The Irreversibility Premium

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Abstract

When investment is irreversible, theory suggests that firms will be “reluctant to invest.” This reluctance will result in a positive difference between the discount rate guiding investment decisions and the standard Jorgensonian user cost (adjusted for risk). We use the intertemporal tradeoff between the benefits and costs of changing the capital stock to estimate this difference, which we label the irreversibility premium. Panel data are carefully analyzed to isolate episodes of negative investment. The latter situation requires a special econometric approach to deliver consistent estimates. Our results provide a readily interpretable measure of the importance of irreversibility constraints (or, more generally, non-convex adjustment costs) and suggest that the irreversibility premium is both economically and statistically significant.

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1. Introduction

How important is investment irreversibility? When capital goods are highly specialized or industry specific, firms may find that reversing an investment decision is impossible or, more generally, costly in that the purchase cost exceeds the selling price of the capital good. If there are fixed costs of investment, firms may be reluctant to undertake investment projects that would otherwise seem desirable. Much recent theoretical work has examined the impact of completely or partly irreversible investment on firm behaviour.1 The fundamental result is that irreversibility generates a “reluctance to invest,” as a forward-looking firm hesitates to invest today because of the possibility that it may wish to sell capital in the uncertain future but will be able to reclaim little if any of the undepreciated value.2

The impact of irreversibility – or, more generally, non-convex adjustments costs – has been assessed in several studies examining its role in determining the current level of investment expenditures.3 Caballero, Engel, and Haltiwanger (1995, U.S. plant data) and Goolsbee and Gross (1997, airplane data) show that the adjustment rate of investment is asymmetric, being much larger for expanding

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1 Among other studies, see Bernanke (1983), Abel and Eberly (1994), Bertola and Caballero (1994), and Dixit and Pindyck (1994), as well as the early work of Arrow (1968). See Caballero (1999) for a recent survey.

2 Throughout the paper, we use “irreversible” to refer to either complete or partial irreversibility that results in unrecoverable sunk costs. The theoretical model and estimating equation allow for fixed costs and the more general case of partial irreversibility (which includes complete irreversibility as a special case when the resale value of capital is zero).

3 Tests examining the level of the capital stock are ambiguous. When there are irreversibility constraint, firms face a “user cost effect” that has a negative impact on the desired capital stock, but they also face a “hangover effect” capturing the fact that firms will occasionally have more capital than is desired and the irreversibility constraint will prevent them from making the appropriate reduction. Thus, the observed capital stock can be higher or lower under irreversibility. See, e.g., Abel and Eberly (1999).
than contracting plants or airlines. These results are consistent with irreversibility constraints and at odds with the familiar convex adjustment cost model. Abel and Eberly (1996a, U.S. firm data), Eberly (1997, firm data for 11 industrialized countries), and Cooper and Halitwanger (2000, U.S. plant data) find that the addition of non-convex adjustment costs to a model with convex adjustment costs significantly improves the fit. In Guiso and Parigi (1999, Italian firm data), investment is negatively affected by uncertainty, and this effect is greater for firms which cannot easily reverse their investment decisions because of limited resale markets for capital goods. Barnett and Sakellaris (1998, U.S. firm data) examine the sensitivity of investment to Tobin’s Q over different regimes defined by Q. They document differential sensitivity across three regimes but, in contrast to the irreversibility model of Abel and Eberly (1994), do not find that the sensitivity is lower in the regime where Q equals its long-run equilibrium value of unity. Abel and Eberly (1996a) show that this result could be consistent with their model when it includes heterogeneous capital goods.4

This paper takes a new approach to assessing the impact of irreversibility by focusing on the intertemporal pattern of investment, rather than its current level. The “reluctance to invest” result can be characterized by a difference between the discount rate guiding investment decisions and the Jorgensonian user cost of capital. In effect, the firm uses a higher discount rate. The “irreversibility premium” is the difference between this discount rate and the risk-adjusted market interest rate and enters the intertemporal tradeoff between the costs and benefits of

4 Some other studies of irreversibility do not explain investment expenditures. Pindyck and Soliananos (1993, aggregate data for 30 countries) and Caballero and Pindyck (1996, U.S. industry data) estimate the relationship between proxies for the investment threshold and variables such as the volatility of the marginal product of capital, reporting results consistent with irreversibility. Ramey and Shapiro (2001) examine capital allocation in the depressed aerospace industry and find that, on average, the estimated replacement cost of new equipment (adjusted for depreciation) is 350% greater than its market value as a used asset.
adjusting the capital stock. We use the intertemporal tradeoff to estimate the irreversibility premium. Our approach focuses on the fundamental theoretical implication of irreversibility and provides a readily interpretable measure of the economic importance of irreversibility constraints.

We derive the irreversibility premium – and our empirical specification – from the optimality conditions for investment. Our analysis is based on the irreversible investment model of Abel and Eberly (1994), which encompasses a variety of frictions which have been prominent in the investment literature – irreversibility (or, more precisely, costly reversibility), convex adjustment costs, and fixed costs. We extract the testable implications associated with the Abel-Eberly model for the discount rate appearing in the Euler equation for capital. We show that irreversibility and fixed costs both raise the discount rate guiding investment decisions and thus obtain the “reluctance to invest” result.

Our test estimates the Euler equation for capital and allows the intertemporal pattern of investment spending to reveal what discount rate is being used by firms. Because there are other potential influences on the discount rate, we calculate the irreversibility premium by estimating the difference in risk-adjusted discount rates between firms which are more likely to face binding irreversibility constraints and the rest of the firms in our data. For example, if corporate governance problems lead some managers to empire-building behaviour, their firms may behave as if they are using low (or even negative) discount rates in evaluating investment projects, thus blurring inferences about the irreversibility premium.

Economic theory suggests a number of factors which should determine how important irreversibility is for a given firm: low growth, low depreciation, high uncertainty, and limited resale markets. We estimate the irreversibility premium by examining the difference in discount rates between, for example, firms with
high and low growth or different degrees of difficulty in reselling capital goods. On their own, low growth and limited resale markets yield economically significant estimates of the irreversibility premium. In combination with other factors, such as low depreciation and high uncertainty, low growth and limited resale markets yield both economically and statistically significant estimates of the irreversibility premium.

A combination of factors – low growth, low depreciation, and high uncertainty (which implies occasional large negative shocks) – will tend to push firms toward the zone of inaction, where investment is zero. In addition to examining these factors, we can identify the irreversibility premium by looking directly at firms with very low investment. These firms are the most likely to be close to the zone of inaction and hence influenced by the potential impact of irreversibility. Estimating the difference between the discount rate used by the firms close to the zone of inaction and the remaining firms, we obtain a preliminary estimate of the irreversibility premium of 670 basis points.\(^5\)

The paper is organized as follows. Section 2 derives the irreversibility premium and our empirical specification from the optimality conditions for non-zero investment. Section 3 briefly describes auxiliary assumptions required for estimation (e.g., rational expectations), data, and related issues. Section 4 presents the empirical results based on a preliminary dataset of 199 Canadian firms. (Future work will use a much larger panel of US firms that is carefully analyzed to isolate periods of negative investment.) Section 5 returns to the issue of zero investment, and describes an econometric method for dealing with this situation. Section 6 offers a brief conclusion and describes some implications.

\(^5\) Those thinking in terms of the IS curve may initially believe that high interest rates would be associated with low investment in a neoclassical investment model without irreversibility constraints. This would be incorrect: in the neoclassical model, the interest rate affects the level
of the capital stock; investment is determined by the change in interest rates.
2. Optimal Investment Behavior With Costly Reversibility

This section derives the irreversibility premium and our empirical specification from the optimality conditions for the firm’s investment problem. We use the Abel and Eberly (1994) model as a point of departure. The Abel-Eberly model encompasses a variety of frictions, including costly reversibility, convex adjustment costs, and fixed costs of changing the capital stock. Equation (12) expresses the intertemporal tradeoff that we use to estimate the irreversibility premium. Readers with less immediate interest in the derivations are encouraged to proceed to Section 2.2., which offers an intuitive explanation of the intertemporal tradeoff and describes our strategy for identifying the irreversibility premium.

An important characteristic in our dataset is that there are virtually no zero investment observations. Consequently, in this and the next two sections, we develop and estimate a model under the assumption that, when formulating investment plans today, the firm is always actively investing or disinvesting. However, the firm does consider the possibility that zero investment may be optimal in the unknown future. Section 5 returns to the issue of zero investment. We assume that some of observed investment represents an ongoing frictionless flow, and, in that section, modify the data and estimation framework accordingly.

6 Other models (such as Bertola and Caballero (1994) and Dixit and Pindyck (1994)) are likely to yield a corresponding irreversibility premium. For example, in equations (10) and (11) in Bertola and Caballero, the expression $\frac{1}{2} \sum A$ corresponds to the irreversibility premium $\theta$ derived below.

7 In our dataset, xx of xx observations are zero. This result is common in COMPUTSTAT data. For example, Barnett and Sakellaris (1998, p. 269) report that investment is zero for three out of their 23,207 observations.
2.1. The Intertemporal Tradeoff

We begin by assuming that a risk-neutral firm selects policies to maximize its expected present value of profits in the face of four constraints. First, output is determined by a technology depending on capital (K), a vector of variable factors of production, and a stochastic technology shock (ε). Second, ε is a diffusion process evolving according to the following equation (time subscripts have been suppressed),

\[ dε = \mu[ε] \, dt + \sigma[ε] \, dz, \quad (1) \]

where \( \mu[ε] \) is the drift term, \( \sigma[ε] \) is the instantaneous variance, and \( z \) is a standard Weiner process. Third, capital depreciates geometrically at rate \( δ \), and evolves according to the following equation,

\[ dK = (1 - δK) \, dt, \quad (2) \]

where \( I \) is the investment rate.

Fourth, the firm is constrained by an augmented adjustment cost function, \( C[I,K] \), that distinguishes between regimes where investment is positive, negative, or zero. \( C[I,K] \) is differentiable for all \( I \), except at \( I=0 \). To identify these different regimes in the optimization problem, we define the following indicator variables,

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8 The \( ε \) variable can also represent stochastic shocks to the demand schedule or to prices for the variable factors.

9 There are two independent sources of non-differentiability at zero: a difference between purchase and resale prices and fixed costs.
\[ v^+ = 1 \quad \text{if } I > 0, \quad 0 \quad \text{otherwise,} \quad (3a) \]
\[ v^- = 1 \quad \text{if } I < 0, \quad 0 \quad \text{otherwise,} \quad (3b) \]
\[ v^0 = 1 \quad \text{if } I = 0, \quad 0 \quad \text{otherwise.} \quad (3c) \]

Note that the three regimes are mutual exclusive and exhaustive. To reflect the partly irreversible nature of investment, we distinguish between the prices at which the firm can purchase \( p^+ \) and resell \( p^- \) a unit of uninstalled capital, where \( p^+ > p^- \geq 0 \). (If \( p^- = 0 \), investment is completely irreversible; if \( p^+ = p^- \), investment is completely reversible.) Additionally, whenever investment is non-zero, the firm incurs convex adjustment costs, \( G[I,K] \). As is standard in the literature, \( G[I,K] \) depends positively on \( I \) and negatively on \( K \), and \( G[0,K] = G_i[0,K] = G_K[0,K] = 0 \). Lastly, the firm faces fixed costs, \( F \), whenever it is investing or disinvesting.\(^{10}\) The augmented adjustment cost function is specified as follows,

\[ C[I,K] = v^+ p^+ I + v^- p^- I + G[I,K] + (1 - v^0) F. \quad (4) \]

Based on the above model, optimal investment can be positive, zero, or negative. Which regimes is operative is determined by the value of \( q \) relative to two thresholds, \( q^+ \) and \( q^- \).\(^{11}\) If optimal investment is non-zero, the following relation holds,

\(^{10}\) To preserve the homogeneity of the cost function and to capture the notion that fixed costs (e.g., planning meetings) increase with firm size, \( F \) might be considered as fixed costs per unit of capital. In this case, the last term in (4) becomes \( F*K \).

\(^{11}\) Specifically, \( I > 0 \) when \( q > q^+ \), \( I < 0 \) when \( q < q^- \), and \( I = 0 \) when \( q^- \leq q \leq q^+ \). The thresholds depend only on the augmented adjustment cost technology. For example, if \( G[I,K] = (\gamma/2) (I^2 / K) \), then \( q^+ = p^+ + (2*\gamma*F)^{1/2} \) and \( q^- = p^- - (2*\gamma*F)^{1/2} \). See Abel and Eberly (1994, Section I and especially Figure 1) for further discussion of regimes and thresholds.
\[ q = C_t[I,K] = v^+ p^+ + v^- p^- + G_t[I,K]. \]  \hspace{1cm} (5)

When \( I > 0 \), \( v^+ = 1 \) and \( v^- = 0 \); when \( I < 0 \), \( v^- = 1 \) and \( v^+ = 0 \). If optimal investment is zero, \( q_t \) is indeterminate but bounded,

\[ 0 \leq q^- \leq q \leq q^+ \]  \hspace{1cm} (6)

The above model of the firm also generates the following one-period return relation (Abel and Eberly, 1994, equation 19) obtained by differentiating the Bellman equation with respect to the capital stock,

\[(r + \delta) q = \pi_K[I_t,K_t,\varepsilon_t] + E\{dq\} / dt, \]  \hspace{1cm} (7)

where \( r \) is the market interest rate, \( q \) is the marginal valuation of a unit of installed capital, \( E\{.\} \) is the expectation operator, and \( \pi_K[I,K,\varepsilon] \) is the marginal revenue product of capital. The latter term includes both the increment to production and the decrement to convex adjustment costs (\( G_K[I,K] \)), and reflects optimal choices of variable factors and investment. The left side of (7) is the required return on a marginal unit of capital, and is equated to capital's expected return, the incremental profit plus expected capital gain.

We recast (7) into a discrete equation to be used in estimation by assuming that \( \pi_K[I,K,\varepsilon] \equiv \pi_K[I_t,K_t,\varepsilon_t] \equiv \pi_{K,t}, dq \equiv q_{t+1} - q_t, \) and \( r \equiv r_t \). Expectations are formed with information available in period \( t \), \( E\{.\} \equiv E_t\{.\} \). With these definitions, equation (7) is rewritten as follows,
\[-q_t(1+r_t+\delta) + (\pi_{K,t} + E_t\{q_{t+1}\}) = 0,\]  

(8)

which has the form of a standard Euler equation appearing frequently in the investment literature.

In order for (8) to be estimable, we need to relate the \(q_t\) and \(E_t\{q_{t+1}\}\) terms to observable variables. Regarding \(q_t\), it is important to observe that our dataset does not contain any \(I_t=0\) observations. Since \(q_t\) is part of the period \(t\) information set, the firm knows the applicable regime and whether investment is positive or negative in period \(t\). Thus, we can replace \(q_t\) with equation (5).

The analysis of \(E_t\{q_{t+1}\}\) is more complicated. We begin by taking the expectation of (5) for period \(t+1\) conditional on period \(t\) information, and form the following set of conditional expectations,

\[E_t\{q_{t+1}\} = h^{+}_{t+1} E_t\{q_{t+1} : v^{+}_{t+1}\} + h^{-}_{t+1} E_t\{q_{t+1} : v^{-}_{t+1}\} + h^{0}_{t+1} E_t\{q_{t+1} : v^{0}_{t+1}\}, \]

(9)

where \(h^{+}_{t+1}\) is the probability of being in the positive investment regime in period \(t+1\) and \(v^{+}_{t+1}\) is the indicator variable equal to one if investment is positive in \(t+1\). Similar definitions apply to \((h^{-}_{t+1}, v^{-}_{t+1})\) and \((h^{0}_{t+1}, v^{0}_{t+1})\). It is important to note that, while the firm knows in period \(t\) that it is not in the zero-investment regime, it must assign some positive probability of being in this regime in period \(t+1\).

Our goal is to obtain an investment equation specified in terms of observable variables. For the non-zero investment regimes, we advance the terms in (5) by one period, and substitute for \(q_{t+1}\) in (8) in terms of the \(p_{t+1}\) and \(G_t[I_{t+1}, K_{t+1}]\),
\[ \begin{align*}
E_t\{q_{t+1}\} &= h^+_{t+1} E_t\{p^+_{t+1} : v^+_{t+1}\} + h^-_{t+1} E_t\{p^-_{t+1} : v^-_{t+1}\} \\
&\quad + h^+_{t+1} E_t\{G_t[I_{t+1,K_{t+1}}] : v^+_{t+1}\} + h^-_{t+1} E_t\{G_t[I_{t+1,K_{t+1}}] : v^-_{t+1}\} \\
&\quad + h^0_{t+1} E_t\{q_{t+1} : v^0_{t+1}\}. 
\end{align*} \] (10)

Adding and subtracting the following two terms -- \( h^-_{t+1} E_t\{p^-_{t+1} : v^-_{t+1}\} \) and \( h^0_{t+1} E_t\{p^+_{t+1} : v^0_{t+1}\} \) -- to (10) and rearranging, we obtain,

\[ \begin{align*}
E_t\{q_{t+1}\} &= h^+_{t+1} E_t\{p^+_{t+1} + h^-_{t+1} E_t\{p^+_{t+1} : v^-_{t+1}\} + h^0_{t+1} E_t\{p^+_{t+1} : v^0_{t+1}\} \\
&\quad + h^+_{t+1} E_t\{G_t[I_{t+1,K_{t+1}}] : v^+_{t+1}\} + h^-_{t+1} E_t\{G_t[I_{t+1,K_{t+1}}] : v^-_{t+1}\} \\
&\quad + h^0_{t+1} E_t\{q_{t+1} - p^+_{t+1} : v^0_{t+1}\}
\end{align*} \] (11)

The first line in (11) is the unconditional expectation of the purchase price of capital, \( E_t\{p^+_{t+1}\} \). Since \( G_t[0,K_{t+1}] = 0 \), the \( E_t\{G_t[0,K_{t+1}] : v^0_{t+1}\} = 0 \), and the second line in (11) represents the unconditional expectation of marginal adjustment costs, \( E_t\{G_t[I_{t+1,K_{t+1}}]\} \). We represent the latter two terms by \( \eta_{t+1} \), which will be discussed in the next sub-section, and rewrite (11) as follows,

\[ \begin{align*}
E_t\{q_{t+1}\} &= E_t\{p^+_{t+1}\} + E_t\{G_t[I_{t+1,K_{t+1}}]\} + \eta_t 
\end{align*} \] (12)

\[ \eta_t = h^-_{t+1} E_t\{p^-_{t+1} - p^+_{t+1} : v^-_{t+1}\} + h^0_{t+1} E_t\{q_{t+1} - p^+_{t+1} : v^0_{t+1}\} \leq 0 \] (13)

The sign of \( \eta_t \) is depends on two terms. The non-positive value of the first term follows from the assumption that the resale price is no greater than the purchase price. The second term is evaluated in the zero-investment regime, and its sign depends on fixed costs. When fixed costs are zero, this term is non-
positive because $q_{t+1} \leq p^+_{t+1}$ in the zero-investment regime. However, when fixed costs are positive, the second term in (13) is positive, and depends positively on fixed costs and adjustment costs. For example, if adjustment costs are quadratic and parameterized by $\gamma$ (see fn. xx), then $E_t\{q_{t+1} - p^+_{t+1} : v^0_{t+1}\} = E_t\{(2*\gamma*F)^{1/2} : v^0_{t+1}\} = (2*\gamma*F)^{1/2} > 0$. In the presence of fixed costs, the sign of $\eta_t$ is ambiguous, and it is more negative (or less positive) the larger the spread between resale and purchase prices, the smaller fixed costs and adjustment costs, and the larger the difference in the probability of being in the negative investment regime relative to being in the zero investment regime. Since $h^-_{t+1}$ is much larger than $h^0_{t+1}$ in our dataset, it seems likely that $\eta_t \leq 0$.

2.2. Frictions And The Irreversibility Premium

Equations (12) and (13) can be interpreted in terms of a perturbation argument. Along the optimal capital accumulation path, the firm is indifferent to an increase in capital by 1 unit in period t and a decrease of 1 unit in t+1, thus leaving the capital stock unaffected from period t+1 onward. The cost of this perturbation is represented by $p^+_{t} + G_t[I_t,K_t]$ -- the marginal purchase cost and marginal convex adjustment costs incurred in period t. In the absence of costly reversibility, perturbing the capital stock creates two benefits, $\pi_{K,t}$ -- the marginal revenue product of capital -- and $E_t\{p^+_{t+1} + G_t[I_{t+1},K_{t+1}]\}$ -- the expected saving in period t+1. This saving arises because the period t investment removes the need to acquire an additional unit of capital in period t+1 to remain on the optimal accumulation path. The Euler equation adjusts for discounting and depreciation.

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12 Without fixed costs, the critical value of q demarcating the zero-investment and positive-investment regimes is $q^+_{t+1} = p^+_{t+1} + G_t[0,K_{t+1}] = p^+_{t+1}$, where $G_t[0,K_{t+1}] = 0$ given the properties of the convex adjustment cost function. See Abel and Eberly (1994, p. 1374, Figure 1) with $c_t[0,K]^+ = p^+_{t+1}$ and $c[0,K] = F = 0$. 
(1+r_t+\delta), and equates benefits and costs expressed in temporally comparable terms.

Frictions due to costly reversibility and fixed costs impede the firm in equating known costs to expected benefits. These frictions manifest themselves by creating three regimes in which optimal investment is positive, negative, or zero. With probability \( h^+_{t+1} + h^-_{t+1} \), the firm will be actively investing (either positive or negative), and the perturbation interpretation in the previous paragraph is applicable (provided the costs of purchasing and reselling capital are identical, an assumption relaxed below). However, with probability \( h^0_{t+1} \), the firm will realize a shock such that it will find itself in the zone of inaction where it is optimal to undertake zero investment. In this case, the period \( t+1 \) saving that was expected in period \( t \) vanishes. This loss is partly compensated by unwanted unit of capital valued at \( q_{t+1} \). Consequently, the firm must "discount" the saving it expects in period \( t+1 \). This discount is embedded in \( \eta_{t+1} \) in equation (13).

The analysis in the immediately preceding paragraph temporarily suspended the impact of costly reversibility by assuming that the purchase price of capital \( (p^+_{t+1}) \) equals its resale price \( (p^-_{t+1}) \). When this restriction is removed and \( p^+_{t+1} > p^-_{t+1} \), \( \eta_{t+1} \) contains a second term. Assume that in period \( t \) the firm was planning on positive investment in \( t+1 \). However, the firm receives a large negative shock so that reselling capital is now optimal in \( t+1 \). The firm anticipates the possibility of not realizing the full saving in \( t+1 \), and must add an additional "discount" to \( \eta_{t+1} \) based on the probability of encountering the disinvestment regime and the difference between purchase and resale prices.

The above interpretation of \( \eta_{t+1} \) was based on the assumption that the firm anticipated undertaking positive investment in period \( t+1 \). This benchmark was chosen because of the need to express the Euler equation in terms of observable
variables. The interpretation of $\eta_{t+1}$ is preserved if the period $t+1$ benchmark is negative investment. That discussion is placed in Appendix A.\textsuperscript{13}

The derived discount wedge $\eta_{t+1}$ reflects the "reluctance to invest" that is a hallmark of the irreversibility literature (Caballero, 1999). In a discrete time model, Bertola and Caballero (1994, Section 2) show that the marginal product of capital under irreversibility exceeds the Jorgensonian user cost applicable when investment is costlessly reversible. In the continuous time model of Abel and Eberly (1996b, Section V; 1999, Section 2), optimal investment occurs only when the marginal revenue product of capital reaches a barrier equal to the Jorgensonian user cost plus a term reflecting irreversibility and uncertainty. Dixit and Pindyck (1994, Chapter 5) analyze the option to invest today versus tomorrow, and show that the marginal product of capital triggering the investment outlay is higher under irreversibility and uncertainty. A similar result holds in our model with $\eta_t$.

To relate $\eta_t$ to the discount rates emphasized in the literature, we normalize the discount wedge by the marginal value of an additional unit of capital,

$$\theta_t \equiv -\eta_t / q_t \geq 0,$$

and rewrite the intertemporal tradeoff as follows,

$$-(p_t + G_t[I_t,K_t]) (1+r_t+\delta+\theta_t) + (\pi_{K,t} + E_t\{p_{t+1}^t\} + E_t\{G_t[I_{t+1}^t,K_{t+1}^t]\}) = 0,$$ (15)

where $p_t$ equals $p_t^+$ if $I_t > 0$ or $p_t^-$ if $I_t < 0$. Thus, costly reversibility and fixed costs under uncertainty raise the effective discount rate guiding investment decisions

\textsuperscript{13} When setting its investment plans, a firm will never find it optimal to plan to undertake zero investment in period $t+1$. 

from \( r_t \) to \( (r_t + \theta_t) \). This extra term, \( \theta_t \), is the "irreversibility premium" estimated in this paper.

### 3. From Theory to Estimation

In order to estimate the intertemporal tradeoff (15), we need to make several assumptions. First, the two variables evaluated with the expectation operator are rewritten based on the rational expectations property: \( E_t\{W_{t+1}\} = W_{t+1} + \omega_t \), where \( \omega_t \) is a forecast error. Second, we assume that the irreversibility premium is constant over time and estimate it as a parameter. Third, we assume that the shock affecting marginal productivity (\( \epsilon_t \)) enters additively.

As noted in the introduction, other economic forces besides irreversibility may affect the discount rate which firms use in evaluating investment projects. Our strategy therefore compares the discount rates used by firms which are more or less likely to face binding irreversibility constraints. To the extent that forces other than irreversibility (which we denote by \( \Psi \)) affect all firms in the sample, we can reduce potential problems by focusing on differences in discount rates between classes of firms. Using the three assumptions noted above and estimating \( \theta \) based on differences, equation (17) can be written as follows,

\[
-(p_t^+ + G_t[I_t,K_t]) (1 + r_t + \delta + \Psi + \Gamma \theta) + (\pi_{K,t} + p_{t+1}^+ + G_t[I_{t+1},K_{t+1}]) = \zeta_t, \tag{16}
\]

where \( \Gamma \) is an indicator variable (1 if a firm falls into the indicated class, 0 otherwise) and \( \zeta_t / (\omega_t + \epsilon_t) \) is an error term.
We assume that the marginal convex adjustment costs $G_{I} [I_{t}, K_{t}]$ depend on the investment/capital ratio and are represented by the following second-order Taylor expansion,

$$
G_{I} [I_{t}, K_{t}] = \alpha_{0} + \alpha_{1} (I_{t}/K_{t}) + \alpha_{2} (I_{t}/K_{t})^{2}.
$$  \hspace{1cm} (17)

The production function is assumed to be homogeneous of degree $\xi$ (where $\xi$ is not necessarily equal to unity). Product markets may be imperfectly competitive. Rewriting the revenue function (which includes both the production and adjustment cost functions), using Euler's Theorem on Homogeneous Functions, and rearranging terms to isolate the marginal revenue product of capital, we obtain the following specification,

$$
\pi_{K,t} = \zeta * (REV_{t}/K_{t}) - (COST_{t}/K_{t}) - G_{I} [I_{t}, K_{t}] *(I_{t}/K_{t}),
$$  \hspace{1cm} (18)

where $(REV_{t}/K_{t})$ and $(COST_{t}/K_{t})$ are revenues and variable costs, respectively, divided by the capital stock, $G_{I} [I_{t}, K_{t}]$ is defined in (17), and $\zeta$ is a parameter capturing the combined effects of non-constant returns to scale and imperfect competition. Decreasing returns to scale and/or non-competitive product markets imply that $\zeta < 1$.

The interest rate $r_{t}$ is adjusted for risk using CAPM. The estimated Euler equation accounts for the relevant features of the Canadian tax system. These details are described in the Data Appendix. We estimate the Euler equation by GMM using as instruments a constant, $(REV_{t-1}/K_{t-1})$, $(COST_{t-1}/K_{t-1})$, $(I_{t-1}/K_{t-1})$, $r_{t-1}$, and the indicator variable for the class of firms.
Summary statistics for I/K, K, growth, uncertainty, and depreciation are presented in the first column of Table 1.\textsuperscript{14}

4. Empirical Results

The Estimates that follow are based on a panel of 199 Canadian Firms. This dataset will be replaced by a much larger dataset of US firms that has been analyzed carefully to isolate episodes of negative investment.

4.A. Initial Estimates of $\theta$

In models of irreversibility, the “Bad News Principle” of Bernanke (1983) holds that only factors affecting the lower tail of outcomes impact the firm’s investment decisions. Consequently, firms that grow slowly, that face high levels of uncertainty, or have slowly depreciating capital are more likely to be affected by the irreversibility constraint.\textsuperscript{15}

We begin by estimating the irreversibility premium based on the specification in (16), using the difference in discount rates between low growth firms and the remaining firms to identify $\theta$.\textsuperscript{16} The model fits well in the sense that the test of

\textsuperscript{14} Uncertainty is measured using the residual from a regression of sales on lagged sales. See Section 5 for details.

\textsuperscript{15} These implications for the irreversibility premium can be derived formally. Bertola and Caballero (1994, p. 228) demonstrate that the marginal product of capital sufficient to induce positive investment (relative to the standard user cost) is affected by the drift rate of the “business conditions” process, the depreciation rate, and uncertainty.

\textsuperscript{16} We calculate the average real sales growth rate for each firm over our sample period and divide firms into those above and below the median. 2 is the estimated difference in discount
overidentifying restrictions (the J test) fails to reject the model. The point estimate of $\theta$ is 300 basis points with a standard error of 270 basis points. This is economically (but not statistically) significant.

A natural question is whether the way in which we adjust the market interest rate for risk has a substantial effect on the estimate of $\theta$. The main estimates in the paper are based on CAPM. An alternative which has enjoyed considerable popularity in the recent finance literature is the Fama-French (1993) three-factor model.\footnote{Compared to CAPM, the Fama-French (1993) technique adds two additional variables, size and the ratio of book to market value (both of which can be interpreted as risk factors), to the excess market return (which is the single risk factor taken into account by CAPM).} In the second panel of Table 1, we present estimates based on this alternative technique for adjusting for risk. The estimated values of $\theta$ based on CAPM and the Fama-French three factor model are similar, so we focus on CAPM in subsequent tables.\footnote{The only substantive difference is the estimate of $\theta$ based on a comparison of low and high growth firms. In this case, the Fama-French risk adjustment yields an estimate of $\theta$ which is within two standard deviations of the CAPM estimate, but the Fama-French-based estimate is significantly different from 0.}

In the second and third columns of Table 2, we use differences in uncertainty and depreciation, respectively, to estimate $\gamma$. Specifically, we assign a dummy variable equal to 1 for firms with uncertainty above the median for firms in our sample.\footnote{We regress the sales/capital ratio on the lagged sales/capital ratio for each firm and use the root mean square error from this regression as a measure of uncertainty.} Similarly, we divide firms into high- and low-depreciation firms, assigning a dummy variable equal to 1 for firms with a depreciation rate below the median. This yields an estimate of $\theta$ which is close to 0 in both cases.

\begin{table}
\centering
\begin{tabular}{|c|c|c|}
\hline
1 & 2 & 3 \\
\hline
\end{tabular}
\caption{Estimates of $\theta$}
\end{table}
In order for the problem of irreversibility to arise, firms must face some difficulty in reselling previously acquired capital goods. In some theoretical work, this constraint is modeled as a complete inability to sell capital goods (e.g., Bertola and Caballero, 1994; Dixit and Pindyck, 1994). More generally, irreversibility can be modeled as a gap between the purchase and resale prices of capital goods, as we have done in Section 2. Unfortunately, we are not aware of any study which has found a way to directly measure purchase and resale prices for a broad range of industries.\textsuperscript{20}

Instead, we use two approaches to measuring the difficulty firms have in disposing of capital goods. Our first approach to identifying limited resale markets is somewhat subjective. Firms in the utility industry\textsuperscript{21} are likely to have considerable difficulty in ridding themselves of excess capital goods, especially in Canada during the time period covered by our sample. For example, pipeline companies would have found it very difficult to rip pipelines out of the ground and sell them to pipeline companies in other jurisdictions. The point estimate of 2 is 430 basis points, based on sorting firms with this measure of limited resale markets. The standard error is 150 basis points, so the null hypothesis that the irreversibility premium is less than or equal to 0 is strongly rejected.

Our second measure of limited resale markets is based on the work of Guiso and Parigi (1999), who had access to survey data in which firms were asked to

\textsuperscript{20} The one notable exception is Ramey and Shapiro (2001). Their data are for capital assets used in the aerospace industry and do not provide sufficient coverage of the larger set of assets used by firms in our sample. Asplund (2001) focus on a limited number of Swedish firms selling metalworking machinery.

\textsuperscript{21} The utility industry is defined as Toronto Stock Exchange sectors 900-1003, which includes gas and oil pipelines and gas, electrical, and telephone utilities.
select one of four categories best describing the ease with which their capital could be resold:

(i) It is relatively easy to find a buyer in a short time who is willing to pay a reasonable price.
(ii) It takes time to find a buyer, and selling prices are not very rewarding.
(iii) It is very difficult to find a buyer, and selling prices can become very low.
(iv) There is no resale market for capital goods.\textsuperscript{22}

Following Guiso and Parigi, we use affirmative responses to categories (iii) and (iv) as indicating limited resale markets. We then rank the industries by the percentage of firms selecting (iii) or (iv). The Guiso and Parigi survey was for Italian firms. We assume that the technological characteristics of capital goods which inhibit resale are reasonably similar across advanced industrial economies (an assumption used by Rajan and Zingales, 1998), and draw a correspondence between the Italian SIM industries and our Canadian TSE industries (see the Data Appendix). A dummy variable is created for firms which are above the median when their industries are ranked by the percentage of firms selecting (iii) or (iv) (66 firms are classified as having limited resale markets). Based on the Guiso-Parigi measure of limited resale markets, the point estimate of $\theta$ is $-480$ basis points.

\textsuperscript{22} See Guiso and Parigi (1999, Section VI and Appendix 2) for the exact wording of the questionnaire and further details about the survey. We are grateful to Luigi Guiso for kindly providing us with the industry aggregated data (thus preserving the confidential nature of the
individual responses).
4.B. Jensen, Canada, and Limited Resale Markets

The most surprising result in Table 2 is that firms with limited resale markets (based on the Guiso-Parigi technique) have a negative 2. This result may be attributable to the concentration of Canadian firms in resource industries. Much of our sample period (the 1970s and early 1980s) were boom times for natural resource firms. In several cases, these commodity price booms were not sustainable over the longer run. Oil prices, for example, tended to fall substantially after their peak in the early 1980s. As pointed out by Jensen (1986), the combination of large amounts of free cash flow and limited long-run investment opportunities can exacerbate agency problems within a firm. Agency problems of the type analyzed by Jensen lead firms to use lower discount rates. Intuitively, managers whose utility depends on the size of their firm (i.e., who are empire building) tend to use too low a discount rate in evaluating cash flows from investment projects.

Jensen’s original analysis of the agency costs of free cash flow was prompted by the behaviour of resource firms in the early 1980s. Agency problems may partially explain the lower discount rate for firms which are judged to have limited resale markets by the Guiso-Parigi technique, since 50 of these 66 firms are in resource industries. To explore this possibility, we estimate the difference in discount rates between resource firms and other firms and find that the discount rate for resource firms is 750 basis points below that of other firms. (The difference is highly significant.) To directly examine the Jensen hypothesis in our data, we estimate the difference in discount rates between firms with high free cash flow and poor investment opportunities and other firms. The results

---

23 These are defined as firms in Toronto Stock Exchange sectors 100-401.
24 We define firms as having high free cash flow if their free cash flow (cash flow minus dividends, normalized by total assets) for the previous three years is above the median for the
reinforce the view that agency problems lower a firm’s discount rate: firms with high free cash flow and poor investment opportunities have a discount rate that is 410 basis points below other firms. With a standard error of 140 basis points, this difference is highly significant.

4.C. A Second Look at Limited Resale Markets and \( \theta \)

In view of the evidence that corporate governance problems affect the discount rate, we consider two other measures of limited resale markets. The first focuses on firms that have limited resale markets (as measured by the Guiso-Parigi technique) but which are not in resource industries. As shown in the first column of Table 3, the point estimate of \( \theta \) is 300 basis points with a standard error of 190 basis points.

In an effort to improve the precision of the estimates, the second measure combines utility firms with the firms identified by the Guiso-Parigi technique which are not in resource industries. As the second column of Table 3 shows, the point estimate of \( \theta \), 400 basis points (with a standard error of 150 basis points), is both economically and statistically significant.

4.D. Identifying \( \theta \) with Combinations of Factors

Any one of the factors we have been examining in Tables 2 and 3 may not be sufficient by itself to make it likely that a firm will bump up against the irreversibility constraint. For example, a firm could face limited resale markets, but, if its growth rate is high, it might seldom if ever find itself with excess capital.

---

firms in our sample. Poor investment opportunities are measured by a Tobin’s Q below the median for the industry in a given year.
This suggests that it might be useful to examine the interaction of the factors – low growth, low depreciation, high uncertainty, and limited resale markets.25

We begin by examining interactions of low growth with the other factors. The first column of Table 4 shows that the point estimate of $\theta$ is 440 basis points based on a comparison between firms with low growth and high uncertainty and the remaining firms. The null hypothesis that $\theta$ is less than or equal to 0 is rejected at the .05 level. The second column of Table 4 estimates $\theta$ based on a comparison between firms with low growth and low depreciation rates and other firms. The point estimate of $\theta$ is 510 basis points (with a standard error of 190 basis points).

A second possible set of combinations starts with low depreciation. We have already seen in column 2 that the point estimate of $\theta$ is 510 basis points, based on a comparison between firms with low depreciation and low growth and other firms. If we focus on low depreciation rates and high uncertainty, the point estimate of $\theta$ is negative but small (210 basis points) and insignificantly different from 0. However, if we estimate $\theta$ based on low depreciation and limited resale markets, the point estimate is 330 basis points (with a standard error of 140 basis points), as reported in the fourth column of Table 4.

There are two remaining combinations of the four factors. Unfortunately, there are only four firms with low growth and limited resale markets and only one firm with limited resale markets and high uncertainty. The estimated $\theta$’s based on these combinations (evaluated separately) are positive. Given the small number of firms for these two combinations, we do not include these estimates in Table 4. Nonetheless, the results for the other combinations reported in Table 4 suggest that the irreversibility premium is both economically and statistically significant.

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25 Throughout this subsection, we define limited resale markets using the combination of utility firms and non-resource firms identified by the Guiso-Parigi technique.
4.E. Identifying $\theta$ Based on Firms Close to the Zone of Inaction

Up to this point, we have used factors suggested by economic theory, both individually and in combination, to identify the irreversibility premium. We now turn to another approach, which can be thought of as combining all the factors which push firms toward the zone of inaction. We simply focus on firms which are close to the zone of inaction where investment is zero, and hence are likely to be influenced by the potential impact of the irreversibility constraint. In Table 5, we estimate $\theta$, using a comparison between firms with very low investment (normalized by the capital stock) and the remaining firms in the sample. As the first column of Table 5 shows, the point estimate of $\theta$ is 670 basis points, based on a cutoff at the lowest 10% of firms. The standard error is 170 basis points, so the null hypothesis that $\theta$ is less than or equal to 0 is strongly rejected. As the second column of Table 5 shows, this result is not sensitive to the exact point at which we set the cutoff for firms close to the zone of inaction. As the third column of Table 5 shows, the effect of irreversibility on the discount rate attenuates as firms move further away from the zone of inaction. The point estimate of $\theta$ is still positive if the cutoff is set at the median level of investment, but it is much smaller – only 200 basis points – and no longer significantly different from 0.

5. Zero Investment

Future Work

5.1. Creating Zeros
As noted above, it is rarely the case that firms have zero investment. Based on this fact, some economists are inclined to dismiss irreversibility models. However, as Caballero (1999, p. 823) points out, actual investment may involve three types of adjustments: (a) ongoing frictionless flow, (b) gradual adjustments, and (c) major and infrequent adjustments. Convex adjustment cost models tend to ignore (c), while some models of irreversibility neglect (a). In practice, firms may require a small amount of replacement investment for crucial components of the firm’s capital. For example, a pulp and paper plant may need to replace a few worn-out parts even in a year when it finds itself with excess capacity.

We use the concept of the frictionless flow of investment (FFt) to transform investment data that are "close" to zero into zeros. Specifically, we define FFt as some small fraction (ϕ) of the existing capital stock at the beginning of the period,

$$FF_t = \phi * K_{t-1},$$

and create the transformed investment series (IT_t) from the original investment data as follows,

If $0 \leq I_t \leq FF_t$, $I^T_t = 0$

Otherwise, $I^T_t = I_t$

5.2. Incorporating Zero Investment In Period t

The existence of a substantial number of observations for which $I_t = 0$ can be estimated with the model developed in Sections 2 and 3 with one important modification. That derivation was based on the firm knowing in period t that investment was non-zero. This permitted us to specify q_t in equation (8) in terms of the marginal price and adjustment costs given by equation (5). When $I_t = 0$, 
equation (5) is no longer useful, and the q_t's associated with zero-investment remain unobservable.

We proceed with estimation by modeling the unobservable q_t's as nuisance parameters, and partial them out of the regression model by an appropriate definition of dummy variables. Specifically, we define dummy variables, D_{i,t}, for those firm/year observations with IT_t = 0, and then regress each of the model variables and instrumental variables on this set of dummies. The fitted values from these auxiliary regressions become the model and instrumental variables.

This estimation strategy does not place any restrictions on the estimated coefficients, \( \Omega_{i,t} \). These coefficients represent the q_{i,t}'s for periods of zero investment and, according to the theory, these are bounded between the thresholds, \( q^-_{i,t} \) and \( q^+_{i,t} \) (equation 6). Since the thresholds depend on unknown parameters, they do not place any restrictions on the \( \Omega_{i,t} \)'s. However, since q cannot be negative, there is an effective lower bound at 0. We thus estimate the auxiliary equations with the following exponential transformation,

\[
\Omega_{i,t} = \exp[\omega_{i,t}].
\]
6. **Summary And Conclusions**

Irreversibility – or, more generally, non-convex adjustment costs – has important economic implications. Firm dynamics become more complicated. With irreversible capital and other non-convexities, investment behaviour becomes path dependent (Dixit, 1992), and aggregate investment depends on the higher moments of firm characteristics (Caballero, 1999). Many tax distortions are amplified by irreversibilities (Faig and Shum, 1999; Panteghini, forthcoming). Irreversible investment decisions change the nature of the inefficiencies arising from asymmetric information in capital markets (Lensik and Sterken, 2001). Insofar as higher rates of inflation increase uncertainty, irreversible capital creates an additional channel through which monetary policy and inflationary shocks impact the real economy (Pindyck and Soliamanos, 1993).

As noted in the introduction, a variety of empirical studies have examined whether irreversibility exists. In this paper, we make a distinctive contribution to this evidence by estimating the irreversibility premium, a readily interpretable measure of the economic importance of non-convex adjustment costs. The evidence presented in this paper suggests that the irreversibility premium is both economically and statistically significant.
References


Data Appendix

This Appendix provides data sources and detailed descriptions for the variables used in the econometric analysis. The main data sources are the CANSIM, Laval, and Financial Post databases. (The latter two are comparable to the often used CRSP and COMPUSTAT databases, respectively, in the United States.) The sample covers 199 firms over the period 1973-86. The Appendix is divided into three sections: The Risk-Adjusted Real Market Interest Rate (which includes a discussion of the CAPM and Fama-French three-factor models for constructing the equity risk premium), Limited Resale Markets, and Other Variables.

The Risk-Adjusted Real Market Interest Rate

The risk-adjusted real market interest rate, $r_t$, can be written as defined in the text (20) as follows,

$$r_t = \frac{(1+i_t)}{(1+\pi^e_t)} - 1.0. \quad (B1)$$

This rate discounts annual cash flows from the middle-of-period $t$ to the middle-of-period $t+1$. The equity risk premium is estimated by the CAPM ($r_{CAPM}^t$) or the Fama-French three-factor ($r_{FF3}^t$) models. The components of $r_t$ are defined and constructed as follows,

\[
\begin{align*}
  i_t &= \text{Nominal, short-term, risk adjusted company cost of capital from the middle-of-period (MOP) $t$ to the MOP $t+1$,} \\
       &= \lambda(1-\tau_t)i^D_t + (1-\lambda)i^E_t. \\
  i^D_t &= \text{Nominal, one year, Commercial Paper rate from the MOP $t$ to the}
\end{align*}
\]
MOP \( t+1 \). This rate is constructed from monthly data for the Canadian 90-day Commercial Paper Rate, average of daily rates (CANSIM series B14017). These monthly data are converted into beginning-of-period (BOP) monthly data for January, April, July, and October by averaging the monthly data in the preceding and current months. For example, the January BOP rate is the arithmetic average of the monthly December and January rates. The BOP rates are represented as \( i_{t,\text{JAN}}^D, i_{t,\text{APR}}^D, i_{t,\text{JUL}}^D, \) and \( i_{t,\text{OCT}}^D \). The one year MOP rate is constructed from these BOP rates as follows,

\[
\left[ (1+i_{t,\text{JUL}}^D) (1+i_{t,\text{OCT}}^D) (1+i_{t+1,\text{JAN}}^D) (1+i_{t+1,\text{APR}}^D) \right]^{25} - 1.
\]

\( i_t^E \) = Nominal, short-term, risk adjusted cost of equity capital from the MOP \( t \) to the MOP \( t+1 \).

\[ i_t^E = i_t^F + \sigma \]

\( i_t^F \) = Nominal, one year, risk free Treasury Bill rate from the MOP \( t \) to the MOP \( t+1 \). This rate is constructed from monthly data for the Canadian 90-day Treasury Bill Rate, average yields at weekly auctions (CANSIM series B14007). These monthly data are converted into BOP monthly data for January, April, July, and October by averaging the monthly data in the preceding and current months. For example, the January BOP rate is the arithmetic average of the monthly December and January rates. The BOP rates are represented as \( i_{t,\text{JAN}}^F, i_{t,\text{APR}}^F, i_{t,\text{JUL}}^F, \) and \( i_{t,\text{OCT}}^F \). The one year MOP rate is constructed from these BOP rates as follows,

\[
\left[ (1+i_{t,\text{JUL}}^F) (1+i_{t,\text{OCT}}^F) (1+i_{t+1,\text{JAN}}^F) (1+i_{t+1,\text{APR}}^F) \right]^{25} - 1.
\]

\( \pi_t^e \) = One year expected inflation rate from the MOP \( t \) to the MOP \( t+1 \). We assume expected inflation equals actual inflation up to an additive, orthogonal error. The inflation rate is constructed from monthly All Items Consumer Price Index data (CANSIM series P700000). These monthly data are converted into BOP
monthly data for July by averaging the monthly data in June and July, and are represented as CPI_{t,JUL}. The one year MOP inflation rate is constructed from these BOP rates as follows:

\[
\sigma = \left( \frac{\text{CPI}_{t+1,JUL}}{\text{CPI}_{t,JUL}} \right) - 1.0.
\]

\(\sigma\) = Equity risk premium. The methods used to estimate \(\sigma\) are discussed below.

\(\tau_t\) = Marginal rate of corporate income taxation (federal and provincial) incorporating variation across time and industries.

\(\lambda\) = Firm specific leverage ratio calculated as the time-average of bonds / (bonds + equity), where bonds are at book value and equity at market value.

Two methods are used to estimate the equity risk premium, \(\sigma\). Under the CAPM,

\[
\sigma = \beta (\mu^E - \mu^F),
\]

(B2)

where

\(\beta\) = CAPM \(\beta\) from Hatch and White (1988, Table 5-36). These industry \(\beta\)'s are assigned to firms on the basis of the Toronto Stock Exchange's industry classification.

\(\mu^E\) = Total return on equities from 1950-1986: 0.1148. The source is Hatch and White (1988, Table 5-15).

\(\mu^F\) = Total return on risk free Treasury bills from 1950-1986: 0.0584. The source is Hatch and White (1988, Table 5-5).

We also measure the equity risk premium using the Fama and French (1993) three-factor model (FF3),

\[
\sigma = \beta^{EMR} \mu^{EMR} + \beta^{SMB} \mu^{SMB} + \beta^{HML} \mu^{HML},
\]

(B3)
where $\mu$ is a mathematical expectation, EMR is the excess market return (value-weighted market return minus risk-free rate), SMB is the size risk factor, and HML is the book-to-market risk factor. $\beta_{EMR}^{EMR}$, $\beta_{SMB}^{SMB}$, and $\beta_{HML}^{HML}$ are the firm-specific factor loadings on these three risk factors.

The portfolios are constructed as follows: firms were included in the sample in a given year if: 1) book equity (common stock capital, plus deferred income taxes if available) and market equity for the end of the previous year were available in the Financial Post dataset; and 2) returns data for the current year were available from the TSE-Western dataset. All firms in the sample in a given year were ranked on size (using market equity) and split into small and big (S and B) depending on whether they were above or below the median. All firms in the sample in a given year were then ranked by the ratio of book equity to market equity with breakpoints for the bottom 30% (Low), middle 40% (Medium), and top 30% (High). Six portfolios (S/L, S/M, S/H, B/L, B/M, B/H) were constructed from the intersection of the two size and three book-to-market categories, and monthly value-weighted returns on the six portfolios were calculated for each calendar year.

SMB is defined as the difference, each month, between the simple average of the returns on the three small firm portfolios (S/L, S/M, and S/H) and the corresponding simple average of the returns on the three big firm portfolios. HML is defined as the difference, each month, between the simple average of the returns on the two high-book-to-market portfolios and the corresponding simple average of the returns on the two low-book-to-market portfolios.

The factor loadings were estimated from a regression of excess returns for firm $i$ on the three risk factors at a monthly frequency. $\mu_{EMR}^{EMR}$, $\mu_{SMB}^{SMB}$, and $\mu_{HML}^{HML}$ are
the annualized geometric means of the three risk factors over all months from 1973 to 1986 inclusive.

**Limited Resale Markets**

Section 4.B of the paper describes how we use the Guiso-Parigi survey data to identify firms with limited resale markets. The following table provides the details of the correspondence between the Italian SIM and Canadian TSE industry codes.
Correspondence Between The Italian Survey Of Investment In Manufacturing (SIM) Industry And The Canadian Toronto Stock Exchange (TSE) Industry Classifications

<table>
<thead>
<tr>
<th>Italian SIM Industries</th>
<th>Canadian TSE Industry Code</th>
<th>Rankings</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) (2) (3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Metallurgy (1)</td>
<td>100,101,102,200,201</td>
<td>6</td>
</tr>
<tr>
<td>Production Of Non-Metal Mineral Goods (2)</td>
<td>103,104,300,301,302</td>
<td>5</td>
</tr>
<tr>
<td>Chemical Products, Including Pharmaceuticals (3)</td>
<td>607</td>
<td>2</td>
</tr>
<tr>
<td>Metal Products (4)</td>
<td>601,602</td>
<td>9</td>
</tr>
<tr>
<td>Industrial And Agricultural Machines (5)</td>
<td>603</td>
<td>13</td>
</tr>
<tr>
<td>Computers and Precision Tools (6)</td>
<td>699</td>
<td>4</td>
</tr>
<tr>
<td>Electric and Electronic Materials (7)</td>
<td>598,605</td>
<td>3</td>
</tr>
<tr>
<td>Cars and Motor Vehicles (8)</td>
<td>506</td>
<td>7</td>
</tr>
<tr>
<td>Construction of Trains, Airplanes, and Ships (9)</td>
<td>604</td>
<td>1</td>
</tr>
<tr>
<td>Food and Tobacco (10)</td>
<td>501,502,503,504,599</td>
<td>8</td>
</tr>
<tr>
<td>Textiles (11)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+Leather and Shoes (12)</td>
<td>500, 597</td>
<td></td>
</tr>
<tr>
<td>+Clothes (13)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Timber and Furniture (14)</td>
<td>400,401</td>
<td>10</td>
</tr>
<tr>
<td>Papers, Journals, Newspapers, Books (15)</td>
<td>609,698</td>
<td>11</td>
</tr>
<tr>
<td>Plastics and Rubber (16)</td>
<td>Omitted</td>
<td></td>
</tr>
<tr>
<td>Miscellaneous (17)</td>
<td>Omitted</td>
<td></td>
</tr>
</tbody>
</table>

The rankings in column 3 are determined by the percentage of respondents in an Italian industry answering yes to questions (iii) or (iv), which are described in Section 4.B of the paper. A low number in column 3 indicates limited resale markets. The top six industries are classified as having limited resale markets for capital goods (and are highlighted in bold). This assumption was effectively used by Guiso and Parigi (1999) and approximately halves the number of firms which can be classified using the above table. Firms with rankings above the median are classified as facing limited resale markets for capital goods. Seventy-two firms cannot be classified using this approach. We retain these firms in the sample (to improve estimation of the technological parameters) and assign a value of 0 to the indicator variable ($\mu$) for these firms.
**Other Variables**

We require data for investment, revenues, costs, and the physical capital stock. Three series are drawn directly from the Financial Post Annual Corporate Database. Investment (I) is gross capital expenditures on property, plant and equipment. Revenue (REV) is net sales. Cost (COST) is revenue minus operating income, where operating income is the income derived from the principal activities of a business after deducting all operating expenses except depreciation, depletion and amortization, interest expense and other expenses included in the other income/expense category. In the Euler equation, all three series are divided by the capital stock.

The capital stock (K) for a given firm is constructed in two steps. The first step estimates the depreciation rate (\(\delta\)). This paper uses firm-specific depreciation rates based on the firms' reported depreciation, using the procedure described in Salinger and Summers (1983). A recursive formula is then used to calculate the replacement value of the capital stock, which evolves as

\[ K_t = (K_{t-1} + I_t) \left( \frac{p^{m}_{t-1}}{p^{m}_{t-1}} \right) (1 - \delta) \]

where \(p^{m}_{t}\) is the implicit price index for business investment in machinery and equipment (CANSIM series D11123) relative to the implicit price index for final domestic demand (CANSIM series 11130). The tax-adjusted relative price ratio, \(p^{r}_{t}\), equals \((1-k_t^{-1}z_t) / (1-t_t))\*p^{m}_{t}\), where \(t_t\) is the sum of the federal and provincial corporate income tax rates (incorporating variation across time and industries), \(k_t\) is the investment tax credit rate, and \(z_t\) is the present value of tax depreciation allowances.
### Table 1
**Summary Statistics**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Full Sample</th>
<th>Low Growth Firms</th>
<th>High Uncertainty Firms</th>
<th>Low Depreciation Firms</th>
<th>Utilities</th>
<th>Guiso-Parigi</th>
</tr>
</thead>
<tbody>
<tr>
<td>I/K</td>
<td>0.172</td>
<td>0.149</td>
<td>0.198</td>
<td>0.137</td>
<td>0.122</td>
<td>0.172</td>
</tr>
<tr>
<td></td>
<td>[0.251]</td>
<td>[0.188]</td>
<td>[0.295]</td>
<td>[0.211]</td>
<td>[0.154]</td>
<td>[0.261]</td>
</tr>
<tr>
<td></td>
<td>(0.340)</td>
<td>(0.179)</td>
<td>(0.421)</td>
<td>(0.368)</td>
<td>(0.150)</td>
<td>(0.426)</td>
</tr>
<tr>
<td>K</td>
<td>58758</td>
<td>73855</td>
<td>24463</td>
<td>79618</td>
<td>380002</td>
<td>71882</td>
</tr>
<tr>
<td></td>
<td>[321567]</td>
<td>[397412]</td>
<td>[84978]</td>
<td>[445586]</td>
<td>[636639]</td>
<td>[442742]</td>
</tr>
<tr>
<td></td>
<td>(726623)</td>
<td>(821060)</td>
<td>(171650)</td>
<td>(912696)</td>
<td>(730257)</td>
<td>(921378)</td>
</tr>
<tr>
<td>Real Sales</td>
<td>0.056</td>
<td>0.026</td>
<td>0.060</td>
<td>0.069</td>
<td>0.098</td>
<td>0.076</td>
</tr>
<tr>
<td></td>
<td>[0.087]</td>
<td>[0.016]</td>
<td>[0.089]</td>
<td>[0.104]</td>
<td>[0.089]</td>
<td>[0.128]</td>
</tr>
<tr>
<td></td>
<td>(0.127)</td>
<td>(0.035)</td>
<td>(0.145)</td>
<td>(0.162)</td>
<td>(0.072)</td>
<td>(0.189)</td>
</tr>
<tr>
<td>Uncertainty</td>
<td>0.574</td>
<td>0.539</td>
<td>1.148</td>
<td>0.383</td>
<td>0.055</td>
<td>0.411</td>
</tr>
<tr>
<td></td>
<td>[1.141]</td>
<td>[0.861]</td>
<td>[2.002]</td>
<td>[0.910]</td>
<td>[0.163]</td>
<td>[0.813]</td>
</tr>
<tr>
<td></td>
<td>(2.026)</td>
<td>(1.236)</td>
<td>(2.591)</td>
<td>(1.865)</td>
<td>(0.345)</td>
<td>(1.219)</td>
</tr>
<tr>
<td>Depreciation Rate</td>
<td>0.079</td>
<td>0.081</td>
<td>0.092</td>
<td>0.045</td>
<td>0.051</td>
<td>0.068</td>
</tr>
<tr>
<td></td>
<td>[0.079]</td>
<td>[0.081]</td>
<td>[0.094]</td>
<td>[0.043]</td>
<td>[0.055]</td>
<td>[0.069]</td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td>(0.046)</td>
<td>(0.051)</td>
<td>(0.025)</td>
<td>(0.030)</td>
<td>(0.045)</td>
</tr>
<tr>
<td>N</td>
<td>199</td>
<td>99</td>
<td>100</td>
<td>99</td>
<td>14</td>
<td>66</td>
</tr>
</tbody>
</table>

Median, [mean], (standard deviation). See Section 4 for precise definitions of the classes of firms and the Data Appendix for data sources and variable construction.
### Table 2
Initial Estimates of the Irreversibility Premium

#### Panel A
CAPM Risk Adjustment

<table>
<thead>
<tr>
<th></th>
<th>Low Growth</th>
<th>High Uncertainty</th>
<th>Low Depreciation</th>
<th>Limited Resale Markets (Utilities)</th>
<th>Limited Resale Markets (Guiso-Parigi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>θ</td>
<td>.030 (.027)</td>
<td>-.001 (.020)</td>
<td>.002 (.019)</td>
<td>.043 (.015)</td>
<td>-.048 (.019)</td>
</tr>
<tr>
<td>P</td>
<td>-.100 (.027)</td>
<td>-.080 (.012)</td>
<td>-.082 (.021)</td>
<td>-.086 (.015)</td>
<td>-.063 (.015)</td>
</tr>
<tr>
<td>γ</td>
<td>.950 (.005)</td>
<td>.950 (.005)</td>
<td>.950 (.005)</td>
<td>.950 (.005)</td>
<td>.952 (.005)</td>
</tr>
<tr>
<td>α₁</td>
<td>-1.435 (.386)</td>
<td>-1.594 (.365)</td>
<td>-1.600 (.369)</td>
<td>-1.611 (.365)</td>
<td>-1.626 (.367)</td>
</tr>
<tr>
<td>α₂</td>
<td>.065 (.097)</td>
<td>.084 (.095)</td>
<td>.085 (.096)</td>
<td>.086 (.095)</td>
<td>.088 (.095)</td>
</tr>
<tr>
<td>J</td>
<td>0.008 [.931]</td>
<td>0.015 [.903]</td>
<td>0.010 [.922]</td>
<td>0.000 [1.000]</td>
<td>0.021 (.884)</td>
</tr>
</tbody>
</table>
### Panel B

**Fama-French Three-Factor Risk Adjustment**

<table>
<thead>
<tr>
<th></th>
<th>Low Growth</th>
<th>High Uncertainty</th>
<th>Low Depreciation</th>
<th>Limited Resale Markets (Utilities)</th>
<th>Limited Resale Markets (Guiso-Parigi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>.081 (.035)</td>
<td>.016 (.023)</td>
<td>.007 (.020)</td>
<td>.045 (.016)</td>
<td>-.049 (.020)</td>
</tr>
<tr>
<td>( P )</td>
<td>-.084 (.014)</td>
<td>-.080 (.014)</td>
<td>-.080 (.022)</td>
<td>-.081 (.014)</td>
<td>-.057 (.015)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>.950 (.005)</td>
<td>.950 (.005)</td>
<td>.951 (.005)</td>
<td>.950 (.005)</td>
<td>.952 (.006)</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>-1.998 (.376)</td>
<td>-2.025 (.380)</td>
<td>-2.244 (.396)</td>
<td>-2.245 (.388)</td>
<td>-2.271 (.393)</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>.136 (.089)</td>
<td>.140 (.089)</td>
<td>.166 (.088)</td>
<td>.166 (.087)</td>
<td>.170 (.087)</td>
</tr>
<tr>
<td>( J )</td>
<td>.000 [.985]</td>
<td>.009 [.926]</td>
<td>.026 [.871]</td>
<td>.022 [.882]</td>
<td>.013 [.909]</td>
</tr>
</tbody>
</table>

Parameter estimates are based on equation (18) with the definitions in equations (19) and (20). The estimation method is GMM with a constant, \((REV_{t-1}/K_{t-1}),\ (COST_{t-1}/K_{t-1}),\ (I_{t-1}/K_{t-1}),\ r_{t-1}\), and a dummy variable representing the class of firms listed at the top of the column as the instruments. Standard errors are in parentheses under the parameter estimates. The p-value is listed in brackets under the J statistic (the test of overidentifying restrictions). See Section 4 for precise definitions of the classes of firms and the Data Appendix for data sources and variable construction.
Table 3
A Second Look at Resale Markets

<table>
<thead>
<tr>
<th></th>
<th>Guiso-Parigi, Non-Resource</th>
<th>Guiso-Parigi, Non-Resource Plus Utilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>.030</td>
<td>.040</td>
</tr>
<tr>
<td></td>
<td>(.019)</td>
<td>(.015)</td>
</tr>
<tr>
<td>$P$</td>
<td>-.083</td>
<td>-.089</td>
</tr>
<tr>
<td></td>
<td>(.014)</td>
<td>(.016)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>.951</td>
<td>.950</td>
</tr>
<tr>
<td></td>
<td>(.005)</td>
<td>(.005)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>-1.603</td>
<td>-1.621</td>
</tr>
<tr>
<td></td>
<td>(.366)</td>
<td>(.366)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>.085</td>
<td>.088</td>
</tr>
<tr>
<td></td>
<td>(.095)</td>
<td>(.095)</td>
</tr>
<tr>
<td>$J$</td>
<td>0.023</td>
<td>0.001</td>
</tr>
<tr>
<td></td>
<td>[.881]</td>
<td>[.969]</td>
</tr>
</tbody>
</table>

Parameter estimates are based on equation (18) with the definitions in equations (19) and (20). The estimation method is GMM with a constant, $(REV_{t-1}/K_{t-1})$, $(COST_{t-1}/K_{t-1})$, $(I_{t-1}/K_{t-1})$, $r_{t-1}$, and a dummy variable representing the class of firms listed at the top of the column as the instruments. Standard errors are in parentheses under the parameter estimates. The p-value is listed in brackets under the J statistic (the test of overidentifying restrictions). See Section 4 for precise definitions of the classes of firms and the Data Appendix for data sources and variable construction.
Table 4
Identifying 2 with Combinations of Factors

<table>
<thead>
<tr>
<th></th>
<th>Low Growth, High Uncertainty</th>
<th>Low Growth, Low Depreciation</th>
<th>Low Depreciation, High Uncertainty</th>
<th>Low Depreciation, Limited Resale Markets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>.044 (.025)</td>
<td>.051 (.019)</td>
<td>-.021 (.026)</td>
<td>.033 (.014)</td>
</tr>
<tr>
<td>$P$</td>
<td>-.092 (.016)</td>
<td>-.096 (.019)</td>
<td>-.078 (.014)</td>
<td>-.086 (.015)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>.951 (.005)</td>
<td>.951 (.005)</td>
<td>.949 (.005)</td>
<td>.950 (.005)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>-1.421 (.371)</td>
<td>-1.914 (.388)</td>
<td>-1.35 (.356)</td>
<td>-1.620 (.369)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>.063 (.097)</td>
<td>.125 (.093)</td>
<td>.054 (.097)</td>
<td>.878 (.095)</td>
</tr>
<tr>
<td>$J$</td>
<td>0.008 [.927]</td>
<td>0.005 [.944]</td>
<td>0.006 [.941]</td>
<td>0.001 [.980]</td>
</tr>
</tbody>
</table>

Parameter estimates are based on equation (18) with the definitions in equations (19) and (20). The estimation method is GMM with a constant, $(\text{REV}_{t-1}/K_{t-1})$, $(\text{COST}_{t-1}/K_{t-1})$, $(\text{It}_{t-1}/K_{t-1})$, $r_{t-1}$, and a dummy variable representing the class of firms listed at the top of the column as the instruments. Standard errors are in parentheses under the parameter estimates. The p-value is listen in brackets under the J statistic (the test of overidentifying restrictions). In this table, firms with limited resale markets are defined as utilities plus firms identified by the Guiso-Parigi technique which are not in resource industries. See Section 4 for precise definitions of the classes of firms and the Data Appendix for data sources and variable construction.
### Table 5
**Identifying \( \theta \) Based on Firms Close to the Zone of Inaction**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lowest 10%</th>
<th>Lowest 15%</th>
<th>Lowest 50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \theta )</td>
<td>.067 (.017)</td>
<td>.067 (.018)</td>
<td>.020 (.030)</td>
</tr>
<tr>
<td>( \tau )</td>
<td>-0.093 (.017)</td>
<td>-0.099 (.019)</td>
<td>-0.096 (.034)</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>.951 (.005)</td>
<td>.950 (.005)</td>
<td>.950 (.005)</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>-1.922 (.386)</td>
<td>-1.949 (.391)</td>
<td>-1.660 (.418)</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>.126 (.092)</td>
<td>.130 (.092)</td>
<td>.093 (.099)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.016 [.900]</td>
<td>0.000 [1.000]</td>
<td>.011 [.918]</td>
</tr>
</tbody>
</table>

Parameter estimates are based on equation (18) with the definitions in equations (19) and (20). The estimation method is GMM with \( \text{REV}_{t-1}/K_{t-1} \), \( \text{COST}_{t-1}/K_{t-1} \), \( \text{I}_{t-1}/K_{t-1} \), \( r_{t-1} \), and a dummy variable representing the class of firms listed at the top of the column as the instruments. Standard errors are in parentheses under the parameter estimates. The p-value is listed in brackets under the \( J \) statistic (the test of overidentifying restrictions). See Section 4 for precise definitions of the classes of firms and the Data Appendix for data sources and variable construction.