Competition as a Discovery Procedure:
Schumpeter Meets Hayek in a Model of Innovation

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Abstract

Recent empirical studies have identified two seemingly contradictory relationships between competition, innovation, and productivity growth — a positive relationship between competition and industry-level productivity growth, and an inverted-U relationship between competition and firm-level innovation. To account for these phenomena I incorporate an insight of Friedrich Hayek (2002) — that competition acts to discover the optimal path of innovation — into a standard model of innovation wherein firm entry, firm-level research, and markups are all endogenously determined. When firms are uncertain about the optimal direction in which to innovate, more firms implies more innovations, resulting in a higher expected value of the ‘best’ innovation. When this Hayekian uncertainty meets the traditional Schumpeterian mechanism, whereby lower rents discourage firm-level research, the model generates the relationships emphasized in empirical studies. I use the model to evaluate the effects of antitrust policy. Notwithstanding the positive relationship between competition and growth, I find antitrust policy to be detrimental to industry-level productivity growth.

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1 Introduction

Schumpeter (1942) argues that the expectation of monopoly power is necessary to induce innovation. Although his argument is more nuanced, a simplistic implication is that a higher level of competitive rivalry should translate into lower levels of innovation, and thus lower productivity growth. Early models of innovation include this Schumpeterian mechanism and share the conclusion that competition is harmful to growth.¹ Recent empirical studies, however, come to very different conclusions.² Productivity growth at the industry level has been shown to be positively correlated with competition, while the average level of innovation per firm in an industry exhibits an inverted-U relationship with competition — that is, a positive relationship when competition is relatively low, and a negative relationship when competition is relatively high. These two stylized facts suggest the need for a model that both explains how an increase in competition can be accompanied by higher growth, and allows for industry-level outcomes to differ from those of the average firm. The present paper accomplishes this by introducing uncertainty into a general equilibrium model of quality-improving innovations. I assume that entrepreneurs are uncertain about the relative value of each possible direction of innovation, until an innovation has actually been introduced. In such an economy, a greater number of firms implies a greater number of innovations tried, which in turn implies a higher expected value of the ‘best’ innovation. By further assuming the best innovation can capture a market, it becomes possible that measured productivity growth for an industry can be increasing in the number of firms, even if the average level of research per firm is declining (due to the usual Schumpeterian mechanism).

Imposing uncertainty about the optimal direction of innovation (call it Hayekian uncertainty) captures an insight of F.A.Hayek (2002). In Hayek’s words, competition is (p.9):

> a procedure for discovering facts which, if the procedure did not exist, would

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¹For example, see Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992).
²For example, see Nickell (1996), Blundell et al. (1999), Dutz and Hayri (2000), Aghion et al. (2005) and Aghion et al. (2008)
remain unknown or at least would not be used.

When there is nothing to discover, the only benefit of competition is static efficiency—identical firms compete for inputs and customers, driving price to marginal cost and unwittingly attaining allocative efficiency. When firms are uncertain about which new products or production processes will turn out to be the most valuable, however, competition acts to sort the best from the worst ideas tried.

Why are entrepreneurs so uncertain about the value of innovations? When setting out to improve a product or production process, there is no one-dimensional measure of quality to progress along. One firm may improve the carrying capacity and handling of its car, only to lose market share to a competitor’s more stylish alternative. A fast-food chain may increase the speed of its drive-thru service, only to lose market share to a competitor who introduces fresher ingredients.

Incorporating Hayekian uncertainty into a model of quality innovations creates a positive link between industry-level productivity growth and the endogenously-determined number of firms in an industry. By allowing the quality of its rivals to affect a firm’s pricing decision, I create a link between a firm’s markup over marginal cost (a standard measure of competition) and the number of firms in its industry. By further allowing each firm to decide the magnitude of its quality improvement through research, I can examine the interplay between competition, industry-level productivity growth, and firm-level innovation in general equilibrium. Throughout the paper the Schumpeterian mechanism, whereby lower rents discourage research, is always present. But when Schumpeter meets Hayek in this model of innovation, more competition is accompanied by higher industry-level productivity growth, even if firm-level research is reduced.

An advantage of modeling competition and innovation as endogenous outcomes is it allows for policy evaluation that is less susceptible to the Lucas Critique. When competition is endogenous, the evaluation of antitrust policy requires one to model a policy interven-
tion as a change in the parameters of the model or in the constraints faced by firms, rather than as an assumption that more enforcement is equivalent to more competition. Whereas studies like Geroski (1990) and Dutz and Hayri (2000) interpret the positive empirical relationship between competition and growth as implying greater antitrust enforcement is good for growth, I find that both ceilings on markups and restrictions on collusion between firms have a detrimental effect on industry-level productivity growth.

Other papers have developed models to address either the firm-level or industry-level relationships found in empirical studies. Aghion et al. (2005) model an economy with a continuum of industries, each structured as a duopoly, where industries differ in the potential for collusion between competitors. The authors show their model can generate an inverted-U between competition and firm-level innovation, when variation in competition is driven by variation in the potential for collusion. The present paper instead focuses on variation in the economic and regulatory costs faced by innovators. By incorporating free entry into the market, I also retain a connection between the conventional definition of ‘competition’ — more firms in the market — and the measure of competition used in recent empirical studies. Further, I allow for the differing relationships between competition and growth at the firm and industry levels suggested by studies like Aghion et al. (2008). Vives (2008) analyzes the relationship between competition and productivity growth by modeling differentiated products with cost-reducing innovations. Although his model can generate correlations between competition and growth that differ at the firm and economy level, Vives finds the model cannot explain an inverted-U relationship. Boldrin and Levine (2002, 2005, 2008) study the conditions under which innovation can occur under perfect competition, while the present paper considers variation in the level of competition.

In the next section, I present the model. I follow this with a short survey of the empirical evidence that supports the implications of the model. In Section 4 I extend the model to evaluate the effects of antitrust policies on growth. Section 5 concludes.
2 The Model

2.1 Environment

Consider a closed economy where time is discrete and indexed by $t$. There is a final-good ($y$) sector, and an intermediate-good ($x$) sector made up of a measure 1 of intermediate-good industries. The final-good market is perfectly competitive, with a representative final-good firm producing according to the following standard CES production function:\(^3\)

$$ y = \left( \int_0^1 X^\alpha(j) dj \right)^{\frac{1}{\alpha}}, $$

where $j$ indexes intermediate-good industries and $\frac{1}{1-\alpha}$ is the elasticity of substitution between industries. I define $X(j)$ in the following way;

$$ X(j) \equiv \sum_{i=1}^{e(j)} A_i(j)x_i(j), $$

where $x_i(j)$ denotes the amount of good-$j$ demanded from firm-$i$, $A_i(j)$ is a measure of the quality of $x_i(j)$, and $e(j)$ is the (endogenously-determined) number of firms in industry-$j$.

Any firm-$i$ in industry-$j$ can produce according to

$$ x_i(j) = L_i(j), $$

where $x_i(j)$ is the amount of good-$j$ produced by firm-$i$ of quality $A_i(j)$, and $L_i(j)$ is its labor input. Each period, firms in industry-$j$ can choose to invest in innovation, thereby acquiring a quality of $A_i(j) > 0$ for output produced in the subsequent period. I assume any firm-$i$ that does not innovate in $t-1$ receives a quality of $A_i(j) = 0$.\(^4\) The total investment required to innovate is made up of a fixed cost of introducing an innovation, $zy\psi(j)$, and

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\(^3\)Throughout the paper, I omit the time subscript unless clarity requires it.

\(^4\)Relaxing this assumption complicates the model without changing any qualitative results.
the cost of research, \( m \psi(j)n_i(j) \), where \( z \) and \( m \) are exogenous parameters, and \( n_i \) is the level of research chosen by firm-\( i \). The research cost is the cost of actually improving the quality of a firm’s product, while the fixed cost can be interpreted as the cost of introducing a new product to the market. Introducing a new product to the market might entail holding an inventory for a time before demand is realized, or fulfilling regulatory requirements like efficacy trials for drugs. The term \( \psi(j) \) is equal to \( A^{\alpha_{[1]}}[1](j) \left( \int_{0}^{1} A^{\alpha_{[1]}}[1](j')dj' \right)^{-1} \), where \( A^{[1]}[1](j) \) is the first-best (\([1]\)) quality in industry-\( j \). The presence of \( \psi(j) \) can be rationalized as reflecting the phenomenon that each incremental quality improvement is more costly than the last (this phenomenon is modeled in a similar fashion in Aghion and Howitt (2005)), but \( \psi(j) \) also serves to make research and innovation decisions independent of past innovations. Finally, firms finance investments by issuing equity to households.

At the beginning of each period, the quality \( A_i(j) \) associated with each firm-\( i \) that invested in innovation in the previous period becomes known to all agents. For each firm-\( i \) the introduction of an innovation in period \( t \) results in a quality of

\[
A_{it}(j) = A_{i-1[1]}(j)h_{it}(j),
\]

where \( A_{i-1[1]}(j) \) is the highest (\([1]\)) quality in industry-\( j \) in period \( t - 1 \), and \( h_i(j) \) is the realized value of a random draw. I assume \( h_i(j) \) is distributed according to some continuously differentiable distribution \( F_i(h) \), bounded by 0 and \( n_i^{[1]}(j), \theta \in (0, 1), \) where \( n_i(j) \) is the level of research invested in by firm-\( i \). The randomness of the draw represents each firm’s uncertainty about the value of the direction in which it has chosen to innovate relative to other directions, while the level of research \( n \) determines the magnitude of the improvement in that direction. Note the lower bound of zero implies an innovator may misread its potential customers so badly that its new product is less valuable than the previous period’s best. I call this the New Coke Phenomenon.\(^5\)

\(^5\)Changing the support of \( F_i(h) \) to \([1, 1 + n_i(j)]\) complicates the analysis without changing any of the
There is a mass of households of measure 1, each of which supplies $L$ units of labor to each intermediate-good industry.\footnote{Making labor mobile across industries complicates the analysis without changing any of the qualitative results.} Households only value consumption, and have a constant discount rate $\beta \in (0, 1)$. Income can be consumed or saved, and the only vehicle for savings is the purchase of equity in innovating intermediate firms, earning a gross rate of return of $R$. With a continuum of intermediate industries and no aggregate uncertainty, each household faces a deterministic interest rate and economy-wide growth rate. The household’s problem is therefore to choose consumption $\{c_t\}_{t=0}^{\infty}$ and savings $\{s_t\}_{t=0}^{\infty}$ to solve
\begin{equation}
\max_{\{c_t, s_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln(c_t), \quad \text{s.t.} \quad c_t + s_t \leq L \int_0^1 w_t(j) dj + s_{t-1} R_t,
\end{equation}
where $L \int_0^1 w_t(j) dj$ is the total wage income from labor supplied to intermediate firms.

\section*{2.2 Market Structure}

The final-good sector is perfectly competitive, and so the representative final-good producer takes all prices as given and chooses $x_i(j)$ from each firm-$i$ in each industry-$j$ to solve
\begin{equation}
\max_{\{x_i(j)\}_{i,j}} \left( \int_0^1 \left( \sum_{i=1}^{e(j)} A_i(j) x_i(j) \right)^{\frac{1}{\alpha}} \right) - \int_0^1 \sum_{i=1}^{e(j)} P_i(j) x_i(j) dj,
\end{equation}
where $P_i(j)$ is the price of $x_i(j)$ in terms of the final good, $A_i(j)$ is the quality of $x_i(j)$, and $e(j)$ is the number of firms in industry-$j$. The combination of perfect substitutability between firms in the same industry and a constant-returns-to-scale intermediate production function ensures that only one firm in each industry produces in equilibrium. As a result,
the final-good firm demands \( x_i(j) \), such that

\[
P_i(j) = A_i^\alpha(j)x_i^{1-\alpha}y^{1-\alpha}, \quad \text{if} \quad \frac{A_i(j)}{P_i(j)} > \frac{A_k(j)}{P_k(j)}, \forall k \neq i \in \{1, ..., e(j)\};
\]

(3)

\( x_i(j) = 0 \) otherwise.

In each industry-\( j \), each firm-\( i \) faces a given wage \( w(j) \) and chooses \( x_i(j) \) to solve

\[
\begin{align*}
\max_{x_i(j)} & \quad P_i(j)x_i(j) - w(j)x_i(j), \\
\text{s.t.} & \quad P_i(j) = A_i^\alpha(j)x_i^{1-\alpha}y^{1-\alpha}, \\
& \quad \frac{A_i(j)}{P_i(j)} > \frac{A_k(j)}{P_k(j)}, \forall k \neq i \in \{1, ..., e(j)\}.
\end{align*}
\]

(4)

As a result only the best firm in each industry-\( j \) produces, charging the following price for \( x_{[1]}(j) \);

\[
P_i(j) = \begin{cases} 
w(j)/\alpha, & \text{if } \frac{A_{[2]}(j)}{A_{[1]}(j)} < \alpha \\
w(j) \frac{A_{[1]}(j)^{\alpha}}{A_{[2]}(j)^{1-\alpha}}, & \text{if } \frac{A_{[2]}(j)}{A_{[1]}(j)} > \alpha
\end{cases}
\]

(5)

where \( A_{[\ell]}(j) \) is the \( \ell \)th-highest quality in industry-\( j \).

Labor-market clearing (\( L = x_{[1]}(j) \)) combines with the demand for \( x_{[1]}(j) \) (3) and the price of \( x_{[1]}(j) \) (5) to determine the wage rate in industry-\( j \);

\[
w(j) = \begin{cases} 
\alpha A_{[1]}^\alpha(j)L^{\alpha-1}y_1^{1-\alpha}, & \text{if } \frac{A_{[2]}(j)}{A_{[1]}(j)} < \alpha \\
\frac{A_{[2]}(j)}{A_{[1]}^{1-\alpha}(j)}L^{\alpha-1}y_1^{1-\alpha}, & \text{if } \frac{A_{[2]}(j)}{A_{[1]}(j)} > \alpha.
\end{cases}
\]
Any firm-\(i\) innovating in \(t - 1\) will therefore face the following expected discounted profits;\(^7\)

\[
E_{t-1}\left(\frac{\pi_{it}}{R_t}\right) = \text{Prob} \left[ A_{it} = A_{t[1]} > \frac{A_{t[2]}}{\alpha} \right] \cdot E_{t-1} \left[ (1 - \alpha) A_{t[1]}^{\alpha} \frac{L^a y_{t-1}^{1-\alpha}}{R_t} \mid A_{it} = A_{t[1]} > \frac{A_{t[2]}}{\alpha} \right] \\
+ \text{Prob} \left[ A_{it} = A_{t[1]} < \frac{A_{t[2]}}{\alpha} \right] \cdot E_{t-1} \left[ \left( \frac{A_{t[1]} - A_{t[2]}}{A_{t[1]}^{1-\alpha}} \right) \frac{L^a y_{t-1}^{1-\alpha}}{R_t} \mid A_{it} = A_{t[1]} < \frac{A_{t[2]}}{\alpha} \right] \\
- y_{t-1} \psi_{t-1} (z + mn_{i,t-1}),
\]

where \(zy\psi\) is the cost of introducing an innovation, \(mn_{i}y\psi\) is the cost of firm-\(i\)'s research \(n_{i}\), and \(\psi(j) \equiv A_{[1]}^{a}(j) \left( \int_0^1 A_{[1]}^{a}(j')dj' \right)^{-1}\). Substituting for \(A_{t[\ell]} = A_{t-1[1]} h_{t[\ell]}\) and letting \(\Delta_{t-1} \equiv y_{t-1} \psi_{t-1},\) expected discounted profits are

\[
E_{t-1}\left(\frac{\pi_{it}}{R_t}\right) = \text{(6)}
\]

\[
(1 - \alpha) \frac{\Delta_{t-1}(1 + g_t)^{1-\alpha}}{R_t} \text{Prob} \left[ h_{it} = h_{t[1]} > \frac{h_{t[2]}}{\alpha} \right] \cdot E_{t-1} \left[ h_{t[1]}^{\alpha} \mid h_{it} = h_{t[1]} > \frac{h_{t[2]}}{\alpha} \right] \\
+ \frac{\Delta_{t-1}(1 + g_t)^{1-\alpha}}{R_t} \text{Prob} \left[ h_{it} = h_{t[1]} < \frac{h_{t[2]}}{\alpha} \right] \cdot E_{t-1} \left[ \frac{h_{t[1]} - h_{t[2]}}{h_{t[1]}^{1-\alpha}} \mid h_{it} = h_{t[1]} < \frac{h_{t[2]}}{\alpha} \right] \\
- \Delta_{t-1}(z + mn_{i,t-1}),
\]

where \(h_{t[\ell]}\) denotes the \(\ell\)th-highest realized draw in period \(t\), and \(g_t\) is the growth rate of final-good output \(\left(\frac{y_t}{y_{t-1}} - 1\right)\).

Each firm-\(i\) draws from the continuously differentiable distribution function \(F_t(h)\), which is bounded from above by \(n_i^0\). For ease of notation (and without loss of generality), I assume firm-\(i\) believes every other firm will choose identical levels of research \(n_k(j) = n_{\ell}(j),\) \(\forall k, \ell \neq i \in \{1, ..., e(j)\}\). It follows that the joint density function of \(h_{t[1]}, h_{t[2]}\), and \(h_{it}\),

\(^7\)The industry subscript \(j\) will henceforth be dropped unless required for the sake of clarity.
conditional on \( n_{i,t-1}, n_{k\neq i,t-1}, \) and \( e_{t-1} \), is

\[
\begin{align*}
  f(h_{t[1]} = v, h_{t[2]} = u, h_{it} = h_{t[1]} | n_{i,t-1}, n_{k\neq i,t-1}, e_{t-1}) &= (e_{t-1} - 1)f_i(v)f_k(u)F_k(u)^{e_{t-1} - 2}, \quad (7)
\end{align*}
\]
defined over the relevant intervals of \( u \) and \( v \), where \( F_\ell(\omega) \) is the probability that \( h_\ell \) is less than \( \omega \) given an upper bound of \( n_\ell \), and \( f_\ell(\omega) = F_\ell'(\omega) \).

### 2.3 Competitive Equilibrium

A competitive equilibrium is defined as a sequence of allocations \( \{c_t, s_t, y_t, x_{it}(j), L_{it}(j), e_t(j), n_{it}(j)\} \) and prices \( \{P_{it}(j), w_{it}(j), R_t\} \), \( \forall t \geq 0, \forall i \in \{1, \ldots, e_t(j)\}, \forall j \in [0, 1] \), such that:

(i) households take prices as given and choose \( \{c_t\}_{t=0}^{t=\infty} \) and \( \{s_t\}_{t=0}^{t=\infty} \) to solve (1);
(ii) in each period \( t \), each innovating firm-\( i \) in each industry-\( j \) takes wages as given and chooses \( x_{it}(j) \)
to solve (4);
(iii) in each period \( t \), the final-good firm takes prices as given and chooses \( x_{it}(j), \forall i \in \{1, \ldots, e_t(j)\}, \forall j \in [0, 1] \), to solve (2);
(iv) in each period \( t \), each innovating firm-\( i \) in each industry-\( j \) takes \( e_t(j), n_{k\neq i,t}(j) \), and \( R_{t+1} \) as given, and chooses \( n_{it}(j) \) to maximize (6);
(v) free entry is satisfied, which implies \( E_t(\pi_t + 1(j) \frac{R_t}{R_{t+1}}) = 0 \) in each industry-\( j \) in each period \( t \), where \( E_t(\pi_t + 1(j) \frac{R_t}{R_{t+1}}) \) is given by (6);
(vi) in each period \( t \), all goods, labor, and equity markets clear.

The cost function for innovation faced by innovating firms ensures decisions about whether to innovate and what level of research to undertake are independent of all past innovations. Combined with the lack of any aggregate uncertainty, this implies competitive equilibrium is stationary, in the sense that \( e_t(j) = e(j) \) and \( n_t(j) = n(j), \forall t, \) for each industry-\( j \). The number of firms \( e(j) \), the level of research per firm \( n(j) \), and the gross rate of return on equity \( R \) can be solved for by using the following three conditions;

\( ^8 \)This joint distribution is derived in Appendix A.1.1.
The free-entry condition;
\[ E \left( \frac{\pi_i(j)}{R} \right) = 0, \]

The research condition;
\[ \frac{\partial}{\partial n_i(j)} E \left( \frac{\pi_i(j)}{R} \right) = 0, \]

The Euler condition;
\[ 1 = \frac{\beta R}{1 + g}, \]

where the Euler condition is derived from the household’s problem, and \( g \) refers to the growth rate of the economy.

For the rest of the paper, I assume firms draw from a uniform distribution - i.e., \( h_i \sim U(0, n_i^\theta) \), where \( n_i \) is the level of research chosen by firm-\( i \), and \( h_i \) is the realization of its random quality draw. This allows equilibrium to be solved for analytically. I discuss the use of other distributions in the Section 2.4. Appendix A.2 derives firm-\( i \)'s expected discounted profits and the corresponding equilibrium conditions for a uniform distribution.\(^9\) Given the gross interest rate \( R \) and the growth rate of the economy \( g \), the equilibrium values of \( e(j) \) and \( n(j) \) for any industry-\( j \) satisfy the following two equations;

\[ n = \frac{z \theta(e + \alpha - 1)}{m[1 - \theta(e + \alpha - 1)]} \tag{8} \]

\[ z = \frac{(1 - \alpha^e)(1 + g)^{1-\alpha} n^\alpha \theta}{Re(e + \alpha)} - mn, \tag{9} \]

where the number of firms \( e \), research per firm \( n \), and all parameters should be understood to be specific to industry-\( j \). \( z \) and \( m \) are the fixed cost of innovation and the marginal cost of research.

\(^9\)Equations 8 through 13 are also derived in Appendix A.2.
The growth rate of the economy $g$ is equal to

$$g = \frac{y}{y_{-1}} - 1 = \left( \frac{\int_{0}^{1} A_{-1[1]}^\alpha(j)h_{[1]}^\alpha(j) dj}{\int_{0}^{1} A_{-1[1]}^\alpha(j) dj} \right)^{\frac{1}{\alpha}} - 1,$$

where $A_{[1]}(j)$ is the highest quality of industry-$j$ in the current period, equal to $A_{-1[1]}(j)h_{[1]}(j)$, and $h_{[1]}(j)$ is the realization of the best quality draw in industry-$j$ in the current period. In a symmetric economy where parameter values are common across industries, the growth rate can be shown to reduce to

$$g = E(h_{[1]}^\alpha)^{\frac{1}{\alpha}} - 1 = \left( \frac{en^\alpha}{e + \alpha} \right)^{\frac{1}{\alpha}} - 1. \quad (10)$$

Given the economy’s growth rate $g$, the gross interest rate is

$$R = \frac{1+g}{\beta}. \quad (11)$$

Using the free-entry and research conditions above, I can now characterize the number of firms $e$ in each industry and the level of research per firm $n$ in a symmetric economy as satisfying the following two equations;

$$n = \frac{z\theta(e + \alpha + 1)}{m[1 - \theta(e + \alpha - 1)]}, \quad (12)$$

$$z = \frac{(1 - \alpha^e)\beta [1 - \theta(e + \alpha - 1)]}{e^2}, \quad (13)$$

where all variables and parameters are common across industries.
2.4 Results

The primary goal of the present paper is to explain the relationships between competition, innovation, and productivity growth reported in recent empirical studies. The model developed in the preceding subsections implies that differences in these three endogenous variables across observations (whether across industries or over time) are driven by differences in the underlying parameters of the economy. In this section, I discuss how equilibrium is affected by changes in the values of exogenous parameters. I focus on the effect of changes in $z$, the cost of introducing an innovation, as the most plausible driver of the relationships identified in the data.\footnote{In Section 3, I cite evidence that supports this presumption.} Each of the graphs below plot the equilibrium values of a variable associated with different values of $z$, keeping all other parameters constant. This can be interpreted as an analysis of a cross-section of industries, controlling for all parameters except $z$, or as a time-series analysis of one industry where only $z$ is changing over time.

![Graphs showing equilibrium values of variables associated with different $z$ values](image)

Figure 1: Number of Firms and Research per Firm with variation in $z$ for industry-$j$

Figure 1a plots the equilibrium values of the number of firms (equivalently, innovations) $e(j)$ for a particular industry-$j$, for different values of $z$, the fixed cost of introducing an
innovation. Figure 1b does the same for the level of research per firm \( n(j) \). Not surprisingly, free entry ensures the number of firms \( e(j) \) adjusts downwards as the fixed cost of innovating increases. The equilibrium level of research per firm, on the other hand, has an inverted-U relationship with \( z \). To understand this relationship, it is useful to note that a decrease in the number of firms (i) increases a firm-\( i \)'s probability of winning, thus increasing the return to research; (ii) decreases the marginal effect of research on the probability firm-\( i \) wins; and (iii) decreases the expected value of the best innovation, given that firm-\( i \) wins. Now consider two variations of the model – one in which the winning firm can charge a monopoly price regardless of how close the second-best firm is, and another in which the winning firm must charge a limit price regardless of how far the second-best firm is. In the monopoly-only model, effects (ii) and (iii) above dominate, so that research per firm will always decrease with the number of firms \( e(j) \). In the limit-price-only model, however, a decrease in \( e(j) \) has the additional effect of increasing the expected gap between the best and second-best firms. This increases profits enough that a decrease in the number of firms will now be associated with an increase in research per firm. In the full model, a decrease in the number of firms \( e(j) \) makes a monopoly price more likely. If \( e(j) \) starts high, a decrease in \( e(j) \) will be associated with an increase in research per firm \( n(j) \). As the number of firms continues to drop and a monopoly price becomes more likely, however, \( n(j) \) eventually decreases along with \( e(j) \). An important caveat here is that the inverted-U relationship is only present when \( \alpha \) is close to 1 – otherwise, monopoly is too unlikely even with a small number of firms.

To analyze the relationships between productivity growth, innovation, and competition, I first find the (expected) equilibrium values of each variable for different values of \( z \), the cost of introducing an innovation. I then plot these variables against each other, where each point represents the set of equilibrium values associated with a particular \( z \). The expected rate of productivity growth in an industry-\( j \) is simply the expected value of the highest draw,
minus one;¹¹

\[
Productivity Growth = E_{t-1} \left( \frac{A_{t}[j]}{A_{t-1}[1](j)} \right) - 1 = E(h_{t}[j]) - 1 = \frac{e(j)n^\theta(j)}{e(j) + 1} - 1. \tag{14}
\]

The average level of innovation per firm is the expected value of each draw;

\[
Average Innovation = E_{t-1} \left( \frac{A_{t}(j)}{A_{t-1}[1](j)} \right) - 1 = E(h_{t}(j)) - 1 = \frac{n^\theta(j)}{2} - 1. \tag{15}
\]

Following Aghion et al. (2005), I use one minus the average Lerner Index in an industry as the measure of competition in that industry. The Lerner Index is equal to the ratio of price minus marginal cost over price for the best firm, and zero for all other firms. The expected value of this measure of competition for an industry-\(j\) is thus

\[
Competition = 1 - \frac{1}{e(j)} E \left( \frac{Price(j) - Marginal Cost(j)}{Price(j)} \right) = \frac{e^2(j) + \alpha e(j) - 1}{e^2(j)}, \tag{16}
\]

which is monotonically increasing in the number of firms \(e(j)\).

Figure 2a plots the expected level of innovation per firm against the expected value of competition. Each point represents the values of both variables associated with a particular value of \(z\), the fixed cost of introducing an innovation, for industry-\(j\). Competition is monotonically decreasing in \(z\), so average innovation per firm has an inverted-U relationship with competition just as research per firm \(n(j)\) has as an inverted-U relationship with \(z\) in Figure 1b.

Figure 2b plots expected industry-level productivity growth against competition as the cost of introducing an innovation varies for industry-\(j\). Even while firm-level innovation follows an inverted-U, industry-level productivity growth always increases with competition. The decrease in the expected markup of the winning firm from more competition tends to discourage research, all else equal, since a drop in the fraction of value captured by the win-

¹¹Equations 14 and 16 are derived in Appendix A.3.
Figure 2: Competition, Productivity Growth, and Average Innovation per Firm, with variation in \( z \) for industry-\( j \)

Figure 2 illustrates the relationship between competition, productivity growth, and average innovation per firm. The graphs show how competition affects these variables, with variation in \( z \) for industry-\( j \).

The experiment illustrated in Figures 1 and 2 (changing equilibrium outcomes by changing \( z \)) can be repeated for each of the exogenous parameters in the model. Given that the number of firms \( e(j) \) is a decreasing function of \( z \) (as illustrated in Figure 1a), (8) and (9) show \( e(j) \) is decreasing in the marginal cost of research \( m \), as is research per firm \( n(j) \). Both \( e(j) \) and \( n(j) \) have an inverted-U relationship with the elasticity of substitution. The parameter \( \theta \) governs the elasticity of each firm’s upper bound with respect to research, as
each firm-$i$ draws from a distribution with support $[0, n_i^\theta]$. Research per firm $n(j)$ is increasing in $\theta$, while the number of firms $e(j)$ is decreasing. An increase in the interest rate $R$ (exogenous with respect to a single industry) lowers discounted profits and therefore results in fewer firms and less research per firm in equilibrium.

I now consider how equilibrium in a symmetric economy (where parameter values are common across industries) is affected by changes in exogenous parameters. Figures 3 and 4 illustrate how equilibrium outcomes change when the cost of introducing an innovation $z$ changes for all industries. Figure 3a shows the number of firms in each industry decreases as $z$ increases, analogous to the relationship in Figure 1a. In Figure 3b research per firm $n$ is strictly increasing in $z$, and the inverted-U apparent in Figure 1b is no longer present. When the cost of introducing an innovation increases, the economy-wide growth rate decreases and brings the interest rate $R$ down with it. This offsets the downward pressure on research discussed in Figure 1b, so that research per firm always increases if the cost of introducing an innovation increases for all industries. Figure 4 plots the growth rate of a symmetric economy against measured competition as the cost of introducing an innovation increases.
$z$ is varied. As in Figure 2b, both growth and competition always decrease with $z$, making the relationship between them strictly positive. Changes to other parameters in a symmetric economy generally affect the number of firms $e$ and research per firm $n$ in the same way as changes to parameters for one industry. Finally, an increase in the discount rate of households $\beta$ increases both $e$ and $n$ by lowering the interest rate $R$, thus lowering the total cost of innovation for firms.

Figure 4: Competition and Growth in a Symmetric Economy for varying $z$

Throughout this section, firms have been assumed to be drawing from a uniform distribution. Although not presented here, the model has been solved for several other distributions.\footnote{Results for other distributions are available from the author upon request.} All of the qualitative relationships discussed here hold more generally, except for the inverted-U relationship between research per firm $n$ and the cost of introducing an innovation $z$. Many distributions generate the inverted-U, while others result in a strictly positive relationship between $n$ and $z$, even at high elasticities of substitution between industries.
3 Empirical Support

In the preceding section, I focus on the effects of a change in the cost of introducing an innovation $z$ on equilibrium. This focus is appropriate if changes in $z$ are actually driving the relationships reported in empirical studies. In fact, each of the studies cited throughout this paper control for both year and industry fixed effects, and so are presumably reporting relationships driven by within-industry variation in an exogenous parameter over time. The regulatory portion of the cost of introducing an innovation is at least a plausible candidate for the source of this variation. Evidence from Aghion et al. (2005) more directly supports a focus on regulatory costs. In an attempt to control for endogeneity in their test of the effect of competition on firm-level innovation, Aghion et al. employ a series of stuttered waves of deregulation in different industries as a source of exogenous changes to competition. In the end, their IV results are almost identical to their OLS results (controlling for time and fixed effects) with a reduced-form $R^2$ of 0.8, suggesting that much of the correlation between competition and firm-level innovation can be explained using the present model with variation in $z$ over time.

The primary implication of the model developed in this paper, that competition and productivity growth are positively related, is well supported in empirical studies of industry-level growth like Nickell (1996), Blundell et al. (1999), and Aghion et al. (2008). To my knowledge, no empirical studies have provided evidence to the contrary. In addition, studies like Graham et al. (1983) and Nicoletti and Scarpetta (2003) have suggested both higher marketing costs and more burdensome regulations are associated with lower growth, which is consistent with the model. Aghion et al. (2005) test the relationship between competition and the average level of innovation per firm and report an inverted-U relationship, consistent with the present model.

In recent publications, economists have interpreted evidence of an inverted-U relationship between firm-level innovation and competition as evidence of a similar relationship between
industry-level productivity growth and competition.\textsuperscript{13} Aghion et al. (2008) present evidence to the contrary. Their results are consistent with those of the model presented here, where variation in the cost of innovating induces positively correlated differences in competition and productivity growth at the industry level, but an inverted-U relationship between competition and firm-level innovation.

4 Applications

In this section, I use extensions of the baseline model to evaluate the effects of antitrust policies on innovation and productivity growth. An advantage of using a model in which competition is endogenous is the ability to specify exactly how various policies affect exogenous constraints or parameters in the model and evaluate the resulting outcomes, rather than merely assume antitrust policy exogenously increases competition.

The baseline model developed in Section 2 implies that deregulation should increase competition and productivity growth. In the following two subsections, I extend the model to evaluate the effects of a cap on markups, and the effects of legal restrictions on collusion or (equivalently) ‘anti-competitive’ mergers. Whereas empirical studies of competition and productivity growth generally conclude with calls for greater antitrust enforcement (on the presumption that greater enforcement implies more competition, which is found to be positively correlated with growth)\textsuperscript{14}, I find that both caps on markups and restrictions on collusion lead to lower productivity growth in equilibrium.

4.1 Ceiling on Markups

I start by considering the effect of a legislated cap on a firm’s markup of price over marginal cost. I assume this cap is known \textit{ex ante}, so there is no uncertainty about its level

\textsuperscript{13}See Bianco (2007), for example. Indeed, this view seems to have become the conventional wisdom.

\textsuperscript{14}Examples of this particular policy proposal can be found in the concluding sections of Geroski (1990) and Dutz and Hayri (2000).
or about whether it will be enforced.

Whereas in the competitive model the winning firm will charge a monopoly price when
\[ \frac{A_{[2]}(j)}{A_{[1]}(j)} < \alpha, \]
firms will now be forced to charge a markup less than or equal to \( q < \frac{1}{\alpha} \). The only change in the model is therefore that the price of the winning firm in industry-\( j \) will be

\[
P(j) = \begin{cases} 
qw(j), & \text{if } \frac{A_{[2]}(j)}{A_{[1]}(j)} < \frac{1}{q}, \\
w(j)\frac{A_{[1]}(j)}{A_{[2]}(j)}, & \text{if } \frac{A_{[2]}(j)}{A_{[1]}(j)} > \frac{1}{q},
\end{cases}
\]

where \( A_{[\ell]}(j) \) is the \( \ell \)th-highest quality in industry-\( j \) and \( w(j) \) is the wage faced by the firm.

Using the same procedure followed in Section 2, expected discounted profits for any innovationg firm-\( i \) can be expressed as

\[
E \left( \frac{\pi_i}{R} \right) = \frac{(1 - q^{-e})\Delta(1 + g)^{1-\alpha}n_i^{\theta(e+\alpha-1)}}{Re(e + \alpha)n_k^{\theta(e-1)}} - \Delta(z + mn_i),
\]

where \( n_i \) is firm-\( i \)'s level of research, \( e \) is the number of firms, and \( R \) is the gross interest rate. \( z \) and \( m \) continue to denote the fixed cost of introducing an innovation and the marginal cost of research.\(^{15}\)

Using the free-entry and research conditions, equilibrium in an industry with a cap on markups \( q \) can be characterized by the following two equations;

\[
n = \frac{z\theta(e + \alpha - 1)}{m[1 - \theta(e + \alpha - 1)]}
\]

\[
z = \frac{(1 - q^{-e})(1 + g)^{1-\alpha}n^{\alpha\theta}}{Re(e + \alpha)} - mn.
\]

For all feasible parameter values, both the number of innovations \( e \) and the level of research per innovation \( n \) are lower when markups are capped, relative to the competitive

\(^{15}\)Industry subscripts have been dropped for ease of notation.
equilibrium. Figure 5 compares the rates of productivity growth for two otherwise identical industries for different values of $z$, the cost of introducing an innovation. Just as with its competitive counterpart, productivity growth in the capped industry is declining in $z$. Furthermore, growth is always higher in the competitive industry, for any $q < \frac{1}{\alpha}$.

The results of this experiment should not be surprising. If innovators face a positive probability of making less profit than they would absent this policy, we should expect less innovation, notwithstanding the positive relationship between competition and productivity growth in the data.

4.2 Price-Fixing and Anti-Competitive Mergers

While ceilings on markups have perhaps been commonly understood to have potentially negative consequences for innovation and investment, the consequences for growth and innovation of restrictions on collusion and ‘anti-competitive’ mergers have received little atten-
This section extends the baseline model by allowing the winning firm in any period to either purchase or collude with any other firm that might constrain the winning firm’s pricing decision. Williamson (1968), Mathewson and Winter (1987), and others have considered situations in which price-fixing and mergers thought to be anti-competitive might actually improve allocative efficiency. The possible benefits of joint research ventures with respect to technology diffusion have also been analyzed in studies like Katz (1986). The contribution of this section is to evaluate the effects of restrictions on price-fixing and mergers for which no efficiency defence exists - that is, behavior undertaken by firms that increases profits at the expense of consumers ex post, and provides no benefit to allocative efficiency.

I model the cost of purchasing or colluding with all firms that pose a threat as an exogenous fraction $\tau^C$ of the additional profit firm-[1] (the winning firm) stands to gain by charging a monopoly price, rather than a limit price. In addition, I assume firm-[1] must incur a cost to monitor its co-conspirators (or manage its larger size after merging), which I model in a similar fashion as a fraction $\tau^M$.

The total cost of achieving a monopoly price when $\frac{h_{[2]}}{h_{[1]}} > \alpha$ is thus

$$(\tau^C + \tau^M) \frac{\Delta(h_{[2]} - \alpha h_{[1]})}{h_{[1]}^{1-\alpha}},$$

where $\Delta \equiv A_{-1[1]}^\alpha L^{\alpha}y_{-1}[1]^{-\alpha}$, $A_{-1[1]}$ is the first-best quality of the previous period, and $h_{[\ell]}$ is the $\ell$th-best draw of the current period. The payoff for each firm-[r] for which $\frac{h_{[r]}}{h_{[1]}} > \alpha$ is equal to the fraction $\tau^C$ of the additional profit the winning firm receives by colluding with firm-[r]. The payoff for an eligible firm-[r] is therefore

$$\begin{cases}
\tau^C \frac{\Delta(h_{[r]} - h_{[r+1]})}{h_{[1]}^{1-\alpha}}, & \text{if } \frac{h_{[r+1]}}{h_{[1]}} > \alpha \\
\tau^C \frac{\Delta(h_{[r]} - \alpha h_{[1]})}{h_{[1]}^{1-\alpha}}, & \text{if } \frac{h_{[r+1]}}{h_{[1]}} < \alpha.
\end{cases}$$

When $\frac{h_{[2]}}{h_{[1]}} > \alpha$, the operating profits of a winning firm choosing a limit price are $\frac{\Delta(h_{[1]} - h_{[2]})}{h_{[1]}^{1-\alpha}}$,.
while those of a winning firm choosing to collude are \((1 - \alpha)\Delta h_{[1]}^\alpha - (\tau^C + \tau^M) \frac{\Delta(h_{[2]} - \alpha h_{[1]})}{h_{[1]}^{1-\alpha}}\).

Firm-[1] will therefore choose to collude if \(\frac{h_{[2]}}{h_{[1]}} > \alpha\) and

\[
(1 - \alpha)h_{[1]}^\alpha - \frac{h_{[1]} - h_{[2]}}{h_{[1]}^{1-\alpha}} \geq (\tau^C + \tau^M) \frac{h_{[2]} - \alpha h_{[1]}}{h_{[1]}^{1-\alpha}},
\]

or if \(\tau^C + \tau^M \leq 1\). Since the decision to collude or not depends only on \(\tau^C\) and \(\tau^M\), and not on the realization of any random variables, I assume \(\tau^C + \tau^M \leq 1\).

In Appendix A.4, I derive the expected discounted profits facing each innovating firm (22). Using the free-entry and research conditions, the collusive equilibrium can be characterized by the following two equations:

\[
nm = \frac{\theta n^\alpha}{Re(e + \alpha)} (1 - \alpha)(e(1 - \alpha) - \tau^M[e(1 - \alpha) + \alpha^e] - 1)
\]

\[
- \frac{\theta n^\alpha}{Re(e + \alpha)} (e(1 - \alpha) - \tau^C[e(1 - \alpha) + \alpha^e] - 1)
\]

\[
z + mn = \frac{n^\alpha}{Re(e + \alpha)} (1 - \alpha) - \tau^M[e(1 - \alpha) + \alpha^e] - 1).
\]

If total payoffs are zero \((\tau^C = 0)\) and monitoring costs eat up the entire benefit of collusion \((\tau^M = 1)\), then these conditions reduce to those of a competitive equilibrium.

Figures 6a through 7b plot the number of innovations and the level of research per firm, for various values of \(z\) (the cost of introducing an innovation), and for both a collusive and a competitive industry. The interest rate and all parameters besides \(z\) are held constant throughout. Figures 6a and 6b show how outcomes change as monitoring/management costs \((\tau^M)\) increase, while keeping \(\tau^C\) equal to zero. Both \(e\) and \(n\) are decreasing in \(\tau^M\), but remain above the associated competitive equilibrium values until \(\tau^M = 1\).

Figures 7a and 7b illustrate the same experiment with respect to \(\tau^C\), keeping monitoring costs at zero. As payouts to competing firms increase, it becomes less profitable to be the
Figure 6: Number of Firms and Research per Firm with varying monitoring costs $\tau^M$

Figure 7: Number of Firms and Research per Firm with varying payouts $\tau^C$
best firm, but *more* profitable to be a lower-ranked firm. This results in lower research per firm, but encourages more firms to enter in an effort to capture some of the payouts. While the number of innovations always remains above the competitive level, research per firm drops below this level once total payouts increase past a certain threshold. This pattern holds when monitoring costs are positive.

If payoffs to colluding firms were zero, figures 6a and 6b imply that growth must be higher when collusion is possible, relative to the competitive outcome, for any level of $\tau^M$ lower than one. Since collusion increases the profits of the winning firm without otherwise distorting incentives, growth is higher in this case when collusion is permitted. On the other hand, when the cost of buying-out or buying-off competitors ($\tau^C$) is positive, a higher number of innovations comes at the cost of less research per innovation. Ultimately, however, growth is still higher with collusion in this case. Figure 8 shows how the expected growth rate varies with $\tau^C$, keeping $\tau^M$ equal to zero. Growth remains higher with collusion for any value of $\tau^C$ lower than 0.95. As long as the best firm retains enough profit to make winning more profitable than losing, the negative effect of payoffs on research is more than offset by the increase in the number of innovations.

![Figure 8: Expected Productivity Growth with varying payouts $\tau^C$](image)

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Testing these results against real outcomes is difficult, due to the ubiquitousness of antitrust laws throughout the last century. The story of Standard Oil in the late nineteenth and early twentieth centuries, however, does provide a case study that seems consistent with the implications of the present model. Over a forty-year period, the company used a combination of process and product innovations, acquisitions, and price-fixing agreements to dominate the market, maintaining a market share of close to 90% until both politics and more able rivals began to drag them back down to earth. Just as the model predicts, a large number of new entrants appeared each year, many seemingly with the intention of becoming just competitive enough to get bought-out by Standard Oil. While some refineries were used to add to capacity, many were just bought and then shut down in an effort to reduce competition. Nevertheless, the period saw an enormous number of new process innovations, as well as new ways to turn the ‘waste’ from the production of kerosene into products like gasoline, paving tar, and petroleum jelly. Over the course of thirty years, just as Standard Oil first acquired and then maintained its monopoly, the deflation-adjusted price of kerosene dropped by over 65%, even while petroleum output increased by more than a factor of three.

It is important to point out that the model is ill-suited for evaluating the welfare implications of collusion, as it does not allow for any static inefficiency resulting from a monopoly price. That collusion may be welfare-reducing even while increasing growth is most obvious in the limiting case where monitoring costs erode the entire benefit of charging a monopoly price. When $\tau_M$ is equal to one, growth will be no higher than in a competitive industry, but resources will nonetheless be wasted enforcing the collusive agreement. The policy implication of this model is that regulators should either allow price-fixing agreements to be enforced, thereby lowering monitoring costs, or else raise monitoring costs to a prohibitive level. The appropriate choice depends on the trade-off between dynamic and static efficiency.

\textsuperscript{16}McGee (1958) provides a detailed analysis of the competitive behavior of Standard Oil and its rivals. Boudreaux and Folsom (1999) provide a short summary of the innovations and price reductions that accompanied J.D. Rockefeller’s entertaining quest to dominate the market.
(as well as on enforcement costs and ideology).

5 Conclusion

Hayek (2002) argued that the competitive process could be thought of as a procedure for discovering and making use of knowledge that would otherwise not emerge. When firms are uncertain about the optimal direction of innovation, the best innovation to emerge will tend to be of higher value when more innovations are tried. Throughout this paper the Schumpeterian mechanism, whereby lower ex post rents discourage lower ex ante research, is always present. But when Schumpeter meets Hayek in this model of innovation, more competition is accompanied by higher industry-level productivity growth, even if research per firm is lower. Combining Hayekian uncertainty with endogenous entry, markups, and firm-level research results in a model able to generate a positive relationship between competition and industry-level productivity growth, and an inverted-U relationship between competition and firm-level innovation.

By treating competition, growth, and innovation as endogenous, I develop a framework for evaluating competition policy that is less susceptible to the Lucas Critique. Two such evaluations of antitrust policies (restrictions on markups and price-fixing) come to conclusions very much at odds with policy prescriptions based on simple interpretations of empirical studies. The results of recent empirical studies notwithstanding, I conclude that one of Schumpeter’s central messages should be taken to heart — regardless of any static benefits of antitrust enforcement and regulation, these policies come with the cost of less innovation and growth. Or alternatively, there ain’t no such thing as a free lunch.
References


A Appendix

A.1 Density Functions

A.1.1 General

To solve the competitive model, it is necessary to derive the joint density function of \( h_{[1]} \), \( h_{[2]} \), and \( h_i \). When deciding its optimal level of research, I assume firm-\( i \) believes all other firms will choose levels of research identical to each other. This makes the notation simpler, without sacrificing any generality. If \( h_\ell \) is distributed according to a continuously differentiable distribution \( F_\ell(h) \) between 0 and \( n_\ell^\theta \), the relevant joint density function is

\[
f(h_{[1]} = u_1, h_{[2]} = u_2, h_i = h_{[1]} \mid e, n_i, n_k \neq i) = \int_0^{u_2} \cdots \int_0^{u_{e-1}} (e - 1)! f(h_i = u_1) \prod_{\ell=2}^{e} f(h_k = u_\ell) du_e \cdots du_3,
\]

where \( e \) is the number of firms (draws), \( n_i \) is firm-\( i \)’s level of research, and \( n_k \) is every other firm’s level of research. Integrating over all \( u_{\ell>2} \), this simplifies to

\[
f(h_{[1]} = u_1, h_{[2]} = u_2, h_i = h_{[1]} \mid e, n_i, n_k \neq i) = (e - 1) f_i(u_1) f_k(u_2) F_k(u_2)^{e-2},
\]

defined over all \( u_1 \) and \( u_2 \), such that \( u_1 \leq n_i^\theta, u_2 \leq n_k^\theta \), and \( u_1 \geq u_2 \geq 0 \).

When calculating the expected value of an equilibrium variable, the relevant joint density function is similar to the one above, but without the qualification that firm-\( i \) has the highest draw. Taking into account \( n_i = n_k = n \) in equilibrium, the required density function is

\[
f(h_{[1]} = u_1, h_{[2]} = u_2 \mid e, n) = \int_0^{u_2} \cdots \int_0^{u_{e-1}} e! \prod_{\ell=1}^{e} f(h = u_\ell) du_e \cdots du_3,
\]

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or
\[ f(h_1 = u_1, h_2 = u_2 \mid e, n) = e(e - 1)f(u_1)f(u_2)F(u_2)^{e-2}, \]
defined over all \( u_1 \) and \( u_2 \), such that \( n \geq u_1 \geq u_2 \geq 0 \).

### A.1.2 Uniform

If each firm-\( \ell \)'s quality draw is from a uniform distribution bounded by 0 and \( n^{\theta}_\ell \), then
\[ f_\ell(u) = \frac{1}{n^{\theta}_\ell} \text{ and } F_\ell(u) = \frac{u}{n^{\theta}_\ell}. \]
It follows that the two joint density functions derived in Appendix A.1.1 become
\[ f(h_1 = u_1, h_2 = u_2, h_i = h_1 \mid e, n_i, n_k \neq i) = \frac{(e - 1)u_2^{e-2}}{n^{\theta}_i n^{\theta}(e - 1)}, \]
and
\[ f(h_1 = u_1, h_2 = u_2 \mid e, n) = \frac{e(e - 1)u_2^{e-2}}{n^{\theta}e}. \]

### A.2 Competitive Equilibrium

Firm-\( i \)'s expected discounted profits (6) are
\[
E^t_{t-1} \left( \frac{\pi_{it}}{R_t} \right) =
\begin{aligned}
(1 - \alpha) & \frac{\Delta_{t-1}(1 + g_t)^{1-\alpha}}{R_t} \text{Prob} \left[ h_{it} = h_{t[1]} \geq \frac{h_{t[2]}}{\alpha} \right] \cdot E^t_{t-1} \left[ \frac{h_{1}}{\alpha} \mid h_{it} = h_{t[1]} \geq \frac{h_{t[2]}}{\alpha} \right] \\
+ & \frac{\Delta_{t-1}(1 + g_t)^{1-\alpha}}{R_t} \text{Prob} \left[ h_{it} = h_{t[1]} < \frac{h_{t[2]}}{\alpha} \right] \cdot E^t_{t-1} \left[ \frac{h_{t[1]} - h_{t[2]}}{h_{t[1]}^{1-\alpha}} \mid h_{it} = h_{t[1]} < \frac{h_{t[2]}}{\alpha} \right] \\
- & \Delta_{t-1}(z + mn_{i,t-1}),
\end{aligned}
\]
where \( h_{t[\ell]} \) denotes the \( \ell \)th-highest realized draw in period \( t \), \( R \) is the gross interest rate, \( g_t \equiv \frac{y_t}{y_{t-1}} - 1 \), and \( \Delta_{t-1} \equiv \gamma_{t-1} A_{t-1[1]}(j) \left( \int_0^1 A_{t-1[1]}(j') dj' \right)^{-1} \). Using the joint density function.
from Appendix A.1.2, profits can be expressed as\textsuperscript{17}

\[
E_{t-1} \left( \frac{\pi_i}{R_t} \right) = \frac{(1 - \alpha) \Delta_{t-1}(1 + g_t)^{1-\alpha} n_i^{\theta(e+\alpha-1)}}{R_t \alpha \theta(e+1)} \int_0^{n_i^{\theta, t-1}} \int_0^{n_k^{\theta, t-1}} \frac{(e_{t-1} - 1)v^{\alpha} u^{e_t - 2}}{n_i^{\theta, t-1} n_k^{\theta, t-1}} \, du \, dv \\
+ \frac{\Delta_{t-1}(1 + g_t)^{1-\alpha}}{R_t} \int_0^{n_i^{\theta, t-1}} \int_0^{v \alpha v_0^{\theta, t-1}} \frac{(e_t - 1)(v - u) u^{e_t - 2}}{v^{1-\alpha} n_i^{\theta, t-1} n_k^{\theta, t-1}} \, du \, dv - \Delta_{t-1}(z + mn_i).
\]

Since it is now obvious that decisions are time-independent, expected discounted profits can be expressed as

\[
E \left( \frac{\pi_i}{R} \right) = \frac{\alpha^{e-1}(1 - \alpha) \Delta(1 + g)^{1-\alpha} n_i^{\theta(e+\alpha-1)}}{R(e + \alpha) n_k^{\theta(e-1)}} \\
+ \frac{\Delta(1 + g)^{1-\alpha} n_i^{\theta(e+\alpha-1)}}{Re(e + \alpha) n_k^{\theta(e-1)}} \left[ 1 - \alpha^e - e^{e-1} (1 - \alpha) e \right] - \Delta(z + mn_i),
\]
or

\[
E \left( \frac{\pi_i}{R} \right) = \frac{(1 - \alpha^e)(1 + g)^{1-\alpha} n_i^{\alpha \theta}}{Re(e + \alpha) n_k^{\theta(e-1)}} - \Delta(z + mn_i).
\]

Each firm-\(i\) chooses its level of research \(n_i\) to maximize \(E \left( \frac{\pi_i}{R} \right)\), given \(n_k \neq i\) and the number of firms \(e\). Given identical firms, this results in the following equilibrium condition;

\[
\frac{mn}{\theta(e + \alpha - 1)} = \frac{(1 - \alpha^e)(1 + g)^{1-\alpha} n^{\alpha \theta}}{Re(e + \alpha)}.
\] \hspace{1cm} (19)

Free entry ensures that the number of firms \(e\) adjusts until \(E \left( \frac{\pi_i}{R} \right) = 0\). Given that \(n_i = n_k = n\), the following condition must also hold in equilibrium;

\[
z + mn = \frac{(1 - \alpha^e)(1 + g)^{1-\alpha} n^{\alpha \theta}}{Re(e + \alpha)}.
\] \hspace{1cm} (20)

\textsuperscript{17}To be precise, the above is true only if \(n_{it} \leq n_{kt}\). But if one were to instead assume \(n_{it} \geq n_{kt}\), the same \textit{free-entry} and \textit{research} conditions would result in equilibrium, where \(n_{it} = n_{kt}\).
Rearranging conditions (19) and (20) above results in equations (8) and (9):

\[
n = \frac{z\theta(e + \alpha - 1)}{m[1 - \theta(e + \alpha - 1)]}
\]

\[
z = \frac{\alpha(1 - \alpha^e)(1 + g)1^{-\alpha}n^{\alpha\theta}}{Re(e + \alpha)} - mn.
\]

The growth rate of the economy \( g_t \) is equal to

\[
g_t = \frac{y_t}{y_{t-1}} - 1 = \left( \frac{\int_0^1 A_{t-1[1]}(j)h_{t[1]}^\alpha(j) dj}{\int_0^1 A_{t-1[1]}^\alpha(j) dj} \right)^{\frac{1}{\alpha}} - 1,
\]

where \( A_{t-1[1]}(j) \) is the highest quality of industry-\( j \) in period \( t - 1 \), and \( h_{t[1]}(j) \) is the realization of the best quality draw in industry-\( j \) in period \( t \). Given arbitrary \( A_0(j) \), \( g_t \) can be expressed as

\[
g_t = \left( \frac{\int_0^1 A_0^\alpha(j) \prod_{s=1}^t h_{s[1]}^\alpha(j) dj}{\int_0^1 A_0^\alpha(j) \prod_{s=1}^{t-1} h_{s[1]}^\alpha(j) dj} \right)^{\frac{1}{\alpha}} - 1.
\]

In a symmetric economy where all parameter values including \( A_0 \) are common across industries, the above expression is equal to

\[
g_t = \left( \frac{E[A_0 \prod_{s=1}^t h_{s[1]}^\alpha]}{E[A_0 \prod_{s=1}^{t-1} h_{s[1]}^\alpha]} \right)^{\frac{1}{\alpha}} - 1.
\]

With independence of draws across time periods, the growth rate can be conveniently expressed as

\[
g_t = \left( \frac{A_0^\alpha E[h_{t[1]}^\alpha]^t}{A_0^\alpha E[h_{t[1]}^\alpha]^{t-1}} \right)^{\frac{1}{\alpha}} - 1 = E[h_{t[1]}^\alpha]^{\frac{1}{\alpha}} - 1.
\]
Using the joint density function from A.1.2, this becomes equation 10;

\[ g = \left( \int_{0}^{n^\theta} \int_{0}^{v} \frac{e(e - 1)v^\alpha u^{e-2}}{n^{\theta e}} \, du \, dv \right)^{\frac{1}{\alpha}} - 1 = \left( \frac{en^{\theta \theta}}{e + \alpha} \right)^{\frac{1}{\alpha}} - 1, \]

where the number of firms \( e \) and the level of research per firm \( n \) are constant across industries.

Given the interest rate \( R \) is equal to \( \frac{1 + g}{\beta} \) from the household’s problem, (10) can be combined with the equilibrium conditions for a single industry (8) and (9) to characterize equilibrium in a symmetric economy (equations 12 and 13).

### A.3 Competition and Productivity Growth

Productivity growth in an industry-\( j \) is equal to

\[ \frac{A_{t[1]}(j)}{A_{t-1[1]}(j)} - 1 = \frac{A_{t-1[1]}(j)h_{t[1]}(j)}{A_{t-1[1]}(j)} - 1 = h_{t[1]}(j) - 1, \]

so expected productivity growth in an industry is simply equal to the expected value of the highest draw:

\[ E[h_{t[1]}(j)] - 1. \]

Given the joint density function from A.1.2, expected industry-level productivity growth for an industry-\( j \) (14) is equal to

\[ \text{Productivity Growth} = \int_{0}^{n^\theta(j)} \int_{0}^{v} \frac{e(j)(e(j) - 1)v^{\alpha(j)}u^{e(j)-2}}{n^{\theta(j)e(j)}} \, du \, dv - 1 = \frac{e(j)n^{\theta(j)}}{e(j) + 1} - 1. \]

Measured competition is equal to

\[ 1 - \frac{1}{e(j)} \left( \frac{\text{Price}(j) - \text{Marginal Cost}(j)}{\text{Price}(j)} \right). \]
Using the pricing strategy given by (5), *ex post* competition is

\[
\begin{cases}
  \frac{e(j)-1+\alpha}{e(j)}, & \text{if } \frac{h[2]}{h[1]} \leq \alpha \\
  \frac{e(j)-1}{e(j)} + \frac{1}{e(j)} \left( \frac{h[2]}{h[1]} \right), & \text{if } \frac{h[2]}{h[1]} > \alpha.
\end{cases}
\]

The expected value of measured competition in an industry-\(j\) is therefore

\[
\text{Competition} = \frac{e(j) - 1}{e(j)}
\]

\[+
\int_0^{n^e(j)} \int_0^{e} \frac{\alpha}{e(j)} f(h[1](j) = v, h[2](j) = u \mid e(j), n(j)) du dv
\]

\[+
\int_0^{n^e(j)} \int_0^{v} \frac{1}{e(j) v} f(h[1](j) = v, h[2](j) = u \mid e(j), n(j)) du dv.
\]

Substituting in the density function from Appendix A.1.2 results in equation (16);

\[
\text{Competition} = \frac{e^2(j) + \alpha e(j) - 1}{e^2(j)}.
\]

### A.4 Collusive Density Functions

To calculate expectations in a model with collusion, two additional density functions are required; \(f(h[1] = u_1, h[r] = u_r, h[r+1] = u_{r+1}, h_i = h[r] \mid e, n_i, n_{k\neq i})\) for \(e > r > 1\), and \(f(h[1] = u_1, h[e] = u_e, h_i = h[e] \mid e, n_i, n_{k\neq i})\) for \(r = e\). I start with the former;

\[
f(h[1] = u_1, h[r] = u_r, h[r+1] = u_{r+1}, h_i = h[r] \mid e, n_i, n_{k\neq i}) =
\int_0^{u_1} \cdots \int_0^{u_{r-2}} \int_0^{u_{r+1}} \cdots \int_0^{u_{e-1}} (e-1)! f(h_i = u_r)
\]

\[\cdot \prod_{\ell \neq r} f(h_k = u_\ell) du_e \cdots du_{r+2} du_{r-1} \cdots du_2, \quad \text{if } r > 2,
\]

or

\[
\int_0^{u_{r+1}} \cdots \int_0^{u_{e-1}} (e-1)! f(h_i = u_r) \prod_{\ell \neq r} f(h_k = u_\ell) du_e \cdots du_{r+2}, \quad \text{if } r = 2.
\]
Given that $f(h_k = u_\ell) = \frac{1}{n_k}$, $\forall k, \ell$, this works out to

$$f(h_{1}] = u_1, h_{[e} = u_e, h_{[r+1} = u_{r+1}, h_i = h_{[i} \mid e, n_i, n_k \neq i)$$

$$= \frac{(e - 1)! (u_1 - u_r)^{r-2} u_{r+1}^{r-1}}{(r - 2)! (e - r - 1) n_i^\theta n_k^\theta}$$

for $e > r > 1$. (21)

The second necessary density function can be derived the same way;

$$f(h_{1}] = u_1, h_{[e} = u_e, h_i = h_{[i} \mid e, n_i, n_k \neq i)$$

$$= \int_0^{u_1} \cdots \int_0^{u_{e-2}} (e - 1) f(h_i = u_e) \prod_{\ell \neq r} f(h_k = u_\ell) du_{e-1} \cdots du_2$$

$$= \frac{(e - 1) (u_1 - u_e)^{e-2}}{n_i^\theta n_k^\theta}.$$

When collusion is possible, each firm takes into account both the benefit of capturing the market and the potential payout from the winning firm. Firm-\(i\)'s expected discounted
where the first term is the expected monopoly profits from production, the second term is the total cost of ensuring the monopoly price, the third term is the payoff for firm-[r] when \( h_{[r+1]} > \alpha h_{[1]} \), the fourth term is the payoff for firm-[r] when \( h_{[r]} > \alpha h_{[1]} \), but \( h_{[r+1]} < \alpha h_{[1]} \), and the fifth term is the payoff for the worst firm when \( h_{[e]} > \alpha h_{[1]} \).

The joint density function for \( h_{[1]}, h_{[r]}, \) and \( h_{[r+1]} \) (21) must be summed over all values of \( r \) between 2 and \( e - 1 \):

\[
\sum_{r=2}^{e-1} f(h_{[1]} = u_1, h_{[r]} = u_r, h_{[r+1]} = u_{r+1}, h_i = h_{[r]} \mid e, n_i, n_k \neq i) =
\]
\[
\frac{(e - 1)(e - 2)(u_1 - u_r + u_{r+1})^e - 3}{n_i^n n_k^\theta (e-1)}.
\]

The expected discounted profits of firm-\(i\) can now be expressed as

\[
E \left( \frac{\pi_i}{R} \right) =
\frac{(1 - \alpha)\Delta}{R} \int_0^{n_i^\theta} \int_0^{u_1^\theta} u_1^\alpha f(h_{[1]} = u_1, h_{[2]} = u_2, h_i = h_{[1]} ) du_2 du_1
- \frac{\Delta(\tau^C + \tau^M)}{R} \int_0^{n_i^\theta} \int_{u_1^\alpha}^{u_1} \frac{(u_2 - \alpha u_1)}{u_1^{1-\alpha}} f(h_{[1]} = u_1, h_{[2]} = u_2, h_i = h_{[1]} ) du_2 du_1
+ \frac{\Delta \tau^C}{R} \int_0^{n_k^\theta} \int_{u_1^\alpha}^{u_1} \frac{(u_r - u_{r+1})}{u_1^{1-\alpha}} \sum_{r=2}^{e-1} f(h_{[1]} = u_1, h_{[r]} = u_r, h_i = h_{[r+1]} = u_{r+1}, h_i = h_{[r+1]} ) du_r du_{r+1} du_1
+ \frac{\Delta \tau^C}{R} \int_0^{n_k^\theta} \int_{u_1^\alpha}^{u_1} \frac{(u_r - \alpha u_1)}{u_1^{1-\alpha}} \sum_{r=2}^{e-1} f(h_{[1]} = u_1, h_{[r]} = u_r, h_i = h_{[r+1]} = u_{r+1}, h_i = h_{[r+1]} ) du_r du_{r+1} du_1
+ \frac{\Delta \tau^C}{R} \int_0^{n_k^\theta} \int_{u_1^\alpha}^{u_1} \frac{(u_e - \alpha u_1)}{u_1^{1-\alpha}} f(h_{[1]} = u_1, h_{[e]} = u_e, h_i = h_{[e]} ) du_e du_1
- \Delta(z + mn_i),
\]

or

\[
E \left( \frac{\pi_i}{R} \right) =
\frac{(1 - \alpha)\Delta n_i^\theta (e+\alpha-1)}{Re(e + \alpha) n_k^\theta (e-1)} - \frac{\tau^C + \tau^M}{Re(e + \alpha) n_k^\theta (e-1)} [e(1 - \alpha) + \alpha^e - 1]
+ \frac{\tau^C n_k^\theta (1+\alpha)}{Re(e + \alpha) n_i^\theta} [e(1 - \alpha) + \alpha^e - 1] - \Delta(z + mn_i).
\]

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