Price Dispersion in a Model with Middlemen and Oligopolistic Market Makers: A Theory and an Application to the North American Natural Gas Market

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Abstract

We develop a market microstructure model in which trade amongst heterogeneous consumers and producers is intermediated by middlemen and oligopolistic market makers (gatekeepers). Market makers post bid and ask prices. All parties can freely view the posted bid and ask prices. Middlemen stand ready to trade at bid and ask prices they quote on a private basis to consumers or producers who identify these intermediaries via a costly search process. We model competition between market makers as a two-stage game: capacity setting in the first stage and bid and ask prices in the second stage. We characterize the equilibrium market structure of intermediaries and the distribution of prices in equilibrium. Our main focus is the effect on prices that emerges following a change in the market structure of intermediation, specifically through the entry or exit of a market maker. Exit of a market maker initially results in a shift of trade from market makers as a whole to middlemen resulting in an increase in price dispersion (variability). Following transition to the new market structure with fewer market makers, price dispersion returns to its pre-exit level when total trade passing through the remaining market makers is roughly equal to the pre-exit level. We present an empirical study of price dispersion in the North American natural gas market for the period before and following the exit of Enron in late 2001. The empirical evidence supports the main propositions of our theory: price dispersion jumped 4-fold immediately following Enron's exit but returned to its pre-exit level within roughly 2 months following the exit date.
1 Introduction

The intermediation of trade in real goods is a significant feature of many markets. Typically, intermediation is facilitated by middlemen and/or market makers. Market makers facilitate trade by posting publicly observable bid and ask prices. On the other hand, middlemen stand ready to trade at bid and ask prices they personally quote on a private basis to consumers or producers who identify them. The implications of such trading structures when market makers are present have however received surprisingly little attention, notable and important exceptions being Rust and Hall (2003) who extend the price-setting middlemen model of Spulber (1996, 1999) by introducing a monoplistic market maker, and Baye and Morgan (2001) who study a monopolist information gatekeeper.\footnote{Related papers based upon search-theoretic structures include Rubinstein and Wolinsky (1987), Yavas (1992, 1994), Gehrig (1993), Li (1998), Camera (2000, 2001), and Shevchenko (2004).}

Some markets however, such as the North American market for natural gas, have exhibited the joint existence of both middlemen as well as multiple market makers. Such an economic setting requires an alternative formulation of the model in order to gain insight into the functioning of such markets. The model developed in this study features both oligopolistic market makers and competitive middlemen thus extending the work of Spulber (1996, 1999) and Rust and Hall (2003), allowing us to capture a facet of intermediated markets for real goods that has not heretofore been investigated.

The study of oligopolistic competition amongst market makers is limited. Rust and Hall (2003) suggest approaching the problem within a Bertrand-
style price competition setting. In an economy in which multiple market
makers survive in equilibrium the Bertrand formulation of the problem re-
sults in zero prof...t and market power vanishes. While interesting in its own
right, such an approach does not permit the implications of market power
amongst oligopolists to be directly investigated. We alternatively formulate
the economic setting to preserve the existence of market power amongst
market makers.

We study a market for a homogenous product in which market maker
capacity is constrained. We formulate a two-stage game in which market
makers compete in capacity choice at the rst stage and in bid and ask
prices in the second stage. Similar to the well known argument made by
Kreps and Scheinkman (1983), we show that the outcome of our two-stage
game in which optimal bid and ask prices are established is equivalent to
the Cournot outcome.

Rust and Hall (2003) and Spulber (1996) focus primarily on the mar-
et equilibrium as a result of the pricing policies of a market maker (Rust
and Hall), or competitive middlemen (Spulber), and the comparison of such
market equilibria with the Walrasian equilibrium of the model. While we
also characterize the market equilibrium of our model, we focus on the dis-

persion of prices in the market, and in particular on how this dispersion
changes when the market structure of intermediation changes. We con-
sider three market structure situations: 1) the market with middlemen but
no market makers, 2) the market with middlemen and oligopolistic market
makers, and, 3) the market during and after a transition phase in which a
market maker exits.
In our model middlemen are heterogenous in the transaction costs they incur when intermediating trade. A higher transaction cost increases the optimal ask price, but reduces the optimal bid price offered by a middleman. In a market with middlemen but no market makers, the spread between the highest and lowest ask (bid) price equals the spread between the most efficient middleman’s ask (bid) price and the highest (lowest) reservation price of all consumers (producers). In the market with middlemen and oligopolistic market makers, all market makers publicly post an unique ask (bid) price, which serves as the high (low) bound for the ask (bid) prices of middlemen. The consumer (producer) who has a reservation price lower (higher) than the price posted by the market makers transacts through middlemen. In this setting the spread between the highest and lowest ask (bid) price equals the spread between the most efficient middleman’s ask (bid) price and the ask (bid) price posted by market makers. As market makers enter or exit the market, the ask and bid prices posted by market makers will change, and therefore the dispersion of ask and bid prices will also change. We show that the exit of a market maker initially results in a shift of trade from market makers as a whole to middlemen resulting in an increase in price dispersion. Following transition to the new market structure with fewer market makers, price dispersion returns to its pre-exit level if total trade passing through the remaining market makers is roughly equal to the pre-exit level.

Our model is close in spirit to recent work by Baye and Morgan (2001, 2004) who also concern themselves with the dispersion of prices. We however reach different conclusions about the source of such dispersion. An important difference between the two studies is the underlying institutional
Baye and Morgan are concerned with a setting in which the market maker faces no capacity constraints. Our model on the other hand directly incorporates capacity constraints. The model of Baye and Morgan (2001, 2004) predicts price dispersion arises because firms which list prices on a monopoly gatekeeper’s screen make their prices strategically unpredictable – to avoid all-out price competition, while firms which do not list prices on the gatekeeper’s screen all charge the same price. The gatekeeper (market maker) thereby becomes the source of price dispersion in the Baye and Morgan model. Our theory alternatively predicts that all prices charged by oligopoly market makers are the same, while prices charged by middlemen with heterogeneous transaction costs are spread over a range. Market makers’ services reduce price dispersion in our model. Our work also differs from theirs in that we study how price dispersion changes with a change in market structure which we also empirically test. To the best of our knowledge there have been no empirical studies that examine how price dispersion varies with the structure of market makers.

The exit of Enron in late 2001, the then largest market maker in the North American natural gas market, provides a unique opportunity for us to examine the effect of this change in market structure on the dispersion of market prices. Enron was a major intermediary handling roughly 20% of the wholesale natural gas volume sold in the domestic United States by the top 10 marketers during the period leading up to December 2001. We present an empirical study of price dispersion in the North American natural gas market for the period before and following the exit of Enron in late 2001. The empirical evidence supports the main propositions of our theory: Natural
gas spot price dispersion jumped 4-fold immediately following Enron's exit at the end of 2001 but returned to its pre-exit level within roughly 2 months following the exit date.

We begin by studying a market with middlemen but no market maker. We then introduce a fixed number of oligopolistic market makers in Section 3. In both sections we highlight the dispersion of prices in equilibrium. In Section 4 we turn to the effect on the dispersion of prices when a market maker exits. In Section 5 we present an empirical study of the behavior of North American natural gas spot prices before and after the exit of Enron in late 2001. Section 6 concludes the paper.

2 Market Settings

The initial market setting consists of a continuum of heterogeneous consumers, producers, and middlemen who trade a homogenous good. We follow Spulber (1996, 1999) and Rust and Hall (2003) and assume the cost incurred by any middleman in conducting a transaction is less than the cost that would be incurred by any consumer or producer if they were forced to search for other consumers or producers with whom to trade. We investigate the equilibrium of this market setting and characterize the distribution of equilibrium prices. We then introduce a finite set of market makers and show how the bid and ask prices that prevail in equilibrium are influenced by the presence of the two types of intermediaries. The primary characteristic differentiating middlemen from market makers is the manner in which the prices at which each are willing to transact are revealed to consumers and producers. Market makers post the price at which they are willing to
buy (bid price) and the price at which they are willing to sell (ask price) for all consumers and producers to freely see at no cost to the consumer or producer. In contrast the bid and ask prices of middlemen are private information. The only way a consumer or producer can discover the bid and ask prices of a middleman is through a costly search process.

3 The Market with Middlemen but No Market Maker

3.1 Consumers and Producers

The population of consumers is represented by a uniform distribution of willingness-to-pay levels on the interval \([v, \bar{v}]\) similar to Spulber (1996, 1999) and Rust and Hall (2003). We assume any particular consumer searches randomly across middlemen. Each middleman therefore faces an equal probability of making a trade. Consumers know the equilibrium distribution of ask prices offered by middlemen \(F(p)\) but not the particular middleman associated with each price. A consumer’s optimal search rule is to compare the net value of current consumption with the returns to search. The consumer purchases the good if and only if the ask price \(p\) is less than or equal to his reservation value. Consumers remain in the market for a random length of time before permanently exiting where \(\lambda \in (0, 1)\) is the probability that a consumer exits the market in period \(t\). In each period a fraction \(\lambda\) of the population of consumers exits the market and is replaced by an equal fraction of new consumers.
Total expected discounted demand is the expected discounted value of the stream of demands in all future periods by the initial population of consumers as well as the stream of demands from each succeeding generation of new consumers entering the market (Spulber, 1996; Rust and Hall, 2003). Let \( p \) be the ask price of a particular middleman and \( D(p) \) be the share of total expected discounted demand serviced by any middleman, then

\[
D(p) = \frac{1}{N} [a_1 + a_2 p]
\]

where \( N \) is the number of middlemen, \( a_1 > 0 \) and \( a_2 > 0 \), and each active middleman receives an equal share of searchers.

Producers have an opportunity cost \( c \) for the good supplied. The population of producers is represented by a uniform distribution of opportunity costs on the interval \([a, c]\). Producers know the equilibrium distribution of bid prices \( G(w) \) but not the particular middleman associated with each price. Each producer searches randomly across middlemen and sells the good to a middleman if and only if the bid price offered, \( w \), is greater than or equal to the producer’s reservation value. Producers also exit the market each period with probability \( \lambda \leq 2 \ (0, 1) \) and are replaced accordingly with new producers. By reasoning similar to the articulation of demand, a middleman’s expected discounted supply function is therefore

\[
S(w) = \frac{1}{N} [a_3 + a_4 w]
\]

where \( a_3 < 0 \) and \( a_4 > 0 \).\(^2\)

\(^2\)Spulber (1999) and Rust and Hall (2003) derive \( a_1, a_2, a_3 \) and \( a_4 \) from the fundamental
The highest reservation value of the population of consumers is equal to \( v = a_1/a_2 \), and the lowest reservation value of the population of producers is equal to \( c = a_3/a_4 \). The equilibrium demand of consumers who participate in the market is given by \( a_2(v, p) = ND(p) \), which is equal to the equilibrium supply of producers who participate in the market, \( a_4(w, c) = NS(w) \).

3.2 Middlemen

Middlemen incur a transaction cost \( k \) per unit of the good purchased from any producer. The population of potentially active middlemen is represented by a uniform distribution of transaction costs \( k \) on the interval \( [\bar{k}, \bar{k}] \). A middleman’s present discounted value of trading profits is given by

\[
\pi(p, w, k) = pD(p) - (w + k)S(w)
\]

Each middleman, indexed by \( k \), solves the following problem

\[
\max_{p, w} \pi(p, w, k) \text{ subject to } D(p) \cdot S(w)
\]

The first order conditions associated with problem (4) yield optimal ask and bid prices, \( p \) and \( w \) respectively, for a middleman with transaction cost \( k \)

\[
p = A_0 + A_1 k
\]

\[
w = B_0 + B_1 k
\]

parameters of the model: \( a_1 = f(1 + (\rho\lambda)/(1 - \rho))g/[1 - \rho(1 - \lambda)] > 0 \), \( a_3 = i f(1 + (\rho\lambda)/(1 - \rho))g/[1 - \rho(1 - \lambda)] < 0 \), and \( a_2 = a_4 = [1 + (\rho\lambda)/(1 - \rho)]/[1 - \rho(1 - \lambda)] > 0 \) where \( \rho \) is the time discount factor. In contrast our formulation is more general requiring only that the coefficients obey the indicated sign conventions.

\(^3\)See Spulber (1999, Ch. 6).
where
\[
A_0 = \frac{a_1a_4 + a_2a_3 + 2a_1a_2}{2a_2(a_2 + a_4)} > 0, \\
A_1 = \frac{a_4}{2(a_2 + a_4)} > 0
\]
\[
B_0 = \frac{a_1a_4 + a_2a_3 + 2a_3a_4}{2a_4(a_2 + a_4)} > 0, \\
B_1 = \frac{a_2}{2(a_2 + a_4)} < 0.
\]

A higher transaction cost increases the optimal ask price, but reduces the optimal bid price. These bid and ask prices also equate supply and demand in every period as has been shown by Spulber (1999). Substitution of the optimal bid and ask prices into the profit function gives
\[
\pi(p, w, k) = \frac{a_2a_4}{4N(a_2 + a_4)} \cdot \frac{a_1a_4 + a_2a_3}{a_2a_4} \cdot k^2
\]

The value for \( k \) which solves \( \pi(p, w, k) = 0 \) gives the highest transaction cost that any middleman can incur while both serving the market and surviving. Denote this value as \( k^\alpha \) where
\[
k^\alpha = \frac{a_1a_4 + a_2a_3}{a_2a_4}
\]

Any middleman with transaction cost \( k > k^\alpha \) will not enter the market. If \( k^\alpha < \bar{k} \), the market is inactive since no middleman has a transaction cost less than \( k^\alpha \) and hence none could survive. We are interested in active markets and so assume \( k^\alpha > \bar{k} \). We will also assume that \( \bar{k} \) is sufficiently large so that \( k^\alpha < \bar{k} \). Thus, the number of middlemen operating in equilibrium is represented by
\[
N = \frac{k^\alpha}{\bar{k}}.\]

Letting \( k \) in equation (5) equal rst \( \bar{k} \) and then

\[\text{Following standard convention we have normalized the sum of the number of middlemen to equal 1. Thus, } k^\alpha \cdot \bar{k} \text{ represents the fraction of all middlemen with transactions costs in the interval } k^\alpha \text{ to } \bar{k}.\]
$k^a$, we obtain the lower and upper bounds of the equilibrium distribution of ask and bid prices. Ask prices are uniformly distributed on the interval $[\underline{p}, \overline{p}]$, and bid prices are uniformly distributed on the interval $[\underline{w}, \overline{w}]$, where

$$p = A_0 + A_1 k^a, \quad \overline{p} = A_0 + A_1 k^a$$

$$w = B_0 + B_1 k^a, \quad \overline{w} = B_0 + B_1 k^a$$

(9)

It can be easily verified that

$$\overline{p} = a_1/a_2 = \overline{v}$$

$$\underline{w} = a_3/a_4 = \underline{c}$$

(10)

That is, the maximum ask price is equal to the maximum price consumers are willing to pay and the minimum bid price is equal to the minimum opportunity cost of producers. We restrict our attention to stationary pricing policies on the equilibrium path. Thus, $D(p)$ and $S(w)$ in the above represent stationary demand and supply functions in each period, and the pair of equilibrium ask and bid prices derived above represent steady-state equilibrium prices in each period.

4 The Market with Oligopolistic Market Makers

We now extend Rust and Hall’s (2003) model by introducing oligopolistic market makers. We begin by assuming there are $m$ market makers. Market maker $j (j = 1, \ldots, m)$ posts publicly observable ask and bid prices $(p_j, w_j)$. 

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There is no cost to a consumer or producer to view these prices.\footnote{We assume the cost of observing the market maker’s ask and bid prices is equal to zero. All that is really required is that this cost be less than the cost of search across middlemen.} We analyze a setting in which both price and quantity decisions by market makers are considered. Competition amongst market makers is modeled as a two-stage game in which market makers compete in quantity bought in the first stage, and compete in bid and ask prices in the second stage. The quantity purchased acts as a capacity constraint for price competition.\footnote{In contrast to Baye and Morgan (2001, 2003) we focus on the price dispersion resulting from the heterogenous transaction costs of the middlemen. Thus, we only discuss the pure strategy Nash equilibrium and do not discuss mixed strategy Nash equilibria in which market makers randomize the price.}

4.1 Two-Stage Market Makers’ Competition

Each consumer will search for the lowest ask price $p$ among all posted ask prices, and buys the good if $p$ is less than or equal to the consumer’s reservation value. Each producer will search for the highest bid price $w$ among all posted bid prices, and sell the good if $w$ is greater than or equal to the producer’s reservation value. Therefore, market makers as a whole face total market demand, $X = a_1 x_2 p$, and total market supply, $Y = a_3 + a_4 w$. The inverse market demand and supply functions are therefore

\begin{align}
   p(X) &= \frac{a_1}{a_2} x_2 \frac{X}{a_2} \\
   w(Y) &= \frac{a_3}{a_4} \frac{Y}{a_4}
\end{align}

Let $y_j$ and $x_j$ denote the quantity bought and sold by market maker $j$, respectively. Market maker $j$ incurs a per unit cost $r_j$ for each transac-
tion processed. We order market makers so that $r_j$ is nondecreasing in $j$. Therefore, market maker $1$ is the most efficient in terms of having the lowest processing cost while market maker $m$ is the least efficient. Assuming efficient rationing, market maker $j$ will face a residual-demand function against other market makers. No market maker will set bid and ask prices such that purchases exceed sales, since otherwise the market maker could increase its profit by reducing purchases in the first stage and correspondingly lowering the bid price. Similar to the well known argument made by Kreps and Scheinkman (1983), we will first construct the Cournot equilibrium in quantity competition, and then show that the outcome of our two-stage game in which optimal bid and ask prices are established is equivalent to the Cournot outcome.

Let $q_j = x_j = y_j$ be the quantity that market maker $j$ trades. Market maker $j$’s profit is

$$
\pi_j = (p - w - r_j)q_j = C_j i b q_i q_j
$$

(12)

where $C_j = \frac{a_1}{a_2} + \frac{a_3}{a_4} i r_j$ and $b = \frac{1}{a_2} + \frac{1}{a_4}$. Recall that we are working with expected discounted demand and supply and that we restrict our attention to stationary pricing policies on the equilibrium path. The first order condition for profit maximization by a market maker is

$$
C_j i b q_i i b q_j = 0 \text{ for } j = 1, \ldots, m
$$

(13)

which gives market maker $j$’s optimal reaction function
Solving the $m$ equations in (13) gives the Cournot equilibrium quantity that the market maker trades

\[ q_j = R_j(q_{\mathbb{I}j}) = \frac{C_j i b}{2b} \frac{P_{\mathbb{I}j} q_i}{i \Phi j} \]  

(14)

It is easy to see $q_1 > q_2 > \ldots > q_m$. We assume the processing cost $r_j$ for any market maker is sufficiently small so that $q_m > 0$. In other words all market makers are active in the market. All market makers will transact at their respective optimal Cournot capacities and charge the same bid and ask prices in the two-stage game.

Proposition 1 Suppose market makers compete in quantity bought at the first stage and compete in ask and bid prices at the second stage. In a pure-strategy subgame perfect Nash equilibrium, all market makers buy their optimal Cournot quantities in the first stage, asking the same price $p_1 = \ldots = p_m = p = p(2) \sum_{j=1}^{m} q_j$, and bidding the same price $w_1 = \ldots = w_m = w = w(2) \sum_{j=1}^{m} q_j$. Each market maker $j$ sells up to its Cournot capacity $q_j^n$. 

Proof. We first show that all market makers charge the same bid and ask prices in equilibrium. Suppose this were not true and suppose we let market maker $j$ bid $w_j < \mathbf{e}$ where $\mathbf{e} = \max_{i} w_i (i = 1, \ldots, j, 1, j + 1, m)$, then market maker $i$ with the highest bid could reduce its bid price and maintain its Cournot capacity; if $i$ did not act to reduce its bid the result would be that $i$ would buy the entire market supply at $\mathbf{e}$. In this case $j$
buys nothing at \( w_j \), whereas it could build up positive capacity by bidding \( \mathcal{e} + \varepsilon \) for small \( \varepsilon \) and earn a positive profit. Similarly, let market maker \( j \) offer the ask price \( p_j > \mathcal{e} \) where \( \mathcal{e} = \min_{p_k(i = 1, \mathcal{f}, j \neq 1, j + 1, m)} \).

Then market maker \( i \) with the lowest ask price could increase its ask price and sell up to its equilibrium capacity. Conversely \( \mathcal{e} \) is \( i \)'s monopoly price and \( i \) will supply the entire market demand at \( \mathcal{e} \). In this case \( j \) sells nothing at \( p_j \), whereas it could earn a positive profit by undercutting to \( \mathcal{e} - \varepsilon \).

Given capacity \( \overline{q}_j(j = 1, \mathcal{f}, M) \), we next show that market makers buy and sell up to capacity. Suppose that \( w_1 = \mathcal{f} = w_m = w < w(\varepsilon) \sum_{j=1}^{m} \overline{q}_j \).

Then the bid price is too low, in that at least some market maker \( j \) cannot realize its capacity. If \( w_1 = \mathcal{f} = w_m = w > w(\varepsilon) \sum_{j=1}^{m} \overline{q}_j \), all market makers strictly ration their suppliers. By reducing its price by \( \varepsilon \), \( j \) would still be able to build up capacity and would earn a larger profit. Suppose that \( p_1 = \mathcal{f} = p_m = p > p(\varepsilon) \sum_{j=1}^{m} \overline{q}_j \).

Then the ask price is too high, so that at least some market maker \( j \) cannot sell up to its capacity: \( q_j < \overline{q}_j \).

By charging \( p \varepsilon \), market maker \( j \) gets all of the market and can sell up to \( \overline{q}_j \). For sufficiently small \( \varepsilon \), we must have \( (p \varepsilon) \overline{q}_j > pq_j \), so that \( j \) would gain from undercutting. If \( p_1 = \mathcal{f} = p_m = p < p(\varepsilon) \sum_{j=1}^{m} \overline{q}_j \), all market makers strictly ration their customers. By raising its price by \( \varepsilon \), \( j \) would still be able to sell its capacity and would earn a larger profit.

Now we show \( \overline{q}_j = q^\mathcal{a}_j(j = 1, \mathcal{f}, M) \) is the Nash equilibrium in the first stage. Suppose that market maker \( i(i \neq j) \) buys \( q^\mathcal{a}_i \). The profit of market

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maker $j$, if it buys $q_j \notin q_j^\nu$, equals the LHS of the following equation

$$4p(q_j + \sum_{i \in S_j} q_i^m) \cdot w(q_j + \sum_{i \in S_j} q_i^m) \cdot r_j \cdot q_j^5 \cdot q_j$$

using the result that profit measured at Cournot capacity $q_j^\nu$ exceeds profit measured at $q_j$ when $q_j \notin q_j^\nu$. Q.E.D. ■

Using equation (15), the total output traded by market makers is given by

$$Q^m = \frac{P_m \sum_{i=1}^C C_i}{(m + 1)b}$$

(16)

Substituting equations (15) and (16) back into the profit function (12), we have

$$\pi_j(p, w, r_j) = \frac{1}{b(m + 1)^2} \sum_{i=1}^\infty \frac{a_1 a_4 + a_2 a_3}{a_2 a_4} \cdot \sum_{i=1}^{mr_j} \frac{X^m}{i} \cdot r_i \cdot r_j$$

(17)

The least efficient market maker $m$ survives in the market if

$$r_m < \frac{a_1 a_4 + a_2 a_3}{a_2 a_4} \cdot \sum_{i=1}^{mr_j} \frac{X^m}{i} \cdot r_i < \frac{a_1 a_4 + a_2 a_3}{a_2 a_4} = k^m$$

(18)

So the transaction cost of the least efficient surviving market maker is lower than that of the marginal middleman in a market without market makers. Market makers’ ask and bid prices are given by

$$p^m = \frac{a_1}{a_2} \cdot \frac{Q^m}{a_2}, \quad w^m = \frac{a_3}{a_4} + \frac{Q^m}{a_4}$$

(19)
The consumer who has a reservation price higher than $p^m$ and the producer who has a reservation price lower than $w^m$ are served by market makers. Note that $p^m < \frac{a_1}{a_2} = \overline{p}$ and $w^m > \frac{a_3}{a_4} = \overline{w}$.

The market maker in our model can alternatively be viewed as an information gatekeeper similar to the gatekeeper studied by Baye and Morgan (2001), (see also Rust and Hall 2003). An oligopolistic gatekeeper charges buyers and sellers a commission fee rather than buying and selling on its own account and quoting its own bid and ask prices. Suppose that gatekeepers charge a per unit commission $\tau_b$ to buyers and $\tau_s$ to sellers for posting prices on the site. If a buyer posts a bid of $w$ and transacts at this price, the total per unit cost he actually pays will be $w + \tau_b$; if a seller posts an ask price of $p$ and transacts at this price, the seller’s actual gain per unit will be $p - \tau_s$.

Since there are continuums of buyers and sellers, the only pure strategy Nash equilibrium outcome for buyers and sellers is to post a common bid price and a common ask price. Gatekeepers can use a variety of different commission structures to implement the same outcome as market makers. For example, suppose consumers and producers post perfectly competitive equilibrium prices $p = w = (a_1 - a_3)/(a_2 + a_4)$. Gatekeepers in this case would charge commissions $\tau_b = p^m - w$ and $\tau_s = p - w^m$.7

4.2 Middlemen

The consumer who has a reservation price lower than $p^m$ and the producer who has a reservation price higher than $w^m$ transact through mid-

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7Later we examine the behavior of spot prices in the North American Natural Gas market. That market was characterized by the presence of market makers as well as middlemen during the period of time we study, as is also true at the current time.
middlemen rather than through a market maker. Each consumer randomly searches across middlemen until a middleman is identified who offers an ask price lower than the consumer’s reservation value. Similarly, each producer randomly searches across middlemen until a middleman is identified who offers a bid price greater than the producer’s reservation value. The residual demand and supply faced by middlemen equals

\[
X = a_1 + a_2 p + Q^m \quad \text{and} \quad Y = a_3 + a_4 w + Q^m
\]

where \( Q^m \) equals the total demand (supply) serviced by market makers. The shares of total demand and supply for any single middleman when market makers exist equal

\[
D^\hat{\gamma}(p) = \frac{1}{N^\hat{\gamma}} [b_1 + a_2 p] \quad \text{and} \quad S^\hat{\gamma}(w) = \frac{1}{N^\hat{\gamma}} [b_3 + a_4 w].
\]

where \( b_1 = (a_1 + Q^m) \) and \( b_3 = (a_3 + Q^m) \). The above equations indicate that the highest reservation value of the consumers served by middlemen is \( b_1/a_2 = p^m \), while the lowest reservation value of the consumer served by middlemen is \( b_3/a_4 = w^m \). Similar to the analysis in Section 3.2, the optimal ask and bid prices for middlemen are

\[
p^\hat{\gamma} = A_0^\hat{\gamma} + A_1 k
\]
\[
w^\hat{\gamma} = B_0^\hat{\gamma} + B_1 k
\]
where

\[ A_0^\wedge = \frac{b_1 a_4 i \ a_2 b_3 + 2 b_1 a_2}{2 a_2 (a_2 + a_4)} = A_0 \ i \ \frac{Q^m}{2 a_2} \]

\[ B_0^\wedge = \frac{b_1 a_4 i \ a_2 b_3 + 2 b_3 a_4}{2 a_4 (a_2 + a_4)} = B_0 + \frac{Q^m}{2 a_4} \]  

(23)

Solving for \( \pi(p^\wedge, w^\wedge, k) = 0 \) gives the highest transaction cost a middleman can bear and also survive,

\[ \frac{\hat{k}}{k} = \frac{b_1 a_4 + a_2 b_3}{a_2 a_4} = \frac{\mu}{a_2 + a_4} \frac{Q^m}{k} \]  

(24)

It is clear that \( A_0^\wedge < A_0, B_0^\wedge > B_0 \) and \( \frac{\hat{k}}{k} < \frac{k}{\mu} \). The number of middlemen operating in the equilibrium is \( N^\wedge = \frac{\hat{k}}{k} \ i \ \frac{k}{\mu} < N \). Ask prices are uniformly distributed on the interval \([p^\wedge, p]\), and bid prices are uniformly distributed on the interval \([w^\wedge, w]\), where

\[ \hat{p} = A_0^\wedge + A_1 \hat{k} \]

\[ \bar{p} = A_0 + A_1 \hat{k} \]  

\[ \hat{w} = B_0^\wedge + B_1 \hat{k} \]

\[ \bar{w} = B_0 + B_1 \hat{k} \]  

(25)

The fraction of consumers and producers served by middlemen is

\[ Q^\wedge = a_2 (p^m_i \hat{p}) = a_2 \ \frac{a_1}{a_2} \ i \ \frac{Q^m}{a_2} \]  

\[ = a_4 \ \frac{w^m}{\bar{w} \ i} \ i \ \frac{a_3}{a_4} \ ]  

\[ = b_3 + a_4 \hat{w} \]  

(26)
4.3 Dispersion of Ask and Bid Prices

Using equations (6), (8), (10), (23), (24) and (25), we have

\[ \hat{p} = p \cdot \left(1/2\right) [p \cdot p^m], \quad \hat{p} = p^m \]

\[ \hat{w} = w^m, \quad \bar{w} = \overline{w} + (1/2) [w^m \cdot w] \]

Total consumer demand served by market makers and middlemen is \( \bar{Q} = Q^m + Q^\hat{\cdot} \). Using equations (19) and (26), we have

\[ \bar{Q} = a_2 \cdot \mu \cdot a_3 \cdot p^m + (p^m \cdot \hat{p}) \]

\[ = a_4 \cdot \frac{a_3}{a_4} + w^m + (\hat{w} \cdot w^m) = a_4 \cdot \hat{w} \cdot \hat{w} \]

A consumer (or producer) is served by middlemen with probability \( \alpha \) and is served by market makers with probability \( 1 - \alpha \)

\[ \alpha = \frac{Q^\hat{\cdot}}{\bar{Q}} = \frac{p^m \cdot \hat{p}}{\hat{p} \cdot \hat{p}} = \frac{w^m \cdot \hat{w}}{\hat{w} \cdot \hat{w}} 
\]

\[ 1 - \alpha = \frac{Q^m}{\bar{Q}} = \frac{\hat{p} \cdot p^m}{\hat{p} \cdot \hat{p}} = \frac{w^m \cdot w}{\hat{w} \cdot \hat{w}} \]

Let \( F(p \mid p^m) \) denote the distribution function of ask prices when market makers ask \( p^m \). \( F(p \mid p^m) \) is represented by the kinked line \( \hat{p} AB \) in Figure 1. The lower and upper bound of the distribution are \( \hat{p} \) and \( p^m \) respectively. The consumer whose reservation value is greater than or equal to \( \hat{p} \) and less than \( p^m \) is served by middlemen and \( F(p \mid p^m) = \alpha \cdot \frac{p^m \cdot \hat{p}}{p^m \cdot \hat{p}} = \frac{p^m \cdot \hat{p}}{p^m \cdot \hat{p}} \)

where \( p \) is the consumer’s reservation value. From \( \hat{p} \) to \( p^m \), \( F(p \mid p^m) \) is a
straight line represented by $p^A$ in Figure 1. If $p = p^m$, the consumer whose reservation value is greater than or equal to $p^m$ is served by market makers and $F(p|p^m) = 1$. At $p = p^m$, $F(p|p^m)$ is depicted by the line $AB$. Note that $F(p|p^m)$ is completely determined by $p^m$. When $p^m = \bar{p}$, no consumer is served by market makers, and $F(p|\bar{p})$ is the same distribution as that in the market without market makers, which is represented by line $pE$. When $p^m < \bar{p}$, the upper bound of $F(p|p^m)$ is reduced to $p^m$. The lower bound is cut to $p^\wedge$. The distance between $p^\wedge$ and $p$ is half of the distance between $p^m$ and $\bar{p}$ as indicated by equation (27). Therefore, the difference between the lowest and the highest prices in $F(p|p^m)$, $p^m \wedge p^\wedge$, is less than $\bar{p} - p$ by the amount of $(1/2)(\bar{p} - p)$.

Let $G(w|w^m)$ denote the distribution function of bid prices when market makers bid $w^m$. $G(w|w^m)$ is represented by the kinked line $w^mCD$ in Figure 2. The lower and upper bound of the distribution is $w^m$ and $w^\wedge$. If $w = w^m$, the producer whose reservation value is less than or equal to $w^m$ is served by market makers and $G(w|w^m) = 1$ if $\alpha = \frac{w^m - w}{w^m - w^\wedge}$. At $w = w^m$, $G(w|w^m)$ is depicted by the line $w^mC$. If $w^m < w \cdot \wedge$, the producer whose reservation value is greater than $w^m$ and less than or equal to $w^\wedge$ is served by middlemen and $G(w|w^m) = \frac{w^m - w}{w^m - w^\wedge}$. From $w^m$ to $w^\wedge$, $G(w|w^m)$ is a straight line and is represented by $CD$. $G(w|w^m)$ is determined by $w^m$. When $w^m = w$, no producer is served by market makers and $G(w|w)$ is the same distribution as that in the market without
market makers, which is represented by line $\omega H$. When $w_m > \omega$, the lower bound of $G(w \mid w_m)$ is raised to $w_m$. The upper bound is increased to $\bar{w} \hat{\omega}$. The distance between $\bar{w}$ and $\bar{w} \hat{\omega}$ is half of the distance between $\bar{w}$ and $w_m$ as indicated by equation (27). Therefore, the difference between the lowest and the highest prices in $G(w \mid w_m)$, $\bar{w} \hat{\omega} - w_m$, is less than $\bar{w} - \omega$ by the amount of $(1/2) (w_m - \omega)$. When $w_m$ is increased to $w_m^0$, $G(w \mid w_m^0)$ changes to the kinked line $w_m^0 C \hat{D}^0$. It is immediately seen that $G(w \mid w_m^0)$ stochastically dominates $G(w \mid w_m)$. We are now ready to state the result as Proposition 2.

**Proposition 2** As the price asked by market makers $p_m$ decreases, the mean of ask prices in the market decreases and the difference between the lowest and the highest ask prices in the market decreases. As the price bid by market makers $w_m$ increases, the mean of bid prices in the market increases and the difference between the lowest and the highest bid prices in the market decreases. If the market makers serve at least half of the market, the variance of ask prices decreases as $p_m$ decreases, while the variance of bid prices decreases as $w_m$ increases.

**Proof.** Let $p_m$ be reduced to $p_m^0$, then $F(p \mid p_m)$ stochastically dominates $F(p \mid p_m^0)$. Thus, the mean of the ask price is decreasing in $p_m$. Using equation (27), the difference between the lowest and highest ask prices in the market equals

$$p_m \hat{\omega} - \bar{p} = \frac{\bar{p} + p_m}{2}$$
which decreases as $p^m$ decreases. As $w^m$ is increased to $w^m_0$, $G(w^m_0)$ first-order stochastically dominates $G(w^m)$. Thus, the mean of the bid price is increasing in $p^m$. Using equation (27), the difference between the lowest and the highest bid prices in the market is given by

$$\bar{w} - \bar{w} = \frac{w^m}{2} - \frac{w^m}{2}$$

and decreases as $w^m$ increases. We prove in the Appendix that the variance of ask prices decreases as $p^m$ decreases and the variance of bid prices decreases as $w^m$ increases, as long as market makers serve at least half of the market.

Some intuition about the result will be helpful at this point. The total output traded by market makers is given by $Q^m$. When $Q^m$ increases $p^m$ decreases and $w^m$ increases. This results in a reduction in the spread for middlemen and the overall dispersion of prices falls.

5 The Effect of a Change in Market Structure on Price Dispersion

We now consider a change in market structure represented by a change in the number of market makers serving the market. We study the case in which a single market maker $j$ exits from the market. Immediately after the exit of the market maker consumers and producers who would have otherwise traded with that party are forced to search for a new counterparty for their trades. We define a transition period after the exit of market maker $j$ during which the remaining market makers are unable to adjust their capacities.
More middlemen will enter into the market and the consumers and producers who would have transacted with market maker $j$ must implement a search across the middlemen operating in the market to identify trading partners. Consumers and producers who had already planned to transact with the remaining $m-1$ market makers and $N$ middlemen do not deviate from their original optimal choices (prices and quantities). Following the transition period the displaced consumers and producers settle into trading relations with the remaining $m-1$ market makers if their reservation values are respectively higher than the ask price and lower than the bid price offered by those market makers, otherwise they are served by middlemen.

5.1 Price Dispersion during the Transition from $m$ to $m-1$ Market Makers

We assume that consumers who would have been served by the exiting market maker $j$ have reservation values that are evenly spread from $p^m$ to $\bar{p}$. Therefore, during the transition period these consumers, represented by their share of total output traded through market makers $\frac{q^j}{Q^m}$ will be served by middlemen. Using the same argument as in Section 4, we conclude that the highest price asked by middlemen is $\bar{p}$. Thus, the difference between the lowest and the highest ask prices in the market becomes

$$\bar{p} - \hat{p} > p^m - \hat{p}$$

Similarly, the share of total trades of producers handled by market makers $\frac{q^n}{Q^n}$ with reservation values spread from $w$ to $w^m$ will be served by middlemen. So the lowest price bid by any middleman is $\underline{w}$. Thus, the difference
between the lowest and the highest bid prices in the market becomes

\[ \bar{w}^i \bar{w} > \bar{w}^i \bar{w}^m \]

Summarizing we have:

**Proposition 3** The differences between the lowest and the highest bid prices and the lowest and highest ask prices increase during the transition from \( m \) to \( m+1 \) market makers. In other words, both bid and ask prices become more dispersed during the transition period.

At this stage it will be beneficial to articulate the connection between the dispersion of bid and ask prices and the dispersion of the prices at which transactions occur. As equation (11) indicates, the ask price is determined by the inverse demand curve while the bid price is determined by the inverse supply curve at any equilibrium quantity \( Q \). Thus, the ask price is always greater than the bid price. Let ask prices be spread between \( p_1 \) and \( p_2 (p_2 > p_1) \) and bid prices be spread between \( w_1 \) and \( w_2 (w_2 > w_1) \). Therefore, because transactions occur at either an ask or a bid price, transaction prices will be spread between \( w_1 \) and \( p_2 \), which are the lower bound of bid prices and the upper bound of ask prices. The following proposition follows.

**Proposition 4** An increase in the spread of ask prices and an increase in the spread of bid prices results in an increase in the spread of transaction prices. During the transition period from \( m \) to \( m+1 \) market makers, the dispersion of transaction prices increases from \( (w^m, p^m) \) to \( (\bar{w}, \bar{p}) \).
5.2 Price Dispersion Following the Transition Period

The new structure of the market following the transition period is characterized by \( m \) \( 1 \) market makers. Rewriting equation (16), we have

\[
Q^m = \frac{\mathbf{P} \sum_{i=1}^{m} C_i}{(m + 1)b} = \frac{\sum_{i=1}^{m} \left( a_1 + a_2 r_i \right)}{(m + 1)b} \tag{30}
\]

After market maker \( j \)'s exit, \( Q^m \) becomes \( Q^{m_i} \). With \( m \) \( 1 \) market makers, the market becomes less competitive. Using equations (30) and (18), we can show that \( Q^{m_i} < Q^m \), which implies that \( p^{m_i} > p^m \) and \( w^{m_i} < w^m \). Immediately after a market maker exits from the market, the amount of consumer and producer trade volume served by all market makers decreases; the mean of ask prices increases and the mean of bid prices decreases. The differences between the lowest and the highest bid prices and the lowest and highest ask prices increase. Note that the change in price dispersion is determined by the change in \( Q^m \), the total trade volume served by market makers. We offer the following corollary to proposition 4.

Corollary 5 If a change in market structure does not significantly affect the total trade volume served by all market makers, the prediction is neither will it significantly affect price dispersion. Therefore, if for instance the total trade volume serviced by market makers following the transition period recovers to the pre-exit level, then we would expect that price dispersion would also return to its pre-exit level.

25
6 The North American Natural Gas Market

The North American natural gas industry is composed of a distinct chain of businesses and exhibits little vertical integration. The parties involved include producers, gatherers, processors, pipelines, marketers (both market makers and middlemen as we have defined them), distributors and end users. Marketers act as intermediaries. There is an active market for physical gas as well as an active market for gas futures contracts traded on the New York Mercantile.

Table 1 presents the daily trading volume in billions of cubic feet of natural gas for the major North American gas marketers during the third and fourth quarters of 2001 and for comparison, the fourth quarter of 2002. The total trading volume for the top 10 marketers in each quarter is reported at the bottom of the table. During the third quarter of 2001 Enron’s share of the volume done by the top 10 firms was roughly 20%. Enron effectively ceased wholesale trading of natural gas shortly before filing for bankruptcy on December 2, 2001. Thus Enron’s exit represented a large, at least short-run, disruption. It is widely agreed that prior to the collapse, Enron was the largest market maker in the natural gas market. In the aftermath trading still appeared to flow through roughly 20-25 intermediaries. The model developed earlier suggests that in the immediate wake of Enron’s exit total volume should have fallen. The data presented in Table 1 are consistent with this prediction. The total volume of transactions handled by the top 10 marketers fell in the quarter following Enron’s exit. Note however that total volume had returned to the pre-exit level by the fourth quarter of 2002.
6.1 A First-look at Price Dispersion

The model developed earlier makes specific predictions about price dispersion during the period following a market maker's exit. The model predicts that price dispersion should have increased in the immediate wake of the Enron exit. Following the transition to a new market structure, the model predicts that if the total volume served by market makers does not change significantly, price dispersion should not be significantly different from its pre-exit level.

We obtained the daily high, low and midpoint spot prices of natural gas, stated in MMCF, for gas delivered at the Henry Hub for the period 1/2/2001 through 5/14/2002 from records maintained by Platts. Platts is a leading industry monitor and publisher of energy industry information. The Henry Hub is a major delivery point for natural gas and is the basis for the natural gas futures contract traded on the New York Mercantile Exchange (NYMEX).

Our model predicts that during the transition period from \( m \) to \( m - 1 \) market makers, the dispersion of transaction prices increases (Proposition 4). We compute a relative range statistic for each day of the sample period equal to \( R_g = (\text{Daily High Price} - \text{Daily Low Price})/\text{Daily Midpoint Price} \). Figure 3 presents a plot of the range statistic for the sample period. The plot shows that during the period immediately following the Enron exit, beginning on roughly 11/15/01 and extending through roughly 12/10/01, price dispersion jumped significantly. At its peak during this calendar period, natural gas price dispersion was roughly 4-5 times greater than its pre-Enron
exit levels. Conversely, price dispersion had returned to its pre-exit levels by mid-December of 2001 and has remained at that level since. The behavior of the relative range data is consistent with the prediction of Proposition 4, dispersion increases during the transition period immediately following exit of the market maker. The data are also consistent with the prediction of the corollary that if the amount of trade owing through market makers was roughly the same following the transition period as compared to the pre-Enron collapse level then price dispersion in the post-transition period should be similar to price dispersion during the pre-exit period. The trade level condition is supported by the data presented in Table 1. While Figure 3 is very suggestive, we now turn to formal statistical tests of the null hypothesis that dispersion was equal during the pre-exit, transition and post-transition subperiods.

6.2 Statistical Tests for Shifts in Price Dispersion

We divide the total sample period into three subperiods: Sample period 1: 1/2/2001-11/15/2001 (Pre-Enron Exit); Sample period 2: 11/16/2001-12/10/2001 (Transition Period); Sample period 3: 12/11/2001-5/24/2002 (Post-Transition Period). Table 2 presents statistical tests of the null hypothesis that the mean relative range statistics are equal across pairs of subperiods. Panel A presents the sample period means and variances of the relative price range, $R_g$, by subperiod. The point estimates of the mean relative price ranges clearly indicate an increase in the mean occurred during the Transition Period. The mean during the Pre-Enron Exit period is 0.0552 while the mean during the Transition Period is .2190. Assuming un-
equal variances, as confirmed by the test results presented in Panel B, the t-statistic for an equality of means test equals -6.653. Likewise the mean during the Post-Transition Period is equal to .0551. The mean during the Pre-Enron Exit Period is not significantly different from the mean during the Post-Transition Period (t-statistic: 0.036), while the mean during the Transition Period is significantly different from the Post-Transition Period (t-statistic: 6.667). These results are consistent with the predictions of Propositions 4 and support the visual evidence presented in Figure 1.

Table 3 presents a final set of tests. We estimate a linear model of the relative range statistic ratio in which we allow for autoregressive behavior as well as day-of-the-week effects. Two dummy variables are introduced to test the propositions that dispersion increased during the Transition Period and fell back to its Pre-Enron Exit level following the Transition Period. Specifically, $D_1$ takes the value 1 during the period 11/16/2001 to 12/10/2001 and 0 otherwise, and, $D_2$ takes the value 1 during the period following 12/10/2001. The model estimated is given by

$$R_{gt} = \beta_0 + \beta_1 R_{gt-1} + \beta_2 D_1 t + \beta_3 D_2 t + \beta_4 T_1 + \beta_5 W_1 + \beta_6 T_2 + \beta_7 F_1 + \epsilon_t \quad (31)$$

where $R_{gt}$ is the relative price range, $T_1$ takes the value 1 if day $t$ is a Tuesday and 0 otherwise, and corresponding dummies are identified for $W$ (Wednesday), $T_2$ (Thursday) and $F$ (Friday). We estimate the model using the total time series spanning the period 1/2/2001-5/24/2002. The estimated coefficients for the model are reported in Table 3. The results are consistent with what has heretofore been suggested. The prediction from
Proposition 4 is that $\beta_2$ should be positive and significantly different from 0. The point estimate of $\beta_2$ is 0.119 (t-statistic: 10.23). The prediction from the corollary is that $\beta_3$ should not be significantly different from zero if following the Transition Period the trade volume served by market makers settles back to its Pre-Enron Exit level. The point estimate of $\beta_3$ is -0.00 (t-statistic: -.009).

We conclude from the statistical tests that the behavior of natural gas spot prices during the period prior to Enron’s exit from the natural gas market through and after the market’s transition to a new configuration of intermediaries is consistent with the predictions of the theory developed in this study.

7 Conclusions

Neoclassical economics leaves open the question of how actual markets attain equilibrium prices. The theory of intermediation and the microstructure of markets developed by Spulber (1996, 1999) and Rust and Hall (2003), amongst others, has made significant progress in addressing this question in markets served by middlemen and a monopoly market maker. We observe however that many markets are served by multiple market makers and these market makers have significant market power even in homogeneous product markets. An extension of the models of Spulber and Rust and Hall is required to accommodate this market feature. We fill this gap in the literature by developing a market microstructure model in which trade amongst heterogeneous consumers and producers is intermediated by middlemen and
oligopolistic market makers. Market makers' intermediation behavior is decomposed into a two-stage process: capacity setting in the first stage and bid and ask price setting in the second stage. It is shown that the two-stage competition among market makers is equivalent to Cournot competition. Thus, oligoplooly market makers retain their market power in our model.

Our theory has much to say about homogeneous product markets where capacity choice plays an important role. The North American natural gas market is such a market and provides an ideal setting to test the implications of our theory. The exit of Enron, the largest market maker in the North American natural gas market, in late 2001, provides an unique opportunity for us to examine the effect of a change in market structure on prices, through the exit of a market maker, and in particular the dispersion of prices. Our model predicts price dispersion will increase temporarily following a market maker's exit and then settle back to at or near the level it held prior to the market maker's exist if trade volume handled by all market makers is restored to its pre-exit level. The empirical evidence supports the main propositions of our theory: Natural gas spot price dispersion jumped 4-fold immediately following Enron's exit at the end of 2001 but returned to its pre-exit level within roughly 2 months following the exit date, with trade volume of all market makers rst falling and the recovering to its pre-exit level.

In our model oligopolistic market makers charge the same ask (bid) price which imposes an upper (lower) bound on the ask (bid) prices of middlemen. Further the prices charged by middlemen with heterogenous transaction costs are characterized by spread. Thus, the presence of market makers will
tend to reduce price dispersion. When a major market maker exits from the market, consumers and producers who had been intermediated through that market maker are temporarily forced to transact instead through middlemen until the market settles back to a new equilibrium with a smaller number of market makers. As a result, during this transition period, price dispersion increases. An alternative view has been proposed by Baye and Morgan (2001, 2004). In their model, firms which list their prices on a market maker’s screen choose to randomize the price listed in an attempt to avoid all-out price competition in the online market. Conversely, firms in their model which do not list prices on the market maker’s screen all charge the same price. The existence of the market maker therefore becomes the source of price dispersion in the Baye and Morgan model. We believe both models have merit and highlight how the implications of the intermediation of trade for price dispersion depend upon the context within which intermediation occurs. A fruitful extension of this work will be the development of a theory combining features of the assumptions made by Baye and Morgan with these we make herein, which we leave for future research.

References


Forthcoming.


8 Appendix

In this Appendix, we prove that the variance of ask prices decreases as $p^m$ decreases, and the variance of bid prices decreases as $w^m$ increases, provided that market makers serve at least half of the market. We only prove the result for ask prices. The result for bid prices can be proved in the same manner. The mean of equilibrium ask prices in the market is

$$E(p | p^m) = \alpha E(p) + \left( \frac{1}{2} \right) \alpha p^m$$

Integrating by parts yields

$$E(p | p^m) = \frac{p^m}{\bar{p}} \frac{\bar{p}}{p} ! dp$$

Noting that $\bar{p} = p | (1/2) [\bar{p} | p^m]$, we have

$$\frac{dE(p | p^m)}{dp^m} = \frac{\bar{p} i}{\bar{p} i} | p^m \frac{p^m}{\bar{p} i} ! dp + \frac{1}{2} \frac{Z}{\bar{p} i} \frac{p^m}{\bar{p} i} ! dp$$

The variance of ask prices is
\[ V(p, m) = \alpha Z_p^m \left[ \frac{\bar{p} \cdot p}{\bar{p}^m} \right]^2 + (1 - \alpha) \left[ \frac{\bar{p} \cdot p}{\bar{p}^m} \right]^2 \]

Integrating by parts yields

\[ V(p, m) = [p \cdot E(p, m)]^2 \left[ \frac{\bar{p} \cdot p}{\bar{p}^m} \right]^2 \]

Differentiating the above with respect to \( p^m \) and using the equation (33), we have

\[ \frac{dV(p, m)}{dp^m} = -2 \left( \frac{\bar{p} \cdot p}{\bar{p}^m} \right) \left[ \frac{\bar{p} \cdot p}{\bar{p}^m} \right] + 2 \left( \frac{\bar{p} \cdot p}{\bar{p}^m} \right)^2 \]

The assumption that market makers serve at least half of the market implies

\[ 2(\bar{p} \cdot p^m) \leq \bar{p} \cdot \bar{p}, \]

which is used to obtain the last inequality. Q.E.D.
Figure 1: Behavior of Bid Prices
Figure 2: Behavior of Ask Prices
Figure 3: Spot Natural Gas Price Dispersion
Table 1
North American Natural Gas Marketers
by Wholesale Physical Volumes Sold
3rd and 4th Quarters 2001 and 3rd Quarter 2002.¹

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<tr>
<th>Company</th>
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<th>4th Q 2001</th>
<th>3rd Q 2002</th>
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<td>Enron²</td>
<td>26.7</td>
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<td>0</td>
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<td>Mirant</td>
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<td><strong>131.2</strong></td>
<td><strong>121.1</strong></td>
<td><strong>134</strong></td>
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¹Source: Platts Gas Daily, Vol. 19, Nos. 40 (February 28, 2002) and 234 (December 30, 2002)
Table 2
Tests of Equality of Price Range

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<td>Mean</td>
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<td>Variance</td>
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B. Tests of equality of variance

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<th>2 vs. 3</th>
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<td>F-stat</td>
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<td>3.330</td>
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<td>F-Critical</td>
<td>1.737</td>
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C. Tests of equality of means assuming unequal variances

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<th>1 vs. 3</th>
<th>2 vs. 3</th>
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<tbody>
<tr>
<td>t-stat</td>
<td>-6.653</td>
<td>0.036</td>
<td>6.667</td>
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<tr>
<td>t-Critical</td>
<td>-2.145</td>
<td>1.967</td>
<td>2.145</td>
</tr>
</tbody>
</table>

6, 2002). Bcf/day: Billions of cubic feet per day.

2Enron effectively stopped wholesale trading of natural gas shortly before filing for bankruptcy on December 2, 2001.

1Sample period 1: 1/2/2001-11/15/2001; Sample period 2: 11/16/2001-12/10/2001;
Table 3
Regression Analysis of Henry Hub Spot Price Range

Model: \[ R_t = \beta_0 + \beta_1 R_{t-1} + \beta_2 D_1 + \beta_3 D_2 + \beta_4 T_t + \beta_5 W_t + \beta_6 Th_t + \beta_7 F_t + \epsilon_t \]

Where \( R_t \) is the price range defined as \((\text{Daily High Price - Daily Low Price})/\text{Daily Midpoint Price}\)

- \( D_1 \) takes the value 1 during the period 11/16/2001 to 12/10/2001 and 0 otherwise
- \( D_2 \) takes the value 1 during the period following 12/10/2001
- Weekday dummies include: T, Tuesday; W, Wednesday; Th, Thursday; F, Friday

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \beta_0 )</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
<th>( \beta_3 )</th>
<th>( \beta_4 )</th>
<th>( \beta_5 )</th>
<th>( \beta_6 )</th>
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</thead>
<tbody>
<tr>
<td>Estimates</td>
<td>0.0346</td>
<td>0.28</td>
<td>0.119</td>
<td>-0.00</td>
<td>0.005</td>
<td>0.0008</td>
<td>0.005</td>
</tr>
<tr>
<td>t-statistics</td>
<td>6.68***</td>
<td>5.55***</td>
<td>10.23***</td>
<td>-0.009</td>
<td>1.00</td>
<td>0.16</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Sample period 3: 12/11/2001-5/24/2002. The Relative Price Range is defined as \( R_g = (\text{Daily High Price - Daily Low Price})/\text{Daily Midpoint Price} \). The spot natural gas price data were obtained from Platts. ADD PLATTS WEBSITE ADDRESS.
$p < .001; \text{np} < .01$