Testing for Serial Correlation in Fixed-Effects Panel Data Models

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In this paper, we propose three new tests for serial correlation in the disturbances of fixed-effects panel data models. First, a modified Bhargava, Franzini and Narendranathan (1982) panel Durbin-Watson statistic that does not need to be tabulated as it follows a standard normal distribution. Second, a modified Baltagi and Li (1991) LM statistic with limit distribution independent of T, and, third, a test using an unbiased estimator for the autocorrelation coefficient to achieve robustness against temporal heteroskedasticity. The first two tests are robust against cross-sectional but not time dependent heteroskedasticity and the third statistic is robust against both forms of heteroskedasticity. Furthermore, all test statistics can be easily adapted to unbalanced data. Monte Carlo simulations suggest that our new tests have good size and power properties compared to the often used Wooldridge (2002)-Drukker (2003) test.

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1 Introduction

Panel data models are increasingly being used in applied work as they have many advantages over cross-sectional approaches (see e.g. Hsiao 2003). The classical error component panel data model assumes serially uncorrelated disturbances, which might be too restrictive. For example, Baltagi (2008) argues that unobserved shocks to economic relationships like investment or consumption will often have an effect for more than one period. Therefore, it

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is important to test for serial correlation in the disturbances as ignoring this issue would lead to inefficient estimates and biased standard errors.

A number of tests for the presence of serial error correlation in a fixed effects panel data model have been proposed in the literature. Bhargava et al. (1982) generalize the Durbin-Watson statistic (Durbin and Watson 1950, 1971) to the fixed effects panel model. Baltagi and Li (1991, 1995) derive an LM statistic that tests for first order serial correlation. Drukker (2003), using an idea originally proposed by Wooldridge (2002), proposes an easily implementable test for serial correlation based on the OLS residuals of the first-differenced model.

But these tests all have their deficiencies. A serious problem of the Bhargava et al. (1982) statistic is that the distribution depends on N and T and, therefore, the critical values have to be provided in large tables depending on both dimensions. Baltagi and Li (1995) note that, for fixed T, their test statistic does not possess the usual χ^2 limiting distribution due to the (Nickell) bias in the estimation of the autocorrelation coefficient. The Wooldridge-Drukker test is not derived from the usual test principles (like LM, LR or Wald) and, therefore, it is not clear whether the test has desirable properties. Furthermore, all test suggested in the literature are not robust against heteroskedasticity.

In this paper, we propose new test statistics and modifications of existing test statistics that correct some of the deficiencies. In Section 2 we first present the model framework and briefly review the existing tests. Our new test procedures are suggested in Section 3 and the relative small sample properties of the tests are studied in Section 4. Section 5 concludes.

2 Existing tests

Consider the usual fixed effects panel data model with serially correlated disturbances

$$y_{it} = x_{it}'\beta + \mu_i + u_{it} \tag{1}$$

$$u_{it} = \rho u_{i,t-1} + \varepsilon_{it} \ , \tag{2}$$

where $i = 1, \ldots, N$ denotes the cross-section dimension and $t = 1, \ldots, T$ is the time dimension. The $K \times 1$ vector of explanatory variables is assumed to be strictly exogenous, i.e. $E(x_{it}u_{it}) = 0$, β is the associated $K \times 1$ parameter vector, and μ_i is a fixed individual specific effect. In our benchmark situation we assume that the innovations are i.i.d. with $E(\varepsilon_{it}) = 0$, $E(\varepsilon_{it}^2) = \sigma_\varepsilon^2$. However, we are also interested in constructing test statistics that are robust against heteroskedasticity across i and t.

To test the null hypothesis $\rho = 0$, Bhargava et al. (1982) propose a pooled Durbin-Watson statistic given by

$$p\text{DW} = \frac{\sum_{i=1}^{N}\sum_{t=2}^{T}(\widehat{u}_{it} - \widehat{u}_{i,t-1})^2}{(\sum_{i=1}^{N}\sum_{t=1}^{T}\widehat{u}_{it} - \overline{u}_i)^2} ,$$

where $\overline{u}_i = T^{-1}\sum_{t=1}^{T} u_{it}$. A serious problem of this test is that its null distribution depends on N and T and, therefore, the critical values are provided in large tables depending on both dimensions in Bhargava et al. (1982).

Baltagi and Li (1991) derive the LM test statistic for the hypothesis $\rho = 0$ assuming normally distributed errors. The resulting test statistic is equivalent to (the LM version of) the t-statistic of ϱ in the regression

$$\widehat{u}_{it} - \overline{\overline{u}}_i = \varrho(\widehat{u}_{i,t-1} - \overline{\overline{u}}_{i,-1}) + \nu_{it} , \tag{3}$$

where $\overline{\overline{u}}_i = (T-1)^{-1}\sum_{t=2}^{T}\widehat{u}_{it}$ and $\overline{\overline{u}}_{i,-1} = (T-1)^{-1}\sum_{t=2}^{T}\widehat{u}_{i,t-1}$.

It is convenient to introduce the $T \times 1$ vector $\widehat{u}_i = [\widehat{u}_{i1}, \ldots, \widehat{u}_{iT}]'$ and the matrices

$$M_0 = [0, M_{T-1}] \quad \text{and} \quad M_1 = [M_{T-1}, 0] ,$$

where $M_{T-1} = I_{T-1} - (T-1)^{-1}\iota_{T-1}\iota_{T-1}'$, and ι_{T-1} is a $(T-1) \times 1$ vector of ones. The LM test statistic can be written as

$$\mathrm{LM}_{NT} = \frac{\left(\sum\limits_{i=1}^{N} \widehat{u}_i' M_0' M_1 \widehat{u}_i\right)^2}{\left(\frac{1}{T}\sum\limits_{i=1}^{N} \widehat{u}_i' M_0' M_0 \widehat{u}_i\right)\left(\sum\limits_{i=1}^{N} \widehat{u}_i' M_1' M_1 \widehat{u}_i\right)} ,$$

where $T^{-1}\sum_{i=1}^{N} \widehat{u}_i' M_0' M_0 \widehat{u}_i$ is the estimator for σ_ε^2 under the null hypothesis. Baltagi and Li (1995) show that if $N \to \infty$ and $T \to \infty$, the LM statistic is χ^2 distributed with one degree of freedom. However, if T is fixed and $N \to \infty$, the test statistic does not possess a χ^2 limiting distribution due to the (Nickell) bias of the least-squares estimator for ϱ.

To obtain a valid test statistic for fixed T, Wooldridge (2002) suggests to run a least squares regression of the first differences $\Delta\widehat{u}_{it} = \widehat{u}_{it} - \widehat{u}_{i,t-1}$ on the lagged differences $\Delta\widehat{u}_{i,t-1}$. Under the null hypothesis $\rho = 0$ the first order autocorrelation of the first differences converges to -0.5. Since Δu_{it} is serially autocorrelated, Drukker (2003) computes the test statistic based on heteroskedasticity and autocorrelation consistent (HAC) standard errors[1] yielding the test statistic

$$\mathrm{WD}_{NT} \;=\; \frac{(\widehat{\theta} + 0.5)^2}{\widehat{s}_\theta^2} ,$$

where $\widehat{\theta}$ denotes the least-squares estimator of θ in the regression

$$\Delta\widehat{u}_{it} = \theta\Delta\widehat{u}_{i,t-1} + e_{it} \;. \tag{4}$$

$\widehat{s}_\theta^2$ is the HAC estimator of the standard errors given by

$$\widehat{s}_{\widehat{\rho}}^2 = \frac{\sum\limits_{i=1}^{N} (\Delta\widehat{u}_{i,-1}' \widehat{e}_i)^2}{\sum\limits_{i=1}^{N}\sum\limits_{t=2}^{T} (\Delta\widehat{u}_{i,t-1})^2} ,$$

where $\Delta\widehat{u}_{i,-1} = [\Delta\widehat{u}_{i1}, \ldots, \Delta\widehat{u}_{i,T-1}]'$, $\widehat{e}_i = [\widehat{e}_{i2}, \ldots, \widehat{e}_{iT}]'$, and $\widehat{e}_{it}$ is the pooled OLS residual from the autoregression (4).

Note that, due to employing robust standard errors, this test is robust against cross-

[1]This approach is also known as “robust cluster” or “panel corrected” standard errors.

sectional heteroskedasticity. However, using $\theta = -0.5$ requires that the variance of u_{it} is identical for all time periods. Thus this test rules out time dependent heteroskedasticity.

3 New test procedures

In this section, we modify the existing approaches to obtain test procedures that are valid for fixed T and $N \to \infty$. The first two tests are robust against cross-sectional but *not* time dependent heteroskedasticity and the third statistic is robust against cross-sectional and temporal heteroskedasticity. Furthermore, all test statistics can be easily adapted to unbalanced data although for the ease of exposition we focus on balanced panels.

To simplify the discussion, we ignore the estimation error $\widehat{\beta} - \beta$ as this error does not play any role in our asymptotic analysis. Indeed, for fixed T and strictly exogenous regressors we have $\beta - \widehat{\beta} = O_p(N^{-1/2})$, $\sum_{i=1}^{N} x_{it} u_{i,t+k} = O_p(N^{1/2})$ (for all$k = 0, \pm 1, \pm 2, \ldots$), and

$$\begin{aligned}\sum_{i=1}^{N} \widehat{u}_{it}\widehat{u}_{i,t-k} =& \sum_{i=1}^{N} u_{it}u_{i,t-k} + \sum_{i=1}^{N} u_{it}x_{i,t-k}(\beta - \widehat{\beta}) \\ &+ \sum_{i=1}^{N} u_{i,t-k}x_{it}(\beta - \widehat{\beta}) + \sum_{i=1}^{N} x_{it}x_{i,t-k}(\beta - \widehat{\beta})^2 \\ =& \sum_{t=2}^{T} u_{it}u_{i,t-1} + O_p(1) \ ,\end{aligned}$$

Therefore, the estimation error of $\widehat{\beta}$ does not affect the asymptotic properties of the test. In practice, the error vector u_i can be replaced by the residual vector $\widehat{u}_i = y_{it} - x_{it}'\widehat{\beta}$. Note that the individual effect μ_i is included in $\widehat{u}_{it}$. Since all test statistics remove the individual effects by some linear transformation (first differences or by subtracting the group-mean), the test statistics are invariant to the individual effects if u_i is replaced by $\widehat{u}_i$.

3.1 A modified Durbin-Watson statistic for fixed T

The pDW statistic suggested by Bhargava et al. (1982) is the ratio of the sum of squared differences and the sum of squared residuals. Instead of the ratio (which complicates the theoretical analysis) our variant of the Durbin-Watson test is based on the linear combination

$$\delta_{Ti} = u_i' M D' D M u_i - 2\, u_i' M u_i \ ,$$

where $M = I_T - \iota_T \iota_T'/T$, ι_T is a $T \times 1$ vector of ones, and D is a $(T-1)\times T$ matrix producing first differences, i.e.

$$D = \begin{pmatrix} -1 & 1 & 0 & \cdots & & 0 \\ 0 & -1 & 1 & \cdots & & 0 \\ & & & \ddots & & \\ 0 & & & & -1 & 1 \end{pmatrix} .$$

Using $tr(MD'DM) = 2(T-1)$ and $tr(M) = T-1$, it is easy to verify that $E(\delta_{Ni}) = 0$ for all i. Furthermore

$$\delta_{Ti} = -2\left[\sum_{t=2}^{T}(u_{it} - \overline{u}_i)(u_{i,t-1} - \overline{u}_i)\right] - \left[(u_{i1} - \overline{u}_i)^2 + (u_{iT} - \overline{u}_i)^2\right]$$

and, therefore, it is obvious that the test is related to the LM test proposed by Baltagi and Li (1991). The main difference is the latter correction term to adjust bias of the first order autocovariance.

The panel test statistic is based on the mean of this statistic:

$$\xi_{NT} = \frac{1}{\widehat{s}_\delta \sqrt{N}} \sum_{i=1}^{N} \delta_{Ti} \ ,$$

where

$$\widehat{s}_\delta^2 = \frac{1}{N}\sum_{i=1}^{N} \delta_{Ti}^2 - \left(\frac{1}{N}\sum_{i=1}^{N} \delta_{Ti}\right)^2 .$$

If it is assumed that the cross section units are independent and the fourth moments of u_{it} exist, the central limit theorem for independent random variables implies that ξ_{NT} has a standard normal limiting distribution. Furthermore, it is important to note that the null distribution is robust against heteroskedasticity across the cross-section units (but not against heteroskedasticity across the time dimension).

3.2 The LM test

An important problem with the LM test of Baltagi and Li (1991) is that the limit distribution for $N \to \infty$ depends on T. This is due to the fact that the least-squares estimator of ϱ in the regression (3) is biased and the errors ν_{it} are autocorrelated. Specifically we obtain

$$\widehat{\varrho} \ \xrightarrow{p} \ \varrho_0 \ = \ \frac{tr(M_0' M_1)}{tr(M_1' M_1)} \ = \ \frac{-(T-2)/(T-1)}{T-2} \ = \ -\frac{1}{T-1}$$

as $N \to \infty$ (see Nickell 1981). To account for this bias, a regression t-statistic is formed for the modified null hypothesis $H_0' : \ \varrho = \varrho_0$. Under this null hypothesis, the vector of residuals is obtained as:

$$\widetilde{e}_i = (M_0 - \varrho_0 M_1) u_i \ .$$

Since

$$E(\widetilde{e}_i \widetilde{e}_i') = \sigma^2 (M_0 - \varrho_0 M_1)(M_0 - \varrho_0 M_1)' \ ,$$

it is seen that the errors in the regression (3) are autocorrelated, although the autocorrelation disappears as $T \to \infty$. To account for this autocorrelation, (HAC) robust standard errors (see e.g. Arellano 1987) are employed, yielding the test statistic

$$\widetilde{\mathrm{LM}}_{NT} = \frac{1}{\widetilde{v}^2} (\widehat{\varrho} - \varrho_0)^2 \ ,$$

where

$$\widetilde{v}^2 = \frac{\sum_{i=1}^{N} \widehat{u}_i' M_1' \widetilde{e}_i \widetilde{e}_i' M_1 \widehat{u}_i}{\left(\sum_{i=1}^{N} \widehat{u}_i' M_1' M_1 \widehat{u}_i \right)^2} \ .$$

This test statistic is asymptotically χ^2 distributed for all T, and $N \to \infty$.

3.3 A heteroskedasticity robust test statistic

An important drawback of all test statistics considered so far is that they are not robust against time dependent heteroskedasticity. This is due to the fact that the implicit or explicit bias correction of the autocovariances depends on the error variances. To overcome this drawback of the previous test statistics, we construct an unbiased estimator of the autocorrelation coefficient. The idea is to apply backward and forward transformations such that the products of the transformed series are uncorrelated under the null hypothesis. Specifically we employ the following transformations for eliminating the individual effects:

$$\begin{aligned}
\widetilde{\eta}^0_{it} &= u_{it} - \frac{1}{T-t+1}\left(u_{it} + \cdots + u_{iT}\right) \\
\widetilde{\eta}^1_{it} &= u_{it} - \frac{1}{t}\left(u_{i1} + \ldots + u_{it}\right) \ .
\end{aligned}$$

The test is then based on the regression

$$\widetilde{\eta}_{it} = \theta\widetilde{\eta}_{i,t-1} + \eta_{it} \quad t = 3, \ldots, T-1 \ ,$$

or, in matrix notation,

$$V_0 u_i = \theta V_1 u_i + \eta_i \ ,$$

where the $(T-3) \times T$ matrices V_0 and V_1 are defined as

$$\begin{bmatrix}
-\frac{1}{2} & \frac{1}{2} & 0 & 0 & \ldots & 0 & 0 & 0 \\
-\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} & 0 & \ldots & 0 & 0 & 0 \\
\vdots & & & & & & & \vdots \\
-\frac{1}{T-2} & -\frac{1}{T-2} & -\frac{1}{T-2} & -\frac{1}{T-2} & \ldots & \frac{T-3}{T-2} & 0 & 0
\end{bmatrix}$$

and

$$\begin{bmatrix}
0 & 0 & \frac{T-3}{T-2} & -\frac{1}{T-2} & -\frac{1}{T-2} & \ldots & -\frac{1}{T-2} & -\frac{1}{T-2} \\
0 & 0 & 0 & \frac{T-4}{T-3} & -\frac{1}{T-3} & \ldots & -\frac{1}{T-3} & -\frac{1}{T-3} \\
\vdots & & & & & & & \vdots \\
0 & 0 & 0 & 0 & 0 & \ldots & \frac{1}{2} & -\frac{1}{2}
\end{bmatrix} .$$

The error term of the test regression exhibits heteroskedasticity in the time dimension since the variance of the mean depends on the number of observations. To account for the heteroskedasticity, we again use robust (HAC) standard errors, yielding the test statistic

$$t_{\widehat{\theta}} = \frac{1}{\widehat{s}_\theta}\widehat{\theta},$$

where

$$\widehat{s}_\theta^2 = \frac{\sum_{i=1}^{N} \widehat{u}_i' V_1' \widehat{\eta}_i \widehat{\eta}_i' \widehat{V}_1 \widehat{u}_i}{\left(\sum_{i=1}^{N} \widehat{u}_i' V_1' V_1 \widehat{u}_i\right)^2} \ .$$

Under the null hypothesis the $t_{\widehat{\theta}}$ statistic has a standard normal limiting distribution.

4 Monte Carlo Simulation

The data generating process for the Monte Carlo simulation is a linear panel data model of the form

$$y_{it} = x_{it}\beta + \mu_i + \nu_{it},$$

where $i = 1, \ldots, N$ and $t = 1, \ldots, T$. We set β to 1 in all simulations and draw the individual effects μ_i from a $\mathcal{N}(0, 2.5^2)$ distribution. To create correlation between the regressor and the individual effect, we follow Drukker (2003) by drawing x_{it} from a $\mathcal{N}(0, 1.8^2)$ distribution and then redefining $x_{it} = x_{it} + 0.5\mu_i$. The regressor is drawn once and then held constant for all experiments. The disturbances term follows an autoregressive process of order 1,

$$\nu_{it} = \rho\nu_{i,t-1} + \varepsilon_{it} \ ,$$

where $\varepsilon_{it} \sim \mathcal{N}(0, 1)$ and we discard the first 100 observations to eliminate the influence of the initial value.

Table (1) reports the simulation results under the null hypothesis of no serial correlation for $N \in \{25, 50\}$ and $T \in \{10, 20, 30, 50\}$. All tests are close to the correct size of 0.05 although especially the Wooldridge-Drukker and the heteroskedasticity robust test tend to be a bit oversized in these small samples.

Figures (1) and (2) present the power curves of the discussed tests. The modified DW and

Table 1: Empirical Size

N	T	Woold.-Drukker	mod. DW	LM	heterosk. robust
25	10	0.083	0.064	0.051	0.074
	20	0.061	0.054	0.04	0.067
	30	0.073	0.066	0.042	0.062
	50	0.079	0.066	0.052	0.067
50	10	0.064	0.062	0.049	0.063
	20	0.070	0.067	0.052	0.072
	30	0.065	0.065	0.053	0.06
	50	0.055	0.063	0.049	0.062

LM statistics show superior power compared to the Wooldridge-Drukker test for all sample sizes. The heteroskedasticity robust test has lower power than the other proposed tests but catches up with increasing N and T.

5 Conclusion

In this paper, we proposed three new tests for serial correlation in the disturbances of fixed-effects panel data models. First, a modified Bhargava et al. (1982) panel Durbin-Watson statistic that does not need to be tabulated as it follows a standard normal distribution. Second, a modified Baltagi and Li (1991) LM statistic with limit distribution independent of T, and, third, a test using an unbiased estimator for the autocorrelation coefficient to achieve robustness against temporal heteroskedasticity. The first two tests are robust against cross-sectional but not time dependent heteroskedasticity and the third statistic is robust against both forms of heteroskedasticity. Furthermore, all test statistics can be easily adapted to unbalanced data. Monte Carlo simulations suggest that our new tests have good size and power properties compared to the often used Wooldridge (2002)-Drukker (2003) test.

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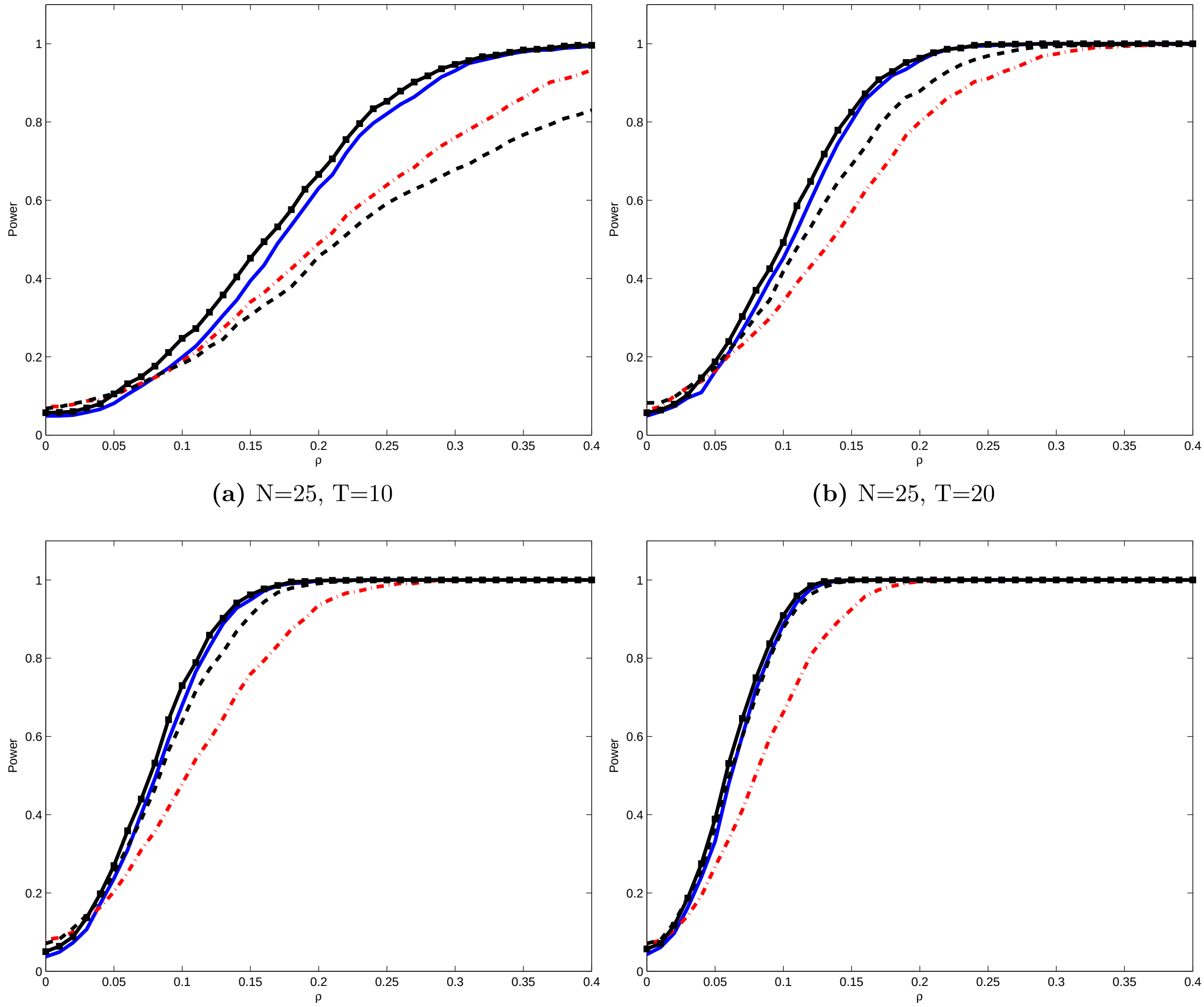


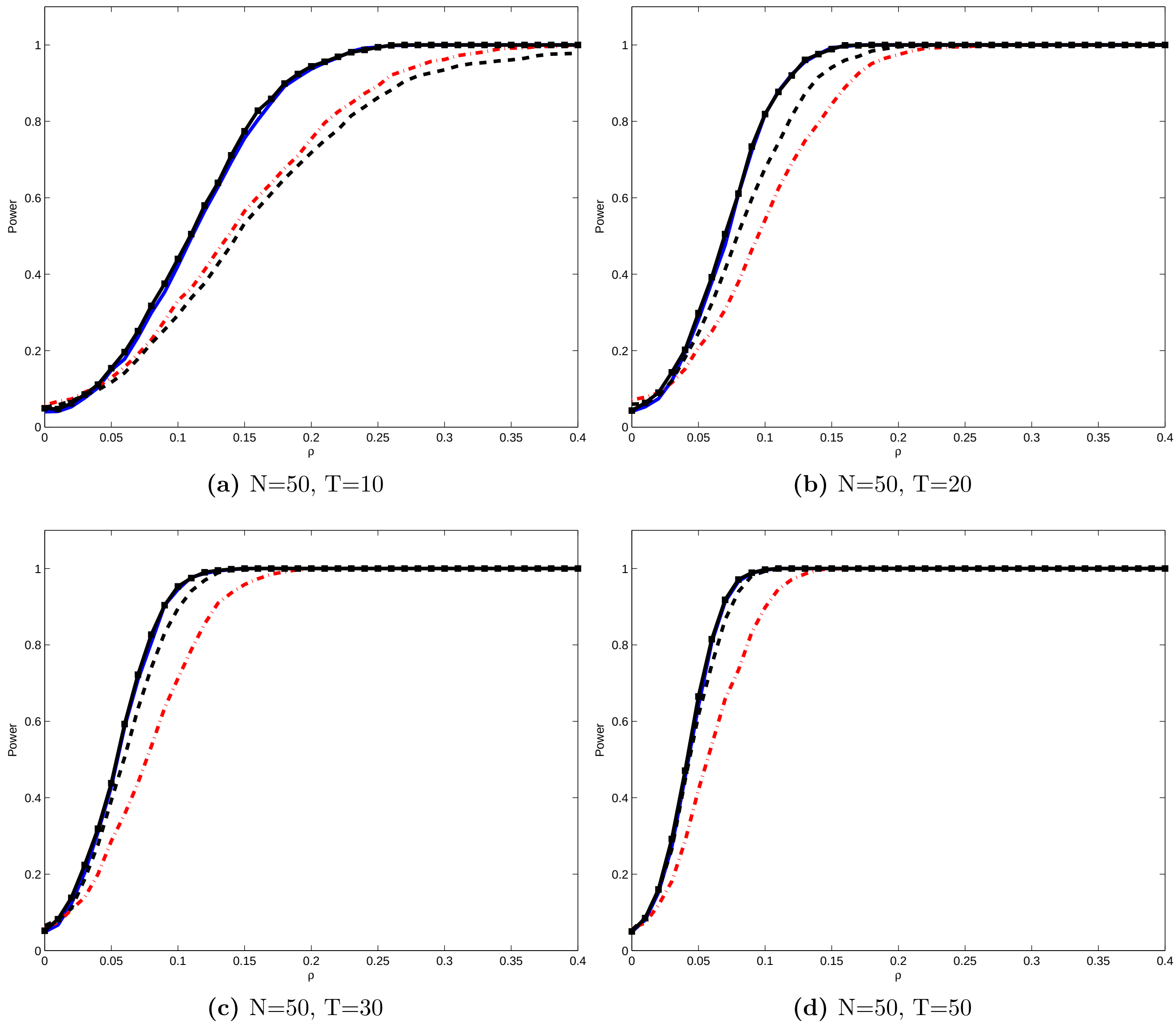
(a) N=25, T=10

(b) N=25, T=20

(c) N=25, T=30

(d) N=25, T=50

Figure 1: Empirical Power for N=25.
Note: blue solid line: mod. LM test; red dashed-dotted line: Wooldridge-Drukker test; black dashed line: robust test; black line with squares: modified Durbin-Watson test.



(a) N=50, T=10

(b) N=50, T=20

(c) N=50, T=30

(d) N=50, T=50

Figure 2: Empirical Power for N=50.
Note: blue solid line: mod. LM test; red dashed-dotted line: Wooldridge-Drukker test; black dashed line: robust test; black line with squares: modified Durbin-Watson test.