The choice of inefficient instrument in a simple retrospective voting model with voter abstention

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Abstract

I consider the problem of why inefficient tariff is used. The incumbent politician can use lump-sum transfer and tariff. After the decision, the incumbent goes to the re-election. Thus the incumbent has to take the re-election into his consideration when he makes trade policy decision. The originality of this paper is the use of voter abstention. The abstention and voter heterogeneity would give opportunity to the incumbent to minimize the political opposition to him at re-election. For the minimization, the incumbent would choose tariff.

Keywords: tariff; lump-sum transfer; election; abstention; choice of instrument

JEL classification: D63; D72; F13

1 Introduction

“unequal participation spells unequal influence” (Arend Lijphart (1997))

The objective of this paper is to propose an explanation about why governments use inefficient instruments. Especially, I consider why governments use tariff to protect domestic manufactures, though the tariff is inefficient compared to subsidy from the viewpoint of economic welfare. The hinge of the explanation is the voter abstention and the minimization of political resistance, or political cost, by the incumbent politician to his policy. As for the problem of tariff choice, there is the literature of Endogenous Tariff Formation (Mayer (1984), Grossman and Helpman (1994), Yang (1995), e.t.c.). However, they generally do not answer to the question of why the inefficient tariff is used. Instead, they answer to the question about the level of tariff with assuming away the subsidy (or lump-sum transfer) from their models. Rodrik (1995) and Dixit and Roemer (2006) criticize such restriction on the set of possible policy instruments. Rodrik

I thank Professor Jay D. Wilson, Steven J. Matusz, Douglas Nelson and Eric Chang for kind and valuable comments on the idea of this paper. Without their comments, the idea could not have developed to the current one. The responsibility for the content and possible mistakes is mine.
(1995) says “the existing literature is largely a literature on the political economy of redistribution, and not of trade policy proper” (pp.1470). In this paper I try to give an explanation to the question in a simple retrospective voting situation with voter abstention. Of course, there are some other papers about why tariff is used. Generally, however, what they do is the comparison of economic welfare between the case that tariff is used and the case that subsidy is used; then they show situations where tariff achieves higher welfare than subsidy. In this paper, an incumbent has both of tariff and subsidy at his discretion. He has to decide how he uses them.

I consider a situation that an incumbent faces an appeal from import competing domestic manufactures for help. The incumbent can use both of tariff and subsidy simultaneously. If the incumbent decides to help domestic manufactures, he has to choose a combination of income tax and tariff to finance the help. After the decision, the incumbent will go to the re-election. The outcome of the re-election depends on the decision he made during his term. The incumbent is office oriented, caring only winning the re-election. For the winning, the incumbent has to take into his consideration of policy decision two things: voter heterogeneity and voter abstention. Voter heterogeneity is the main idea behind the endogenous tariff formation papers with election (e.x. Mayer (1984)). Because of the heterogeneity, different voters prefer different policies. This paper considers the specific factor model, so the heterogeneity is represented as workers with only labor and capitalists with specific capital. Though abstention was also incorporated into a model in Mayer (1984), the idea of abstention has not been explored much in the literature of trade policy determination. In this paper, voter abstention plays the main role and gives the capability of voter manipulation to the incumbent. Because of the voter heterogeneity, different instrument affects different voters differently. Then, since the voter abstention at the re-election is supposed to depend on the size of voter groups and how voters were affected by the decision of the incumbent during his term, the incumbent can choose policy instruments to affect the number of votes against him at the re-election. In other word, he can choose the combination of instruments (subsidy, tax and tariff) to minimize the political opposition to him (votes for his challenger) for any level of help he chooses. Of course, he might choose no help. But, if he chooses to give help, what combination of instruments would he choose? I am going to check if he would choose only efficient lump-sum transfer.

As I wrote above, there are papers about why tariff or inefficient instrument is used. Rodrik (1986) shows that since the tariff has public goods character compared to firm specific subsidies, the lobbying effort for tariff is undersupplied, compared to the lobbying effort for firm specific subsidy. So the distortion under the tariff would be lower than the one under the subsidy. Mayer and Riezman (1987) show that when the country is small and voters are heterogeneous in factor endowment, voters prefer production tax cum subsidy to tariff. So, if voters will vote on trade protection like in Mayer (1984), tariff will not be chosen. But in cases that the country is large or that voters are heterogeneous in both of factor endowment and consumption preference, they say it is possible for all or some voters prefer tariff. Wilson (1990) shows that
efficient instrument could invite more lobbying pressure so the total distortion could be larger under the efficient instrument than under inefficient one. Since politicians care about the level of total distortion that would affect voters’ sentiment toward politicians negatively, there is an incentive for them to restrict the possible policy instrument to the inefficient one. Mayer and Riezman (1990) talk about several cases in which some voters prefer tariff to subsidy, like the case of heterogeneous, the case of risk averse voters in the uncertainty about tariff rate and tax rate, e.t.c, though they do not show how such voter preferences will be reflected in political process of choosing policy instrument. In this paper, preferences of potential voters on instruments are also different among different groups. Since the reason for such difference is similar to the one in the one section of Mayer and Riezman (1990), my paper could be considered as a sort of an extension of their paper\textsuperscript{1}. Coates and Morris (1995) also considers a model with uncertainty. They consider two types of uncertainty; one about the outcome of public project and another about the type of incumbent politician. The incumbent politician has information about the project outcome and will face the re-election. Good type politician will carry on the project only when the information he has tells him that the project has high probability of producing good outcome. Bad incumbent wants to transfer money to special interests. He can do that through direct subsidy or public project. Voters update their reputations on the type of politician based on what the incumbent does and what is its outcome. So, to increase the probability of winning the re-election, bad type politician chooses the project over direct subsidy (this gives him higher utility than the project) so that he can pretend a good type thinking the project has high probability of success.

Above, I mentioned papers in which there is no intrinsic difference of difficulty in implementing different policy instruments. If there is, however, difference of difficulty (for example, cheaper collection cost of tariff and higher cost of other taxes), it is actually no wonder that governments would choose easier one. Gordon and Li (2005a) argues that governments of developing countries rely more on tariff as their revenue source than the ones of developed countries because it is more difficult to monitor economic activities of domestic private firms in developing countries than in developed countries. Gordon and Li (2005b) claim that the model of Gordon and Li (2005a) fits actual data better than Grossman and Helpman (1994). Even in current developed countries, the tax collection was not a easy problem in the past (maybe still). Gardner and Kimbrough (1992) applied to U.S. this line of explanation about tariff\textsuperscript{2}. Though my paper is different in many aspects from this collection cost type explanation, there is a common element; politicians choose the policy instruments which are politically cheaper (or easier). In this paper, the cost is the votes

\textsuperscript{1}The reason is similar, but not same. In the paper of Mayer and Riezman (1990), the preference for tariff comes from the difference of income tax rate and the share of distribution of tariff revenue. In this paper, the preference comes from the difference of income tax rate and the share of spending on importing good of different consumers.

\textsuperscript{2}I thank Professor Wilson and Professor Nelson for putting my attention to those papers and this line of argument.
for the challenger at the re-election, not monitoring and implementation costs of policies. This difference in the nature of cost could have a big implication for welfare (maybe benevolent dictator might choose tariff when the collection costs of other taxes are high. But will the dictator choose tariff for political cost reason when there is no intrinsic cost difference between tariff and other taxes?) But from the viewpoint of each politician, whatever the reason of the cost difference is, anyway they are all political costs for the policy he wants to implement. So, politicians want to minimize those political costs. In this sense, I think this paper and the collection cost explanation are in complimentary, looking at different type of costs, not contradicting each other as the explanation about why tariff is used.

Though not as a paper about why tariff, as a paper with similar idea to this one, Anderson and Zanardi (2004) consider “political pressure deflection”. In their model, politicians hide behind delegations to hide their policy preferences. Though the delegation means giving away some political power, it could deflect political pressure to them.

As another related idea, I would like to mention Acemoglu and Robinson (2001). They consider a two-period model with a special interest group whose power source is the number of voters belong to it. In multi-periods models, policy commitment by politicians is an important problem. If it is not guaranteed by an assumption, it has to be guaranteed by some mechanism (Harrington (1992)), or no policy commitment (Alesina (1988)). In Acemoglu and Robinson model, the special interest prefers the inefficient policy to efficient one since it distorts newcomers’ incentive toward joining to the special interest, that guarantees the power of the special interest in the second period and the commitment by the government to the transferring resource to the special interest in the period.

As I wrote above, the abstention is the hinge of the model in this paper. As the surveys of abstention, Aldrich (1993) and Feddersen (2004). There are many papers about abstention and models about why some potential voters abstain from voting (Harsanyi (1980), Shachar and Nalebuff (1999), Feddersen and Sandroni (2006), e.t.c). However, since my attention is not on the abstention itself but on its potential effect for the choice of policy instruments, I go on an easy way about abstention: a black box approach. I assume a voting participation function and the incumbent knows it. As extensively emphasized in Lijphart (1997), the very important empirical observation of the abstention, or voting participation, is that “socioeconomic status and voting were positively, not, negatively, linked” (Lijphart (1997, pp.1)). Higher income and better educated, more likely a potential voter votes. I do not consider why. In the following, I simply assume that people sometimes do not vote because of the free rider problem and a small voting cost; more population, so less likely one’s vote determines the election, though it is equally costly to the voter. So, why vote? Here I assume that “population” is the population of the group a potential voter

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3 Actually, in this formulation, the problem is not why some potential voters do not vote, but why there is some potential voters who do vote. This is the reason why this problem is called “the paradox of not voting” (Feddersen (2004)).
belongs to. This means that I assume that voters see their socio-economic group they belong to as their reference group to vote or not (for the importance of reference groups for voting decision, see Schram (1992), Franklin (2004)). Thus, higher the population of a group, potential voters in the group are less likely to vote. These assumptions do not imply above empirical observation directly. But with an additional assumption that the population of type X capitalists (assumed to be rich) is less than the one of workers (assumed to be poor or less rich), the assumptions fit with empirical observations in the model. Since my objective is not abstention itself, this is enough.

In the following, I describe the model in section 2, results in section 3. The conclusion is in the section 4. Some proofs of less relevant claims are relegated to the Appendix.

2 Model

Firstly, I am going to explain the economy side of the model, then its political side, and the flow of the game.

2.1 Economy side

I consider a specific factor model of a small country consisting of three groups: type X capitalists, workers and type Y capitalists. The population is continuum and each group has population $N_x, N_w$ and $N_y$. It is assumed that $N_x + N_w = N_y < 1$. There are two goods, good X (exporting good) and good Y (import competing good). The production functions of good X and good Y are usual ones: use labor and capital, differentiable, increasing in each argument, and homogeneous of degree one. But, the capital is specific. The good X is manufactured with capital X and labor, not with capital Y and labor, and vice versa for good Y. I assume the perfect competition. Let $l_a$ be the labor employed in the production of good $a \in \{x, y\}$. Since the total amount of both capital is fixed, I denote the production function of good $a$ as the function of only labor, $f_a(l_a)$ for $a \in \{x, y\}$. Each of type X capitalists has only one unit of capital X, and each of type Y capitalists has only one unit of capital Y. Also, each worker has one unit of labor (so, $l_x + l_y = N_w$). Thus, the incomes they receive come from profit of good X industry (for type X capitalists), profit of good Y industry (for type Y capitalists) and wage (for workers). Let good X be the numeraire. Given wage rate $w$, the total profit from production of good X is $f_x(l_x) - w l_x$. I denote the before tax (and before tariff revenue distribution) income as $z$. Then, since the population of type X capitalists is $N_x$, the income of one type X capitalist is $z^x \equiv \frac{f_x - w l_x}{N_x}$. The incomes of workers and type Y capitalists are $z^w = w$ and $z^y$. Though there is no intrinsic necessary reason in the model, I assume that $z^x > z^w \equiv w$.

Type X capitalists and workers consume good X and good Y. But, to simplify the model, type Y capitalists are assumed to consume only good X. Let $x^a$ and $y^a$ be the consumption of good X and good Y by one type X capitalist or one worker.
The common utility function of type X capitalists and workers is \( U(x, y) \). No dis-utility of labor supply for workers. Of course, this is a strictly concave, derivative and increasing function in both arguments. Let \( p \) be the domestic price of good Y. Denote disposable income as \( I \), which is after-tax and tariff distribution income. Then, let \( y(p, I) \) be the common demand function of good Y for type X capitalists and workers derived from the utility maximization of \( U(x, y) \) with disposable income \( I \) and price \( p \). From this, \( x^a = I^a - py^a(p, I^a) \). Here, deviating from the tradition of international economics, I assume the common preference of type X capitalists and workers is not homothetic. So, the income elasticities of good X and good Y are not one. The indirect utility function for type X capitalists and workers is \( W(I, p) = U(I - py(I, p), y(I, p)) \).

As for the type Y capitalists, since they are supposed to consume only good X, I assume their utility function is their consumption of good X, \( x^y \).

The country used to be an autarky economy. But it starts trading. Let \( \bar{p} \) be the world fixed price of good Y, which is lower than the autarky price of the good Y. Since the income of type Y capitalists come from the profit of good Y production and they consume only good X, the trade definitely hurts type Y capitalists.

### 2.2 Political side

The country has an incumbent politician in office, who faces the appeal for help from type Y capitalists. He has to decide to give help or not, and if give help, he would have to decide how to finance it. Government has two options for the help; tariff rate \( \tau \) (specific tariff) and per unit production subsidy \( s \). After the decision, he will face the re-election, where potential voters decide to re-elect him or not based on utility level they received during his term compared to the case the incumbent did not make the decision. If he decides to help type Y capitalists, it helps them but hurts type X capitalists and workers who have to finance the help. Thus, the incumbent has to take into his consideration the reactions from all potential voters. Though this type of re-election model is a common setting, since my purpose is to consider why inefficient instrument would be used, the re-election in this paper is intentionally very simple one (as shown later) except for one assumption: abstention. Not all potential voters vote. Even though Mayer (1984) incorporates it in some of its models, this is not a common feature in the tariff determination literature. To incorporate the abstention, I assume a common election participation function for all groups. I will explain more about it later.

If the incumbent decides to give help to type Y capitalists, the help has to be financed. Tariff is, of course, financed by tariff itself. The tariff revenue is returned only to type X capitalists and workers. Remember that type Y capitalists do not consume good Y. I will explain more about the distribution of tariff revenue later. The production subsidy \( s \) is financed by income tax \( t \). Notice that since this model lacks the dis-utility of labor supply, the income tax is equivalent to lump-sum tax. Here, again to simplify the model, I assume that type Y capitalists do not pay the income tax.
The objective of the incumbent is to win the election, or more precisely, to maximize the probability of winning. For the election, the only decisions the incumbent can make are the decision to help or not and how to finance the help. The challenger is assumed not to propose anything. His role in this model is just to provide an alternative to potential voters and makes retrospective voting possible for them. He can be considered as a convinced free trader. But it is not that the incumbent can determine everything. As for the participation rate, there is small stochastic elements (this might come from weather, political scandals, campaigns of incumbent and the challenger to influence the election outcome, e.t.c.). If the incumbent decides to give help, this increases the welfare of type Y capitalists but hurts type X capitalists and workers. Thus, the help makes every potential voter of type Y capitalists prefer the incumbent but makes every potential voters of type X capitalists and workers prefer the challenger. So, if every potential voter votes, the incumbent would never give help because if he would the challenger should win the election since \( N_x + N_w = 1 > N_y \).

But not all potential voters vote, because of abstention. As I wrote above, I take an easy way to incorporate this.

**Participation function:** Let \( V(N, d) \) be the function for expected voting participation rate. The function is defined on each group, i.e., giving the participation rate for each group, and same for all group. The argument \( N \) is the population of the group and \( d \) is the utility difference between the utility of no action by the incumbent and the utility by the policy actually taken by the incumbent (the “the policy actually taken” could be no action. In that case \( d = 0 \)). This \( d \) is defined so that \( d \geq 0 \). Then, the group with \( d = 0 \) is indifferent between the incumbent and the challenger. In the case, following the Political Economy tradition, half of the group is supposed to cast their vote to the incumbent and other half cast votes to the challenger. For example, \( V(N_x, d) \) gives the expected participation rate of type X capitalists given the utility difference of \( d \), and \( N_x V(N_x, d) \) gives the expected number of type X capitalists voters. The abstention is supposed to happen because of free rider problem and a small voting cost. Since every member in a group is same, every one would vote for the same candidate if they vote. Thus, larger the population of a group is, more likely a potential voter in the group is not to vote (because others will do that for him/her, anyway). At same time, larger the utility difference, more likely the potential voter to vote. Thus, I assume the following,

\[
V_N \leq 0, \ V_{NN} \leq 0, \ V_d > 0, \ V_{dd} > 0^4.
\]

I also assume that

\[
V_d(0) = 0, \ V_{dN} < 0^5.
\]

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4. \( V_{dd} > 0 \) might be a questionable assumption. Maybe this assumption can be replaced by a weaker and less questionable assumption. But I hold it here for the sake of simplicity.

5. Hill and Leighley (1996) did regressions of participation rates of several income groups on many factors. Their results suggest that higher the income group, more the participation rate responds to the income level change, though the highest income group is an exception. Since the models in their paper and this are different, that is, however, not a direct support for the assumptions on \( V \) here.
To save space, hereafter, I denote $V(N^x, d)$ and $V(N^w, d)$ as $V^x(d)$ and $V^w(d)$, respectively. Since $N^x < N^w$ and $V_{dN} < 0$, $V^x(d) > V^w(d)$ for any level of utility difference $d$. So, type X capitalists who are assumed to be rich than workers are more likely to vote than workers, same as empirical evidences. As for the argument $d$, assuming that the utility difference affects the participation rate is intuitive and common in papers with abstention, though actual forms vary in different papers (Hinich, Ledyard and Ordeshook (1972), Mayer (1984), e.t.c.)

The actual participation rate: The actual rate is the sum of this expected part and stochastic part. The stochastic part is represented by $\varepsilon$ for each group. Thus, the actual participation rate for a group $k$ is $V^k(d^k) + \varepsilon^k$, where $\varepsilon^k$ is a bounded random variable with mean zero and variable $\sigma^2$. $\varepsilon$s are i.i.d. The ranges of these random variables are same for all three groups and small enough so that $V^k(d^k) + \varepsilon^k$ is in $(0, 1)$.

2.3 The flow of the game, and the number of votes

The flow of the game is simple; the incumbent decides if he gives the help asked by type X capitalists or not, and if give, he would decide how to finance it. At the re-election, potential voters vote according to the their utility differences. The winner goes to the government office.

Let $H \equiv \tau + s$ be the sum of the subsidy from government and the price hike by tariff. I consider only the case of $H \geq 0$. As I will explain later, the utility of type Y capitalists depends only on $H$. But utilities of type X capitalists and workers depend on not only $H$ but also $t$ and $\tau$. The decision of the incumbent consists of $H$, income tax rate $t$ and tariff rate $\tau$. The incumbent knows the participation function $V$. But the actual participation rate depends on $V$ and $\varepsilon$s. Let $d^x$, $d^w$ and $d^y$ be the utility differences of type X capitalists, workers and type Y capitalists.

If the incumbent decides not to give the help asked by type Y capitalists, the incumbent and the challenger have same expected number of votes. Both have probability of 0.5 to win the election. If the incumbent decides to give help, since the help hurts type X capitalists and workers, no potential voters of type X capitalists or workers will vote for the incumbent. Only potential type Y capitalist voters might vote for the incumbent. Then, the number of votes the incumbent receives minus the number of votes the challenger receives is,

\[
N^y(V^y(d^y) + \varepsilon^y) - N^x(V^x(d^x) + \varepsilon^x) - N^w(V^w(d^w) + \varepsilon^w)
\]

\[
= N^yV^y(d^y) - N^xV^x(d^x) - N^wV^w(d^w) + N^y\varepsilon^y - N^x\varepsilon^x - N^w\varepsilon^w
\]

This is the random variable with mean

\[
(1) \quad N^yV^y(d^y) - N^xV^x(d^x) - N^wV^w(d^w)
\]

and the variance $(N^{y^2} + N^{x^2} + N^{w^2})\sigma^2$. Since the variance is constant, by maximizing the mean (1), the incumbent can maximize the probability of winning\(^6\). Since $d^x$ depends only on $H$, $d^w$ and $d^y$ depend on $H$, $t$ and $\tau$,

\[^6\text{Here is a potential problem that (1) could be too high so that } N^yV^y(d^y) - N^xV^x(d^x) - N^wV^w(d^w) + N^y\varepsilon^y - N^x\varepsilon^x - N^w\varepsilon^w \text{ cannot be negative in the range of } \varepsilon$s. In such case, the}
for any level of $H$, the incumbent has to minimize $N^x V^x(d^x) + N^w V^w(d^w)$ to maximize (1) by choosing an appropriate combination of $t$ and $\tau$. If there is some combination of $H$, $t$ and $\tau$ such that the value of (1) is positive, clearly the incumbent will choose to give help. That such combination exists or not depends on the parameters of populations, $\sigma^2$ and also $V$. If there is no such combination, the incumbent would choose no help. I am going to check, in the case that such combination exists, if it is possible that $\tau = 0$ would be chosen by the incumbent at equilibrium, so only efficient instrument (production subsidy) would be used. Also, if it is possible, I check what is the condition for it.

3 Results

I am going to consider two systems of income tax: the flat income tax rate system and two-tier tax rates system. As I said above, the income tax in this economy is equivalent to lump-sum tax. The tax revenue is transferred as subsidy to type $Y$ capitalists. Thus, unlike many endogenous tariff formation papers, this paper incorporates lump-sum transfer. However, the flat tax rate means that there is still a restriction on the form of the transfer: no different tax treatment among groups. I consider the flat tax case first. Then I consider the two-tier income tax rates system, which loosens the restriction. I am going to show how such relaxation of the restriction affects the choice of instruments.

Before stating main results, I explain about the tariff revenue distribution.

**Tariff**: Let $\delta^x$ and $\delta^w$ be the distributional shares of tariff revenue of each type $X$ capitalist and each worker (those shares are same within a group). Since only type $X$ capitalists and workers receive tariff revenue, it has to be that $N^x \delta^x + N^w \delta^w = 1$. I am going to consider only three types of distributional methods. These shares are assumed to be fixed ones; tariff rate change will not affect these shares. In the trade literature, the common assumption on distribution share is the distribution proportional to income (e.g., Mayer (1984)). Let $z$ be the sum of before-tax incomes of type $X$ capitalists and workers, which is $N^x z^x + N^w z^w = N^x \frac{f^x - w^x}{z^x} + N^w z^w$. Then the shares for type $X$ capitalists and workers are $\delta^x = \frac{z^x}{z}$ and $\delta^w = \frac{w}{z}$. I also consider two more way of distributions: the distribution proportional to consumption and the simple equal distribution. Let the average and total consumption of good $Y$ be $y \equiv N^x y^x + N^w y^w$. Then, the shares of a type $X$ capitalist and a worker with the distribution proportional to consumption is, $\delta^x \equiv \frac{y^x}{y}$ and $\delta^w \equiv \frac{y^w}{y}$. The simple equal distribution is $\delta^x = \delta^w = 1$, so every type $X$ capitalist and worker receives same amount of tariff revenue.

incumbent does not have incentive to maximize (1). This is problematic for the argument in the following. But anyway, the structure of election itself is not the objective of this paper, so I assume that the incumbent always tries to maximize (1).

Of course, I am still putting a restriction that everyone in same group face same tax burden.
3.1 The flat tax system

In this subsection, I consider the income tax system consisting of only the flat tax rate $t$. Other than this $t$, government has tariff $\tau$ and subsidy $s$ as its policy instruments. Let $w$ be the wage rate in terms of good X. Given the production levels of good X and good Y, the government budget constraint is,

$$sf_y = tz = t\{f_x - wt_x + N^w w\} = t\{f_x + t_y w\}$$

From this, $t = \frac{f_y}{z}$. As for the production levels, given $H \equiv \tau + s$, the following is the equilibrium condition for them.

$$f'_x(t_x) = w = (\bar{p} + H)f'_y(t_y)$$

**Changes in wage and production levels:** Suppose that government is giving positive level of $s$ but $\tau = 0$. Then, suppose that the incumbent decides to increases tariff slightly but reduces the subsidy $s$ by same amount so that the level of $H$ does not change (i.e., $\frac{ds}{d\tau} = -1$). If $w$ does not change in regardless of this change, there would be no reason for change in production levels of good X and good Y. Since type Y capitalists are assumed not to consume good Y, this means the utility level of type Y capitalists depends only on $H$ (as I wrote above), not on $\tau$ and $s$. Since type X capitalists and workers consume good Y, even if $w$ and production level do not change, changes in $\tau$ and $s$ (or $t$) would cause changes in their consumption of goods and utility levels. Since the country was importing good Y and exporting good X before the increase of tariff, if the increase is small, the country would still continue to import good Y and export good X even after the tariff increase. So, the combined effect of changes in demand and no-change in supply should be reflected in changes in export and import if the tariff increase is small. Since the balanced trade is assumed to be an equilibrium condition as usual in trade literature, if the trade after the tariff increase is still balanced with the same wage level, it means that the change in components of $H$ is consistent with no changes in wage and production levels. This is the case. Though this need to be proved for the following arguments, it is not important itself. So I put the proof in the appendix. Then, hereafter in this subsection, I consider the changes of $\tau$ and $s$ such that causes no change in $H$ and no change in wage and production levels.

3.1.1 Main results under flat income tax rate system

As I stated above, the utility of type Y capitalists depends on $H$, not on its components, $\tau$ and $s$. But the utilities of type X capitalists and workers depend on $\tau$ and $s$ (through the domestic price of good Y and the income tax $t$). Though any positive level of $H$ reduces their utilities, different combinations of $\tau$ and $s$ lead to different utility levels for them. Since the voting participation rates of type X capitalists and workers depend on their utility levels and if potential voters of type X capitalists and workers actually vote they will vote for the challenger, the incumbent has to find the combination of $\tau$ and $s$ such that minimizes their combined voting participation rates. Such combination is, for a given level of $H$, the one that minimizes the following,
were after-tax income

capitalists and workers spend their disposable income. Elasticity of good Y is not one, so, they would have opposite signs. Since type X and type Y capitalists and workers have different income elasticities, their indirect utility of type X capitalists and workers depends on their wealth effect of tariff increase on type X capitalists and workers, take the help to type Y capitalists is all financed by the income tax. To check disparity between type X capitalists and workers, look at the help to type Y capitalists is all financed by the income tax. To check disparity between type X capitalists and workers, look at

\[ W((1 - t)z^a + \delta^a \tau Q, p + \tau) \]

For a given level of H, denote this as

\[ W(\tau : H) = W((1 - t)z^a + \delta^a \tau Q, p + \tau). \]

**Disparity between type X capitalists and workers**

Given \( H \), if \( \tau = 0 \), the help to type Y capitalists is all financed by the income tax. To check the welfare effect of tariff increase on type X capitalists and workers, take the derivative of \( W \) w.r.t. \( \tau \) for \( a \in \{ x, w \} \),

\[ W_\tau(\tau : H) = W_I\{-z^a \frac{\partial}{\partial \tau} + (1 - t) \frac{\partial z^a}{\partial \tau} + \delta^a Q + \delta^a \tau Q\} + W_p \]

At \( \tau = 0 \) with the change of \( \tau \) and \( s \) such that \( \frac{\partial s}{\partial \tau} = -1 \), this becomes

\[ W_\tau(0 : H) = W_I\{\frac{f_y}{z} z^a + \delta^a Q\} + W_p \]

\[ = \frac{W_I}{z} \left[ f_y z^a + \delta^a Q z - y^a z \right] \quad \text{(from the Roy's identity)} \]

\[ = \frac{W_I}{z} \left[ (f_y + Q) z^a - y^a z + (\delta^a z - z^a)Q \right] \]

\[ = \frac{W_I}{z} \left[ (N_x y^x + N_w y^w) z^a - y^a (N_x z^x + N_w w) - (z^a - \delta^a z)Q \right] \]

\[ \quad \text{(from } f_y + Q = N_x y^x + N_w y^w \text{ and } z = N_x z^x + N_w w) \]

If \( a = x \), the inside of this bracket becomes,

\[ N_w y^w z^x - N_x y^x w - (z^x - \delta^x z)Q \]

\[ (3a) \]

If \( a = w \), the inside of this bracket becomes,

\[ N_x y^x w - N_w y^w z^x - (z^x - \delta^x z)Q \]

\[ (3b) \]

Looking at (3a) and (3b), it is clear that the first terms of them have opposite signs to each other, or they are zero. If before-tax income is in these first terms were after-tax income is, those first terms would be non-zero because the income elasticity of good Y is not one, so, they would have opposite signs. Since type X capitalists and workers spend their disposable income \( I \), not before-tax income \( z \), the assumption that good X is luxury good makes it likely that they are non-zero but does not guarantee that. The distribution proportional to income, however, guarantees it. The equal distribution in case of the income elasticity of good Y less than one also guarantees the non-zero. Since the proof of these
is simple but irrelevant to the following arguments, I give the proof in the appendix. The distribution proportional to consumption does not guarantee it. Of course, the total effect of tariff increase depends on both of first and second terms. Again, as easily seen in (3a) and (3b), the distribution proportional to income is the easiest case; since shares of distribution proportional to income is $\delta^a = \frac{z^a}{y^a}$, the second terms of (3a) and (3b) disappear. So, (3a) and (3b) are non-zero, and they have opposite signs. If the income elasticity of Y is less than one, type X capitalists would prefer tariff increase from zero tariff since this could reduce their tax burden, and the workers would prefer income tax to positive tariff.

In the case of the consumption proportional distribution, i.e., $\delta^o = \frac{y^o}{y}$, (3a) becomes,

$$N^w y^w w \left( \frac{z^x}{w} - \frac{y^x}{y^w} \right) - \left( z^x - \frac{y^x}{y} \right) Q$$

$$= N^w y^w w \left( \frac{z^x}{w} - \frac{y^x}{y^w} \right) - \frac{N^x y^x z^x + N^w y^w z^x - N^x y^x z^x - N^w y^w w Q}{y}$$

$$= \frac{f_y}{y} N^w y^w w \left( \frac{z^x}{w} - \frac{y^x}{y^w} \right) \left( \frac{z^x}{w} - \frac{y^x}{y^w} \right) (from \ y = f_y + Q)$$

(3b) becomes,

$$-N^x y^w w \left( \frac{z^x}{w} - \frac{y^x}{y^w} \right) - \left( w - \frac{y^w}{y} \right) Q$$

$$= -N^x y^w w \left( \frac{z^x}{w} - \frac{y^x}{y^w} \right) - \frac{N^x y^x z^x + N^w y^w w - N^x y^x z^x - N^w y^w w Q}{y}$$

$$= \frac{-f_y}{y} N^x y^w w \left( \frac{z^x}{w} - \frac{y^x}{y^w} \right)$$

Thus, the signs of total effect of tariff increase are opposite, or zero. Consider, again, the case of less than one income elasticity of good Y. Then, since it means $\frac{i^x}{y^w} > \frac{i^x}{y}$, the necessary condition for (3a) to be non-positive would be, $\frac{i^x}{y^w} > \frac{i^x}{y}$. In other words, in this case, the necessary condition would be that the income inequality between type X capitalists and workers widens after income tax and tariff distribution.

In the case of the equal distribution, (3a) becomes,
\[ N^w y^w w \left( \frac{z^x}{w} - \frac{y^x}{y^w} \right) - (z^x - z)Q \]

\[ = N^w y^w w \left( \frac{z^x}{w} - \frac{y^x}{y^w} \right) - N^w (z^x - w)Q \]

\[ = N^w y^w w \left[ \frac{z^x}{w} - \frac{y^x}{y^w} \frac{Q}{y^w} \left( \frac{z^x}{w} - \frac{y^x}{y^w} \right) \right] \]

The sign of this is same to \[- \frac{z^x}{w} - \frac{y^x}{y^w} \left( 1 + \left( \frac{z^x}{w} - 1 \right) \frac{Q}{y^x} \right)\]. The sign of (3b) is same to \[- \frac{z^x}{w} - \frac{y^x}{y^w} \left( 1 + \left( \frac{z^x}{w} - 1 \right) \frac{Q}{y^x} \right)\]. Notice that \( Q \) is divided by \( y^x \), not by \( y^w \).

Thus, unlike two other cases, both can have same non-zero sign. For example, suppose that the income elasticity of good Y is less than zero, again. Then, sincethis means \( z^x > y^x \) with the equal distribution, the sign of (3a) would be positive if and only if \( \frac{z^x}{w} / \frac{y^x}{y^w} > 1 + \left( \frac{z^x}{w} - 1 \right) \frac{Q}{y^x} \). Since \( z^x > w \), the right hand side is always more than one. This condition is more likely to hold as lower the income elasticity of good Y, as smaller the before-tax income inequality \( \frac{z^x}{w} \), and as smaller \( \frac{Q}{y^w} \) is. Since \( y^x > y^w \), both of (3a) and (3b) can have negative signs. However, if (3a) is positive, (3b) is always negative, so, no double positive sign.

Hereafter I consider only the income proportional distribution because, firstly, it is the easiest in calculations; secondly, it is a common assumption about the tariff revenue distribution; and thirdly, the implication is same as long as (3a) is positive and (3b) is negative; as long as type X capitalists prefer tariff increase, same results could be obtained with any distribution share (in the case that workers prefer tariff increase, things would be more complicated).

### 3.2 The change of the number of votes for the challenger w.r.t. \( \tau \)

For any given level of \( H \), the incumbent wants to minimize

\[ (2) \quad N^x V^x (d^x) + N^w V^w (d^w) \]

So, the incumbent wants to choose the value of \( \tau \) such that \( N^x V^x d^x_\tau + N^w V^w d^w_\tau = 0 \). I need to check if \( \tau = 0 \) achieves it\(^\text{10}\). The utility difference \( d \) is defined in terms of the indirect utility function \( W \) as follows.

**Utility difference** \( d = W(0:0) - W(\tau:H) \)

Since the first term is constant, the derivative of \( d \) w.r.t. \( \tau \) for a given level of \( H \) is,

\[ \frac{\partial d}{\partial \tau} \bigg|_{dH=0} = - \frac{\partial W}{\partial \tau} \bigg|_{dH=0} \]

Then, substituting formulas obtained above into \( N^x V^x d^x_\tau + N^w V^w d^w_\tau \) at \( \tau = 0 \),

\(^{10}\) I do not check the concavity of (2) with respect to \( \tau \) here since if (2) is convex at \( \tau = 0 \) then the incumbent clearly has an incentive to increase \( \tau \), given that he wants to give positive \( H \).
\begin{align*}
&\text{I got,} \\
&\quad -N^x V_d^x \frac{W_d^x}{z} N^w y^w w \left( \frac{z^x}{w} - \frac{y^x}{y^w} \right) + N^w V_d^w \frac{W_i^w}{z} N^x y^w w \left( \frac{z^x}{w} - \frac{y^x}{y^w} \right) \\
&\quad = N^x N^w y^w w \left( \frac{z^x}{w} - \frac{y^x}{y^w} \right) (V_d^w W_i^w - V_d^x W_i^x)
\end{align*}

If the income elasticity of good \( Y \) is fixed to one (like homothetic utility functions), this is zero. So, the incumbent might choose \( \tau = 0 \). Since the elasticity is not one, the result depends on \( \left( \frac{z^x}{w} - \frac{y^x}{y^w} \right) (V_d^w W_i^w - V_d^x W_i^x) \). I state my first Proposition.

**Proposition 1** In the case that the incumbent decides to give help, tariff, which is the inefficient instrument, will be used if \( \left( \frac{z^x}{w} - \frac{y^x}{y^w} \right) (V_d^w W_i^w - V_d^x W_i^x) < 0 \).

If the condition holds, the incumbent could reduce the number of votes his challenger would receive at the election by increasing the tariff. If the left hand side of the condition is positive, the incumbent can reduce the number by reducing tariff from zero, an import subsidy. Since I consider the situation of the incumbent considering help to type X capitalists, I do not consider the case of import subsidy.

To get a clearer result, suppose that the utility function \( U \) is such that the marginal utility of income, \( W_I \), is constant and the income elasticity of good \( Y \) is less than one (so always \( \frac{z^x}{w} - \frac{y^x}{y^w} > 0 \)). Then I only need to check \( V_d^x - V_d^x \).

Because of \( V_d N < 0 \), for any \( d > 0 \), \( V_d^x(d) - V_d^x(d) < 0 \). Since tax rate is same, the tax burden of type X capitalists is larger than that of workers at \( \tau = 0 \). Because the marginal utility of income is constant, this means \( d^x > d^w \), so, \( V_d^x(d^x) - V_d^x(d^w) < 0 \). Thus, if the incumbent gives help at equilibrium, it must not be that \( \tau = 0 \).

**Corollary 2** If the utility is such that the marginal utility is constant and income elasticity of good \( Y \) is less than one, and if the incumbent decides to give help, the incumbent always finance some part of help by inefficient tariff.

This does not mean that the incumbent uses only tariff to finance the help. If the tariff is too high, further increase of tariff will not transfer the tax burden from one group to the other. Both groups will hurt more by the tariff increase, implying more votes for challenger. So it is possible that some combination of tax and tariff will be used. How to finance the help depends on parameters and functions. But what important for my purpose is that if conditions of the corollary are satisfied then \( \tau = 0 \) would not be the equilibrium choice for the incumbent.

Though this corollary depends on a specific type of utility function, the result can be obtained with a wider set of utility functions. The second condition of the corollary says that good \( Y \) is not a luxury good (so good \( X \) is a luxury good). But as long as \( \frac{z^x}{w} - \frac{y^x}{y^w} > 0 \) in the relevant range, it is fine even if good
Y is a luxury good in some income range. The first condition is not a necessary condition for the corollary. Clearly increasing marginal utility of income leads to the same conclusion. Even if the marginal utility of income is decreasing, if the decrease is not very severe so that \( \frac{W^w}{W^f} < \frac{V^x}{V^d} \), the same result could be obtained.

3.3 Two tier tax rates case

In this subsection, I consider the simple extension of the previous subsection: tax system with different income tax rates for type X capitalists and workers. Let \( t^x \) and \( t^w \) be the income tax rates for type X capitalists and workers, respectively. Again, I need to check if the increase of tariff from \( \tau = 0 \) does not affect wage and production levels. It does not, and I give the proof in Appendix. Given more policy instruments, would the incumbent choose \( \tau = 0 \) as his equilibrium strategy? For any level of \( H \), he has to determine levels of \( t^w, t^x \) and \( \tau \). First, I erase \( t^x \) with government budget constraint. Then, given level of \( H \) and \( \tau \), derive the condition for \( t^w \) for the minimization of \( N^x V^x(d^x) + N^w V^w(d^w) \). Finally, I check, given level of \( H \), if \( \tau = 0 \) is the equilibrium choice of not.

3.3.1 Determination of the level of \( t^w \)

The government budget constraint is,
\[
t^x N^x z^x + t^w N^w = sf_y
\]
From this, \( t^x = \frac{sf_y - t^w N^w}{N^x} \). Then the disposable income of type X capitalists is,
\[
\hat{I} = (1 - t^x) z^x + \delta^x \tau Q
\]
\[
= z^x - \frac{sf_y - t^w N^w}{N^x} \frac{N^x}{N^w} + \delta^x \tau Q
\]
Given \( H, t^w \) and \( \tau \), the indirect utility of type X capitalists is,
\[
W^x(t^w, \tau : H) = W(z^x - \frac{sf_y - t^w N^w}{N^x} + \delta^x \tau Q, \bar{p} + \tau)
\]
The indirect utility of workers is,
\[
W^w(t^w, \tau : H) = W((1 - t^w) w + \delta^w \tau Q, \bar{p} + \tau)
\]
For the given level of \( H \), the incumbent has to determine the levels of \( t^w \) and \( \tau \) to minimize \( N^x V^x(d^x) + N^w V^w(d^w) \). To determine such values, the incumbent has to find the value of \( t^w \) such that makes the value of the following zero.
\[
(4) \quad N^x V^x d^x \frac{\partial d^x}{\partial t^w} + N^w V^w d^w \frac{\partial d^w}{\partial t^w} = 0
\]
The \( d^s \) are still defined as \( d = W(0, 0 ; 0) - W(t^w, \tau ; H) \). So, the derivatives of \( d \) w.r.t. \( t^w \) are
\[
\frac{\partial d^x}{\partial t^w} = \frac{\partial W^x}{\partial t^w} = -W^x \frac{N^w}{N^x} \frac{N^x}{N^w}
\]
\[
\frac{\partial d^w}{\partial t^w} = W^w
\]
Substituting these into (4), it becomes,
If \( t^w = 0 \), type X capitalists has to pay all the burden of the help to type Y capitalists (of course, if tariff is being used, type X capitalists would not need to pay all that burden. But I am trying to check if \( \tau = 0 \) at equilibrium is possible or not). Unless the marginal utility of income decreases very rapidly, it is likely that above value is negative. If \( \tau = 0 \), then \( d^w = 0 \), which means \( V^w_d = 0 \). So it is negative. If \( t^w = 1 \), workers has to shoulder all the burden.

Then, unless the marginal utility is increase rapidly, it is likely that the above value is positive. Again, if \( \tau = 0 \), then \( d^w = 0 \), which means \( V^w_d = 0 \). So it is positive. Precisely speaking, unless \( \tau = 0 \), there is no guarantee that there is the level of \( t^w \) that makes it zero. If there is no such \( t^w \) minimizing the value), the optimal level of \( t^w \) for the incumbent is the corner solution, \( t^w = 0 \) or \( t^w = 1 \). If utilities are restricted to quasi-linear types like \( x^z + u(y^z) \), then it is easy to prove that there is such \( t^w \) (given an innocuous assumption on \( V \)). Since what I want to check is if \( \tau = 0 \) is the equilibrium choice when the incumbent decides to help, hereafter, I assume that there is \( t^w \) such that

\[
(5) \quad -N^w w V^f_d \tilde{W}^f + N^w w V^p_d \tilde{W}^w = 0
\]

From this, \( t^w = t^w(\tau, H) \) is defined.

### 3.3.2 The level of \( \tau \)

Given \( t^w = t^w(\tau, H) \), the optimal level of \( \tau \) is determined by,

\[
(6) \quad \frac{\partial (N^z V^r (d^z) + N^w V^r (d^w))}{\partial \tau} = N^z V^r_d \frac{\partial d^z}{\partial \tau} + N^w V^r_d \frac{\partial d^w}{\partial \tau}
\]

To calculate this, first I take the derivatives of \( d^z \) and \( d^w \) with respect to \( \tau \) at \( \tau = 0 \).

\[
(7) \quad \frac{\partial d^z}{\partial \tau}\bigg|_{\tau=0} = -\frac{W^z_f}{N^z}[-f_y \frac{\partial s}{\partial \tau} + N^w w \frac{\partial t^w}{\partial \tau} + N^z \delta^z Q] - W^z_p
\]

\[
(8) \quad \frac{\partial d^w}{\partial \tau}\bigg|_{\tau=0} = -W^w_f[-w \frac{\partial \tau}{\partial \tau} + \delta^w Q] - W^w_p
\]

Substituting these into (6).

\[
N^z V^r_d \frac{\partial d^z}{\partial \tau} + N^w V^r_d \frac{\partial d^w}{\partial \tau}
\]

\[
= -N^z V^r_d \frac{W^z_f}{N^z}[-f_y \frac{\partial s}{\partial \tau} + N^w w \frac{\partial t^w}{\partial \tau} + N^z \delta^z Q - N^z y^z] - N^w V^r_d \tilde{W}^w [-w \frac{\partial \tau}{\partial \tau} + \delta^w Q - y^w]
\]

\[
= -V^r_d W^z_f [-f_y \frac{\partial s}{\partial \tau} + N^w w \frac{\partial t^w}{\partial \tau} + N^z \delta^z Q - N^z y^z] - V^r_d \tilde{W}^w [-w \frac{\partial \tau}{\partial \tau} + \delta^w Q - N^w y^w]
\]

\[
= -V^r_d \tilde{W}^w [-y + N^w w \frac{\partial t^w}{\partial \tau} - N^w w \frac{\partial \tau}{\partial \tau} + (N^z \delta^z + N^w \delta^w) Q - N^z y^z - N^w y^w]
\]

from the condition (5),

\[
= -V^r_d \tilde{W}^w [y + Q - y]
\]
\[ f_y + Q \text{ is total supply of good X to this country and } y \text{ is the total demand} \]

Now I state the first Proposition under the two-tier tax rates system.

**Proposition 3** In the two-tier tax rates system, if the incumbent decides to give help, the incumbent would choose \( \tau = 0 \).

It seems that this Proposition says the two-tier tax rates system is better than the flat tax system. But, unfortunately, here is a catch. The condition for the minimization (2) w.r.t. \( t^w \) is,

\[-N^w w V^w_d W^w_I + N^w w V^w_d W^w_I = 0 \Leftrightarrow V^w_d W^w_I = V^w_d W^w_I\]

**Proposition 4** The necessary condition for no use of inefficient tariff to finance the positive help is \( V^w_d W^w_I = V^w_d W^w_I \).

Since \( V^w_d N < 0 \), unless the marginal utility of income is decreasing fast enough so that \( W^w_I \) is much larger than \( W^x_I \), the condition to satisfy this is that \( d^w > d^x \).

If the utility function is restricted to ones with constant marginal utility of income, this is the case. Moreover, with those utility functions, utility difference is proportional to the sum of the tax and tariff burdens. So, \( d^w > d^x \) means workers pay more burden than type X capitalists. Notice that this condition is not for group, but for each person.

**Corollary 5** If the utility functions is such that the marginal utility of income is constant, the necessary condition for no use of inefficient tariff to finance the help is that each worker pays more tax burden than each type X capitalist.

This necessary condition does not look politically easy to be satisfied. Again, the constant marginal utility of income is not a necessary condition. Unless the marginal utility of income decreases rapidly, the corollary actually holds. Thus, from this corollary, the main Theorem is derived.

**Theorem 6** Suppose that the marginal utility of income does not decrease very rapidly. Then, if the situation that each worker pays more tax burden than each type X capitalist is not politically feasible, no use of inefficient tariff is impossible.

### 4 Conclusion

In this paper, I proposed one explanation about why governments use inefficient tariff for trade protection. Though its logic is simple, interesting results were obtained, especially for the two-tier tax rates case. Because of voter heterogeneity, different instruments affect different voters differently. If a politician wants to maximize the support for him from voters over opposition to him from voters, he needs to minimize support for his opponent for any given level of support for him. This is same to cost minimization of firms or expenditure minimization.
of consumers. Unless in some extreme conditions, the combination of factors or goods can achieve the optimal cost/expenditure. From this, then it seems unlikely that the optimal combination of instruments for the politician consists of only one instrument. But, in the two-tier tax rates case, that is the case. The problem is, then, if it is politically feasible. If it is not feasible, some inefficient instruments would be used.

Of course, this result depends on model specification. Though the government budget in this model consists of help to type X capitalists and tax and tariff to finance it, actual government budget consists of more elements, especially debt. So, the implication of this model looks irrelevant to actual policy decision. Though I do not claim that the model is full of reality, I believe the logic of this paper could be applied for any government spending. If a government is to spend money, politicians have to decide how to finance it, which would invoke anger from voters who have to pick up the final tab. Politicians can hide somewhat behind the institutions and delegations (Anderson and Zanardi (2004)). But they cannot avoid responsibility completely. Moreover, it can be said that the most important for politicians is winning the next election. Even though models with special interests are very popular in Political Economy literature, the importance of the contribution from special interest comes from the fact that it helps politicians in elections (unless they receive it as direct bribe). Then, why do not politicians care how financing the government spending affects voters? Why not minimize the political cost for them? If some voters are less sensitive to the cost of financing (less likely to cast opposing votes), why not put some more cost on them?

If all of politicians and voters act only in a selfish way, then the model says the necessary condition for no use of inefficient instrument is that workers (less sensible potential voters) pay more tax burden than type X capitalists (more sensible potential voters). This result is counter-intuitive and seems politically infeasible. Thus, the necessary condition is not to be satisfied. So, the theorem says that the reason for the political infeasibility is the reason for the use of tariff. What is the reason of political infeasibility? It seems like the sense of fairness. We feel type X capitalists who can shoulder higher cost should shoulder more cost than workers who can shoulder only lower cost. If so, it can be said that the inefficiency of tariff is the cost of fairness.

Back to the trade protection, this paper proposes one possible explanation for the use of tariff. But this paper does not oppose to other explanations, especially to the collection cost explanation. Actually, I think my model could be extended to incorporate the collection cost. If collection of tax from domestic voters is difficult and expensive, why no impose tariff? In the end, foreign exporters are not domestic voters (of course, foreign exporters still can influence domestic politics through other ways).

Clearly, the election in this paper is unrealistic. It is not easy to imagine a real election is been fought on the trade policy. There could be an argument that it is institutions, like International Trade Commission in U.S., not elections, that determine trade protection policies. I agree with this argument. But at same time, still politicians are making political decisions based on political costs (of
which those institutions might determine some parts). Also those institutions have been produced through political processes. In those processes, who can deny that political cost was an important element for the designs of them?

5 Appendix

Proof of Non-zero first terms of (3a) and (3b)

The first terms of (3a) and (3b) are,

\[(3a) \quad N^w y^w w (\frac{z^x}{w} - \frac{y^x}{y^w})\]

\[(3b) \quad -N^x y^w w (\frac{z^x}{w} - \frac{y^x}{y^w})\]

Then, what is needed to show is \(z^x w - y^x y^w \neq 0\). First, suppose that the income elasticity of good Y is less than one, so that \(\frac{y^w}{y^w} > 0 \Leftrightarrow \frac{i^x}{i^w} > \frac{y^x}{y^w} > 0\). Given the distributional share \(\delta^a\), the disposable income \(i^x = (1 - t) z^x + \delta^a \tau Q\). Substituting this, \(i^x = (1 - t) z^x + \delta^a \tau Q\). I consider only two types of distribution.

The distribution proportional to income \(\delta^a = \frac{z^a}{z}\): Substituting this, \(i^x = (1 - t) z^x + \delta^a \tau Q\). Since \(z^x > w\), \(i^x > (1 - t) z^x + \delta^a \tau Q\). Thus, \(\frac{z^x}{w} > \frac{y^x}{y^w}\).

The equal distribution \(\delta^a = 1\): Substituting this, \(i^x = (1 - t) z^x + \tau Q\). Since \(z^x > w\), \(i^x > (1 - t) z^x + \tau Q\). Thus, \(\frac{z^x}{w} > \frac{i^x}{i^w} > \frac{y^x}{y^w}\).

If the income elasticity of good Y is more than one, so that \(\frac{y^x}{y^w} > \frac{i^x}{i^w}\), still \(\frac{z^x}{w} - \frac{y^x}{y^w} \neq 0\) can be proved in the case of income proportional distribution. Unfortunately, the equal distribution does not guarantee non-zero anymore.

Production levels and trade balance under the flat tax

Let \(w\) be the wage rate in terms of good X. Then, given \(H \equiv \tau + s\), the following is the equilibrium condition for production

\[f_x(l_x) = w = (\bar{p} + H) f_y(l_y)\]

Suppose that government is giving positive level of \(s\) but \(\tau = 0\). Then, government changes \(s\) and \(\tau\) so that the level of \(H\) does not change (i.e., \(\frac{ds}{d\tau} = -1\)). Let \(E\) and \(Q\) be the quantities of export and import, respectively. Since the world price of good Y is \(\bar{p}\), the balanced trade condition is, \(E = \bar{p}Q\). Because
the price $\bar{p}$ is fixed, the tariff change can cause changes only in $E$ and $Q$. Then, after the tariff increase, to keep trade balanced, it has to be that $\Delta E - \bar{p}\Delta Q = 0$. Even if no change in wage and production levels happens, higher domestic price of good Y and lower income tax rate would change the demands for goods X and Y from type X capitalists and workers\(^{11}\). Then, given fixed levels of productions, the demand decrease of good Y and demand increase of good X means the decreases in the import of good Y and the export of good X. Since $\tau$ and $s$ change so that $H$ does not change, $\frac{ds}{d\tau} = -1$. Then, if the change of $\tau$ does not affect productions and wage, $\frac{\partial t}{\partial \tau} = \frac{\partial t}{\partial s} \frac{\partial s}{\partial \tau} = f_y \frac{\partial s}{\partial \tau} = -\frac{f_y}{z}$. Given the disposable incomes of a type X capitalist and a worker as $I^x = (1-t)z^x + \delta^x \tau Q$ and $I^w = (1-t)w + \delta^w \tau Q$, the derivatives of disposable incomes w.r.t. tariff $\tau$ at $\tau = 0$ are,

$$\frac{\partial I^x}{\partial \tau}|_{\tau=0} = -z^x \frac{\partial t}{\partial \tau} + \delta^x Q = \frac{z^x}{z} f_y + \delta^x Q$$

$$\frac{\partial I^w}{\partial \tau}|_{\tau=0} = -w \frac{\partial t}{\partial \tau} + \delta^w Q = \frac{w}{z} f_y + \delta^w Q$$

Then, the change in the demand for good X of a type X capitalist is, $\frac{\partial (I^x - p_y(I^x, p))}{\partial \tau}$. The change in demand for good Y of a type X capitalist is, $\frac{\partial y^x}{\partial \tau}$. At $\tau = 0$, the change of export minus import value calculated at world price from a type X capitalist is,

$$-\frac{\partial (I^x - p_y^x)}{\partial \tau}|_{\tau=0} - \bar{p} \frac{\partial y^x}{\partial \tau} = -\frac{\partial I^x}{\partial \tau} + y^x$$

Similarly, the change of export minus import value calculated at the world price from a worker is,

$$-\frac{\partial (I^x - p_y^w)}{\partial \tau}|_{\tau=0} - \bar{p} \frac{\partial y^w}{\partial \tau} = -\frac{\partial I^x}{\partial \tau} + y^w$$

and the change in the demand for good Y of a worker is, $\frac{\partial y^w}{\partial \tau}$. Then the change of export minus import value calculated at world price from this country is,

$$N^x \{ -\frac{\partial I^x}{\partial \tau} + y^x \} + N^w \{ -\frac{\partial I^w}{\partial \tau} + y^w \} = -N^x \frac{z^x}{z} f_y - N^x \delta^x Q + N^x y^x - N^w \frac{w}{z} f_y - N^w \delta^w Q + N^w y^w$$

$$= -N^x \frac{z^x}{z} + N^w \frac{w}{z} f_y - (N^x \delta^x + N^w \delta^w) Q + N^x y^x + N^w y^w$$

The sum of first two terms are negative of total supply of good Y to the country, and the sum of last two terms is the total demand for good Y in this country. That should be zero, so the trade is still balanced even after the tariff increase, if the original state is zero tariff and the increase of tariff is small. Since what I want to show is if $\tau = 0$ would be chosen or not, this is enough.

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\(^{11}\)There is no increase of demand from type Y capitalists. Since they do not consume good Y anyway, there is no substitution effect. Since the level of $H$ is fixed, there is no income effect, either.
Production levels and trade balance under the two income tax rates

The budget constraint for government is,
\[ t^x N^x z^x + t^w N^w w = s f_y \]

The derivative of the budget constraint w.r.t. \( \tau \) with \( dH = 0 \) and no change in wage and production levels is,
\[ N^x z^x \frac{\partial t^x}{\partial \tau} + N^w w \frac{\partial t^w}{\partial \tau} = f_y \frac{\partial s}{\partial \tau} = -f_y \]

The disposable income for group \( a \in \{x, w\} \) is,
\[ I^a = (1 - t^a) z^a + \delta^a \tau Q \]

The derivative of this disposable income is,
\[ \frac{\partial I^a}{\partial \tau} = -z^a \frac{\partial t^a}{\partial \tau} + \delta^a Q \]

Assuming no change of wage and production level for the change of \( s \) and \( \tau \) such that \( dH = 0 \), the change in demand for good X of group \( a \in \{x, w\} \) at \( \tau = 0 \) is,
\[ \frac{\partial (I^a - py^a)}{\partial \tau} \bigg|_{\tau=0, dH=0} = \frac{\partial I^a}{\partial \tau} - y^a - p \frac{\partial y^a}{\partial \tau} \]

Then the change in the trade balance of group \( a \in \{x, w\} \) at \( \tau = 0 \) is,
\[ N^a (-\frac{\partial (I^a - py^a)}{\partial \tau}) - \bar{p} \frac{\partial y^a}{\partial \tau} = N^a (-\frac{\partial I^a}{\partial \tau} + y^a) \]

The change in the trade balance of this country at \( \tau = 0 \) is,
\[ N^x (-\frac{\partial I^x}{\partial \tau} + y^x) + N^w (-\frac{\partial I^w}{\partial \tau} + y^w) \]
\[ = -N^x \frac{\partial I^x}{\partial \tau} - N^w \frac{\partial I^w}{\partial \tau} + y \]
\[ = N^x z^x \frac{\partial t^x}{\partial \tau} + N^w z^w \frac{\partial t^w}{\partial \tau} - (N^x \delta^x + N^w \delta^w) Q + y \]
\[ = -f_y - Q + y \]
\[ = 0. \]

Thus, the change of \( s \) and \( \tau \) such that \( dH = 0 \) is consistent with no change in wage and production levels.

References


