On the Magnet Effect of Foreign Direct Investment

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Abstract

We extend Antrás and Helpman (2004) on firm heterogeneity and choice of production location and organizational form to a dynamic setting with FDI uncertainty, in which the risk of investment failure increases with the sophistication of transplant technology, decreases with the infrastructure level in the host country, and evolves over time with the network generated by the multinational firms of the same origin. We formulate the often noted “magnet effect” of FDI, where an initial wave of industrial migration generates positive externality (information spillover) for subsequent investors, which stimulates consecutive waves of industrial migration. We prove the existence of a stable steady state for this process and characterize the effect of key parameters on the range of industry migration at the steady state.

Keywords: foreign direct investment; dynamic; magnet effect; network effect; externality

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1 Introduction

The FDI Confidence Index constructed by A.T. Kearney since 1998, based on annual surveys of the CEOs and top managers of the world’s largest 1000 companies, reveals a great degree of sensitivity of these executives to political and economic uncertainties when making overseas investment. Given the significant weight placed on risks in practice by firms when making FDI decisions, it is surprising that the risk factor associated with FDI decision making is not well analyzed in the existing FDI literature which focuses on the proximity-concentration trade off argument. This paper addresses this gap in the literature and investigates the incentives underlying firms’ FDI decisions and its timing when firms’ perceived investment uncertainty is incorporated. We extend the seminal paper of Antràs and Helpman (2004), hereafter the AH, on firm heterogeneity and choice of production location and organizational form to a dynamic setting with investment uncertainty. We focus in particular on two types of industrial organizational forms, domestic vertical integration and international production fragmentation via green-field FDI. Production of final goods requires a successful match of headquarter services with intermediate inputs. Headquarter services are always produced in the North by a parent firm; intermediate inputs, however, can be produced either in the South by a subsidiary or in the North by the parent firm.

It is assumed that international fragmentation is associated with higher risk of matching failure than domestic vertical integration. Therefore, in addition to the set up cost associated with FDI, each firm seeking cheap labor in the South through international fragmentation has to take into account the potential higher risk of not being able to appropriate the yield of final good due to unsuccessful matching, which in short will be called investment failure.

In this paper, we explicitly model three main factors determining the degree of uncertainty associated with FDI. They are (1) the quality of infrastructure and institution in the host country, (2) the degree of sophistication of the technology, and (3) the barriers of international information acquisition, or so called, the network effect. The first factor is country specific, the second firm-specific, and the third industry-specific, as to be elaborated below.

First, as shown in the FDI Confidence Index, the top decision makers frequently emphasize that the reliability and quality of infrastructure provision (such as power, water, roadway, and IT), and transparency or the severity of corruption are key factors determining their favorite destination for
investment. Given this stylized fact, we assume in the paper that the probability of investment failure increases with poorer infrastructure or institution in the host country.

Second, quality control is noted by the CEOs and top managers the most critical product-specific risk associated with overseas operations. CEOs cite that these risks are likely to temper the speed at which they undertake FDI in low-cost countries since poor yield rates may substantially erode FDI profits. The risks and damages are especially threatening for firms producing superior products which requiring more advanced technology. The reasons are quite straightforward. Production of goods with superior quality usually requires intensive face-to-face communications throughout the entire designing, manufacturing, and marketing process (Vernon (1966); Antràs (2005); Lu (2006)). Therefore, the yield rate (uncertainty) of producing abroad is thus lower (higher) than that of producing in house. To incorporate this concern by firms regarding the risks of quality control in their FDI decision, we assume that other things being equal, firms with higher productivity or producing better quality goods bear higher investment uncertainty.

Third, as it is well recognized in the trade literature, international business and social networks, such as immigration, long-settled co-ethnic communities, and overseas subsidiaries, facilitate information sharing and promote international trade (Greif (1993); Gould (1994); Belderbos and Sleuwaegen (1998); Rauch and Trindade (2002); Rauch and Casella (2003)). Similarly, the literature on labor migration decision has also rationalized and supported the view that family and community networks are important means by which ”pre-migration information” is conveyed from experienced migrants to potential migrants. Such information influences the expected income and uncertainty and enhances the incentive of the potential migration from the same country origin. (Boyd (1989); Banerjee (1991); Chau (1997); Bauer and Zimmerman (1997); Wilson (1998); Grieco (32); Winters et al. (2001)).

Much in the same spirit, we argue that international network also promotes FDI by helping the prospective investors to overcome informational barriers. Specifically, it is assumed in our paper that through the network formed by firms with FDI experiences, the information about the host country is relayed to the firms operating in the source country, which consequently lowers the investment uncertainty facing the prospective investors.

In sum, the risk of investment failure increases with the degree of sophistication of transplant technology, decreases with the infrastructure level in the host country, and evolves over time with
the network generated by the established multinational firms of the same origin. The incorporation
of these risk factors alters the basic prediction of the AH that more productive firms in a given
industry are more likely to undertake foreign direct investment (FDI). Instead, in this paper, we
show that firms with intermediate productivity levels are the first ones to migrate, while the most
productive and the least productive firms tend to stay behind. Thus, there is a non-monotonic
relationship between firm productivity and FDI propensity.

We show that the migration of Northern firms to the South is not automatic. We pin down the
minimum level of infrastructure that the South must possess in order to attract the first wave of
industry migration from the North. Migrating firms’ identities as reflected by their productivity
levels are traced out. We formalize the often noted “magnet effect” of FDI, where international
industry migration often occurs in clusters and in orders. As shown in the paper, when more firms
have succeeded in FDI and produced in the South, the general investment uncertainty decreases,
which lowers the FDI risk of failure. The first wave of industry migration generates positive extern-
ality for subsequent investors and stimulates a second wave of industry migration, which further
triggers a third wave of migration. We prove the existence of a stable steady state for this pro-
cess and characterize the effect of key parameters on the range of firms undertaking FDI at the
steady state. We then conduct simulations to illustrate the magnet effect of FDI and the structure
of international production at the steady state as introduced above. We illustrate the change in
the structure of international production at the steady state as key parameters of the model vary.
These include the level of infrastructure in the South, the economic distance between the host and
home country of the FDI, the wage of the South, and the fixed management costs of producing in
the South and in the North.

A number of related papers exist that study the timing of investment, entry, and exit under
uncertainty. These include McDonald and Siegel (1986), Dixit (1989), Dixit and Pindyck, Caballero
and Pindyck (1996), and Chawla (2006) among others. Complementing to most of these papers that
assume the uncertainty to follow some deterministic, time-invariant stochastic processes, we provide
explicit economic foundations for the force governing the underlying distribution of uncertainty.
In addition, our paper is also distinct from this literature in that we allow the distribution of
uncertainty to evolve over time with the endogenously formed network.

The rest of the paper is organized as follows. Section 2 demonstrates some empirical observations
that motivate our study. Section 3 presents the model, introduces the FDI uncertainty, and discusses the necessary and sufficient conditions under which non-zero measure of firms would undertake FDI before any network is formed. Section 4 describes the dynamics of industry migration as the magnet effects take place. Section 5 simulates the comparative static results. Section 6 concludes.

2 Empirical Observations

The non-monotonic relationship between firm productivity and FDI propensity is observed in a sample of Taiwanese firms over a period of 15 years since 1991 when the Taiwanese government lifted the ban on westward FDI in mainland China. The details are shown in Figure 1. In the figure, the timing of the first-time FDI undertaken by a Taiwanese firm, measured as the number of quarters elapsed since 1991, is indicated along the vertical axis, and the productivity of a firm along the horizontal axis. Firms in six major industries are sampled, with each panel indicating the pattern for each industry.

The firms in our sample are firms listed on either the Taiwan Stock Exchange Corporation or the Over the Counter market. We compile the information on the timing of FDI for each firm based on the Foreign Direct Investment Database published by Taiwan Economic Journal Data Bank (TEJ) and the Yearly Report published by the Investment Commissions, Ministry of Economic Affairs, Taiwan. The productivity of a firm is measured by the ratio of Return on Assets (ROA). We take the average of the ratios for a firm between 1991 and 1993, as an indication of a firm’s initial productivity level. The ROA data are taken from the TEJ Financial Statement Database.

As shown in the figure, across all industries, the firms with intermediate productivity levels tend to undertake FDI earlier than the most productive and the least productive firms. The phenomenon is most notable in the Electronics industry, where firms are spread over a wider range of productivity levels. This finding contradicts the conventional perception that higher-productivity firms have a higher tendency to undertake FDI than lower-productivity firms, as captured by the theoretical model of Antràs and Helpman (2004) in a static setting. The panel for the Electronics industry also suggests that the range of intermediate firms who undertake FDI expands over time, resembling a ‘cone’. This is an interesting dynamics that a static model could not readily explain. These observations motivate us to develop a theoretical model that accounts for the non-monotonic
relationship between firm productivity and FDI propensity, and a dynamic one that predicts the ‘cone’ dynamics of FDI.

3 The Model

The world has a unit measure of population with the following preference structure:

\[ U = x_0 + \frac{1}{\alpha_j} \sum_{j=1}^{J} X_j^{\alpha_j}, \quad 0 < \alpha_j < 1 \]

\[ X_j = \left( \int x_j(i)^{\alpha_j} di \right)^{1/\alpha_j}, \]

where \( x_0 \) is the numeraire, homogeneous, goods produced both in the North and in the South. The labor is the only factor of production and the production technology of \( x_0 \) exhibits constant returns to scale; thus the labor productivity of a country in producing the numeraire goods determines its wage rate. The North is assumed to have an absolute advantage in producing the numeraire goods, and therefore commands a higher wage: \( w^N > w^S \). \( X_j \) is the aggregate consumption of differentiated products \( x_j(i) \) in sector \( j \). The elasticity of substitution between any two varieties within a sector \( (\sigma_j = \frac{1}{1-\alpha_j}) \) is allowed to differ across sectors.

The derived inverse demand function for each variety \( i \) in sector \( j \) is:

\[ p_j(i) = x_j(i)^{\alpha_i-1} \] (1)

Let \( h_j(i) \) denote the headquarter service input and \( m_j(i) \) the intermediate input used in the production of variety \( i \) in sector \( j \). It is assumed that only the North has the ability to produce the headquarter service, while the intermediate input can be produced either in the North or the South. The production technology of variety \( i \) in sector \( j \) is:

\[ x_j^N(i) = \theta \left[ \frac{h_j(i)}{\eta_j} \right]^{\eta_j} \left[ \frac{m_j(i)}{1-\eta_j} \right]^{1-\eta_j} \] (2)

when the intermediate input is produced in the North. The parameter \( \eta_j \) denotes the intensity of the headquarter service component in the production of the final goods in sector \( j \). The higher \( \eta_j \) is, the
more important is the headquarter service component. The parameter $\theta$ indicates the productivity level of the headquarter producing the variety. We will also use this parameter to indicate the level of technology sophistication employed to produce the product. Thus, it is assumed that a more productive headquarter produces a variety that embodies more sophisticated technology.

On the other hand, if the intermediate input is produced in the South, the headquarter faces a potential risk that the intermediate input produced abroad may fail to match the exact specification designed by the North. In this case, the output is zero. In brief, the production technology in this case is:

$$x^S_j(i) = \begin{cases} \theta \left[ \frac{h_j(i)}{\eta_j} \right]^\eta \left[ \frac{m_j(i)}{1-\eta_j} \right]^{1-\eta_j}, & \text{in case of success;} \\ 0, & \text{in case of failure.} \end{cases}$$

The headquarter chooses between acquiring the ownership of a manufacturing plant in the North (domestic vertical integration) or one in the South (international vertical integration). It is assumed that the organizational fixed cost to coordinate the headquarter and the supplier of intermediate components is higher in the case of international vertical integration than domestic vertical integration:

$$f^S > f^N. \quad (3)$$

In the following, we focus on one sector and suppress the sector index to simplify the notation. By (1) and (2), the revenue function for variety $i$ given the amounts of the inputs is:

$$R^l(i) = \gamma^l \theta^\alpha \left[ \frac{h(i)}{\eta} \right]^\alpha \left[ \frac{m(i)}{1-\eta} \right]^{\alpha(1-\eta)}, \quad l = \{N, S\} \quad (4)$$

where $\gamma^N = 1$, and $0 < \gamma^S < 1$ is the probability of successful matching of the headquarter service and the intermediate input produced in the South.

As the contract between the headquarter and the supplier of intermediate inputs is non-enforceable, the two parties negotiate the division of the sales revenue ex post. The division depends on the relative negotiating powers of the two parties and their outside options. Suppose the negotiation fails. Given the headquarter’s ownership in the manufacturing plant, it can potentially seize the intermediate component at the cost of a fraction $(1 - \delta)$ of the output. Also suppose that the headquarter is able to extract at the ex post negotiation, a fraction $\beta$ of the surplus from
the contract with the supplier. Then, the division of the sales revenue for the headquarter is:

$$\beta^N = \beta^S = (\delta)^\alpha + \beta[1 - (\delta)^\alpha].$$  \hspace{1cm} (5)

In the first stage, the headquarter and the supplier of the intermediate input determine respectively how much headquarter service and intermediate input to produce, taking into account the fraction of the sales revenue they will extract from the negotiation in the second stage. The production of either input is assumed to have a unit labor requirement of one. With profit maximization by both parties, the joint profit of the two parties in either of the two potential organizational forms is:

$$\pi^l(\theta, \eta) = \gamma^l \frac{1}{\alpha} \theta^{\frac{\alpha}{1}} \psi^l(\eta) - w^N f^l, \hspace{0.5cm} l = \{N, S\},$$  \hspace{1cm} (6)

where

$$\psi^l(\eta) = \frac{1 - \alpha[\beta^l \eta + (1 - \beta^l)(1 - \eta)]}{\{1/\alpha\} (w^N/\beta^l) \eta [w^l/(1 - \beta^l)]^{1 - \eta} \eta^{\alpha/(1 - \alpha)}}. \hspace{1cm} (7)

It can be shown that the headquarter will choose the organizational form that maximizes the above joint profit. Given that \(w^S < w^N\), it follows that \(\psi^S(\eta) > \psi^N(\eta)\). Thus, headquarters choosing to engage in FDI in the South enjoy a larger variable profit margin. However, they face a higher fixed organizational cost, and the risk of mismatch involved in FDI further reduces the variable profit gain of producing in the South when compared to the North. We assume that the probability of successful matching takes the following specific form:

$$\gamma^S(\theta) = \left(\frac{b}{\theta}\right)^{\alpha z}, \hspace{0.5cm} \theta \geq b > 0, \hspace{1cm} (8)

z = \frac{1}{K + X^S}, \hspace{0.5cm} K > 1, \hspace{0.5cm} X^S \geq 0, \hspace{1cm} (9)

where the parameter \(b\) represents the lower bound of \(\theta\), \(K\) the level of infrastructure in the South, and \(X^S\) the degree of FDI penetration in the South. The level of infrastructure encompasses various aspects of a nation’s capacity in absorbing FDI. This includes physical infrastructure, social capital, human capital, and governance infrastructure. Equations (8) and (9) imply that the higher the level of infrastructure and the larger the presence of FDI in the South, the higher the probability that a FDI undertaking will succeed for a given \(\theta\). Observe that the FDI success probability is
strictly decreasing in product sophistication, $\theta$, with the probability equal to one for the lowest-tech product and approaching zero as the product sophistication increases toward infinity ($\partial \gamma^S / \partial \theta < 0$, $\gamma^S(b) = 1$, and $\lim_{\theta \to \infty} \gamma^S(\theta) = 0$).

To simplify notation in the following exposition, define $\tilde{\theta} = \theta^{1/\alpha}$, and $\tilde{b} = b^{1/\alpha}$. It follows that

$$\pi^S(\tilde{\theta}, z) = \psi^S \tilde{b}^z \tilde{\theta}^{1-z} - w^N f^S, \quad 0 < z < 1 \quad (10)$$

$$\pi^N(\tilde{\theta}) = \psi^N \tilde{\theta} - w^N f^N. \quad (11)$$

Equation (10) implies that the profit function of undertaking FDI is increasing and concave in $\tilde{\theta}$, in contrast with the linear profit function of producing in the North. Note that $\lim_{z \to 0} \gamma^S = 1$ and $\lim_{z \to 0} \pi^S(\tilde{\theta}, z) \to \tilde{\pi}^S(\tilde{\theta}) \equiv \psi^S \tilde{\theta} - w^N f^S$. Thus, our model includes Antràs and Helpman (2004) as a special case, when $z$ approaches zero and FDI uncertainty disappears. We adopt the following assumption to avoid a taxonomy of cases.

**Assumption 1** The parameters satisfy the following conditions: (i) $\tilde{b} < w^N f^N / \psi^N$; (ii) $f^N / \psi^N < f^S / \psi^S$.

Define $\tilde{\theta}_N \equiv w^N f^N / \psi^N$ and $\tilde{\theta}' \equiv w^N f^S / \psi^S$. Headquarters (hereafter firms) with the productivity level $\tilde{\theta}_N$ break even by producing in the North, while firms with the productivity level $\tilde{\theta}'$ break even by producing in the South, if there is no FDI uncertainty. By Assumption 1(ii), it follows that $\tilde{\theta}_N < \tilde{\theta}'$, and there exists $\tilde{\theta}_S \equiv w^N (f^S - f^N) / (\psi^S - \psi^N) > \tilde{\theta}'$ such that firms with the productivity level $\tilde{\theta}_S$ are indifferent between producing in the North and producing in the South under no FDI uncertainty. Together, Assumptions 1(i) and (ii) imply that the least productive firm $\tilde{b}$ is below the lower threshold to produce in the North, $\tilde{\theta}_N$, which is further below the lower threshold to produce in the South under no uncertainty, $\tilde{\theta}_S$. Firms are partitioned according to their productivity levels into the least productive ones who do not produce, the less productive ones who produce in the North, and the most productive ones who produce in the South. There is no complete specialization by the South in the production of intermediate components even in the best scenario of no FDI uncertainty. This is the benchmark in Antràs and Helpman (2004) and represents the limiting case of our model as the FDI uncertainty becomes negligible.

**Proposition 1** Under Assumption 1, there exists a unique $z^* \in (0, 1)$ such that the curve $\pi^S$ is tangent to $\pi^N$, and
(i) for all \(z \in (0, z^*)\), \(\exists \{\tilde{\theta}_0, \tilde{\theta}_1\}\) with \(\tilde{\theta}_S < \tilde{\theta}_0 < \tilde{\theta}_1\), such that \(\pi^S(\tilde{\theta}, z) > \pi^N(\tilde{\theta}) > 0\) for all \(\tilde{\theta} \in (\tilde{\theta}_0, \tilde{\theta}_1)\), and \(\pi^S(\tilde{\theta}, z) < \pi^N(\tilde{\theta})\) for all \(\tilde{\theta} \in [\tilde{b}, \tilde{\theta}_0) \cup (\tilde{\theta}_1, \infty)\);

(ii) for all \(z \in (z^*, 1)\), \(\pi^S(\tilde{\theta}, z) < \pi^N(\tilde{\theta})\) for all \(\tilde{\theta} \in [\tilde{b}, \infty)\).

Proof of Proposition 1. Define \(\tilde{\theta}^+\) such that \(\frac{\partial \pi^S}{\partial \tilde{\theta}}|_{\tilde{\theta}=\tilde{\theta}^+} = \frac{\partial \pi^N}{\partial \tilde{\theta}}|_{\tilde{\theta}=\tilde{\theta}^+}\); that is, \(\tilde{\theta}^+\) is the productivity level where the two curves \(\pi^S\) and \(\pi^N\) have the same slope. It is straightforward to verify that

\[\tilde{\theta}^+ = \tilde{b} \left[ (1 - z) \frac{\psi^S}{\psi^N} \right]^{1/z}.\]

Thus, \(\tilde{\theta}^+ \geq \tilde{b}\) if and only if \(z \leq 1 - \frac{\psi^N}{\psi^S} \equiv \bar{z}\).

(i) For \(0 < z < \bar{z}\), the difference between the two curves at \(\tilde{\theta}^+\) is

\[\pi^S(\tilde{\theta}^+, z) - \pi^N(\tilde{\theta}^+) = \frac{z}{1 - z} \tilde{\theta}^+ \psi^N - w^N(f^S - f^N).\]

Define \(\phi(z) \equiv \frac{z}{1 - z} \tilde{\theta}^+\) and \(g(z) \equiv \frac{1 - z}{z}\). It is straightforward to show that

\[
\frac{\partial \tilde{\theta}^+}{\partial z} = \frac{\partial \ln \tilde{\theta}^+}{\partial z} \tilde{\theta}^+ = \left\{ -\frac{1}{z^2} \ln \left( 1 - z \right) \frac{\psi^S}{\psi^N} - \frac{1}{z(1 - z)} \right\} \tilde{\theta}^+ < 0,
\]

where the last inequality follows from the fact that \((1 - z)\frac{\psi^S}{\psi^N} > 1\) for \(0 < z < \bar{z}\). Because \(\lim_{z \to 0} \tilde{\theta}^+ = \infty\) and \(\lim_{z \to 0} g(z) = \infty\), by L'Hôpital rule,

\[
\lim_{z \to 0} \phi(z) = \lim_{z \to 0} \frac{\partial \tilde{\theta}^+}{\partial z} \frac{\partial g(z)}{\partial z} = \lim_{z \to 0} \left\{ \ln \left( 1 - z \right) \frac{\psi^S}{\psi^N} + \frac{z}{1 - z} \right\} \tilde{\theta}^+ \to \infty.
\]

Thus,

\[\lim_{z \to 0} \pi^S(\tilde{\theta}^+, z) - \pi^N(\tilde{\theta}^+) = \tilde{b} \left[ (1 - z) \frac{\psi^S}{\psi^N} \right]^{1/z} \psi^S - \psi^N - w^N(f^S - f^N) < 0,
\]

On the other hand, it is straightforward to show that \(\lim_{z \to \bar{z}} \phi(z) = \frac{\bar{z}}{1 - \bar{z}} \tilde{b} = \frac{\psi^S - \psi^N}{\psi^S} \tilde{b}\). Therefore,

\[\lim_{z \to \bar{z}} \pi^S(\tilde{\theta}^+, z) - \pi^N(\tilde{\theta}^+) = \tilde{b}(\psi^S - \psi^N) - w^N(f^S - f^N) < 0,
\]
where the last inequality follows by Assumption 1. Finally, observe that for $0 < z < \bar{z}$,

$$\frac{\partial \phi(z)}{\partial z} = \left[ \frac{\partial \ln \hat{\theta}^+}{\partial z} - \frac{\partial \ln g(z)}{\partial z} \right] \phi(z)$$

$$= \left\{ -\frac{1}{z^2} \ln \left[ (1-z) \frac{\psi^S}{\psi^N} \right] - \frac{1}{z(1-z)} + \frac{1}{z(1-z)} \right\} \phi(z)$$

$$= \left\{ -\frac{1}{z^2} \ln \left[ (1-z) \frac{\psi^S}{\psi^N} \right] \right\} \phi(z) < 0.$$

Therefore,

$$\partial \left[ \pi^S(\hat{\theta}^+, z) - \pi^N(\hat{\theta}^+) \right] / \partial z < 0.$$

In summary, as $z \to 0$, $\pi^S(\hat{\theta}, z)$ becomes linear, and $\hat{\theta}^+ \to \infty$. The difference between $\pi^S(\hat{\theta}, z)$ and $\pi^N(\hat{\theta})$ at $\hat{\theta} = \hat{\theta}^+$ approaches infinity. As $z$ increases, $\hat{\theta}^+$ decreases and the difference between $\pi^S(\hat{\theta}^+, z)$ and $\pi^N(\hat{\theta}^+)$ also decreases monotonically. At $z = \bar{z}$, $\hat{\theta}^+ = \hat{b}$ and the difference between $\pi^S(\hat{\theta}^+, z)$ and $\pi^N(\hat{\theta}^+)$ becomes negative. Therefore, there must exist a unique $z^* \in (0, \bar{z})$ such that $\pi^S(\hat{\theta}^+, z^*) - \pi^N(\hat{\theta}^+) = 0$. In other words, the curve $\pi^S(\hat{\theta}, z)$ is tangent to $\pi^N(\hat{\theta})$ at $z = z^*$.

For $z \in (0, z^*)$, $\pi^S(\hat{\theta}^+, z) - \pi^N(\hat{\theta}^+) > 0$. Thus, the curve $\pi^S(\hat{\theta}, z)$ must have intersected the line $\pi^N(\hat{\theta})$ twice. Label the corresponding productivity levels $\hat{\theta}_0$ and $\hat{\theta}_1$ with $\hat{\theta}_0 < \hat{\theta}_1$, such that $\pi^S(\hat{\theta}, z) - \pi^N(\hat{\theta}) = 0$ at $\hat{\theta} = \{\hat{\theta}_0, \hat{\theta}_1\}$. Then it follows from the concavity of $\pi^S(\hat{\theta}, z)$ that $\pi^S(\hat{\theta}, z) - \pi^N(\hat{\theta}) > 0$ for all $\hat{\theta} \in (\hat{\theta}_0, \hat{\theta}_1)$. Next, note that for $z > 0$, the curve $\pi^S(\hat{\theta}, z)$ falls below the line $\pi^S(\hat{\theta}) = \psi^S(\hat{\theta}) - w^N f^S$, which intersects the line $\pi^N(\hat{\theta})$ at $\hat{\theta}_S$. Thus, it must be the case that $\hat{\theta}_0 > \hat{\theta}_S$ for $z > 0$. Moreover, because $\hat{\theta}_0 > \hat{\theta}_N$, it follows that $\pi^N(\hat{\theta}) > 0$ for $\hat{\theta} \in (\hat{\theta}_0, \hat{\theta}_1)$. Therefore, $\pi^S(\hat{\theta}, z) > \pi^N(\hat{\theta}) > 0$ for all $\hat{\theta} \in (\hat{\theta}_0, \hat{\theta}_1)$, where $\hat{\theta}_S < \hat{\theta}_0 < \hat{\theta}_1$. It also follows from the concavity of $\pi^S(\hat{\theta}, z)$ that $\pi^S(\hat{\theta}, z) < \pi^N(\hat{\theta})$ for all $\hat{\theta} \in [\hat{b}, \hat{\theta}_0) \cup (\hat{\theta}_1, \infty)$.

$(ii)$ For $z^* < z < \bar{z}$, the curve $\pi^S(\hat{\theta}, z)$ falls completely below the line $\pi^N(\hat{\theta})$ for all $\hat{\theta} \in [\hat{b}, \infty)$. Thus, $\pi^S(\hat{\theta}, z) < \pi^N(\hat{\theta})$ for all $\hat{\theta} \in [\hat{b}, \infty)$.

For $\bar{z} \leq z < 1$, $\hat{\theta}^+ \leq \hat{b}$, and because $\frac{\partial w^S}{\partial \theta}$ is decreasing in $\hat{\theta}$, it follows that $\frac{\partial w^S}{\partial \theta} < \frac{\partial w^N}{\partial \theta}$, for all $\hat{\theta} > \hat{b}$. In addition, at $\hat{\theta} = \hat{b}$, $\pi^S(\hat{\theta}, z) < \pi^N(\hat{\theta})$ by Assumption 1. It follows that $\pi^S(\hat{\theta}, z) < \pi^N(\hat{\theta})$ for all $\hat{\theta} \in [\hat{b}, \infty)$. The desired result in $(ii)$ therefore follows.

The intuition of Proposition 1 is illustrated in Figure 2. In the figure, $\hat{\theta}$ is indicated on the horizontal axis and $\pi^S$ (or $\pi^N$) on the vertical axis. Note that regardless of $z$, the curve $\pi^S(\hat{\theta}, z)$
always passes through the point \((\tilde{\theta}, \pi^S) = (\tilde{b}, \psi^S\tilde{b} - w^N f^S)\), denoted Point A in Figure 2, and the vertical intercept point \((\tilde{\theta}, \pi^S) = (0, -w^N f^S)\). As \(z \to 0\), the curve \(\pi^S(\tilde{\theta}, z)\) approaches the line \(\pi^S(\tilde{\theta}) = \psi^S\tilde{b} - w^N f^S\), which intersects the line \(\pi^N(\tilde{\theta})\) at \(\tilde{\theta}_S\) and lies above the line \(\pi^N(\tilde{\theta})\) for all \(\tilde{\theta} > \tilde{\theta}_S\). Define \(\pi^S_x(\tilde{\theta}, z) = \frac{\partial \pi^S(\tilde{\theta}, z)}{\partial z}\), we have

\[
\pi^S_x(\tilde{\theta}, z) = \psi^S\tilde{b}^1-z \ln(\frac{\tilde{b}}{\theta}).
\]

Observe that \(\pi^S_x(\tilde{\theta}, z) \leq 0\) for \(\hat{\theta} \geq \tilde{b}\). Thus, as \(z\) departs from zero and increases, the curve \(\pi^S(\tilde{\theta}, z)\) rotates clockwise around Point A. In the limit as \(z \to 1\), the curve \(\pi^S(\tilde{\theta}, z)\) approaches a step function with \(\pi^S = -w^N f^S\) for \(\tilde{\theta} = 0\) and \(\pi^S = \psi^S\tilde{b} - w^N f^S\) for all \(\tilde{\theta} > 0\). This falls below the line \(\pi^N(\tilde{\theta})\) completely for all \(\tilde{\theta} \geq \tilde{b}\) by Assumption 1. Thus, there exists a unique \(z^* \in (0, 1)\) such that the curve \(\pi^S(\tilde{\theta}, z)\) is tangent to the line \(\pi^N(\tilde{\theta})\). For \(z \in (0, z^*)\), the curve \(\pi^S(\tilde{\theta}, z)\) intersects the line \(\pi^N(\tilde{\theta})\) twice, at \(\tilde{\theta}_0\) and \(\tilde{\theta}_1\), with both of them necessarily larger than \(\tilde{\theta}_S\). By the concavity of the curve \(\pi^S(\tilde{\theta}, z)\), it follows that \(\pi^S(\tilde{\theta}, z) > \pi^N(\tilde{\theta}) > \pi^N(\tilde{\theta}_S) > 0\) for all \(\tilde{\theta} \in (\tilde{\theta}_0, \tilde{\theta}_1)\), and that \(\pi^S(\tilde{\theta}, z) < \pi^N(\tilde{\theta})\) otherwise. For \(z \in (z^*, 1)\), the curve \(\pi^S(\tilde{\theta}, z)\) falls completely below the line \(\pi^N(\tilde{\theta})\), and therefore \(\pi^S(\tilde{\theta}, z) < \pi^N(\tilde{\theta})\) for all \(\tilde{\theta} \in [\tilde{b}, \infty)\).

**Corollary 2** For \(z \in (0, z^*)\), firms are partitioned according to their productivity levels as follows:

(i) for firms with \(\tilde{\theta} \in [\tilde{b}, \tilde{\theta}_N]\), they exit the market; (ii) for firms with \(\tilde{\theta} \in [\tilde{\theta}_N, \tilde{\theta}_0] \cup [\tilde{\theta}_1, \infty)\), they integrate the production in the North; (iii) for firms with \(\tilde{\theta} \in [\tilde{\theta}_0, \tilde{\theta}_1]\), they undertake FDI.

On the other hand, for \(z \in (z^*, 1)\), FDI is not viable: (i) for firms with \(\tilde{\theta} \in [\tilde{b}, \tilde{\theta}_N]\), they exit the market; (ii) for firms with \(\tilde{\theta} \in [\tilde{\theta}_N, \infty)\), they integrate the production in the North.

**Proof of Corollary 2.** The results follow immediately from Proposition 1 and the definition of \(\tilde{\theta}_N\).

**Proposition 3** For \(z \in (0, z^*)\), the range of firms undertaking FDI, as denoted by \((\tilde{\theta}_1 - \tilde{\theta}_0)\), is monotonically decreasing in \(z\):

(i) as \(z \to 0\), \(\tilde{\theta}_1 - \tilde{\theta}_0 \to \infty\);

(ii) as \(z \to z^*\), \(\tilde{\theta}_1 - \tilde{\theta}_0 \to 0\)
Proof of Proposition 3. Recall that by definition,

\[ \pi^S(\tilde{\theta}_1, z) - \pi^N(\tilde{\theta}_1) = 0 \]  
(13)

\[ \pi^S(\tilde{\theta}_0, z) - \pi^N(\tilde{\theta}_0) = 0 \]  
(14)

with \( \tilde{\theta}_0 < \tilde{\theta}_1 \). Take total derivatives of (13) and (14) with respect to \( \tilde{\theta}_1, \tilde{\theta}_0 \) and \( z \) and define \( \pi^S_\theta \equiv \frac{\partial \pi^S(\theta, z)}{\partial \theta} \). We obtain

\[ \frac{d\tilde{\theta}_1}{dz} = -\left[ \pi^S_\theta(\tilde{\theta}_1, z) - \psi^N \right]^{-1} \pi^S_z(\tilde{\theta}_1, z) < 0 \]  
(15)

\[ \frac{d\tilde{\theta}_0}{dz} = -\left[ \pi^S_\theta(\tilde{\theta}_0, z) - \psi^N \right]^{-1} \pi^S_z(\tilde{\theta}_0, z) > 0 \]  
(16)

where the inequalities follow from the fact that \( (\pi^S_\theta(\tilde{\theta}_1, z) - \psi^N) < 0, (\pi^S_\theta(\tilde{\theta}_0, z) - \psi^N) > 0 \) and that \( \pi^S_z(\theta, z) < 0 \) for all \( \theta > \bar{\theta} \). The results (15) and (16) imply that \( \frac{d(\tilde{\theta}_1 - \tilde{\theta}_0)}{dz} < 0 \). As \( z \to 0 \), \( \pi^S(\theta, z) \to \pi^S(\bar{\theta}) = \psi^S - w^N f^S \); thus, \( \tilde{\theta}_0 \to \tilde{\theta}_S \) and \( \tilde{\theta}_1 \to \infty \). On the other hand, as \( z \to z^* \), the curve \( \pi^S(\bar{\theta}, z) \) becomes tangent to the line \( \pi^N(\bar{\theta}) \); thus, \( \tilde{\theta}_0 \to \tilde{\theta}_1 \).

4 The Dynamics of FDI

In this section, we extend the static model introduced above to a dynamic setting with multiple time periods. We formalize the stylized fact that earlier FDI often reveals information regarding the host country’s investment environment, which helps lower the uncertainty faced by later investors. The information spillover is assumed to be external to firms and to affect the whole economy. Specifically, we assume that

\[ \gamma_t^S(\theta) = \left( \frac{b}{\theta} \right)^{\alpha_{\theta t}}, \theta \geq b > 0, \]  
(17)

\[ z_t = \frac{1}{K + X_{t-1}^S}, \quad K > 1, \quad t = 1, 2, \ldots \]  
(18)

where \( X_{t-1}^S \) denotes the degree of FDI penetration in the South in period \( t - 1 \). Equations (17) and (18) imply that a higher degree of FDI penetration in period \( t - 1 \) raises the success probability of FDI in period \( t \) for any given \( \theta \). Thus, earlier FDI creates a positive externality for subsequent FDI. In particular, the degree of FDI penetration in period \( t \) is defined as the effective mass of
firms producing in the South in period $t$:

$$X^S_t = \frac{G(\theta_{1,t}) - G(\theta_{0,t})}{d}, \quad d > 0,$$

(19)

where $G(\cdot)$ is the cumulative distribution function of firm productivity levels and is chosen to be a Pareto distribution with shape $k$, i.e., $G(\theta) = 1 - \left(\frac{b}{\theta}\right)^k$. In (19), the magnitude $G(\theta_{1,t}) - G(\theta_{0,t})$ represents the absolute mass of firms producing in the South in period $t$, which is scaled by the economic distance, $d$, between the host and home country of FDI. The economic distance, $d$, summarizes the barriers to information exchange arising from physical distance, and cultural, language, and institutional differences. Thus, for a given absolute mass of firms transplanted in the South, the effective mass and the extent of information spillover it creates is larger, the closer the two countries are in terms of their economic distance.

We now formulate the dynamics of FDI. Begin with an initial period ($t = 0$) when no FDI is present in the South ($X^S_0 = 0$). It is straightforward to identify the minimum level of infrastructure required of the South to trigger the first wave of FDI. By Proposition 1, it follows that:

**Corollary 4** The minimum level of infrastructure required of the South to trigger the first wave of FDI is $K^* = 1/z^*$.

The importance of the recipient country’s infrastructure in influencing FDI flows has been documented by various empirical contributions. See, for example, Wei (2000) for a study of corruption and its depressing effect on inward FDI. In another study, Globerman and Shapiro (2002) estimated the minimum threshold of infrastructure that a recipient country must achieve to attract positive flows of FDI from the United States.

Let $\theta_{0,t}$ and $\theta_{1,t}$ indicate the lower and upper bound of productivity levels, within which firms undertake FDI in period $t$. If the South meets the minimum threshold of infrastructure ($K > K^*$), FDI takes place at $t = 1$, and by Corollary 2, there exists $\tilde{\theta}_{0,1} < \tilde{\theta}_{1,1}$ such that firms with $\tilde{\theta} \in (\tilde{\theta}_{0,1}, \tilde{\theta}_{1,1})$ undertake FDI. By (17)–(19), it follows that $\gamma^S_2 > \gamma^S_1$, given that $X^S_1 > X^S_0 = 0$. Thus, the first wave of FDI helps raise the success probability of FDI at $t = 2$. This is illustrated in Figure 3, where the curve $\pi^S(\tilde{\theta}; K, X^S_{t-1})$ tilts upward at $t = 2$. Firms with productivity levels in the range of $(\tilde{\theta}_{0,1} - \epsilon, \tilde{\theta}_{0,1})$ or $(\tilde{\theta}_{1,1}, \tilde{\theta}_{1,1} + \epsilon)$, who find it not profitable to undertake FDI at $t = 1$, now
prefer moving the production process to the South, since the risk associated with FDI is lower than before. As a result, the first wave of migration induces a second wave of migration, $(\tilde{\theta}_{0,1}, \tilde{\theta}_{1,1}) \subset (\tilde{\theta}_{0,2}, \tilde{\theta}_{1,2})$, and the effective mass of FDI firms in the South is enlarged, $X^S_2 > X^S_1 > X^S_0 = 0$. The larger mass of FDI firms will further reduce the FDI uncertainty and trigger subsequent waves of FDI. This dynamic scenario is what we call the “magnet” effect of FDI. In summary, the time paths of $\{(\tilde{\theta}_{1,t}, \tilde{\theta}_{0,t}, z_t)\}_{t=1}^{\infty}$ can be represented as: 

$$
\pi^S(\tilde{\theta}_{1,t}, z_t) - \pi^N(\tilde{\theta}_{1,t}) = 0 \quad t = 1, 2, \ldots 
$$

(20)

$$
\pi^S(\tilde{\theta}_{0,t}, z_t) - \pi^N(\tilde{\theta}_{0,t}) = 0 \quad t = 1, 2, \ldots 
$$

(21)

$$
z_t - \frac{1}{K + d^{-1}G(\tilde{\theta}_{1,t-1}) - d^{-1}G(\tilde{\theta}_{0,t-1})} = 0 \quad t = 2, 3, \ldots 
$$

(22)

with an initial value $z_1 = 1/K < z^*$. To facilitate exposition, we introduce the transformed cumulative distribution function $\tilde{G}(\tilde{\theta}) \equiv 1 - \left(\frac{\tilde{b}}{\tilde{\theta}}\right)^{\tilde{k}} = G(\theta)$, where $\tilde{k} \equiv \frac{1-\alpha}{\alpha}$. 

**Proposition 5** There exists a stable steady-state solution $(\tilde{\theta}^c_1, \tilde{\theta}^c_0, z^c)$ to the system (20)–(22). 

**Proof of Proposition 5.** As we are focused on the steady state, the time index will be omitted in the following proof. Let $\tilde{\theta}_1(z)$ and $\tilde{\theta}_0(z)$ indicate the solutions to (20) and (21) as functions of $z$. Thus, (19) can be re-expressed as: 

$$
X^S(z) = \frac{\tilde{G}(\tilde{\theta}_1(z)) - \tilde{G}(\tilde{\theta}_0(z))}{d} 
$$

(23)

On the other hand, at the steady state, (22) reduces to: 

$$
z(X^S) = \frac{1}{K + X^S} 
$$

(24)

These two functions are illustrated in Diagram (a) of Figure 4. Define $\tilde{z} \equiv \frac{1}{K + d^{-1}(1-G(\theta_S))}$. Note that $z(0) = 1/K$, where $1/K < z^*$ by default such that a first wave of FDI occurs and the system (20)–(21) applies. Next, note that $z(\frac{1-\tilde{G}(\tilde{\theta}_S)}{d}) = \tilde{z}$, which is the minimum value that $z$ can possibly take and which occurs if $\tilde{\theta}_1 \to \infty$ and $\tilde{\theta}_0 \to \tilde{\theta}_S$. Finally, note that $z(X^S)$ is a decreasing convex function of $X^S$. 

On the other hand, note that $X^S(z^*) = 0$ and $X^S(\tilde{z}) < \frac{1-\tilde{G}(\tilde{\theta}_S)}{d}$. The former result follows from
Proposition 3 (ii). The latter result follows from the fact that at \( z > 0 \), the curve \( \pi^S(\tilde{\theta}, z) \) is still concave and will hence intersect \( \pi^N(\tilde{\theta}) \) at finite \( \tilde{\theta}_1 \) and at \( \tilde{\theta}_0 > \tilde{\theta}_S \), leading to \( \tilde{G}(\tilde{\theta}_1(z)) < 1 \) and \( \tilde{G}(\tilde{\theta}_0(z)) > \tilde{G}(\tilde{\theta}_S) \). Recall from (15) and (16) that \( d\tilde{\theta}_1/dz < 0 \) and \( d\tilde{\theta}_0/dz > 0 \). This implies that \( dX^S/dz < 0 \). Thus, \( X^S(z) \) is a monotonically decreasing function of \( z \).

The above characterizations of \( X^S(z) \) and \( z(X^S) \) are captured in Diagram (a) of Figure 4, where \( z \) is expressed along the horizontal axis and \( X^S \) the vertical axis. We note that the curve \( X^S(z) \) crosses the curve \( z(X^S) \) from below. The intersection of the two curves defines the steady-state level \( z_c \), which implies the corresponding steady-state level of \( (\tilde{\theta}_c^1, \tilde{\theta}_c^0) \). If there is a deviation from the steady state, for example, at point “1” in Diagram (a), the system converges back to the steady-state level \( z_c \). Thus, the steady state is also stable.

In Diagram (b), a more general scenario is illustrated, where the curve \( X^S(z) \) is still monotonically decreasing in \( z \) but intersects the curve \( z(X^S) \) more than once. In this case, we note that there must be an odd number of steady-state solutions and more than one of them are stable. In fact, if the system starts from \( z = 1/K \) as is the case in our dynamic setting, the stable steady state closer to \( z = 1/K \) is the realized steady state, which is represented by Point I in Diagram (b). To conclude, there always exists a stable steady-state triplet \( (\tilde{\theta}_c^1, \tilde{\theta}_c^0, z_c) \) which solves the system (20)–(22).

Proposition 6 At a stable steady state \( (\tilde{\theta}_c^1, \tilde{\theta}_c^0, z_c) \) of the system (20)–(22), the following holds:

\[
\lambda_c \equiv (z_c)^2 d^{-1} \tilde{G}'(\tilde{\theta}_1) \left[ \pi^S_{\tilde{\theta}}(\tilde{\theta}_c^1, z_c) - \psi_N \right]^{-1} \pi^S_z(\tilde{\theta}_1, z_c) - (z_c)^2 d^{-1} \tilde{G}'(\tilde{\theta}_0) \left[ \pi^S_{\tilde{\theta}}(\tilde{\theta}_c^0, z_c) - \psi_N \right]^{-1} \pi^S_z(\tilde{\theta}_0, z_c) < 1. \tag{25}
\]

Proof of Proposition 6. Refer to Figure 4 again. Note that at a stable steady state, the curve \( X^S(z) \) crosses the curve \( z(X^S) \) from below. Thus, at a stable steady state, the slope of \( X^S(z) \) in absolute value is smaller than the slope of \( z^{-1}(X^S) \) in absolute value, where \( z^{-1}(X^S) \) denotes the inverse function of \( z(X^S) \). It is straightforward to see that \( z^{-1}(X^S) \) is \( X^S = 1/z - K \). It follows that at a
stable steady state \((\tilde{\theta}_1^c, \tilde{\theta}_0^c, z^c)\),

\[
|\text{slope of } X^S(z)| < |\text{slope of } z^{-1}(X^S)|
\]

\[
\Rightarrow \left| d^{-1}\tilde{G}'(\tilde{\theta}_1^c) \left( d\tilde{\theta}_1^c / dz \right) - d^{-1}\tilde{G}'(\tilde{\theta}_0^c) \left( d\tilde{\theta}_0^c / dz \right) \right| < |1/(z^c)^2| \]

\[
\Rightarrow -d^{-1}\tilde{G}'(\tilde{\theta}_1^c) \left( d\tilde{\theta}_1^c / dz \right) + d^{-1}\tilde{G}'(\tilde{\theta}_0^c) \left( d\tilde{\theta}_0^c / dz \right) < 1/(z^c)^2 \]

\[
\Rightarrow -(z^c)^2 d^{-1}\tilde{G}'(\tilde{\theta}_1^c) \left( d\tilde{\theta}_1^c / dz \right) + (z^c)^2 d^{-1}\tilde{G}'(\tilde{\theta}_0^c) \left( d\tilde{\theta}_0^c / dz \right) < 1.
\]

Substitute \(d\tilde{\theta}_1^c / dz\) and \(d\tilde{\theta}_0^c / dz\) with the expressions from (15) and (16), and evaluate them at the steady-state value \((\tilde{\theta}_1^c, \tilde{\theta}_0^c, z^c)\). The desired result (25) follows.

Proposition 7 The following parameters have definite effects on the stable steady state \((\tilde{\theta}_1^c, \tilde{\theta}_0^c, z^c)\):

(i) \(d\tilde{\theta}_1^c / dK > 0, \ d\tilde{\theta}_0^c / dK < 0, \ dz^c / dK < 0\);

(ii) \(d\tilde{\theta}_1^c / dd < 0, \ d\tilde{\theta}_0^c / dd > 0, \ dz^c / dd > 0\);

(iii) \(d\tilde{\theta}_1^c / df^S < 0, \ d\tilde{\theta}_0^c / df^S > 0, \ dz^c / df^S > 0\);

(iv) \(d\tilde{\theta}_1^c / df^N > 0, \ d\tilde{\theta}_0^c / df^N < 0, \ dz^c / df^N < 0\);

(v) \(d\tilde{\theta}_1^c / dw^S < 0, \ d\tilde{\theta}_0^c / dw^S > 0, \ dz^c / dw^S > 0\).

Proof of Proposition 7. At the steady state, the system (20)–(22) becomes

\[
\pi^S(\tilde{\theta}_1^c, z^c) - \pi^N(\tilde{\theta}_1^c) = 0 \tag{26}
\]

\[
\pi^S(\tilde{\theta}_0^c, z^c) - \pi^N(\tilde{\theta}_0^c) = 0 \tag{27}
\]

\[
z^c - \frac{1}{K + d^{-1}\tilde{G}(\tilde{\theta}_1^c) - d^{-1}\tilde{G}(\tilde{\theta}_0^c)} = 0 \tag{28}
\]

(i) and (ii): Let \(q \in \{K, d\}\) represent the parameter of interest. Taking total derivatives of the system (26)–(28) with respect to \((\tilde{\theta}_1^c, \tilde{\theta}_0^c, z^c)\) and \(q\), we obtain:

\[
\begin{bmatrix}
\pi_0^S(\tilde{\theta}_1^c, z^c) - \psi^N \\
0
\end{bmatrix}
\begin{bmatrix}
\frac{d\tilde{\theta}_1^c}{dq} \\
\frac{d\tilde{\theta}_0^c}{dq}
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
(z^c)^2 d^{-1}\tilde{G}'(\tilde{\theta}_1^c) & -(z^c)^2 d^{-1}\tilde{G}'(\tilde{\theta}_0^c) \\
(z^c)^2 d^{-1}\tilde{G}'(\tilde{\theta}_1^c) & -(z^c)^2 d^{-1}\tilde{G}'(\tilde{\theta}_0^c)
\end{bmatrix}
\begin{bmatrix}
\frac{dz^c}{dq}
\end{bmatrix}
= \begin{bmatrix}
(\partial z^c / \partial q) dq
\end{bmatrix} \tag{29}
\]

Express \(d\tilde{\theta}_1^c\) and \(d\tilde{\theta}_0^c\) in terms of \(dz^c\) using the first two rows of (29) and substitute them into the
Express the system (26)–(28) with respect to \( \tilde{\theta} \) by Proposition 6 and Equations (15) and (16). Similarly, given that \( \partial z^c / \partial K = -(z^c)^2 < 0 \), it follows that

\[
\begin{align*}
\frac{dz^c}{dK} &= (1 - \lambda^c)^{-1}(\partial z^c / \partial K) < 0, \\
\frac{d\tilde{\theta}_1}{dK} &= (d\tilde{\theta}_1 / dz^c)(dz^c / dK) > 0, \\
\frac{d\tilde{\theta}_0}{dK} &= (d\tilde{\theta}_0 / dz^c)(dz^c / dK) < 0,
\end{align*}
\]

where we have used Proposition 6 and Equations (15) and (16). Similarly, given that \( \partial z^c / \partial d = (z^c)^2d^{-2} [\tilde{G}(\tilde{\theta}_1) - \tilde{G}(\tilde{\theta}_0)] > 0 \), it follows that

\[
\begin{align*}
\frac{dz^c}{dd} &= (1 - \lambda^c)^{-1}(\partial z^c / \partial d) > 0, \\
\frac{d\tilde{\theta}_1}{dd} &= (d\tilde{\theta}_1 / dz^c)(dz^c / dd) < 0, \\
\frac{d\tilde{\theta}_0}{dd} &= (d\tilde{\theta}_0 / dz^c)(dz^c / dd) > 0,
\end{align*}
\]

by Proposition 6 and Equations (15) and (16).

(iii)–(iv): Let \( q \in \{f^S, f^N, w^S\} \) represent the parameter of interest. Taking total derivatives of the system (26)–(28) with respect to \( (\tilde{\theta}_1, \tilde{\theta}_0, z^c) \) and \( q \), we obtain:

\[
\begin{pmatrix}
\pi^S_\theta(\tilde{\theta}_1, z^c) - \psi^N \\
0 \\
(z^c)^2d^{-1}\tilde{G}'(\tilde{\theta}_1) - (z^c)^2d^{-1}\tilde{G}'(\tilde{\theta}_0)
\end{pmatrix}
\begin{pmatrix}
\frac{d\tilde{\theta}_1}{dz^c} \\
\frac{d\tilde{\theta}_0}{dz^c} \\
\frac{dz^c}{d\Omega_q^c}
\end{pmatrix}
= 
\begin{pmatrix}
-\pi^N_\psi(\tilde{\theta}_1, z^c) - \pi^N_q(\tilde{\theta}_1) \\
-\pi^N_\psi(\tilde{\theta}_0, z^c) - \pi^N_q(\tilde{\theta}_0) \\
0
\end{pmatrix} dq
\]  

(37)

Express \( d\tilde{\theta}_1 \) and \( d\tilde{\theta}_0 \) in terms of \( dz^c \) and \( dq \) using the first two rows of (37) and substitute them into the third row. We obtain

\[
\begin{align*}
(1 - \lambda^c) dz^c &= \left( (z^c)^2d^{-1}\tilde{G}'(\tilde{\theta}_1) \right) [\pi^S_\theta(\tilde{\theta}_1, z^c) - \psi^N]^{-1} \left[ \pi^N_\psi(\tilde{\theta}_1, z^c) - \pi^N_q(\tilde{\theta}_1) \right] dq \\
&\quad - \left( (z^c)^2d^{-1}\tilde{G}'(\tilde{\theta}_0) \right) [\pi^N_\psi(\tilde{\theta}_0, z^c) - \psi^N]^{-1} \left[ \pi^N_\psi(\tilde{\theta}_0, z^c) - \pi^N_q(\tilde{\theta}_0) \right] dq \\
&\equiv \Omega^c_d q
\end{align*}
\]

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In the case of the parameter $f^S$, given that $\left[ \pi_{f^S}^S(\tilde{\theta}, z) - \pi_{f^S}^N(\tilde{\theta}) \right] = -w^N < 0$, it follows that $\Omega_{f^S}^c > 0$. Furthermore, using Proposition 6 and the system (37), we have

$$dz^c/f^S = (1 - \lambda^c)^{-1}\Omega_{f^S}^c > 0$$

$$d\tilde{\theta}_1^c/f^S = - \left[ \pi_\theta^S(\tilde{\theta}_1^c, z^c) - \psi^N \right]^{-1} \pi_{\tilde{\theta}_1^c}^S(\tilde{\theta}_1^c, z^c) dz^c/df^S$$
$$\phantom{d\tilde{\theta}_1^c/f^S} - \left[ \pi_\theta^S(\tilde{\theta}_1^c, z^c) - \psi^N \right]^{-1} \left[ \pi_{f^S}^S(\tilde{\theta}_1^c, z^c) - \pi_{f^S}^N(\tilde{\theta}_1^c) \right] < 0$$

$$d\tilde{\theta}_0^c/f^S = - \left[ \pi_\theta^S(\tilde{\theta}_0^c, z^c) - \psi^N \right]^{-1} \pi_{\tilde{\theta}_0^c}^S(\tilde{\theta}_0^c, z^c) dz^c/df^S$$
$$\phantom{d\tilde{\theta}_0^c/f^S} - \left[ \pi_\theta^S(\tilde{\theta}_0^c, z^c) - \psi^N \right]^{-1} \left[ \pi_{f^S}^S(\tilde{\theta}_0^c, z^c) - \pi_{f^S}^N(\tilde{\theta}_0^c) \right] > 0.$$ 

In the case of the parameter $f^N$, given that $\left[ \pi_{f^N}^S(\tilde{\theta}, z) - \pi_{f^N}^N(\tilde{\theta}) \right] = w^N > 0$, it follows that $\Omega_{f^N}^c < 0$. Similarly, by using Proposition 6 and the system (37), we obtain:

$$dz^c/f^N = (1 - \lambda^c)^{-1}\Omega_{f^N}^c < 0$$

$$d\tilde{\theta}_1^c/f^N = - \left[ \pi_\theta^S(\tilde{\theta}_1^c, z^c) - \psi^N \right]^{-1} \pi_{\tilde{\theta}_1^c}^S(\tilde{\theta}_1^c, z^c) dz^c/df^N$$
$$\phantom{d\tilde{\theta}_1^c/f^N} - \left[ \pi_\theta^S(\tilde{\theta}_1^c, z^c) - \psi^N \right]^{-1} \left[ \pi_{f^N}^S(\tilde{\theta}_1^c, z^c) - \pi_{f^N}^N(\tilde{\theta}_1^c) \right] > 0$$

$$d\tilde{\theta}_0^c/f^N = - \left[ \pi_\theta^S(\tilde{\theta}_0^c, z^c) - \psi^N \right]^{-1} \pi_{\tilde{\theta}_0^c}^S(\tilde{\theta}_0^c, z^c) dz^c/df^N$$
$$\phantom{d\tilde{\theta}_0^c/f^N} - \left[ \pi_\theta^S(\tilde{\theta}_0^c, z^c) - \psi^N \right]^{-1} \left[ \pi_{f^N}^S(\tilde{\theta}_0^c, z^c) - \pi_{f^N}^N(\tilde{\theta}_0^c) \right] < 0.$$ 

In the case of the parameter $w^S$, given that $\left[ \pi_{w^S}^S(\tilde{\theta}, z) - \pi_{w^S}^N(\tilde{\theta}) \right] = -\frac{(1-n^N)}{w^S(1-(1-n))} \psi^S \tilde{\theta} (1-z) < 0$, it follows that $\Omega_{w^S}^c > 0$. Using Proposition 6 and the system (37) again, we have:

$$dz^c/dw^S = (1 - \lambda^c)^{-1}\Omega_{w^S}^c > 0$$

$$d\tilde{\theta}_1^c/dw^S = - \left[ \pi_\theta^S(\tilde{\theta}_1^c, z^c) - \psi^N \right]^{-1} \pi_{\tilde{\theta}_1^c}^S(\tilde{\theta}_1^c, z^c) dz^c/dw^S$$
$$\phantom{d\tilde{\theta}_1^c/dw^S} - \left[ \pi_\theta^S(\tilde{\theta}_1^c, z^c) - \psi^N \right]^{-1} \left[ \pi_{w^S}^S(\tilde{\theta}_1^c, z^c) - \pi_{w^S}^N(\tilde{\theta}_1^c) \right] < 0$$

$$d\tilde{\theta}_0^c/dw^S = - \left[ \pi_\theta^S(\tilde{\theta}_0^c, z^c) - \psi^N \right]^{-1} \pi_{\tilde{\theta}_0^c}^S(\tilde{\theta}_0^c, z^c) dz^c/dw^S$$
$$\phantom{d\tilde{\theta}_0^c/dw^S} - \left[ \pi_\theta^S(\tilde{\theta}_0^c, z^c) - \psi^N \right]^{-1} \left[ \pi_{w^S}^S(\tilde{\theta}_0^c, z^c) - \pi_{w^S}^N(\tilde{\theta}_0^c) \right] > 0.$$ 

It is straightforward to verify that $\partial z^c/\partial q$ in (29) is indefinite for $q \in \{k\}$ and that $\pi_q^S(\tilde{\theta}, z) - \pi_q^N(\tilde{\theta})$ in (37) is indefinite for $q \in \{w^N, \eta, \delta, \beta, \alpha\}$. Thus, their effects on $(\tilde{\theta}_1^c, \tilde{\theta}_0^c, z^c)$ are indefinite. 


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5 Simulation

In this section, we conduct simulations to illustrate the magnet effect of industry migration introduced above. By varying the parameter values of the model, we will also illustrate the effects of such changes on the steady-state level \((\tilde{\theta}_c^1, \tilde{\theta}_c^0)\) as proposed in Proposition 7. We choose the following parameter values for our benchmark case: \(w^N = 4\), \(w^S = 1\), \(f^N = 1\), \(f^S = 2\), \(\alpha = 0.5\), \(\delta = 0.64\), \(\beta = 0.5\), \(\eta = 0.6\), \(b = 100\), \(K = 5\), \(k = 1\), and \(d = 1\). Substituting them into (7), one can derive \(\psi^N\) and \(\psi^S\) and verify that Assumption 1 holds. Given the parameter values, the time paths of \(\{(\theta_{0,t}, \theta_{1,t})\}_{t=1}^\infty\) can be derived by iteration according to the system (20)–(22). The result for the benchmark case is shown in the middle panel of Figures 5–9.

We perturb the value of each of the following parameters \(q \in \{K, d, f^S, f^N, w^S\}\) with respect to the benchmark case and derive the corresponding time paths of \((\theta_{0,t}, \theta_{1,t})\). For each parameter, two alternative values are tried to illustrate the effect of an increase and a decrease in the corresponding parameter value. We discuss the results for each of the parameters in turn.

Figure 5 illustrates the effect of the level of infrastructure in the South on the transition dynamics and the steady-state level of FDI. The parameter \(K\) is perturbed as follows: \(K = \{5.25, 5, 4.75\}\). The results indicate that a better infrastructure in the South attracts a wider range of intermediate firms in the first wave of industry migration, which in turn creates a larger externality and leads to a bigger second wave of industry migration. At the steady state, a wider range of intermediate firms undertake FDI in the South which has a higher level of infrastructure. It is straightforward to see that if the level of infrastructure in the South falls significantly below \(K = 4.75\), the first wave of FDI will not start at all.

The next case demonstrates the effect of the economic distance between the host and the home country of FDI. We vary the parameter \(d\) according to: \(d = \{10, 1, 0.5\}\). The results are shown in Figure 6. Conditional on the same level of infrastructure in the South, \(K\), the same range of firms undertake FDI in the first wave of industry migration. However, given the same absolute mass of firms producing in the South in the first period, a shorter distance between the host and the home country facilitates faster information spillover and leads to a larger subsequent wave of industry migration. At the steady state, a wider range of intermediate firms produce in the South which is closer to the North.
The next two experiments look at how the relative management cost affects the extent of FDI. This is reflected by the fixed organizational cost $f^S$ when the intermediate input is produced in the South and $f^N$ when it is produced in the North. The experiments are $f^S = \{2.05, 2, 1.95\}$ and $f^N = \{1.08, 1, 0.92\}$, respectively. The results in Figure 7 demonstrate that the required fixed organizational cost to produce the intermediate input in the South is negatively correlated with the extent of FDI, consistent with Proposition 7. As $f^S$ increases, the curve $\pi^S$ shifts down uniformly, thus intersecting the schedule $\pi^N$ at a larger lower bound and a smaller upper bound. Thus, a smaller range of intermediate firms undertake FDI in the first wave of migration. The effect then propagates to the subsequent waves of migration. At the steady state, a smaller range of intermediate firms produce in the South that requires a higher fixed organizational cost. Figure 8 shows that the fixed organizational cost of producing the intermediate input in the North has exactly the opposite effect. As $f^N$ increases, a larger range of intermediate firms relocate their production of intermediate inputs to the South in the first wave of migration. The trend continues until the steady state is reached.

Figure 9 illustrates the effect of the wage of the South, $w^S$, in determining the extent of FDI. The range of variation for this parameter is: $w^S = \{1.02, 1, 0.9\}$. As shown by the figure, the higher the wage of the South, the smaller the range of the first wave of industry migration, and the steady-state range of intermediate firms located in the South. This is as predicted by Proposition 7. The intuition is straightforward. As $w^S$ increases, the curve $\pi^S$ rotates downward clockwise, leading to intersections with the schedule $\pi^N$ at a larger lower bound and a smaller upper bound. The smaller initial range of firms undertaking FDI leads to a smaller range of firms producing in the South eventually at the steady state.

6 Conclusion

[To revise: In this paper, we develop a dynamic theoretical model of FDI with firm heterogeneity in productivity. We show that with the presence of FDI uncertainty, firms with intermediate productivity levels will undertake FDI ahead of firms in the lower tier and upper tier of productivity levels. This is contrary to the results of Antràs and Helpman (2004), where FDI propensity rises with firm productivity. Given a first wave of industry migration, we then demonstrate the magnet...]

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effect of FDI, where the presence of some FDI in the South creates positive externality for the later comers by reducing the general uncertainty. This triggers a second wave of industry migration; a wider range of intermediate firms now find it profitable to produce in the South. The enlarged mass of firms located in the South further reduce FDI uncertainty and attract a third wave of migration. We prove the existence of a stable steady state for this process, and characterize the effects of key parameters on the range of firms undertaking FDI at the steady state. We find that at the steady state, the extent of FDI undertaken by firms from the North (measured by the range of productivity levels of intermediate firms) is positively correlated with the level of infrastructure in the South and the fixed organizational cost of producing in the North. On the other hand, it is negatively correlated with the economic distance between the North and the South, the fixed organizational cost of producing in the South, and the wage of the South. Simulations are conducted to illustrate the above propositions.]

References


Dixit, A., Pindyck, R., .


Figure 1: Firm Productivity and FDI Timing: FDI in China by Taiwanese Firms

Note: This figure shows the relationship between the timing that a Taiwanese firm undertakes its first FDI in mainland China and its productivity. The timing is measured as the number of quarters elapsed since the Taiwanese government lifted the ban in 1991, and the productivity of a firm is measured by the ratio of Return on Assets. We take the average of the ratios for a firm between 1991 and 1993. The data source is described in the text.
Figure 2: $\pi^S(\tilde{\theta}, z)$ as $z$ varies
Figure 3: Magnet Effect of FDI

\[ \pi^S(\tilde{\theta}^{i}; K, X^S_i) - w^N N \]

First Wave of Migration $\Rightarrow X^S_1$

Second Wave of Migration $\Rightarrow X^S_2$
Figure 4: Existence and Stability of Steady State

(a) 

(b) 

$z(X^S)$: always convex

$z^c$: stable

$z^*$: always convex

$X^S(z)$: stable

$X^S(z)$: unstable

I. stable

II. unstable

III. stable
Figure 5: Effects of $K$ on the Dynamics of FDI

Case 1: $K = 5.25$

Case 2: $K = 5$

Case 3: $K = 4.75$
Figure 6: Effects of $d$ on the Dynamics of FDI

Case 1: $d = 10$

Case 2: $d = 1$

Case 3: $d = 0.5$
Figure 7: Effects of $f^S$ on the Dynamics of FDI

Case 1: $f^S = 2.05$

Case 2: $f^S = 2$

Case 3: $f^S = 1.95$
Figure 8: Effects of $f^N$ on the Dynamics of FDI

Case 1: $f^N = 1.08$

Case 2: $f^N = 1$

Case 3: $f^N = 0.92$
Figure 9: Effects of $w^S$ on the Dynamics of FDI

Case 1: $w^S = 1.02$

Case 2: $w^S = 1$

Case 3: $w^S = 0.9$