Home Production, Price Stickiness, and Economic Fluctuations

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Abstract

I analyze the consequences of including home production in a New Keynesian model with staggered price setting. Home production is a significant amplification mechanism to technology and monetary policy shocks. Compared with a model without home production, the model generates twice the output response to a monetary policy shock. I consider the implications of several nominal interest rate rules and show that a traditional Taylor rule lacks its usual attractive properties. Finally, I use the model to develop an empirical framework for measuring home hours and output.

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1. Introduction

A substantial amount of individuals' time endowments are allocated to neither work on the market nor leisure, but to work at home. Home work includes activities such as meal preparation, laundry, and grocery shopping. The amount of time devoted to home work is not only large, but also persistent over time. Aguiar and Hurst (2006), using evidence from time use surveys spanning five decades, show that adults spend approximately 20 hours per week working at home (Table 1). This compares to an average of 34 hours per week working in the labor market. Goods and services produced at home are, at least in principle, substitutable for market produced goods and services. This suggests that the relative price between home and market consumption influences the labor allocation, and hence production, in both sectors. Consequently, to the degree that market prices are sticky, the allocation of labor across sectors may be distorted. What are the consequences of including a more realistic market structure in an economy with sticky prices? Does home production alter monetary policy tradeoffs? If not, economists studying monetary policy can safely abstract from it. However, if home production changes the consequences of various monetary policies, it becomes an empirical question whether these effects can be safely neglected.

This paper investigates the dynamic properties of an economy with nominal rigidities and home production. More specifically, I incorporate household production into a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model with staggered price setting and analyze the responses of the endogenous variables to technology and money supply shocks. Allowing households to substitute between market and home production provides an amplification mechanism to changes in the money supply. In particular, including home production generates a market output response that is twice as large as in a model without it.

Next, I ask what are the consequences of following different nominal interest rate rules in an economy with home production. The attractive properties of a conventionally defined Taylor rule, which include minimizing the output gap and inflation, no longer hold in an economy with home production. Intuitively, if there is a reallocation of labor away from the market sector, due to say a favorable technology shock in the home sector, a central bank following the Taylor rule would cut interest rates, which would stimulate market output and hinders the efficient allocation of resources. In contrast, a nominal interest rate rule with a slight modification to the Taylor rule preserves its stabilizing properties.

Despite its quantitative importance in the data and significance for monetary policy, data on home production is collected only sporadically and never at quarterly frequencies. Following an approach pioneered by Ingram, Kocherlakota and Savin (1997), I use identification restrictions implied by theory to back out measures of home hours and output at a quarterly frequency. I decompose the constructed series into its trend and cyclical components and document how the properties of the cyclical component compare to the properties of the cyclical components of other aggregate series.

This paper contributes to the work incorporating home production in DSGE models of business cycles, which started with Greenwood and Hercowitz (1991) and Benhabibib, Rogerson, and Wright (1991) and continues with more recent contributions such as Fisher (2007) and Aruoba, Morris and Wright (2011). Generally speaking, including home production improves the model's ability to match business cycle moments.

Another strand of related literature is multiple sector New Keynesian models like Kilian, Ohanian and Stockman (1995), and more recently, Barsky, House and Kimball (2007). The latter paper shows that in an economy where only some prices are sticky, namely those of durable goods, monetary shocks generate large effects. In this paper, the market sector has sticky prices and the home sector, at least implicitly, has flexible prices. Closest to this paper is Ngouana (2009), who considers home production for services in a model with staggered price setting. In contrast to Ngouana (2009), I generalize to one market sector and extend his work by considering different monetary policy rules.

The paper runs as follows. The next section introduces and solves the model. Section 3 discusses the intuition of the model and presents analytic results. Section 4 includes quantitative analysis. Section 5 shows the inadequacy of the conventionally defined Taylor rule and proposes a simple modification. Section 6 uses the theory and data on observable quantities to back out measures of home hours and production. The final section concludes.

2. The Model

The baseline model has a textbook New Keynesian structure[2], but is augmented with home production. To keep the mechanism as straightforward as possible, there is no capital accumulation. The model includes an infinitely lived representative household, intermediate good firms, final good firms, and a central bank. The firms are owned by the household, the final goods firm operates in a perfectly competitive market, and the intermediate good firms are monopolistic competitors. As is standard, staggered price setting is introduced as in Calvo (1983).

Households

A representative household has preferences over total consumption, leisure, and real money balances. Total consumption is an aggregate of market produced goods and home

[2]See for example, Walsh (2010) or Gali (2008).

produced goods. Lifetime expected utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t U \left(C_t, l_t, \frac{M_{t+1}}{p_t} \right) \tag{1}$$

$$C_t = g(c_{m,t}, c_{h,t})$$

where p_t is the price level, $c_{m,t}$ is consumption of market produced goods, $c_{h,t}$ is consumption of home produced goods, l_t is leisure, M_{t+1} is nominal money holdings, and C_t is a consumption aggregator. The aggregator function $g(\cdot)$ is increasing in both arguments and quasiconcave. Within a period the household decides how to allocate time between leisure, home production and market production. The time endowment is normalized to one. Note, since we take the numeraire to be money, the price level is equivalent to the money price of the market produced good. Flow income is given by the households wage income, profit from ownership of firms, and interest from a riskless bond. Flow expenditures are given by the change in nominal bond holdings, the change in nominal money balances, and expenditure on the market produced good. Finally, consumption of home produced goods equals the amount of goods produced at home. These constraints are summarized by

$$l_t + n_{m,t} + n_{h,t} = 1 \tag{2}$$

$$p_t c_{m,t} + \triangle B_{t+1} + \triangle M_{t+1} \leq W_t n_{m,t} + i_t B_t + \Pi_t \tag{3}$$

$$c_{h,t} = z_{h,t} f^h(n_{h,t}) \tag{4}$$

where $n_{m,t}$ is labor allocated to market production, $n_{h,t}$ is labor allocated to home production, W_t is nominal wage, $\triangle B_{t+1}$ and $\triangle M_{t+1}$ are changes in bond holdings and nominal money balances going from period t to $t+1$ respectively, i_t is the nominal interest rate, Π_t is nominal profits, $f^h(n_{h,t})$ is the production function for home goods and $z_{h,t}$ is total factor productivity (TFP) in the home sector. Also, I represent real wage and real money balances by $w_t = \frac{W_t}{p_t}$ and $m_t = \frac{M_{t+1}}{p_t}$ The household maximizes lifetime utility subject to equations 2-4. An interior solution is characterized by

$$\frac{MU_{c_m,t}}{MU_{c_h,t}} = \frac{z_{h,t} \frac{\partial f^h}{\partial n_{h,t}}}{w_t} \tag{5}$$

$$\frac{MU_{l,t}}{MU_{c_m,t}} = w_t \tag{6}$$

$$\frac{MU_{l,t}}{MU_{c_h,t}} = z_{h,t}\frac{\partial f^h}{\partial n_h, t} \tag{7}$$

$$\frac{MU_{M,t}}{MU_{c_m,t}} = \frac{i_{t+1}}{1+i_{t+1}} \tag{8}$$

$$MU_{c_m,t} = \beta E_t MU_{c_m,t+1}\frac{1+i_{t+1}}{1+\pi_{t+1}} \tag{9}$$

where $1+\pi_{t+1} = \frac{p_{t+1}}{p_t}$. The first-order conditions describe the household's relevant trade offs. Equation (5) says the marginal rate of substitution of market produced goods for home produced goods equals the marginal product of home production divided by the real wage. Equations (6) and (7) state that the marginal rate of substitution of leisure for market (home) consumption equals the marginal benefit of more time allocated to market (home) production. Equation (8) describes the trade off between nominal money holdings and consumption of market produced goods. Finally, equation (9) is the Euler equation which governs optimal intertemporal behavior. Throughout I assume the following functional forms:

$$U\left(C_t, l_t, \frac{M_{t+1}}{p_t}\right) = \log(C_t) + \theta\log(l_t) + \frac{\left(\frac{M_{t+1}}{p_t}\right)^{1-v} - 1}{1-v}$$

$$C_t = \left[\xi c_{m,t}^e + (1-\xi)c_{h,t}^e\right]^{1/e}$$

$$z_{h,t}f(n_{h,t}) = z_{h,t}n_{h,t}$$

Where $\frac{1}{1-e}$ is the elasticity of substitution between home and market produced goods. As $e \to 1$, the goods become perfect substitutes, as $e \to 0$, the aggregator converges to Cobb Douglas, and as $e \to -\infty$ the aggregator converges to perfect complements

Final Goods Firms

The final goods firms purchase intermediate inputs and bundle them utilizing a CES technology. There are a continuum of intermediaries on the interval $[0,1]$. There are no additional costs to bundling and since there is perfect competition, unit price for the final good equals marginal cost. The individual firm's problem is

$$\underset{y_{j,t}}{\text{Max}} \left(p_t \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} - \int_0^1 p_{j,t} y_{j,t} dj \right)$$

where $p_{j,t}$ and $y_{j,t}$ represent the price and quantity purchased from intermediate producer j.

The firm's first order condition yields the input demands

$$y_{j,t} = y_{m,t}\left(\frac{p_{j,t}}{p_t}\right)^{-\epsilon} \tag{10}$$

where $y_{m,t} = \left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$. Hence, the firm's demand for input j is increasing in total output and decreasing in its price. Here, ϵ represents the price elasticity of demand. A high value of ϵ indicates greater substitution possibilities available to the final goods firm, and consequently, less market power for the intermediaries. Using the input demands and the zero-profit condition, I obtain the price index

$$p_t = \left(\int_0^1 p_{j,t}^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}} \tag{11}$$

Given perfect competition, there are no profits to rebate to households.

Intermediate Goods Firms

Intermediate goods firms derive market power by producing inputs that are imperfect substitutes. It is helpful to break the firm's problem into two parts. First, acting as price takers in the labor market, they minimize costs subject to producing a given amount of output. Then, given the cost function, they choose price and output subject to the input demand curve (10). The cost minimization problem is

$$\text{Min } W_t n_{j,t}$$

$$s.t.\ z_{m,t} f(n_{j,t}) \geq y_{j,t}$$

I assume $z_{m,t} f(n_{j,t}) = z_{m,t} n_{m,t}$. Solving this problem gives an expression for real marginal cost, $mc_{j,t}$,

$$mc_{j,t} = \frac{w_t}{z_{m,t}} \tag{12}$$

Since TFP and the real wage are identical across firms, each intermediary has the same real marginal cost, so $mc_{j,t} = mc_t$.

Following Calvo (1983) let the probability the firm gets to update prices be equal to ϕ. The firm's pricing decision is dynamic. It must account for the chance that it does not get to update prices and therefore chooses price to maximize a weighted sum of discounted profits. The problem is

$$\underset{p_{j,t}, y_{j,t}}{\text{Max}}\ E_t \sum_{s=0}^{\infty} \phi^s \Lambda_{t+s}\left[\frac{p_{j,t}}{p_{t+s}} y_{j,t+s} - mc_{t+s} y_{j,t+s}\right] \tag{13}$$

subject to (10). $\Lambda_{t+s} = \beta^s U_c(C_{t+s}, l_{t+s}, m_{t+s})$ is the rate at which the firm discounts profits, where U_c is the partial derivative of the utility function with respect to total consumption. This gives the first order condition for optimal reset price, $p^{\#}_{j,t}$,

$$p^{\#}_{j,t} = p^{\#}_t = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^{\infty} \phi^s \Lambda_{t+s} p^{\epsilon}_{t+s} y_{t+s} mc_{t+s}}{E_t \sum_{s=0}^{\infty} \phi^s \Lambda_{t+s} p^{\epsilon-1}_{t+s} y_{t+s}} \tag{14}$$

Since the right hand side is independent of j, all firms with the opportunity to update choose the same reset price, $p^{\#}_t$. Once a price is chosen, each firm's demand curve, given by (10), determines the quantity produced. If there was no price stickiness, i.e. $\phi = 0$, I would have $p^{\#}_t = \frac{\epsilon}{\epsilon-1} mc_t$. That is, price would be a constant markup over marginal cost. Given that a proportion $(1-\phi)$ can update their price and the other proportion ϕ of firms was selected randomly, I use the price index, equation (11), to get

$$p_t = \left[(1-\phi) p^{\#^{1-\epsilon}}_t + \phi p^{1-\epsilon}_{t-1}\right]^{\frac{1}{1-\epsilon}} \tag{15}$$

Central Bank

In the baseline model, nominal money balances follow an AR(1) in growth rates,

$$\Delta \log(M_{t+1}) = (1-\rho_{mon})\pi^{ss} + \rho_{mon} \Delta \log(M_t) + \epsilon_{m,t}$$

where π^{ss} is steady state inflation. I can transform this into a stationary series in terms of growth rates of real money balances

$$\Delta \log(m_t) + \pi_t = (1-\rho_{mon})\pi^{ss} + p_{mon} \Delta \log(m_{t-1}) + \rho_{mon} \pi_{t-1} + \epsilon_{m,t} \tag{16}$$

In the extensions, I consider various monetary policy rules.

Aggregation, Stochastic Processes and Equilibrium

TFP in both sectors follows an AR(1) in logs. That is,

$$\ln(z_{h,t}) = \rho_h \ln(z_{h,t-1}) + \epsilon_{h,t} \tag{17}$$

$$\ln(z_{m,t}) = \rho_m \ln(z_{m,t-1}) + \epsilon_{m,t} \tag{18}$$

and $(\epsilon_{h,t}, \epsilon_{m,t})$have a covariance matrix of V. The market clearing conditions for labor and intermediate output are

$$\int_0^1 n_{j,t} dj = n_{m,t}$$

$$\int_0^1 y_{j,t} dj = z_{m,t} n_{m,t}$$

Using (11), I have

$$y_{m,t} = \frac{z_{m,t} n_{m,t}}{\nu_t} \tag{19}$$

where $\nu_t = \int_0^1 \left(\frac{p_t}{p_{j,t}}\right)^\epsilon dj$ is a measure of the deadweight loss due to price dispersion. Equilibrium real profits and bond holdings are

$$\text{profits}_t = \int_0^1 \left(\frac{p_{j,t}}{p_t} y_{j,t} - w_t n_{j,t}\right) dj = y_{m,t} - w_t n_t = z_{m,t} n_{m,t} (1 - mc_t)$$

$$B_t = 0$$

where the former follows from (10) and (11). Also, I have that everything produced on the market sector is consumed, so

$$y_{m,t} = c_{m,t} \tag{20}$$

Finally, (15) can be rewritten in terms of inflation

$$1 + \pi_t = [(1 - \phi)(1 + \pi_t^{\#})^{1-\epsilon} + \phi]^{\frac{1}{1-\epsilon}} \tag{21}$$

where $1 + \pi_t^{\#} = \frac{p_t^{\#}}{p_{t-1}}$ is reset price inflation. I have the following definition of equilibrium.

Definition: A *recursive equilibrium* consists of :

(1) A transition equation for the state variables $\Omega_{t+1} = H(\Omega_t)$, where
$\Omega_t = (m_{t-1}, m_{t-2}, z_{m,t}, z_{h,t}, \pi_{t-1})$

(2) A set of policy functions for the households
$c_m(\Omega_t), c_h(\Omega_t), n_h(\Omega_t), n_m(\Omega_t), B(\Omega_t), m(\Omega_t)$

(3) A set of policy functions for the intermediate good firms $p_j^{\#}(\Omega_t), l_j(\Omega_t)\ y_j(\Omega_t)$

(4) A policy function for the final goods firm $y(\Omega_t)$

(5) Aggregate Prices $i(\Omega_t), w(\Omega_t), p(\Omega_t)$

such that: (i) household policy functions solve their optimization problem; (ii) final goods producing firm's policy function and intermediate goods firm's policy functions solve their maximization problem; (iii) aggregate prices clear the bond, labor, and goods markets and (iv) the law of motion $H(\Omega_t)$ is consistent with individual decisions and stochastic processes for $z_{m,t}$, $z_{h,t}$ and m_t.

3. Discussion of the Model

As will be shown in the quantitative experiments, the key highlight of the model relative to a baseline NK is the amplification of monetary policy shocks. That is, the response of real variables to nominal shocks is significantly bigger on impact in the model with home production. What is the mechanism behind this amplification effect? When households can substitute between market and home production, they are more price sensitive. Because of this additional substitution margin, intermediate goods firms who can update their prices don't raise them by as much as they would in the baseline case. Since the nominal shocks aren't as quickly transmitted into price increases, there is a greater response in real variables. In New Keynesian nomenclature,[3] this is referred to as a "real rigidity". Price setters are particularly adverse to dispersion in relative prices and, consequently, do not allow their prices to differ from those already set. A model with more real rigidity will have a smaller response in inflation for a given change in real quantities, or, equivalently, a bigger response in real quantities for a given change in inflation.

In the model at hand this can be seen by looking at the Phillips Curve, which relates current inflation to the current output gap and expected future inflation. The log linearized Phillips Curve is derived in Appendix 1 and is given by

$$\pi_t = \kappa(1-e)(y_{m,t} - y^f_{m,t}) - k(1-e)(y_{h,t} - y^f_{h,t}) + \beta E_t \pi_{t+1} \tag{22}$$

where $\kappa = \frac{(1-\phi\beta)(1-\phi)}{\phi}$. The $(1-e)$ multiplying the market output gap represents the change in slope and the middle term a change in intercept from the canonical Phillips Curve. The canonical Phillips Curve has an intercept term equal to 0 and a slope equal to $\frac{\kappa}{1-n^*_m}$ .For given home and market gaps, an increase in e leads to a flatter Phillips curve, makes the shift term smaller. Relative to the standard Phillips curve, inflation is smaller for a given output gap.

The Phillips Curve explicitly shows how the response of real variables depends on the substitutability between home and market output. Recall that in every period a fraction ϕ of the firms get to update their prices. As in the standard model, an unexpected increase in money supply results in a low relative price of market output, p_t, since some firms can't update their prices. Basically, households substitute away from leisure and towards consumption. In the model with home production, an unexpected increase in money supply leads households to substitute away from leisure and home production and into market production. The real rigidity effect deters intermediate good producers from raising their prices too much on impact.

[3] Woodford (2003).

I have hinted that the results depend critically on the degree of substitutability between home and market goods. In fact, with the functional forms I can analytically derive closed form solutions of the lower and upper bounds of the effects of monetary policy. First, consider the case when $e = 0$, meaning the consumption aggregator is Cobb Douglas. The within-period utility function is

$$U = \xi \log c_{m,t} + (1-\xi)\log c_{h,t} + \theta \log(1 - n_m - n_h) + \frac{\left(\frac{M_{t+1}}{P_t}\right)^{1-\nu} - 1}{1-\nu}$$

Recall, the home production constraint is $c_{h,t} = z_{h,t} n_{h,t}$. In the case of an interior solution, the FOC with respect to home labor is

$$n_{h,t} = \frac{(1-\xi)(1-n_m)}{\theta + 1 - \xi}$$

The reduced form utility function is

$$V(c_{m,t}, n_{m,t}, m_t) = \xi log c_{m,t} + (1 - \xi + \theta)\log(1 - n_{m,t}) + \frac{\left(\frac{M_{t+1}}{P_t}\right)^{1-\nu} - 1}{1-\nu} + c$$

$$c = \theta \log\theta - \theta\log(\theta + 1 - \xi) - (1-\xi)\log(\theta + 1 - \xi)$$

In a standard New Keynesian model with log utility over consumption and labor supply, the within period utility function is

$$U = \log c_{m,t} + \theta\log(1 - n_{m,t}) + \frac{\left(\frac{M_{t+1}}{P_t}\right)^{1-\nu} - 1}{1-\nu}$$

Comparing the standard utility function to the reduced form utility function in the home production model, one can see that they differ by the scaling parameters on the arguments and a constant, c. What are the implications for the model? In both cases the log linearized first order conditions for labor and consumption are

$$-\hat{c}_{m,t} = \hat{\lambda}_t$$

$$\frac{n^*}{1-n^*}\hat{n}_{m,t} = \hat{\lambda}_t + \hat{w}_t$$

where hatted variables denote percentage deviation from steady state and n^* is steady state labor supply. In addition, both models produce the same FOCs for bonds and money and have identical market structures on the production side. Therefore, the model with home production is observationally equivalent to a model without it. Intuitively, in the absence of some additional intertemporal link[4], home production only matters insofar as the goods are substitutable. With Cobb Douglas preferences, the cross price effects are nil. That is, if x_j is the demand curve, $\frac{\partial x_j}{\partial p_i} = 0$ for $j \neq i$.

Going to the opposite extreme, when the goods are perfect substitutes, a necessary condition for an interior solution is that

$$\frac{\xi}{1-\xi} = \frac{p_{m,t}}{p_{h,t}}$$

where $p_{h,t}$ is the opportunity cost of devoting one more unit time to home production and $p_{m,t}$ is the price of a final good from equation (15). By construction, prices in the market sector are sticky so any shock that changes relative prices there will induce the household to consume at a corner solution. For example, an increase in home TFP lowers $p_{h,t}$. For an interior solution, $p_{m,t}$ must drop in proportion, but due to price stickiness it can't. Consequently, only home produced goods will be consumed. It follows that output deviations are the largest when the goods are perfect substitutes.

4. Quantitative Analysis

Calibration

The following calibration exercise is highly stylized. The point of this section is to compare the baseline New Keynesian model to the extended model. A more serious empirical procedure is implemented in the next section. I take steady state-hours in the market and home to be $n_m^{ss} = 0.255$ and $n_h^{ss} = 0.24$. This matches data from the American Time Use Survey as reported by Ruppert and Gomme (2007). I take $\pi^{ss} = 0$, which implies $\pi^{\#^{ss}} = 0$. Since $\beta = \frac{1}{1+r^{ss}}$, β is set to match the quarterly real interest rate on a riskless asset. This implies $\beta = 0.99$. The TFP shocks have standard deviations of $\sigma_m = \sigma_h = 0.07$, autocorrelations of $\rho_h = \rho_m = 0.95$ and a correlation $\gamma = 0$. Models in the home production literature typically assume $\gamma > 0$. However, to compare dynamic responses to a market technology shock in the New Keynesian model with and without home production, it is most natural to orthogonalize the TFP shocks. The results are robust to $\gamma > 0$. Real money balances are set to have a standard deviation $\sigma_{mon} = 0.05$ and autocorrelation coefficient of $\rho_{mon} = 0.5$. The elasticity, ϵ is set to 6, implying a steady-state markup of 20 percent, and ϕ is set to $\frac{5}{6}$ implying that the average duration of price rigidity is six quarters. Finally, given a value of the elasticity between home and market goods, $\frac{1}{1-e}$, the household's first order conditions in conjunction with the steady-state hours targets imply particular values for ξ and θ. The results are quite sensitive to the choice of e, and therefore, I consider various

[4] Including capital accumulation, for instance, breaks the equivalence.

specifications below. Benhabib, Rogerson, and Wright (1991) use $e = 0.8$, McGrattan, Rogerson and Wright (1997) econometrically estimate it at $e = 0.326$, and Ruppert and Gomme consider $e = 0$. As a baseline I take $e = 0.6$. Given this value of e, evaluating the first order conditions in nonstochastic steady state gives $\xi = 0.5615$ and $\theta = 0.9038$. The calibrated values are summarized in Table 1.

Following the calibration exercise, I log linearize the model and solve by the method of Blanchard and Kahn (1980). The impulse responses to a one standard deviation change in market technology and money supply are depicted in figures 1 and 2 respectively. For the sake of comparison, I include the analogous responses from a baseline New Keynesian (NK) model without home production. The baseline NK model is nested in the home production model and corresponds to the case of $\xi = 1$. I recalibrate $\theta = 2.337$ so that steady state market hours, and therefore market output, are the same in both models. All of the other parameters are the same across models. The next section provides an analytic description of the results.

Simulations

In response to the technology shock, the output series displays a modest increase in both the model with and without home production. In the benchmark (no home production) log linearized NK model, the flexible level of output, y_t^f, is equal to the technology shock, $z_{m,t}$. Given that the money supply is constant, the economy has the same amount of money, but an increase in aggregate supply, therefore necessitating a price drop. However, when prices are less than fully flexible, some intermediate good producers are not allowed to adjust their prices downwards. Relative prices are distorted and the non-updating firms are punished with a lower demand for their product. Consequently, output does not rise to its fully flexible level. Over time, as firms are allowed to update, market output rises. This causes the hump shaped impulse response. Including home production shifts the hump up. Intuitively, in addition to market intermediaries cutting prices, there is a reallocation effect as workers flow out of the relatively unproductive home sector and into the relatively productive market sector. This causes market output to rise by more than in the case of no home production. Since the shocks are orthoganilized, the effect of a market technology shock on home TFP is nil, and consequently, the response in home output is identical to the response in home hours. Given that labor is flowing from home to market, home output drops by necessity. This drop accelerates in the early periods as more firms are able to adjust their price, but levels off as the shock dissipates. The responses to the other variables are approximately the same as in the baseline model.

The impulse responses for a one standard deviation change in money supply are displayed in Figure 2. When the money supply increases, households allocate less time to home production and leisure, but more time to market production. Market output increases on impact and slowly dissipates over several quarters. The response of market output relative to the baseline case is stark. On impact, the market output response is 1.44%, while it is only 0.78% in the baseline model. However, the interest rate and inflation responses are identical in the two models. That is, the increase in the output gap just offsets the dampening effect of the positive elasticity. The responses of real money balances and the real wage are also quite similar.

Instead of asking what is the difference in magnitude of the output gaps on impact for a given degree of price rigidity, i.e. value of ϕ, I can ask how much less price rigidity is needed in the home production model to achieve the same response on impact as the baseline. It turns out $\phi = \frac{4}{7}$, produces the same output gap on impact as the benchmark case. A $\phi = \frac{4}{7}$ implies prices are sticky for an average of two and one third periods. In the benchmark case the average duration of price stickiness is six quarters. The former is more consistent with the available micro evidence on the frequency of price changes, such as in Bils and Klenow (2004).

Figure 3 demonstrates how the output gap is affected by the elasticity of substitution parameter. Consistent with the analytic results from section 3, one sees that the output response to a monetary shock is increasing in e.

5. Nominal Interest Rate Rules

Monetary policy is typically thought to be conducted in terms of interest rates with the necessary adjustments to money supply happening in the background. To that end, this section compares different nominal interest rate rules. Taylor (1993) describes a rule that is both prescriptive and descriptive of monetary policy in the post WWII US economy. Although it exists in many incarnations, the basic partial adjustment Taylor rule takes the form

$$\tilde{i}_{t+1} = \rho\tilde{i}_t + \phi_\pi(1-\rho)\tilde{\pi}_t + \phi_y(1-\rho)\left(\tilde{y}_{m,t} - \tilde{y}^f_{m,t}\right) + \epsilon_{t+1}$$

where $(\tilde{y}_t - \tilde{y}^f_t)$ is the log deviation of the output gap, and both coefficients are positive. The log linearized flexible rate of output in the benchmark New Keynesian model is equal to log linearized TFP. In the baseline model, following a nominal interest rate rule similar to the one specified above decreases the output gap in response to a change in TFP. The intuition is that when TFP increases, intermediate goods firms want to cut their prices but are prevented because of price stickiness. The final goods firm buys less intermediate goods than if prices were flexible, and consequently, aggregate output doesn't rise by as much as its flexible price level. By cutting nominal interest rates the central bank ameliorates the situation. A decrease in nominal interest rates necessitates an increase in money supply to meet money demand. This raises inflation and induces updating intermediate firms to cut prices by less. The net effect is to limit the relative price distortion and narrow the output gap. Unfortunately, if the home production sector is subject to its own TFP shocks, the conventional Taylor rule lacks its usual stabilizing effects.

To see this, first log linearize equation (12) around its steady state to obtain

$$\tilde{mc}_t = \tilde{w}_t - \tilde{z}_{m,t}$$

In flexible price equilibrium, marginal cost is constant so $\tilde{mc}_t^f = 0$, and $\tilde{w}_t^f = \tilde{z}_{m,t}$. Log linearizing equation (5) around its steady state gives

$$\tilde{w}_t = (1-e)(\tilde{y}_{m,t} - \tilde{y}_{h,t}) + \tilde{z}_{h,t}$$

Evaluating the last equation in flexible price equilibrium gives

$$\tilde{z}_{m,t} - \tilde{z}_{h,t} = (1-e)(\tilde{y}_{m,t}^f - \tilde{y}_{h,t}^f)$$

This suggests that the central bank should track a different output gap than the one defined in the baseline. Consider a modified Taylor rule

$$\tilde{i}_{t+1} = \rho\tilde{i}_t + \phi_\pi(1-\rho)\tilde{\pi}_t + \phi_x(1-\rho)\left(\tilde{x}_t - \tilde{x}_t^f\right) + \epsilon_{t+1}$$

where $\tilde{x}_t = (1-e)(\tilde{y}_{m,t} - \tilde{y}_{h,t})$. Figure 4 displays the impulse responses of a technology shock in the home and market sectors when $\rho = 0.5$, $\phi_\pi = 1.5$, and $\phi_y = \phi_x = 0.5$. The shocks are orthoganilized. Using the conventional Taylor rule dampens the effects of shocks relative to their flexible price counterparts. Consider the shock in the home sector first. An increase in home TFP causes the implicit relative price of home produced goods to drop and labor to be reallocated away from the market sector to the home sector. Following the conventional Taylor rule, the central bank cuts nominal interest rates stimulating the market sector, thereby impeding the efficient reallocation of labor. Conversely, in the modified Taylor rule the partial effect of the output gap is negative if $\frac{\tilde{z}_{h,t}}{1-e} < \tilde{y}_{h,t} - \tilde{y}_{m,t}$. Due to the price stickiness, this inequality holds. However, the partial effect isn't as in the conventional Taylor rule. The actual output gap $\tilde{x}_t$ in the modified Taylor rule is closer to the flexible output gap, $\tilde{x}_t^f$, than the conventional Taylor rule. The intuition for the market technology shock is similar, but the details are different. The explanation for why intermediaries want to cut price is the same as in the benchmark model with no home production. However, due to the labor reallocation effect the flexible level of market output increases by even more when home production is included than the benchmark case. Here, the central bank when following a conventional Taylor rule doesn't cut rates rates sufficiently following the shock, which widens the spread of $\tilde{x}_t - \tilde{x}_t^f$. The modified rule cuts rates more initially, limiting the size of the gap. Figure 4 displays the deviations of marginal cost from following the two different Taylor rules. Additionally, Figure 5 displays the output gaps $\tilde{y}_{m,t} - \tilde{y}_{m,t}^f$ and $\tilde{y}_{h,t} - \tilde{y}_{h,t}^f$. The flexible deviations are obtained from simulating the model with no price stickiness. As conjectured above, the modified Taylor rule narrows the output gaps in response to technology shocks originating in both the home and market sector.

One surprising result of the benchmark NK model is that, in the absence of cost push terms, there is no tradeoff between inflation and output variability.[5] That is, extreme inflation targeting minimizes the welfare loss due to price stickiness. An implication is that for a monetary authority responding sufficiently to inflation, including an output gap term in its reaction function is redundant. Since the output gap is minimized under the extreme inflation targeting, including the output gap term is irrelevant. If this Divine Coincidence result holds in the NK model with home production, setting monetary policy would be much easier, because the central bank could ignore home hours.

To compare the welfare implications of following different monetary policy rules I derive a second-order approximation of the household's value function using the approach of Schmitt-Grohe and Uribe (2004). I use the same notation as Faia and Monacelli (2008). Specifically, the households lifetime expected utility, ignoring real money balances, is

$$W_{0,t} = E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, l_t)$$

where C_t, the consumption aggregator, is the same as above. The problem can be written recursively,

$$W_{0,t} = U(C_t l_t) + \beta E_t W_{0,t+1}$$

To gain any insight into the welfare properties of different monetary policy rules, a second-order approximation is necessary. A first order approximation of expected lifetime utility is equal to the nonstochastic steady state of lifetime utility, which doesn't change as the policy rule changes. It can be shown that a second-order approximation of the value function is given by

$$W_{0,t} = \bar{W} + \frac{1}{2} \triangle W_0$$

where $\bar{W} = \frac{U(C^{ss}, l^{ss})}{1-\beta}$ and Δ is a vector of constant correction terms corresponding to the

[5] See Clarida, Gali and Gertler (1999).

deviations of the expected value from its steady state. In addition to the two contrasting Taylor rules described above, I also include the strict inflation targeting rule

$$\log(1 + i_{t+1}) = (1 - \rho)i^* + \rho\log(1 + i_t) + \phi_\pi(1 - \rho)\log\left(\tfrac{1+\pi_t}{1+\pi^*}\right) + \epsilon_{t+1}$$

where i^* is the steady state interest rate and $\phi_\pi = 10$. I go back and specify the other rules in levels rather than first order log deviations to compute the second order approximations. The results for following the different rules, as well as expected utility in an economy with flexible prices are displayed in Table 3. The results suggest a corollary to the Divine Coincidence as found in the standard NK model. The central bank maximizes welfare by aggressively responding to inflation. The welfare consequences of following the modified Taylor rule versus the strict inflation rule are negligible. However, following a traditional Taylor rule results in a reduction in expected lifetime utility of approximately 0.4%. The implication is that the central bank needs to rely only on observable quantities, i.e. inflation to maximize welfare. This is good news since, in addition to not knowing the flexible rate of output, home output is not measured reliably or at quarterly frequencies in the data. Additionally, the numerical results support the analytic reasoning above. Following a TFP shock in the home sector, the traditional Taylor rule counteracts the efficient reallocation of resources.

How strong does the response to inflation need to be? The last row in Table 3 shows that the answer is not that strong. Keeping the inflation coefficient at $\phi_\pi = 1.5$ and dropping all output gap terms from the policy function results in almost identical expected lifetime utility as the extreme inflation response rule. The take away is this: in the basic NK model, provided the central bank responds aggressively enough to inflation, its response to the output gap is irrelevant. In the NK model with home production responding to the market output gap is welfare reducing.

Optimal policy aside, many central banks devise policy to respond to output and inflation gaps. For instance, in the United States the Federal Reserve has a "dual mandate" to respond to both inflation and employment. If operating procedures require a central bank to respond to some sort of output gap, it at least makes sense to respond to the correct gap. This means that the monetary authority needs to have data on home output. The next section addresses how to recover home output from existing data.

6. Estimating Home Production at Quarterly Frequencies

The purpose of this section is to devise a method for backing out home hours in the data. I use identification restrictions that come from theory and data on market variables to construct measures of home I follow the approach pioneered by Ingram, Kocherlakota and Savin (IKS) (1997), who construct home hours and a measure of welfare under the assumption of no price rigidity. In what follows, I allow for nominal rigidity. I make two exceptions from the baseline model outlined in the second section. First, production is a function of capital, as well as labor, and household's decide how much capital to accumulate. Second, I abstract from money. That is I imagine that the central bank sets nominal interest rates, and given some household demand for money there is an implied nominal money supply. Money is operating in the background, but is irrelevant for the model.

Household preferences are similar to before. In particular, I have

$$U(c_{m,t}, c_{h,t}, l_t) = E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{e} \log(\xi c_{m,t}^e + (1-\xi) c_{h,t}^e) + \theta \log(1 - n_{m,t} - n_{h,t}) \right]$$

Households maximize lifetime utility subject to the constraints

$$1 = l_t + n_{m,t} + n_{h,t}$$

$$c_{m,t} + \frac{\triangle B_{t+1}}{P_t} + I_t = \frac{B_t}{P_t} i_t + w_t n_{m,t} + R_t k_{m,t} + Profits_t$$

$$I_t = k_{m,t+1} + k_{h,t+1} - (1-\delta_m) k_{m,t} - (1-\delta_h) k_{h,t}$$

$$c_{h,t} = z_{h,t} k_{h,t}^{\eta} n_{h,t}^{1-\eta}$$

Here I_t denotes investment, $k_{m,t}$ and $k_{h,t}$ the market and home capital stocks respectively and R_t the gross interest rate on market capital. All the other variables are defined as before. Household maximization implies that the marginal rate of substitution between leisure and consumption equals the wage and that the marginal rate of substitution between market and home consumption equals the ratio of the marginal product of labor in the home sector to the wage. These conditions are given by

$$\theta \left[1 + \frac{1-\xi}{\xi} \left(\frac{c_{h,t}}{c_{m,t}} \right)^e \right] \frac{c_{m,t}}{l_t} = w_t \tag{23}$$

$$\frac{1-\xi}{\xi} \left(\frac{c_{h,t}}{c_{m,t}} \right)^{e-1} = \frac{w_t}{(1-\eta) y_{h,t} / n_{h,t}} \tag{24}$$

Equation (24) can be manipulated to give

$$\left(\frac{c_{h,t}}{c_{m,t}} \right)^e = \frac{\xi}{1-\xi} \frac{w_t n_{h,t}}{c_{m,t}(1-\eta)} \tag{25}$$

Substituting this into equation (23) and doing some straight forward algebra one obtains

$$n_{h,t} = \frac{1-\eta}{1-\eta+\theta}\left(1 - n_{m,t} - \theta\frac{c_{m,t}}{w_t}\right) \tag{26}$$

With the exception of the parameters η and θ, the right hand side of equation (26) contains observable quantities. Once the home labor supply is known I can recover home production using (25).

On the producer's side, it can be seen from the firm's first-order condition that

$$w_t = \zeta_t(1-\alpha)\frac{y_{m,t}}{n_{m,t}} \tag{27}$$

where ζ_t is the inverse of the price markup over marginal cost. Manipulating the above equation, one can express the inverse markup as a function of labor's share of income $s_{l,t} = \frac{w_t n_{m,t}}{y_{m,t}}$.

$$\zeta_t = \frac{s_{l,t}}{1-\alpha} \tag{28}$$

Using the variation in labor's share of income to measure changes in the markup has been used extensively in macroeconomics and is summarized by a 1999 survey article by Rotemberg and Woodford. After estimating the markup, I substitute (28) into (26) and obtain

$$n_{h,t} = \frac{1-\eta}{1-\eta+\theta}\left(1 - n_{m,t} - \theta\frac{c_{m,t}}{(1-\alpha)\zeta_t y_{m,t}/n_{m,t}}\right) \tag{29}$$

The right hand side of equation (29) contains parameters, market quantities and the estimated markup. This is the equation I use going forward.

I take market consumption to be the sum of services and non-durable consumption available from the National Income and Product Accounts (NIPA). Investment is the sum of residential and residential fixed investment plus consumer durables, also available from the NIPA. Market output is the sum of market consumption and investment. All variables are in real terms, deflated by the GDP deflator with 2005 as the base year. Additionally, variables are divided by the non-institutionalized population age 16 and over. For market hours, I take the average number of hours per week in the non-farm business sector multiplied by the ratio of civilian employment to population ratio, available from the Bureau of Labor Statistics (BLS), divided by the number of hours in a week, 168. Labor's share of income is taken to be labor's share in the non-farm sector. The share and hours data is available from the (BLS). The series start in the first quarter of 1964 and go through the third quarter of 2011.

In terms of parameter values, I take the IKS values as a baseline, using $(\alpha, \eta, \theta, \xi, e) = (0.28, 0.14, 2.7, .4, 0.5)$. The measure for capital's share in the market sector, α, is reasonably conventional and is used by Gomme and Ruppert (2007) in their detailed article on calibration in modern macro models. The measure for capital's share in home production η, is more difficult. On one hand, if home capital includes both residential fixed capital and consumer durables then the share is close to 0.35. However, if one views housing as a consumption good, not being used for the production of other goods, the share is less than 0.1 as in Benhabib, Rogerson, and Wright (1991). The intermediate cases would consider a fraction of both housing and consumer durables as productive capital. I don't advocate one interpretation over another, but will take the IKS case as the baseline and consider robustness checks later. Finally, the preference parameters θ, ξ, and e are taken from IKS.

Results

Once home hours and output are constructed I take logs of each variable and detrend with the HP filter using a smoothing parameter of 1600. Figure 5 compares home hours and consumption to their market counterparts and various business cycle moments are reported in table 2. Relative to market hours, home hours are less volatile and are countercyclical. That is, in periods when market output is high, market hours are also high, but home hours are low. Moreover, as evidenced by its low volatility, a rise in market hours comes at the expense of market hours rather than leisure. Despite being very countercyclical, leisure does not change by much over the cycle. Assuming no markup and analyzing a different time period, IKS obtain qualitatively similar results for their hours series.

Turning to the consumption series, home consumption is almost twice as volatile as market output. Since it is usually thought that households smooth consumption, this finding is at first paradoxical. However, households want to smooth aggregate consumption $C_t = (\xi c^e_{m,t} + (1-\xi) c^e_{h,t})^{1/e}$, not the individual components. The relative standard deviation of the consumption aggregator to output is approximately 0.9. Contributing to the high volatility of home consumption is the absence of a smoothing mechanism. Unlike market consumption where households can smooth by investing in capital, there is no such smoothing mechanism for home consumption. Rather, households consume what they produce. Consequently, there is substantially more variation in home production/consumption than market consumption. In terms of comovement, home consumption is weakly positively correlated with output.

As previously mentioned, there is no consensus on what capital's share should be in the home production function. Increasing both consumer durables and residential fixed investment in home's capital stock gives an upper bound of $\eta = 0.35$. Redoing the same exercise as above with the new value of η reduces the volatility of home hours to 0.46, but does not affect the summary statistics of the home output series out to one decimal place.

In contrast, varying e from 0.3 to 0.7 makes the volatility relative to output range from 1 to 3 times as volatile.

7. Conclusion

As documented in the introduction, households devote a nontrivial amount of time to home work. I show that including home production in an otherwise standard New Keynesian model provides a substantial amplification mechanism for monetary policy shocks and an amplification and propagation mechanism for technology shocks. Despite its intentions, a conventional Taylor rule does not stabilize the economy, but a simple modification restores its appealing properties. Strict inflation targeting results in a "divine coincidence" where maximizing welfare is achieved by minimizing inflation variability. For central banks that insist, perhaps because of a legislated mandate, to target an output gap, they must amend the output gap they target. To that end, I use the elaborated theory to back out home hours and home production as a function of parameters and observable market quantities. Needless to say, this is a second best outcome, but in the absence of quarterly data on these quantities, alternatives are limited. Potential future work includes incorporating home production in medium scale DSGE models and/or formulating optimal monetary policy in more general settings.

References

Aguiar, Mark and Erik Hurst (2007): "Measuring Trends in Leisure: The Allocation of Time Over Five Decades," *Quarterly Journal of Economics*, 122: 969-1006.

Arouba, S. Boragan, Morris Davis and Randall Wright (2011): "Homework in Monetary Economics: Inflation, Home Production and the Production of Homes," manuscript, University of Maryland.

Benhabib, Jess, Richard Rogerson and Randall Wright (1991): "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy*, 99: 1166-1187.

Barsky, Robert, Christopher House, and Miles Kimball (2007): "Sticky-Price Models and Durable Goods," *American Economic Review*, 97: 984:998.

Becker, Gary (1965): "A Theory of the Allocation of Time," *Economic Journal*, 75:493-517.

Bils, Mark and Peter Klenow (2004): "Some Evidence on the Importance of Sticky Prices," *Journal of Political Economy*, 112: 947-985.

Calvo, Guillermo (1983): "Staggered Prices in a Utility Maximization Framework," *Journal of Monetary Economics*, 12: 383-398.

Chari, V.V., Patrick Kehoe and Ellen McGrattan (2007): "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem?" *Econometrica* 68: 1151-1179.

Clarida, Richard, Jordi Galí, and Mark Gertler (1999): "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, 37: 1661-1707.

Cogley, Timothy and James Nason (1995): "Output Dynamics in a Real Business Cycle Model," *American Economic Review*, 85: 492-511.

Dixit, Avinash and Joseph Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67: 297-308.

Faia, Ester and Tommaso Monacelli (2007): "Optimal interest rate rules, asset prices, and credit frictions," *Journal of Economic Dynamics and Control,* 31: 3228-3254.

Fisher, Jonas (2007): "Why Does Household Investment Lead Business Investment Over the Business Cycle?" *Journal of Political Economy* 114: 413-451.

Galí, Jordi (2008): *Monetary Policy, Inflation and the Business Cycle: An Introduction to the New Keynesian Framework*, Princeton, NJ: Princeton University Press.

Galí, Jordi, Mark Gertler and J. David López-Salido (2007): "Markups, Gaps, and the Welfare Costs of Business Fluctuations," *The Review of Economics and Statistics*, 89: 44-59.

Gomme, Paul, Fynn Kydland and Peter Rupert (2001): "Home Production Meets Time to Build," *Journal of Political Economy*, 109: 1115-1131.

Gomme, Paul and Peter Rupert (2007): "Theory, Measurement and Calibration of Macroeconomic Models," *Journal of Monetary Economics*, 54: 460-497.

Greenwood, Jeremy and Zvi Hercowitz (1991): "The Allocation of Capital and Time Over the Business Cycle," *Journal of Political Economy*, 99: 1188-1214.

Greenwood, Jeremy, Richard Rogerson and Randall Wright (1995): "Household Production in Real Business Cycle Theory," in *Frontiers of Business Cycle Theory* by Tomas Cooley, Princeton.

Ingram, Beth, Narayana Kocherlakota, and N. Savin (1997): "An Analysis of the Cyclical Behavior of Home Production," *Journal of Monetary Economics*, 40: 435-456.

Kilian, Litz, Lee Ohanian and Alan Stockman (1995): "The Effects of Real and Monetary Shocks in a Business Cycle Model wit Some Sticky Prices," *Journal of Money, Credit and Banking*, 27: 1209-1234.

Ngouana, C.L. (2009): "Household Production, Services and Monetary Policy," manuscript, University of Montreal.

McGrattan, Ellen, Richard Rogerson and Randall Wright (1997): "An Equilibrium Model of the Business Cycle with Household Production and Fiscal Policy," *Journal of International Economics*, 38: 267-290.

Ríos Rull, José-Victor (1993): "Working in the Market, Working at Home, and the Acquisition of Skills: A General Equilibrium Approach," *American Economic Review*, 83: 893-907.

Rotemberg Julio and Michael Woodford (1999): "The Cyclical Behavior of Prices and Costs," *Handbook of Macroeconomics*, 16 1051-1131.

Schmitt-Grohè, Stephanie and Martín Uribe (2004): "Solving dynamic stochastic general equilibrium models using a second-order approximation to the policy function," *Journal of Economic Dynamics and Control*, 28: 755-775.

Taylor, John (1993): "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conferences Series on Public Policy* 39 (Dec.): 195-214.

Walsh, Carl (2010): *Monetary Theory and Policy*, Cambridge, MA: MIT Press.

Woodford, Michael (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton, NJ: Princeton University Press.

Appendix 1: Derivation of Phillips Curve

Here we derive the Phillips curve stated in the text. Start by writing inflation as a function of reset price inflation

$$\pi_t = \left[(1-\epsilon)(1+\pi_t^{\#})^{1-\epsilon} + \phi\right]^{\frac{1}{1-\epsilon}} \tag{30}$$

Linearizing around the steady state $\pi = \pi^{\#} = 0$ gives

$$\hat{\pi}_t = (1-\phi)\hat{\pi}_t^{\#} \tag{31}$$

Hats denote linearized variables. Recall the optimal reset price is

$$p_t^{\#} = \frac{\epsilon E_t \sum_{s=0}^{\infty} \phi^s \Lambda_{t+s} p_{t+s}^{\epsilon} y_{t+s} mc_{t+s}}{(\epsilon-1)E_t \sum_{s=0}^{\infty} \phi^s \Lambda_{t+s} p_{t+s}^{\epsilon-1} y_{t+s}}$$

We can rewrite this in terms of inflation and reset price inflation. Defining the auxiliary variables $A_t = \Lambda_t y_t mc_t + \beta\phi(1+\pi_t)^{\epsilon} E_t A_{t+1}$ and $B_t = \Lambda_t y_t + \beta\phi(1+\pi_t)^{\epsilon} E_t B_{t+1}$we have

$$\pi_t^{\#} = \frac{\epsilon A_t}{(\epsilon-1)B_t}(1+\pi_t) \tag{32}$$

Linearization around the steady state gives

$$\hat{\pi}_t^{\#} = \hat{\pi}_t + \hat{A}_t - \hat{B}_t \tag{33}$$

and

$$\hat{A}_t = \frac{y\Lambda mc\hat{\Lambda}_t + y\Lambda mc(\hat{mc}_t) + y\Lambda mc\hat{y}_t + \epsilon\beta\phi A E_t\hat{\pi}_{t+1} + \beta\phi A E_t \hat{A}_{t+1}}{A}$$

$$\hat{B}_t = \frac{y\Lambda\hat{\Lambda}_t + y\Lambda\hat{y}_t + \epsilon\beta\phi B(\epsilon-1)E\hat{\pi}_{t+1} + \beta\phi B E_t \hat{B}_{t+1}}{B}$$

Substituting these into 4 and making use of 3 we have

$$\frac{\phi}{1-\phi}\hat{\pi}_t = (1-\phi\beta)\hat{mc}_t + \phi\beta E_t\hat{\pi}_{t+1} + \phi\beta E_t(\hat{A}_{t+1} - \hat{B}_{t+1})$$

Straight forward manipulation gives

$$\hat{\pi}_t = \frac{(1-\phi\beta)(1-\phi)}{\phi}\hat{mc}_t + \beta E_t\hat{\pi}_{t+1} \tag{34}$$

Then using the defining of real marginal cost we have $\hat{mc}_t = \hat{w}_t - \hat{z}_{m,t}$. Then using the household's first order condition 5 and linearizing around the steady state we have

$$\hat{w}_t = \hat{z}_{h,t} + (1-e)(\hat{y}_{m,t} - \hat{y}_{h,t})$$

This implies

$$\hat{\pi}_t = \frac{(1-\phi\beta)(1-\phi)}{\phi}(\hat{z}_{h,t} - \hat{z}_{m,t} + (1-e)(\hat{y}_{m,t} - \hat{y}_{h,t})) + \beta E_t \hat{\pi}_{t+1} \tag{35}$$

Finally, when prices are flexible we have $\hat{mc}_t = 0$. This implies $\hat{z}_{h,t} = \hat{z}_{m,t} + (1-e)(\hat{y}^f_{h,t} - \hat{y}^f_{m,t})$ where the superscript f denotes the flexible price deviation from steady state. Then,

$$\hat{\pi} = \frac{(1-\phi\beta)(1-\phi)(1-e)}{\phi}((\hat{y}_{m,t} - \hat{y}^f_{m,t}) - (\hat{y}_{h,t} - \hat{y}^f_{h,t})) + \beta E_t \hat{\pi}_{t+1} \tag{36}$$

gives us the desired equation.

Appendix 2: Log-linearized System

$$(e-1)\hat{y}_{m,t} - \hat{C}_t = \hat{\lambda}_t \tag{37}$$

$$\hat{z}_{h,t} + (e-1)\hat{y}_{h,t} - \hat{C}_t = \frac{n_m\hat{n}_{m,t} + n_h\hat{n}_{h,t}}{1 - n_m - n_h} \tag{38}$$

$$\hat{\lambda}_t + \hat{w}_t = \frac{n_m\hat{n}_{m,t} + n_h\hat{n}_{h,t}}{1 - n_m - n_h} \tag{39}$$

$$-\nu\hat{m}_{t+1} = E_t\hat{\lambda}_t + \frac{\hat{i}_{t+1}}{i(1+i)} \tag{40}$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + E_t\hat{r}_{t+1} \tag{41}$$

$$\hat{mc}_t = \hat{w}_t - \hat{z}_{m,t} \tag{42}$$

$$\hat{y}_{m,t} = \hat{z}_{m,t} + \hat{n}_{m,t} \tag{43}$$

$$\hat{y}_{h,t} = \hat{z}_{h,t} + \hat{n}_{h,t} \tag{44}$$

$$\hat{\pi}_t = \frac{(1-\phi\beta)(1-\phi)}{\phi}\hat{mc}_t + \beta E_t\hat{\pi}_{t+1} \tag{45}$$

$$\hat{i}_{t+1} = \hat{r}_{t+1} + E_t\hat{\pi}_{t+1} \tag{46}$$

$$\hat{C}_t = \frac{ay_m^e\hat{y}_{m,t} + (1-a)y_h^e\hat{y}_{h,t}}{ay_m^e + (1-a)y_h^e} \tag{47}$$

$$\hat{z}_{m,t} = \rho_m\hat{z}_{m,t-1} + \epsilon_{m,t} \tag{48}$$

$$\hat{z}_{h,t} = \rho_h\hat{z}_{h,t-1} + \epsilon_{h,t} \tag{49}$$

$$\Delta\hat{m}_t + \hat{\pi}_t = (1-\rho_{mon})\pi + \rho_{mon}\Delta\hat{m}_{t-1} + \rho_{mon}\hat{\pi}_{t-1} + \epsilon_{mon,t} \tag{50}$$

With the exception of inflation and the interest rates, the hatted variables denote percentage deviation. The hatted variables for interest rates and inflation denote percentage point deviation. Endogenous variables without hats and time subscripts are in steady state. The above system has 14 equations and 14 unknowns.

Appendix 3: Tables and Figures

Figure 1: Response to a Technology Shock

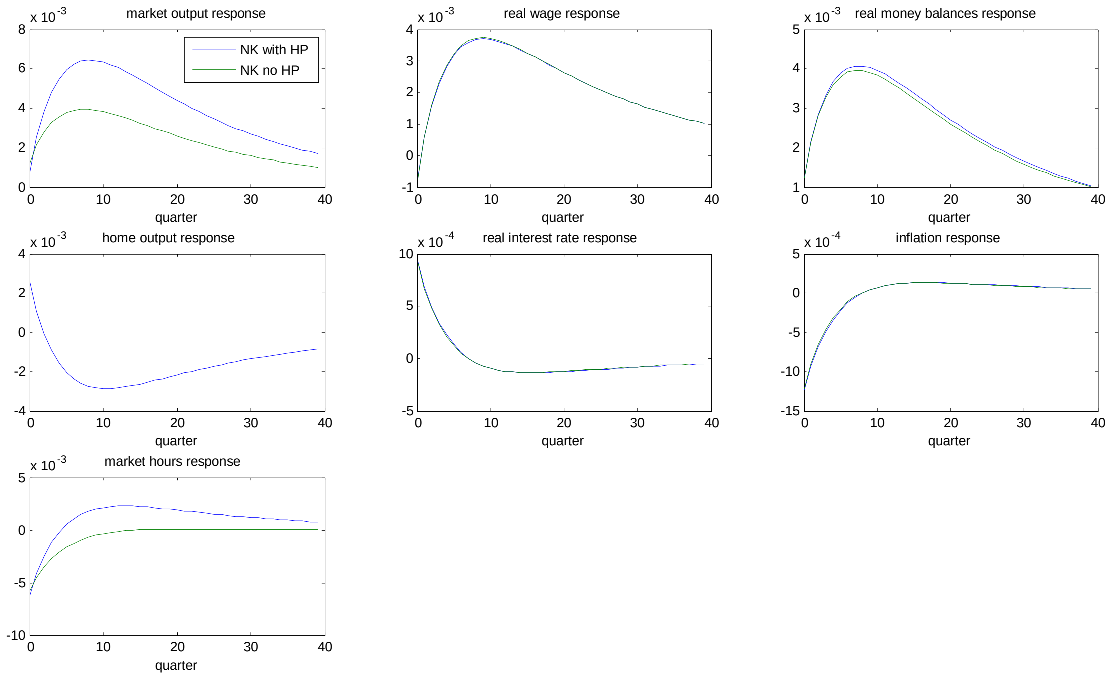


Figure 2: Response to a Monetary Policy Shock

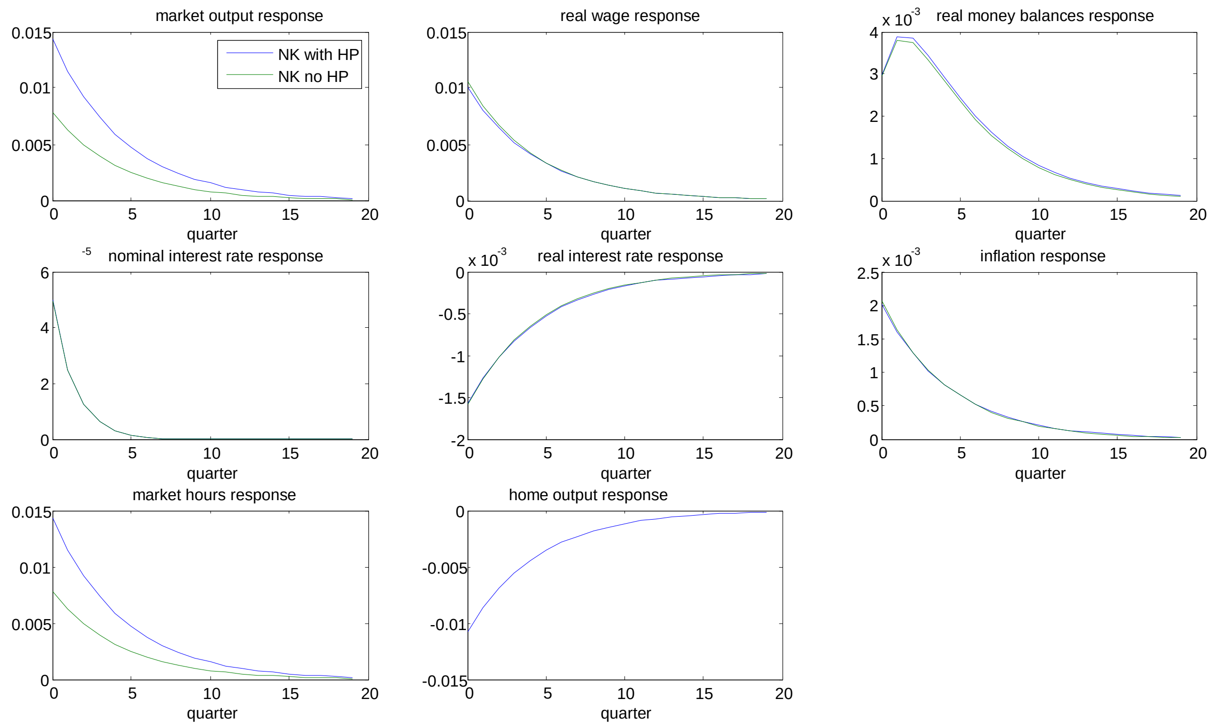


Figure 3: Market Output Response to a Technology Shock

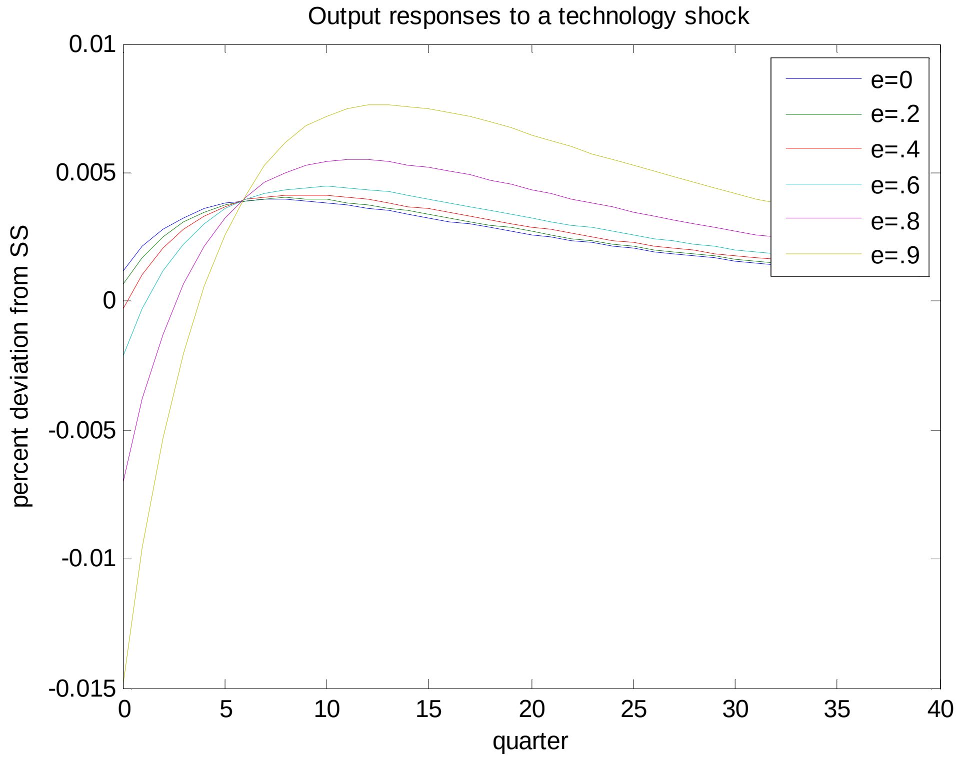
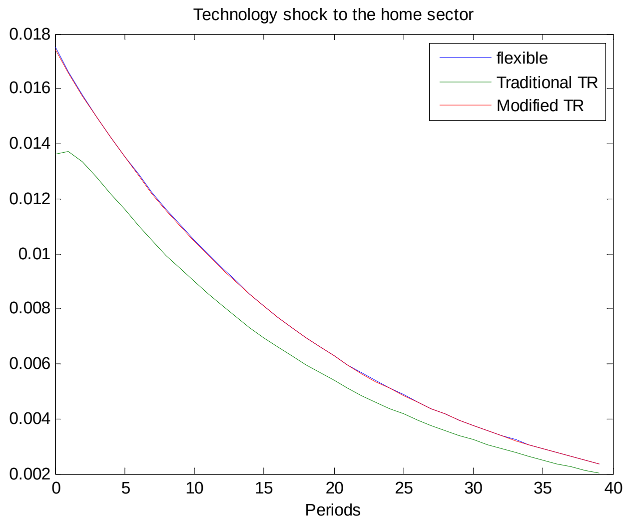


Figure 4: Comparing Taylor Rules



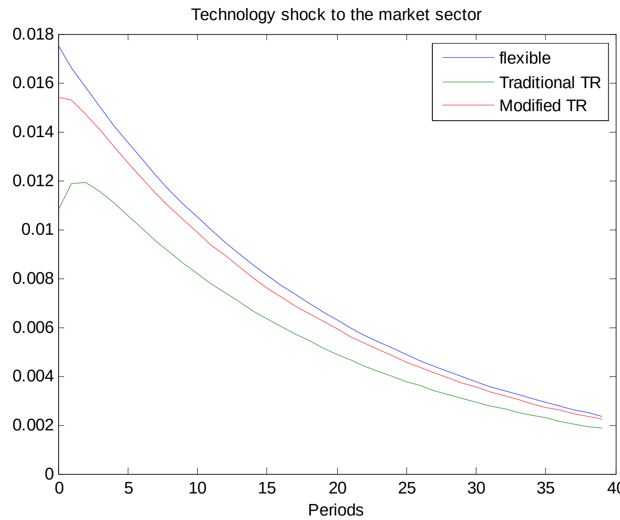


Figure 5: Output Gaps

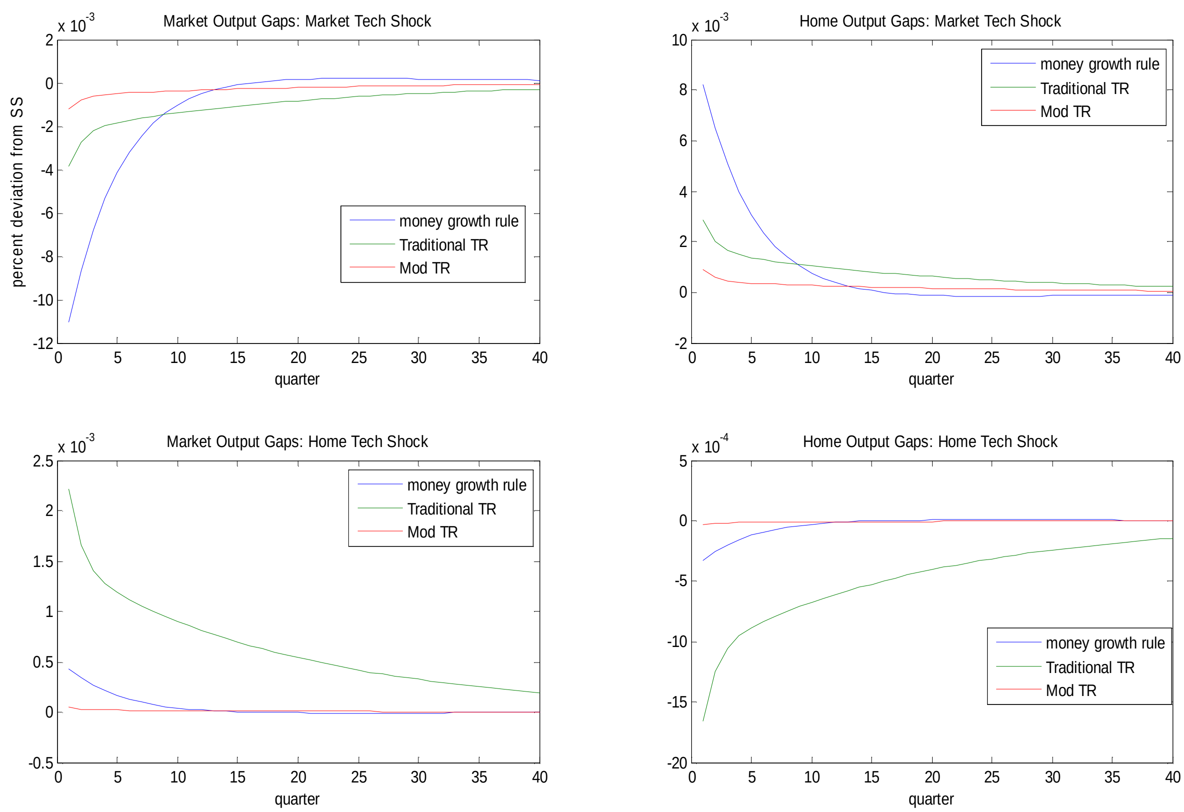


Figure 6: Hours and Output Series

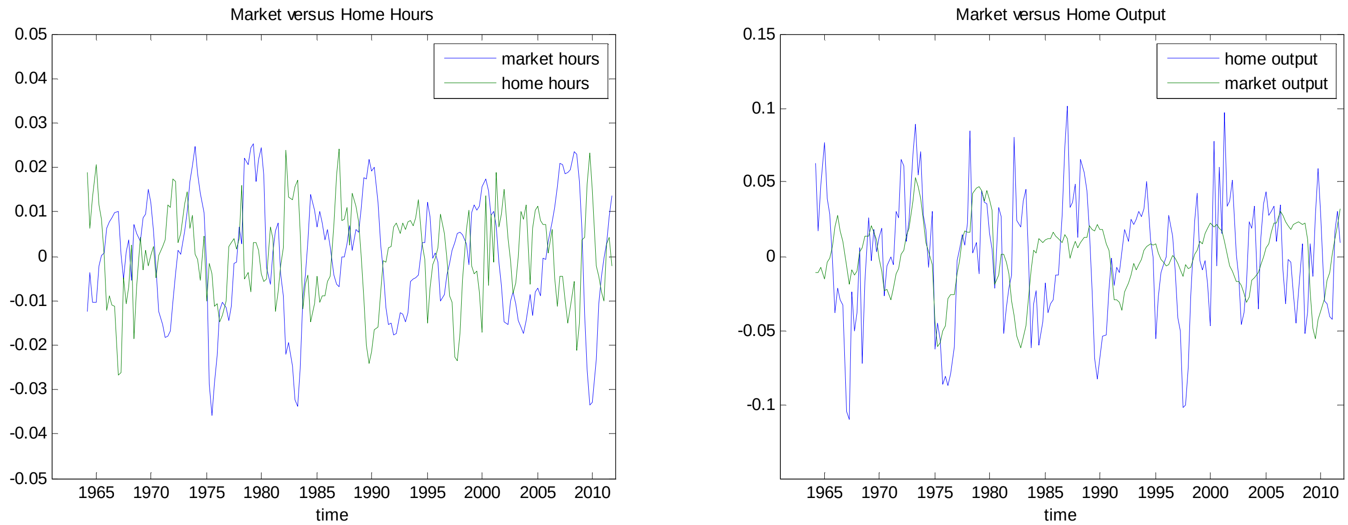


Table 1: Home Versus Market Hours Over Time

Year	1965	1975	1985	1993	2003
Market Hours	34.24	32.13	32.13	34.02	33.01
Home Hours	23.52	20.30	20.64	17.94	18.00
Sample Size	1,862	1,712	3,283	5,465	15,244

* Taken from Aguire and Hurst (2006): "The Allocation of Time Over Five Decades."

Table 2: Parameter Values

Parameter	Interpretation	Value
ξ	relative weight of market consumption	0.5815
β	discount factor	0.99
θ	disutility of labor	0.9038
υ	curvature of money demand	1
e	elasticity of substitution	0.5
ρ_{mon}	persistence of money shock	0.5
ρ_h, ρ_m	persistence of technology shocks	0.95
σ_{mon}	volatility of money shock	0.005
σ_m, σ_h	volatility of technology shock	0.007
ϕ	frequency firms update their price	5/6
γ	correlation between tech shocks	0
ϵ	intermediaries pricing power	6

Table 3: Welfare Approximations

Rule	$W_{o,t}$
flexible prices	-201.01
$\phi_y = 1.5,\ \phi_\pi = 0.5$	-201.73
$\phi_x = 1.5,\ \phi_\pi = 0.5$	-201.03
$\phi_\pi = 10$	-201.02
$\phi_\pi = 1.5$	-201.07

Table 4: Business Cycle Moments

	Relative Std to output	Correlation with output
market output	1	1
market consumption	0.4	0.83
home consumption	1.84	0.19
market hours	0.71	0.81
home hours	0.46	-0.16
investment	2.36	0.98
markup	0.46	0.20
leisure	0.09	-0.80

* All moments are HP filtered with a smoothing parameter of 1600