Effects of Technology Improvement in the Ricardian Model

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Abstract

We generalize Samuelson's numerical examples (2004) by analyzing nine cases of technology improvements in the Ricardian model. It is shown that the export-biased technical progress at home benefits the foreign country, but the import-biased technical progress at home hurts the foreign country as long as the comparative advantage is not reversed. We then study optimal strategies of technology improvement and show that bigger countries should invest more in R&D per capita. For a small country it is optimal to choose export-biased technical progress. For a big country it is optimal to improve technology in both sectors at the rate proportional to the consumers' expenditure share. Therefore, if the expenditure share of import sector is larger than that of export sector, the big country will choose a relatively import-biased technology improvement, which hurts its trade partner. If both countries are fully specialized, it is optimal for a bigger country to choose the "catching-up" strategy at earlier development stage than a smaller country should do.

Keywords: Export-biased, Import-biased, Ricardian model, Technology improvement

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1 Introduction

The purpose of this paper is twofold. First, it presents a comprehensive analysis of technology improvement in the standard Ricardian model. Our analysis confirms the understanding in the literature for the age-old question often made by informal writings and incomplete analyses, and reconciles recent opposite results found by Samuelson (2004) and Ruffin and Jones (2005). Second, we study optimal strategies of technology improvement and point out the sharp difference between strategies taken by small countries versus big countries.

There is much concern in economic policy making over the effects of technology improvement (through technology innovation or transfer) in developing countries such as China and India on the welfare of technology advanced countries such as the United States. “These effects on trade of technology and technological change are at the heart of the debate on international economic policy,” as pointed out by Krugman (1986, pp 152). However, “to a remarkable extent the treatment of technology in formal trade theory has failed to connect with policy concerns (Krugman, 1986, pp 152).” Considering the simplest two-good, two-country Ricardian model, “the problem with this model as a vehicle for discussing technical change is that too many things can happen (Krugman, 1986, pp 153).”

Among these “too many things”, Samuelson (2004) in his recent article points out two opposite effects with two elegant numerical examples. In his Act I, it is shown that China’s technology improvement in its export sector raises U.S. welfare. In his Act II, it is shown that China’s technology improvement in its import sector reduces U.S. welfare. In the example Samuelson used in the Act II, China improves technology in its import sector to the point that postinvention relative labor productivity in China is identical to that in U.S., which emasculates all comparative advantages and therefore eliminates all of the United States’ previous enjoyments from free trade. Samuelson’s Act II, however, is challenged by the
“technology transfer paradox” discussed by Ruffin and Jones (2005) and Jones and Ruffin (2005). They show that U.S. will, nonetheless, gain from China’s technology improvement in its import sector if such technology improvement is sufficiently large to reverse the comparative advantage.

In this paper, we study the technology change in a two-good, two-country Ricardian model. Following the literature, we first assume that the technology improvement is costless. There are indeed “many things” that could happen. First, there are three possible types of equilibria: Type B is characterized by the big home country producing both goods, Type S has fully specialized home and foreign countries, or Type B* which is the reverse of Type B where foreign country is the big country and produces both goods. Second, as for the technical progress, it could be one of the following three cases: taking place only in export sector which is labeled as export-biased technology improvement, taking place only in import sector which is labeled as import-biased technology improvement,1 or taking place in both sectors. The improvement in technology at home not only changes equilibrium prices and outputs, but also shifts trade patterns. Together, there are nine cases of technology changes to be analyzed.2 Nevertheless, under the assumption of Cobb-Douglas utility function, we are able to generalize Samuelson’s results and conclude that:

1. Under the assumption of Cobb-Douglas utility function, the increase in real income which results from the technology improvement always dominates possible adverse terms of trade effect. Thus, any technical progress at home improves the home country’s welfare.

2. The export-biased technical progress at home always improves foreign country’s terms of trade and therefore benefits the foreign country.

3. If the import-biased technical progress does not reverse the comparative advantage,

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1The distinction between “export-biased” and “import-biased” technology improvements was first discussed by Hicks (1953).

2Without considering the change in types of equilibria, Grossman and Helpman (1995) discussed two cases of technology improvement.
it always deteriorates foreign country’s terms of trade and therefore hurts the foreign country.

4. If the technology improves at the same rate in both sectors at home, which is labeled as uniform technology improvement, it must benefit the foreign country.

The “technology transfer paradox” of Ruffin and Jones (2005) follows from a composition of the export-biased technical progress and the import-biased technical progress. Let home import good 2. The import-biased technical progress that occurs in sector 2 until the reversal of comparative advantage hurts the foreign country. After the reversal of comparative advantage the technical progress in sector 2 becomes export-biased, and therefore benefits the foreign country. In the example of Ruffin and Jones (2005), the latter effect dominates the former so that the overall effect of technical progress in the initial import sector benefits the foreign country.

The question that immediately emerges is when will a country choose export/import-biased technology improvement? The answer depends on the size of the country. Assuming costly improvements now, we show that it is optimal for a small country (Type B∗) to choose export-biased technical progress, which benefits the partner country. A big country (Type B) produces both goods and improves technology in both sectors. Interestingly, the optimal rate of technology improvement in each sector is proportional to the consumers’ expenditure share. Therefore, if the expenditure share of import sector is larger than that of export sector, the big country will choose a relatively import-biased technology improvement, which hurts its trade partner. When both countries are fully specialized (Type S), the home country may choose either an export-biased technical progress, or a “catching-up” strategy in which the country also improves technology in import sector and be “self-sufficient” in both goods. The “catching-up” strategy is shown to be optimal if the expenditure share on the importable good is sufficiently large, the country is relatively big and the
technology gap to advanced country in import sector is relatively small. Ceteris paribus, therefore, it is optimal for a larger country to choose the “catching-up” strategy at earlier development stage than a smaller country should do.

This paper is related to the theoretical literature that investigates technology and trade. Grossman and Helpman (1995) provide an excellent survey of the literature. Helpman (1993) analyzes the welfare effect of intellectual property rights policy and argues that faster diffusion will stimulate research in the innovating country. Eaton and Kortum (2001) examine the link among innovation, technology, trade and growth and conclude that research intensity does not depend on country size, research productivity, or openness. Eaton and Kortum (2002) show technology improvement in one country benefits all other countries when the trade is costless. Using a dynamic Ricardian model, Eaton and Kortum (2006) show that faster technology diffusion shifts research activity to the countries that are better at R&D, but openness to trade does not change the research specialization. Alvarez and Lucas (2005) analyze a variation of the Eaton-Kortum model (2002) and get similar implications. In all these papers, technology improvement is modeled to be uniform in which the technology in import sector and export sector is improved at the same rate. We show in this paper, however, that the export/import-biased technology improvement may have very different welfare impact from that of uniform technology improvement. In the absence of terms of trade effect, Demidova’s (2005) also shows that technology improvement hurts the innovator’s partner if the specialization does not occur.

The rest of paper is organized as follows. Section 2 sets up the Ricardian model with three types of equilibria. Section 3 examines welfare effects in nine cases of technology improvement. Section 4 studies the optimal strategy of technology improvement, and Section 5 concludes.
Our analysis is based on the standard Ricardian model, which has two goods and one factor (labor). The market is perfectly competitive and the labor is perfectly mobile between industries in each country, but immobile across countries. As usual, variables of foreign country are denoted by superscript “*”. Let \( a_i(a_i^*) \) be the labor needed per unit of production at home (abroad) before technology change in sector \( i \) where \( i = 1, 2 \). We assume
\[
a_1/a_2 < a_1^*/a_2^* \quad (1)
\]

Thus, the home country has a comparative advantage in producing good 1 before technology change. The total labor force at home (abroad) is \( L(L^*) \). Let \( p_i \) be the price in each industry and \( p = p_1/p_2 \) denote the relative price of good 1. The output in sector \( i \) is denoted by \( y_i \). Perfect competition implies that \( p^a = a_1/a_2 \) and \( p^{a*} = a_1^*/a_2^* \) in autarky. Let good 2 be the numeraire good and we normalize \( p_2 = 1 \) thereafter.

Let home and foreign countries open to trade. The world relative supply curve has a “stair-up” shape, which is depicted in Figure 1. The vertical and horizontal axes represent the relative price \( p \) and the relative supply of good 1, \( y = (y_1 + y_1^*) / (y_2 + y_2^*) \), respectively. For the world relative price \( p < p^a = a_1/a_2 < p^{a*} = a_1^*/a_2^* \), both countries are specialized in good 2 so the world relative supply of good 1 is zero. For \( p^a < p < p^{a*} \), the home country is specialized in good 1, while the foreign country is specialized in good 2, and the world relative supply is \( (L/a_1) / (L^*/a_2^*) \). Finally, if \( p > p^{a*} > p^a \), both countries are specialized in good 1 so the world relative supply of good 1 is infinity.

The utility functions of the representative consumer in two countries are identical and can be represented by
\[
u(x_1, x_2) = x_1^\beta x_2^{1-\beta} \quad (2)
\]
Thus, the relative demand is:

\[
x(p) = \frac{x_1}{x_2} = \frac{\beta}{(1 - \beta)} p \quad \Leftrightarrow \quad p(x) = \frac{\beta}{(1 - \beta)} x
\]

(3)

The type of equilibria is determined by the relative positions of demand and supply. There are three possibilities. The first type is that the demand curve cuts the lower horizontal line of supply curve at \( p = a_1/a_2 \), which is represented by equilibrium B in Figure 1. This is denoted as Type B of trade in which the home country is big, producing both goods and exporting good 1. The second one is that the demand curve cuts the vertical line of supply curve at \( y = (y_1 + y_1^*) / (y_2 + y_2^*) \), which is denoted as Type S of trade. Both countries are fully specialized in Type S and the equilibrium is indicated by S in Figure 1. The third type is that the demand curve cuts the upper horizontal line of supply curve at \( p = a_1^*/a_2^* \), which is represented by equilibrium B* in Figure 1. It is denoted as Type B* of trade in which the foreign country is big and produces both goods.

It is easily seen that in Figure 1 at \( x^* = (L/a_1)/(L^*/a_2^*) \), the inverse demand \( p(x^*) = \frac{\beta_L^*}{(1 - \beta)L^*} \) must be less than \( a_1/a_2 \) in Type B, greater than \( a_1/a_2 \) but less than \( a_1^*/a_2^* \) in Type S, and greater than \( a_1^*/a_2^* \) in Type B*, respectively. As shown in Figure 2, therefore, if \( 0 \leq \frac{\beta L^*}{(1 - \beta)L} < \frac{a_1^*}{a_2^*} \) two countries are engaged into Type B trade; if \( \frac{a_2}{a_2^*} \leq \frac{\beta L^*}{(1 - \beta)L} < \frac{a_1}{a_1^*} \) two countries are engaged into Type S trade, and if \( \frac{\beta L^*}{(1 - \beta)L} \geq \frac{a_1}{a_1^*} \) two countries are engaged into Type S* trade. The type of trade relies on countries’ relative size and productivity. The technical progress changes relative productivity between countries and shifts the type of equilibria as well. We now turn to the welfare analysis.

## 3 Welfare Effects of the Technology Improvement

Following the literature we assume that the technology improvement is costless in this section, and we will relax this assumption in next section. All technology
improvements are assumed to take place at the home country. We will begin with the analysis of technology improvement in Type B, then Type S, and Type B* in the end.

3.1 The Technology Improvement in a Big Country

The home country is a big country in Type B trade. World relative price is equal to the autarky relative price at home, \( a_1/a_2 \), as indicated by point B in Figure 1. The relative country size, \( \frac{\beta L^*}{(1-\beta)L} \), is in the interval of Type B in Figure 2, as indicated by point \( E_1 \). The home country produces both goods, exporting good 1 while importing good 2. We will analyze the effect of export-biased, import-biased, and uniform technology improvements sequentially.

3.1.1 The Export-Biased Technology Improvement

The export-biased technology improvement is represented by the decrease in \( a_1 \). In Figure 2, the cutoff point between types B and S is point \( e_1 \), which equals \( \frac{a_1^*}{a_2} \). The cutoff point between types S and B* is point \( e_2 \) and equals \( \frac{a_2^*}{a_1^*} \). The decrease in \( a_1 \) shifts the cutoff point \( e_2 \) to the right, but has no effect on the cutoff point \( e_1 \). Thus, \( E_1 = \frac{\beta L^*}{(1-\beta)L} \) is still within the interval of Type B as \( a_1 \) decreases. In other words, the export-biased technology improvement does not change the type of trade.

The welfare effects are illustrated in Figure 3 where \( O \) and \( O^* \) represent the origins for home and foreign countries. The outputs of good 1 and good 2 are represented by horizontal and vertical axes, respectively. \( AB \) is the production possibility frontier (PPF) at home before the technology change. It is also the budget line since the world relative price is equal to \( a_1/a_2 \), the slope of the PPF at home. \( A^*B^* \) is the PPF of the foreign country. The budget line abroad is represented by \( A^*D^* \), which is parallel to \( AB \).

When \( a_1 \) decreases to \( a_1' \), the PPF at home shifts out to \( AB' \) at home. The real income at home increases due to the technology improvement. The budget
line shifts out and the optimal consumption bundle at home shifts out from \( C \) to \( C' \). Hence, the welfare of the home country improves. As \( a_1 \) decreases to \( a'_1 \), the world relative price \( p \) decreases from \( a_1 / a_2 \) to \( a'_1 / a_2 \). The budget line abroad shifts out to \( A^*D^{**} \) due to the improvement of terms of trade, which moves the optimal consumption from \( C^* \) out to \( C^{**} \). Thus, the export-biased technology improvement at home benefits the foreign country.

### 3.1.2 The Import-Biased Technology Improvement

The import-biased technology improvement is represented by the decrease from \( a_2 \) to \( a'_2 \). Let \( a'_2 = \frac{a_2 a_1}{a_2} \), which is the switching point for the comparative advantage reversal. As long as \( a'_2 \geq a'_2 \) which implies that \( \frac{a'_2}{a_2} \leq \frac{a'_2}{a_1} \), the initial comparative advantages in both countries are maintained. The point \( E_1 = \frac{\beta L^*}{(1-\beta)L} \) in Figure 2 is still within the interval of Type B after the technical change.

The PPF changes from \( AB \) to \( A'B \) as depicted in Figure 3. Again, the real income at home increases due to the technology improvement. The optimal consumption bundle at home shifts out from \( C \) to \( C'' \) and welfare at home improves. As \( a_2 \) decreases to \( a'_2 \), the world relative price \( p \) increases from \( a_1 / a_2 \) to \( a'_1 / a_2 \). The budget line abroad shifts in to \( A^*D^{**} \) due to worsening of terms of trade and the optimal consumption bundle moves from \( C^* \) back to \( C^{**} \). Therefore, the import-biased technology improvement at home hurts the foreign country as long as the comparative advantage is not reversed.

If \( a'_2 < a'_2 \), the comparative advantage is reversed. In Figure 2 \( e_1 \) shifts to the right of \( e_2 \) at \( e'_1 \). The home country still produces both goods, but with the comparative advantage in producing good 2 now. The home country exports good 2 instead of good 1 and the foreign country is specialized in producing good 1. A further decrease in \( a_2 \) now becomes the export-biased technology improvement, which improves welfare both at home and abroad as we have discussed in Section 3.1.1.
3.1.3 The Uniform Technology Improvement

Both $a_1$ and $a_2$ decrease at the same rate in the uniform technology improvement. Since $e_1$ and $e_2$ shift to the right at the same rate in Figure 2, the type of trade is not affected. In Figure 3 the PPF (budget line) at home $AB$ shifts out parallel. The welfare at home must be improved. The budget line in the foreign country, $A^*D^*$, is not affected by the uniform technology improvement. Thus, the welfare abroad is unchanged. Summarizing we have:

Lemma 1 Suppose the equilibrium of trade is Type B. Any technology improvement at home improves the welfare at home. The export-biased technology improvement at home benefits the foreign country. The import-biased technology improvement at home hurts the foreign country as long as the comparative advantage is not reversed. The uniform technology improvement at home (weakly) benefits the foreign country.

3.2 The Technology Improvement When Countries are Fully Specialized

We now turn to Type S in which the home country is specialized in good 1 and the foreign country is specialized in good 2. In Figure 2, Type S corresponds to the point $E_2 = \frac{\beta L^*}{1-\beta}L$ which indicates that the relative country size is within the interval of $[e_1, e_2]$. Like the above subsection, we will analyze the effect of export-biased, import-biased, and uniform technology improvements case by case.

3.2.1 The Export-Biased Technology Improvement

The decrease in $a_1$ shifts the cutoff point $e_2$ to the right in Figure 2, but has no effect on the cutoff point $e_1$. Thus, $E_2 = \frac{\beta L^*}{1-\beta}L$ is within the interval of Type S as $a_1$ decreases and the type of trade does not change.

The effect of the technical progress on the home welfare follows from a composition of the real income effect and the terms of trade effect. These two effects are reflected by the change of budget line. When both countries are fully specialized, the world
relative supply is \( y = \frac{(L/a_1)}{(L^*/a_2^*)} \), as indicated by point S in Figure 1. Using the inverse demand function (3), the world relative price at S is \( p(x^s) = p(y) = \frac{\beta a_1 L^*}{(1-\beta)a_2^* L} \).

The income at home

\[
I = wL = (p/a_1) L = \frac{\beta L^*}{(1-\beta) a_2^*}
\]

(4)

Thus, the budget constraint at home is:

\[
px_1 + x_2 = I \iff \left[ \frac{\beta a_1 L^*}{(1-\beta) a_2^* L} \right] x_1 + x_2 = \frac{\beta L^*}{(1-\beta) a_2^*},
\]

(5)

which is shown by \( BD \) line in Figure 4. The horizontal intercept of the budget line is equal to \( L/a_1 \), which is the same as that of PPF line \( AB \). As \( a_1 \) decreases to \( a_1' \), the real income increases and the horizontal intercept of both PPF and budget line shift to the right to \( B' \), which we identify as the real income effect. On the other hand, the terms of trade represented by the slope of the budget line decline, which is labeled as the terms of trade effect. Had the terms of trade not changed, the budget line would be parallel to \( BD \) and indicated by \( B'E \). Note, however, the vertical intercept of the budget line, \( \frac{\beta L^*}{(1-\beta) a_2^*} \), is not affected by the change of \( a_1 \) which is indicated by \( D \). Therefore, combining the real income effect and the terms of trade effect together, the budget line shifts out from \( BD \) to \( B'D \) as \( a_1 \) decreases. The real income effect dominates the terms of trade effect and domestic welfare increases.

The above result relies on the assumption of Cobb-Douglas utility function. If the demand is less elastic, as noted by both Samuelson (2004) and Ruffin and Jones (2005), the case of immiserizing growth may occur. For example, the budget line may change to \( B'F \) instead of \( BD \) in Figure 4, which could reduce the domestic welfare.

The foreign country is specialized in producing good 2. As the relative price of good 1 declines, the budget line abroad shifts out from \( A^*D^* \) to \( A^*D'^* \) in Figure 4: the terms of trade abroad improves and therefore the welfare in the foreign country increases.
3.2.2 The Import-Biased Technology Improvement

Let $a_2^{II}$ be the cutoff point between Type S and Type B in Figure 1. That is,

$$
\frac{a_1}{a_2^{II}} = p = \frac{\beta a_1 L^*}{(1 - \beta) a_2^{II} L} \Leftrightarrow a_2^{II} = \frac{(1 - \beta) a_2^{II} L}{\beta L^*}
$$

(6)

When $a_2' \geq a_2^{II}$, $e_1 \leq E_2$ and we are within the Type S equilibrium as depicted in Figure 2. It is clearly seen in Figure 1 that the decrease of $a_2$ up to $a_2^{II}$ has no effect on the economy. If $a_2' < a_2^{II}$, however, the type of trade switches to Type B, and therefore the effect of the decrease in $a_2$ is the same as our analysis in Section 3.1.2. The import-biased technology improvement at home benefits home but hurts the foreign country as long as $a_2^{II} > a_2' \geq a_2$. If $a_2' < a_2'$, the comparative advantage is reversed. Further decreases in $a_2$ becomes the export-biased technology improvement, and improves the welfare both at home and abroad.

3.2.3 The Uniform Technology Improvement

Whenever $a_2' \geq a_2^{II}$, it is easy to see in Figure 2 that the uniform reductions in both $a_1$ and $a_2$ are reduced to the export-biased technology improvement as we have discussed in Section 3.2.1 since the reduction in $a_2$ has no effect on the economy. Both the home and the foreign countries benefit from it. When $a_2' < a_2^{II}$, the type of trade switches to Type B. Uniform reductions in $a_1$ and $a_2$ represents the case of Type B uniform technology improvement, which we have discussed in Section 3.1.3. Again, welfare both at home and abroad are improved. Summarizing we have:

Lemma 2 Suppose the utility function is Cobb-Douglas and both countries are fully specialized before the technology change. Any technology improvement at home improves the welfare at home. The export-biased technology improvement at home benefits the foreign country. The import-biased technology improvement at home hurts the foreign country as long as the comparative advantage is not reversed. The uniform technology improvement at home benefits the foreign country.
3.3 The Technology Improvement in a Small Country

Finally, we examine the case of Type B* trade. The world relative price is equal to the autarky relative price in the foreign country, $a_1^* / a_2^*$, as indicated by point B* in Figure 1. The country relative size, $\frac{\beta L^*}{(1-\beta)L}$, is in the interval of Type B* in Figure 2, as indicated by point $E_3$. The home country specializes in good 1 and exports it.

3.3.1 The Export-Biased Technology Improvement

Let $a_{1II}^{II}$ be the cutoff point between Type S and Type B* in Figure 2, which is given by

$$e_2 = \frac{a_1^*}{a_{1II}^{II}} = E_3 = \frac{\beta L^*}{(1-\beta)L} \leftrightarrow a_{1II}^{II} = \frac{a_1^* (1 - \beta) L}{\beta L^*} \quad (7)$$

When $a'_1 \geq a_{1II}^{II}$, $e_2 \leq E_3$ in Figure 2 and we are within the Type B* trade. The welfare effect can be seen in Figure 5. The decrease in $a_1$ shifts the PPF $AB$ to $AB'$ and the budget line $BD$ to $B'D'$, respectively. The slope of the budget line for both home and foreign countries, $a_1^* / a_2^*$, is not affected as long as $a'_1 \geq a_{1II}^{II}$. The welfare in the home country is improved, while the foreign country is not affected.

When $a'_1 < a_{1II}^{II}$, the type of trade switches to Type S. The cutoff point between Type S and B* in Figure 2, $e'_2$, now is on the right side of $E_3$. Further decreases in $a_1$ produces the Type S export-biased technology improvement, which is analyzed in Section 3.2.1. The welfare in both home and foreign countries are improved.

3.3.2 The Import-Biased Technology Improvement

As long as the comparative advantage is not reversed, that is, $a'_2 \geq a'_2$, the decrease in $a_2$ has no effect on the economy. If $a'_2 < a'_2$, $e_1$ shifts to $e'_1$ in Figure 2. Now $e'_1$ is the cutoff point between types S and B* and good 2 becomes the export sector. Thus, further decreases in $a_2$ becomes the export-biased technology improvement, which is analyzed in Section 3.3.1.
3.3.3 The Uniform Technology Improvement

Whenever $a_1' \geq a_1^{III}$, it is immediately seen in Figure 2 that the uniform reductions in both $a_1$ and $a_2$ are reduced to the export-biased technology improvement in Type $B^*$ as we have discussed in Section 3.3.1 since the reduction in $a_2$ has no effect on the economy. Both home and foreign countries benefit from it. When $a_1' < a_1^{III}$, the type of trade switches to Type S. Uniform reductions in both $a_1$ and $a_2$ are equivalent to the Type S uniform technology improvement mentioned in Section 3.2.3. The welfare in both home and foreign countries are improved. Summarizing the above results we have:

**Lemma 3** Suppose the type of equilibria is Type $B^*$. Any technology improvement at home improves the welfare at home. The export-biased technology improvement at home benefits the foreign country. The import-biased technology improvement at home (weakly) hurts the foreign country as long as the comparative advantage is not reversed. The uniform technology improvement at home benefits the foreign country.

We are now ready to put all results in above nine cases together, summarizing as the following theorem.

**Theorem 1** If the utility function is Cobb-Douglas, any technology improvement at home improves the welfare at home. The export-biased technology improvement at home benefits the foreign country. The import-biased technology improvement at home hurts the foreign country as long as the comparative advantage is not reversed. The uniform technology improvement at home benefits the foreign country.

4 Optimal Strategy of Technology Improvement

The effects of export-biased versus import-biased technology improvements on the welfare of the partner country are strikingly opposite. A crucial question that follows is when a country will choose the export/import-biased technology improvement. To
answer this question, we investigate the optimal strategy of technology improvement in this section.

Departing from the costless technology improvement assumption, we assume that after the technical progress the labor needed to produce one unit of output in sector \( i \) is:

\[
a'_i = \frac{a_i}{1 + \theta d_i}
\]  

(8)

where \( d_i > 0 \) is the labor used in sector \( i \) for the technology improvement. \( \theta > 0 \) measures the efficiency of R&D in sector \( i \), which is assumed to be the same in both sectors for simplicity. The costs of technology improvement can be interpreted in two ways. First, home pays \( w(d_1 + d_2) \) of income to the foreign country for the technology transfer, which is typically the case for developing countries. Second, \( \frac{d_i}{L_i} \) share of labor per worker in sector \( i \) are spent to improve the working efficiency. Either way, the total income left for the consumption after the technology improvement at home is \( w(L - d_1 - d_2) \). A social planner will choose optimal \( (d_1, d_2) \) to maximize the utility of the representative consumer.

### 4.1 The Case of Big Country

We start from Type B equilibrium in which the home country is a big country. The home country produces both goods and the world price \( p \) equals the autarky price at home, \( a'_1/a'_2 \). In Figure 2 decreases in \( a_1 \) and \( a_2 \) enlarge the length of the interval of Type B, but do not change the type of trade. Note that the wage rate at home \( w = \frac{p}{a'_1} = \frac{1}{a'_2} \), which will be used to derive the optimal consumption bundle,

\[
x_1 = \frac{\beta w}{p} (L - d_1 - d_2) = \frac{\beta (1 + \theta d_1)}{a'_1} (L - d_1 - d_2)
\]

and

\[
x_2 = (1 - \beta) w (L - d_1 - d_2) = \frac{(1 - \beta) (1 + \theta d_2)}{a'_2} (L - d_1 - d_2)
\]  

(9)
Substituting (9) into the utility function (2), the social planner’s problem becomes

$$\max_{d_1, d_2} u^B(d_1, d_2) = \frac{\beta^\beta (1 - \beta)^{1-\beta}}{a_1^\beta a_2^{1-\beta}} (L - d_1 - d_2) (1 + \theta d_1)^\beta (1 + \theta d_2)^{1-\beta}$$ \hspace{1cm} (10)

Solving the first order condition gives the optimal amounts of labor used in technology improvement:

$$d_1^B = \frac{\beta \theta L - 2 (1 - \beta)}{2 \theta}, \quad d_2^B = \frac{\theta L (1 - \beta) - 2 \beta}{2 \theta}$$ \hspace{1cm} (11)

Assuming that

$$L\theta > \max\left\{ \frac{2 (1 - \beta)}{\beta}, \frac{2 \beta}{(1 - \beta)} \right\},$$ \hspace{1cm} (12)

which implies $d_i > 0$, we then substitute (11) into (8) and obtain the optimal technology:

$$a_1' = \frac{2a_1}{(\theta L + 2) \beta} \quad \text{and} \quad a_2' = \frac{2a_2}{(\theta L + 2) (1 - \beta)}$$ \hspace{1cm} (13)

The optimal technology improvement rate, defined as $(a_1/a_1', a_2/a_2')$, are proportional to consumer’s expenditure share $(\beta, 1 - \beta)$. That is, it is optimal to improve the technology at higher rate in the sector that the consumer spends more.

In Type B trade, sector 2 is the import sector at home. If $\beta < \frac{1}{2}$, the optimal strategy at home is to improve the technology in import sector at higher rate than that in export sector. So the optimal strategy is relatively import-biased. More precisely, the world relative price after the optimal technical progress at home,

$$\frac{a_1'}{a_2'} = \left( \frac{a_1}{a_2} \right) \left( \frac{1 - \beta}{\beta} \right) > \frac{a_1}{a_2}$$ \hspace{1cm} (14)

which implies that terms of trade in the foreign country become worse. As we discussed in Section 3.1.2, the welfare in the foreign country decreases due to such optimal technology improvement at home.

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3With some computations we show that the second order condition for the maximization problem (10) holds. The proof is available from the authors upon the request.
4.2 The Case of Small Country

We now turn to Type B* equilibrium in which the home country is a small country. For simplicity, we assume that technology improvement does not change the type of trade in this case. The condition of which will be discussed later. Since the home country is small and always specialized in producing one good, it will not make the effort to improve the technology in the sector not produced. The optimal strategy, therefore, is determined by the comparison between reducing $a_1$, the export-biased strategy, and reducing $a_2$, the import-biased strategy.

If the export-biased technology improvement is chosen, the home country is specialized in producing good 1 and the wage rate $w = p/a_1' = a_1^*(1 + \theta d_1) / (a_2^* a_1)$.

The optimal consumptions are,

\[
x_1 = \frac{\beta w (L - d_1)}{p} = \frac{\beta (1 + \theta d_1) (L - d_1)}{a_1} \quad \text{and} \quad x_2 = (1 - \beta) w (L - d_1) = \frac{a_1^* (1 - \beta) (1 + \theta d_1) (L - d_1)}{a_2^* a_1}
\]

Substituting (15) into the utility function (2), the social planner’s problem becomes

\[
\max_{d_1} u^E(d_1) = \frac{\beta^\beta (1 - \beta)^{1-\beta} a_1^{1-\beta} a_2^{1-\beta} (L - d_1) (1 + \theta d_1)}{a_1 a_2 a_1}
\]

Solving the first order condition gives the optimal amount of labor used in technology improvement:

\[
d_1^E = \frac{\theta L - 1}{2\theta}
\]

If the import-biased technology improvement is chosen, the home country would be specialized in producing good 2 and the wage rate $w = 1/a_2' = (1 + \theta d_2)/a_2$. The world price is still $p = a_1^*/a_2^*$. Similar to the above, the social planner solves the
maximization problem

$$\max_{d_2} u^I(d_2) = \frac{\beta^\beta (1 - \beta)^{1-\beta} a_2^* \beta}{a_2 a_1^* \beta} (L - d_2)(1 + \theta d_2)$$  \hspace{1cm} (18)$$

and the solution is:

$$d_2^I = \frac{\theta L - 1}{2\theta}$$  \hspace{1cm} (19)$$

d_1^E and d_2^I need to be positive, which requires \(\theta L > 1\). Furthermore, in order to stay within Type B* equilibrium, \(E_3\) should be greater than both \(a_2^*/a_2^*\) and \(a_1^*/a_1^*\). Combining these two conditions we have

$$L(\theta L + 1) < \min \left\{ \frac{2\beta L^* a_1}{(1 - \beta) a_1^*}, \frac{2(1 - \beta) L^* a_2}{\beta a_2^*} \right\},$$  \hspace{1cm} (20)$$

which is the condition assumed to hold for the small country case.

For the import-biased technology improvement to be realized, the home country must reverse the comparative advantage. If \(d_2^I\) is not sufficiently large to reverse the comparative advantage, the home country needs to input \(d_2 > d_2^I\) to improve technology in sector 2. Nevertheless, \(u^I(d_2^I)\) is the highest utility that could be reached had the import-biased technology improvement been chosen.

Substituting (17) and (19) into (16) and (18), respectively, we have:

$$\frac{u^E(d_1^E)}{u^I(d_2^I)} = \frac{a_2 a_1^*}{a_1 a_2^*} = \frac{a_1^*/a_2^*}{a_1/a_2} > 1$$  \hspace{1cm} (21)$$

The inequality (21) comes from the comparative advantage assumption (1), which proves that the export-biased technology improvement is optimal for the small country.

\(d_i/L\) represents R&D per capita in the country. It is interesting to note that \(d_i^B/L\) in the big country case and \(d_i^E/L\) in the small country case are all increasing in \(L\). Therefore, it is optimal for bigger countries to invest more in R&D per capita.

We summarize our results as the following theorem:
Theorem 2 Bigger countries should invest more in R&D per capita. For a small country it is optimal to choose the export-biased technology improvement, which benefits the foreign country. The optimal strategy for a big country, however, is to improve the technology at a higher rate in the sector that consumers spend more, which hurts the foreign country if consumers at home spend more on the importable good.

4.3 When Will a “Catching-Up” Strategy Be Optimal?

Our final investigation examines the case of Type S equilibrium, where both countries are fully specialized. The home country can choose to reduce $a_1$, which is an export-biased technology improvement. On the other hand, if the home country chooses to reduce $a_2$, the magnitude of such technical progress must be sufficiently large. As we have discussed in Section 3.2.2, the reduction of $a_2$ will not be effective until $a'_2 \leq a''_2 = \frac{(1-\beta)a^*_{L\beta L}}{\beta L}$. The effective technology improvement in sector 2, however, shifts the type of trade to Type B. Within the Type B trade, as we have discussed in Section 4.1, the home country then must choose to improve technology in both sectors following the rule of (13), which is labeled as a “catching-up” strategy.

Instead of completely relying on imports in sector 2, the home country may try to “catch up” in import sector and be “self-sufficient” in both goods. Our model allows us to study conditions under which the catching-up strategy is optimal, which

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4The catching-up strategy has been a controversial policy in economic development. Implementations of the catching-up strategy in 1950s and 1960s by many developing countries are hardly viewed as successful, while the catching-up strategy in automobile industry by Japan and South Korea is impressive.
are stated as follows:

\[
\frac{a_2^*}{a_2} < \frac{\beta L^*}{(1 - \beta) L} < \frac{a_1^*}{a_1} \quad (22)
\]

\[
a_2' < a_2^H \iff a_2 < \frac{(1 - \beta)^2 (L\theta + 2)^2 a_2^*}{4\beta L^* \theta} \quad (23)
\]

\[
L\theta > \max\left\{ \frac{2(1 - \beta)}{\beta}, \frac{2\beta}{(1 - \beta)} \right\} \quad (24)
\]

\[
u^B(d_B^1, d_B^2) > \nu^E(d_E^1) \quad (25)
\]

Inequality (22) is the condition to ensure the type S trade, as indicated in Figure 2. To make sure that the catching-up strategy is effective, the technical progress in sector 2 should be sufficiently large so that the equilibrium switches from S to B, which is stated as the condition (23) as we have discussed in Section 3.2.2. Once (23) is satisfied and we are in Type B, and condition (24) restates the condition (12) which ensures that the country improves technology in both sectors. Finally, the condition (25) requires that the utility of catching-up strategy is larger than that of export-biased strategy. In the appendix, we derive a set of sufficient conditions under which (22), (23), (24), and (25) are satisfied. They are:

\[
\beta < 0.3494 \quad (26)
\]

\[
L\theta > \frac{2(1 - \beta)}{\beta} \quad (27)
\]

\[
a_2(L) < a_2 < \overline{a}_2(L) \quad (28)
\]

where

\[
\overline{a}_2(L) = \frac{(1 - \beta) L a_2^*}{\beta L^*} \quad (29)
\]

\[
\overline{\pi}_2(L) = \frac{(1 - \beta)^2 a_2^*}{4\beta L^* \theta} \left[ \frac{\beta^2 (L\theta + 2)^2}{(L\theta + 1)^{2\beta}} \right]^{1/(1 - \beta)} \quad (30)
\]

The condition \(a_2(L) < a_2\) in (28) ensures that the type of trade is S. The
catching-up strategy results in the increase in real income in terms of importable good and the improvement in terms of trade, while the export-biased strategy brings about the increase in real income in terms of exportable good and the adverse effect in terms of trade. Moreover, the resource spent in reducing labor usage per unit of production in sector 2 from $a_2$ to $a_2'1$ has no effect on economy and therefore is viewed as the cost of catching-up strategy. For the catching-up strategy to be effective, the condition (23) must be satisfied, which requires $\beta$ to be small, $L$ to be large, and $a_2$ to be small. The condition (25) is transformed to the second inequality in (28), again requiring $a_2$ to be small. Those are what sufficient conditions (26), (27), and (28) state: when the expenditure share of the import sector is sufficiently large ($\beta < 0.3494$), for the catching-up strategy to be optimal, the country should be relatively big and the technology in the import sector should be relatively advanced.

A numerical example can help to check our conclusion. Let $\beta = 1/4$, $\theta = 1/5$, $a_2^* = 1$, $L^* = 100$ and $L = 100$. Then if $3 < a_2 < 8.84545$, the sufficient conditions are satisfied. Further assuming $a_2 = 5$, then $u^B(d_1^B, d_2^B)/u^E(d_1^E) = 1.5339$. The welfare level generated by the catching-up strategy is about 50% higher than that generated by the export-biased strategy.

The sufficient conditions are depicted in Figure 6. Both $\alpha_2(L)$ and $\pi_2(L)$ are increasing in $L$ when (26) and (27) are satisfied. At any point of $(a_2, L)$ in the shaded area the country should choose the catching-up strategy. Consider two possible sizes of home country, $L^1 < L^2$. If the size were $L^2$, the country should begin to choose the catching-up strategy when $a_2 = \bar{a}_2^2$; were the size $L^1$, the country should begin to choose the catching-up strategy when $a_2 = \bar{a}_2^1$, and $\bar{a}_2^2 > \bar{a}_2^1$. If a less advanced technology (larger $a_2$) represents an earlier stage of the development, then we have shown that a larger country should choose the catching-up strategy in earlier stage of development. Summarizing we have:

**Theorem 3** When both countries are fully specialized, the catching-up strategy is optimal if the expenditure share on import sector is sufficiently large, the country is
relatively big, and the technology in import sector is relatively advanced. A larger country should choose the catching-up strategy in earlier stage of development than a smaller country should do.

5 Conclusion

Two central questions for international economic policy are what the optimal strategy of technology improvement for a country is, and what the impact of such optimal strategy will have on living standards, both at home and abroad. Answers to these questions are remarkably absent even in the simplest two-good, two-country Ricardian model. This paper fills this gap in the existing literature. We show that the size of country, the expenditure share on importable good, and the level of technology are three crucial factors to determine the optimal strategy of technology improvement and its effect on welfare.

We show that as the size becomes larger, the country should invest more in R&D per capita. This result differs from the conclusion that “research intensity does not depend on country size” made by Eaton and Kortum (2001, 2002, and 2006) in a series of papers. It is shown in this paper that it is optimal for a small country to choose the export-biased technical progress, which benefits both home and foreign countries. For a big country it is optimal to improve technology in both sectors at the rate proportional to the consumers’ expenditure share. Therefore, if the expenditure share of import sector is larger than that of export sector, the big country will choose a relatively import-biased technology improvement, which hurts its trade partner.

The “catching-up” strategy that the country also improves technology in import sector and becomes “self-sufficient” in all goods has been a controversial issue in economic development. Asides from the obvious security consideration of such a policy, when will the “catching-up” strategy be optimal economically? We provide
sufficient conditions for the “catching-up” strategy to be optimal and show that the strategy is economically sound if the expenditure share on import sector is sufficiently large, the country being relatively big and the technology in import sector being relatively advanced. Furthermore, a bigger country should choose the “catching-up” strategy at earlier development stage than a smaller country will do.

To simplify the analysis we make some restrictive assumptions. The utility function is Cobb-Douglas. The effect of R&D on production efficiency is taken as a linear form and the efficiency of R&D is assumed identical across all sectors. The analysis is based on the standard Ricardian model which ignores imperfect competition, product differentiation, externality, and firms heterogeneity. Nevertheless, the results found in this paper may provide a different perspective to examine the technology issue than the current literature.

6 Appendix

In this appendix we formally derive sufficient conditions for the catching-up strategy to be optimal in Pattern S trade. Using (4) and noting that \( w = p/a_1 \), the utility of export-biased strategy can be derived as

\[
U^E(d_1) = \frac{\beta L^{s_1-\beta}}{a_1^\beta a_2^{1-\beta}} (L - d_1)^\beta (1 + \theta d_1)^\beta
\]  

Maximizing \( U^E(d_1) \) we obtain that the optimal \( d_1^E = \frac{\theta L - 1}{2\theta} \). Now using (10), the condition (25) can be written as

\[
\frac{U^B(d_1^B, d_2^B)}{U^E(d_1^E)} = \frac{\beta^{2-1}(1 - \beta)^{2-2\beta} a_2^{1-\beta} (L\theta + 2)^2}{(L\theta)^{1-\beta} a_2^{1-\beta} 2^{2-2\beta} (L\theta + 1)^{2\beta}} > 1
\]  

Conditions (22), (23) and (25) are combined as
\[
\frac{(1 - \beta)La_2^*}{\beta L^*} < a_2 < \min\left\{ \frac{(1 - \beta)^2(L\theta + 2)^2a_2^*}{4\beta L^*\theta}, \frac{(1 - \beta)^2a_2^*}{4\beta L^*\theta} \frac{\beta^2(L\theta + 2)^2}{(L\theta + 1)^{2\beta}} \right\}^{1/(1-\beta)}
\]

We can show that the first term in the above curly brackets is larger than the second one, so a sufficient condition for the above inequality is

\[
\frac{(1 - \beta)La_2^*}{\beta L^*} < a_2 < \frac{(1 - \beta)^2a_2^*}{4\beta L^*\theta} \frac{\beta^2(L\theta + 2)^2}{(L\theta + 1)^{2\beta}}^{1/(1-\beta)}
\]
\[
\Rightarrow a_2(L) < a_2 < \sigma_2(L) \quad (33)
\]

In order for (33) to hold, we must have \(a_2(L) < \sigma_2(L)\). Computations reveal that \(a_2(L) < \sigma_2(L)\) if \(\beta < 0.3494\), which proves that (26), (27), and (28) are sufficient conditions for (22), (23), (24), and (25). When \(\beta < 0.3494\), it can be easily shown that \(\frac{\partial a_2(L)}{\partial L} > 0\).

References


Figure 1

Relative demand
Relative demand
Relative demand
Figure 2
Note: the expressions in brackets are the values at the corresponding points.
Figure 5
Note: the expressions in brackets are the values at the corresponding points

Figure 6