, such that as $N, T \to \infty$,*

$$(i) \qquad \sum_{t=1}^{T} E\xi_{NT,t} \longrightarrow 0,$$

$$(ii) \qquad \sum_{t=1}^{T} var\xi_{NT,t} \longrightarrow s^2 < \infty,$$

$$(iii) \qquad \sum_{t=1}^{T} E\left|\xi_{NT,t}\right|^3 \longrightarrow 0.$$

Then, $\sum_{t=1}^{T} \xi_{NT,t} \longrightarrow_{\mathcal{L}} \mathcal{N}(0, s^2)$ *as* $N, T \to \infty$. □

Similarly, Theorem 1.2 is the double-index version of Theorem 19 of Chapter 7, p.179 in Pollard (2002). The proof is exactly the same as that in Pollard (2002) and thus it is skipped.

This paper is organized as follows. The main text is in section 2, in which we derive the asymptotic distribution of the test for cross-sectional correlation under the following cases: (1) both time-series dimension (T) and cross-sectional dimension (N) go to ∞ (*jointly*), (2) $T \to \infty$ while N is fixed, and (3) $N \to \infty$ while T is fixed. The distributions under (1) and (2) are normal, while that under (3) is not. The critical values under (3) can be easily approximated by Monte Carlo simulations though and the details can be found in section 3. A Monte Carlo experiment is performed and the results are reported in section 4. It aims to throw light on the choice among the critical values suggested in the three cases, given a T and an N. We close the paper with a discussion on the diagnostic testing, after the cross-sectional correlation is taken out with one of the four approaches, and a discussion on the future research in section 5. All the proofs are relegated to the Appendix. Throughout, the norm of a matrix A is defined as $\|A\| = (tr\,(A'A))^{1/2}$.

2 Testing for cross-sectional correlation

We first consider the following cross-sectional correlation test statistic:

$$\begin{aligned} CSC_0 &= \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \left[\left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} \hat{\nu}_{it} \right)^2 - \hat{\sigma}_{NT}^2 \right] / \sqrt{2} \hat{\sigma}_{NT}^2 \\ &= \frac{1}{\sqrt{T}} \sum_{t=1}^{T} \left[\frac{1}{N} \sum_{i=1}^{N} \hat{\nu}_{it} \sum_{\substack{j=1 \\ j \neq i}}^{N} \hat{\nu}_{jt} \right] / \sqrt{2} \hat{\sigma}_{NT}^2, \qquad (2.1) \end{aligned}$$

where $\hat{\sigma}_{NT}^2$ and $\hat{\nu}_{it}$ are defined around (1.3) above.

Breusch and Pagan (1980) claim that the CSC_0 in (2.1) is asymptotically $\mathcal{N}(0,1)$ distributed. However, it is unclear if this is a sequential limit, a diagonal path limit ($T = T(N)$ and $N \to \infty$, or $N = N(T)$ and $T \to \infty$), or a joint limit ($N, T \to \infty$ jointly) (see the discussion on the *double-indexed* CLT in Theorems 1.1 and 1.2). In Theorem 2.1 below, we derive the asymptotic $\mathcal{N}(0,1)$ distribution under the joint limit. In addition, we also derive the *different* asymptotic distributions under two other cases, one is that $T \to \infty$ and N is fixed; and the other is that $N \to \infty$ and T is fixed.

It should be noted that the CSC_0 test in (2.1) is essentially a *special* case of the CD test considered in Pesaran (2004) (see also a succinct summary on p.284 of Baltagi, 2008), who allows for heteroskedasticity across i. In particular, Pesaran (2004) assumes $var(\nu_{it}) \neq var(\nu_{jt})$ for $i \neq j$ in general. Each of the $var(\nu_{it})$'s is thus estimated by:

$$T^{-1}\sum_{t=1}^{T}\hat{\nu}_{it}^2,$$

which asymptotic properties are unclear when T is fixed. To focus on the asymptotic properties under various assumptions on N and T in this paper, we leave cases with this more *realistic* assumption to future research. For the same reason, the LM test considered in Pesaran (2004) and Hsiao, Pesaran and Pick (2007) and the bias-adjusted LM test considered in Pesaran, Ullah and Yamagata (2008) will be discussed in our future research.

Throughout this paper, we consider the tests for cross-sectional correlation. That said, as pointed out by Breusch and Pagan (1980), we may consider the following statistic for testing time-series correlation,

$$TSC_0 = \frac{1}{\sqrt{N}}\sum_{i=1}^{N}\left[\left(\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\hat{\nu}_{it}\right)^2 - \hat{\sigma}_{NT}^2\right]/\sqrt{2}\hat{\sigma}_{NT}^2. \tag{2.2}$$

The analysis is exactly the same as that for CSC_0 test for cross-sectional correlation.

We will briefly discuss this in a subsequent section.

Theorem 2.1. *Suppose Assumptions (?)-(?) hold. Let* $\delta > 0$.

(1) $\sup_{i,t} E|\nu_{it}|^{2+\delta} = c < \infty$. *Then*

$$CSC_0 \longrightarrow_{\mathcal{L}} \mathcal{N}(0,1), \text{ as } N, T \to \infty \text{ jointly.} \tag{2.3}$$

(2) *For each* i, $\sup_{t} E|\nu_{it}|^{2+\delta} = c_i < \infty$. *Then*

$$V^{-1/2} CSC_0 \longrightarrow_{\mathcal{L}} \mathcal{N}(0,1), \text{ as } T \to \infty \text{ and } N \geq 2 \text{ is fixed,} \tag{2.4}$$

where $$V = \frac{N-1}{N} + \frac{2}{N\sigma^2} E\left(\frac{1}{N}\sum_{j=1}^{N}\nu_{j1}\sum_{i=1}^{N}x'_{i1}\right)\left(Ex_{11}x'_{11}\right)^{-1} E\left(\frac{1}{N}\sum_{i=1}^{N}x_{i1}\sum_{j=1}^{N}\nu_{j1}\right).$$

(3) *For each* t, $\sup_{i} E|\nu_{it}|^{2+\delta} = c_t < \infty$. *Then*

$$CSC_0 \longrightarrow_{\mathcal{L}} \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left[\left(u_t - \frac{1}{T}\sum_{s=1}^{T} z_s\right)^2 - 1\right]/\sqrt{2}, \text{ as } N \to \infty \text{ and } T \text{ is fixed,} \tag{2.5}$$

$$\text{where } z_t = a u_t + \sqrt{a(1-a)}v_t, \ a = E\left(x'_{11}\right)\left(Ex_{11}x'_{11}\right)^{-1} E\left(x_{11}\right),$$

$$\text{and } (u_t, v_t)' \sim_{i.i.d.} \mathcal{N}(0, I_2). \qquad \square$$

Remarks:

(1) In Case (3), if $m = 1$ and $x_{is} = 1$, $a = 1$ and $z_s = u_s$. On the other hand, when $a = 0$, the distribution boils down to:

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left[\chi_t^2 - 1\right]/\sqrt{2}, \tag{2.6}$$

where χ_t^2's are independently Chi-square distributed random variables with 1 degree of freedom.

(2) In fact, we can generalize Case (3) to heterogenous x_{it}'s.

(3) The unbalanced panel may cause some complexity, especially when (a) some of the cross-sectional units carry long-time series ($T_i \to \infty$) while some carry short time-series (T_i is fixed); (b) some of the time-series units carry long cross-section ($N_t \to \infty$) while some carry short cross-section (N_t is fixed). We leave this case to future study.

(4) The condition for Case (1) is stronger than that for Case (2) or that for Case (3). These different assumptions highlight the difference between the *joint limit* and the *sequential limit*.

(5) Theorem 2.1 also suggests some strategies in bootstrapping. For Case (1), one may bootstrap with N and T; for Case (2), one may bootstrap with T; while for Case (3), one may bootstrap with N.

(6) It should be noted that if $E\left(\frac{1}{N}\sum_{j=1}^{N}\nu_{j1}\sum_{i=1}^{N}x_{i1}'\right)=0$ in Case (2), $V^{-1/2}CSC_0$ is exactly the CD test suggested in Pesaran (2004) with $\hat{\sigma}_i^2=\hat{\sigma}_j^2=\hat{\sigma}_{NT}^2$, for all i,j.

(7) Applying Theorems 1.1 and 1.2, when $T\to\infty$, the denominator in our test can be consistently estimated by:

$$\left(\frac{1}{T}\sum_{t=1}^{T}\left(\sum_{i=1}^{N}\frac{\hat{\nu}_{it}}{\sqrt{NT}}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{\hat{\nu}_{jt}}{\sqrt{N}}\right)^2\right)^{1/2}.$$

The performance of this estimator, given an N and a T, is unclear though.

The critical values of the asymptotic distribution in Case (3) can be approximated by Monte Carlo simulation. Some of them can be found in section 3 below. In empirical applications, the choice among Cases (1), (2) and (3) depends on the *given* T and N as well as the specific data generating process (DGP). See the Monte Carlo experiment in section 4.

The somehow involved asymptotic variance in Case (2), or the inclusion of z_t in the asymptotic distribution in Case (3), is due to the *estimation error*. Alternatively, we may consider the following modified test statistic:

$$\begin{aligned} CSC_1 &= \frac{1}{\sqrt{S}}\sum_{t=1}^{S}\left[\left(\frac{1}{\sqrt{M}}\sum_{i=1}^{M}\hat{\nu}_{it}\right)^2-\hat{\sigma}_{NT}^2\right]/\sqrt{2}\hat{\sigma}_{NT}^2 \qquad (2.7)\\ &= \frac{1}{\sqrt{S}}\sum_{t=1}^{S}\left[\frac{1}{M}\sum_{i=1}^{M}\nu_{it}\sum_{\substack{j=1\\ j\neq i}}^{M}\nu_{jt}\right]/\sqrt{2}\sigma^2+o_p(1), \end{aligned}$$

where $S\leq T$, $M\leq N$. The asymptotic distribution of CSC_1 is shown in the following corollary:

Corollary 2.1. *Suppose Assumptions (?)-(?) hold. Let $\delta > 0$.*

(1) $\sup_{i,t} E|\nu_{it}|^{2+\delta} < \infty$.

Then $CSC_1 \longrightarrow_{\mathcal{L}} \mathcal{N}(0,1)$, *as* $M, S \to \infty$ *jointly.* (2.8)

(2) *For each* i, $\sup_t E|\nu_{it}|^{2+\delta} = c_i < \infty$. *Either* $S = o(T)$ *or* $E\left[\frac{1}{M}\sum_{i=1}^{M} x_{i1} \sum_{j=1}^{M} \nu_{j1}\right] = 0$.

Then $\sqrt{\frac{M}{M-1}} CSC_1 \longrightarrow_{\mathcal{L}} \mathcal{N}(0,1)$, *as* $S \to \infty$ *and* $M \geq 2$ *is fixed.* (2.9)

(3) *For each* t, $\sup_i E|\nu_{it}|^{2+\delta} = c_t < \infty$. *Either* $M = o(N)$ *or* $E(x_{11}) = 0$.

Then $CSC_1 \longrightarrow_{\mathcal{L}} \frac{1}{\sqrt{S}} \sum_{t=1}^{S} \left[\chi_t^2 - 1\right] / \sqrt{2}$, *as* $M \to \infty$ *and* S *is fixed,* (2.10)

where χ_t^2*'s are independently Chi-square distributed random variables with* 1 *degree of freedom.* □

Remarks:

(1) In practice, if we are sure that Case (1) is applicable, we may let $M = N \geq 2$, $S = T$ and increase the power of the test.

(2) In practice, if we are sure that Case (2) is applicable, we may let $M = N \geq 2$ and increase the power of the test.

(3) In practice, if we are sure that Case (3) is applicable, we may let $S = T$ and increase the power of the test.

(4) In Breusch and Pagan (1980) and Honda (1985), it is assumed that x_{it}'s are *exogenous* across i in the sense that $E(x_{it}\nu_{jt}) = 0$ for all i, j. As shown in Theorem 2.1(1) and Corollary 2.1(1), the asymptotic distribution does not hinge on this assumption if $M, S \to \infty$.

(5) Similarly, as shown in Theorem 2.1(3) and Corollary 2.1(3), the asymptotic distribution does not hinge on this *exogeneity* assumption if $M \to \infty$.

(6) One may see from Theorem 2.1(2) and Corollary 2.1(2), in order to have a simple asymptotic variance, we may either (i) invoke this *exogeneity* assumption, or (ii) let $S = o(T)$.

Theorem 2.2. *Consider the assumptions in Theorem 2.1. The conclusions in Cases (1), (2) and (3) hold if the* $2+\delta$*-th moment condition is replaced by the following:* ν_{it}*'s are i.i.d. across* i *and across* t *with* $E[\nu_{11}^2] = \sigma^2 < \infty$. □

Remarks:

(1) The proof of Theorem 2.2 follows that of Theorem 21, p.180 of Pollard (2002). That proof is not *directly* applicable, as the term $\sum_{i=1}^N \frac{\nu_{it}}{\sqrt{NT}} \sum_{\substack{j=1 \\ j\neq i}}^N \frac{\nu_{jt}}{\sqrt{N}}$ is *not i.i.d.* across t, though ν_{it} is *i.i.d.* across i and across t.

(2) In order to apply Theorem 1.2 to the proof of Theorem 2.2, we need to assume that $T \to \infty$. In fact, as shown in Theorem 2.2(3), the asymptotic distribution is *non-normal* when $N \to \infty$ and T is fixed.

3 Simulating the critical values

TABLE 3.1: 5% Simulated Critical Values (for one-sided negative correlation)

T	$a = 0.0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	-.704	-.705	-.705	-.705	-.705	-.706	-.706	-.706	-.707	-.707	-.707
2	-.948	-.951	-.953	-.956	-.960	-.963	-.967	-.971	-.977	-.983	-.998
3	-1.080	-1.085	-1.090	-1.095	-1.102	-1.109	-1.117	-1.126	-1.138	-1.154	-1.183
4	-1.162	-1.167	-1.174	-1.183	-1.191	-1.201	-1.211	-1.223	-1.239	-1.259	-1.289
5	-1.216	-1.223	-1.232	-1.241	-1.250	-1.261	-1.274	-1.288	-1.304	-1.326	-1.355
6	-1.257	-1.264	-1.274	-1.283	-1.293	-1.305	-1.318	-1.332	-1.350	-1.372	-1.399
7	-1.289	-1.298	-1.308	-1.319	-1.329	-1.340	-1.354	-1.369	-1.385	-1.406	-1.430
8	-1.314	-1.324	-1.333	-1.344	-1.354	-1.366	-1.381	-1.396	-1.413	-1.433	-1.456
9	-1.337	-1.346	-1.355	-1.365	-1.376	-1.389	-1.403	-1.417	-1.435	-1.454	-1.476
10	-1.354	-1.362	-1.372	-1.383	-1.393	-1.405	-1.420	-1.435	-1.452	-1.469	-1.490
11	-1.368	-1.379	-1.388	-1.397	-1.409	-1.422	-1.436	-1.451	-1.466	-1.483	-1.504
12	-1.383	-1.392	-1.402	-1.411	-1.422	-1.433	-1.447	-1.460	-1.476	-1.496	-1.515
13	-1.394	-1.405	-1.413	-1.425	-1.436	-1.448	-1.461	-1.475	-1.491	-1.508	-1.526
14	-1.404	-1.413	-1.423	-1.433	-1.444	-1.456	-1.468	-1.481	-1.497	-1.514	-1.533
15	-1.413	-1.421	-1.430	-1.442	-1.452	-1.463	-1.476	-1.490	-1.505	-1.522	-1.540
16	-1.420	-1.430	-1.440	-1.450	-1.461	-1.472	-1.484	-1.497	-1.512	-1.527	-1.543
17	-1.427	-1.436	-1.446	-1.458	-1.468	-1.480	-1.491	-1.504	-1.518	-1.533	-1.549
18	-1.434	-1.443	-1.454	-1.464	-1.474	-1.485	-1.497	-1.510	-1.522	-1.538	-1.554
19	-1.443	-1.450	-1.460	-1.470	-1.480	-1.491	-1.503	-1.516	-1.529	-1.543	-1.558
20	-1.447	-1.456	-1.466	-1.476	-1.486	-1.496	-1.508	-1.521	-1.534	-1.548	-1.563
30	-1.484	-1.494	-1.503	-1.512	-1.522	-1.533	-1.543	-1.553	-1.564	-1.576	-1.588
40	-1.507	-1.513	-1.522	-1.531	-1.540	-1.549	-1.558	-1.568	-1.579	-1.589	-1.600
50	-1.524	-1.529	-1.537	-1.545	-1.552	-1.561	-1.569	-1.578	-1.587	-1.597	-1.607
60	-1.536	-1.539	-1.545	-1.553	-1.560	-1.568	-1.578	-1.586	-1.595	-1.604	-1.613
70	-1.544	-1.549	-1.557	-1.564	-1.571	-1.579	-1.586	-1.594	-1.602	-1.611	-1.619
80	-1.556	-1.561	-1.566	-1.573	-1.579	-1.585	-1.591	-1.598	-1.607	-1.614	-1.621
90	-1.559	-1.564	-1.570	-1.577	-1.584	-1.590	-1.596	-1.603	-1.610	-1.618	-1.625
100	-1.560	-1.564	-1.570	-1.576	-1.583	-1.590	-1.597	-1.604	-1.610	-1.617	-1.623
200	-1.588	-1.592	-1.596	-1.600	-1.605	-1.609	-1.614	-1.619	-1.624	-1.629	-1.634
500	-1.612	-1.615	-1.617	-1.619	-1.623	-1.625	-1.628	-1.631	-1.634	-1.638	-1.641
2000	-1.626	-1.627	-1.629	-1.631	-1.633	-1.635	-1.637	-1.639	-1.640	-1.641	-1.643
∞	-1.645	-1.645	-1.645	-1.645	-1.645	-1.645	-1.645	-1.645	-1.645	-1.645	-1.645

TABLE 3.2: 5% Simulated Critical Values (for one-sided positive correlation)

T	$a = 0.0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	2.029	1.757	1.482	1.207	.939	.669	.393	.116	-.159	-.434	-.707
2	2.008	1.867	1.724	1.583	1.454	1.330	1.223	1.132	1.051	.987	.933
3	1.970	1.873	1.771	1.678	1.594	1.511	1.443	1.382	1.330	1.279	1.236
4	1.942	1.866	1.791	1.717	1.652	1.588	1.536	1.486	1.439	1.400	1.363
5	1.922	1.860	1.797	1.736	1.680	1.624	1.573	1.530	1.488	1.452	1.420
6	1.907	1.851	1.797	1.742	1.694	1.644	1.599	1.562	1.525	1.492	1.461
7	1.883	1.834	1.785	1.742	1.701	1.660	1.621	1.583	1.550	1.521	1.491
8	1.876	1.832	1.785	1.744	1.703	1.667	1.632	1.601	1.569	1.541	1.514
9	1.861	1.821	1.784	1.746	1.709	1.677	1.641	1.608	1.580	1.557	1.532
10	1.854	1.819	1.780	1.745	1.709	1.677	1.647	1.618	1.590	1.566	1.542
11	1.847	1.810	1.778	1.747	1.713	1.682	1.653	1.626	1.601	1.576	1.553
12	1.826	1.799	1.765	1.736	1.708	1.679	1.653	1.631	1.606	1.583	1.563
13	1.825	1.796	1.767	1.738	1.709	1.682	1.656	1.632	1.610	1.589	1.568
14	1.832	1.799	1.771	1.744	1.716	1.691	1.665	1.643	1.621	1.600	1.578
15	1.819	1.793	1.764	1.740	1.712	1.686	1.664	1.643	1.622	1.603	1.584
16	1.816	1.790	1.764	1.736	1.711	1.689	1.667	1.646	1.626	1.606	1.587
17	1.811	1.786	1.762	1.740	1.717	1.693	1.669	1.648	1.630	1.611	1.594
18	1.812	1.788	1.761	1.738	1.716	1.696	1.674	1.654	1.634	1.616	1.599
19	1.803	1.777	1.754	1.734	1.713	1.692	1.670	1.650	1.630	1.614	1.596
20	1.798	1.772	1.751	1.729	1.707	1.687	1.667	1.649	1.630	1.613	1.597
30	1.771	1.754	1.739	1.723	1.708	1.691	1.675	1.660	1.645	1.631	1.618
40	1.756	1.741	1.727	1.711	1.699	1.686	1.675	1.662	1.651	1.640	1.629
50	1.748	1.735	1.723	1.710	1.699	1.687	1.675	1.663	1.652	1.641	1.631
60	1.737	1.724	1.713	1.703	1.691	1.681	1.672	1.661	1.650	1.641	1.632
70	1.725	1.714	1.702	1.693	1.682	1.672	1.664	1.655	1.646	1.637	1.629
80	1.724	1.714	1.704	1.696	1.687	1.678	1.669	1.661	1.653	1.645	1.635
90	1.719	1.710	1.701	1.691	1.684	1.676	1.667	1.659	1.651	1.644	1.635
100	1.719	1.708	1.699	1.692	1.683	1.674	1.666	1.659	1.651	1.644	1.636
200	1.695	1.688	1.682	1.676	1.671	1.666	1.661	1.656	1.650	1.645	1.640
500	1.676	1.673	1.669	1.666	1.663	1.660	1.657	1.653	1.649	1.646	1.643
2000	1.659	1.657	1.655	1.654	1.653	1.652	1.650	1.648	1.646	1.644	1.643
∞	1.645	1.645	1.645	1.645	1.645	1.645	1.645	1.645	1.645	1.645	1.645

TABLE 3.3: 5% Simulated Critical Values (for two-sided correlation)

T	$a = 0.0$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1	2.029	1.757	1.482	1.207	.939	.707	.707	.707	.707	.707	.707
2	2.008	1.867	1.724	1.583	1.454	1.330	1.223	1.132	1.051	1.000	1.000
3	1.970	1.873	1.771	1.678	1.594	1.511	1.443	1.382	1.330	1.279	1.236
4	1.942	1.866	1.791	1.717	1.652	1.588	1.536	1.486	1.439	1.403	1.400
5	1.922	1.860	1.797	1.736	1.680	1.624	1.573	1.539	1.518	1.507	1.510
6	1.907	1.851	1.797	1.742	1.695	1.652	1.623	1.603	1.590	1.584	1.587
7	1.883	1.835	1.788	1.749	1.719	1.691	1.670	1.653	1.642	1.639	1.644
8	1.878	1.838	1.799	1.767	1.740	1.721	1.706	1.693	1.685	1.683	1.684
9	1.873	1.841	1.812	1.785	1.763	1.746	1.731	1.721	1.716	1.714	1.717
10	1.875	1.850	1.824	1.802	1.783	1.767	1.755	1.747	1.741	1.740	1.743
11	1.880	1.856	1.834	1.817	1.799	1.784	1.775	1.765	1.762	1.759	1.761
12	1.875	1.854	1.834	1.818	1.804	1.792	1.784	1.777	1.774	1.773	1.775
13	1.882	1.863	1.845	1.831	1.818	1.806	1.799	1.794	1.790	1.788	1.791
14	1.892	1.873	1.859	1.845	1.832	1.822	1.814	1.809	1.805	1.805	1.806
15	1.893	1.877	1.863	1.850	1.839	1.830	1.823	1.817	1.814	1.812	1.813
16	1.896	1.882	1.870	1.859	1.848	1.839	1.832	1.827	1.824	1.824	1.825
17	1.900	1.886	1.873	1.864	1.852	1.844	1.836	1.832	1.828	1.828	1.830
18	1.906	1.894	1.883	1.871	1.861	1.854	1.846	1.842	1.840	1.839	1.840
19	1.911	1.892	1.882	1.871	1.862	1.855	1.851	1.847	1.843	1.841	1.843
20	1.906	1.893	1.883	1.874	1.864	1.857	1.853	1.848	1.846	1.847	1.849
30	1.927	1.918	1.910	1.904	1.899	1.894	1.891	1.888	1.888	1.888	1.889
40	1.939	1.930	1.924	1.920	1.915	1.913	1.912	1.910	1.911	1.911	1.911
50	1.939	1.935	1.932	1.927	1.924	1.922	1.922	1.921	1.921	1.922	1.922
60	1.945	1.940	1.936	1.933	1.930	1.926	1.925	1.924	1.923	1.924	1.924
70	1.941	1.938	1.935	1.933	1.931	1.929	1.928	1.927	1.926	1.927	1.928
80	1.945	1.941	1.938	1.936	1.934	1.934	1.933	1.932	1.933	1.933	1.933
90	1.942	1.942	1.940	1.938	1.936	1.935	1.933	1.932	1.931	1.931	1.932
100	1.946	1.942	1.940	1.938	1.937	1.936	1.936	1.935	1.934	1.935	1.935
200	1.954	1.952	1.951	1.951	1.950	1.950	1.949	1.948	1.949	1.949	1.948
500	1.954	1.954	1.954	1.954	1.953	1.953	1.953	1.952	1.952	1.952	1.952
2000	1.963	1.962	1.962	1.962	1.962	1.962	1.962	1.962	1.961	1.961	1.961
∞	1.960	1.960	1.960	1.960	1.960	1.960	1.960	1.960	1.960	1.960	1.960

4 Monte Carlo experiment

TABLE 4.1(a) Rejection percentage (size) of various cases of tests: DGP(A)

			1%			5%			10%		
N	T	$Case$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$
2	1	(1)	.000	.000	.000	.000	.000	.000	.000	.000	.000
		(2)	.000	.000	.000	.000	.000	.000	.000	.000	.000
		(3)	.600	.000	.000	3.095	.000	.000	5.705	.000	.000
5	1	(1)	.000	.830	.180	.000	4.605	2.540	.000	8.365	4.605
		(2)	.000	1.830	.880	.000	6.280	3.680	.000	10.095	6.280
		(3)	.835	.000	.000	4.670	2.125	2.125	8.720	9.295	9.295
10	1	(1)	.000	2.890	2.095	.000	6.120	4.395	.000	9.070	6.120
		(2)	.000	3.330	2.475	.000	6.755	4.995	.000	9.705	6.755
		(3)	.900	.215	.215	4.895	4.105	4.105	9.095	9.850	9.850
20	1	(1)	.000	3.350	2.670	.000	6.520	4.770	.000	9.260	6.520
		(2)	.000	3.535	2.830	.000	6.845	5.045	.000	9.445	6.845
		(3)	.920	.510	.510	5.170	4.500	4.500	9.805	9.845	9.845
30	1	(1)	.000	3.715	3.000	.000	6.780	5.075	.000	9.315	6.780
		(2)	.000	3.860	3.145	.000	6.940	5.185	.000	9.480	6.940
		(3)	.950	.725	.725	5.075	4.790	4.790	9.615	10.090	10.090
50	1	(1)	.000	3.685	2.980	.000	6.715	5.115	.000	9.430	6.715
		(2)	.000	3.735	3.040	.000	6.840	5.185	.000	9.535	6.840
		(3)	1.005	.840	.840	5.245	4.825	4.825	9.840	10.145	10.145
100	1	(1)	.000	3.920	3.195	.000	7.060	5.265	.000	9.835	7.060
		(2)	.000	3.970	3.240	.000	7.120	5.350	.000	9.895	7.120
		(3)	.910	.935	.935	4.865	4.960	4.960	9.480	10.465	10.465
200	1	(1)	.000	3.680	3.045	.000	6.775	5.035	.000	9.355	6.775
		(2)	.000	3.690	3.055	.000	6.795	5.060	.000	9.380	6.795
		(3)	.960	1.015	1.015	5.385	4.775	4.775	9.975	10.175	10.175

TABLE 4.1(b) Rejection percentage (size) of various cases of tests: DGP(A)

			1%			5%			10%		
N	T	$Case$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$
2	5	(1)	.000	.000	.000	.000	.000	.000	1.320	1.310	.000
		(2)	.000	.000	.000	2.770	3.035	.815	8.815	9.175	5.805
		(3)	.360	.000	.000	2.020	.000	.000	4.425	.740	.545
5	5	(1)	.000	1.295	.775	.000	5.015	2.800	2.480	9.050	5.015
		(2)	.000	2.280	1.355	.245	6.725	4.080	5.640	11.105	6.970
		(3)	.730	.130	.130	3.860	2.970	2.970	7.925	8.320	8.005
10	5	(1)	.000	2.085	1.435	.000	5.840	3.855	2.620	9.690	5.840
		(2)	.000	2.540	1.750	.010	6.580	4.370	4.100	10.735	6.590
		(3)	.775	.465	.465	4.105	4.070	4.070	8.525	8.945	8.760
20	5	(1)	.000	2.475	1.690	.000	6.685	4.185	3.190	10.425	6.685
		(2)	.000	2.755	1.855	.000	7.030	4.555	3.915	10.880	7.030
		(3)	.950	.640	.640	4.720	4.465	4.465	9.675	9.705	9.600
30	5	(1)	.000	2.580	1.840	.000	6.820	4.445	3.190	10.690	6.820
		(2)	.000	2.710	1.965	.000	7.050	4.625	3.745	10.870	7.050
		(3)	.945	.715	.715	5.020	4.635	4.635	9.805	9.955	9.770
50	5	(1)	.000	2.830	2.040	.000	6.950	4.600	3.230	10.795	6.950
		(2)	.000	2.900	2.080	.000	7.145	4.700	3.520	10.940	7.145
		(3)	.970	.825	.825	4.855	4.795	4.795	9.740	10.120	10.135
100	5	(1)	.000	3.055	2.255	.000	7.245	4.845	3.275	10.690	7.245
		(2)	.000	3.110	2.300	.000	7.295	4.910	3.465	10.750	7.295
		(3)	.995	1.025	1.025	5.060	5.095	5.095	9.910	10.080	10.075
200	5	(1)	.000	3.010	2.150	.000	7.235	4.780	3.345	10.895	7.235
		(2)	.000	3.025	2.165	.000	7.265	4.825	3.410	10.945	7.265
		(3)	.980	.880	.880	4.990	5.050	5.050	9.890	10.230	10.195

TABLE 4.1(c) Rejection percentage (size) of various cases of tests: DGP(A)

			1%			5%			10%		
N	T	$Case$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$
2	10	(1)	.000	.000	.000	.260	.290	.010	2.560	2.520	.550
		(2)	.265	.290	.110	4.200	4.220	2.995	9.545	9.560	8.420
		(3)	.205	.000	.000	1.820	.040	.070	4.415	1.960	1.330
5	10	(1)	.000	1.070	.580	.810	4.525	2.525	4.695	8.655	5.335
		(2)	.000	1.920	1.125	2.060	6.215	4.020	7.595	10.720	8.275
		(3)	.715	.190	.190	3.510	3.035	3.025	7.560	7.965	7.200
10	10	(1)	.000	1.780	1.110	.860	5.855	3.495	5.560	10.060	6.715
		(2)	.000	2.215	1.430	1.605	6.660	4.325	6.975	10.915	8.265
		(3)	.775	.530	.530	4.190	4.270	4.195	8.830	9.370	8.920
20	10	(1)	.000	2.275	1.465	1.150	6.190	4.000	6.230	10.265	7.340
		(2)	.000	2.475	1.640	1.465	6.595	4.300	7.030	10.710	8.060
		(3)	1.020	.795	.795	4.770	4.595	4.565	9.715	9.580	9.505
30	10	(1)	.000	2.270	1.515	1.110	6.205	3.885	6.650	10.405	7.315
		(2)	.000	2.445	1.600	1.290	6.445	4.140	7.140	10.705	7.735
		(3)	.995	.835	.835	5.090	4.495	4.495	10.000	9.740	9.565
50	10	(1)	.000	2.360	1.710	1.095	6.290	4.020	6.360	10.325	7.385
		(2)	.000	2.440	1.745	1.230	6.455	4.145	6.695	10.530	7.685
		(3)	.975	.860	.860	4.775	4.650	4.725	9.685	9.600	9.585
100	10	(1)	.000	2.125	1.370	1.215	6.205	3.965	6.670	10.245	7.420
		(2)	.000	2.175	1.395	1.295	6.325	4.080	6.815	10.325	7.620
		(3)	1.070	.795	.795	5.055	4.640	4.675	10.135	9.615	9.735
200	10	(1)	.000	2.470	1.675	1.090	6.455	4.275	6.575	10.610	7.545
		(2)	.000	2.480	1.685	1.135	6.505	4.310	6.675	10.640	7.640
		(3)	.975	.995	.995	4.915	4.925	4.910	10.190	9.850	9.790

TABLE 4.1(d) Rejection percentage (size) of various cases of tests: DGP(A)

			1%			5%			10%		
N	T	$Case$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$
2	50	(1)	.010	.010	.000	.860	.835	.375	3.240	3.080	1.695
		(2)	.865	.835	.665	4.795	4.575	4.520	9.890	9.420	9.370
		(3)	.155	.000	.000	1.335	.425	.410	3.850	2.835	1.860
5	50	(1)	.160	.750	.480	2.360	3.820	2.710	6.635	8.055	6.180
		(2)	.430	1.445	1.030	3.975	5.505	4.725	9.215	10.290	9.480
		(3)	.545	.350	.335	3.330	3.070	2.845	7.585	7.610	6.595
10	50	(1)	.230	1.340	.855	3.185	4.955	3.810	8.100	9.455	8.140
		(2)	.420	1.735	1.210	4.025	5.780	4.910	9.350	10.420	9.805
		(3)	.815	.700	.705	4.385	4.105	3.990	9.080	9.020	8.570
20	50	(1)	.235	1.520	.970	3.325	5.220	4.350	8.435	9.620	8.545
		(2)	.325	1.735	1.095	3.710	5.595	4.840	9.115	10.145	9.305
		(3)	.975	.835	.835	4.500	4.360	4.600	9.575	9.160	8.980
30	50	(1)	.330	1.555	1.010	3.445	5.565	4.330	8.680	9.990	9.010
		(2)	.370	1.695	1.130	3.705	5.815	4.725	9.140	10.315	9.520
		(3)	.900	.825	.815	4.850	4.570	4.555	9.735	9.500	9.455
50	50	(1)	.195	1.715	1.085	3.470	5.750	4.655	8.820	10.515	9.220
		(2)	.230	1.790	1.145	3.675	5.935	4.855	9.070	10.675	9.610
		(3)	.900	.910	.890	4.865	4.805	4.860	9.855	9.985	9.850
100	50	(1)	.255	1.700	1.140	3.445	5.945	4.785	8.415	10.445	9.390
		(2)	.270	1.720	1.155	3.520	6.030	4.885	8.510	10.565	9.550
		(3)	.930	.970	.960	4.745	5.015	5.035	9.405	9.935	9.845
200	50	(1)	.245	1.635	1.015	3.410	6.020	4.700	9.045	10.640	9.430
		(2)	.250	1.640	1.030	3.455	6.075	4.735	9.115	10.685	9.530
		(3)	1.015	.840	.845	4.855	4.950	4.910	10.135	10.195	9.875

TABLE 4.2(a) Rejection percentage (size) of various cases of tests: DGP(B)

			1%			5%			10%		
N	T	*Case*	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$
2	1	(1)	.000	.000	.000	.000	.000	.000	.000	.000	.000
		(2)	.000	.000	.000	.000	.000	.000	.000	.000	.000
		(3)	.600	.000	.000	3.095	.000	.000	5.705	.000	.000
5	1	(1)	.000	4.660	1.920	.000	12.735	8.820	.000	17.660	12.735
		(2)	.000	7.495	4.915	.000	15.170	11.370	.000	19.550	15.170
		(3)	.615	.000	.000	3.930	8.045	8.045	7.430	18.735	18.735
10	1	(1)	.000	10.235	9.070	.000	14.380	12.250	.000	17.320	14.380
		(2)	.000	10.885	9.685	.000	15.070	12.935	.000	17.905	15.070
		(3)	.910	3.920	3.920	4.595	11.725	11.725	8.605	18.050	18.050
20	1	(1)	.000	9.025	8.120	.000	12.445	10.555	.000	14.925	12.445
		(2)	.000	9.235	8.330	.000	12.710	10.900	.000	15.195	12.710
		(3)	.915	4.570	4.570	4.970	10.285	10.285	9.340	15.510	15.510
30	1	(1)	.000	7.720	6.860	.000	10.850	9.305	.000	13.260	10.850
		(2)	.000	7.855	6.985	.000	11.005	9.425	.000	13.460	11.005
		(3)	.840	3.925	3.925	4.890	8.955	8.955	9.135	14.015	14.015
50	1	(1)	.000	6.490	5.705	.000	9.470	7.925	.000	11.850	9.470
		(2)	.000	6.620	5.765	.000	9.560	8.000	.000	11.990	9.560
		(3)	.885	3.060	3.060	4.830	7.655	7.655	9.345	12.480	12.480
100	1	(1)	.000	5.340	4.630	.000	8.545	6.800	.000	11.025	8.545
		(2)	.000	5.375	4.675	.000	8.630	6.875	.000	11.060	8.630
		(3)	.970	2.240	2.240	5.300	6.455	6.455	9.775	11.810	11.810
200	1	(1)	.000	4.470	3.705	.000	7.440	5.830	.000	9.905	7.440
		(2)	.000	4.480	3.725	.000	7.465	5.865	.000	9.940	7.465
		(3)	.915	1.515	1.515	5.020	5.570	5.570	9.335	10.620	10.620

TABLE 4.2(b) Rejection percentage (size) of various cases of tests: DGP(B)

			1%			5%			10%		
N	T	*Case*	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$
2	5	(1)	.000	.000	.000	.000	.000	.000	.390	3.620	.000
		(2)	.000	.000	.000	.895	5.960	1.965	4.070	12.725	6.855
		(3)	.075	.000	.000	.665	.000	.000	1.805	2.575	1.430
5	5	(1)	.000	2.905	2.120	.000	6.875	4.680	1.355	10.635	6.875
		(2)	.000	4.115	3.005	.100	8.460	5.950	3.440	12.540	8.560
		(3)	.340	.930	.930	2.300	4.955	4.955	5.130	10.000	9.330
10	5	(1)	.000	3.255	2.465	.000	7.115	4.970	2.000	10.765	7.115
		(2)	.000	3.735	2.845	.000	7.845	5.595	3.235	11.625	7.845
		(3)	.645	1.180	1.180	3.245	5.190	5.190	7.040	10.025	9.890
20	5	(1)	.000	3.245	2.395	.000	7.375	5.045	2.365	10.765	7.375
		(2)	.000	3.455	2.550	.000	7.695	5.350	3.045	11.120	7.695
		(3)	.635	1.140	1.140	3.870	5.280	5.280	8.195	10.115	9.835
30	5	(1)	.000	3.130	2.375	.000	7.225	4.940	2.835	10.795	7.225
		(2)	.000	3.285	2.495	.000	7.430	5.130	3.295	11.095	7.430
		(3)	.940	1.060	1.060	4.315	5.145	5.145	9.010	10.120	10.105
50	5	(1)	.000	3.100	2.275	.000	6.720	4.650	2.970	10.575	6.720
		(2)	.000	3.155	2.345	.000	6.860	4.755	3.235	10.735	6.860
		(3)	.930	1.080	1.080	4.760	4.855	4.855	9.565	9.850	9.800
100	5	(1)	.000	3.025	2.100	.000	7.045	4.745	3.100	10.745	7.045
		(2)	.000	3.070	2.175	.000	7.085	4.815	3.245	10.805	7.085
		(3)	.955	1.015	1.015	4.745	5.010	5.010	9.420	10.140	10.045
200	5	(1)	.000	3.020	2.175	.000	7.005	4.755	3.310	10.935	7.005
		(2)	.000	3.035	2.215	.000	7.050	4.780	3.395	10.970	7.050
		(3)	1.000	1.005	1.005	5.045	4.960	4.960	9.845	10.235	10.240

TABLE 4.2(c) Rejection percentage (size) of various cases of tests: DGP(B)

			1%			5%			10%		
N	T	$Case$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$
2	10	(1)	.000	.000	.000	.015	1.300	.125	.385	4.760	1.315
		(2)	.015	1.300	.445	.875	6.460	3.740	3.860	11.310	7.335
		(3)	.015	.000	.000	.235	.315	.270	.995	4.110	2.255
5	10	(1)	.000	1.845	1.285	.280	5.370	3.225	2.715	9.160	5.650
		(2)	.000	2.670	1.905	.910	6.965	4.605	5.035	11.085	7.875
		(3)	.240	.595	.595	1.825	3.885	3.770	5.000	8.515	7.060
10	10	(1)	.000	2.245	1.570	.545	6.000	3.960	4.070	9.790	6.545
		(2)	.000	2.700	1.880	.930	6.725	4.580	5.245	10.840	7.655
		(3)	.470	.755	.755	2.890	4.530	4.455	6.760	9.135	8.095
20	10	(1)	.000	2.450	1.715	.840	6.425	4.160	5.410	10.510	7.265
		(2)	.000	2.660	1.900	1.145	6.720	4.505	6.135	10.995	7.865
		(3)	.745	.990	.990	4.145	4.855	4.810	8.570	9.760	9.255
30	10	(1)	.000	2.470	1.785	.910	6.485	4.290	5.930	10.705	7.395
		(2)	.000	2.600	1.880	1.125	6.700	4.520	6.375	11.065	7.825
		(3)	.800	1.080	1.080	4.505	4.860	4.875	9.100	9.910	9.480
50	10	(1)	.000	2.610	1.820	.995	6.780	4.440	5.970	10.705	7.775
		(2)	.000	2.700	1.885	1.095	6.915	4.580	6.175	10.880	8.010
		(3)	.875	1.030	1.030	4.460	5.065	5.045	8.965	10.050	9.790
100	10	(1)	.000	2.635	1.765	1.115	6.820	4.535	6.380	10.900	7.935
		(2)	.000	2.670	1.800	1.185	6.870	4.610	6.590	10.980	8.055
		(3)	.985	.980	.980	4.855	5.250	5.235	9.510	10.150	10.170
200	10	(1)	.000	2.540	1.745	1.025	6.695	4.300	6.280	10.520	7.720
		(2)	.000	2.550	1.760	1.050	6.710	4.330	6.345	10.545	7.760
		(3)	.925	.985	.985	4.765	4.985	4.945	9.865	9.810	9.815

TABLE 4.2(d) Rejection percentage (size) of various cases of tests: DGP(B)

			1%			5%			10%		
N	T	$Case$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$
2	50	(1)	.000	.380	.180	.025	2.205	.985	.750	4.745	2.230
		(2)	.025	2.210	1.490	1.520	5.955	4.245	6.160	9.880	7.475
		(3)	.000	.140	.125	.110	1.725	1.045	1.015	4.485	2.345
5	50	(1)	.030	1.070	.625	1.560	4.195	2.690	5.365	7.970	5.755
		(2)	.205	1.735	1.165	3.090	5.700	4.440	7.925	9.940	8.790
		(3)	.265	.555	.535	2.600	3.495	2.840	6.235	7.585	6.105
10	50	(1)	.115	1.260	.915	2.485	4.700	3.380	7.140	8.875	7.185
		(2)	.220	1.595	1.125	3.305	5.475	4.330	8.475	9.815	8.780
		(3)	.595	.775	.760	3.630	3.895	3.530	8.205	8.430	7.600
20	50	(1)	.165	1.555	.990	3.055	5.470	4.120	8.185	9.775	8.525
		(2)	.220	1.750	1.095	3.485	5.850	4.695	8.935	10.310	9.335
		(3)	.685	.840	.815	4.335	4.605	4.375	9.260	9.340	8.985
30	50	(1)	.220	1.615	1.065	3.165	5.565	4.220	8.540	9.900	8.730
		(2)	.250	1.740	1.215	3.505	5.870	4.565	9.035	10.290	9.375
		(3)	.805	.855	.845	4.670	4.595	4.430	9.580	9.495	9.295
50	50	(1)	.235	1.820	1.190	3.340	5.995	4.590	8.670	10.515	9.335
		(2)	.245	1.890	1.250	3.535	6.165	4.785	8.910	10.715	9.700
		(3)	.825	1.035	.980	4.650	5.005	4.800	9.675	10.125	9.810
100	50	(1)	.285	1.765	1.145	3.245	5.710	4.655	8.205	10.000	8.955
		(2)	.305	1.815	1.180	3.380	5.780	4.725	8.325	10.080	9.160
		(3)	.915	.970	1.010	4.645	4.785	4.835	9.205	9.660	9.515
200	50	(1)	.275	1.955	1.285	3.295	6.020	4.675	8.550	10.660	9.315
		(2)	.290	1.955	1.320	3.370	6.055	4.715	8.585	10.675	9.425
		(3)	.890	1.035	1.015	4.590	5.055	4.870	9.720	10.175	9.850

TABLE 4.3(a) Rejection percentage (power) of various cases of tests: DGP(C)

			1%			5%			10%		
N	T	*Case*	<0	>0	$\neq 0$	<0	>0	$\neq 0$	<0	>0	$\neq 0$
2	1	(1)	.000	.000	.000	.000	.000	.000	.000	.000	.000
		(2)	.000	.000	.000	.000	.000	.000	.000	.000	.000
		(3)	.555	.000	.000	3.010	.000	.000	5.720	.000	.000
5	1	(1)	.000	3.470	1.060	.000	13.060	8.350	.000	19.785	13.060
		(2)	.000	6.655	3.725	.000	16.195	11.360	.000	22.570	16.195
		(3)	.725	.000	.000	3.210	7.365	7.365	6.210	21.310	21.310
10	1	(1)	.000	23.580	21.165	.000	29.865	26.945	.000	33.675	29.865
		(2)	.000	24.680	22.450	.000	30.790	27.700	.000	34.525	30.790
		(3)	.640	8.630	8.630	3.420	26.365	26.365	6.530	34.655	34.655
20	1	(1)	.000	33.420	32.825	.000	36.070	34.825	.000	38.050	36.070
		(2)	.000	33.625	32.985	.000	36.300	35.085	.000	38.305	36.300
		(3)	.585	28.770	28.770	3.400	34.545	34.545	6.650	38.620	38.620
30	1	(1)	.000	34.580	34.010	.000	36.585	35.480	.000	38.365	36.585
		(2)	.000	34.730	34.090	.000	36.675	35.575	.000	38.515	36.675
		(3)	.675	32.375	32.375	3.425	35.275	35.275	6.635	38.865	38.865
50	1	(1)	.000	33.645	33.100	.000	35.795	34.570	.000	37.675	35.795
		(2)	.000	33.695	33.140	.000	35.865	34.660	.000	37.755	35.865
		(3)	.585	31.685	31.685	3.655	34.365	34.365	6.850	38.180	38.180
100	1	(1)	.000	34.705	34.270	.000	36.890	35.750	.000	38.710	36.890
		(2)	.000	34.745	34.295	.000	36.910	35.785	.000	38.765	36.910
		(3)	.725	32.710	32.710	3.825	35.550	35.550	6.865	39.235	39.235
200	1	(1)	.000	34.570	34.110	.000	36.505	35.420	.000	38.390	36.505
		(2)	.000	34.580	34.110	.000	36.530	35.440	.000	38.410	36.530
		(3)	.635	32.710	32.710	3.640	35.240	35.240	7.160	38.840	38.840

TABLE 4.3(b) Rejection percentage (power) of various cases of tests: DGP(C)

			1%			5%			10%		
N	T	*Case*	<0	>0	$\neq 0$	<0	>0	$\neq 0$	<0	>0	$\neq 0$
2	5	(1)	.000	.000	.000	.000	.000	.000	.540	4.810	.000
		(2)	.000	.000	.000	1.110	9.155	2.070	3.570	22.265	10.265
		(3)	.145	.000	.000	.875	.000	.000	1.820	2.845	1.485
5	5	(1)	.000	32.155	26.650	.000	47.960	40.610	.550	56.600	47.960
		(2)	.000	37.680	32.530	.040	51.995	45.475	1.130	59.885	52.035
		(3)	.125	15.230	15.230	.780	41.380	41.380	1.555	55.160	53.495
10	5	(1)	.000	65.360	62.270	.000	73.090	69.600	.460	77.095	73.090
		(2)	.000	66.680	63.810	.000	74.020	70.580	.690	77.765	74.020
		(3)	.155	54.440	54.440	.690	69.960	69.960	1.390	76.485	75.720
20	5	(1)	.000	81.850	80.930	.000	84.250	83.120	.430	85.310	84.250
		(2)	.000	82.060	81.140	.000	84.345	83.320	.540	85.430	84.345
		(3)	.130	78.255	78.255	.710	83.250	83.250	1.470	85.130	85.010
30	5	(1)	.000	85.230	85.000	.000	86.100	85.630	.460	86.750	86.100
		(2)	.000	85.260	85.025	.000	86.130	85.685	.510	86.805	86.130
		(3)	.105	84.275	84.275	.625	85.690	85.690	1.365	86.650	86.630
50	5	(1)	.000	85.250	85.180	.000	85.940	85.610	.510	86.495	85.940
		(2)	.000	85.260	85.180	.000	85.965	85.620	.550	86.535	85.965
		(3)	.170	85.010	85.010	.775	85.645	85.645	1.480	86.415	86.435
100	5	(1)	.000	86.025	85.875	.000	86.680	86.310	.500	87.215	86.680
		(2)	.000	86.025	85.875	.000	86.695	86.315	.520	87.225	86.695
		(3)	.155	85.710	85.710	.770	86.320	86.320	1.430	87.125	87.160
200	5	(1)	.000	85.675	85.595	.000	86.335	85.935	.435	86.970	86.335
		(2)	.000	85.675	85.595	.000	86.335	85.940	.450	86.985	86.335
		(3)	.135	85.405	85.405	.645	86.005	86.005	1.315	86.845	86.815

TABLE 4.3(c) Rejection percentage (power) of various cases of tests: DGP(C)

			1%			5%			10%		
N	T	*Case*	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$
2	10	(1)	.000	.000	.000	.060	3.325	.160	.415	15.200	3.385
		(2)	.060	3.330	.885	.715	20.975	11.115	1.990	35.160	21.690
		(3)	.055	.000	.000	.290	.650	.530	.765	12.865	6.115
5	10	(1)	.000	53.910	47.880	.035	69.765	62.615	.205	77.050	69.800
		(2)	.000	59.785	54.405	.120	73.325	67.315	.350	79.540	73.445
		(3)	.030	39.045	39.045	.165	65.145	64.635	.345	75.995	72.250
10	10	(1)	.000	86.210	84.125	.035	91.005	88.890	.185	93.075	91.040
		(2)	.000	87.060	85.300	.060	91.500	89.555	.225	93.415	91.560
		(3)	.030	80.860	80.860	.140	89.590	89.470	.275	92.730	91.740
20	10	(1)	.000	95.750	95.320	.025	96.835	96.390	.160	97.340	96.860
		(2)	.000	95.850	95.465	.030	96.905	96.455	.195	97.380	96.935
		(3)	.020	94.500	94.500	.115	96.550	96.525	.250	97.260	97.070
30	10	(1)	.000	97.550	97.395	.000	97.900	97.745	.080	98.035	97.900
		(2)	.000	97.555	97.430	.005	97.915	97.760	.115	98.065	97.920
		(3)	.000	97.165	97.165	.060	97.790	97.780	.170	98.015	97.960
50	10	(1)	.000	97.890	97.880	.015	98.010	97.940	.135	98.105	98.025
		(2)	.000	97.895	97.880	.035	98.010	97.940	.135	98.105	98.045
		(3)	.015	97.835	97.835	.115	97.940	97.945	.165	98.085	98.095
100	10	(1)	.000	97.820	97.810	.020	97.925	97.860	.210	98.000	97.945
		(2)	.000	97.820	97.810	.020	97.925	97.860	.210	98.005	97.945
		(3)	.020	97.790	97.790	.145	97.880	97.875	.255	97.985	97.985
200	10	(1)	.000	97.945	97.925	.030	98.040	97.995	.165	98.105	98.070
		(2)	.000	97.945	97.925	.030	98.040	97.995	.165	98.105	98.070
		(3)	.020	97.920	97.920	.135	98.015	98.020	.250	98.090	98.135

TABLE 4.3(d) Rejection percentage (power) of various cases of tests: DGP(C)

			1%			5%			10%		
N	T	*Case*	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$
2	50	(1)	.000	11.050	4.820	.000	45.080	27.200	.005	65.400	45.080
		(2)	.000	45.110	34.630	.010	71.460	59.775	.065	82.725	71.470
		(3)	.000	3.950	3.675	.000	38.750	28.165	.005	63.730	46.480
5	50	(1)	.000	99.040	98.605	.000	99.710	99.475	.000	99.885	99.710
		(2)	.000	99.370	99.085	.000	99.800	99.650	.000	99.915	99.800
		(3)	.000	98.450	98.390	.000	99.650	99.495	.000	99.860	99.735
10	50	(1)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
		(2)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
		(3)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
20	50	(1)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
		(2)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
		(3)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
30	50	(1)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
		(2)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
		(3)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
50	50	(1)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
		(2)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
		(3)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
100	50	(1)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
		(2)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
		(3)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
200	50	(1)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
		(2)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000
		(3)	.000	100.000	100.000	.000	100.000	100.000	.000	100.000	100.000

TABLE 4.4(a) Rejection percentage (power) of various cases of tests: DGP(D)

			1%			5%			10%		
N	T	*Case*	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$
2	1	(1)	.000	.000	.000	.000	.000	.000	.000	.000	.000
		(2)	.000	.000	.000	.000	.000	.000	.000	.000	.000
		(3)	.600	.000	.000	3.095	.000	.000	5.705	.000	.000
5	1	(1)	.000	.205	.050	.000	1.220	.595	.000	2.470	1.220
		(2)	.000	.425	.205	.000	1.765	.965	.000	3.220	1.765
		(3)	1.300	.000	.000	6.665	.495	.495	12.710	2.915	2.915
10	1	(1)	.000	.175	.105	.000	.475	.285	.000	.970	.475
		(2)	.000	.185	.130	.000	.570	.325	.000	1.135	.570
		(3)	1.580	.005	.005	8.415	.240	.240	16.040	1.145	1.145
20	1	(1)	.000	.035	.020	.000	.130	.065	.000	.335	.130
		(2)	.000	.035	.025	.000	.165	.070	.000	.345	.165
		(3)	1.815	.005	.005	10.055	.060	.060	18.390	.390	.390
30	1	(1)	.000	.025	.015	.000	.110	.045	.000	.185	.110
		(2)	.000	.030	.015	.000	.115	.060	.000	.190	.115
		(3)	2.015	.000	.000	10.395	.045	.045	19.125	.240	.240
50	1	(1)	.000	.000	.000	.000	.030	.000	.000	.085	.030
		(2)	.000	.000	.000	.000	.030	.000	.000	.085	.030
		(3)	2.255	.000	.000	10.730	.000	.000	19.920	.095	.095
100	1	(1)	.000	.000	.000	.000	.010	.000	.000	.045	.010
		(2)	.000	.000	.000	.000	.010	.000	.000	.045	.010
		(3)	2.045	.000	.000	11.205	.000	.000	20.905	.065	.065
200	1	(1)	.000	.000	.000	.000	.005	.005	.000	.025	.005
		(2)	.000	.000	.000	.000	.005	.005	.000	.025	.005
		(3)	2.330	.000	.000	11.490	.005	.005	21.485	.040	.040

TABLE 4.4(b) Rejection percentage (power) of various cases of tests: DGP(D)

			1%			5%			10%		
N	T	*Case*	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$
2	5	(1)	.000	.000	.000	.000	.000	.000	6.065	.290	.000
		(2)	.000	.000	.000	11.840	.575	2.580	27.155	2.000	12.415
		(3)	2.045	.000	.000	8.945	.000	.000	17.130	.170	1.780
5	5	(1)	.000	.005	.000	.000	.020	.005	20.230	.060	.020
		(2)	.000	.005	.005	3.145	.035	.010	35.980	.070	3.180
		(3)	8.000	.000	.000	27.755	.010	.010	44.735	.050	7.070
10	5	(1)	.000	.000	.000	.000	.005	.000	32.365	.005	.005
		(2)	.000	.000	.000	.110	.005	.000	42.480	.005	.115
		(3)	13.450	.000	.000	42.505	.000	.000	62.565	.005	11.815
20	5	(1)	.000	.000	.000	.000	.000	.000	42.650	.000	.000
		(2)	.000	.000	.000	.000	.000	.000	48.490	.000	.000
		(3)	18.895	.000	.000	54.035	.000	.000	73.005	.000	16.545
30	5	(1)	.000	.000	.000	.000	.000	.000	46.320	.000	.000
		(2)	.000	.000	.000	.000	.000	.000	50.265	.000	.000
		(3)	20.885	.000	.000	57.680	.000	.000	76.705	.000	18.255
50	5	(1)	.000	.000	.000	.000	.000	.000	48.685	.000	.000
		(2)	.000	.000	.000	.000	.000	.000	51.135	.000	.000
		(3)	22.855	.000	.000	60.825	.000	.000	79.605	.000	20.365
100	5	(1)	.000	.000	.000	.000	.000	.000	52.160	.000	.000
		(2)	.000	.000	.000	.000	.000	.000	53.540	.000	.000
		(3)	24.750	.000	.000	63.880	.000	.000	82.110	.000	22.000
200	5	(1)	.000	.000	.000	.000	.000	.000	53.120	.000	.000
		(2)	.000	.000	.000	.000	.000	.000	53.820	.000	.000
		(3)	25.940	.000	.000	65.320	.000	.000	83.300	.000	23.325

TABLE 4.4(c) Rejection percentage (power) of various cases of tests: DGP(D)

			1%			5%			10%		
N	T	$Case$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$
2	10	(1)	.000	.000	.000	5.160	.010	.280	21.705	.100	5.170
		(2)	5.170	.010	1.410	28.890	.200	15.935	46.155	.665	29.090
		(3)	4.570	.000	.000	17.460	.000	.740	29.815	.075	9.330
5	10	(1)	.000	.000	.000	27.595	.000	2.620	65.435	.000	27.595
		(2)	.295	.000	.000	46.940	.000	16.270	75.715	.000	46.940
		(3)	25.705	.000	.000	58.815	.000	6.725	75.655	.000	40.665
10	10	(1)	.000	.000	.000	47.790	.000	6.770	84.495	.000	47.790
		(2)	.000	.000	.000	58.945	.000	17.310	88.020	.000	58.945
		(3)	45.400	.000	.000	79.695	.000	15.360	91.005	.000	62.835
20	10	(1)	.000	.000	.000	62.465	.000	11.755	92.835	.000	62.465
		(2)	.000	.000	.000	68.145	.000	18.715	94.000	.000	68.145
		(3)	59.860	.000	.000	89.710	.000	24.240	96.530	.000	76.815
30	10	(1)	.000	.000	.000	67.945	.000	14.000	95.215	.000	67.945
		(2)	.000	.000	.000	71.855	.000	18.995	95.790	.000	71.855
		(3)	65.395	.000	.000	92.600	.000	27.890	97.805	.000	81.320
50	10	(1)	.000	.000	.000	72.715	.000	15.600	96.615	.000	72.715
		(2)	.000	.000	.000	74.925	.000	19.080	96.910	.000	74.925
		(3)	70.175	.000	.000	94.590	.000	31.490	98.570	.000	84.640
100	10	(1)	.000	.000	.000	75.990	.000	17.940	97.805	.000	75.990
		(2)	.000	.000	.000	77.000	.000	19.815	97.890	.000	77.000
		(3)	73.600	.000	.000	96.105	.000	34.860	99.110	.000	87.535
200	10	(1)	.000	.000	.000	77.075	.000	19.200	97.830	.000	77.075
		(2)	.000	.000	.000	77.635	.000	20.180	97.860	.000	77.635
		(3)	74.765	.000	.000	96.350	.000	35.715	99.150	.000	88.360

TABLE 4.4(d) Rejection percentage (power) of various cases of tests: DGP(D)

			1%			5%			10%		
N	T	$Case$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$	< 0	> 0	$\neq 0$
2	50	(1)	27.975	.000	14.930	70.720	.000	51.555	86.510	.000	70.720
		(2)	70.755	.000	60.460	89.885	.000	82.740	95.130	.005	89.885
		(3)	46.950	.000	11.315	76.970	.000	53.115	88.110	.000	72.125
5	50	(1)	98.070	.000	93.480	99.965	.000	99.645	100.000	.000	99.965
		(2)	99.450	.000	98.265	99.995	.000	99.950	100.000	.000	99.995
		(3)	99.520	.000	90.800	99.990	.000	99.665	100.000	.000	99.980
10	50	(1)	99.960	.000	99.790	100.000	.000	100.000	100.000	.000	100.000
		(2)	99.990	.000	99.920	100.000	.000	100.000	100.000	.000	100.000
		(3)	100.000	.000	99.615	100.000	.000	100.000	100.000	.000	100.000
20	50	(1)	100.000	.000	99.990	100.000	.000	100.000	100.000	.000	100.000
		(2)	100.000	.000	99.990	100.000	.000	100.000	100.000	.000	100.000
		(3)	100.000	.000	99.975	100.000	.000	100.000	100.000	.000	100.000
30	50	(1)	100.000	.000	100.000	100.000	.000	100.000	100.000	.000	100.000
		(2)	100.000	.000	100.000	100.000	.000	100.000	100.000	.000	100.000
		(3)	100.000	.000	100.000	100.000	.000	100.000	100.000	.000	100.000
50	50	(1)	100.000	.000	100.000	100.000	.000	100.000	100.000	.000	100.000
		(2)	100.000	.000	100.000	100.000	.000	100.000	100.000	.000	100.000
		(3)	100.000	.000	100.000	100.000	.000	100.000	100.000	.000	100.000
100	50	(1)	100.000	.000	100.000	100.000	.000	100.000	100.000	.000	100.000
		(2)	100.000	.000	100.000	100.000	.000	100.000	100.000	.000	100.000
		(3)	100.000	.000	100.000	100.000	.000	100.000	100.000	.000	100.000
200	50	(1)	100.000	.000	100.000	100.000	.000	100.000	100.000	.000	100.000
		(2)	100.000	.000	100.000	100.000	.000	100.000	100.000	.000	100.000
		(3)	100.000	.000	100.000	100.000	.000	100.000	100.000	.000	100.000

5 Conclusions

A Appendix: Technical proofs

Proof of Theorem 1.1. See Theorem 4 of Section 21 (p.103); and Theorem 4 of Section 28 (p.143) in Gnedenko and Kolmogorov (1954). □

Proof of Theorem 1.2. See Theorem 19 of Chapter 7, p.179 in Pollard (2002).

□

Proof of Theorem 2.1.

By Chebyshev's inequality, it is not difficult to show that in all three cases,

$$\hat{\sigma}^2_{NT} \longrightarrow_p \sigma^2. \tag{A. 1}$$

We consider Case (1) first. Note for all t,

$$N^{-1/2}\sum_{i=1}^{N}\hat{\nu}_{it} = N^{-1/2}\sum_{i=1}^{N}\nu_{it} - T^{-1/2}N^{-1}\sum_{i=1}^{N}x'_{it}\sqrt{NT}\left(\hat{\beta}_{NT}-\beta\right). \tag{A. 2}$$

By (2.1) and (A.2), we can write the numerator of the CSC_0,

$$\begin{aligned}
&\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(\frac{1}{N}\sum_{i=1}^{N}\hat{\nu}_{it}\sum_{\substack{j=1\\ j\neq i}}^{N}\hat{\nu}_{jt}\right)\\
=\;&\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(\frac{1}{N}\sum_{i=1}^{N}\nu_{it}\sum_{\substack{j=1\\ j\neq i}}^{N}\nu_{jt}\right) + \sqrt{NT}\left(\hat{\beta}_{NT}-\beta\right)'\frac{2}{TN^{3/2}}\sum_{t=1}^{T}\sum_{i=1}^{N}x_{it}\nu_{it}\\
&-\sqrt{NT}\left(\hat{\beta}_{NT}-\beta\right)'\frac{2}{TN^{3/2}}\sum_{t=1}^{T}\left(\sum_{i=1}^{N}x_{it}\sum_{j=1}^{N}\nu_{jt}\right)\\
&-\sqrt{NT}\left(\hat{\beta}_{NT}-\beta\right)'\frac{1}{T^{3/2}N^2}\sum_{t=1}^{T}\left(\sum_{i=1}^{N}x_{it}x'_{it}\right)\sqrt{NT}\left(\hat{\beta}_{NT}-\beta\right)\\
&+\sqrt{NT}\left(\hat{\beta}_{NT}-\beta\right)'\frac{1}{T^{3/2}N^2}\sum_{t=1}^{T}\left(\sum_{i=1}^{N}x_{it}\sum_{j=1}^{N}x'_{jt}\right)\sqrt{NT}\left(\hat{\beta}_{NT}-\beta\right)\\
=\;&\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(\frac{1}{N}\sum_{i=1}^{N}\nu_{it}\sum_{\substack{j=1\\ j\neq i}}^{N}\nu_{jt}\right) + (I)+(II)+(III)+(IV).
\end{aligned} \tag{A. 3}$$

First consider (I). It is not difficult to show that $\sqrt{NT}\left(\hat{\beta}_{NT}-\beta\right) = O_p(1)$ and $\frac{1}{\sqrt{TN}}\sum_{t=1}^{T}\sum_{i=1}^{N}x_{it}\nu_{it} = O_p(1)$. Thus $(I) = O_p(T^{-1/2}N^{-1}) = o_p(1)$.

Next consider (II). Using the fact $\sup_{N,t}E\|N^{-1/2}\sum_{i=1}^{N}x_{it}\|^2 < \infty$ and $\sup_{N,t}E\|N^{-1/2}\sum_{i=1}^{N}\nu_{it}\|^2 < \infty$,

$$\begin{aligned}
&E\left\|\frac{1}{TN^{3/2}}\sum_{t=1}^{T}\left(\sum_{i=1}^{N}x_{it}\sum_{j=1}^{N}\nu_{jt}\right)\right\|\\
\leq\;& T^{-1}N^{-1/2}\sum_{t=1}^{T}\left(E\left\|N^{-1/2}\sum_{i=1}^{N}x_{it}\right\|^2\right)^{1/2}\left(E\left(N^{-1/2}\sum_{j=1}^{N}\nu_{jt}\right)^2\right)^{1/2} = O(N^{-1/2}) = o(1).
\end{aligned}$$

Thus by Chebyshev's inequality, $\frac{1}{TN^{3/2}}\sum_{t=1}^{T}\left(\sum_{i=1}^{N}x_{it}\sum_{j=1}^{N}\nu_{jt}\right) = o_p(1)$. $(II) = o_p(1)$. Finally consider (III) and (IV). Once again using the fact that $\sup_{N,t}E\|N^{-1/2}\sum_{i=1}^{N}x_{it}\|^2 <$

∞,

$$E\left\|\frac{1}{T^{3/2}N^2}\sum_{t=1}^{T}\left(\sum_{i=1}^{N}x_{it}x_{it}'\right)\right\| \le T^{-3/2}N^{-2}\sum_{t=1}^{T}\sum_{i=1}^{N}E\left\|x_{it}\right\|^2 = O(T^{-1/2}N^{-1}) = o(1).$$

$$E\left\|\frac{1}{T^{3/2}N^2}\sum_{t=1}^{T}\left(\sum_{i=1}^{N}x_{it}\sum_{j=1}^{N}x_{jt}'\right)\right\| \le T^{-3/2}N^{-1}\sum_{t=1}^{T}E\left\|N^{-1/2}\sum_{i=1}^{N}x_{it}\right\|^2 = O(T^{-1/2}N^{-1}) = o(1).$$

Again by Chebyshev's inequality, both $\frac{1}{T^{3/2}N^2}\sum_{t=1}^{T}\left(\sum_{i=1}^{N}x_{it}x_{it}'\right)$ and $\frac{1}{T^{3/2}N^2}\sum_{t=1}^{T}\left(\sum_{i=1}^{N}x_{it}\sum_{j=1}^{N}x_{jt}'\right)$ are $o_p(1)$ and so are (III) and (IV). All in all,

$$\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(\frac{1}{N}\sum_{i=1}^{N}\hat{\nu}_{it}\sum_{\substack{j=1\\j\neq i}}^{N}\hat{\nu}_{jt}\right) = \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(\frac{1}{N}\sum_{i=1}^{N}\nu_{it}\sum_{\substack{j=1\\j\neq i}}^{N}\nu_{jt}\right) + o_p(1). \qquad \text{(A. 4)}$$

And due to the independence between ν_{it} and ν_{jt} for $i \neq j$,

$$\begin{aligned} & E\left(\frac{1}{N}\sum_{i=1}^{N}\nu_{it}\sum_{\substack{j=1\\j\neq i}}^{N}\nu_{jt}\right)^2 \\ = \; & E\left(\frac{2}{N^2}\sum_{i=1}^{N}\sum_{\substack{j=1\\j\neq i}}^{N}\nu_{it}^2\nu_{jt}^2\right) + E\left(\frac{4}{N^2}\sum_{i=1}^{N}\sum_{\substack{j=1\\j\neq i}}^{N}\sum_{\substack{k=1\\k\neq j,k\neq i}}^{N}\nu_{it}^2\nu_{jt}\nu_{kt} + \frac{1}{N^2}\sum_{i=1}^{N}\sum_{\substack{j=1\\j\neq i}}^{N}\sum_{\substack{k=1\\k\neq j,k\neq i}}^{N}\sum_{\substack{l=1,l\neq k\\l\neq j,l\neq i}}^{N}\nu_{it}\nu_{jt}\nu_{kt}\nu_{lt}\right) \\ = \; & 2\left(1-\frac{1}{N}\right)\sigma^4. \end{aligned} \qquad \text{(A. 5)}$$

Thus, by (2.1), (A.1), (A.4), (A.5) and the fact that $\frac{1}{N} \to 0$, the conclusion follows by the assumption $\sup_{i,t} E|\nu_{it}|^{2+\delta} < \infty$ (the Liapounov condition) and Theorem 1.1.

Next we consider Case (2). The proof is the same as that for Case (1), except that $(II) \neq o_p(1)$ in general.

$$\begin{aligned} (II) \; &= \; -\frac{2}{N^{1/2}}\frac{1}{T}\sum_{t=1}^{T}\left(\frac{1}{N}\sum_{j=1}^{N}\nu_{jt}\sum_{i=1}^{N}x_{it}'\right)\sqrt{NT}\left(\hat{\beta}_{NT}-\beta\right) \\ &= \; -\frac{2}{N^{1/2}}E\left(\frac{1}{N}\sum_{j=1}^{N}\nu_{j1}\sum_{i=1}^{N}x_{i1}'\right)E\left(x_{11}x_{11}'\right)^{-1}\frac{1}{\sqrt{NT}}\sum_{t=1}^{T}\sum_{i=1}^{N}x_{it}\nu_{it} + o_p(1). \end{aligned} \qquad \text{(A. 6)}$$

Note the asymptotic variance of (II),

$$avar[(II)] = \frac{4\sigma^2}{N}E\left(\frac{1}{N}\sum_{j=1}^{N}\nu_{j1}\sum_{i=1}^{N}x_{i1}'\right)E\left(x_{11}x_{11}'\right)^{-1}E\left(\frac{1}{N}\sum_{i=1}^{N}x_{i1}\sum_{j=1}^{N}\nu_{j1}\right). \qquad \text{(A. 7)}$$

Moreover, the asymptotic covariance,

$$acov\left[\frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left(\frac{1}{N}\sum_{i=1}^{N}\nu_{it}\sum_{\substack{j=1\\j\neq i}}^{N}\nu_{jt}\right), (II)\right] = 0, \text{ as} \qquad \text{(A. 8)}$$

$$E\left(\sum_{i=1}^{N}\nu_{it}\sum_{\substack{j=1\\j\neq i}}^{N}\nu_{jt}\sum_{k=1}^{N}\nu_{kt}x_{kt}'\right)$$

$$= E\left(\sum_{i=1}^{N}x_{it}\nu_{it}\sum_{\substack{j=1\\j\neq i}}^{N}\nu_{jt}\right)' + E\left(\sum_{i=1}^{N}\nu_{it}\sum_{\substack{j=1\\j\neq i}}^{N}\nu_{jt}x_{jt}'\right) + E\left(\sum_{i=1}^{N}\nu_{it}\sum_{\substack{j=1\\j\neq i}}^{N}\nu_{jt}\sum_{\substack{k=1,k\neq j\\k\neq i}}^{N}\nu_{kt}x_{kt}'\right)$$

$$= 0.$$

Thus by (2.1), (A.5), (A.7), (A.8) and the continuous mapping theorem, the asymptotic variance of CSC_0 when $T\to\infty$ and N is fixed,

$$V = \frac{N-1}{N} + \frac{2}{N\sigma^2}E\left(\frac{1}{N}\sum_{j=1}^{N}\nu_{j1}\sum_{i=1}^{N}x_{i1}'\right)E\left(x_{11}x_{11}'\right)^{-1}E\left(\frac{1}{N}\sum_{i=1}^{N}x_{i1}\sum_{j=1}^{N}\nu_{j1}\right).$$

Thus Case (2) is proved, given the fact that for each i, $\sup_t E|\nu_{it}|^{2+\delta} = c_i < \infty$ (the Liapounov condition) and a CLT for the triangular-array process. See, for instance, Theorem 4, p.103, Section 21 of Gnedenko and Kolmogorov (1954).

Finally we turn to Case (3). Given the fact that for each t, $\sup_i E|\nu_{it}|^{2+\delta} = c_t < \infty$ (the Liapounov condition) and a CLT for the triangular-array process,

$$\begin{aligned}
\frac{1}{\sigma\sqrt{N}}\sum_{i=1}^{N}\hat{\nu}_{it} &= \frac{1}{\sigma\sqrt{N}}\sum_{i=1}^{N}\nu_{it} - \left(\frac{1}{N}\sum_{i=1}^{N}x_{it}'\right)\left(\frac{1}{NT}\sum_{i=1}^{N}\sum_{s=1}^{T}x_{is}x_{is}'\right)^{-1}\frac{1}{T}\sum_{s=1}^{T}\left(\frac{1}{\sigma\sqrt{N}}\sum_{i=1}^{N}x_{is}\nu_{is}\right)\\
&= \frac{1}{\sigma\sqrt{N}}\sum_{i=1}^{N}\nu_{it} - \frac{1}{T}\sum_{s=1}^{T}E\left(x_{11}'\right)E\left(x_{11}x_{11}'\right)^{-1}\left(\frac{1}{\sigma\sqrt{N}}\sum_{i=1}^{N}x_{is}\nu_{is}\right) + o_p(1)\\
&\longrightarrow_{\mathcal{L}} u_t - \frac{1}{T}\sum_{s=1}^{T}z_s, \qquad \text{(A. 9)}
\end{aligned}$$

where the last two equalities are due to the appropriate $WLLN$s applied to $\{x_{it}\}$ and $\{T^{-1}\sum_{s=1}^{T}x_{is}x_{is}'\}$, where

$$\begin{pmatrix}u_t\\z_t\end{pmatrix}\sim_{iid}\mathcal{N}\left(\begin{pmatrix}0\\0\end{pmatrix},\begin{pmatrix}1 & E\left(x_{11}'\right)\left[E\left(x_{11}x_{11}'\right)\right]^{-1}E\left(x_{11}\right)\\ E\left(x_{11}'\right)\left[E\left(x_{11}x_{11}'\right)\right]^{-1}E\left(x_{11}\right) & E\left(x_{11}'\right)\left[E\left(x_{11}x_{11}'\right)\right]^{-1}E\left(x_{11}\right)\end{pmatrix}\right).$$

Define $a\equiv E\left(x_{11}'\right)\left[E\left(x_{11}x_{11}'\right)\right]^{-1}E\left(x_{11}\right)$. As $(u_t, z_t)'$ is jointly normal, we can write:

$$z_t = au_t + \sqrt{a(1-a)}v_t, \text{ with } (u_t, v_t)' \sim_{i.i.d.} \mathcal{N}\left(0, I_2\right).$$

Thus by (A.1), (A.9) and the continuous mapping theorem,

$$\begin{aligned}
CSC_0 &= \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left[\left(\frac{1}{\sqrt{N}}\sum_{i=1}^{N}\hat{\nu}_{it}\right)^2 - \sigma^2\right]/\sqrt{2}\sigma^2 + o_p(1)\\
&\longrightarrow_{\mathcal{L}} \frac{1}{\sqrt{T}}\sum_{t=1}^{T}\left[\left(u_t - \frac{1}{T}\sum_{s=1}^{T}z_s\right)^2 - 1\right]/\sqrt{2}.
\end{aligned}$$

Case (3) is also proved. □

Proof of Theorem 2.2. As the proofs are similar, we only show Case (1). For $i = 1, \ldots, N$ and $t = 1, \ldots, T$, define the truncation of ν_{it}, $\mu_{NT,it} := \nu_{it} I_{\{|\nu_{it}| \le N^{1/2}T^{1/4}\}}$, and let $\xi_{NT,t} := \sum_{i=1}^{N} \frac{\mu_{NT,it}}{\sqrt{NT}} \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{\mu_{NT,jt}}{\sqrt{N}}$.

We verify the first sufficient condition in Theorem 1.2. As $\mu_{NT,it}$ is *i.i.d.* across i and across t,

$$
\begin{aligned}
\left| E \sum_{t=1}^{T} \sum_{i=1}^{N} \frac{\mu_{NT,it}}{\sqrt{NT}} \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{\mu_{NT,jt}}{\sqrt{N}} \right| &= \left| \sum_{t=1}^{T} \sum_{i=1}^{N} E\left(\frac{\mu_{NT,it}}{\sqrt{NT}}\right) \sum_{\substack{j=1 \\ j\neq i}}^{N} E\left(\frac{\mu_{NT,jt}}{\sqrt{N}}\right) \right| \\
&\le \frac{NT}{N^{1/2}T^{3/4}} \left|E\mu_{NT,11}\right| \frac{N}{N^{1/2}} T^{1/4} \left|E\mu_{NT,11}\right| \\
&\le N^{1/2}T^{1/4} \left|E\mu_{NT,11}\right| N^{1/2}T^{1/4} \left|E\mu_{NT,11}\right|.
\end{aligned}
$$

As $E\nu_{11} = 0$,

$$
\begin{aligned}
N^{1/2}T^{1/4} \left|E\mu_{NT,11}\right| &= N^{1/2}T^{1/4} \left| -E\nu_{11} I_{\{|\nu_{11}| > N^{1/2}T^{1/4}\}} \right| \\
&\le N^{1/2}T^{1/4} E|\nu_{11}| I_{\{|\nu_{11}| > N^{1/2}T^{1/4}\}} \\
&\le E|\nu_{11}|^2 I_{\{|\nu_{11}| > N^{1/2}T^{1/4}\}} \\
&\longrightarrow 0, \text{ as } N, T \to \infty. \qquad \text{(A. 10)}
\end{aligned}
$$

$$
\text{Thus, } \sum_{t=1}^{T} E\xi_{NT,t} \longrightarrow 0, \text{ as } N, T \to \infty. \qquad \text{(A. 11)}
$$

Next we verify the second sufficient condition in Theorem 1.2. As $\xi_{NT,t}$ is identically distributed across t,

$$
\begin{aligned}
\sum_{t=1}^{T} var\xi_{NT,t} &= \sum_{t=1}^{T} E\xi_{NT,t}^2 - \sum_{t=1}^{T} \left(E\xi_{NT,t}\right)^2 \\
&= E\left(\sum_{i=1}^{N} \frac{\mu_{NT,i1}}{\sqrt{N}} \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{\mu_{NT,j1}}{\sqrt{N}} \right)^2 - T\left(E\xi_{NT,1}\right)^2.
\end{aligned}
$$

By (A.11), $E\xi_{NT,1} = o(\frac{1}{T})$ and thus $T\left(E\xi_{NT,1}\right)^2 = o(\frac{1}{T})$. For ease of notation, write $\mu_i = \mu_{NT,i1}$. Similar to (A.5) in the proof of Theorem 2.1, and in virtue of the fact that μ_i is $i.i.d.$ across i,

$$
\begin{aligned}
& E\left(\sum_{i=1}^{N}\frac{\mu_i}{\sqrt{N}}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{\mu_j}{\sqrt{N}}\right)^2 \\
= \; & E\left(\frac{2}{N^2}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\mu_i^2\mu_j^2\right) + E\left(\frac{4}{N^2}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\sum_{\substack{k=1\\ k\neq j,k\neq i}}^{N}\mu_i^2\mu_j\mu_k + \frac{1}{N^2}\sum_{i=1}^{N}\sum_{\substack{j=1\\ j\neq i}}^{N}\sum_{\substack{k=1\\ k\neq j,k\neq i}}^{N}\sum_{\substack{l=1,l\neq k\\ l\neq j,l\neq i}}^{N}\mu_i\mu_j\mu_k\mu_l\right) \\
\leq \; & 2\left(\frac{1}{N}\sum_{i=1}^{N}E\mu_i^2\frac{1}{N}\sum_{\substack{j=1\\ j\neq i}}^{N}E\mu_j^2\right) + 4\left(\frac{1}{N}\sum_{i=1}^{N}E\mu_i^2\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{|E\mu_j|}{\sqrt{N}}\sum_{\substack{k=1\\ k\neq j,k\neq i}}^{N}\frac{|E\mu_k|}{\sqrt{N}}\right) \\
& + \left(\sum_{i=1}^{N}\frac{|E\mu_i|}{\sqrt{N}}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{|E\mu_j|}{\sqrt{N}}\sum_{\substack{k=1\\ k\neq j,k\neq i}}^{N}\frac{|E\mu_k|}{\sqrt{N}}\sum_{\substack{l=1,l\neq k\\ l\neq j,l\neq i}}^{N}\frac{|E\mu_l|}{\sqrt{N}}\right) \\
\leq \; & 2\left(E\mu_1^2\right)^2 + 4\left(E\mu_1^2\right)\left(\sqrt{N}|E\mu_1|\right)^2 + \left(\sqrt{N}|E\mu_1|\right)^4.
\end{aligned}
$$

By (A.10), $\sqrt{N}|E\mu_1| = o(\frac{1}{T^{1/4}})$. On the other hand,

$$
E\mu_1^2 = E\nu_{11}^2 - E\nu_{11}^2 I_{\{|\nu_{11}|>N^{1/2}T^{1/4}\}} \longrightarrow E\nu_{11}^2 = \sigma^2, \text{ as } N,T\to\infty.
$$

All in all,

$$
\sum_{t=1}^{T} var\xi_{NT,t} \longrightarrow 2\sigma^4 < \infty, \text{ as } N,T\to\infty. \tag{A. 12}
$$

We then verify the third sufficient condition in Theorem 1.2. As $\xi_{NT,t}$ is identically distributed across t, it remains to show that:

$$
TE\xi_{NT,1}^3 = \frac{1}{N^3T^{1/2}}E\left(\sum_{i=1}^{N}\mu_{NT,i1}\sum_{\substack{j=1\\ j\neq i}}^{N}\mu_{NT,j1}\right)^3 = o(1). \tag{A. 13}
$$

For ease of notation, we write $\mu_i = \mu_{NT,i1}$ and $S_i = \sum_{\substack{j=1\\ j\neq i}}^{N}\mu_j$. Some algebra yields:

$$
\begin{aligned}
& \frac{1}{N^3T^{1/2}}E\left(\sum_{i=1}^{N}\mu_{NT,i1}\sum_{\substack{j=1\\ j\neq i}}^{N}\mu_{NT,j1}\right)^3 \\
= \; & \frac{1}{N^3T^{1/2}}E\left(\sum_{i=1}^{N}\mu_i S_i\right)^3
\end{aligned}
$$

$$= \frac{1}{N^3T^{1/2}}E\left(\sum_{i=1}^N \mu_i^3 S_i^3\right) + \frac{2}{N^3T^{1/2}}E\left(\sum_{i=1}^N \mu_i^2 S_i^2\left(\sum_{\substack{j=1\\j\neq i}}^N \mu_j S_j\right)\right)$$

$$+\frac{1}{N^3T^{1/2}}E\left(\sum_{i=1}^N \mu_i S_i\left(\sum_{\substack{j=1\\j\neq i}}^N \mu_j^2 S_j^2\right)\right) + \frac{1}{N^3T^{1/2}}E\left(\sum_{i=1}^N \mu_i S_i\left(\sum_{\substack{j=1\\j\neq i}}^N \mu_j S_j\left(\sum_{\substack{k=1\\k\neq j,k\neq i}}^N \mu_k S_k\right)\right)\right).$$

Thus, it suffices to show all the above four terms are $o(1)$. As the other terms can be shown in a similar way, we only show the first term. Some algebra yields:

$$\frac{1}{N^3T^{1/2}}E\left(\sum_{i=1}^N \mu_i^3 S_i^3\right)$$

$$= \frac{1}{N^3T^{1/2}}E\left(\sum_{i=1}^N \mu_i^3 \sum_{\substack{j=1\\j\neq i}}^N \mu_j^3\right) + \frac{2}{N^3T^{1/2}}E\left(\sum_{i=1}^N \mu_i^3 \sum_{\substack{j=1\\j\neq i}}^N \mu_j^2 \sum_{\substack{k=1\\k\neq j,k\neq i}}^N \mu_k\right)$$

$$+\frac{1}{N^3T^{1/2}}E\left(\sum_{i=1}^N \mu_i^3 \sum_{\substack{j=1\\j\neq i}}^N \mu_j \sum_{\substack{k=1\\k\neq j,k\neq i}}^N \mu_k^2\right) + \frac{1}{N^3T^{1/2}}E\left(\sum_{i=1}^N \mu_i^3 \sum_{\substack{j=1\\j\neq i}}^N \mu_j \sum_{\substack{k=1\\k\neq j,k\neq i}}^N \mu_k \sum_{\substack{l=1,l\neq k\\l\neq j,l\neq i}}^N \mu_l\right).$$

Thus, it suffices to show all the above four terms are $o(1)$. As the other terms can be shown in a similar way, we only show the second term. As μ_i is i.i.d. across i, in view of (A.10),

$$\frac{2}{N^3T^{1/2}}E\left(\sum_{i=1}^N \mu_i^3 \sum_{\substack{j=1\\j\neq i}}^N \mu_j^2 \sum_{\substack{k=1\\k\neq j,k\neq i}}^N \mu_k\right)$$

$$\leq \frac{2}{T^{1/2}}\left(\frac{1}{N^{3/2}T^{1/4}}\sum_{i=1}^N E\left|\mu_i\right|^3\right)\left(\frac{1}{N}\sum_{\substack{j=1\\j\neq i}}^N E\left|\mu_i\right|^2\right)\left(\frac{T^{1/4}}{N^{1/2}}\sum_{\substack{k=1\\k\neq j,k\neq i}}^N E\left|\mu_i\right|\right)$$

$$\leq \frac{2}{T^{1/2}}E\left(\left|\nu_{11}\right|^2 I_{\left\{1\wedge\frac{|\nu_{11}|}{N^{1/2}T^{1/4}}\right\}}\right)E\left(\left|\nu_{11}\right|^2 I_{\{|\nu_{11}|\leq N^{1/2}T^{1/4}\}}\right)N^{1/2}T^{1/4}E|\mu_{NT,11}|$$

$$\longrightarrow 0.$$

All in all, (A.13) is proved and the third sufficient condition in Theorem 1.2 is also verified.

Lastly it remains to show:

$$P\left(\sum_{t=1}^T\sum_{i=1}^N \frac{\nu_{it}}{\sqrt{NT}}\sum_{\substack{j=1\\j\neq i}}^N \frac{\nu_{jt}}{\sqrt{N}} \neq \sum_{t=1}^T \xi_{NT,t}\right) \longrightarrow 0. \qquad \text{(A. 14)}$$

In virtue of Lemma 1, pp.206-207 in Chung (1974), it suffices to show that for a fixed $\eta > 0$,

$$P\left(\cup_{t=1}^{T}\cup_{i=1}^{N}\left\{\left|\frac{\nu_{it}}{\sqrt{NT}}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{\nu_{jt}I_{\{|\nu_{jt}|>N^{1/2}T^{1/4}\}}}{\sqrt{N}}\right|>\eta\right\}\right)\longrightarrow 0,\ \text{and} \tag{A. 15}$$

$$P\left(\cup_{t=1}^{T}\cup_{i=1}^{N}\left\{\left|\frac{\nu_{it}I_{\{|\nu_{it}|>N^{1/2}T^{1/4}\}}}{\sqrt{NT}}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{\nu_{jt}I_{\{|\nu_{jt}|\leq N^{1/2}T^{1/4}\}}}{\sqrt{N}}\right|>\eta\right\}\right)\longrightarrow 0. \tag{A. 16}$$

As the proofs are similar, we only show (A.15).

$$\begin{aligned}
& P\left(\cup_{t=1}^{T}\cup_{i=1}^{N}\left\{\left|\frac{\nu_{it}}{\sqrt{NT}}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{\nu_{jt}I_{\{|\nu_{jt}|>N^{1/2}T^{1/4}\}}}{\sqrt{N}}\right|>\eta\right\}\right)\\
\leq\ & \sum_{t=1}^{T}\sum_{i=1}^{N}P\left(\left\{\left|\frac{\nu_{it}}{\sqrt{NT}}\sum_{\substack{j=1\\ j\neq i}}^{N}\frac{\nu_{jt}I_{\{|\nu_{jt}|>N^{1/2}T^{1/4}\}}}{\sqrt{N}}\right|>\eta\right\}\right)\\
\leq\ & \frac{1}{\eta N^2 T}\sum_{t=1}^{T}\sum_{i=1}^{N}E\nu_{it}^2\sum_{\substack{j=1\\ j\neq i}}^{N}E\nu_{jt}^2 I_{\{|\nu_{jt}|>N^{1/2}T^{1/4}\}}\\
\leq\ & \frac{1}{\eta}E\nu_{11}^2\left(E\nu_{11}^2 I_{\{|\nu_{11}|>N^{1/2}T^{1/4}\}}\right)\longrightarrow 0.
\end{aligned}$$

Thus (A.14) is also shown and the proof is complete. □

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