Testing for breaking cointegration in dependent panels via residual-based bootstrap methods

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Motivating example: the Feldstein-Horioka puzzle

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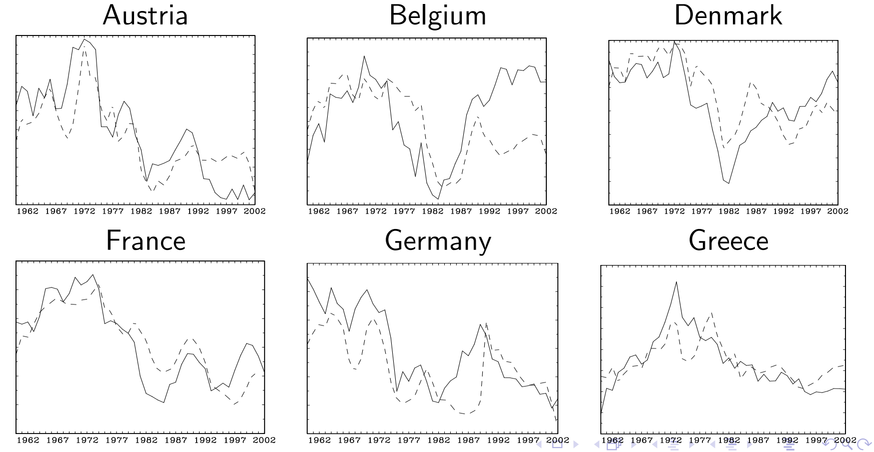
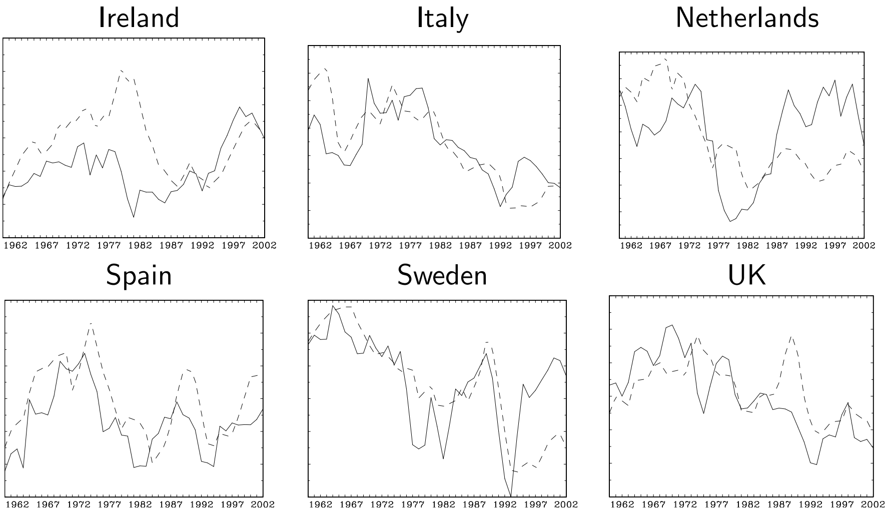
- But the graphical evidence often tells a different story:
- S/GDP (solid line) and I/GDP (dotted line), 1960-2002



The Feldstein-Horioka puzzle



S/GDP (solid line) and I/GDP (dotted line), 1960-2002.

Testing the Feldstein-Horioka puzzle

- *Key point*: capital movements strongly regulated up to early '80's - test must allow for possible breaks
- *Standard cointegration test with breaks*: Gregory-Hansen (1996)

1. compute a no-cointegration statistic $\theta(t_i^b)$ for all possible break points t_i^b
2. take the minimum of all statistics (=the least favourable to H_0, as rejection region is left tail)

$$\Theta = \underset{t_i^b \in [\delta T, (1-\delta) T]}{Min} (\theta(t_i^b))$$

- *Remark #1*: break point implicitly estimated as

$$\widehat{t}_i^b = \underset{t_i^b \in [\delta T, (1-\delta) T]}{Arg \min} (\theta(t_i^b)).$$

- *Remark #2*: simulation evidence show test may have very low power.

Testing the Feldstein-Horioka puzzle

- *Results of Gregory-Hansen tests*: Contrary to graphical evidence, tests on individual countries mostly fail to reject H_0 : no cointegration

Investment and Savings, 1960-2002
Min(ADF) Cointegration Tests with Unknown Break

Austria	Belgium	Denmark	France	Germany	Greece
-4.28	-4.44	$-\mathbf{5.96}^{***}$	$-\mathbf{4.95}^{***}$	-4.22	-4.45
Ireland	Italy	Netherlands	Spain	Sweden	UK
-3.36	-4.25	$-\mathbf{5.50}^{***}$	$-\mathbf{6.01}^{***}$	-3.97	$-\mathbf{4.92}^{*}$

- **Are results reliable,** or **only the consequence of low power?**
- **A panel cointegration test (with breaks?)** may help

RSB Panel Cointegration test, Di Iorio-Fachin (2008):

Mean ($p^* \times 100$)	*Median* ($p^* \times 100$)
-2.55 (**15.3**)	-2.51 (**20.2**)

panel: Austria, Belgium, Denmark, France, Germany, Greece, Ireland, Italy, Netherlands, Spain, Sweden, UK; *Mean/Median*: mean/median of the individual statistics; p^*: bootstrap p-values $\times 100$, 1000 redrawings.

No puzzle - but perhaps because we ignored breaks.
Further work needed.

Cointegration with breaks - Set-up

- A variable X, known to be I(1), is believed to be linked to a "dependent variable" Y, also I(1), by a linear *breaking* relationship. This may, but does not need to be, a long-run equilibrium relationship.
- Data: N units, T time observations
- We are then interested in investigating the properties of

$$y_{it} = \begin{cases} \mu_{0i} + \beta_{0i} x_{it} + \epsilon_{it}, & t \leq t_i^b \\ \mu_{1i} + \beta_{1i} x_{it} + \epsilon_{it}, & t > t_i^b \end{cases}$$

- **Key points**:

1. Locating the break
2. handling possible short- and long-run dependence across units

Locating the break

- Gregory-Hansen: *Argmin(no cointegration statistic).* Known *not* to be consistent.
- Alternative option (Westerlund-Edgerton, 2008, building upon Bai-Perron, 1998):

1. estimate break point as

$$\hat{t}_i^b = \underset{t_i^b \in [\delta T, (1-\delta)T]}{Arg \min} \left(\sum_{t=1}^{T} \epsilon_{it}^2 (t_i^b) \right).$$

2. compute no cointegration statistic Θ on breaking cointegrating model with $\hat{t}_i^b$

- *Remark* $\Theta =$ statistic associated with the model with the best fit - not necessarily the smallest, as in Gregory-Hansen.

How to test for breaking cointegration in dependent panels?

- In panels with possible common factors (CF) only one option currently available:
- modelling and removing the dependence through Principal Componentes (PC) methods (Bai-Ng, 2004) applied on the cointegrating residuals (Banerjee-Carrion, 2006, Westerlund-Edgerton, 2008).
- Shortcomings of PC tests (1-2 same as in no break set-up)

1. If interest centres on panel cointegration rather than CF structure itself the analysis can became unecessarily complex (e.g. H_0: panel cointegration can became a multiple comparison)
2. Only special forms of dependence allowed
3. Power in small samples rather poor (e.g., around 50% for T=100, N=40)

This paper

- Extend a **Residual based Stationary Bootstrap (RSB) panel cointegration test** put forth in separate work [in turn based upon the unit root tests by Parker, Paparoditis and Politis, *J.of Econometrics*, 2006]
- Evaluation: by simulation

A Bootstrap breaking no cointegration test

A - compute the test on the data

1. Estimate breaking cointegrating model on the dataset $\{y_{it}, x_{it}\}$, $i = 1, \ldots, N$, obtaining for each unit estimates
   - of the coefficients $(\widehat{\mu}_{ji}, \widehat{\beta}_{ji}, i, j = 0, 1)$
   - of the cointegrating residuals $\left\{\widehat{\epsilon}(\widehat{t}_i^b)_{it}\right\}$;
2. Compute the N individual no cointegration statistics $\widehat{\theta}_i$ on the cointegrating residuals $\left\{\widehat{\epsilon}(\widehat{t}_i^b)_{it}\right\}$;
3. Compute the summary statistics of interest, e.g., $\widehat{\theta}_{mean} = N^{-1} \sum_{i=1}^{N} \widehat{\theta}_i$, $\widehat{\theta}_{median} = median(\widehat{\theta}_1 \ldots \widehat{\theta}_N)$;

A Bootstrap breaking no cointegration test

B - construct the bootstrap pseudodata under H_0 : no cointegration

4. Compute $\widehat{\nu}_{it} = \widehat{\epsilon}(\widehat{t}_i^b)_{it} - \widehat{\rho}_i \widehat{\epsilon}(\widehat{t}_i^b)_{it-1}$, where $\widehat{\rho}_i$ is a consistent estimate (e.g., OLS) estimate of ρ_i;
5. Resample the series $\{\widehat{\nu}_{it}\}$ via the stationary bootstrap:

- generate $L_1, \ldots, L_T$ i.i.d. from a geometric distribution with parameter $\xi = 1/(1+B)$, B=mean block size;
- for each $t \in [1, T-1]$ let $K_t = \inf\{k : L_1 + \ldots + L_T \geq t\}$ and $M_t = L_1 + \ldots + L_{K_t}$;
- generate $i_1, \ldots, i_K$ i.i.d. from a uniform distribution on $\{2, \ldots, T\}$;
- for all $t \in [1, K]$ set $\nu_t^* = \widehat{\nu}_{\left[(i_{K_t} + (t - M_t)) \bmod (T-1)\right] + 2}$.

A Bootstrap breaking no cointegration test

B - construct the bootstrap pseudodata under H_0 : no cointegration [continued]

6. Cumulate $\{\nu^*_{it}\}$ obtaining pseudoresiduals $\{\epsilon^*_{it}\}$ obeying H_0 :no cointegration;
7. For each unit i compute the pseudodata under H_0 :no cointegration and break in $\widehat{t}_i^b$:

$$y^*_{it} = \begin{cases} \widehat{\mu}_{0i} + \widehat{\beta}_{0i} x_{it} + \epsilon^*_{it}, & t \leq \widehat{t}_i^b \\ \widehat{\mu}_{1i} + \widehat{\beta}_{1i} x_{it} + \epsilon^*_{it}, & t > \widehat{t}_i^b \end{cases}$$

A Bootstrap breaking no cointegration test

C - compute the test on the bootstrap pseudodata

8. Using the datasets $\{y^*_{it}, x_{it}\}$ estimate breaking cointegrating model with breakpoints $\widehat{t}_i^{b*}$, estimated as $\widehat{t}_i^{b}$, obtaining cointegrating residuals $\left\{\widehat{\epsilon}^*(\widehat{t}_i^{b*})_{it}\right\}$;
9. Compute the individual no cointegration statistics $\widehat{\theta}_i^*$ on $\left\{\widehat{\epsilon}^*(\widehat{t}_i^{b*})_{it}\right\}$;
10. Compute the Group statistics $\widehat{\theta}_h^*$ ($h =$ mean, median, maximum, depending on H_1)
11. Repeat 5-11 B times;

A Bootstrap breaking no cointegration test

D - compute the p-values

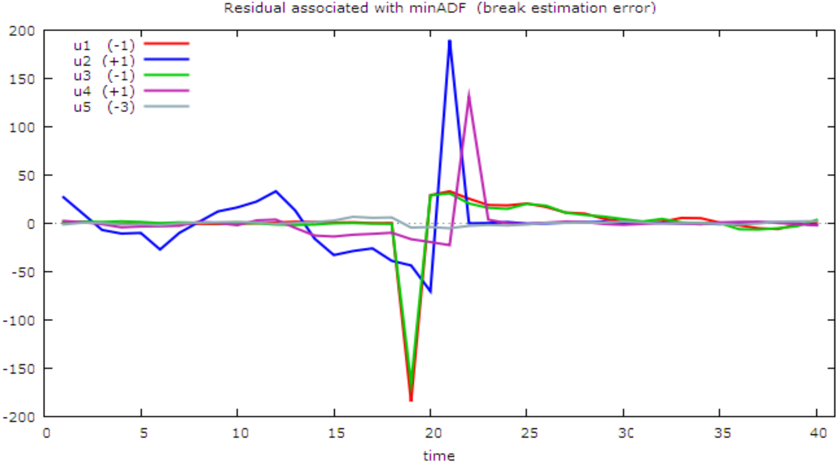
12. Compute the boostrap significance level of the statistics:
$p(\widehat{\theta}_h)^* = prop(\widehat{\theta}_h^* < \widehat{\theta}_h), \;\; h = mean, \; median, \; maximum.$

Question: which breakpoint estimator?

- Key point: in step 4 AR(1) fitted to the breaking cointegrating residuals in order to obtain under both H_0 and H_1 weakly dependent noises to resample.
- We thus need "nice" residuals
- Let us look at the residuals obtained with *Argmin(ADF)* and *Argmin(RSS)* breakpoints estimation.
- Remark: we know *Argmin(ADF)* is not consistent. Question is on small sample results.

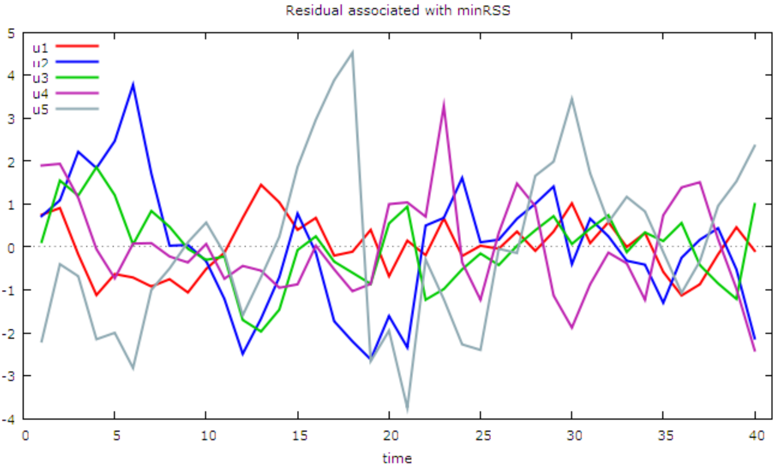
Argmin(ADF) breakpoint estimation: an example. N=5, T=40, break at 20

Plot of residuals for each unit



Argmin(RSS) breakpoint estimation: an example. N=5, T=40, break at 20

Plot of residuals for each unit



Breakpoint estimation rules: bottom line

- *Argmin(ADF)'s* poor performance in locating the break (in fact, largely expected on the basis of Gregory-Hansen's simulations) results in residuals with large outliers: **AR(1) modelling invalid.**
- *Argmin(RSS)* to be preferred.

Introducing the panel dimension in the bootstrap algorithms

- Data: X, Y: T(=time)$\times N$(=units) matrices
- Key requirement: reproducing cross-unit=cross-columns linkages
- Stationary Bootstrap implementation: resample entire rows of the data matrix - e.g. first block:

$$\begin{bmatrix} x^*_{11} & \cdots & x^*_{1N} \\ \vdots & \ddots & \vdots \\ x^*_{b1} & \cdots & x^*_{bN} \end{bmatrix} = \begin{bmatrix} x_{i1} & \cdots & x_{iN} \\ \vdots & \ddots & \vdots \\ x_{(i+b)1} & \cdots & x_{(i+b)N} \end{bmatrix}$$

Monte Carlo Experiment

Main design

- Bivariate DGP related to Gegenbach *et al.* (2006), Di Iorio-Fachin (2008)
- Conditional modelling: Y linked by a linear, possibly breaking cointegrating, relationship to X

$$
\begin{aligned}
y_{it} &= \begin{cases} 1 + x_{it} + \epsilon^{y}_{it}, & t \leq t^{b}_{i} \\ 5 + 5x_{it} + \epsilon^{y}_{it}, & t > t^{b}_{i} \end{cases} \\
t^{b}_{i} &\sim Uni(0.5T - 3, 0.5T + 3) \\
\epsilon^{y}_{it} &= \rho_i \epsilon^{y}_{it-1} + e^{y}_{it} \\
e^{y}_{it} &\sim N(0, \sigma^2_{iy}),\ \sigma^2_{iy} \sim Uni(0.5, 1.5)
\end{aligned}
$$

- X sum of I(1) and I)0) common factors (F_1, F_2) and an idiosyncratic I(0) noise (ϵ^{x}_{it}) :

$$x_{it} = \gamma_1 F_{1t} + \gamma_2 F_{2t} + \epsilon^{x}_{it}$$

Monte Carlo Experiment

Common Factor Structure

- Factors DGP:

$$\begin{bmatrix} F_{1t} \\ F_{2t} \end{bmatrix} = \begin{bmatrix} F_{1t-1} \\ 0.4F_{2t-1} \end{bmatrix} + \begin{bmatrix} f_{1t} \\ f_{2t} \end{bmatrix}$$

- Where

$$\begin{aligned} \begin{bmatrix} f_{1t} \\ f_{2t} \end{bmatrix} &= \begin{bmatrix} \eta_{1t} \\ \eta_{2t} \end{bmatrix} + \begin{bmatrix} \vartheta_1 & 0 \\ 0 & \vartheta_2 \end{bmatrix} \begin{bmatrix} \eta_{1t-1} \\ \eta_{2t-1} \end{bmatrix} \\ \epsilon_{it}^x &= e_{it}^x + \varphi e_{it-1}^x, \end{aligned}$$

- $\eta_{it} \sim N(0,1), i = 1,2$
- $e_{it}^x \sim N(0, \sigma_{ix}^2)$, with $\sigma_{ix}^2 \sim Uniform(1, 1.4)$.
- $\varphi, \vartheta's \sim Uniform[0.5, 0.7]$.

Tests

To evaluate the impact of studentisation we compute

- *ADF*
- $T(\rho - 1)$ [*Coefficient test*]

Results
Summary

1. Very little differences across

- tests (ADF and coefficient, group mean and median)
- block sizes

1. Type I errors: smaller than nominal for small T, but converge rapidly
2. Power: disappointing for T and N both small, but increases rapidly with both (in this design 100% for $T = 80$)
3. *Key message*: with small time samples panel procedure may grant considerable power gains, while ensuring good size control.

$\alpha = 0.05$

$T = 40$

	N							
	5	10	20	40	5	10	20	40
Mean block	*Boot-Mean*				*Boot-Median*			
	ADF							
4	0.03	0.02	0.01	0.01	0.03	0.02	0.01	0.01
6	0.02	0.01	0.01	0.00	0.02	0.02	0.01	0.01
	$T(\rho - 1)$							
4	0.03	0.03	0.02	0.01	0.04	0.03	0.02	0.01
6	0.03	0.02	0.01	0.00	0.03	0.02	0.01	0.01

Bootstrap: 500 redrawings; block size: $4 = 0.10T$, $6 = 1.75\sqrt[3]{T}$.
Montecarlo: 1000 replications.
Approximate confidence interval: $[0.036, 0.064]$

Results

Size

$\alpha = 0.05$

$T = 160$

	N			
	5	10	5	10
Mean block	*Boot-Mean*		*Boot-Median*	
	A. *ADF*			
16	0.01	0.02	0.04	0.02
10	0.02	0.02	0.03	0.03
	B. $T(\rho - 1)$			
16	0.04	0.02	0.05	0.05
10	0.04	0.02	0.06	0.05

approx confidence interval: $[0.036, 0.064]$

$\alpha = 0.05$

$T = 40$

	N							
	5	10	20	40	5	10	20	40
Mean block	*Boot-Mean*				*Boot-Median*			
	ADF							
4	0.59	0.83	0.97	1.00	0.43	0.71	0.92	0.98
6	0.56	0.79	0.95	1.00	0.40	0.66	0.88	0.98
	$T(\rho - 1)$							
4	0.65	0.89	0.98	1.00	0.54	0.82	0.97	1.00
6	0.62	0.86	0.98	1.00	0.50	0.78	0.95	1.00

$\rho_i \sim Uniform(0.6, 0.8)$

Back to the Feldstein-Horioka puzzle

- Panel cointegration tests with unknown breaks mostly agree with graphical evidence, **rejecting** H_0 : no panel cointegration - expected power gains indeed obtained

Investment and Savings 1960-2002
Bootstrap Panel Cointegration Tests with Unknown Breaks

	Mean $(p^* \times 100)$	*Median* $(p^* \times 100)$
ADF	$-\mathbf{4.02}$ $(\mathbf{1.2})$	$-\mathbf{18.41}$ $(\mathbf{7.6})$
$T(\rho-1)$	-3.53 (36.5)	$-\mathbf{18.40}$ $(\mathbf{3.8})$

panel: Austria, Belgium, Denmark, France, Germany, Greece, Ireland, Italy, Netherlands, Spain, Sweden, UK

Conclusions

- A RSB breaking no panel cointegration test based on

1. estimating the break by *Argmin(RSS)* and
2. RSB keeping the units fixed

- can handle well

1. *unknown breaks*
2. *very general dependence structures* across units, hence

- grant considerable power gains with respect to pure time series tests, which applied to the FH puzzle
- lead to conclude that (contrary to theoretical predictions, but consistently with graphical evidence) investements do appear to be linked to savings in a panel of european economies